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Renewable resource and harvesting cost in a simple monetary overlapping generation economy

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Abstract

The literature has pointed out that renewable resource preservation in the long run is ensured in two situations: (1) when resource can serve as a store of value and (2) when households are altruistic. The following paper shows that a third situation also implies resource preservation, that is, when harvesting is costly. To do so, a simple monetary overlapping generation economy is developed in which a renewable resource is privately owned by a representative household who lives for two periods. During her youth, she inherits the resource from her parents and decides how much to harvest. Income obtained from the resource selling is fully saved in fiat money to finance old age consumption. Harvesting is assumed to take time (harvesting cost) and then, the young household has to arbitrate between leisure or harvesting. In this simple context, we show that the resource level can be non zero in the long run (resource preservation). In particular, we observe that two steady states can coexist: one with a low resource level (overexploitation) and the other with a high resource level (underexploitation). Moreover, under a sufficiently productive harvesting technology, two-period cycles can emerge around each steady state through a flip bifurcation as well as local indeterminacy around the higher steady state. In this sense, costly harvesting appears to be both a source of resource preservation and a source of resource fluctuations.

JEL Classification: E32, O44, Q20.

Keywords: Flip bifurcation, local indeterminacy, overlapping generation model, renewable resource.

1 Introduction

Management of renewable resources is a major concern for mankind. History gives examples of ancient civilizations who have collapsed because of full resource depletion. In one of his seminal book, Diamond (2005) points out, for instance, that one of the main explanation for the collapse of the ancient Easter Island civilization is due to a massive deforestation. As reported by Diamond (2005), the complete forest depletion has led to a loss of raw materials, to a loss of wild-caught foods and to a decrease of crop yields, rendering the island unsuitable for human life. If natural resources appear to be essential for production of goods, it is fundamental to manage them in order to avoid their complete depletion.

From a theoretical ground, the economic literature has tried to shed in light the drivers for renewable resources preservation, in particular within the overlapping generation model (OLG model hereafter). To the best of our knowledge, the first attempt to discuss renewable resource dynamics in an OLG framework is Kemp and Van Long (1979). They consider an economy without physical capital and where saving occurs through the resource who serves as the unique store of value. More precisely, the resource stock is sold by the old agent to the young one in order to finance her consumption. The young household uses her labour force to produce both the consumption good and the resource[1]. Concerning the resource, the young household has to decide how much to use as an input of production and how much to save to finance her old age consumption. This particular form of saving implies that the resource is never fully used as a productive input and hence, is preserved in the long run. Koskela et al. (2002) have also pointed out that a renewable resource is not fully depleted in the long run when it serves as a store of value. They also consider a simple OLG economy without capital and in which both labour and resource are inputs of production. Two major differences have to be noted with respect to Kemp and Van Long (1979). First of all, to finance her consumption, the old agent sells the resource stock to the representative firm who decides how much to extract and use to produce the consumption good. After that, the firm sells the remaining resource stock to the young who saves it in order to sell it when old to finance her consumption and so on. Beside this transfer of resource property between households and the firm, Koskela et al. (2002) consider an autonomous growth for the renewable resource[2]. Because of the bell shape of the resource growth function, the economy possesses two steady states: one with a low resource level and the other possesses a high resource level. Analyzing the dynamics, Koskela et al. (2002) point out that those two steady states have opposite stability properties: one is stable while the other is unstable. In a very similar framework but with more general preferences[3], Koskela et al. (2008) show that a low value of intertemporal elasticity of consumption leads to complex dynamics (two-period cycles through a flip bifurcation as well as local indeterminacy).

The second driver for renewable resource preservation pointed out in the OLG literature is altruism. In particular, Bréchet and Lambrecht (2011) have considered an OLG economy with capital accumulation in which the (young agent) household inherits the renewable resource stock from her parents (old

[1]In Kemp and Van Long (1979), the resource is renewable because agents are able to produce it.

[2]Conversely to Kemp and Van Long (1979), human intervention is not needed for resource growth in Koskela et al. (2002).

