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Two halves don't make a whole: instability and idleness emerging from the co-evolution of the production and innovation processes

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Abstract

We propose a disaggregated representation of production using an agent-based fund-flow model that emphasizes inefficiencies, such as factor idleness and production instability, and allows us to explore their emergence through simulations. The model incorporates productivity dynamics (learning and depreciation) and is extended with time-saving process innovations. Specifically, we assume workers possess inherent creativity that flourishes during idle periods. The firm, rather than laying off idle workers, is assumed to harness this potential by involving them in the innovation process. Results show that a firm's organizational and managerial decisions, the temporal structure of the production system, the degree of workers' learning and forgetting, and the pace of innovation are critical factors influencing production efficiency in both the short and long term. The co-evolution of production and innovation processes emerges in our model through the two-sided effects of idleness: the loss of skills through forgetting and the deflection of time from the production of goods to the production of ideas giving birth to idleness-driven innovations. In doing so, it allows us to question the status of labour as an adjustment variable in a productive organisation. The paper concludes by discussing potential solutions to this issue and suggesting avenues for future research.

Keywords: Production Theory; Firm Theory; Agent-based model; Idleness; Innovation; Fund-flow

JEL: D21, D24, D83, J24, L25, O31, O33

1 Introduction

On Idleness Active idleness[1] was praised long before Russell (1932). Ancient Greek and Roman philosophers considered it central to philosophical contemplation, the pursuit of virtue, and happiness. Plato and Aristotle believed it was necessary for contemplation and for achieving the highest form of human flourishing, *eudaimonia* (Samaras, 2017). Nonetheless, their perspectives differed significantly on who idleness was meant for. In the *Republic*, Plato argued that the working class (*producers*) should focus on productive activity, enabling the ruling class to engage in idleness. Although he respected work, he believed that each class should remain devoted to its role; if workers engaged in contemplation and virtue,

[1] Active idleness refers to leisure and self-realization activities that, unlike inactivity, contribute to personal flourishing. It is related to the concepts of *schole* in Ancient Greek and *otium* in Latin.

this would create societal disequilibrium (*injustice*).[2] Aristotle, in contrast, viewed work not as an end in itself, but as a means to happiness (Owens, 1981, Balme, 1984), a perspective later adopted by thinkers like Russell.

The role of idleness in the workplace has also been a subject of debate. Seneca emphasized the importance of inactive idleness for rest and relaxation.[3] However, the pursuit of profit led factory and mine owners to prioritize productivity, often at the expense of workers, who commonly worked long hours. This included women and children (Tuttle, 2001), who were more easily employed due to the Smithian division of labor that simplified tasks.

The culture of efficiency reached its zenith in the late 19th and early 20th centuries with Frederick Taylor's optimization of factory processes. Taylor promoted the repetition of efficient moves in a highly specialized, divided labor process, all under the strict monitoring of the chronometer (Taylor, 1911).[4]

In stark contrast to Taylorism, Russell praised idleness (Russell, 1932). His essay combined a critique of historical labor conditions (and their unethical foundations) with an argument for the benefits of active idleness and leisure for individuals and society. Russell called for the establishment of shorter working hours—roughly equivalent to Keynes' prediction of 15-hour work weeks (Keynes, 1930). Russell observed that, in response to productivity gains and stable demand, firms tended to lay off *unnecessary* workers rather than reduce working hours, a strategy benefiting capital owners, shareholders, and white-collar workers at the expense of the working class. Instead, he suggested that extra productivity should lead to reduced working hours, allowing all individuals to engage in active idleness, which he saw as the key to a fulfilling life.

In this paper, we propose a different approach to the management of idleness, one more compatible with perpetual profit-seeking. Rather than firing workers—whose efforts made the process efficient—firms could involve them in the innovation process, thereby fostering further efficiency. Assuming active idle time is key to creativity, firms could harness innovative ideas by encouraging workers to make constructive use of their idle time.

Production and Firm In economics, there is a classical divide between market and production, with an increasing gap favoring the market over production. Typically, the theory of production is restricted to discussions surrounding the production function and the theory of the firm as an entity that maximizes profit under constraints, with the production function being one of those constraints. Dreze (1985) aptly describes this diagnosis: "The firm fits into general equilibrium theory (GET) as a balloon fits into an envelope: flattened out! Try with a blown-up balloon: the envelope may tear, or fly away: at best, it will be hard to seal and impossible to mail… instead, burst the balloon flat, and everything becomes easy."

Since at least the 1930s, particularly with Coase's seminal work (Coase, 1937), numerous contributions have attempted to conceptualize and model both production and the firm (though many have done so in incomplete or unsatisfactory ways). Among the various attempts to open the black box of the firm and production, two primary trends can be identified in the literature: the first views the firm as an entity that solves inherent issues related to organization, performance, and change within the firm, which only incidentally happens to produce and interact with its environment. The second focuses on the process of production, whose *raison d'être* is the transformation and combination of commodities, labor, and machinery into other commodities, artifacts, or services, which incidentally happen to occur within a firm.

More precisely, the first one deals with the organizational aspects of the production–from transactions to learning and routines–the most recent development being the evolutionary and knowledge-based approaches to firms (Winter, 2006, Nelson, 2006, Dosi and Marengo, 2007, Marengo, 2020). The main perspective is to emphasize the importance and the role of productive knowledge and learning processes. However, strong critiques have been raised against knowledge-based approaches, particularly their ability to address contemporary economic contexts, such as the rise of disruptive digital technologies or creative behavior (Alvarez et al., 2020, Cohendet et al., 2024).

The second approach focuses on the analysis of production itself, centering on the production function as a formalized means to combine production factors. It critiques the simplistic versions of the production function offered by neoclassical economics regarding the representation of factors, their interaction, and how they are combined to yield the final product. The Cambridge Controversy, which emerged after

[2] "that one man should practise one thing only, the thing to which his nature was best adapted [...] justice [is] doing one's own business [...] there are three distinct classes, any meddling of one with another, or the change of one into another, is the greatest harm to the State, and may be most justly termed evil-doing" (Plato, 2000).

[3] "our spirits should have time for relaxation: they return from rest better and keener than before. [...] constant hard work breaks the power of spirits. They gain back their strength after taking a break and resting for a while" (Seneca, 2015)

[4] Most historical facts and interpretations are extracted from the documentary *Le Temps des ouvriers* by Stan Neumann.

WWII, questioned the nature of production factors—particularly capital, whose heterogeneous nature requires more complex integration into production than mere accumulation and combination with labor (Robinson, 1953). This line of thought also raises concerns about the dynamics of production factor combinations and their effects on efficiency. For instance, traditional assumptions, such as substitutability between factors, face inconsistencies with empirical evidence (Kaldor, 1957). Scholars like (Sraffa, 1961) further highlighted the interdependence within and between production processes, stressing the physical constraints that shape production at the firm, sector, and economy-wide levels (Pasinetti, 1973).

More recently, this literature has evolved into two streams. The first, following Hildenbrand (1981), critiques short-run production functions and their relevance to representing the technological characteristics of an industry through aggregation.[5] The second stream focuses on directly representing production processes as sequences of productive activities and tasks, each characterized by factors such as duration and productivity. Notably Georgescu-Roegen (1970) provided a first interesting framework for this with his fund-flow model of production.

One contribution of our paper is to propose a reconciled version of these theories, encapsulating both views by integrating production processes and learning and innovation processes. Starting with a representation of production processes *à la* Georgescu-Roegen, where knowledge and skills are considered funds, we introduce learning processes that transform these funds and innovation processes that transform productive activities. Our model thus enables the analysis of co-evolution and interactions between production and knowledge.

The Fund-Flow Approach Half a century ago, Nicholas Georgescu-Roegen (henceforth NGR) criticized classical quantitative and flow-based[6] production functions for obscuring the underlying process, specifically the role of processing durations (Georgescu-Roegen, 1970). This abstraction conceals inefficiencies inherent in the organization of production, which, somewhat independently of factor availability (productive power), can substantially affect the output rate. In the *fund-flow model* originally proposed by Georgescu-Roegen (1970, 1971) and later studied and refined by Tani (1988), Morroni (1992), Piacentini (1995), Mir-Artigues and González-Calvet (2007), among others, these inefficiencies primarily manifest as fund idleness, i.e., periods when production factors, differentiated between funds and flows, are not working (funds) or being processed (flows).

Funds are defined as factors that remain unaltered by the production process and serve to process flows, which are altered to create the final product. Funds typically include land, machines, and labor, while flows encompass any input or combination of inputs meant to be transformed, from raw materials to commodities, to be marketed. However, because we relax the strict assumption of fund *sameness*[7]—allowing the productivity of workers and machines to change dynamically in response to their service—we redefine funds as production factors that uphold, store, and act upon flows without being physically incorporated into the final good.[8]

NGR advocated for arranging production systems *in-line*, i.e., in a modern factory fashion, which seeks to eliminate the idleness of production factors often found in sequential (or *in-series*) organizations. In sequential systems, idleness arises because subprocesses (also referred to as phases, stages, or tasks) have different durations, causing some phases to wait for others to complete. Production lines are designed to increase the productive capacity of slower phases by allocating relatively more funds to them. By equalizing durations, the process should ideally stabilize and run continuously and efficiently by eliminating idleness. This approach aligns closely with Taylorist views on industrial production. More precisely, NGR's fund-flow model has demonstrated the economic superiority of factory-style production processes from an organizational perspective (assuming no additional frictions).

Outline The paper is composed of two main parts: Sections 2 and 3 constitute the first part, while Section 4 forms the second. In the first part, we present an agent-based model (ABM) of the fund-flow framework, retaining its core principles while introducing several key departures. We discretize the model, introduce

[5]"short-run efficient production functions do not enjoy the well-known properties which are frequently assumed in production theory. For example, constant returns to scale never prevail, the production functions are never homothetic, and the elasticities of substitution are never constant. On the other hand, the competitive factor demand and product supply functions [...] will always have definite comparative static properties which cannot be derived from the standard theory of production." (Hildenbrand, 1981).

[6]I.e., relating inputs $\mathbf{I}$ and outputs Q in quantities, $Q = F(\mathbf{I})$, or input rates $\mathbf{i} = \mathbf{I}/T$ to output rates $q = Q/T$, $q = f(\mathbf{i})$.

[7]The definition and treatment of funds, notably machines and workers, has been criticized by Lager (2000) and Kurz and Salvadori (2007). Despite NGR's awareness of the limiting nature of the constant productivity assumption, he chose to abstract from it.

[8]A service may entail some form of incorporation and alteration of the workers, but we apply this definition only to the production of commodities.

productivity dynamics, and emphasize the principles of factor indivisibility and complementarity, giving rise to the NGR-ADAPT model. We then conduct several experiments to explore the core emerging dynamics of factory systems. In the second part, we augment the base model with mechanics that allow for idleness-driven process innovations, which are assumed to be time-saving, primarily affecting production duration. We investigate the interplay between idleness-driven innovations and productivity dynamics in determining process efficiency, especially in scenarios where workers are assumed to forget as they remain idle for extended periods (skill decay). The results are mixed, and the controversial feasibility of this factory paradigm opens new avenues for research. In Section 5, we outline promising directions for further exploration of this topic and propose additional ways the NGR-ADAPT model can contribute to research in both microeconomic and macroeconomic theory.

2 Base Model: NGR-ADAPT

The original fund-flow model is a time-continuous representation in which production results from the cumulative service of funds and the processing of flows Georgescu-Roegen (1970). NGR highlighted several key characteristics shared by many production systems. First, the production of a commodity often requires the execution of multiple phases that usually differ in the required skills and in duration. Second, he observed that this sequence of subprocesses often leads to the idleness of production factors and he recognized how in-line organizations could eliminate such idleness. In this sense, process and organizational innovations could be deemed time-saving (Morroni, 1992, Piacentini, 1997, von Tunzelmann, 1995). NGR understood that economies of time are central to the development of industries, hence the express representation of durations in his model. A fairly comprehensive survey of the fund-flow model can be found in Marzetti (2013).

In our implementation, we build on these core principles while also departing from some of NGR's assumptions. First of all, we adopt a discrete dynamical framework that can be simulated with computers. Beyond the technicality, the adoption of discrete time also allows us to explicitly describe and discuss the adaptive processes at stake to organise and/or transform the production process. Additionally, we introduce frictions that affect both the productivity and idleness of funds. While NGR acknowledged that the complete elimination of idleness is not always feasible, he assumed that, when it is possible, a flawless process, rid of idleness, would emerge. Production systems, however, are rarely frictionless and the arrangement of phases alone is not sufficient to remove idleness. Specifically, we will consider (i) the existence of a tempo[9], corresponding to the time unit, that limits the pace of operations, (ii) productivity dynamics for machines and workers (the only funds we consider), and (iii) explicit strong indivisibility of production factors. In these settings short-term instabilities endogenously arise and impede production efficiency. These frictions can also disrupt NGR's assumption of constant returns to durations: as slowdowns propagate through the system, rather than sparking randomly, the output of $2n$ periods may not necessarily equal twice the output of n periods.[10]

We shall name the proposed model NGR-ADAPT, in honor of NGR, and to emphasize the firm's need to adapt to local slowdowns in the production process, as will become clear. ADAPT stands for Agent-based DisAggregated Production with Time.

2.1 Sequential Process and Idleness

A unit of the final good is produced through the sequential completion of H production phases, coined *elementary process*. Assuming that funds operate at their maximum productivity, completing the entire process requires $T = \sum_h T_h$ periods, where each phase h is completed in $T_h < T$ periods. Each phase is executed by a duo consisting of a worker i and a machine j of type h, denoted $\left(i, j^h\right)$.

The quantity of final goods produced after the completion of T periods is normalized to $Q_H = 1$. Similarly, intermediate goods Q_h (outflows) are also standardized to 1, and the productivity levels of funds range between 0 and 1. Hence, to produce one unit of Q_h one duo with a combined productivity of 1 must work the task h for T_h periods. Note that beyond the technical simplification allowed by such an assumption, this corresponds to production processes for indivisible goods, that could only be marketed, once fully completed, in units.

