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Auteurs

Verónica Acurio Vásconez, Mónica Pereira Henriques

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Contact :
jaoulgrammare@beta-cnrs.unistra.fr

Unraveling the Interplay of Substitution Elasticities and the Green Energy Rebound Effect

Verónica Acurio Vásconez*

Mónica Pereira Henriques†

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Abstract

This paper explores the nuanced interplay between the green energy rebound effect and the elasticity of substitution within a Solow growth model. Our study characterizes how the rebound effect varies with the elasticity of substitution between production factors. Our findings demonstrate that technological progress in green or fossil energy leads to different consumption patterns depending on the elasticity of substitution. For substitution elasticities below one, technological advances result in slower increases in energy consumption relative to labor growth, while for elasticities above one, energy consumption accelerates. These insights underscore the critical role of substitution elasticities in shaping effective energy policies, highlighting opportunities to mitigate the rebound effect and promote sustainable energy transitions.

JEL Codes: E13, O41, O44, Q43, Q55

Keywords: rebound effect, energy transition, Solow model, elasticity of substitution.

*Corresponding author. Email: veronica.acurio-vasconez@univ-lorraine.fr,Université de Lorraine, Université de Strasbourg, Centre National de la Recherche Scientifique (CNRS), Bureau d'Economie Théorique et Appliquée (BETA), 13 Place Carnot, CO No.026, 54035 Nancy Cedex, France.

†Email: monica.pereira-henriques4@etu.univ-lorraine.fr,Université de Lorraine

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1 Introduction

The global push for renewable and green energy solutions has led to significant progress in integrating green energy into our economy. The share of energy production from renewable sources in the European Union (27 countries) has increased from 14.4% in 2010 to 24.55% in 2023 (Eurostat (2004)).

It is believed that investing in green and/or renewable energy, which contributes to reducing greenhouse gas emissions and decreasing the reliance on fossil fuels, is critical to mitigate climate change (Edenhofer et al. (2011)), protect human health and the environment, promote sustainable development, and promote global cooperation (UNEP (2024)). Thus, by transitioning to cleaner and more sustainable energy sources, we can build a safer, healthier, and more prosperous future for all.

The European Union has set ambitious goals for 2030: 32% renewable energy consumption, 40% reduction in greenhouse gas emissions, and 32. 5% improvement in energy efficiency (European-Commission (2014)). Similarly, countries such as Germany, China, and the United States, at least before the second Trump administration, have established renewable energy goals to drive the transition. These ambitious goals demonstrate a commitment to reducing greenhouse gas emissions and shifting towards sustainable energy sources. Policy support, technological advances, and international collaboration are essential to achieve these objectives and accelerate the transition to sustainable energy.

In this process, embracing technological progress is vital for a greener economy (Hye et al. (2023), Sampene et al. (2024)). However, we should not overlook the importance of the *rebound effect*.

The rebound effect refers to a set of economic mechanisms that diminish the "energy savings" attained from enhanced energy efficiency. It highlights the phenomenon in which gains in energy efficiency are partially offset by increased energy consumption, which limits the overall effectiveness of efficiency improvements.

As energy efficiency improves, it is reasonable to expect a decrease in energy consumption. For example, with solar panels, the idea is that generating 100kW of solar electricity would lead to a corresponding reduction in grid consumption. However, this expectation is not always met, resulting in a rise in overall electricity consumption. In Galvin et al. (2022), the rebound effect associated with solar panels is defined as the fraction of the increase in total energy services consumption and energy production from photovoltaics. To investigate this phenomenon, the authors analyzed electricity consumption patterns between prosumer and non-prosumer households that installed solar systems in or after 2011. Prosumers, who both produce and use energy, were found to consume an average of 675.57 kWh/year more electricity compared to their matched non-prosumers, who rely solely on grid electricity consumption. With an average consumption of 4787 kWh/year of non-prosumers, this suggests a rebound effect of approximately 14% for prosumers.[1]

Understanding and studying the green energy rebound effect is essential for developing effective energy policies, achieving sustainability targets, enhancing policy effectiveness, enabling accurate impact assessments, aligning policies with sustainability goals, optimizing resource allocation, and, overall, contributing to long-term sustainability efforts. By comprehending and addressing rebound effects, policymakers can design holistic and robust energy policies that facilitate a smooth transition to a sustainable and low-carbon future.

The rebound effect is not, however, a novel concept. It traces its roots back to the introduction of the steam engine when Jevons (1865) noticed a significant increase in coal demand in England. He proposed that as energy systems became more efficient, the cost per unit of energy would decrease, stimulating higher energy demand.

In the 1980s, after oil shocks, the notion of the rebound effect was further explored by Khazzoom (1980) and Brookes (1990), and later on by Berkhout et al. (2000),

[1] Here, the rebound effect is calculated as the fraction of what a prosumer consumes on average with respect to non-prosumer average consumption $\frac{657.57}{4787}$=14%.

Herring (2004), Frondel (2004), and Sorrell (2007). They all refined Jevons (1865) original concept, developing a modern analytical framework to understand the effect.

Although research on the rebound effect has predominantly centered around the fossil fuel sector, there is, however, a recent shift towards investigating the rebound effect within the green energy sector, with focused studies commencing in 2021. More concretely, this type of research centers around the concept of "solar rebound," as it is one of the few sectors where empirical evidence has demonstrated the existence of a rebound effect.

Similar to the rebound effect observed in fossil fuel production, the rebound effect of solar energy in the energy sector can be attributed to factors such as price effect, substitution effect, income effect, and price elasticity. However, in the realm of green energy, the rebound effect can also be influenced by a phenomenon known as moral license (Dütschke et al. (2021)). Moral license suggests that households, upon installing photovoltaic panels, may perceive themselves as having fulfilled their environmental responsibility, thus justifying increased consumption of energy services.

Current research on "solar rebound" has predominantly focused on empirical experiments, providing evidence of the rebound effect in solar energy. For instance, Qiu et al. (2019) conducted a study that quantified the rebound effect by analyzing household electricity meter data, along with hourly solar panel electricity production data, from 277 homes with solar panels and 4,000 homes without solar panels in Phoenix, Arizona, USA, between 2013 and 2017. Their findings estimated a rebound effect of 18%, indicating that for every 1 kWh of solar energy produced by a household, there was an additional 0.18 kWh increase in total energy consumption. This empirical study sheds light on the presence of the rebound effect within the context of solar energy.

Qiu et al. (2019) are not the sole researchers who have sought to quantify the rebound effect. Other scholars have also conducted studies to measure and understand this phenomenon, like Aydın et al. (2023), who found a solar photovoltaic rebound effect of 7.7%.

Other than moral license, the rebound effect in the energy sector has been closely linked to the concept of elasticity of substitution (Saunders (1992) and Saunders (2000)). The elasticity of substitution between factors refers to the extent to which one input can be substituted for another. According to Saunders (2000), the greater the ease of substitution, the higher the rebound effect. In other words, the more readily one input can be substituted for another, the greater the rebound effect observed. This suggests that when it is easier to switch between different energy sources or technologies, there is a higher likelihood of rebound.

This idea builds on the earlier work of Saunders (1992), who used a Solow growth model with three production factors (capital, labor, and energy) to examine the relationship between technological improvements and energy consumption. He demonstrated that the impact of energy efficiency gains depends on the elasticity of substitution. Specifically, when the elasticity of substitution between energy and the other two factors is less than one, technological advancements lead to a decrease in energy consumption. Conversely, when the elasticity of substitution is greater than one, energy consumption increases.

In this paper, we aim to analyze the impact of technological shocks on the consumption of both green energy and fossil fuels. To achieve this, we follow Saunders (1992) and construct a classical growth model *à la* Solow. We assume that production needs four factors: capital, labor, green energy and fossil energy. These production factors are linked in several ways, including a Cobb-Douglas framework to a constant elasticity of substitution (CES) one. Under this framework, we study the effects of different technological shocks on green and fossil energy dynamics.

Our results suggest that technological progress in either green or fossil energy within the Cobb-Douglas function leads to a rebound effect. This suggests that a technological shock in either type of energy leads to a more rapid increase in its consumption. In contrast, for other functional forms, the impact of technological changes in green and fossil energy depends on the elasticity of substitution. We start by assuming a CES production function between two composite factors, one composed of capital and labor, the other of green and fossil energy. For each composite, we assume that their respective factors are linked in a Cobb-Douglas function. In this case, we show that a green (fossil) technological shock provokes a slower increase in green (fossil) energy consumption compared to labor growth when the elasticity of substitution between composites is below one. On the contrary, if the elasticity is above one, a green (fossil) technological shock provokes a faster increase of green (fossil) consumption. We then use a production function where capital, labor, and energy are linked in a Cobb-Douglas function, but we assume a CES one for fossil and green energy. In this case, when the elasticity is below one, an increase in green technology provokes a slower increase in both energy consumption, compared to the labor growth, whereas with an elasticity above one, the green technological shock results in a faster increased consumption for both types of energy.

The findings of this study underscore the importance of tailoring energy policies to the elasticity of substitution between green and fossil energy. When dealing with a Cobb-Douglas production function with CES energy, policymakers should adopt a policy mix approach that strategically promotes green energy technological advancements in sectors where the elasticity of substitution exceeds one, as this ensures a stronger shift towards renewable energy without simultaneously increasing fossil fuel consumption. Conversely, in sectors where the elasticity of substitution is below one, directing technological progress towards fossil energy can help curb its consumption while maintaining economic stability. Additionally, policies should account for the potential rebound effects observed under this production function, where efficiency gains may inadvertently increase overall energy use. To mitigate such risks, complementary measures such as carbon pricing, subsidies for renewable energy, and regulations limiting fossil fuel expansion should be considered. Finally, empirical research on sector- and country-specific substitution elasticities is crucial for designing targeted policies that effectively balance economic growth and environmental sustainability.

Through our research, we aim to contribute to understanding of the complex dynamics between green energy efficiency, demand for energy, the role of elasticities of substitution, and the amplitude of the rebound effect. We hope that this knowledge will be valuable in shaping effective energy policies and strategies that promote sustainability while accounting for potential rebound effects in energy consumption.

This paper is organized as follows: In Section 2, we describe the Solow growth model with energy. Section 3 discusses the results obtained. Finally, Section 4, concludes the paper.

2 A neoclassical growth model with energy

We study the rebound effect by delving into a neoclassical growth model *à la Solow* that includes energy (NGME). In this framework, the rebound effect manifests as an increase in consumption of energy resulting from diverse forms of technical progress.

Our model is based on the one developed by Saunders (1992). We assume that capital accumulates without depreciation as follows:

$$K_{t+1} = I_t + K_t \tag{1}$$

We model a closed economy, meaning that:

$$Y_t = C_t + I_t \tag{2}$$

There is a constant save rate such that:

$$s = \frac{I_t}{Y_t} \tag{3}$$

We assume that labor grows at a constant rate.

Under these hypotheses, it can be shown that capital and production grow at the same rate (cf. Appendix A.1):

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} \tag{4}$$

Within this framework, we introduce two energy sources, categorizing them into green energy $E_{g,t}$ and fossil energy $E_{b,t}$, which are sold at real prices $P_{g,t}$ and $P_{b,t}$, respectively. As in Saunders (1992), we assume that those prices are also constant over time, meaning that:

$$\frac{\dot{p}_{g,t}}{p_{g,t}} = \frac{\dot{p}_{b,t}}{p_{b,t}} = 0 \tag{5}$$

Our analysis diverges from Saunders (1992) as we distinguish between our energy sources, categorizing them into green energy E_g and fossil energy E_f.

To facilitate meaningful comparisons between scenarios, we begin by analyzing a scenario where production factors are easily substitutable and no efficiency gains are present.