[3]Indeed, in Koskela et al. (2002), the lifetime utility function is separable and quasi-linear. Koskela et al. (2008) rather consider a concave (separable) utility function.

agent) and decides how much to harvest and to sell to the firm. The particularity of their approach is to consider that the lifetime household's utility depends not only on her own consumption but also on the amount of resource bequeted to her children (altruism). This is a transposition in the renewable resource context of the so called joy-of-giving bequest motive introduced by Andreoni (1989). They show that when resource and capital are poor substitutes to produce, then a strong altruism is necessary to preserve the resource in the long. More recently, Desmarchelier and Mayol (2022) have reconsidered the joy-of-giving bequest motive concerning the resource. As in Bréchet and Lambrecht (2011), they assume that the young representative household has the possibility to harvest and to sell this harvesting to a firm. However, they differ from Bréchet and Lambrecht (2011) by considering that the young household can use her time to contribute to the resource growth (forestry) as in Kemp and Van Long (1979). Concerning the long run, they observe that altruism always ensures the natural resource preservation. By analyzing the dynamics, they point out that under a high forestry productivity, complex dynamics can arise (two-period cycles through a flip bifurcation) as in Koskela et al. (2008).

A third driver for renewable resource preservation could be harvesting costs. Indeed, previous mentioned contributions assume a costless harvesting. However, intuitively, a cost to harvest will eventually moderates harvesting and then, promotes resource preservation in the long run. As reported by Bednar-Friedl and Farmer (2013), general equilibrium literature has largely ignored those costs and to the best of our knowledge, one of the only attempt to introduce those costs in an OLG context is precisely Bednar-Friedl and Farmer (2013). In their paper, they consider both physical capital and resource as stores of value to finance old age consumption. In particular, when old, the agent sells the resource to the young agent. During youth, the household harvests the resource and sells this harvesting to the firm who uses it as an input of production. Interestingly harvesting activities take time (harvesting cost) and then, the young agent has to decide how to allocate time between working in the firm or in harvesting. Concerning the long run, Bednar-Friedl and Farmer (2013) point out the existence of a unique (non-trivial) steady state which is saddle-path stable, avoiding the possibility of complex dynamics. Similar conclusions are also observed by Bednar-Friedl and Farmer (2014) when harvesting cost is resource dependent.

By considering at the same time harvesting cost and renewable resource as a store of value, it is not possible to fully observe the role of harvesting cost in the resource preservation in Bednar-Friedl and Farmer (2013, 2014) since, as discussed before, considering the resource as a store of value is sufficient to prevent from a complete resource depletion in the long run (Kemp and Van Long, 1979, Koskela et al. 2002 and 2008). That is, the present paper proposes to develop a simple OLG model is order to fill this gap and fully observe the way harvesting costs allow to preserve the renewable resource in the long run.

In order to discuss the ability of harvesting costs to preserve a renewable resource in the long run, the following paper develops a simple monetary OLG economy in which fiat money is the only store of value. The renewable resource is the property of households and is transmitted freely from generation to generation as in Bréchet and Lambrecht (2011) but without any altruism. The only reason for which the first generation does not deplete the entire resource stock is because of harvesting cost. As in Bednar-Friedl and Farmer (2013), harvesting takes time, but differently from them, the young household has not to arbitrate between working in the production sector or harvesting but between leisure and harvesting. More precisely, the only source of income for the young household is to harvest the resource and to sell this harvesting to the firm. However, the more is the harvesting time, the lower is the household's utility (labour disutility). The whole income obtained from the resource selling is invested in fiat money to finance old age consumption. By considering a logistic growth function for the natural resource, it appears that two steady states can coexist in the long run: one with a low resource level (overexploitation) and the other with a high resource level (underexploitation). This result clearly points out that the sole introduction of harvesting cost is sufficient for long run resource preservation. Analyzing the dynamics, we show that a sufficiently productive harvesting technology is able to generate two-period cycles around each steady state through a flip bifurcation as well as local indeterminacy around the higher steady state. In this sense, the dynamics is closely related to the one observed by Koskela et al. (2008).