[9]The assumption of a tempo, or cadence, simplifies the discretization of the model as we need not pass flows onto the next phase within a unitary period. Nonetheless, it is a fair assumption given that in a production process intermediary products take time to travel across the plant, if not multiple sites. Besides, machines and software might need to be reset, workers might need to move and grab raw materials and tools, etc.

[10]NGR did not assume constant returns to scale, i.e., to the quantity of employed funds and injected flows.

In phase $h = 1$, a duo takes one unit of inflow, $I_1 = 1$, and produces one unit of outflow, $Q_1 = 1$, upon completion of the task. This outflow becomes the inflow for phase 2. All subsequent phases, $h = 2, ..., H-1$, thus process an inflow $I_h = Q_{h-1} = 1$ to produce $Q_h = 1$. The outflow of the last phase constitutes the final good. Figure 1 provides a diagrammatic representation of a sequential process.

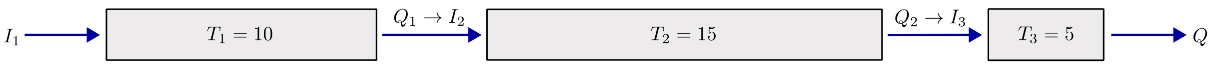

Figure 1: Sequential Production Process

Running this process in-series implies that funds experience idleness as long as there exists a phase h such that $T_h < T_g$ for some $g < h$. The reason is intuitive: if a phase is quicker to complete than any of the preceding phases, it will have to wait for the difference in time, $T_g - T_h$. Consider the example in Figure 1. We assume that phase 1 is always supplied with raw materials, so it never becomes idle. Phase 2 is also never idle, as the outflow from phase 1 is produced faster ($T_2 > T_1$), resulting in an accumulation of flows between the two phases ($Q_1 \equiv I_2$). However, phase 3 remains idle for 10 periods after producing for only 5 periods as it is faster than phase 2. The idleness rate for phase 3 is therefore $(T_2 - T_3)/T_2 = 2/3$. Adding more phases that satisfy $T_h < T_2$ will result in a similar mechanism: these phases will take T_2 periods to produce one unit of outflow and will thus be idle at a rate of $(T_2 - T_h)/T_2$. However, if a phase characterized by $T_h \geq T_2$ is added, it will not experience idleness, as it will receive an inflow every T_2 periods. In general, under an in-series organization and maximum funds' productivity, the idleness rate of any phase h is given by:

$$\mathcal{I}_h = \frac{T_h^{\max} - T_h}{T_h^{\max}}, \quad T_h^{\max} = \max_{g \leq h} \{T_g\}$$

However, this does not hold when the idleness of flows is to be eliminated as well, that is, when the firm wants to prevent the accumulation of inflows, which naturally arises from a process with heterogeneous-duration (Piacentini, 1995). If the firms does so, then quicker phases have to wait as well and the pace is dictated by the slowest phase. As a result, only the latter is continuously working and all other phases have an idle rate of:

$$\mathcal{I}_h = \frac{T^{\max} - T_h}{T^{\max}}, \quad T^{\max} = \max_{h} \{T_h\}$$

2.2 Production Planning

In-line Organization The firm can eliminate, or at least mitigate, idleness by organizing the production process in a modern (hear Fordist) factory fashion, i.e., in-line. Given the set of durations $\mathbf{T_h} = T_1, \ldots, T_H$ (assumed to be integers thus far) idleness can be eliminated by running $C_k = T/\delta$ elementary processes every T periods. The resulting quantity of outflows produced, C_k, represents the *minimum efficient size* (MES) required to eliminate idleness. The firm cannot achieve this degree of efficiency by producing less. The factor δ is termed *elementary lag* and corresponds to the inverse frequency at which elementary processes are run. It is equal to the greatest common divisor (GCD) of the durations $\mathbf{T_h}$. It follows that the number of duos continuously working becomes $C_h = T_h/\delta$. As a result, the firm produces one outflow of any type h every δ periods and C_k outflows every T periods. In the example of Figure 1 $T = T_1 + T_2 + T_3 = 30$. The GCD of $(10, 15, 5)$ is $\delta = 5$, meaning that by creating a production line, one unit of outflows is produced every 5 periods. This corresponds to $C_k = 6$ elementary processes running every 30 periods, achieved by continuously operating $C_1 = 2$, $C_2 = 3$, and $C_3 = 1$ duos in phases 1, 2, and 3, respectively.

The MES matters: first of all, a higher MES means that more funds are required to run the process efficiently. Secondly, it determines the *flexibility* in the organization of an in-line production system. To eliminate idleness, the firm should rely on this MES and target only multiples of it. Oftentimes a higher MES (lower δ) induces lower accuracy in reaching a production target, though it is not necessarily true. For example, if we seek to produce 1.5 units of good every period then a MES of 1/4, 1/5, 1/6, ..., is preferred over 1/3 and 1, while 1/2 would fit perfectly. This becomes an important matter when introducing time-saving innovations. Considering an in-line organization such innovations are not necessarily beneficial for these affect the degree of flexibility of the organization. We elaborate on this specific point in the discussion of the simulation results. In the previous example the firm could produce multiples of $C_k = 6$ every $T = 30$ periods. If we jumped to $T_1 = 9$ following an innovation on the first phase then the elementary lag would become $\delta = \text{GCD}(9, 15, 5) = 1$ for $T = 29$. In this scenario the firm could then efficiently produce only multiples of $C_k = 29$ every 29 periods, which is substantially worse.

To generalize the lines' organization we consider $T_h \in \mathbb{R}_+^*$ instead of integers. In this scase, eliminating idleness completely is rarely feasible.[11] We thus assume that the firm picks the most convenient elementary lag, calculated in the neighborhood of the actual durations $\mathbf{T_h}$. Also, the firm opts for the highest flexibility among the candidates. Doing so, the firm is keen to accept some more idleness but, as this often comes down to choosing between $\delta = 1$ and $\delta = 2$, it is also more flexible. Let denote by $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ the floor and ceiling operators, respectively. The firm considers the neighborhood of integer duration sets:

$$\mathbb{T}_h = \{\mathbf{T'_h} = \{T'_1, \dots, T_h, \dots, T'_H\}|_{0\to 1} \ : \ T'_h \in \{\lfloor T_h \rfloor, \ \lceil T_h \rceil\} \ \forall h\} \tag{1}$$

The operation $|_{0\to 1}$ replaces all 0s with 1s since the GCD does not exist for sets including 0s. Then the firm picks the highest elementary lag derived in this neighborhood:

$$\delta^* = \max\{\boldsymbol{\delta'}\} = \max_{\mathbf{T'_h}\in\mathbb{T}_h} \{\mathrm{GCD}(\mathbf{T'_h})\} \tag{2}$$

The corresponding duration set is denoted $\mathbf{T_h^*}$. Note that if all durations are real numbers greater than 1 then this approach implies $\delta^* \geq 2$ since the neighborhood contains at least one set of even numbers. It follows a natural approximation of the firm's production capacity: about $1/\delta^*$ units can be produced every period. Letting $T^* = \sum_h T_h^*$, the MES and the number of duos per phase become:

$$\mathbf{C'_h} \approx \frac{\mathbf{T_h^*}}{\delta^*}, \qquad C'_k \approx \frac{T^*}{\delta^*} \tag{3}$$

Demand and Parallel Lines We assume constant demand $\mu_d > 0$ as the effect of demand dynamics on production efficiency is not be studied in this paper.

$$Y_t^d = \mu_d \tag{4}$$

The firm simply targets a production level of μ_d per period, or equivalently $T\mu_d$ every T periods. In the ideal case the firm wants to match demand while running an efficient production process, implying to run several lines in parallel (yet we assume that they are physically connected). Specifically, the parallelization of the process must satisfy:

$$\frac{n}{\delta^*} = \mu_d \iff n = \delta^* \mu_d \tag{5}$$

μ_d may not be an integer while n must be, so we set the number of lines n^* to the nearest greater integer of n.

$$n^* = \lceil n \rceil = \lceil \delta^* \mu_d \rceil \tag{6}$$

Notice that if $n \in [0, 1]$ then n^* is set to 1, i.e., the firm prefers to produce in excess instead of nothing. We obtain the final number of duos $C_h^* = n^* C'_h$ to allocate to each phase h and the size of the process is $C_k^* = n^* C'_k$.

The firm revises the production process organization every τ periods. This parameter can be seen as an indicator of the system's timeframes. For example, we set $\tau = 50$ for most of the simulations, which would roughly correspond to weekly planning if the time unit is of the order of an hour.

2.3 Funds Productivity and Management

Funds Productivity The working time of funds within a unitary period, i.e., in $(t-1, t]$, is ranging between 0 and 1. When null, this would mean that the funds are not allocated to the process or that they are allocated yet no flow is available for them to work. A working time of 1 on the other hand describes funds that have been working continuously within the period. Finally, the working time can be fractional of the unitary period if the production phase (i.e processing of the flow) has been completed before the end of the period. In this scenario, and due to the tempo assumption, the duo of funds are both working (for a time $f_{i/j} \in (0, 1)$) and idle (for a time $1 - f_{i/j}$) in the same period. The fractional working time of the duo (i, j^h) can be formalized as follows:

$$f_{i/j,h,t} = \begin{cases} 1 & \text{if working and } q_{ij,h,t} \leq 1, \\ \dfrac{1 - q_{ij,h,t-1}}{q_{i,j,h,t} - q_{ij,h,t-1}} & \text{if working and } q_{ij,h,t} > 1, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

[11] In order to keep a reasonable MES, the GCD can be generalized to real numbers but it often leads to values close to zero, implying a high MES and the use of a larger number of funds.

Where $q_{ij,h,t} \geq 0$ is the current completion of the flow processed by the duo as defined in equation (16). Working time affects the funds composing the duos in two very opposite way: while workers gain experience through practice, hence time affecting positively their productivity, machine productivity decays with use. Working time, hence, has a non-trivial effect on the efficiency of each duo and of the production process as a whole.

Workers' productivity level starts at $a_0 \in (0, 1]$ and grows as they practice a task, i.e., via *learning-by-doing*.[12] For each worker, a_0 is set to a_u for phases they are not familiar with and to $a_s \geq a_u$ for the task h_i^* the worker was initially recruited for.[13] Additionally, a parameter $\gamma_a > 0$ controls the learning rate of the worker. The learning curve is assumed S-shaped:

$$a_{i,h,t+1} = 1 - (1 - a_{i,h,t})^{1+\gamma_a f_{i,h,t+1}}$$

Several adjustments are made to this equation.[14] First, we assume a lower-bound for productivity denoted that a_u. Second, we introduce a *forgetting* mechanism: workers not practicing a task for a long time tend to forget (Argote et al., 1990, Arthur Jr et al., 1998, Shafer et al., 2001, Besanko et al., 2010). This mechanism is controlled by a threshold parameter θ_a such that if $f_{i,h,t+1} > \theta_a$ the worker learns, if $f_{i,h,t+1} < \theta_a$ the worker forgets, and equality is neutral. We define:

$$a_{i,h,t+1} = \max\left\{a_u,\ \min\left\{1,\ 1.01 - (1.01 - a_{i,h,t})^{1+\gamma_a(f_{i,h,t+1} - \theta_a)}\right\}\right\} \tag{8}$$

The combination of these two mechanisms[15] reflects the emerging differentiation in skills among the workers depending on their experience on the various tasks.

In contrast, machines depreciate as they are used. Below is defined the productivity level of a machine j of type h at time t, where θ_b is the depreciation rate and $\mathcal{F}_{j,h,t}$ is the accumulated working time of the machine since last repair.

$$b_{j,h,t} = e^{-\theta_b \mathcal{F}_{j,h,t}} \tag{9}$$

The firm may decide to repair machines when their productivity level falls below some value $\underline{b} \in (0, 1)$. This operation induces a cost e_h expressed in time during which the machine is not available. The firm waits for the next planning (occurring every τ periods) to start the maintenance. We assume that more efficient technologies (lower duration) are more costly to be repaired.

$$e_{h,t} = \frac{\omega\tau}{T_{h,t}} \tag{10}$$

For instance, if $\omega = 10$, then the machines characterized by $T_1 = 5,\ T_2 = 10$ and $T_3 = 20$ will respectively require $2\tau,\ \tau$ and $\tau/2$ periods to be repaired.

Allocation At any instant t the firm is hiring N workers and J machines, split into $J_1 ... J_h ... J_H$ machines of type $1...h...H$. We can represent the pool of funds by a $N \times H$ matrix $\mathbf{L_t^P}$ of workers' productivity levels and by H vectors $\mathbf{K^P_{h,t}}$ of machines' productivity of dimension J_h, respectively:[16]

$$\mathbf{L_t^P} = \begin{pmatrix} a_{1,1,t} & \cdots & a_{1,h,t} & \cdots & a_{1,H,t} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ a_{i,1,t} & \cdots & a_{i,h,t} & \cdots & a_{i,H,t} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ a_{N,1,t} & \cdots & a_{N,h,t} & \cdots & a_{N,H,t} \end{pmatrix} \qquad \mathbf{K^P_{h,t}} = \begin{pmatrix} b_{1,h,t} \\ \vdots \\ b_{j,h,t} \\ \vdots \\ b_{J_h,h,t} \end{pmatrix} \tag{11}$$

[12] Learning-by-doing has been associated with other sources of productivity gains such as organizational and process innovation that do not target the workers routines directly (Wright, 1936, Newell and Rosenbloom, 2013). It has also been modeled through a single proxy embedding many forms of learning (Arrow, 1962, Solow, 1997), be it at the firm or at the country level. In contrast, as did De Jong (1957) we separate workers' skill improvement (to which we reserve the term learning-by-doing), organizational choices, and process innovations.

[13] The parameter a_s can thus be interpreted as a recruitment skill requirement, hence possibly controlled by the firm.

[14] Note that in order to ensure that the productivity level is not stuck at 1 we artificially add a small value to the 1's making the upper-bound.

[15] In recent years, similar mechanisms of differentiation in workers' skill have been used both in macroeconomic models focusing on the dynamics of the labour market as in Dosi et al. (2018) and Bordot and Lorentz (2021) and in an evolutionary production model that tackles the adaptation of production to demand cycles (Llerena et al., 2014).