2.1 An NGME without efficiency gains

Let us assume that output Y_t is generated using four production factors: capital K_t, labor L_t, green energy $E_{g,t}$, and fossil energy $E_{b,t}$, whose real prices are r_t, w_t, $p_{g,t}$, and $p_{b,t}$, respectively. We assume that these production factors are interrelated thought a Cobb-Douglas production function as follows:

$$Y_t = K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \tag{6}$$

where each $\alpha_i, i \in \{k, l, g, b\}$ represents the elasticity of production with respect to factor i. Since firms operate in a perfectly competitive market, it can be shown that these elasticities correspond to factor shares. This equality allows us to properly calibrate these parameters for the simulations (see Appendix A.2 for details).

Assuming a labor growth rate of 3% per year, it can be shown that the growth rates of endogenous variables are those of Table 1 (see Appendix A.2 for derivation).

Table 1: Factor Growth Rates without Technological Progress

	Annual Growth Rate*					Annual Growth Rate*		
	$\frac{\dot{Y}_t}{Y_t}$	$\frac{\dot{K}_t}{K_t}$	$\frac{\dot{L}_t}{L_t}$	$\frac{\dot{E}_{g,t}}{E_{g,t}}$	$\frac{\dot{E}_{b,t}}{E_{b,t}}$	$\frac{\dot{w}_t}{w_t}$	$\frac{\dot{p}_{e,t}}{p_{e,t}}$	$\frac{\dot{\frac{C_t}{L_t}}}{\frac{C_t}{L_t}}$
None	3.0	3.0	3.0	3.0	3.0	0.0	0.0	0.0

* : % per year. $\alpha_l = 0.6188$, $\alpha_k = 0.3314$, $\alpha_g = 0.00018$, $\alpha_b = 0.0495$.

By assuming a Cobb-Douglas function without technical progress and fixed energy prices, we observe uniform growth rates across energy, capital, and real output, equal to the growth rate of labor. The simultaneous growth of all factors and output leads to the stabilization of the E_g/Y and E_b/Y ratios, maintaining a constant energy intensity in both sectors. Furthermore, as all real prices are constants over time, it can be shown that the real consumption per worker remains constant.

Having established our baseline NGME without technological progress, we proceed to contrast this growth pattern with alternative scenarios involving technological progress and the application of CES functions.

2.2 An NGME with efficiency gains

2.2.1 The Cobb-Douglas production function

In contrast to the previous scenario, let us assume here that the production function incorporates technological progress and is expressed as follows:

$$Y_t = \tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b} \tag{7}$$

In this context, τ_i, with $i \in \{k, l, g, b\}$ denotes the level of technological progress for the respective factor i. Specifically, τ_n signifies neutral technological progress—a development that uniformly affects all production factors without favoring one input over another. This underscores the diverse forms that technological advancements can take, whether they augment capital, labor, green energy, or fossil energy or exhibit neutrality.

Please note that in each scenario, we incorporate only one form of technological progress at a time. For instance, when we study the neutral technological progress case, we assume a constant growth rate for this variable and hold the other constants at a value of one over time.

Having incorporated technological progress, we define the rebound effect as in Saunders (1992): Under a fixed real energy price, a rebound effect appears when efficiency gains propel energy consumption to levels surpassing those achievable without such improvements.

As before, we assume that labor grows at a constant rate of 3% per year. Additionally, we assume that each technology type grows at a constant rate of 1.2% per year. The derivation of results can be found in Appendix A.3.

Table 2 summarizes our results. We observe that production, capital, green energy, and fossil energy all exhibit parallel growth rates above the growth rate of labor, in instances of efficiency gains resulting from technological progress. For

Table 2: Factor Growth Rates with Technological Progress: Cobb-Douglas

Type of Techno-logical Progress	Annual Growth Rate*					Annual Growth Rate*			Technology-to-non-technology Energy Ratio
	$\frac{\dot{Y}_t}{Y_t}$	$\frac{\dot{K}_t}{K_t}$	$\frac{\dot{L}_t}{L_t}$	$\frac{\dot{E}_{g,t}}{E_{g,t}}$	$\frac{\dot{E}_{b,t}}{E_{b,t}}$	$\frac{\dot{w}_t}{w_t}$	$\frac{\dot{p}_{e,t}}{p_{e,t}}$	$\frac{\dot{\frac{C_t}{L_t}}}{\frac{C_t}{L_t}}$	$\frac{E_{b,t}}{E_{b,t,\text{no tech}}}$
None	3.0	3.0	3.0	3.0	3.0	0.0	0.0	0.0	1.0
Neutral	4.9	4.9	3.0	4.9	4.9	1.9	0.0	1.9	8.3
Capital	3.6	3.6	3.0	3.6	3.6	0.6	0.0	0.6	2.1
Labor	4.2	4.2	3.0	4.2	4.2	1.2	0.0	1.2	3.6
Green	3.03	3.03	3.0	3.03	3.03	0.0	0.0	0.0	1.0
Fossil	3.1	3.1	3.0	3.1	3.1	0.1	0.0	0.1	1.1

* : % per year. $\alpha_l = 0.6214$, $\alpha_k = 0.316$, $\alpha_g = 0.0141$, $\alpha_b = 0.0484$.

instance, with a neutral-augmenting gain, production, capital, green energy and fossil energy experience a growth rate of 4.9%, surpassing the rates observed in the absence of technological progress (3%).

The critical observation here is that"pure" energy efficiency gains also lead to an uptick in energy consumption. In the case of fossil energy-augmenting technological progress, we note an increase in both types of energy consumption (3.1% compared to 3%), indicating that efficiency gains in fossil energy contribute to an actual rise in fossil and green energy usage. In the case of green energy-augmenting technological progress, we also note an increase in both types of energy consumption, but this increase is much more smaller than the one observed with the fossil energy-augmenting technology.

This result can be explained by the predominant role of fossil energy in energy consumption relative to green energy ($\alpha_{e,b} > \alpha_{e,g}$). This disparity underscores the greater influence of fossil energy over green energy in the production process. These observations are more clearly depicted in Figures 1 through 4. Solid lines represent variable trajectory without technological shock, and discontinued lines represent variable trajectory with technological change in each scenario.