The rest of this paper is organized as follow. Section 2 presents the model. The intertemporal equilibrium and the steady states existence are discussed through a third section. Section 4 analyzes the local dynamics and finally, section 5 provides numerical simulations while section 6 concludes the paper.

2 The model

2.1 Resource dynamics

Let z_t be the stock of a renewable resource at date t and assume the following evolution:

$$z_{t+1} = z_t + g\left(z_t\right) - h_t$$

where $g\left(z_t\right)$ represents the reproduction function of the renewable resource while h_t depicts the harvesting level at date t. The initial resource level is given by $z_0 > 0$. To keep things as simple as possible, we assume that $g\left(z_t\right)$ is logistic[4], that is:

$$g\left(z_t\right) = z_t\left(1 - z_t\right) \tag{1}$$

By definition, $g\left(z_t\right) \geq 0$, implying that $z_t \in (0, 1)$. The value of z_t which maximizes $g\left(z_t\right)$ is known as the Maximal Sustainable Yield (MSY hereafter) in the literature[5]. Interestingly,

[4] This functional form is commonly used in the renewable resource literature, see for instence Koskela et al. (2002), Wirl (2004) or Desmarchelier and Mayol (2022).

[5] See Koskela et al. (2002) among others.

$$g'(z_t) = 1 - 2z_t$$

and then, the MSY is reached if and only if $z_t = \hat{z} \equiv 1/2$. More precisely, $g'(z_t) > (<)\, 0$ if and only if $z_t < (>)\, 1/2$. That is, the renewable resource is said to be overexploited if $z_t < 1/2$ and underexploited if $z_t > 1/2$. Interestingly, $g(\hat{z}) = 1/4$.

2.2 Household

The representative household is assumed to live two periods: youth and old age. During her youth, the representative household born at date t inherits the resource stock z_t from her parents and she decides how much to harvest h_t. She sells the harvesting to the production sector at a price q_t and saves the earned income in fiat money m_t. During her old age, she uses the fiat money saved to consume c_{t+1}. The budget constraints for youth and old age are then simply given by:

$$q_t h_t = m_t \tag{2}$$

$$m_t = p_{t+1} c_{t+1} \tag{3}$$

Where p_{t+1} represents the price of the consumption good at date $t+1$. Combining (2) and (3) gives the life cycle budget constraint of the household born at date t:

$$c_{t+1} = \frac{q_t}{p_{t+1}} h_t \tag{4}$$

Harvesting is assumed to be time consuming, namely,

$$h_t = \varphi(l_t) \tag{5}$$

where l_t represents the time devoted to harvest and φ captures the harvesting technology with positive and decreasing marginal productivity. The following assumption sums up its properties.

Assumption 1 *Function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is C^2 such that $\varphi''(l) < 0 < \varphi'(l)$. Additional boundary conditions hold: $\varphi(0) = 0$, $\lim_{l \rightarrow 0} \varphi'(l) = +\infty$ and $\lim_{l \rightarrow +\infty} \varphi'(l) = 0$.*

For further reference, let us introduce the following elasticities:

$$\gamma \equiv \frac{l\varphi'(l)}{\varphi(l)} > 0 \text{ and } \mu \equiv \frac{l\varphi''(l)}{\varphi'(l)} < 0$$

Let $\hat{l} > 0$ be the labour endowment. As in Grandmont et al. (1998), $\hat{l}$ may be finite or infinite and $\hat{l} - l$ represents the leisure time. Household's preferences are rationalized by a separable felicity function U:

$$U(c_{t+1}, l_t) = u(c_{t+1}) - v(l_t) \tag{6}$$

$u(c_{t+1})$ and $v(l_t)$ represents respectively the utility from consumption and the disutility of labour. Assumption 2 sums up properties of both u and v.