[16] In practice the firm is unlikely to know the actual productivity of funds at any given time, especially since funds of different nature co-operate. In the simulation model we assume that the firm knows the productivity dynamics of machines and infer workers' productivity. However, this comes down to knowing both perfectly when the dynamics are noise-free, which is assumed throughout this paper.

The targeted number of machines of type h is given by $C_h^* = n^* C_h'$. To reach this level the firm assigns to each phase h the most productive available funds first. Therefore, ideally the first duo assigned to some phase h should be:

$$(i, j^h)^1 = \left(\underset{i}{\operatorname{argmax}}\, a_{i,h}, \quad \underset{j}{\operatorname{argmax}}\, b_{j,h} \right) = \left(\max_i \left\{ \mathbf{L^P_{h,t}} \right\}, \quad \max_j \left\{ \mathbf{K^P_{h,t}} \right\} \right)$$

Where $\mathbf{L^P_{h,t}}$ is the h-th column of $\mathbf{L^P_t}$. If we approach this iteratively, the firm repeats this operation on the remaining available funds until the objective is reached. Formally, letting M^h be the amount of duos allocated to phase h and $(i, j^h) \in \mathcal{A}$ be the latent pool of available funds, we can write:

$$\text{Increment } M_h \text{ until } \sum_{(i,j^h)\in\mathcal{A}} a_{i,h} b_{j,h} \geq C^*_{h,t}$$

Adaptation In this version of the model demand is exogenous and unsatisfied demand accumulates without maximum delay.[17] The firm increases its stock of raw materials I_1 by $n^*/\delta^* \approx \mu_d$ every period. Importantly, all phases can experience an accumulation of inflows stocks and thus phases that lag behind then need to be reinforced with additional workforce. Let $I_{h,t}$ be the stock of inflows for phase h at time t. The term τ/T_h represents the maximum number of flows that can be processed in τ periods by one duo of funds. Hence the firm may need to allocate $I_h/(\tau/T_h)$ additional duos to phase h. This mechanism shall be deemed *reactive adaptation*.

Besides, the firm may decide to produce at a faster rate in order to offset slowdowns. For this sake, the firm wants to produce $r \geq 1$ times its production target Q^*. To avoid heavy excess production the firm automatically sets $r = 1$ when excess starts being visible (i.e. when $V_{H,t} < 0$, see eq. (21)). While facing delays, however, the firm wants to allocate rC_h^* duos to phase h. This mechanism shall be deemed *proactive planning*.

Accounting for these mechanisms, the allocation of funds becomes:

$$\text{Increment } M_h \text{ until } \sum_{(i,j^h)\in\mathcal{A}} a_{i,h} b_{j,h} \geq rC^*_{h,t} + \frac{I_{h,t}}{\tau/T_{h,t}} \tag{12}$$

Initial Pool of Funds So as to prevent (i) flows from accumulating excessively and (ii) some phases from going short-staffing, the firm seeks to employ a sufficient amount of workers and machines in the first place. We assume that this choice accounts for the optimal number of duos C_h^*, augmented by the proactive target factor r and adjusted for the maintenance threshold $\underline{b}$. The number of workers shall not be too high though. If we set $N = J$ then many workers would be idle when the pool of machines is not deprecated. Machines are more numerous, based on the maintenance threshold, because of the need for backup machines in periods of maintenance. We define:

$$J_h = \left\lceil \frac{rC_h^*}{\underline{b}} \right\rceil \qquad\qquad N_h = \lceil rC_h^* \rceil \tag{13}$$

Sequential Allocation Phases that are more likely to accumulate delay may benefit from being assigned funds first so that they can catch up rapidly. We introduce a lexicographic heuristic to define the priority order. The firm first allocates funds to the phase with the highest delay in terms of unsatisfied demand[18](V_h), and proceed likewise for picking the next phase to prioritize, and so on. If several phases show the same delay, however, the natural order is preserved, i.e., the earlier phase gets priority. This choice lies in another important consideration: slowdowns occurring at earlier stages are more harmful for the production system due to the sequential nature of the system (delays in one phase impact all the subsequent phases). Hence the importance of accounting for both the urgency and the natural order when setting priority.

[17] The firm can therefore be seen as a monopolist

[18] In contrast, the targeted number of duos C_h^* was augmented by the number of inflows stocks I_h. In fact, both I_h and V_h are reasonable proxies for production delay. However, using either for determining both the priority order and the allocation target C_h^* can lead to bottlenecks. For example, using I_h to set the priority order may cause the early phases to capture most or all of the available funds whenever inflows accumulate faster there, which is likely occurring in the early life of the plant or when the firm is proactive ($r > 1$). On the other hand, using V_h to compute C_h^* can cause the later phases to be provided with many funds although few or no inflows are ready to be processed.

We can finally design the allocation mechanism. Let us vectorize the operations. Starting from the highest-priority phase, say h, let $\tilde{\mathbf{L}}^{\mathbf{p}}_{\mathbf{h,t}}$ be the h-th workers' productivity vector sorted in descending and $\tilde{\mathbf{K}}^{\mathbf{p}}_{\mathbf{h,t}}$ be the counterpart for the machines' productivity. We compute the combined productivity vector for phase h:

$$\tilde{\mathbf{P}}_{\mathbf{h,t}} = \tilde{\mathbf{L}}^{\mathbf{p}}_{\mathbf{h,t}} \underline{\odot} \tilde{\mathbf{K}}^{\mathbf{p}}_{\mathbf{h,t}} \tag{14}$$

Where $\underline{\odot}$ is the Hadamard product applied after truncation of the longest vector such that both have the same dimension. Let $\tilde{\mathbf{P}}^{\mathbf{c}}_{\mathbf{h,t}}$ be the cumulative sum of the productivity vector. M_h is then equal to the index of the first element of $\tilde{\mathbf{P}}^{\mathbf{c}}_{\mathbf{h,t}}$ that is greater or equal than the threshold defined in (12). Then the productivity vectors are recomputed after dropping the previously allocated workers and the exact same process is applied to the next phases.

2.4 Production Mechanisms

Indivisibility The low level of abstraction in the NGR-ADAPT model stems from the intention to allow instabilities to emerge endogenously in a dis-aggregated production model. A key feature that distinguishes our model from NGR's original framework, and most production models in general, is the explicit representation of indivisibility of flows and funds. Although they were recognized and discussed by NGR and later contributors to the fund-flow model (see notably Morroni (1992)), indivisibility were not so central in the analysis. Indeed, under the hypothesis that the production line runs flawlessly the identity of funds and flows does not add contrast to the model and aggregation is simple.

On the contrary, in our framework aggregation is way more arduous for that duos operate at different paces. There, indivisibility truly matter and the term *two halves don't make a whole* takes full meaning. Notably, the following characteristics are embedded in the NGR-ADAPT model:

1. Oftentimes, independently processed fractions of the final good cannot be added and sold as one unit, i.e., the output results from the completion of an elementary process involving all phases. For instance, a car worth \$20k is a whole and no customer is willing to acquire two half-processed cars missing a pedal, two doors, the engine and the tires. Similarly, they usually do not want to purchase \$20k worth of random components of the car.

2. Any phase must complete the processing of a unit before an outflow can be passed onto the next phase. Two bottom halves of a car's door cannot be assembled into one ready-to-use door.

3. Funds (workers and machines) use their productive power for at most one elementary process at any given time.[19]

These indivisibility properties imply *strong complementarity* between production factors. Although the economic literature recognizes the complementarity of production factors, the mechanisms underlying complementarity are typically obscured. Here, simply having sufficient hypothetical production capacity from machines and workers is not enough. The mapping from input factors to output products is non-monotonous and, in fact, it is not even unique. That is, returns to scale and duration are variable over time due to inefficiencies.

Production Consider the vector of duos $\mathbf{F}^*_{\mathbf{h,t}}$ allocated to phase h and the corresponding duos' productivity vector $\mathbf{P}^*_{\mathbf{h,t}}$. Both are sorted in descending order according just after allocation so as to guarantee in the following computations that the inflows are passed on to the most productive duos first.

$$\mathbf{F}^*_{\mathbf{h,t}} = \begin{pmatrix} (i, j^h)^1_\tau \\ \vdots \\ (i, j^h)^n_\tau \\ \vdots \\ (i, j^h)^{N_h}_\tau \end{pmatrix} \qquad \mathbf{P}^*_{\mathbf{h,t}} = \begin{pmatrix} p^1_{h,t} \\ \vdots \\ p^n_{h,t} \\ \vdots \\ p^{N_h}_{h,t} \end{pmatrix} = \begin{pmatrix} a^1_{i,h,t} \cdot b^1_{j,h,t} \\ \vdots \\ a^n_{i,h,t} \cdot b^n_{j,h,t} \\ \vdots \\ a^{N_h}_{i,h,t} \cdot b^{N_h}_{j,h,t} \end{pmatrix} \tag{15}$$

[19] This property of the model is sometimes restrictive. While it is common that workers are individually paired with one workstation, computer, or vehicle for example, it can also be true that a duo contributes to several outputs at once (e.g., cheese production) or that a machine is shared (e.g., an oven). Heterogeneity in the nature and interaction of funds is left for future work.

To ensure the independence of duos production is represented by a latent vector $\tilde{\mathbf{Q}}_{\mathbf{h,t}}$. At every period t the latent production of an activated duo corresponds to the cumulative completion of the flow. Letting $\tilde{I}^n_{h,t}$ be the latent number of available inflows, we propose:

$$\tilde{\mathbf{Q}}_{\mathbf{h,t}} = \begin{pmatrix} q^1_{h,t} \\ \vdots \\ q^n_{h,t} \\ \vdots \\ q^{N_h}_{h,t} \end{pmatrix} \quad \text{where} \quad q^n_{h,t} = \begin{cases} q^n_{h,t-1} + \dfrac{p^n_{h,t}}{T_h} & \text{if } q^n_{h,t-1} \in (0,1), \\ \dfrac{p^n_{h,t}}{T_h} & \text{if } q^n_{h,t-1} \notin (0,1) \text{ and } \tilde{I}_{h,t} \geq 1, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Let us vectorize equation (16). Define a vector of latent inflows stock $\tilde{\mathbf{I}}_{\mathbf{h,t}}$ of dimension N_h corresponding to inflows availability: $\tilde{I}_{h,t} \geq 1$. It is constructed so that more productive duos are prioritized, upon availability of the duos. Because 1s are set for available funds only, $\tilde{\mathbf{I}}_{\mathbf{h,t}}$ represents the whole condition of the second row in equation (16), i.e. $q^n_{h,t-1} \notin (0,1)$ and $\tilde{I}_{h,t} \geq 1$. We also consider the binary vector $\mathbf{X}_{\mathbf{h,t}}$ of dimension N_h which entries take on the value 1 if $q^n_{h,t-1} \in (0,1)$ and 0 otherwise. We can write:

$$\tilde{\mathbf{Q}}_{\mathbf{h,t}} = \left[\mathbf{X}_{\mathbf{h,t}} \odot (\tilde{\mathbf{Q}}_{\mathbf{h,t-1}} + \mathbf{P}^*_{\mathbf{h,t}}/T_h)\right] + \left[\tilde{\mathbf{I}}_{\mathbf{h,t}} \odot \left(\mathbf{P}^*_{\mathbf{h,t}}/T_h\right)\right] \tag{17}$$

Where $\odot$ is the Hadamard product. When a duo reaches a latent production of 1 or greater, it means that an outflow is available to be sold or used as an inflow for the next phase. Hence the phase-wise production function is the sum of duos that have reached a latent production of 1 or greater:

$$Q_{h,t} = \sum_{n=1}^{N_h} \mathbb{1}\left(q^n_{h,t} \geq 1\right) \tag{18}$$

Stocks of Flows The conditions in the second line of equation (16) ensure that a new inflow is being processed by phase h only when available. If so, the latent stock of inflows $\tilde{I}_{h,t}$ is decremented. At the beginning of each period the new stocks of inflows should then become:

$$I_{h,t} = \begin{cases} I_{1,t-1} + \mu_d - \sum_* \tilde{\mathbf{I}}_{\mathbf{1,t-1}} & \text{if } h = 1 \\ I_{h,t-1} + Q_{h-1,t-1} - \sum_* \tilde{\mathbf{I}}_{\mathbf{h,t-1}} & \text{if } h > 1 \end{cases}$$

Where $\sum_*$ stands for the sum of vector entries. In order to reach the proactive production onjective the firm needs to inject $r\mu_d$ raw materials every period. Because the quantities of flows need to be integers we define a latent supplement:

$$\tilde{R}_t = \tilde{R}_{t-1} - \lfloor \tilde{R}_{t-1} \rfloor + (r-1)\mu_d, \qquad R_t = \lfloor \tilde{R}_t \rfloor \tag{19}$$

At every period the firm increases a latent variable $\tilde{R}_t$ by an amount $(r-1)\mu_d$ of supplemetary inputs. When this variable becomes greater than 1, the integer part is stored in $R_t = \lfloor \tilde{R}_t \rfloor$ and the latent variable $\tilde{R}_t$ is decremented by the same amount. We can define the stocks dynamics as follows:

$$I_{h,t} = \begin{cases} I_{1,t-1} + (Q^*_t + R_t) - \sum \tilde{\mathbf{I}}_{\mathbf{1,t-1}} & \text{if } h = 1 \ \& \ V_{H,t} \geq 0 \\ I_{1,t-1} - \sum \tilde{\mathbf{I}}_{\mathbf{1,t-1}} & \text{if } h = 1 \ \& \ V_{H,t} < 0 \\ I_{h,t-1} + Q_{h-1,t-1} - \sum \tilde{\mathbf{I}}_{\mathbf{h,t-1}} & \text{if } h > 1 \end{cases} \tag{20}$$

Where the condition $V_{H,t} < 0$ (equation (21)) corresponds to a regime of excess production, in response to which the firm stops injecting raw materials to avoid stocks to accumulate in large quantities in the plant.

3 Emerging Inefficiencies: Baseline Experiments

In this first set of simulation experiments, we explore the system dynamics for different configurations of managerial reactivity and pro-activity, under various temporal structures, as well as for different degrees of learning and forgetting. In doing so, we first focus on the emergence of inefficiencies, namely production slowdowns and funds idleness, and ways to reduce these.