Figure 1: Cobb-Douglas: Fossil Energy-Augmenting Tech. Prog. on K and Y

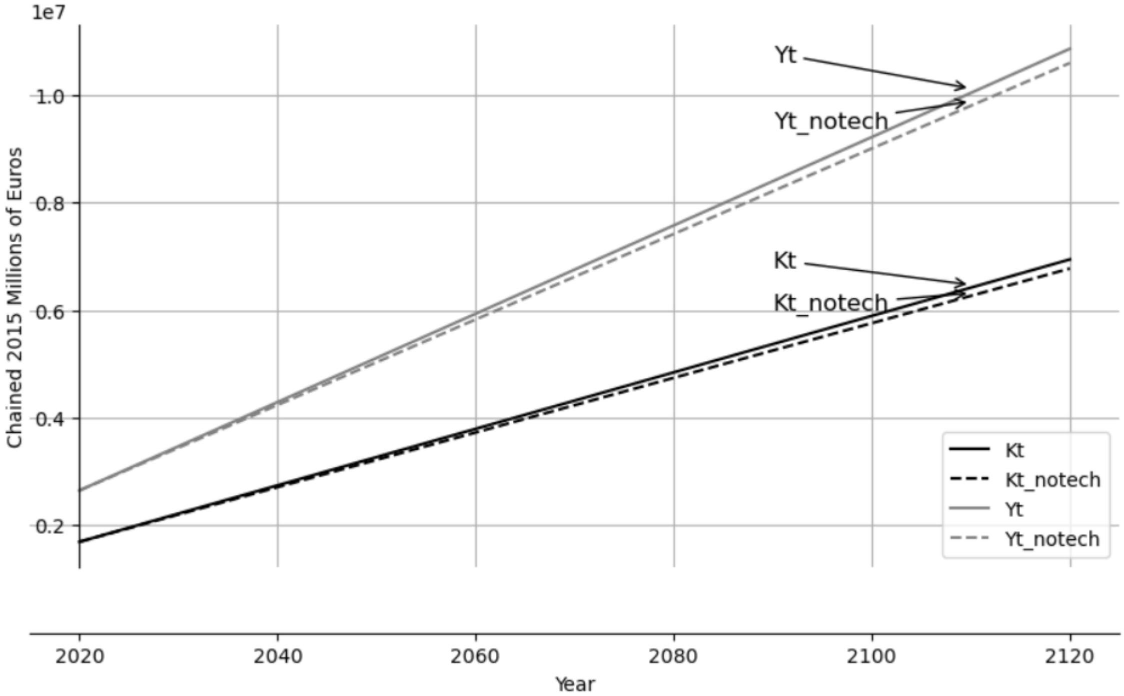

Figure 2: Cobb-Douglas: Fossil Energy-Augmenting Tech. Prog. on E_g and E_f

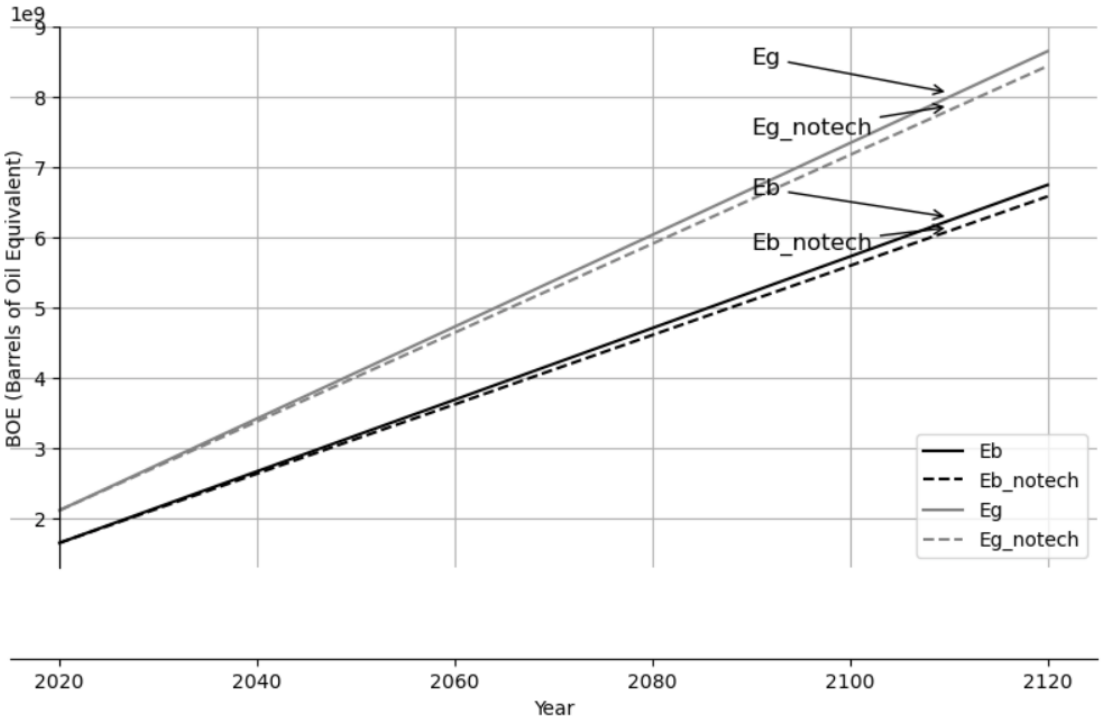

Figure 3: Cobb-Douglas: Green Energy-Augmenting Tech. Prog. on K and Y

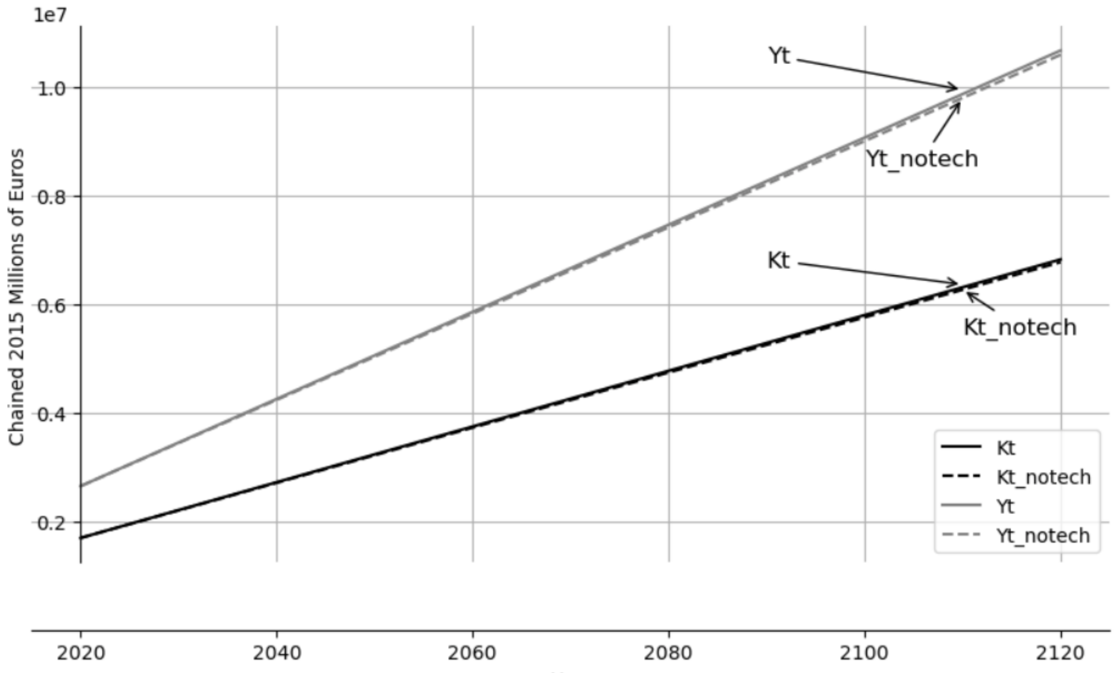

Figure 4: Cobb-Douglas: Green Energy-Augmenting Tech. Prog. on E_g and E_f

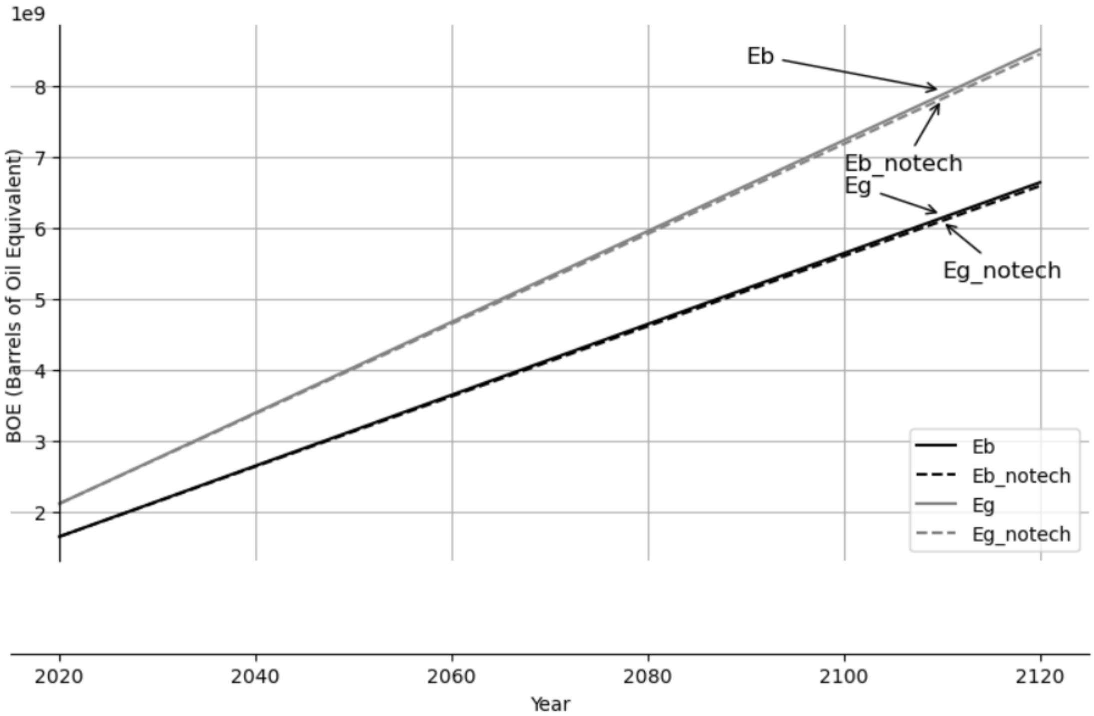

Figure 2 particularly illustrates that enhancements in fossil energy efficiency lead to heightened consumption of both fossil and green energy (as well as capital and production) compared to the no technological case, depicted by the slight gap between the solid and dotted lines. In contrast, when there is an improvement in green energy efficiency, Figure 4, the graphs depict a steady level of consumption over the years. Here, it is evident that the solid and dotted lines are combined.

2.2.2 CES function with nested (K, L) and (Eg, Eb) in Cobb-Douglas functions

Assume now that the production function is given by:

$$Y_t = \tau_n(a((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l})^{\rho} + (1-a)((\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_v})^{\rho})^{1/\rho} \tag{8}$$

where parameter a represents the share parameter between two composite factors, one combining capital and labor and the other combining green and fossil energy, and $\rho = \frac{\sigma-1}{\sigma}$, with σ denoting the elasticity of substitution between the two composites.

In this section, we assume as well that real capital prices are constant as energy prices (i.e. $\frac{\dot{r_t}}{r_t} = 0$).

As shown in Appendix A.4, in this case, both types of energies must have the same growth rates for the existence of a steady state, meaning that:

$$\frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\dot{E_{b,t}}}{E_{b,t}} \tag{9}$$

As shown in Appendix A.4, in this framework, powers α_i no longer equalize the factor's share. It can be shown however that:

$$\alpha_l = \frac{w_t L_t}{r_t K_t + w_t L_t} \tag{10}$$

$$\alpha_k = 1 - \alpha_l \tag{11}$$

$$\alpha_b = \frac{p_{b,t} E_{b,t}}{p_{g,t} E_{g,t} + p_{b,t} E_{b,t}} \tag{12}$$

$$\alpha_g = 1 - \alpha_b \tag{13}$$

Then in this framework, α_i are cost shares of input i in their respective composite. These equations help us with the calibration of these parameters for the simulations.

Results are summarized in Table 3. As before, for each case we assume that only one technological type grows at a constant rate of 1.2%, while the other rest are constant at a value of one. Labor growth at 3%.

Table 3: Factor Growth Rates with Technological Progress: CES Type 1

Type of Technological Progress	Annual Growth Rate*			Annual Growth Rate*			Annual Growth Rate*		
	$\frac{\dot{Y}_t}{Y_t}$	$\frac{\dot{K}_t}{K_t}$	$\frac{\dot{L}_t}{L_t}$	$\frac{\dot{E}_{g,t}}{E_{g,t}}$			$\frac{\dot{E}_{b,t}}{E_{b,t}}$		
				$\sigma =$ 0.5	$\sigma =$ 1	$\sigma =$ 1.5	$\sigma =$ 0.5	$\sigma =$ 1	$\sigma =$ 1.5
None	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Neutral	4.9	4.9	3.0	4.3	4.9	5.5	4.3	4.9	5.5
Capital	3.7	3.7	3.0	3.7	3.7	3.7	3.7	3.7	3.7
Labor	4.2	4.2	3.0	4.2	4.2	4.2	4.2	4.2	4.2
Green	3.0	3.0	3.0	2.9	3	3.1	2.9	3.0	3.1
Fossil	3.0	3.0	3.0	2.5	3	3.5	2.5	3.0	3.5

* : % per year. $\alpha_l = 0.618$, $\alpha_k = 0.382$, $\alpha_g = 0.2324$, $\alpha_b = 0.7676$

Just like the Cobb-Douglas production function, when there is technological progress favoring capital or labor, it results in uniform growth across all inputs and output at the same rate, and the growth rates are the same as those observed in the Cobb-Douglas case. However, in scenarios involving neutral, green energy-augmenting, and fossil energy-augmenting technological shocks, its impact depends on the value of the elasticity of substitution between composites. Interestingly, this dependency is apparent only in the growth rates of green and fossil energy, while other variables remain unaffected by the elasticity of substitution. This emphasizes the dynamic nature of energy growth rates influenced by the substitutability between inputs.

In these scenarios, we observe that the higher the elasticity of substitution, the greater the growth rate of green and fossil energy after a technological shock. A higher σ implies greater flexibility in substituting capital and labor to green and fossil energy in the production process. This increased flexibility enables the economy to allocate resources more efficiently to inputs experiencing technological progress, resulting in higher growth rates for green and fossil energy production relative to overall output and capital.

As explained, we observe a rebound effect, wherein efficiency gains in fossil or green energy lead to increased consumption of the respective energy sources above 3%, which is the growth rate of labor. Under our definition, a rebound effect is evident only when the elasticity of substitution (σ) exceeds one. Conversely, when σ is lower than one, the consumption of either energy type increases at a lower rate than that of labor growth, indicating that technological progress results in reduced energy requirements for the same output level.

A rebound effect is also noticeable in scenarios involving neutral efficiency gains, and it is visible for any value of the elasticity of substitution.

Additionally, the growth rate for fossil energy remains higher than the growth rate of green energy, which again can be attributed to its predominant usage compared to green energy within the energy balance ($\alpha_b > \alpha_g$).

2.2.3 Cobb-Douglas function with CES energy

Let us take the following form for the production function:

$$Y_t = \tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}} \tag{14}$$

Remark that in this functional form, capital, labor, and total energy are linked in a Cobb-Douglas function, while both forms of energy are linked in a CES function. Accordingly, here, a denotes the share parameter between the two types of energy, and $\rho = \frac{\sigma-1}{\sigma}$, with σ indicating the elasticity of substitution between energy types.

Furthermore, it is essential to note that, in this case, achieving a steady state does not require imposing the same growth rate for both types of energy.

The α_i parameters are equal to factor shares.

Table 4: Factor Growth Rates with Technological Progress: CES Type 2

Type of Technological Progress	Annual Growth Rate*			Annual Growth Rate*			Annual Growth Rate*		
	$\frac{\dot{Y}_t}{Y_t}$	$\frac{\dot{K}_t}{K_t}$	$\frac{\dot{L}_t}{L_t}$	$\frac{\dot{E}_{g,t}}{E_{g,t}}$			$\frac{\dot{E}_{b,t}}{E_{b,t}}$		
				$\sigma =$ 0.5	$\sigma =$ 1	$\sigma =$ 1.5	$\sigma =$ 0.5	$\sigma =$ 1	$\sigma =$ 1.5
None	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Neutral	4.9	4.9	3.0	4.9	4.9	4.9	4.9	4.9	4.9
Capital	3.6	3.6	3.0	3.6	3.6	3.6	3.6	3.6	3.6
Labor	4.2	4.2	3.0	4.2	4.2	4.2	4.2	4.2	4.2
Green	3.0	3.0	3.0	2.4	3.0	3.6	3.0	3.0	3.0
Fossil	3.0	3.0	3.0	3.0	3.0	3.0	2.4	3.0	3.6

* : % per year. $\alpha_l = 0.6214$, $\alpha_k = 0.316$, $\alpha_e = 0.0626$.

Results are shown in Table 4. When technological progress in labor increases, production, capital, and both fossil and green energy grow at a uniform rate of 4.2%. This outcome is consistent with those obtained using the Cobb-Douglas function and the CES function with nested (K_t, L_t) and $(E_{g,t}, E_{b,t})$.