Assumption 2 *Functions $u, v : \mathbb{R}_+ \to \mathbb{R}$ are C^2 such that $u''(c) < 0 < u'(c)$ while $v'(l) > 0$ and $v''(l) > 0$. Additional boundary conditions hold: $\lim_{c\to 0} u'(c) = \lim_{l\to+\infty} v'(l) = +\infty$, $\lim_{c\to+\infty} u'(c) = \lim_{l\to 0} v'(l) = 0$.*

Considering (5) and assumption 1, it follows that, for a given consumption level, the more time is spent in harvesting, the lower is the utility level. In this sense, harvesting is costly for the representative household. To the best of our knowledge, the only other contribution introducing a costly harvesting in an OLG context is Bednar-Friedl and Farmer (2013). In their framework, the young household has to choose between working in the production sector or harvesting. In the present paper, the trade-off rather concerns harvesting and leisure.

As usual in an OLG economy, the representative household born at date t chooses the time devoted to harvest l_t in order to maximize the intertemporal utility (6) subject to the intertemporal budget constraint (4). First order condition gives:

$$u'(c_{t+1}) \frac{q_t}{p_{t+1}} = \frac{v'(l_t)}{\varphi'(l_t)} \tag{7}$$

(7) implies that the representative household behaves in order to equalize the (relative) marginal cost of harvesting ($v'(l_t)/\varphi'(l_t)$) to the marginal benefit of harvesting[6] ($u'(c_{t+1})$) taking as given prices.

Before going further, let us introduce two elasticities:

$$\sigma \equiv \frac{cu''(c)}{u'(c)} < 0 \text{ and } \omega \equiv \frac{lv''(l)}{v'(l)} > 0$$

2.3 Firm

The production sector consists of a representative firm who behaves competitively. To keep things as simple as possible, we assume that the natural resource is the only productive input. Let H_t and Y_t represent respectively resource demand and the production level at date t. It follows that:

$$Y_t = f(H_t) \tag{8}$$

f is a standard production function with positive and decreasing marginal productivity. Its properties are summarized in the following assumption.

Assumption 3 *Function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is C^2 such that $f''(H) < 0 < f'(H)$. Moreover, $f(0) = 0$ while $\lim_{H\to 0} f'(H) = +\infty$ and $\lim_{H\to+\infty} f'(H) = 0$.*

At each date in time, the representative firm chooses H_t in order to maximize its profit, taking as given prices, namely:

[6] Indeed, considering (5) and (4), a higher l_t increases c_{t+1}.

$$\max_{H_t} p_t f(H_t) - q_t H_t$$

First order condition gives:

$$f'(H_t) = \frac{q_t}{p_t} \tag{9}$$

As usual, (9) means that the amount of resource used by the representative firm is such that the marginal productivity equalizes the relative marginal resource cost. For further reference, let us introduce the following elasticities:

$$\alpha \equiv \frac{Hf'(H)}{f(H)} > 0 \text{ and } \theta \equiv \frac{Hf''(H)}{f'(H)} < 0$$

3 Intertemporal equilibrium and steady state

3.1 Intertemporal equilibrium

This economy is composed by three markets: the money market, the natural resource market and the good market. At the equilibrium, supply is just equal to demand on each market. To keep things as simple as possible, money supply is assumed to be constant over time ($M > 0$). That is, at the equilibrium:

$$m_t = M \tag{10}$$
$$H_t = \varphi(l_t) \tag{11}$$
$$Y_t = c_t \tag{12}$$

Relation (10) means that money demand at date t (i.e. m_t) is just equal to money supply M. Concerning the resource market, (11) implies that demand at date t (i.e. H_t) is just equal to supply at date t (i.e. $\varphi(l_t)$) and finally, relation (12) means that the amount of good produced at date t (namely, Y_t) is fully consumed (c_t). Considering (11), (12) and (8), it follows that:

$$c_t = f(\varphi(l_t)) \tag{13}$$

Considering jointly (3), (10) and (13), we obtain that:

$$M = p_{t+1} f(\varphi(l_{t+1})) \tag{14}$$

Considering jointly (9), (13) and (14), equation (7) becomes:

$$u'(f(\varphi(l_{t+1}))) f(\varphi(l_{t+1})) = \frac{f(\varphi(l_t)) v'(l_t)}{\varphi'(l_t) f'(\varphi(l_t))}$$

The next proposition follows.