3.1 Performance Evaluation

The flexibility in planning and the adaptation mechanisms are meant to let the firm demonstrate intelligence with the ultimate goal of converging to a relatively stable and efficient production regime. We therefore want to assess the stability and the efficiency of the process. We do so by considering the firm's ability to satisfy demand on the one hand and the idleness of workers on the other. Ideal conditions for a firm are to *quickly converge* to a regime of *fast demand satisfaction with low variability* (i.e., little delays or production excess) while *maintaining workers' idleness low* in the long run.

Demand Satisfaction First of all, we measure the firm's ability to fulfill demand through the difference between production and demand, denoted D_h. We define the accumulated production differential as the sum of these differences and denote it V_h. Specifically, for phase H (the final good), a positive value indicates a lagging production while a positive value amounts to excess production.

$$D_{h,t} = \mu_d - Q_{h,t}, \qquad V_{h,t} = \sum_{z=1}^{t} D_{h,z} \tag{21}$$

Idleness The idleness of a fund is given by $t^-_{i/j,t} = 1 - f_{i/j,h,t}$. It is ranging between 0 and 1 if the fund is allocated and null otherwise. The overall idle rate of workers is:

$$\text{IRW}_t = \frac{1}{N} \sum_i t^-_{i,t} \tag{22}$$

We can distinguish intentional and unintentional idle rates. Let denote by $\mathcal{A}_t$ the set of allocated duos. Its cardinality is the number of allocated duos, i.e., $|\mathcal{A}_t| = \sum_i \mathbb{1}(\text{allocated } i \text{ at } t)$. The overall intentional idle rate of workers corresponds to the fraction of non-allocated workers:

$$\text{IRW}^i_t = 1 - \frac{1}{N} |\mathcal{A}_t| \tag{23}$$

The overall unintentional idle rate incorporates the idleness of allocated funds only:

$$\text{IR}^u_t = \frac{\sum_i t^-_{i,t}}{|\mathcal{A}_t|} \tag{24}$$

3.2 Model Configuration

To isolate the results of experiments and perform *ceteris paribus* analyses parameters are fixed to a unique value unless they are part of the experiment under study. The default environment is parametrized as follows:

- Simulations are run for 50,000 periods. If 50 periods represent about a week, then this corresponds to about 20 years.
- $H = 5$ production phases with equal duration $T_h = 6, \ \forall h$, hence $T = 30$.[20]
- Demand is fixed to $\mu_d = 1$ per period.
- Planning and allocation are operated every $\tau = 50$ periods.
- The initial productivity levels are $a_u = 0.2, \ a_s = 1$.
- The learning (and forgetting) rate is set to $\gamma_a = 0.001$ and the forgetting threshold is set to $\theta_a = 0.2$. See note.[21]
- The depreciation rate is set to $\theta_b = 0.0002$.
- Machine maintenance is triggered when the productivity of a machine goes under $\underline{b} = .8$. As in the related example $\omega = 10$.
- The proactivity parameter is set to $r = 1.5$.

[20] In this scenario durations are integers and the flexibility is high ($\delta = 6$).

[21] Notice that it implies that workers should spend at least 20% of their time working on a task to prevent forgetting. So, unless they distribute their time equitably between the five tasks, they will tend forget at least one skill.

Figure 2 illustrates the hypothetical productivity dynamics of funds in the baseline. Specifically, it assumes that the fractional working time on the given phase is constant and so is a simplified representation. Nevertheless it shows how the productivity of a worker is impacted by the working time relative to the forgetting threshold. In this case, productivity declines or remain minimum if the worker spends less than 20% of their time practicing the task, otherwise, either it remains steady (exactly 20%) or it improves or remains maximum (more than 20%). Moreover, the time required to mastering a task is of the order of thousand of hours for workers consistently practicing, which corresponds to hundreds or thousands of repetitions of the task and would then be consistent with both empirical findings (Newell and Rosenbloom, 2013) and the popular 10,000-hour rule (Simon and Chase, 1988, Gladwell, 2008).

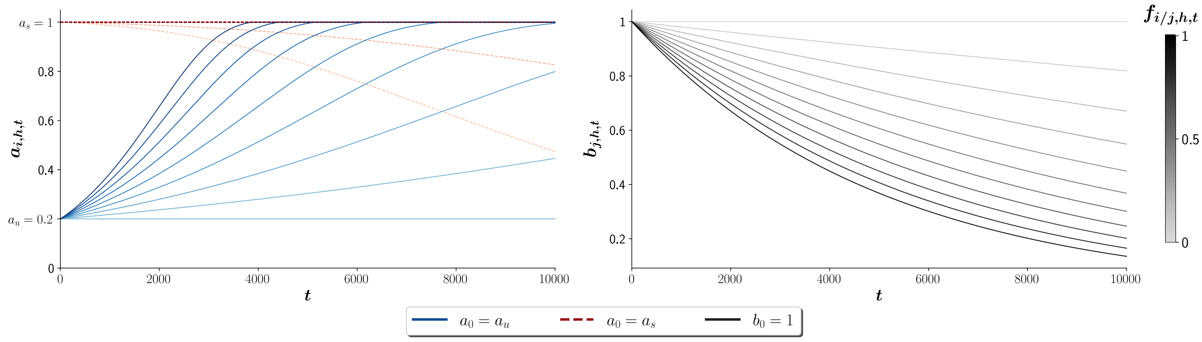

Figure 2: Productivity Dynamics for Baseline Calibration. The left graph shows the productivity of a worker who would spend some time $f_{i,h,t}$ working on task h every period t. The color and style distinguishes between a skilled and an unskilled worker for the task, and shade is a cue for the working time. The right graph shows the depreciation of a machine and follows the same logic. Maintenance is omitted.

3.3 Experiment 1: Managerial Choices

Several parameters can be considered as adjustable by the firm. Three, in particular, hold special significance: the periodicity of organizational updates, denoted by τ, the maintenance threshold $\underline{b}$, and the proactivity parameter r. We let $\tau \in \{10, 50, 1000\}$, $\underline{b} \in \{0.2, 0.5, 0.65, 0.8, 0.95\}$, and $r \in \{1, 1.25, 1.5, 1.75, 2\}$.

Figure 3 presents the evolution of the accumulated production differential indicator $V_{H,t}$ across these 75 configurations. The first clear observation is the impact of proactivity: higher values of r lead to less accumulated delay and a faster convergence to a stable, low-volatility production process. Second, more frequent organizational updates, reflected by lower values of τ, are also beneficial. Additionally, more frequent maintenance proves advantageous. However, this effect appears to hold only up to a certain point. Specifically, we observe increased delayed demand when both τ and $\underline{b}$ are high, potentially indicating suboptimal or "lazy" management practices (little reactivity).

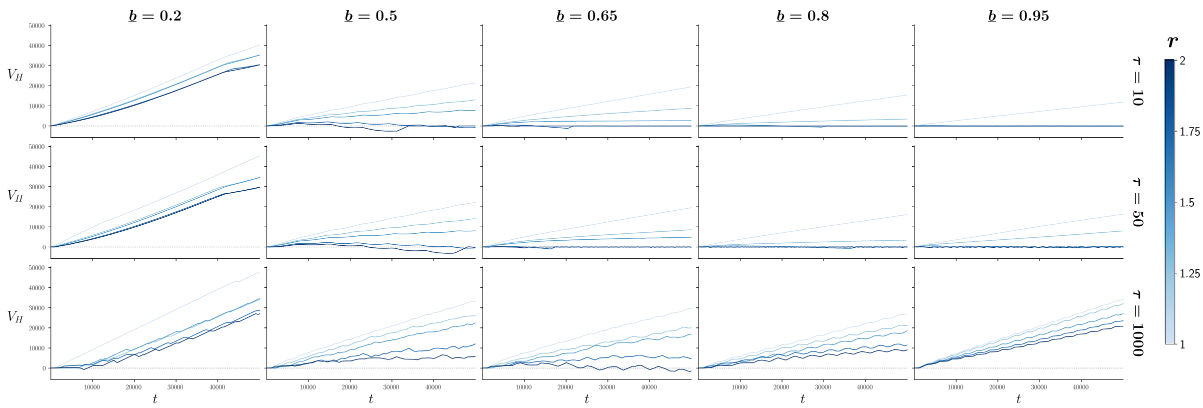

Figure 3: Production Dynamics against Managerial Choices. Is depicted the evolution over time of the variable V_H, smoothed over 100 periods (moving average). Rows let $\underline{b}$ vary, columns let $\tau vary$, the shades let r vary.

The evolution of average idleness is depicted in Figure 4. More frequent maintenance generally leads to greater intentional idleness as it is more likely that large batches of machines are being repaired in the meantime, preventing the allocation of all workers. Regarding unintentional idleness, the association is less clear but this is expected as the maintenance parameter only affects the allocation of funds.

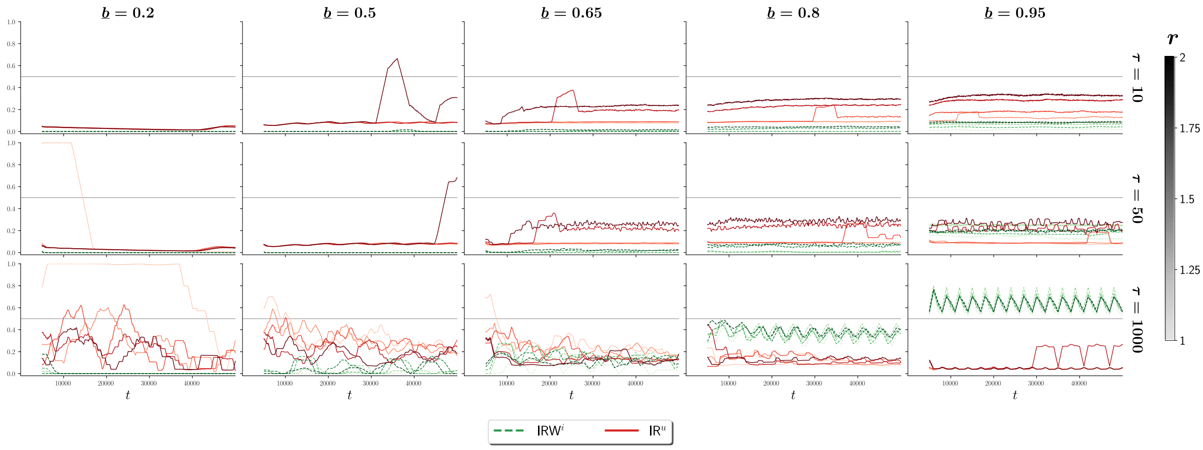

Figure 4: Idleness against Managerial Choices. Is depicted the evolution over time of the variables IRW^i and IR^u, smoothed over 5,000 periods (moving average), and differentiatied by colors and line style. Rows let $\underline{b}$ vary, columns let τ vary, and the shades let r vary.

The frequency of organizational adjustments has a pronounced effect on the production dynamics. Typically, higher values of τ (indicating less frequent adjustments) lead to more intentional idleness and less unintentional idleness. This trend likely interacts with the maintenance threshold $\underline{b}$. Because the firm repairs machines every τ periods, a higher τ results in larger batches of machines requiring repair. With fewer operable machines, fewer workers can be allocated, thus increasing the rate of intentional idleness.

For moderate values of τ (as seen in the first two rows), a more proactive behaviour (higher r) is associated with increased unintentional idleness. This is because, once stable, production oscillates between latent demand and excess production phases, and the latter are likely more important when r is high. Since inputs stop flowing in these periods of excess more funds are affected. Proactivity is thus to be moderate, for that although a larger pool of funds helps to accelerate convergence to stable production dynamics it also induces idleness in the long term.

3.4 Experiment 2: Temporal Structures

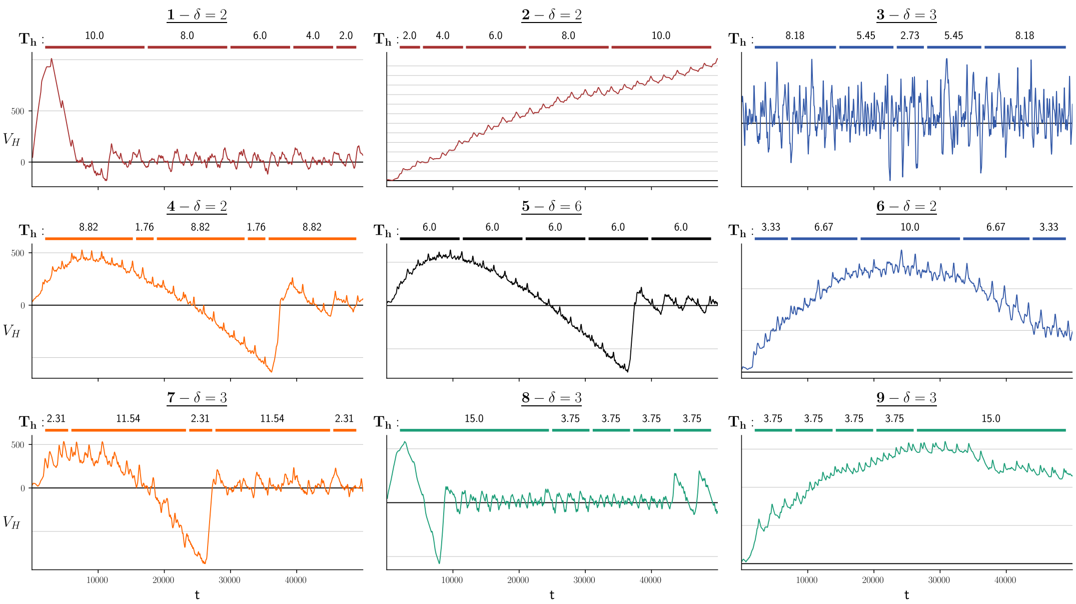

Figure 5: Production Dynamics against Temporal Structures for $T = 30$. Is depicted the evolution over time of the variable V_H, smoothed over 100 periods (moving average). A black horizontal line is set at 0 and grey lines are repeated every ± 500 and serve as cues for comparing the magnitude across plots. The elementary lag δ is displayed in the subplots' title and the temporal structures $\mathbf{T_h}$ are shown under titles. Colors are used to highlight symmetrical structures.