However, it is noteworthy that under a scenario of neutral technological change, the growth rates of production, capital, fossil energy, and green energy are not influenced by the elasticity of substitution between energies, unlike the previous case, with a CES function with nested (K_t, L_t) and $(E_{g,t},\ E_{b,t})$ within a Cobb-Douglas function. Instead, all these variables grow uniformly at a rate of 4.9%.

The main distinction of this particular function compared to both the Cobb-Douglas and the CES function with composites (K_t, L_t) and $(E_{g,t}, E_{b,t})$ lies in the scenario of technological advancements in fossil or green energy. In the case of efficiency gains in fossil energy, production, capital, labor, and green energy all experience identical growth rates. However, only the growth of fossil energy is affected by the elasticity of substitution (σ) and a rebound effect is observed when this elasticity is higher than one. Consequently, higher substitution prompts increased consumption of fossil energy, while green energy consumption remains unchanged.

As an illustration of this case, consider a manufacturing facility that primarily relies on fossil fuels but incorporates a small amount of renewable energy whenever feasible, without being required to do so. Ignoring population growth, upgrading fossil fuel machinery would lead to increased fossil fuel consumption, as this factor becomes more productive and its use is not tied to green energy. Meanwhile, the utilization of green energy remains unchanged.

Alternatively, consider a region where industries rely primarily on natural gas for energy, but a newly introduced policy mandates a gradual increase in the share of renewable energy. In this setting, advancements in natural gas technology that improve efficiency may not necessarily lead to higher natural gas consumption, as firms face regulatory constraints and societal expectations pushing for a greener transition. However, the adoption of renewable energy sources such as wind and solar is unlikely to increase significantly, as existing infrastructure and financial incentives remain insufficient to support a large-scale shift away from fossil fuels.

Similarly, in the case of a green energy technological change, only the dynamics of green energy consumption depend upon the elasticity of substitution. Once again, higher substitution leads to increased consumption of green energy.

Consider a scenario where a city government encourages the adoption of renewable energy, leading to widespread installation of solar panels and wind turbines. In an environment with high energy substitutability, the increase in green energy consumption can occur despite the prevailing reliance on fossil fuels. This is because high substitutability allows for greater flexibility in replacing fossil fuels with green alternatives.

Conversely, in a situation with low substitution elasticity, even significant technological advancements in green energy production may not result in substantial reductions in fossil fuel usage. For instance, the development of more efficient solar panels or wind turbines might not lead to significant decreases in fossil fuel consumption if the potential for substitution is limited. As a result, despite improvements in green energy efficiency, constrained substitution elasticity may lead to stagnant or reduced green energy consumption, while fossil fuel consumption remains unchanged.

This highlights the significant impact of low substitution elasticity on consumption patterns.

3 Results and discussion

These findings provide valuable insights for the development of effective policies aimed at improving our economy and transitioning to a greener economic model. Understanding the implications of the elasticity of substitution is crucial in anticipating the outcomes of efficiency gains in green energy. In a general equilibrium framework, the form of the production function is crucial. In cases where the Cobb-Douglas function applies, efficiency gains in green or fossil energy lead to an increase in overall energy consumption due to technological advancements. Therefore, policymakers should exercise caution when confronted with a Cobb-Douglas production function as improving the efficiency of fossil energy may lead to a rebound effect, where the consumption of both fossil and green energy increases.

These results may change, however, when assuming an imperfect substitutability between factors in different forms.

For our initial analysis, we use a CES function with two composite factors: one combining capital and labor, and the other combining green and fossil energy. When the elasticity of substitution between these composites falls below one, technological progress in either energy source results in lower growth rates for the consumption of both green and fossil energy compared to scenarios without technological progress. Consequently, the consumption of both green and fossil energy grows at a slower rate than labor. While a reduction in fossil energy consumption may be beneficial for society, a significant slowdown in green energy consumption could impede the transition to a more sustainable future.

Nevertheless, if the substitution elasticity between composites exceeds one, an increase in any type of energy technology leads to an increase in the use of both energy types. This gives us a mixed outcome: while green energy rises after energy technological shocks, so does fossil energy.

It is crucial to note, however, that these last results stem from our assumption of constant energy prices, which implies in this case an equal growth rate for both types of energy.

For our next analysis, we use a Cobb-Douglas function for capital, labor, and energy but assume that green and fossil energy are linked in a CES one.

Using this particular production function, we find that when there is technological advancement in green energy and the elasticity of substitution is below one, the technological green energy shock results in an increase in green energy consumption that is smaller than labor growth, while fossil energy consumption grows at the same rate as labor. This slower increase in green energy consumption warrants caution among policymakers. However, when the elasticity of substitution exceeds one, we witness a faster increase in green energy consumption while fossil energy grows at the same rate as labor. This is a promising outcome, as it signifies progress towards a greener future with increased green energy usage, without pushing fossil energy consumption.

Regarding technological advancements in fossil energy, we observe similar results. If the elasticity exceeds one, fossil energy consumption increases at a faster rate than labor, which is undesirable. However, when the elasticity is below one, fossil energy increases at a slower rate than green energy, with the latter growing at the rate of labor. This finding is in line with sustainability objectives, particularly the goal of reducing reliance on fossil fuels and promoting the growth of renewable energy sources.

In view of these results, policymakers should consider adopting a "policy mix" approach when dealing with these types of functions. This entails focusing on technological progress in green energy sectors where the elasticity of substitution exceeds one, leading to increased green energy usage. Conversely, in sectors with an elasticity of substitution below one, policymakers should prioritize technological advancements in fossil energy, resulting in reduced fossil energy consumption. By combining these two strategies, policymakers can achieve the dual objective of boosting green energy while curbing fossil energy consumption, thus promoting a more sustainable energy landscape, while labor, capital and output growth stay constant.

In scenarios where the growth of fossil or green energy is influenced by the elasticity of substitution, our findings align with those of Saunders (1992). When the elasticity of substitution falls below one, it leads to a decrease in the consumption of the respective energy source or both energies. Conversely, if the elasticity of substitution exceeds one, it results in an increase in the consumption of the relevant energy source or both energies.

We hope that, armed with this knowledge, policymakers can design targeted policies to facilitate the transition towards a greener economy. However, it is equally important to consider scenarios where the inputs are not substitutable. In such cases, an increase in green energy efficiency may not be beneficial. Instead of achieving a greener economy, this can lead to an undesirable outcome where fossil energy consumption grows at a faster rate than green energy consumption.

4 Conclusion

In summary, the findings of this study provide valuable insights for shaping effective policies aimed at transitioning to a greener economic model. By understanding the implications of the elasticity of substitution, policymakers can anticipate the outcomes of efficiency gains in green energy.

In a general equilibrium framework, it is also crucial to acknowledge that the nature of energy consumption dynamics varies depending on the production function employed. When factors are linked in a Cobb-Douglas function, a rebound effect, is evident: Any increase in energy efficiency leads to an increase in consumption of both types of energy. However, when functions incorporate elasticities of substitutions different to one, the scenario shifts. When the elasticity of substitution is below one, technological progress in green or fossil energy leads to slower increases in consumption in the respective sector, while a faster increase is observed when the elasticity of substitution exceeds one.

The role of substitution elasticities in energy consumption dynamics is paramount, influencing the outcomes of technological advancements and policy interventions. The elasticity of substitution determines the extent to which consumers and industries can switch between different energy sources in response to changes in prices or efficiency. A higher elasticity implies greater flexibility in substitution, allowing for more significant shifts in consumption patterns. Conversely, a lower elasticity suggests a limited ability to substitute one energy source for another.

Understanding its role is crucial for policymakers as it shapes the effectiveness of policies aimed at promoting renewable energy adoption, reducing reliance on fossil fuels, and mitigating environmental impacts (i.e., like the "policy mix" mentioned above). By considering substitution elasticity in energy models and policy frameworks, policymakers can develop more targeted and effective strategies to navigate the transition towards sustainable energy systems.

However, to ensure the best policy design, it is imperative to also conduct empirical studies on the elasticities of substitution among the factors outlined in the current research. By doing so, we can accurately determine the economic conditions of a specific sector or country and devise appropriate energy policies.

From a theoretical perspective, it's crucial to reevaluate a key assumption within the general equilibrium framework: the notion of constant energy prices over time. Indeed, permitting price variability over time will undoubtedly yield substantial impacts on our outcomes.

It is also important to note that this study primarily focuses on the perspective of producers and their energy consumption. However, households also play a significant role as energy consumers in the economy and should be incorporated into the analysis. A natural extension of this paper could involve modeling household energy consumption within a Ramsey or Real Business Cycle (RBC) framework.

We leave all of this for future work.

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A Appendix A: Derivations for Solow Models

A.1 Capital and production balance growth path

Given equations (1) and (3), which are valid whatever the form of the production function, we can show that:

$$\frac{\dot{K}_t}{K_t} = s\frac{Y_t}{K_t} \tag{15}$$

If the rate of growth of capital is constant over time, then a steady state exists if and only if Y growth at the same speed than K, meaning that:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} \tag{16}$$

A.2 NGME without efficiency gains

For the scenario without efficiency gains, we will use a Cobb-Douglas production function. The maximization problem in real terms is given by:

$$\max_{Y_t;K_t;L_t;E_{g,t};E_{b,t}} \quad Y_t - w_tL_t - r_tK_t - p_{g,t}E_{g,t} - p_{b,t}E_{b,t} \tag{17}$$

$$\text{s.c } Y_t = K_t^{\alpha_k}L_t^{\alpha_l}E_{g,t}^{\alpha_g}E_{b,t}^{\alpha_b} \tag{18}$$

The Lagrangian is as follows:

$$\mathcal{L}_t = Y_t - w_tL_t - r_tK_t - p_{g,t}E_{g,t} - p_{b,t}E_{b,t} - \lambda_t(Y_t - K_t^{\alpha_k}L_t^{\alpha_l}E_{g,t}^{\alpha_g}E_{b,t}^{\alpha_b}) \tag{19}$$

A.2.1 First-order conditions

First order conditions are as follows:

$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t} = 0 \Leftrightarrow Y_t - K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} = 0 \tag{20}$$

$$\Leftrightarrow Y_t = K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \tag{21}$$

$$\frac{\partial \mathcal{L}_t}{\partial Y_t} = 0 \Leftrightarrow 1 - \lambda_t = 0 \Leftrightarrow 1 = \lambda_t \tag{22}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = 0 \Leftrightarrow -r_t - \lambda_t(-L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \alpha_k K_t^{\alpha_k - 1}) = 0 \tag{23}$$

$$\Leftrightarrow r_t = \lambda_t(L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \alpha_k K_t^{\alpha_k - 1}) \tag{24}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = 0 \Leftrightarrow -w_t - \lambda_t(-K_t^{\alpha_k} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \alpha_l L_t^{\alpha_l - 1}) = 0 \tag{25}$$

$$\Leftrightarrow w_t = \lambda_t(K_t^{\alpha_k} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \alpha_l L_t^{\alpha_l - 1}) \tag{26}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{g,t}} = 0 \Leftrightarrow -p_{g,t} - \lambda_t(-K_t^{\alpha_k} L_t^{\alpha_l} E_{b,t}^{\alpha_b} \alpha_g E_{g,t}^{\alpha_g - 1}) = 0 \tag{27}$$

$$\Leftrightarrow p_{g,t} = \lambda_t(K_t^{\alpha_k} L_t^{\alpha_l} E_{b,t}^{\alpha_b} \alpha_g E_{g,t}^{\alpha_g - 1}) \tag{28}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{b,t}} = 0 \Leftrightarrow -p_{b,t} - \lambda_t(-K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} \alpha_b E_{b,t}^{\alpha_b} - 1) = 0 \tag{29}$$

$$\Leftrightarrow p_{b,t} = \lambda_t(K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} \alpha_b E_{b,t}^{\alpha_b} - 1) \tag{30}$$