Proposition 1 *An intertemporal equilibrium is a non-negative sequence* $(l_t, z_t)_0^{+\infty}$ *satisfying equations:*

$$u'\left(f\left(\varphi\left(l_{t+1}\right)\right)\right) f\left(\varphi\left(l_{t+1}\right)\right) = \frac{f\left(\varphi\left(l_t\right)\right) v'\left(l_t\right)}{\varphi'\left(l_t\right) f'\left(\varphi\left(l_t\right)\right)} \tag{15}$$

$$z_{t+1} = z_t + z_t\left(1 - z_t\right) - \varphi\left(l_t\right) \tag{16}$$

We observe that z_t is a predetermined variable while l_t is a jump variable. It is interesting to remark that labour dynamics is not altered by the resource dynamics while the resource dynamics depends on labour.

3.2 Steady state

At the steady state, $l_t = l_{t+1} = l$ and $z_t = z_{t+1} = z$. That is, at the steady state, (15) writes:

$$\pi\left(l\right) = \phi\left(l\right) \tag{17}$$

with $\pi\left(l\right) \equiv u'\left(f\left(\varphi\left(l\right)\right)\right) > 0$ and $\phi\left(l\right) \equiv v'\left(l\right) / \left[\varphi'\left(l\right) f'\left(\varphi\left(l\right)\right)\right] > 0$. Let us prove that there exists a unique $l > 0$ such that (17) is verified. Considering jointly assumptions 1, 2 and 3, it appears that:

$$\lim_{l \to 0} \pi\left(l\right) = +\infty \text{ and } \lim_{l \to +\infty} \pi\left(l\right) = 0 \tag{18}$$

as well as:

$$\lim_{l \to 0} \phi\left(l\right) = 0 \text{ and } \lim_{l \to +\infty} \phi\left(l\right) = +\infty \tag{19}$$

Considering jointly (18) and (19), it follows from the intermediate value theorem that there exists, at least, one positive value of l satisfying (17). Unicity requires in addition monotonicity of both side of (17). Interestingly,

$$\frac{l\pi'\left(l\right)}{\pi\left(l\right)} = \alpha\gamma\sigma < 0$$

$$\frac{l\phi'\left(l\right)}{\phi\left(l\right)} = \omega - \left(\mu + \gamma\theta\right) > 0$$

That is, monotonicity of $\pi\left(l\right)$ and $\phi\left(l\right)$ is always verified. It follows that there always exists a unique value of $l > 0$ such that (17) is verified. Let us call this value l^*. Moreover, taking into account that $l = l^*$ at the steady state, equation (16) writes:

$$\varphi\left(l^*\right) = z\left(1 - z\right) \tag{20}$$

Since it was observed before that $g\left(\hat{z}\right) = 1/4$, it follows that (20) is verified if and only if $\varphi\left(l^*\right) \leq 1/4$. More precisely, if $\varphi\left(l^*\right) < 1/4$, there is two values of z satisfying (20), namely z_1 and z_2 such that $0 < z_1 < \hat{z} < z_2 < 1$. Finally, if

$\varphi(l^*) = 1/4$, then there is a unique value of z satisfying (20) and this value is precisely the MSY, that is, in this case, $z = \hat{z} = 1/2$. This discussion leads to the following proposition.

Proposition 2 *Let assumptions 1, 2 and 3 hold.*

-If $\varphi(l^*) < 1/4$, this economy possesses two steady states, namely (l^*, z_1) and (l^*, z_2) with $0 < z_1 < 1/2 < z_2 < 1$.

-If $\varphi(l^*) = 1/4$, this economy possesses a unique steady state, namely $(l^*, \hat{z})$ with $\hat{z} = 1/2$.