The baseline temporal structure, i.e., the arrangement of durations, is uniform: $T_h = 6\ \forall h$. We now consider eight alternative structures, each with the same total duration $T = 30$, organized into four pairs

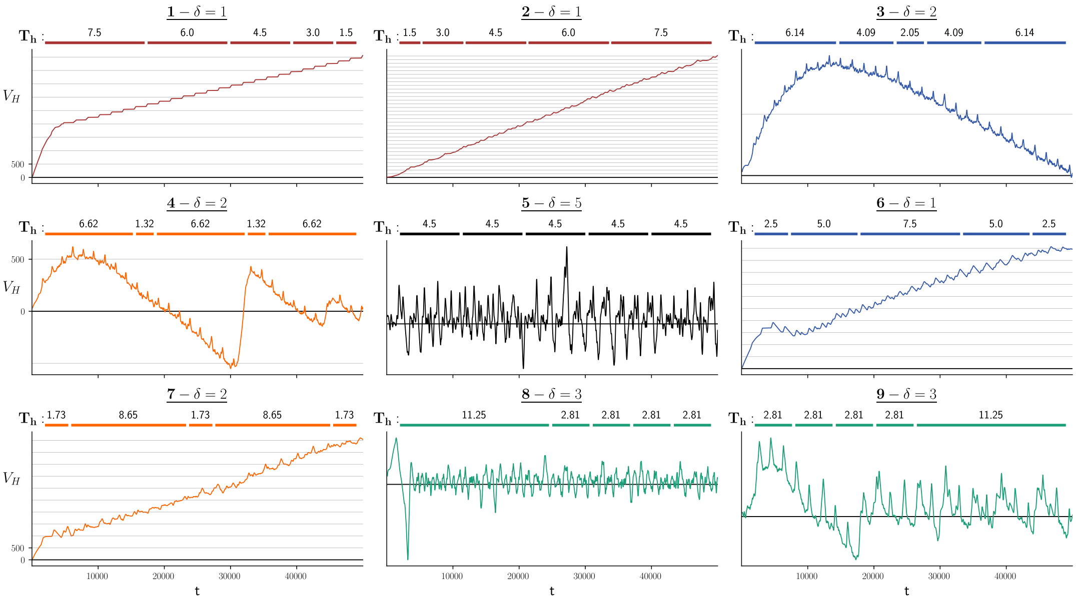

Figure 6: Production Dynamics against Temporal Structures for $T = 22.5$. See the caption of Figure 5.

of symmetrical arrangements. These structures are depicted at the top of each subplot in Figure 5. The graphs display the evolution of the accumulated production differential V_H.

The pair that yields results most similar to the baseline structure (5), which requires no arrangement to be efficient, is the (4-7) pair, which alternates between shorter and longer phases. Other pairs exhibit more divergent dynamics. Interestingly, structure (2) performs significantly worse, while structure (3) rapidly converges to relatively stable, albeit erratic, dynamics around zero with low variance.

Examining the pairs more closely, it appears that the most favorable scenarios may correspond to the structure that would yield the highest idleness if the production process were organized in series. For instance, structure (1), in which all phases except the first are idle in-series, performs better than the idleness-free structure (2). This pattern holds for pairs (8-9) and (3-6), though the difference in idleness for the latter is less pronounced. As for the (4-7) pair, both structures exhibit similar idleness rates in-series, which may explain their comparable performance.[22] This observation requires further verification and the underlying mechanisms to be investigated. For now, all we can assert is that dynamics can differ significantly across temporal structures.

In the peculiar case of the (4-7) pair, a possible determinant of the difference in outcomes is the elementary lag δ. Structure (4) has a shorter elementary lag, implying lower flexibility compared to structure (7). This hypothesis is supported by analyzing these same structures with a reduced total duration, $T = 22.5$, as shown in Figure 6. One might expect faster stabilization in these settings, as seen in the baseline structure (5), but the effects are mixed. While pairwise comparisons yield similar results, the pairs are not uniformly affected. Notably, pairs (1-2) and (3-6) are negatively impacted, whereas pair (8-9) exhibits significantly improved dynamics. In parallel, flexibility decreases for the first two pairs but increases for the last one. This suggests that flexibility and total duration exert opposing influences. Furthermore, in the (4-7) pair, only structure (7) experiences a notable decline in performance, associated with reduced flexibility, while structure (4) remains relatively unaffected in both regards. Finally, although the baseline structure (5) shows less flexibility, it demonstrates faster (instant) convergence. However, the proportional decrease in flexibility is smaller than in previous cases, which may indicate that the negative effect of reduced flexibility diminishes as the initial flexibility level increases.

A deeper investigation into these discrepancies between temporal structures could be a promising direction for future research.

3.5 Experiment 3: Learning

Workers can learn by doing and also forget if they stop practicing a task. To observe the significance of learning and forgetting dynamics we simulate the model in diverse settings by varying the parameters $\theta_a \in \{0, 0.1, 0.2, 0.4\}$, $\gamma_a \in \{0, 0.001, 0.003, 0.01\}$, and $a_s \in \{a_u = 0.2, 0.4, 0.6, 0.8, 1\}$. The results for the

[22]It is worth noting that the better pairwise structures are arranged in-line as well to prevent flows stocks to accumulate.

accumulated production differential are depicted in Figure 7 and idleness dynamics are presented in 9. Moreover, 8 summarizes the distribution of skills as of the end of the simulation.

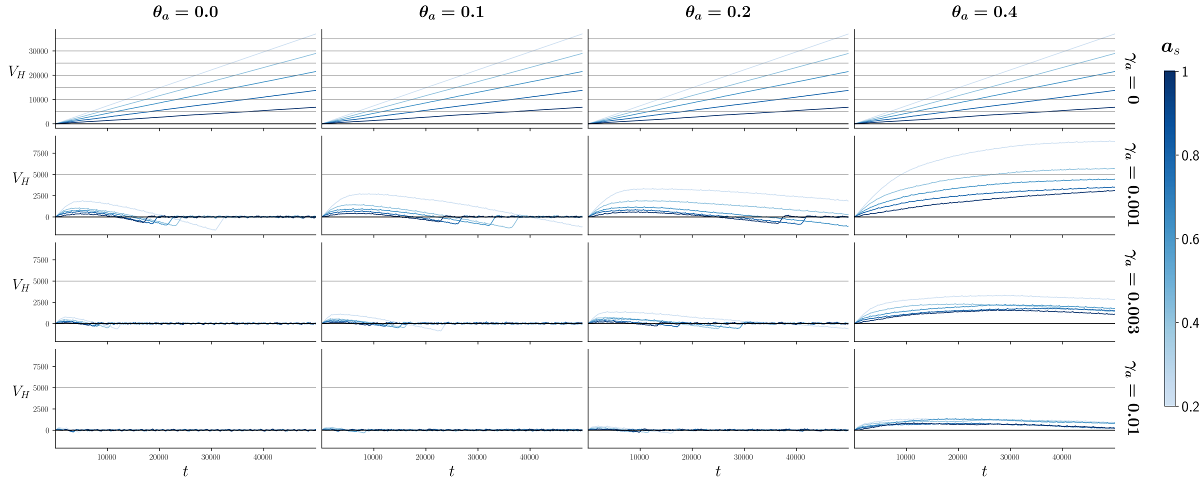

Figure 7: Production Dynamics against Workers' Productivity. The variable V_H smoothed over 100 periods is represented. The parameters γ_a, θ_a and a_s are differentiated in rows, columns, and shade, respectively. The y-axis range is common only for the last three rows ($\gamma_a > 0$). Horizontal grey lines are drawn every 500 to highlight the magnitudes.

All three parameters have a seemingly important effect on the production dynamics. The least of the three is from the hiring productivity a_s. The higher it is the lesser the accumulated delay and the faster the convergence. Starting with more skilled workers indeed places the firm ahead of the game from the beginning. The other two parameters have sound effects on production dynamics as well. A higher forgetting threshold θ_a adds struggle and delays the convergence to a stable regime. Besides, the learning (and forgetting) rate γ_a has a striking impact on the firm's ability to consistently fulfill demand: the higher the better. Most importantly, they have an effect as well when $a_s = 1$, indicating that skill retention matters and that, moreover, the acquisition of new skills required to run other tasks could be important as well.

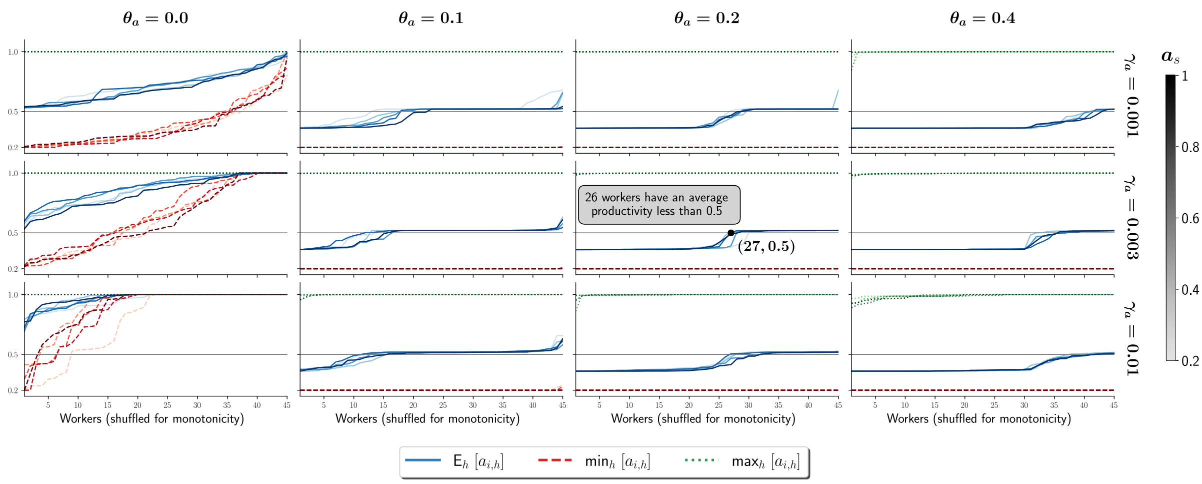

Figure 8: Workers' Specialization. At the last period we compute the mean, minimum, and maximum of each worker's productivity levels. The parameters γ_a, θ_a and a_s are differentiated in rows, columns, and shade, respectively. The resulting series (along workers) are independently sorted and graphed. The graphed series inform on the distribution of productivity levels across workers. An example of the adequate interpretation is given in the subplot for ($\theta_a = 0.2, \gamma_a = 0.001$).

We can learn more on this by looking at Figure 8. We omitted the row for $\gamma_a = 0$ as the outcomes are trivial.[23] First of all, the hiring productivity has little impact on the final productivity distributions. Therefore, in the scenarios with lower a_s, it may be easier for unskilled workers to catch up than it is for the firm to stabilize the process. Second, a higher learning rate, as expected, results in more workers having secondary skills. However, this effect is quickly dampened as the forgetting threshold increases.

[23]Specifically, the average workers' productivity level would be equal to $(4a_u + a_s)/5$. Depending on a_s this would range between 0.2 and 0.36.

Indeed, the effect is very neat for $\theta_a = 0$ and more modest in the other cases. In fact, we can safely state that the effect of the forgetting threshold is predominant. In the extreme case ($\theta_a = 0.4$) no more than a third of the workers have managed to gain productivity in secondary tasks.

What seems to emerge from this analysis is that the reversal of V_H, which is preceding the convergence to a stable regime, may require workers with a larger skill diversity. Workers are not necessarily allocated to the task they have been hired for in the first place. This is confirmed by the persistence of skill diversity. So they can be allocated elsewhere to reinforce a phase that has accumulated delay. This reinforcement is effective if the supporting extra workers are skilled enough to process the flows quickly (i.e. more productive). Furthermore, the easier it is to find highly productive workers to allocate to the needy phases, the fewer of them will be required for the reinforcement. This outcome, although theoretical, seems particularly relevant as it suggests that the specialization of the workers, e.g. at the heart of the division-of-labor paradigm prevailing in Fordist factories, might not be the most adequate in the long run.

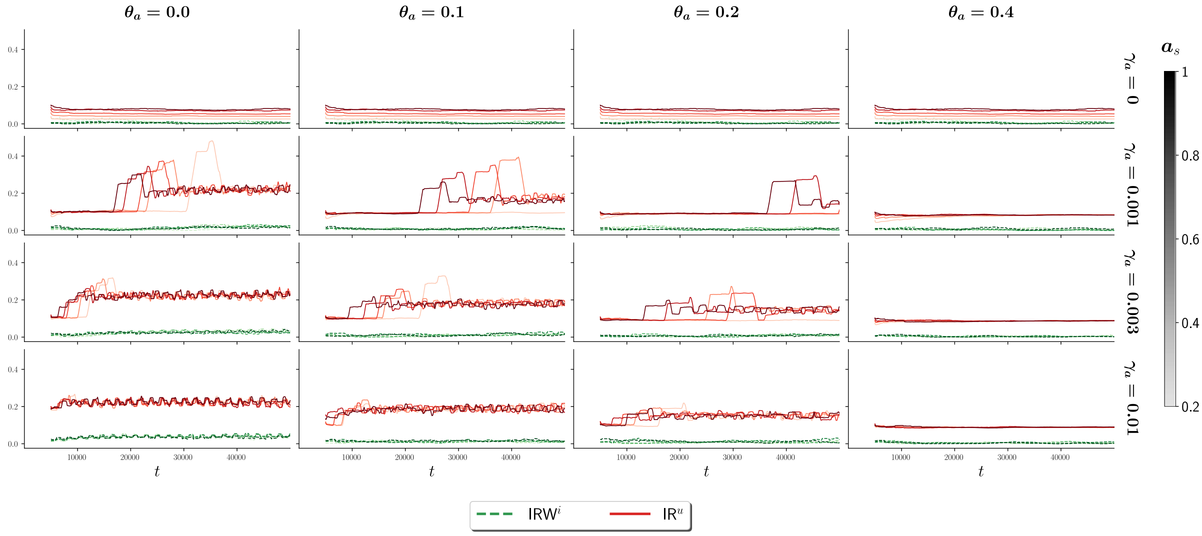

Figure 9: Idleness Rates against Workers' Productivity. The variables IRW^i and IR^u smoothed over 5,000 periods are represented. The parameters γ_a, θ_a and a_s are differentiated in rows, columns, and shade, respectively.