A.2.2 Factor shares

We substitute equation (22) into equations (24) to (30) and use the definition of the production function to obtain:

$$r_t = L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \alpha_k K_t^{\alpha_k - 1} \Leftrightarrow \alpha_k = \frac{r_t K_t}{K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b}} \Leftrightarrow \alpha_k = \frac{r_t K_t}{Y_t} \tag{31}$$

$$w_t = K_t^{\alpha_k} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b} \alpha_l L_t^{\alpha_l - 1} \Leftrightarrow \alpha_l = \frac{w_t L_t}{K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b - 1}} \Leftrightarrow \alpha_l = \frac{w_t L_t}{Y_t} \tag{32}$$

$$p_{g,t} = K_t^{\alpha_k} L_t^{\alpha_l} E_{b,t}^{\alpha_b} \alpha_g E_{g,t}^{\alpha_g - 1} \Leftrightarrow \alpha_g = \frac{p_{g,t} E_{g,t}}{K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b}} \Leftrightarrow \alpha_g = \frac{p_{g,t} E_{g,t}}{Y_t} \tag{33}$$

$$p_{b,t} = K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} \alpha_b E_{b,t}^{\alpha_b - 1} \Leftrightarrow \alpha_b = \frac{p_{b,t} E_{b,t}}{K_t^{\alpha_k} L_t^{\alpha_l} E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b}} \Leftrightarrow \alpha_b = \frac{p_{b,t} E_{b,t}}{Y_t} \tag{34}$$

A.2.3 Growth rates of factors

The growth rate of production per capita is given by:

$$\frac{\dot{y}_t}{y_t} = \alpha_k \frac{\dot{k}_t}{k_t} + \alpha_{e_g} \frac{\dot{e}_{g,t}}{e_{g,t}} + \alpha_{e_b} \frac{\dot{e}_{b,t}}{e_{b,t}} \tag{35}$$

Remark that a steady state exists if the growth rate of capital, green energy and fossil energy per capital are zero.

Furthermore, assuming that the rate of growth of labor is n, then we have:

$$\frac{\dot{k}_t}{k_t} = 0 \Leftrightarrow \frac{\dot{K}_t}{K_t} - \frac{\dot{L}_t}{L_t} = 0 \Leftrightarrow \frac{\dot{K}_t}{K_t} = n \tag{36}$$

$$\frac{\dot{e}_{g,t}}{e_{g,t}} = 0 \Leftrightarrow \frac{\dot{E}_{g,t}}{E_{g,t}} - \frac{\dot{L}_t}{L_t} = 0 \Leftrightarrow \frac{\dot{E}_{g,t}}{E_{g,t}} = n \tag{37}$$

$$\frac{\dot{e}_{b,t}}{e_{b,t}} = 0 \Leftrightarrow \frac{\dot{E}_{b,t}}{E_{b,t}} - \frac{\dot{L}_t}{L_t} = 0 \Leftrightarrow \frac{\dot{E}_{b,t}}{E_{b,t}} = n \tag{38}$$

With these three equations, we can ascertain the growth rate of the output:

$$\frac{\dot{Y}_t}{Y_t} = \alpha_k \frac{\dot{K}_t}{K_t} + \alpha_l \frac{\dot{L}_t}{L_t} + \alpha_{e_g} \frac{\dot{E}_{g,t}}{E_{g,t}} + \alpha_{e_b} \frac{\dot{E}_{b,t}}{E_{b,t}} \Leftrightarrow \frac{\dot{Y}_t}{Y_t} = n \tag{39}$$

Let us analyze the various growth rates of prices using their respective equations found in (31), (32), (33), and (34):

$$\frac{\dot{r}_t}{r_t} = \alpha_l \frac{\dot{L}_t}{L_t} + \alpha_g \frac{\dot{E}_{g,t}}{E_{g,t}} + \alpha_b \frac{\dot{E}_{b,t}}{E_{b,t}} + (\alpha_k - 1)\frac{\dot{K}_t}{K_t} \Leftrightarrow \frac{\dot{r}_t}{r_t} = n(\alpha_l + \alpha_g + \alpha_b + \alpha_k - 1) \Leftrightarrow \frac{\dot{r}_t}{r_t} = 0 \tag{40}$$

$$\frac{\dot{w}_t}{w_t} = \alpha_k \frac{\dot{K}_t}{K_t} + \alpha_g \frac{\dot{E}_{g,t}}{E_{g,t}} + \alpha_b \frac{\dot{E}_{b,t}}{E_{b,t}} + (\alpha_l - 1)\frac{\dot{L}_t}{L_t} \Leftrightarrow \frac{\dot{w}_t}{w_t} = n(\alpha_k + \alpha_g + \alpha_b + \alpha_l - 1) \Leftrightarrow \frac{\dot{w}_t}{w_t} = 0 \tag{41}$$

$$\frac{\dot{p}_{g,t}}{p_{g,t}} = \alpha_k \frac{\dot{K}_t}{K_t} + \alpha_l \frac{\dot{L}_t}{L_t} + \alpha_b \frac{\dot{E}_{b,t}}{E_{b,t}} + (\alpha_g - 1)\frac{\dot{E}_{g,t}}{E_{g,t}} \Leftrightarrow \frac{\dot{p}_{g,t}}{p_{g,t}} = n(\alpha_k + \alpha_l + \alpha_b + \alpha_g - 1) \Leftrightarrow \frac{\dot{p}_{g,t}}{p_{g,t}} = 0 \tag{42}$$

$$\frac{\dot{p}_{b,t}}{p_{b,t}} = \alpha_k \frac{\dot{K}_t}{K_t} + \alpha_l \frac{\dot{L}_t}{L_t} + \alpha_g \frac{\dot{E}_{g,t}}{E_{g,t}} + (\alpha_b - 1)\frac{\dot{E}_{b,t}}{E_{b,t}} \Leftrightarrow \frac{\dot{p}_{b,t}}{p_{b,t}} = n(\alpha_k + \alpha_l + \alpha_g + \alpha_b - 1) \Leftrightarrow \frac{\dot{p}_{b,t}}{p_{b,t}} = 0 \tag{43}$$

The growth of consumption per capita:

$$\frac{\frac{\dot{C}_t}{L_t}}{\frac{C_t}{L_t}} = \frac{\mathrm{d}}{\mathrm{d}t} ln\left(\frac{Y_t}{L_t}\right) = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{L}_t}{L_t} = n - n = 0 \tag{44}$$

Assuming that $n = 3\%$, the growth rates of production, green energy, and fossil energy all exhibit a 3% increase. Then, the energy intensity in each sector is given by:

$$\frac{E_{g,t+1}}{Y_t + 1} = \frac{E_{g,t} \times 1.03}{Y_t \times 1.03} = \frac{E_{g,t}}{Y_t} \tag{45}$$

$$\frac{E_{b,t+1}}{Y_t + 1} = \frac{E_{b,t} \times 1.03}{Y_t \times 1.03} = \frac{E_{b,t}}{Y_t} \tag{46}$$

This indicates the stabilization of both ratios.

481 ## A.3 NGME With efficiency gains: Cobb-Douglas function

The maximization problem in real terms is given by:

$$\max_{Y_t;K_t;L_t;E_{g,t};E_{b,t}} \quad Y_t - W_t L_t - r_t K_t - p_{g,t}E_{g,t} - p_{b,t}E_{b,t} \tag{47}$$

$$\text{s.c } Y_t = \tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}) \tag{48}$$

The associated Lagrangian is given by:

$$\begin{aligned}\mathcal{L}_t =& Y_t - w_t L_t - r_t K_t - p_{g,t}E_{g,t} - p_{b,t}E_{b,t} - \\ & \lambda_t(Y_t - \tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}))\end{aligned} \tag{49}$$

A.3.1 First-order conditions

First order conditions are as follows:

$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t} = 0 \Leftrightarrow Y_t - \tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}) = 0 \tag{50}$$

$$\Leftrightarrow Y_t = \tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}) \tag{51}$$

$$\frac{\partial \mathcal{L}_t}{\partial Y_t} = 0 \Leftrightarrow 1 - \lambda_t = 0 \Leftrightarrow \lambda_t = 1 \tag{52}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = 0 \Leftrightarrow - r_t - \lambda_t(\tau_n((\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_k \tau_k^{\alpha_k} K^{\alpha_k - 1})) = 0 \tag{53}$$

$$\Leftrightarrow r_t = \lambda_t(\tau_n((\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_k \tau_k^{\alpha_k} K^{\alpha_k - 1})) \tag{54}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = 0 \Leftrightarrow - w_t - \lambda_t(\tau_n((\tau_k K_t)^{\alpha_k}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_l \tau_l^{\alpha_l} L^{\alpha_l - 1})) = 0 \tag{55}$$

$$\Leftrightarrow w_t = \lambda_t(\tau_n((\tau_k K_t)^{\alpha_k}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_l \tau_l^{\alpha_l} L^{\alpha_l - 1})) \tag{56}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{g,t}} = 0 \Leftrightarrow - p_{g,t} - \lambda_t(\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_b E_{b,t})^{\alpha_b}\alpha_g \tau_g^{\alpha_g} E_{g,t}^{\alpha_g - 1})) = 0 \tag{57}$$

$$\Leftrightarrow p_{g,t} = \lambda_t(\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_b E_{b,t})^{\alpha_b}\alpha_g \tau_g^{\alpha_g} E_{g,t}^{\alpha_g - 1})) \tag{58}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{b,t}} = 0 \Leftrightarrow - p_{b,t} - \lambda_t(\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}\alpha_b \tau_b^{\alpha_b} E_{g,t}^{\alpha_b - 1})) = 0 \tag{59}$$

$$\Leftrightarrow p_{b,t} = \lambda_t(\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}\alpha_b \tau_b^{\alpha_b} E_{b,t}^{\alpha_b - 1})) \tag{60}$$

483 A.3.2 Factor shares

We substitute equation (52) into equations (54) to (60) and use the definition of the production function to obtain:

$$r_t = \tau_n((\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_k \tau_k^{\alpha_k} K^{\alpha_k - 1}) \tag{61}$$

$$\Leftrightarrow \frac{r_t K_t}{\tau_n((\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}(K_t^{\alpha_k}))} = \alpha_k \Leftrightarrow \alpha_k = \frac{r_t K_t}{Y} \tag{62}$$

$$w_t = \tau_n((\tau_k K_t)^{\alpha_k}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_l \tau_l^{\alpha_l} L^{\alpha_l - 1}) \tag{63}$$

$$\Leftrightarrow \alpha_l = \frac{w_t L_t}{\tau_n((\tau_k K_t)^{\alpha_k}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}(\tau_l L_t)^{\alpha_l}} \Leftrightarrow \alpha_l = \frac{w_t L_t}{Y} \tag{64}$$

$$p_{g,t} = \tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_b E_{b,t})^{\alpha_b}\alpha_g \tau_g^{\alpha_g} E_{g,t}^{\alpha_g - 1} \tag{65}$$

$$\Leftrightarrow \alpha_g = \frac{p_{g,t} E_{g,t}}{\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_b E_{b,t})^{\alpha_b}(\tau_g E_{g,t})^{\alpha_g}} \Leftrightarrow \alpha_g = \frac{p_{g,t} E_{g,t}}{Y} \tag{66}$$

$$p_{b,t} = \tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}\alpha_b \tau_b^{\alpha_b} E_{b,t}^{\alpha_b - 1}) \tag{67}$$

$$\Leftrightarrow \alpha_b = \frac{p_{b,t} E_{b,t}}{\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}} \Leftrightarrow \alpha_b = \frac{p_{b,t} E_{b,t}}{Y} \tag{68}$$

484 A.3.3 Growth rate of factors

Neutral-augmenting technology:

$$Y_t = ((K_t)^{\alpha_k}(\tau_n^{\frac{1}{\alpha_l}} L_t)^{\alpha_l}(E_{g,t})^{\alpha_g}(E_{b,t})^{\alpha_b}) \tag{69}$$

Let:

$$y_t = \frac{Y_t}{(\tau_n)^{\frac{1}{\alpha_l}} L_t} \tag{70}$$

$$k_t = \frac{K_t}{(\tau_n)^{\frac{1}{\alpha_l}} L_t} \tag{71}$$

$$e_{g,t} = \frac{E_{g,t}}{(\tau_n)^{\frac{1}{\alpha_l}} L_t} \tag{72}$$

$$e_{b,t} = \frac{E_{b,t}}{(\tau_n)^{\frac{1}{\alpha_l}} L_t} \tag{73}$$

be the intensity of each variable.