-If $\varphi(l^*) > 1/4$, this economy has no steady state.

The previous proposition is very interesting concerning the resource preservation. A resource z is said to be preserved in the long run if and only if z exists such that $z > 0$. Following proposition 2, we observe that resource preservation is ensured if and only if harvesting is not too high (i.e. $\varphi(l^*) \leq 1/4$). The possibility of multiple steady states was already stressed by Koskela et al. (2002) when the resource reproduction function has a bell shape. This contrasts with the steady state unicity observed by Bednar-Friedl and Farmer (2013) and by Desmarchelier and Mayol (2022) when physical capital is introduced. Since $0 < z_1 < 1/2 < z_2 < 1$ such that $z = 1/2$ represents the MSY, the resource is said to be overexploited (underexploited) in the long run if $z = z_1$ ($z = z_2$).

4 Local dynamics

The previous section has pointed out the possible existence of two steady states located on both side of the MSY. In order to know if the economy converges toward the low or the high steady state in the long run, it is needed to study the local dynamics of each steady state.

To ensure that all elasticities are fully parametric, let us consider the following functional forms:

$$u(c) = \frac{c^{1-\varepsilon}}{1-\varepsilon},\ v(l) = \frac{l^{1+\eta}}{1+\eta} \tag{21}$$

$$\varphi(l) = Al^{\gamma},\ f(H) = BH^{\alpha} \tag{22}$$

To satisfy properties presented within propositions 1, 2 and 3, $\varepsilon > 0$[7], $\eta > 0$, $\gamma \in (0,1)$ and $\alpha \in (0,1)$. Interestingly, functional forms (21) and (22) imply that previous elasticities become:

$$\begin{bmatrix} \frac{cu''(c)}{u'(c)} & \frac{lv''(l)}{v'(l)} \\ \frac{l\varphi'(l)}{\varphi(l)} & \frac{l\varphi''(l)}{\varphi'(l)} \\ \frac{Hf'(H)}{f(H)} & \frac{Hf''(H)}{f'(H)} \end{bmatrix} = \begin{bmatrix} -\varepsilon & \eta \\ \gamma & \gamma-1 \\ \alpha & \alpha-1 \end{bmatrix}$$

[7] As usual, the reader can remark that $\lim_{\varepsilon\to 1}\left[\left(c^{1-\varepsilon}\right)/(1-\varepsilon)\right] = \ln c$.

Linearizing equations (15) and (16) by considering relations (17) and (20), we obtain:

$$\begin{aligned}
\frac{dl_{t+1}}{l_{t+1}} &= \frac{(1+\eta)}{\alpha(1-\varepsilon)\gamma}\frac{dl_t}{l_t} \\
\frac{dz_{t+1}}{z_{t+1}} &= \gamma(z_j-1)\frac{dl_t}{l_t}+2(1-z_j)\frac{dz_t}{z_t}
\end{aligned}$$

with $j=1,2$.

The Jacobian matrix J evaluated at the steady states is then given by:

$$J \equiv \begin{bmatrix} \frac{1+\eta}{\alpha(1-\varepsilon)\gamma} & 0 \\ \gamma(z_j-1) & 2(1-z_j) \end{bmatrix}$$

J is triangular due to the fact that labour dynamics is not affected by the resource dynamics. We directly read the two eigenvalues λ_1 and λ_2 of J on its diagonal:

$$\begin{aligned}
\lambda_1 &= \frac{1+\eta}{\alpha(1-\varepsilon)\gamma} \\
\lambda_2 &= 2(1-z_j)
\end{aligned}$$