The average intentional idleness remains fairly low in all configurations so we shall focus on the analysis of unintentional idleness. First, the hiring productivity a_s has little effect on the long-run idleness rates when the learning rate γ_a is non-null. Second, more forgetting θ_a entails less idleness. And third, to a lesser extent, a higher learning rate seems to induce more idleness. Besides, a common pattern is the synchronicity of the increase in unintentional idleness and the process stabilization, i.e., an effective production process is typically marked by more idleness in the long-run.

This additional idleness is most likely the outcome of the initial proactive behaviour of the firm that fostered the employment of an extra amount of funds, as discussed in the first experiment. This initial effort allows for a faster convergence, shorter delays, and a lower volatility of production. But this short term strategy is offset by some long-run residual inefficiency. As the firm employs more funds to avoid long production delays (periods of shortage), it faces periods of production excess in which production needs to be slowed down to avoid the accumulation of unsold final products. The firm then stops to feed the process with the raw materials required in the first phase, and this induces some temporary idleness of the workers in this phase. For instance, the large jump of IR^u corresponds to the period preceding stabilization in which V_H has reversed and then decreased substantially in the negative domain (i.e. excess production).

4 Making Idleness Productive via Workers' Creativity

Once stabilized, the production process tends to experience higher levels of idleness. A possible response to this from the firm could be to rationalize production by reducing the amount of funds employed. In contrast to situations of economic downturns, a firm might opt to dismiss workers even though production objectivesare consistently achieved. Setting aside the ethical aspects, in line with Smith, Taylor as well as NGR's calls for the efficient optimization of production processes, some layoffs might be an effective solution to reduce residual idleness. Such a solution reflects the idea that idleness is a cost to be reduced (either a cost for the employment of funds or a cost in terms of productivity loss).

Idleness is not necessarily to be considered solely as a cost. Russell (1932), among others, argues that idleness is required to ensure the preservation of both workers' and machines' capabilities through periods of inactivity. An extensive use of funds can lead to breakdowns, whether it be an overheating machine or an exhausted worker who loses focus and risks injury. While these considerations are not directly modelled here, they are worth acknowledging. In this paper and in line with some elements in Cohendet et al. (2024), we focus on an hypothetical positive outcome of workers' idleness: the generation of innovative ideas.

In this respect, low-skilled workers might be well-suited to identifying sources of inefficiency and proposing ideas that, when implemented as innovations, could improve their tasks. We assume these innovations to be time-saving, reducing the duration T_h for specific phases. The firm manages the implementation of these ideas through its R&D department. While idleness-driven innovations might not be as disruptive as those from high-skilled R&D teams, they should be unlikely to harm the production process since the workers generating them are in direct contact with the production system. Equally important, here, is the idea that workers in contact with the production process are less likely to request innovative ideas that could impede the safety of the task. We also consider here that such ideas are to imply only incremental improvements rather than radical transformations.

4.1 Modeling of Innovation

Idea Generation In direct line with the Schumpeterian, evolutionary literature, the process of innovation is to be understood as a dynamic, uneven but endogenous mechanism of improvement of the technological characteristics of the firm (here the production phases) requiring prior investments in the resources dedicated to the innovation process. These improvements more particularly concern the productivity of a given production phase, while the resource required to generate novelty (ideas here) is time.

Let $t^-_{i,t}$ represents the accumulated idle time of worker i at time t and t^*_i takes on the last period at which the worker has had a creative idea and 1 if it has never happened yet. The probability for idleness-driven ideas to be generated by any worker i is an increasing function of their accumulated idle time $\mathcal{T}^-_{i,t}$:

$$\mathcal{T}^-_{i,t} = \sum_{z=t^*_i}^{t} t^-_{i,z} = \sum_{z=t^*_i}^{t} (1 - f_{i,z}) \tag{25}$$

Workers generate innovative ideas at time t with probability:

$$\mathrm{P}_{i,t}(\text{idea}) = \frac{T_{h^*_i,t}}{g} \left(1 - e^{-\kappa \mathcal{T}^-_{i,t}}\right) \tag{26}$$

The probability is increasing in $T_{h^*_i,t}$, with h^*_i being the the phase the worker has been hired for, so as to account for the struggle to improve more efficient tasks. η controls the curvature, $T_{h^*_i,t}/g$ is the highest probability the worker can reach, and κ acts on the idle time it takes to attain it.

Impact When implemented, a process innovation originating from the worker i's idea has an impact that depends on their productivity level at the moment t^*_i they produce the idea, denoted a_{i,h^*,t^*_i}. Innovation for any phase h is complete and halted as soon as the firm achieves $T_h \leq 1$, i.e, close to the tempo. Letting $\zeta \in (0,1)$ control the step size of innovations, we define:

$$T_{h^*_i,t} = (1 - \zeta \cdot \tilde{\alpha}_{i,h^*})\, T_{h^*_i,t-1}, \quad \tilde{\alpha}_{i,h^*} \sim \mathcal{U}\left(0,\ a_{i,h^*,t^*_i}\right)$$

Each process innovation introduces some level of disruption. As a result, workers must adapt to new features, which temporarily reduces their productivity. We update the productivity levels of workers on the targeted phase, with the reduction proportional to the degree of change and the forgetting threshold:

$$a_{i,h^*,t} = \max\left\{a_0,\ \left(1 - \frac{\theta_a\, |\Delta T_{h^*,t}|}{T_{h^*,t-1}}\right) a_{i,h^*,t}\right\} \tag{27}$$

Management and Implementation The firm considers implementing an idea if the worker's productivity for the targeted task exceeds a threshold $\underline{a} \geq a_s$. Thus, an idea is added to the pile of possible innovation if:

$$a_{i,h^*,t} > \underline{a} \tag{28}$$

This pile of possible innovation serves as the firm's memory for potential innovations. However, this memory is cleared if ideas are assumed to be interdependent and tied to the current state of the production process. In this case, when an innovation is implemented in a phase, all pending ideas for that phase are discarded to avoid conflicting modifications. We assume that such interdependences exist.

Every τ periods, if no R&D activity is ongoing, the firm reviews the stack and selects the next innovation to implement. The selection is based on the idea submitted by the worker with the highest productivity at the time of submission. The chosen innovation is then forwarded to the R&D department, which requires time to develop it. Once developed, the innovation is implemented in the next planning period, allowing the process to be rescheduled and resources reallocated.

The time to build, $B_{h^*,t}$, the innovation is assumed increasing in the innovation's impact: the more significant the innovation the longer it takes to develop. Besides, $B_{h^*,t}$ is also a function of T_{h^*}, again representing the struggle to improve efficient technologies. We propose:

$$B_{h^*,t} = \beta \cdot \frac{|\Delta T_{h^*}|}{\left(T_{h^*,t-1}\right)^2} \tag{29}$$

4.2 Configuration

Unless stated otherwise, as part of the sensitivity analysis, the parameters related to the innovation process are set as follows:

- We run 50 Monte Carlo simulations per configuration.
- $g = 10000$ and $\kappa = 0.002$.
- $\zeta = 0.1$.
- $\underline{a} = 0.2$.
- $\beta = 10000$.

4.3 Experiment 4: Idleness-driven Innovation

In order to isolate the effect of process innovation, we set $\gamma_a = 0$ and $\theta_a = 0$, neutralizing both learning and forgetting. This also implies that innovations do not affect the workers' productivity. We choose to focus this experiment on the effect of both the frequency and the impact of ideas through the settings of $g \in \{1000, 10000, 100000\}$ and $\zeta \in \{0, 0.01, 0.05, 0.1, 0.5\}$.

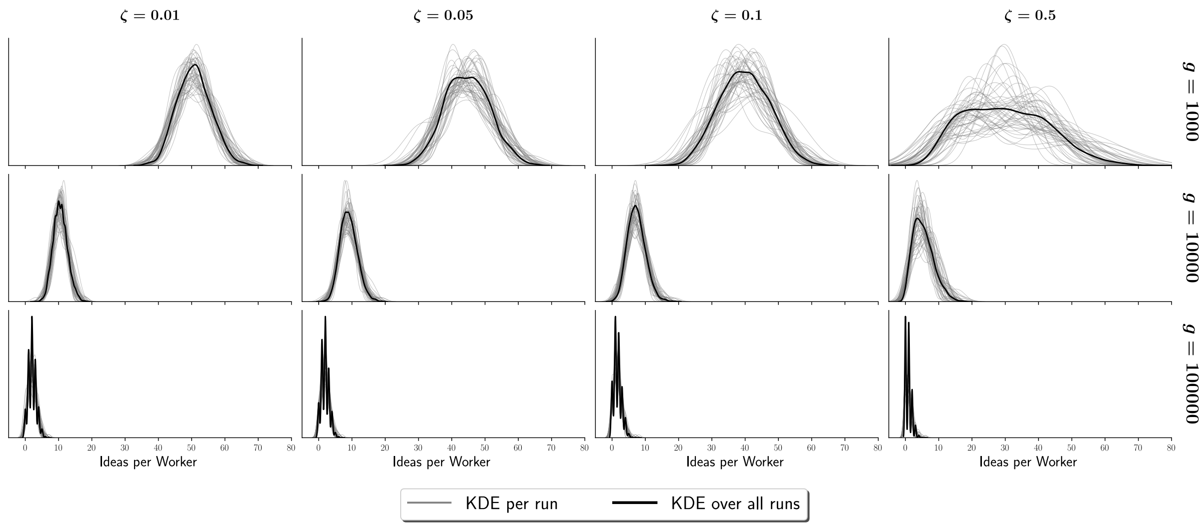

Figure 10: Ideas Distribution across Innovation Regimes. The Kernel Density Estimates (KDEs) are plotted for the 50 Monte Carlo runs. The KDE computed over all runs is shown in bold. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The scales and the densities are not differentiates so as the keep the eye's attention on the distribution shapes.

Figure 10 depicts the distribution of idea generation across workers. Lower values of g imply a higher probability of generating ideas. This mechanically increases the number of ideas per worker. A jump in the potential improvements induced by the new ideas has the opposite effect and seems to make the distribution more uniform. On the one hand, this can be explained by the fact that the larger the step size, the fewer ideas are required to reach the most efficient technology (lowest duration) of a given phase. On the other hand, larger improvements also induce a decrease in the probability of generating new ideas. Both mechanisms together limit the number of ideas when exacerbated by higher jumps in the output of the idea-generation process.

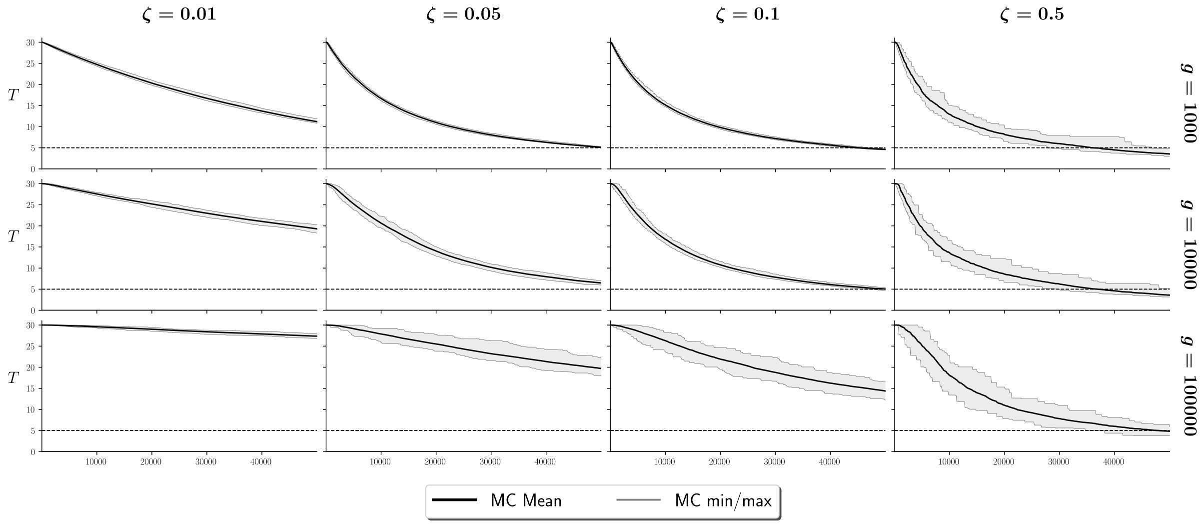

Figure 11: Innovation Pace across Innovation Regimes. Is represented the evolution over time of the total duration $T = \sum_h T_h$ through the Monte Carlo mean, minimum, and maximum values. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The y-axis is shared across all subplots and a horizontal line is drawn at $T = 5$, approximately corresponding to the minimum attainable duration.

Note that ideas do not necessarily lead to an innovation. Figure 11 presents the evolution of the total duration T of the production process. Even though both frequency and step size have an expected accelerating effect on the innovation pace, the difference between $g = 1000$ and $g = 10000$ is not significant. When the frequency of ideas is high enough, the firm is constantly developing upgrades and thus the ideas pile up too fast. Note, moreover, that a higher jump ζ is associated with a wider Monte Carlo min-max range. This extra variability arises from the underlying variability and uncertainty of the impacts of innovation. Furthermore, a higher frequency of ideas, though it does not foster better ideas (productivity is homogeneous), is also associated with lower variability.