Then:

$$\frac{Y_t}{(\tau_n)^{\frac{1}{\alpha_l}} L_t} = \frac{((K_t)^{\alpha_k} (\tau_n^{\frac{1}{\alpha_l}} L_t)^{\alpha_l} (E_{g,t})^{\alpha_g} (E_{b,t})^{\alpha_b})}{(\tau_n)^{\frac{1}{\alpha_l}} L_t} \tag{74}$$

$$y_t = k_t^{\alpha_k} {e_{g,t}}^{\alpha_g} {e_{b,t}}^{\alpha_b} \tag{75}$$

And so, the growth rate of production per capita is given by:

$$\frac{\dot{y_t}}{y_t} = \alpha_k \frac{\dot{k_t}}{k_t} + \alpha_g \frac{\dot{e_{g,t}}}{e_{g,t}} + \alpha_b \frac{\dot{e_{b,t}}}{e_{b,t}} \tag{76}$$

Remark that a steady state exists if the growth rate of capital, green energy and fossil energy per capital are zero.

We can show then that:

$$\frac{\dot{K_t}}{K_t} = \frac{1}{\alpha_l} \frac{\dot{\tau_n}}{\tau_n} + n \tag{77}$$

$$\frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{1}{\alpha_l} \frac{\dot{\tau_n}}{\tau_n} + n \tag{78}$$

$$\frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{1}{\alpha_l} \frac{\dot{\tau_n}}{\tau_n} + n \tag{79}$$

$$\frac{\dot{Y_t}}{Y_t} = \frac{1}{\alpha_l} \frac{\dot{\tau_n}}{\tau_n} + n \tag{80}$$

The same line of reasoning extends to alternative forms of technological advancement.

Capital-augmenting :

$$Y_t = (K_t)^{\alpha_k} (\tau_k^{\frac{\alpha_k}{\alpha_l}} L_t)^{\alpha_l} (E_{g,t})^{\alpha_g} (E_{b,t})^{\alpha_b} \tag{81}$$

$$\frac{\dot{Y_t}}{Y_t} = \frac{\dot{K_t}}{K_t} = \frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\alpha_k}{\alpha_l} \frac{\dot{\tau_k}}{\tau_k} + n \tag{82}$$

Labor-augmenting :

$$Y_t = (K_t)^{\alpha_k} (\tau_l L_t)^{\alpha_l} (E_{g,t})^{\alpha_g} (E_{b,t})^{\alpha_b} \tag{83}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{E}_{b,t}}{E_{b,t}} = \frac{\dot{E}_{g,t}}{E_{g,t}} = \frac{\dot{\tau}_l}{\tau_l} + n \tag{84}$$

Fossil-energy augmenting :

$$Y_t = (K_t)^{\alpha_k} (\tau_{e_{b,t}}^{\frac{\alpha_b}{\alpha_l}} L_t)^{\alpha_l} (E_{g,t})^{\alpha_g} (E_{b,t})^{\alpha_b} \tag{85}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{E}_{b,t}}{E_{b,t}} = \frac{\dot{E}_{g,t}}{E_{g,t}} = \frac{\alpha_b}{\alpha_l} \frac{\dot{\tau}_b}{\tau_b} + n \tag{86}$$

Green-energy augmenting :

$$Y_t = (K_t)^{\alpha_k} (\tau_g^{\frac{\alpha_g}{\alpha_l}} L_t)^{\alpha_l} (E_{g,t})^{\alpha_g} (E_{b,t})^{\alpha_b} \tag{87}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{E}_{b,t}}{E_{b,t}} = \frac{\dot{E}_{g,t}}{E_{g,t}} = \frac{\alpha_g}{\alpha_l} \frac{\dot{\tau}_g}{\tau_g} + n \tag{88}$$

Let us investigate the growth rates of the prices utilizing their respective equations (62), (64), (66), and (68):

$$r_t = \tau_n((\tau_l L_t)^{\alpha_l} (\tau_g E_{g,t})^{\alpha_g} (\tau_b E_{b,t})^{\alpha_b} \alpha_k \tau_k^{\alpha_k} K^{\alpha_k - 1}) \tag{89}$$

$$\Leftrightarrow r_t = Y_t \alpha_k K_t^{-1} \Leftrightarrow \frac{\dot{r}_t}{r_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \tag{90}$$

Under all forms of technological progress, the growth rates of production and capital are equal, then:

$$\frac{\dot{r}_t}{r_t} = 0 \tag{91}$$

Let us establish the growth rate of the price of green energy :

$$p_{g,t} =\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_b E_{b,t})^{\alpha_b}\alpha_g \tau_g^{\alpha_g} E_{g,t}^{\alpha_g-1}) \tag{92}$$

$$\Leftrightarrow p_{g,t} =Y\alpha_g E_{g,t}^{-1} \Leftrightarrow \frac{\dot{p_{g,t}}}{p_{g,t}} = \frac{\dot{Y_t}}{Y_t} - \frac{\dot{E_{g,t}}}{E_{g,t}} \tag{93}$$

Under all technological advancements, production and green energy grow at the same rate, then:

$$\frac{\dot{p_{g,t}}}{p_{g,t}} =0 \tag{94}$$

Let us determine the growth rate for fossil energy prices :

$$p_{b,t} =\tau_n((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(\tau_g E_{g,t})^{\alpha_g}\alpha_b \tau_b^{\alpha_b} E_{b,t}^{\alpha_b-1}) \tag{95}$$

$$\Leftrightarrow p_{b,t} =Y\alpha_b E_{b,t}^{-1} \Leftrightarrow \frac{\dot{p_{b,t}}}{p_{b,t}} = \frac{\dot{Y_t}}{Y_t} - \frac{\dot{E_{b,t}}}{E_{b,t}} \tag{96}$$

The growth rates of fossil energy and production are equal under all forms of technological progress, thus:

$$\frac{\dot{p_{b,t}}}{p_{b,t}} =0 \tag{97}$$

Let us determine the growth rate of the labor price:

$$w_t =\tau_n((\tau_k K_t)^{\alpha_k}(\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_b}\alpha_l \tau_l^{\alpha_l} L^{\alpha_l-1}) \tag{98}$$

$$\Leftrightarrow w_t =Y_t\alpha_l L_t^{-1} \Leftrightarrow \frac{\dot{w_t}}{w_t} = \frac{\dot{Y_t}}{Y_t} - \frac{\dot{L_t}}{L_t} \tag{99}$$

Let us also calculate the growth rate of per capita consumption.

$$\frac{\frac{\dot{C_t}}{L_t}}{\frac{C_t}{L_t}} = \frac{\mathrm{d}}{\mathrm{d}t}ln\left(\frac{Y_t}{L_t}\right) =\frac{\dot{Y_t}}{Y_t} - \frac{\dot{L_t}}{L_t} \tag{100}$$

Here, we will illustrate how to calculate the technology-to-nontechnology fossil energy ratio.

$$ratio = \frac{1.031^{100}}{1.03^{100}} = 1.01 \tag{101}$$

490 A.4 NGME With efficiency gains: CES type 1

The maximization problem in real terms is given by:

$$\max_{Y_t;K_t;L_t;E_{g,t};E_{b,t}} \quad Y_t - w_t L_t - r_t K_t - p_{g,t} E_{g,t} - p_{b,t} E_{b,t} \tag{102}$$

$$\text{s.c } Y_t = \tau_n (a((\tau_k K_t)^{\alpha_k} (\tau_l L_t)^{\alpha_l})^\rho + (1-a)((\tau_g E_{g,t})^{\alpha_g} (\tau_b E_{b,t})^{\alpha_b})^\rho)^{1/\rho} \tag{103}$$

The associated Lagrangian is given by:

$$\begin{aligned} \mathcal{L}_t = & Y_t - w_t L_t - r_t K_t - p_{g,t} E_{g,t} - p_{b,t} E_{b,t} - \\ & \lambda_t (Y_t - \tau_n (a((\tau_k K_t)^{\alpha_k} (\tau_l L_t)^{\alpha_l})^\rho + (1-a)((\tau_g E_{g,t})^{\alpha_g} (\tau_b E_{b,t})^{\alpha_f})^\rho)^{1/\rho}) = 0 \end{aligned} \tag{104}$$

491 ### A.4.1 First-order conditions

First order conditions are as follows:

$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t} = 0 \Leftrightarrow Y_t - \tau_n(a((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l})^{\rho} + (1-a)((\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_f})^{\rho})^{1/\rho} = 0 \tag{105}$$

$$\Leftrightarrow Y_t = \tau_n(a((\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l})^{\rho} + (1-a)((\tau_g E_{g,t})^{\alpha_g}(\tau_b E_{b,t})^{\alpha_f})^{\rho})^{1/\rho} \tag{106}$$

$$\frac{\partial \mathcal{L}_t}{\partial Y_t} = 0 \Leftrightarrow 1 - \lambda_t = 0 \Leftrightarrow \lambda_t = 1 \tag{107}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = 0 \Leftrightarrow -r_t - \lambda_t(-\tau_n^{\rho}\frac{1}{\rho}Y^{1-\rho}a\rho\alpha_k\tau^{\alpha_k\rho}K_t^{\alpha_k\rho-1}(\tau_l L_t)^{\alpha_l\rho}) = 0 \tag{108}$$

$$\Leftrightarrow r_t = \lambda_t(\tau_n^{\rho}Y^{1-\rho}a\alpha_k\tau^{\alpha_k\rho}K_t^{\alpha_k\rho-1}(\tau_l L_t)^{\alpha_l\rho}) \tag{109}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = 0 \Leftrightarrow -w_t - \lambda_t(-\tau_n^{\rho}\frac{1}{\rho}Y^{1-\rho}a\rho\alpha_l\tau^{\alpha_l\rho}L_t^{\alpha_l\rho-1}(\tau_k K_t)^{\alpha_k\rho}) = 0 \tag{110}$$

$$\Leftrightarrow w_t = \lambda_t(\tau_n^{\rho}Y^{1-\rho}a\alpha_l\tau^{\alpha_l\rho}L_t^{\alpha_l\rho-1}(\tau_k K_t)^{\alpha_k\rho}) \tag{111}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{g,t}} = 0 \Leftrightarrow -p_{g,t} - \lambda_t(\tau_n^{\rho}Y_t^{1-\rho}(1-a)\alpha_g\tau_g^{\alpha_g\rho}E_{g,t}^{\alpha_g\rho-1}(\tau_b E_{b,t}^{\alpha_b\rho}) = 0 \tag{112}$$

$$\Leftrightarrow p_{g,t} = \lambda_t(\tau_n^{\rho}Y_t^{1-\rho}(1-a)\alpha_g\tau_g^{\alpha_g\rho}E_{g,t}^{\alpha_g\rho-1}(\tau_b E_{b,t}^{\alpha_b\rho}) \tag{113}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{b,t}} = 0 \Leftrightarrow -p_{g,t} - \lambda_t(\tau_n^{\rho}Y_t^{1-\rho}(1-a)\alpha_b\tau_b^{\alpha_b\rho}E_{b,t}^{\alpha_b\rho-1}(\tau_g E_{g,t}^{\alpha_g\rho}) = 0 \tag{114}$$

$$\Leftrightarrow p_{g,t} = \lambda_t(\tau_n^{\rho}Y_t^{1-\rho}(1-a)\alpha_b\tau_b^{\alpha_b\rho}E_{b,t}^{\alpha_b\rho-1}(\tau_g E_{g,t}^{\alpha_g\rho}) \tag{115}$$