An eigenvalue λ_k, with $k=1,2$, is said to be stable (unstable)[8] if and only if $|\lambda_k|<1\,(>1)$. Moreover, when $|\lambda_k|=1$, a local bifurcation occurs. Let us focus on codimension one[9] local bifurcations, that is, in bifurcations where only one eigenvalue looses (gains) its stability. In case where λ_k is a real number, which is obviously the case here (see λ_1 and λ_2), there is only two possibilities: $\lambda_k=1$ or $\lambda_k=-1$. When $\lambda_k=1$, the two steady states z_1 and z_2 collide and disappear through a saddle-node bifurcation[10]. When $\lambda_k=-1$ the dynamical system converges towards a periodic cycle of order two which appears near a steady state (flip bifurcation[11]). To be as concise as possible, we restrict our analysis to the case where $\varphi(l^*)<1/4$ (see Proposition 2), that is, to cases where the dynamical system (15)-(16) possesses two distinct steady states z_1 and z_2 such that $0<z_1<1/2<z_2<1$. Interestingly, since $z_j\in(0,1)$, then $\lambda_2>0$. Moreover, $\lambda_2<1$ if and only if $z_j>1/2$ which means that λ_2 is stable (unstable) when evaluated at z_2 (at z_1). Moreover, if $\varepsilon<1$, $\gamma\in(0,1)$ and $\alpha\in(0,1)$ ensure that $\lambda_1>1$. If $\varepsilon>1$, then $\lambda_1<0$ and more precisely, $\lambda_1<(>)-1$ if and only if $\eta>(<)\,\eta^f$ such that:

$$\eta^f \equiv \alpha(\varepsilon-1)\gamma-1$$

[8] See, among others, Grandmont (2008, section 2.3.1).

[9] When two eigenvalues loose simultaneously their stability, the bifurcation taking place is said to be of codimension two. The reader interested in a concise presentation of those bifurcations and to economic examples is referred to Bosi and Desmarchelier (2019).

[10] See Grandmont (2008, Section 3.2) among others.

[11] See Grandmont (2008, section 3.3) among others.

One can remark that $\eta^f \geq 0$ if and only if $\varepsilon \geq 1 + 1/(\alpha\gamma)$. Interestingly, if $1 < \varepsilon \leq 1 + 1/(\alpha\gamma)$, $\eta^f \leq 0$ implying that $\eta > \eta^f$ (i.e.$\lambda_1 < -1$) is always true. However, when $\varepsilon > 1 + 1/(\alpha\gamma)$, $\eta = \eta^f$ implies that $\lambda_1 = -1$ inducing the occurrence of a two-periods cycle (flip bifurcation) near the two steady states simultaneously since λ_1 is not affected by the steady state considered (z_1 or z_2). This comes from the fact that the evolution of l is not affected by the evolution of z (see system (15)-(16)). Finally, since the dynamical system (15)-(16) possesses one predetermined variable (i.e. z) and one jump variable (i.e. l), two stable eigenvalues imply local indeterminacy (sunspot equilibria) and then the occurrence of self-fulfilling prophecies[12]. Interpretations for the occurrence of local indeterminacy as well as for the occurrence of the flip bifurcation will be given later. The next proposition follows.

Proposition 3 *Let assumptions 1, 2 and 3 hold, consider functional forms (21) and (22) and assume that* $\varphi(l^*) < 1/4$*:*

(1) If $\varepsilon < 1 + 1/(\alpha\gamma)$*, then* (l^*, z_1) *is locally unstable while* (l^*, z_2) *is a saddle-point.*

(2) If $\varepsilon > 1 + 1/(\alpha\gamma)$*:*

(2.1) such that $\eta > \eta^f$*, then* (l^*, z_1) *is locally unstable while* (l^*, z_2) *is a saddle-point.*

(2.2) such that $\eta < \eta^f$*, then* (l^*, z_1) *is a saddle-point while* (l^*, z_2) *is locally indeterminate*

(2.3) such that $\eta = \eta^f$*, then a two-period cycle emerge near* (l^*, z_1) *and* (l^*, z_2) *through the occurrence of a flip bifurcation.*