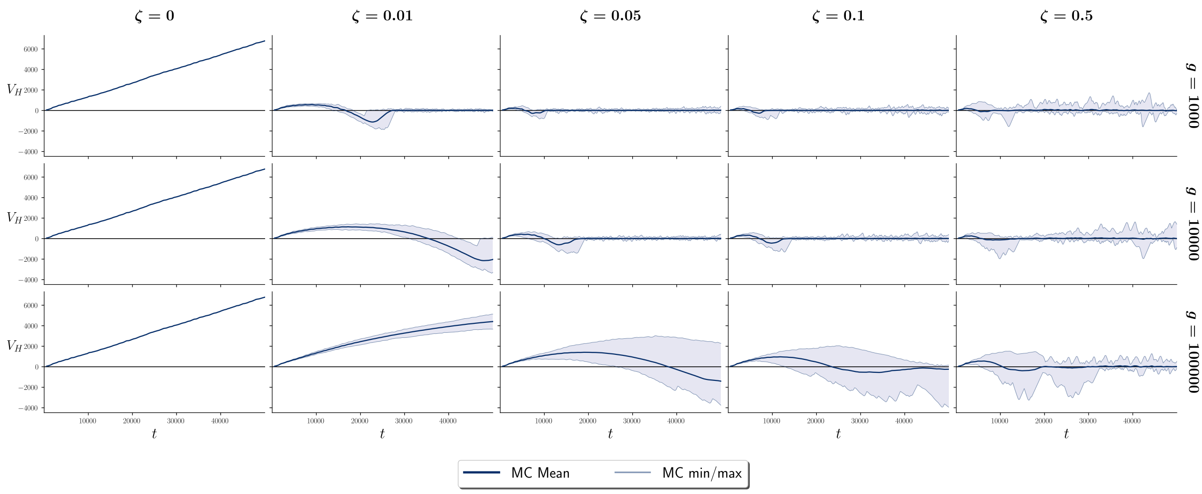

Figure 12: Production Dynamics across Innovation Regimes. The variable V_H smoothed over 100 periods is represented through the Monte Carlo mean, minimum, and maximum values. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The y-axis is shared across all subplots and a horizontal line is drawn at $V_H = 0$.

Similar effects on the accumulated production differential are observed (Figure 12). A larger step size ζ allows for faster convergence but also increases long-term variability. A higher frequency of ideas g entails quicker stabilization, up to a certain point. This effect is particularly visible as ζ increases: a larger step size implies longer development times, so ideas accumulate relatively faster compared to the

R&D capacity. To summarize, more frequent but moderate innovations seem to be the best combination to ensure both a rapid stabilization of the production process and a long-term persistence in its stability (i.e., low variability).

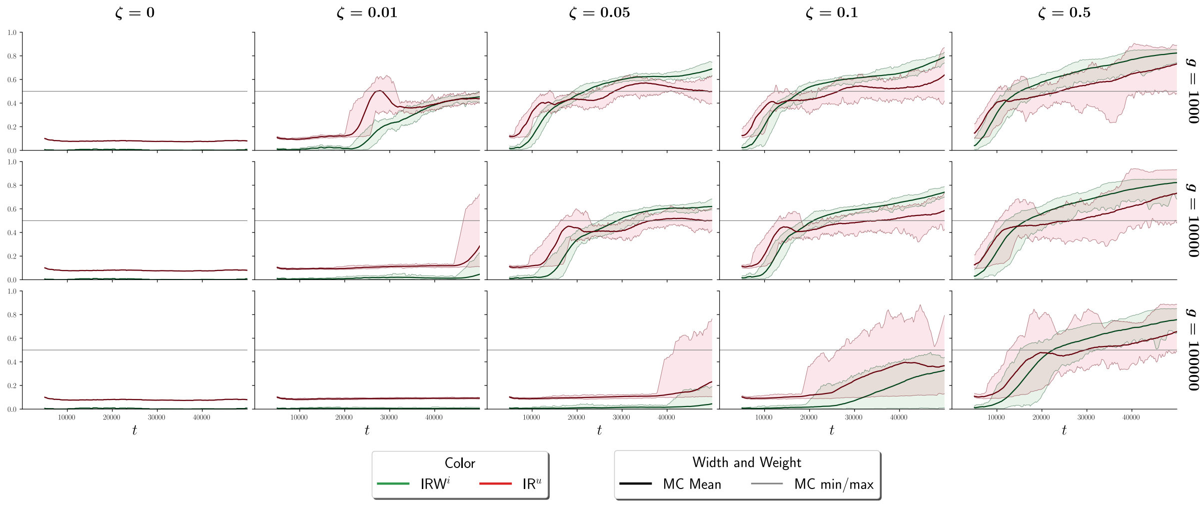

Figure 13: Idleness Rates across Innovation Regimes. The variables IRW^i and IR^u smoothed over 5,000 periods are represented through their Monte Carlo mean, minimum, and maximum values. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The y-axis is shared across all subplots and a horizontal line is drawn at IR= 0.5.

As shown in Figure 13, the intentional idleness rate is increasing as the firm innovates. This result is rather trivial as shorter phase duration and unchanged demand reduce the need for funds (both workforce and capital). Unintentional idleness also increases as innovations are implemented as reflected in the Monte Carlo mean, and its variability, most notably, rises. Since idleness rates are smoothed over 5,000 periods, this variability occurs between runs, reflecting the diversity of innovation trajectories. Variability is also larger for a higher average jump size ζ.

Note that this is intertwined with the variability in production discussed earlier. While both intentional and unintentional idleness grow as the firm innovates and the process converges to a stable regime, it is also apparent that Monte Carlo variability in idleness is connected to the same degree of variability in the production differential. The former phenomenon may result from efficient production leading to periods of excess output, compensated by reactive idleness. The latter, however, is less trivial. This emerging variability may be related to shifts in the temporal structures, which, as shown in the previous section, can yield highly divergent dynamics. Some trajectories of process duration may indeed be more beneficial for the firm than others.

4.4 Experiment 5: Innovation and Learning

Now that we have a sense of the effects of idleness-driven innovation, we can explore their interaction with workers' learning mechanisms. More formally, we allow the forgetting threshold θ_a to vary; it controls the differentiation in the learning path of workers, as observed in the dedicated experiment, and affects the impact of innovation on workers' productivity. The learning rate γ_a is again set to 0.001. On the idleness-driven innovation side we aim to include both the frequency of ideas and their impact. However, these two parameters are tightly related; for instance, a longer average maturation period for ideas is likely to yield more disruptive process innovations. Therefore, instead of letting both parameters vary independently, we select three scenarios: $(g, \zeta) \in \{(1000, 0.01), (10000, 0.1), (100000, 0.5)\}$, respectively referred to as the 'fast-paced', 'intermediate', and 'disruptive' scenarios (rows 1, 2, and 3 in the graphs). Another advantage of treating these parameters as entangled is that the scenarios can be considered chosen by the firm. On the one hand, the fast-paced scenario can be interpreted as a situation in which the firm encourages workers to submit as many incremental ideas as possible. On the other hand, the disruptive scenario would correspond to a firm that advises workers to submit only elaborated, mature ideas. Finally, we differentiate between two managerial approaches: the firm either allows all ideas to accumulate on the stack ($\underline{a} = 0$) or filters out ideas from workers who have started to regress ($\underline{a} = 0.95$), referred to as the 'loose' and 'tight' policies, respectively.

The effect of the forgetting threshold θ_a on the pace of innovation is barely noticeable under the loose policy (Figure 14). However, as the forgetting threshold grows, the gap between the duration dynamics

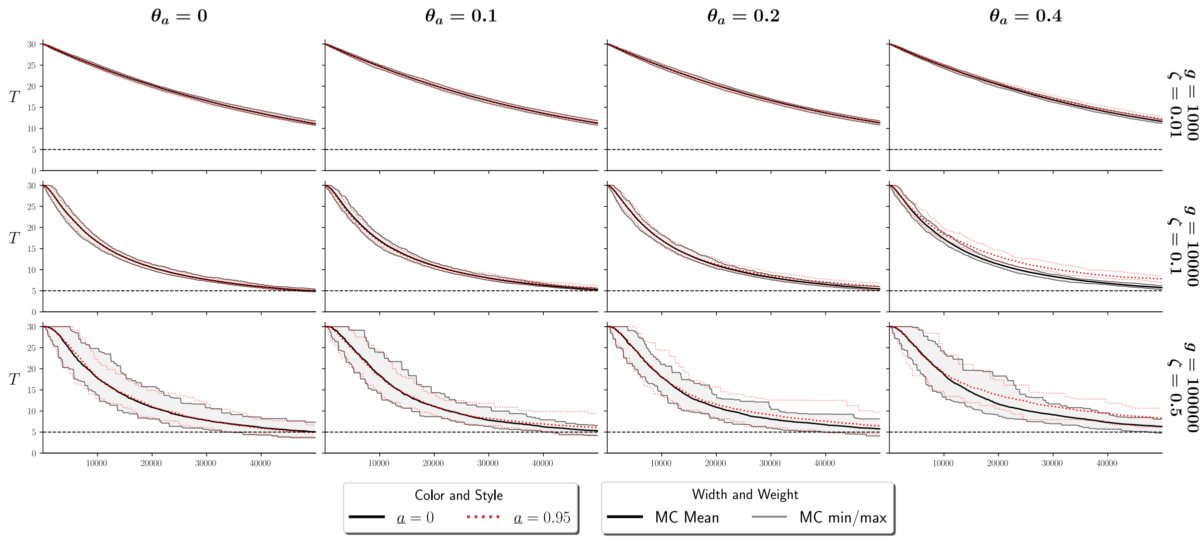

Figure 14: Innovation Pace across Innovation and Forgetting Scenarios. Is represented the evolution over time of the total duration $T = \sum_h T_h$ through the Monte Carlo mean, minimum, and maximum values. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The two stack policies are differentiate in color and line style. The y-axis is shared across all subplots and a horizontal line is drawn at $T = 5$, approximately corresponding to the minimum attainable duration.

under the two policies widens. Specifically, under the tight policy, innovation visibly slows down as forgetting becomes more pronounced, and this effect is especially noticeable when ζ is larger. Based on the innovation pace alone, the intermediate scenario seems, nonetheless, the most desirable, as it fosters quick innovation with minimal variability, and both policies appear suitable.

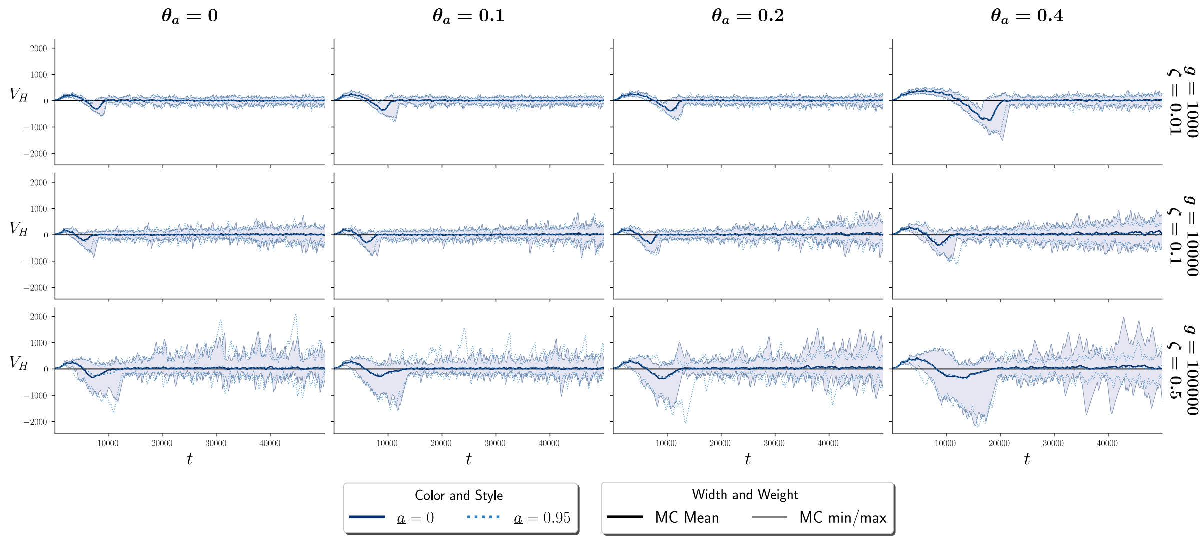

Figure 15: Production Dynamics across Innovation and Forgetting Scenarios. The variable V_H smoothed over 100 periods is represented through the Monte Carlo mean, minimum, and maximum values. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The two stack policies are differentiate in line style. The y-axis is shared across all subplots and a horizontal line is drawn at $V_H = 0$.

Similar observations hold for production dynamics. Examining the cumulative production differential in Figure 15, it becomes evident that a higher forgetting threshold slows the convergence to a stable regime and increases Monte Carlo variability. Variability is also amplified when the innovation step size is larger. Once again, more forgetting and larger innovations highlight the effects of a tight policy. Although the Monte Carlo mean is barely affected, strict filtering dampens variability in these settings. This suggests that, in the presence of more disruptive innovations, the firm should prioritize the most promising ideas and filter out the rest, leading to fewer innovations and, consequently, fewer unnecessary shocks to the process. The impact of the tight policy is more visible when forgetting is higher because lower-quality ideas, which are responsible for these unnecessary shocks, would be more frequent.

In the previous section, we observed that forgetting keeps unintentional innovation low in the long

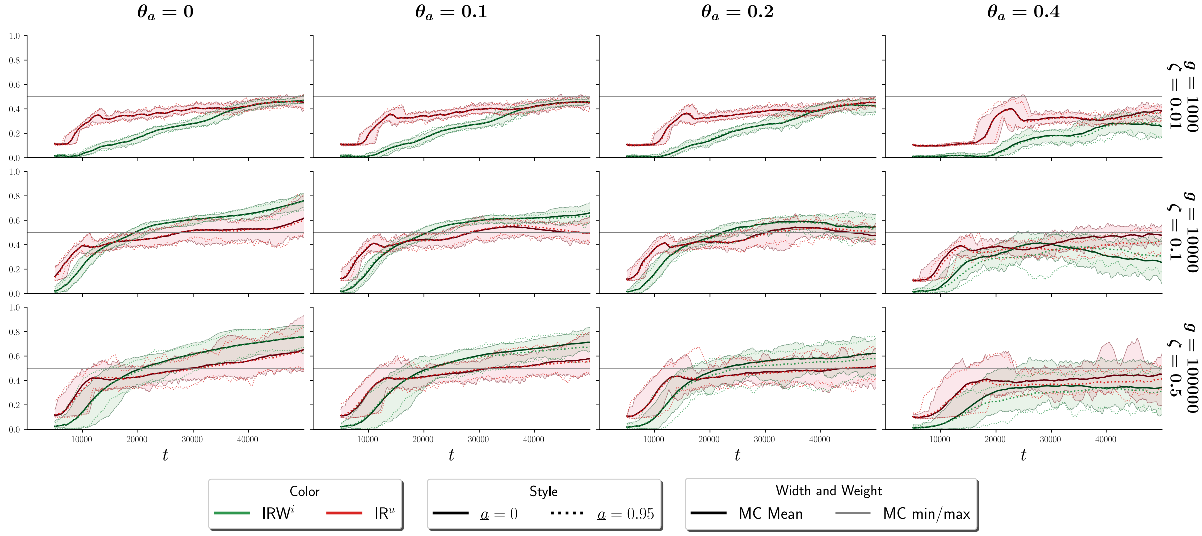

Figure 16: Idleness Rates across Innovation and Forgetting Scenarios. The variables IRW^i and IR^u smoothed over 5,000 periods are represented through their Monte Carlo mean, minimum, and maximum values. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The two stack policies are differentiate in line style. The y-axis is shared across all subplots and a horizontal line is drawn at IR= 0.5.