492 ### A.4.2 Optimal conditions

By substituting equation (107) into equations (109), (111), (113) and (115), we derive:

$$r_t = \tau_n^{\rho}Y^{1-\rho}a\alpha_k\tau^{\alpha_k\rho}K_t^{\alpha_k\rho-1}(\tau_l L_t)^{\alpha_l\rho} \tag{116}$$

$$w_t = \tau_n^{\rho}Y^{1-\rho}a\alpha_l\tau^{\alpha_l\rho}L_t^{\alpha_l\rho-1}(\tau_k K_t)^{\alpha_k\rho} \tag{117}$$

$$p_{g,t} = \tau_n^{\rho}Y_t^{1-\rho}(1-a)\alpha_g\tau_g^{\alpha_g\rho}E_{g,t}^{\alpha_g\rho-1}(\tau_b E_{b,t})^{\alpha_b\rho} \tag{118}$$

$$p_{b,t} = \tau_n^{\rho}Y_t^{1-\rho}(1-a)\alpha_b\tau_b^{\alpha_b\rho}E_{b,t}^{\alpha_b\rho-1}(\tau_g E_{g,t})^{\alpha_g\rho} \tag{119}$$

493 A.4.3 Factor shares

Let us assume that we are in the presence of only neutral technological progress. We substitute equation (107) into equations (109) to (115) and use the definition of the production function to obtain:

$$p_{g,t} = \tau_n^{\rho} Y_t^{1-\rho}(1-a)\alpha_g E_{g,t}^{\alpha_g\rho-1} E_{b,t}^{\alpha_b\rho} \Leftrightarrow \frac{p_{g,t}E_{g,t}^{1-\alpha_g\rho}}{Y_t^{\rho-1}} = \tau_n^{\rho}(1-a)\alpha_g E_{b,t}^{\alpha_b\rho} \tag{120}$$

$$p_{b,t} = \tau_n^{\rho} Y_t^{1-\rho}(1-a)\alpha_b E_{b,t}^{\alpha_b\rho-1} E_{g,t}^{\alpha_g\rho} \Leftrightarrow \frac{p_{b,t}E_{b,t}^{1-\alpha_b\rho}}{Y_t^{\rho-1}} = \tau_n^{\rho}(1-a)\alpha_b E_{g,t}^{\alpha_g\rho} \tag{121}$$

$$r_t = \tau_n^{\rho} Y^{1-\rho} a\alpha_k \tau^{\alpha_k\rho} K_t^{\alpha_k\rho-1}(\tau_l L_t)^{\alpha_l\rho} \Leftrightarrow \frac{r_t K_t^{1-\alpha_k\rho}}{p_t Y^{\rho-1}} = \tau_n^{\rho} a\alpha_k L_t^{\alpha_l\rho} \tag{122}$$

$$w_t = \tau_n^{\rho}\frac{1}{\rho} Y^{1-\rho} a\rho\alpha_l \tau^{\alpha_l\rho} L_t^{\alpha_l\rho-1}(\tau_k K_t)^{\alpha_k\rho} \Leftrightarrow \frac{W_t L_t^{1-\alpha_l\rho}}{Y^{\rho-1}} = \tau_n^{\rho} a\alpha_l K_t^{\alpha_k\rho} \tag{123}$$

494 Dividing equation (121) by equation (120) we get:

$$\frac{p_{b,t}E_{b,t}^{1-\alpha_b\rho}}{p_{g,t}E_{g,t}^{1-\alpha_g\rho}} = \frac{\alpha_b E_{g,t}^{\alpha_g\rho}}{\alpha_g E_{b,t}^{\alpha_b\rho}} \Leftrightarrow \alpha_b = \frac{p_{b,t}E_{b,t}}{p_{g,t}E_{g,t}}(1-\alpha_b) \Leftrightarrow \alpha_b = \frac{\frac{p_{b,t}E_{b,t}}{p_{g,t}E_{g,t}}}{1+\frac{p_{b,t}E_{b,t}}{p_{g,t}E_{g,t}}} \tag{124}$$

$$\alpha_g = 1-\alpha_b \tag{125}$$

Dividing equation (122) by equation (123), we obtain:

$$\frac{w_t L_t^{1-\alpha_l\rho}}{r_t K_t^{1-\alpha_k\rho}} = \frac{\alpha_l K_t^{\alpha_k\rho}}{\alpha_k L^{\alpha_l\rho}} \Leftrightarrow \alpha_l = (1-\alpha_l)\frac{w_t L_t}{r_t K_t} \Leftrightarrow \alpha_l = \frac{\frac{w_t L_t}{r_t K_t}}{1+\frac{w_t L_t}{r_t K_t}} \tag{126}$$

$$\alpha_k = 1-\alpha_l \tag{127}$$

495 A.4.4 Growth rate of factors

Neutral augmenting :

$$Y_t = \tau_n\left(a(K_t^{\alpha_k}L_t^{\alpha_l})^{\rho} + (1-a)(E_{g,t}^{\alpha_g}E_{b,t}^{\alpha_b})^{\rho}\right)^{1/\rho} \tag{128}$$

Assume that prices remain constants over time, meaning:

$$\frac{\dot{r}_t}{r_t} = \frac{\dot{p}_{g,t}}{p_{g,t}} = \frac{\dot{p}_{b,t}}{p_{b,t}} = 0 \tag{129}$$

Using equations (116), (118) and (119), we have:

$$\frac{\dot{p}_{g,t}}{p_{g,t}} = 0 \Leftrightarrow \rho\frac{\dot{\tau}_n}{\tau_n} + (1-\rho)\frac{\dot{Y}_t}{Y_t} + (\alpha_g\rho - 1)\frac{\dot{E}_{g,t}}{E_{g,t}} + \alpha_b\rho\frac{\dot{E}_{b,t}}{E_{b,t}} = 0 \tag{130}$$

$$\frac{\dot{p}_{g,t}}{p_{g,t}} = 0 \Leftrightarrow \rho\frac{\dot{\tau}_n}{\tau_n} + (1-\rho)\frac{\dot{Y}_t}{Y_t} + \alpha_g\rho\frac{\dot{E}_{g,t}}{E_{g,t}} + (\alpha_b\rho - 1)\frac{\dot{E}_{b,t}}{E_{b,t}} = 0 \tag{131}$$

$$\frac{\dot{r}_t}{r_t} = 0 \Leftrightarrow \rho\frac{\dot{\tau}_n}{\tau_n} + (1-\rho)\frac{\dot{Y}_t}{Y_t} + \alpha_l\rho\frac{\dot{L}_t}{L_t} + (\alpha_k\rho - 1)\frac{\dot{K}_t}{K_t} \tag{132}$$

We isolate $\frac{\dot{Y}_t}{Y_t}$ in both equations (130) and (131) and we have :

$$\frac{\dot{Y}_t}{Y_t} = \left(\rho\frac{\dot{\tau}_n}{\tau_n} + (\alpha_g\rho - 1)\frac{\dot{E}_{g,t}}{E_{g,t}} + \alpha_b\rho\frac{\dot{E}_{b,t}}{E_{b,t}}\right)\frac{1}{\rho - 1} \tag{133}$$

$$\frac{\dot{Y}_t}{Y_t} = \left(\rho\frac{\dot{\tau}_n}{\tau_n} + \alpha_g\rho\frac{\dot{E}_{g,t}}{E_{g,t}} + (\alpha_b\rho - 1)\frac{\dot{E}_{b,t}}{E_{b,t}}\right)\frac{1}{\rho - 1} \tag{134}$$

We proceed by equating the last two equations and obtain:

$$\frac{\dot{E}_{g,t}}{E_{g,t}} = \frac{\dot{E}_{b,t}}{E_{b,t}} \tag{135}$$

Remark than in this framework, both energies forms growth at the exactly same rate.

According to equation (130) and considering the relationship established in equation (135), we obtain:

$$\frac{\dot{E}_{b,t}}{E_{b,t}} = \frac{\dot{E}_{g,t}}{E_{g,t}} = \frac{\rho}{1-\rho}\frac{\dot{\tau}_n}{\tau_n} + \frac{\dot{Y}_t}{Y_t} \tag{136}$$

Using equation (132) and given the fact that a steady state exists if output and capital grow at the same rate, we have :

$$\frac{\dot{Y}_t}{Y_t} = \frac{1}{\alpha_l}\frac{\dot{\tau_n}}{\tau_n} + n \tag{137}$$

and so,

$$\frac{\dot{K}_t}{K_t} = \frac{1}{\alpha_l}\frac{\dot{\tau_n}}{\tau_n} + n \tag{138}$$

$$\frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\rho}{1-\rho}\frac{\dot{\tau_n}}{\tau_n} + \frac{\dot{Y}_t}{Y_t} \tag{139}$$

498 Similar logic applies to alternative modes of technological progress.

Capital-augmenting :

$$Y_t = (a((\tau_k K_t)^{\alpha_k} L_t^{\alpha_l})^\rho + (1-a)(E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b})^\rho)^{1/\rho} \tag{140}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\alpha_k}{\alpha_l}\frac{\dot{\tau_k}}{\tau_k} + n \tag{141}$$

Labor-augmenting :

$$Y_t = (aK_t^{\alpha_k}(\tau_l L_t)^{\alpha_l})^\rho + (1-a)(E_{g,t}^{\alpha_g} E_{b,t}^{\alpha_b})^\rho)^{1/\rho} \tag{142}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\dot{\tau_l}}{\tau_l} + n \tag{143}$$

Green-energy augmenting :

$$Y_t = (a(K_t^{\alpha_k} L_t^{\alpha_l})^\rho + (1-a)((\tau_g E_{g,t})^{\alpha_g} E_{b,t}^{\alpha_b})^\rho)^{1/\rho} \tag{144}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = n \tag{145}$$

$$\frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\dot{E_{g,t}}}{E_{g,t}} = \alpha_g\frac{\rho}{1-\rho}\frac{\dot{\tau_g}}{\tau_g} + \frac{\dot{Y}_t}{Y_t} \tag{146}$$

499 **Fossil-energy augmenting :**

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = n \tag{147}$$

$$\frac{\dot{E}_{b,t}}{E_{b,t}} = \frac{\dot{E}_{g,t}}{E_{g,t}} = \alpha_b \frac{\rho}{1-\rho} \frac{\dot{\tau}_b}{\tau_b} + \frac{\dot{Y}_t}{Y_t} \tag{148}$$

A.5 NGME With efficiency gains: CES type 2

The maximization problem in real terms is given by:

$$\max_{Y_t;K_t;L_t;E_{g,t};E_{b,t}} \quad Y_t - w_t L_t - r_t K_t - p_{g,t} E_{g,t} - p_{b,t} E_{b,t} \tag{149}$$

$$\text{s.c } Y_t = \tau_n (\tau_k K)^{\alpha_k} (\tau_l L)^{\alpha_l} (a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}} \tag{150}$$

The associated Lagrangian is as follows:

$$\begin{aligned} \mathcal{L}_t =& Y_t - w_t L_t - r_t K_t - p_{g,t} E_{g,t} - p_{b,t} E_{b,t} - \\ & \lambda_t (Y_t - \tau_n (\tau_k K_t)^{\alpha_k} (\tau_l L_t)^{\alpha_l} (a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}}) \end{aligned} \tag{151}$$

501 ### A.5.1 First-order conditions

First order conditions are given by:

$$\frac{\partial \mathcal{L}_t}{\partial \lambda_t} = 0 \Leftrightarrow Y_t - \tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}} \tag{152}$$

$$\Leftrightarrow Y_t = \tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}} \tag{153}$$

$$\frac{\partial \mathcal{L}_t}{\partial Y_t} = 0 \Leftrightarrow 1 - \lambda_t = 0 \Leftrightarrow 1 = \lambda_t \tag{154}$$

$$\frac{\partial \mathcal{L}_t}{\partial K_t} = 0 \Leftrightarrow r_t - \lambda_t(-\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}})\alpha_k K_t^{-1} = 0 \tag{155}$$

$$\Leftrightarrow r_t = \lambda_t(\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}}\alpha_k K_t^{-1}) \tag{156}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = 0 \Leftrightarrow -w_t - \lambda_t(-\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}})\alpha_l L_t^{-1} = 0 \tag{157}$$

$$\Leftrightarrow W_t = \lambda_t(\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}})\alpha_l L^{-1} \tag{158}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{g,t}} = 0 \Leftrightarrow -p_{g,t} - \lambda_t(-\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}-1})\frac{\alpha_e}{\rho}\rho E_{g,t}^{\rho-1}\tau_g^{\rho} = 0 \tag{159}$$

$$\Leftrightarrow p_{g,t} = \lambda_t(\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}-1})\frac{\alpha_e}{\rho}\rho E_{g,t}^{\rho-1}\tau_g^{\rho} \tag{160}$$

$$\frac{\partial \mathcal{L}_t}{\partial E_{b,t}} = 0 \Leftrightarrow -p_{b,t} - \lambda_t(-\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}-1})\frac{\alpha_e}{\rho}\rho E_{b,t}^{\rho-1}\tau_b^{\rho} = 0 \tag{161}$$

$$\Leftrightarrow p_{b,t} = \lambda_t(\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^{\rho} + (1-a)(\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}-1})\frac{\alpha_e}{\rho}\rho E_{b,t}^{\rho-1}\tau_b^{\rho} \tag{162}$$

A.5.2 Factor shares

We substitute equation (154) into equations (156) to (162) and use the definition of the production function to obtain:

$$r_t = Y_t \alpha_k K^{-1} \Leftrightarrow \alpha_k = \frac{r_t K_t}{Y_t} \tag{163}$$

$$w_t = Y_t \alpha_l L^{-1} \Leftrightarrow \alpha_l = \frac{w_t L_t}{Y_t} \tag{164}$$

To obtain α_e, we adopt a different approach. Define:

$$E_t = (a(\tau_g E_{g,t})^\rho + (1-a)(\tau_b E_{b,t})^\rho)^{\frac{1}{\rho}}) \tag{165}$$

Then, by substituting $(a(\tau_g E_{g,t})^\rho + (1-a)(\tau_b E_{b,t})^\rho)^{\frac{1}{\rho}}$ and $-p_{g,t}Eg,t - p_{b,t}E_{b,t}$ with $-p_{e,t}E_t$ in the Lagrangian and performing the first-order condition (FOC) with respect to E_t, we arrive at:

$$\frac{\partial \mathcal{L}_t}{\partial E_t} = 0 \Leftrightarrow -p_{e_t} - \lambda_t(-\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}\alpha_e E_t^{\alpha_e - 1}) \tag{166}$$

$$\Leftrightarrow p_{e,t} = \lambda(\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}\alpha_e E_t^{\alpha_e - 1}) \tag{167}$$

Subsequently, by substituting FOC (154) into the equation above, we obtain:

$$p_{e,t} = p_t(\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}\alpha_e E_t^{\alpha_e - 1}) \tag{168}$$

Now, we can determine the energy share by utilizing the equation above:

$$p_{e,t} = (\tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}\alpha_e E_t^{\alpha_e - 1}) \Leftrightarrow \alpha_e = \frac{p_{e,t} E_t}{Y_t} \tag{169}$$

Let us now determine the growth rate of the capital price by employing equation (163):

$$\frac{\dot{r}_t}{r_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \tag{170}$$

In order to achieve a steady state, production and capital must grow at the same rate, thus :

$$\frac{\dot{r}_t}{r_t} = 0 \tag{171}$$

A.5.3 Growth rate of factors

$$Y_t = \tau_n(\tau_k K_t)^{\alpha_k}(\tau_l L_t)^{\alpha_l}(a(\tau_g E_{g,t})^\rho + (1-a)(\tau_b E_{b,t})^\rho)^{\alpha_e/\rho} \tag{172}$$

Here, to solve for the growth rates of neutral-augmenting, capital-augmenting, and labor-augmenting, we employ the same resolution method as for the Cobb-Douglas function (Appendix A.3). For fossil and green augmenting, we use the same approach as for the CES Function with Nested (K, L) and (Eg, Eb) in Cobb-Douglas functions (Appendix A.4). In the context of this production function, it's important to note that both energies don't need to grow at the same rate. Taking these factors into account, we derive the following results:

Neutral :

$$Y_t = \tau_n K_t^{\alpha_k} L_t^{\alpha_l}(aE_{g,t}^\rho + (1-a)E_{b,t}^\rho)^{\frac{\alpha_e}{\rho}} \tag{173}$$

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{E}_{b,t}}{E_{b,t}} = \frac{\dot{E}_{g,t}}{E_{g,t}} = \frac{1}{\alpha_l}\frac{\dot{\tau}_n}{\tau_n} + n \tag{174}$$

Capital-augmenting :

$$Y_t = (\tau_k K_t)^{\alpha_k} L_t^{\alpha_l} (a E_{g,t}^{\rho} + (1-a) E_{b,t}^{\rho})^{\frac{\alpha_e}{\rho}} \tag{175}$$

$$\frac{\dot{Y_t}}{Y_t} = \frac{\dot{K_t}}{K_t} = \frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\alpha_k}{\alpha_l}\frac{\dot{\tau_k}}{\tau_k} + n \tag{176}$$

Labor-augmenting :

$$Y_t = K_t^{\alpha_k} (\tau_l L_t)^{\alpha_l} (a E_{g,t}^{\rho} + (1-a) E_{b,t}^{\rho})^{\frac{\alpha_e}{\rho}} \tag{177}$$

$$\frac{\dot{Y_t}}{Y_t} = \frac{\dot{K_t}}{K_t} = \frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{\dot{\tau_l}}{\tau_l} + n \tag{178}$$

Green-energy augmenting :

$$Y_t = K_t^{\alpha_k} L_t^{\alpha_l} (a (\tau_g E_{g,t})^{\rho} + (1-a) E_{b,t}^{\rho})^{\frac{\alpha_e}{\rho}} \tag{179}$$

$$\frac{\dot{Y_t}}{Y_t} = \frac{\dot{K_t}}{K_t} = \frac{\dot{E_{b,t}}}{E_{b,t}} = n \tag{180}$$

$$\frac{\dot{E_{g,t}}}{E_{g,t}} = \frac{p}{1-p}\frac{\dot{\tau_g}}{\tau_g} + \frac{\dot{Y_t}}{Y_t} \tag{181}$$

Fossil-energy augmenting :

$$Y_t = K_t^{\alpha_k} L_t^{\alpha_l} (a E_{g,t}^{\rho} + (1-a) (\tau_b E_{b,t})^{\rho})^{\frac{\alpha_e}{\rho}} \tag{182}$$

$$\frac{\dot{Y_t}}{Y_t} = \frac{\dot{K_t}}{K_t} = \frac{\dot{E_{g,t}}}{E_{g,t}} = n \tag{183}$$

$$\frac{\dot{E_{b,t}}}{E_{b,t}} = \frac{p}{1-p}\frac{\dot{\tau_b}}{\tau_b} + \frac{\dot{Y_t}}{Y_t} \tag{184}$$

B Calibration

In this part, we will explain the procedure that we follow for the calibration of different model parameters.

We calibrate the model to the France country, with yearly data.

A summary of data used can be found on Table 5. We use yearly data from 1996 to 2022.

Table 5: Original Sources

Serie	**Description**	**Source**
KTOE (all energy)	Thousand tons of oil equivalent, for all types of energy	nrg_bal_s Eurostat
p_b	Europe Brent Spot Price Free on Board (FOB) in US Dollars per Barrel for Crude Oil	U.S. Energy Information Administration
GDPCUS	GDP (constant 2015 US$) refers to Gross Domestic Product values adjusted for inflation, with 2015 as the base year	NY.GDP.MKTP.KD World Bank national accounts data and OECD National Accounts data files
KTOE (biofuels and renewable energy (E_g))	Thousand tons of oil equivalent, for biofuels and renewable energy	nrg_bal_s Eurostat
LCOE (Levelized Cost of Energy - Onshore Wind (p_g))	2023 USD/kWh	IRENA, Renewable Power Generation Costs in 2023, International Renewable Energy Agency
Annual Consumer Price Index	Base 2015 - All Households - France - All Items	Identifier 001759970, INSEE
Total Labor Force (L_t)	Total labor in France	SL.TLF.TOTL.IN World Bank
Average Annual Wages	Average annual wages in France at constant prices (Euro, 2023)	OECD Average Annual Wages
α_l CES I	Average labor share percentage in France	LABSHPFRA156NRUG Federal Reserve Economic Data

To compute the energy share (fossil or green), we converted energy quantities from thousand tonnes of oil equivalent (KTOE) to barrels of oil equivalent (BOE) using a fixed conversion factor of 1 KTOE = 7.33 BOE[2].

To ensure consistency in real terms, we adjusted both energy prices and wages

[2] `https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2022-approximate-conversion-factors.pdf`, accessed on February 14, 2025.

for inflation, using the Consumer Price Index (CPI) with base year 2015 (1 = 2015) as a deflator. Given that the Levelized Cost of Energy (LCOE) is expressed in 2023 USD/kWh, we deflated its value accordingly. Additionally, we converted our quantity of green energy from BOE to kWh using the standard conversion rate of 1 BOE = 1,700 kWh[3].

For the price of green energy, we employed the best available proxy: the Levelized Cost of Energy (LCOE). While not identical to market prices, LCOE provides a valuable approximation as it is closely correlated with actual prices.

For the Cobb-Douglas function and the CES type 1 function, the α_i power represents cost shares of input i in their respective composite, meaning that:

$$\alpha_k = \text{capital share} = 0.3160 \tag{185}$$

$$\alpha_l = \text{labor share} = 0.6214 \tag{186}$$

$$\alpha_{e_g} = \text{green energy share} = 0.0141 \tag{187}$$

$$\alpha_{e_b} = \text{fossil energy share} = 0.0484 \tag{188}$$

$$\alpha_e = \text{energy share} = 0.0524 \tag{189}$$

For the Cobb-Douglas case and the CES type 2 function, the α_i power represents factor share, meaning that:

Cobb-Douglas:

$$\alpha_k = \text{capital share} = 0.382 \tag{190}$$

$$\alpha_l = \text{labor share} = 0.618 \tag{191}$$

$$\alpha_{e_g} = \text{green energy share} = 0.2324 \tag{192}$$

$$\alpha_{e_f} = \text{fossil energy share} = 0.7676 \tag{193}$$

[3] `https://atee.fr/energies-renouvelables/club-biogaz/conversion-des-energies?utm_source=chatgpt.com`, accessed on February 14, 2025.

CES type 2:

$$\alpha_k = \text{capital share} = 0.382 \tag{194}$$

$$\alpha_l = \text{labor share} = 0.618 \tag{195}$$

$$\alpha_{e_g} = \text{green energy share} = 0.2324 \tag{196}$$

$$\alpha_{e_f} = \text{fossil energy share} = 0.7676 \tag{197}$$