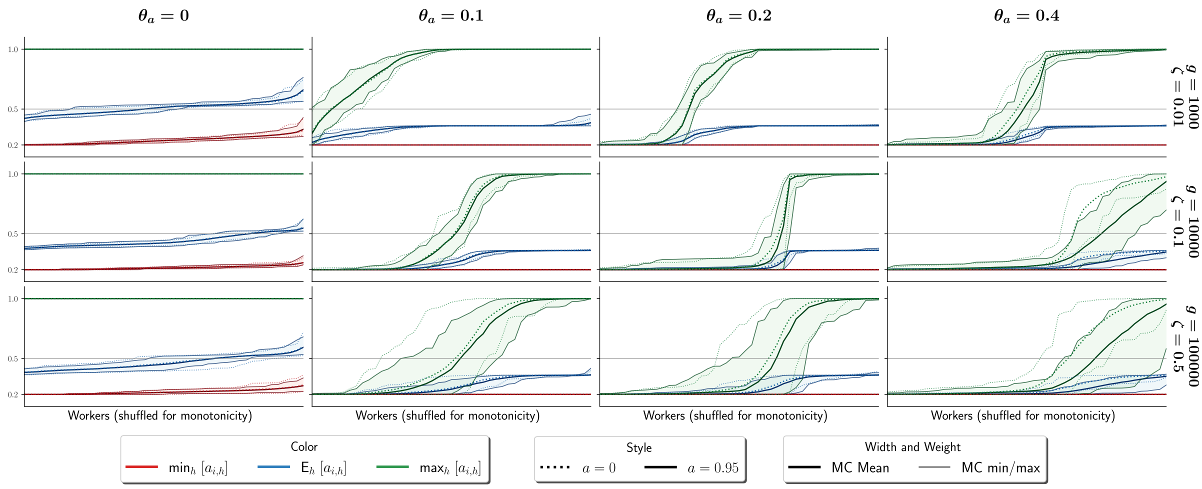

Figure 17: Workers' Specialization across Innovation and Forgetting Scenarios. At the last period we compute the mean, minimum, and maximum of each worker's productivity levels. The resulting series (along workers) are independently sorted and we then graph their Monte Carlo mean, minimum, and maximum values. The graphed series inform on the distribution of productivity levels across workers. The step-size parameter ζ varies in columns and the inverse frequency of ideas g varies in rows. The two stack policies are differentiate in line style. The y-axis is shared across all subplots and a horizontal line is drawn at $a = 0.5$.

run and has little impact on the proportion of workers allocated. Furthermore, innovation without forgetting increased both intentional and unintentional idleness rates. The outcomes with both learning-and-forgetting and innovation are thus quite surprising. Both intentional and unintentional idleness rates seem to be lower in the long run as the forgetting threshold increases. In Figure 16, we can even observe cases where average idleness rates decrease over somewhat extended periods. A straightforward explanation is the effect of θ_a on workers' productivity. Each time an innovation is implemented, workers experience a slight drop in productivity, requiring them to adapt. Consequently, the firm may allocate more workers to the improved phase shortly after the innovation, even though the duration of this phase is reduced. Moreover, skill forgetting due to temporary reinforcements and long-run intentional idleness offsets the increase in unintentional idleness induced by time-saving innovations, resulting in more stable rates.

These results are especially interesting in that the pace of innovation is sensibly the same across values of θ_a. Moreover, we observe a higher variability as forgetting becomes stronger. As in the experiment with innovation only, this variability in idleness rates is congruent with the variability in the production differential, tightening up the link between the two. Finally, the effect of a tight policy is positive for the firm and again most visible in the disruptive scenario and with high forgetting: the long run idleness rates are lowered and the additional variability is dampened.

Examining the portfolios of skills (Figure 17) provides further insight. As expected, and consistent with prior observations, higher forgetting leads to less-skilled workers at the end of the simulations. More interestingly, the disruptive effect of innovation shows a similar, albeit smaller, effect. The same patterns of variability and the benefits of a tight policy are present. Notably, the tight policy results in a more skilled workforce at the end, which aligns with the lower idleness rates.

In a production process where workers contribute to process improvements, there is an inherent feedback loop: idleness fosters innovation, and innovations enhance efficiency, potentially leading to more idleness. More idle workers, however, may experience a drop in productivity, increasing the likelihood of generating ideas of a lower quality. This risk appears higher in the most disruptive settings and when forgetting is more pronounced. These results suggest that if the firm stresses the production process by adopting innovations, it should do so at a moderate frequency, focusing on the most promising ideas. This approach would allow workers to adapt, reducing uncertainty and instability.

Figure 17 shows that a significant proportion of workers remain low-skilled by the end of the simulation. This outcome arises for some of the workers who are not allocated for extended periods of time following the adoption of innovations. As this fosters forgetting, they are left out of the process, losing their skills, and thus generate less relevant, outdated ideas. Under a tight policy, these workers are excluded from the innovation process as well, resulting in a high proportion of unskilled workers at the end of the simulation who are indeed useless to the firm.

This phenomenon reveals an important flaw in the proposed setup and raises questions about the feasibility of the proposed idleness management strategy. To some extent, workers may be *cursed* because, although they remain employed for a time, their contribution to innovation may ultimately render their service obsolete, both in terms of their productivity and their creativity potential. If workers generate low-quality ideas (loose policy) or fail to produce good-enough ideas (tight policy), the firm in fact has more incentives to lay them off.

5 Discussion

Summary To reconsider the role of idleness in the context of the co-evolution of production and innovation mechanisms in the firm, we proposed an ABM of the production process, building on the fund-flow approach along the lines of NGR augmented by productivity dynamics and indivisibility in line with the Schumpeterian and evolutionary literature on firm and innovation dynamics. The firm arranges the production system in-line with the goal of mitigating the idleness of production factors. First, we examined the effects of various organizational choices, and the results indicated that the involvement and reactivity of managerial resources play a significant role. Production converges more rapidly to a stable, low-volatility regime when funds are reallocated frequently, machinery is maintained at a high productivity level, and production objectives are augmented (pro-activity). These increased degree of reactivity and pro-activity, however, also lead to higher rates of idleness in the long run. A trade-off emerges between the speed of stabilization of the production process and the efficiency of the fund utilization.

Next, we explored the differences in production dynamics across various temporal structures of the system. A significant degree of heterogeneity has been observed, which could be associated with the initial degree of idleness, prior to line arrangement, the organizational flexibility, as well as the duration of the production process.

We also investigated the role of workers' learning dynamics. The impact on production is straightforward: hiring more proficient workers and increasing the learning rate both positively influence production, while forgetting has negative effects. Additionally, the diversity of skills may play a crucial role: certain phases of the production process may experience slowdowns, which can be mitigated by reallocating workers to reinforce those phases, even if they were not originally hired for that task. While swiftly addressing slowdowns becomes key to performance, it follows that a versatile workforce capable of reinforcing other phases turns out to be an essential asset. Therefore, specialization should not be viewed as a binary switch but as a spectrum with potential pitfalls at both extremes: while a highly-skilled workforce that retains its expertise is important, it must be balanced with a diversified skillset.

We then shifted our focus to idleness-driven innovation within the firm, which we present as an idleness management strategy with two objectives: leveraging workers' creativity to discover time-saving process innovations and maintaining employment levels by avoiding layoffs when fewer workers are needed to run the process efficiently. To isolate the effects of innovation, we first disabled workers' learning mechanisms. We observed that more frequent but moderate innovations provide the best combination for ensuring both a fast stabilization of the production process and the long-term stability of it. Innovation, however, also leads to increased idleness, as increasing the efficiency of the production process reduces the need for workers. Additionally, the Monte Carlo variability in both production and idleness dynamics is strongly related to the size of the average innovation jump. Larger innovations (i.e., larger gains in the time-saving these allow) result in greater uncertainty in duration trajectories and more heterogeneous temporal structures, which in turn influence the efficiency of the production process.

Next, we analysed the combined effects of individual productivity dynamics, through learning and forgetting, and the idleness-driven innovation process, distinguishing between (i) the firm's strategy regarding the acceptance of creative ideas (tight or loose) and (ii) three scenarios characterizing the process of idea generation in terms of frequency and impact (fast-paced, intermediate, and disruptive). A recurring observation across these experiments is that both stronger forgetting effects and larger innovation jumps induce greater variability in the dynamics. In such cases, firms are better of in adopting a tight policy, filtering out all but the most promising ideas. This aligns with the hypothesis that larger innovations have more disruptive effects on the production system, requiring workers to adapt to new technologies, affecting the temporal structure, and possibly necessitating a rearrangement of the process. Thus, less frequent but more impactful innovations are preferable, and this is all the more important when the decay in skills is high given that ideas are then less likely to be very relevant.

Interestingly, idleness does not increase as much when individual productivity dynamics through learning and forgetting are included alongside the innovation process. This is likely because innovations reduce workers' productivity temporarily, prompting the firm to allocate more funds after each implementation. This lowers intentional idleness, whereas unintentional idleness also decreases or remains stable due to skill decay. However, this strategy presents a potential drawback: as the process becomes more efficient, some workers are excluded, leading to the decay of their skillset and turning their ideas less valuable for the firm. These workers may be sidelined from both the production and innovation processes, or at least, their ideas only accepted under a loosen strategy of ideas adoption by the firm, albeit at the cost of greater uncertainty and instability for the firm.

Future Research Addressing this issue is essential to ensure that both workers and the firm benefit from idleness-driven innovation. We propose two potential solutions. First, introducing a turnover mechanism could maintain regular activity for more workers. For example, instead of consistently assigning the most productive workers first, the firm could periodically assign slightly less productive workers to ensure they consistently practice without significantly disrupting production. Second, dedicating R&D to product innovation could provide a solution. Firms often diversify their products to maintain or increase market share, sometimes requiring additional production capacity. In this case, workers could be reassigned to new tasks or apply their skills to a new product line. This approach, combined with idleness-driven process innovations, could benefit the firm while keeping workers engaged in both productive and innovative tasks. To fully account for competition and the outcomes of product innovation, these scenarios could be integrated into an explicit market model, as previously suggested. Again, different innovation strategies could be compared within this context.

Besides, we identify several avenues for future research building up on the proposed framework.

First, economic models are most effective when calibrated to empirical data. With a preliminary understanding of the model's behavior, several parameters—particularly those not easily adjustable by the firm—should be set to empirically estimated values. For instance, this would assist in calibrating the forgetting threshold θ_a, the minimum productivity a_u, and the innovation step size ζ, which are difficult to estimate. Similarly, parameters such as demand μ_d and duration $\mathbf{T_h}$ could be tailored to specific real-world cases. In contrast, computational calibration could be employed for the remaining parameters. Additionally, empirical validation of the model is necessary to assess how well the model reflects *stylized facts*[24], i.e., how realistic the emergent dynamics are. Empirical validation would also help identify flawed or missing mechanics, thereby improving the model.

Another significant avenue for development involves relaxing the monopolist and constant demand assumptions. Most firms operate in competitive industries, where demand dynamics are endogenously determined. Initial approaches could include (i) exogenous demand shocks and complex dynamics, or (ii) allowing demand to react endogenously to the firm's ability to meet demand and deliver promptly. Further, the market could be modeled explicitly by incorporating multiple firms into a single market. A logical extension would be to study heterogeneous markets with firms that vary in their management strategies or in their initial temporal structures (which represent their technologies).

The substantial effects of temporal structures can also be analyzed within this agent-based market

[24] Micro-economic facts are less likely stylized and more likely highly heterogeneous. Yet some common factory characteristics could be used to validate and calibrate the model.

framework, although an ad-hoc analysis based on the single-firm model presented here may be more suitable as a first step. The variability in temporal structures produced some of the most unexpected results, and identifying the determinants of convergence speed could (i) enhance our understanding of the heterogeneity in firm performance and (ii) guide less arbitrary decisions regarding process innovation by identifying the most promising directions within the duration space.

Regarding innovation, we might want to extend the disruptive effect of ideas. Although processed independently, production phases are likely more interdependent than what has been assumed in this paper. And this implies that innovations targeting one phase might also affect others. For example, the execution of one task h may depend on the quality of the execution of the preceding phases $h-1, h-2, \ldots$. Time-saving innovations do not necessarily imply better processing. Or it may be that multiple phases are connected with common equipment or software, that, when modified for the enhancement of one phase will also affect the others. It follows that workers with limited knowledge of the entire process may inadvertently cause negative disruptions. In this context, a diverse skill set may prove more advantageous than specialization. While specialization is beneficial for production, broader skills may be more useful for innovation, which ultimately enhances productivity. In these settings, management strategies such as a well-calibrated turnover of the workers may prove helpful in fostering balanced portfolios of skills that ensure both stable production and high-quality process innovations.

Finally, embedding firms represented by disaggregated processes into macroeconomic models offers another promising direction. For example, supply chains are known for their instability, and micro-founding supply chain models could provide insight into how instabilities emerge and propagate. A key objective could be identifying the firm-level instabilities that are most disruptive and long-lasting within the chain.

Overall, we hope that this model, or refined versions of it, will contribute to a deeper understanding of the inefficiencies and instabilities in production at both firm and market levels, and ultimately shed light on how these factors impact our societies both economically and socially.

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Code Availability Simulations have been modeled and run using LSD (Pereira and Valente, 2023), an open free software for ABM development (GNU General Public License, Copyright Marcelo Pereira and Marco Valente). The code of the simulation model and the analyses are available upon request from the referees and the readers.

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