Proposition 3 deserves some comments. In cases (1) and (2.1), (l^*, z_1) is locally unstable which means that if the initial resource stock z_0 is such that $z_0 < z_1$, then $\lim_{t\to+\infty} z_t = 0$. In this case, the resource is fully depleted in the long run. Moreover, (l^*, z_2) is a saddle-point. That is, if l is set at its steady state value from the first generation ($l_0 = l^*$), then if $z_1 < z_0 < z_2$ or if $z_0 > z_2$, $\lim_{t\to+\infty} z_t = z_2$. Those cases are very interesting because they mean that the resource is underexploited in the long run as in Desmarchelier and Mayol (2022) despite the absence of altruism in the present context. Moreover, in case (2.2), (l^*, z_1) is a saddle-point, however, the configuration is different from the one of (l^*, z_2) in cases (1) and (2.1) because the unstable arm comes from z. That is, in case (2.2), for $z_0 \neq z_1$, it exists a unique $l_0 \neq l^*$ such that $\lim_{t\to+\infty} z_t = z_1$. Moreover, since (l^*, z_2) is locally stable (indeterminacy), then there exists a continuum of value for l such that $\lim_{t\to+\infty} z_t = z_2$. Following Azariadis (1981), this indicates also the existence of endogenous cycles due to self-fulfilling prophecies while a two-period cycle emerges in case (2.3).

To illustrate the previous discussion, let us construct the phase diagrams in the $(z, \varphi(l))$ space. Considering (16), $z_{t+1} > (<)\, z_t$ implies

$$\varphi(l_t) < (>)\, z_t (1 - z_t) \equiv g(z_t)$$

[12] See Azariadis (1981).

Moreover, as discussed within section 3.2, there exists a unique positive steady state value of l, namely l^* which is independent of z. Interestingly, we have observed that $|\lambda_1| > 1$ in cases (1) and (2.1) which means that $\forall l > l^*$ (i.e. $\varphi(l) > \varphi(l^*)$), $l_{t+1} > l_t$ (i.e. $\varphi(l_{t+1}) > \varphi(l_t)$) and conversely, $\forall l < l^*$ (i.e. $\varphi(l) < \varphi(l^*)$), $l_{t+1} < l_t$ (i.e. $\varphi(l_{t+1}) < \varphi(l_t)$). This discussion leads to Fig.1.

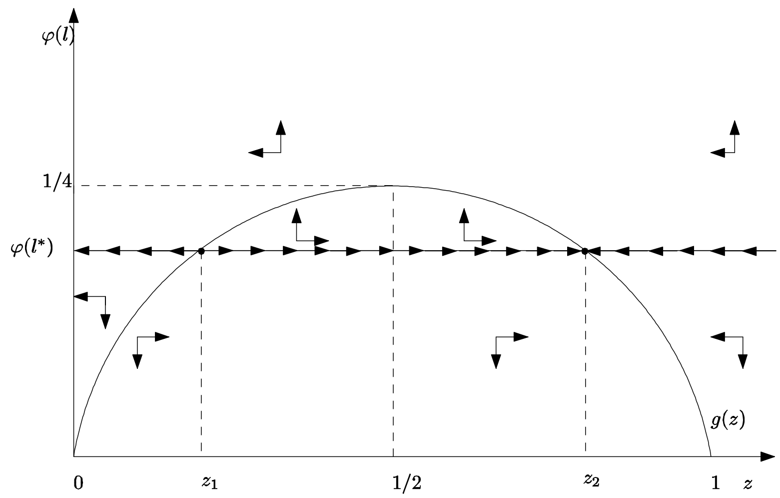

Fig. 1. Phase diagram in cases (1) and (2.1)

In case (2.2), $-1 < \lambda_1 < 0$ implying that $\forall l > l^*$ (i.e. $\varphi(l) > \varphi(l^*)$), $l_{t+1} < l_t$ (i.e. $\varphi(l_{t+1}) < \varphi(l_t)$) and conversely, $\forall l < l^*$ (i.e. $\varphi(l) < \varphi(l^*)$), $l_{t+1} > l_t$ (i.e. $\varphi(l_{t+1}) > \varphi(l_t)$). This discussion leads to Fig.2.

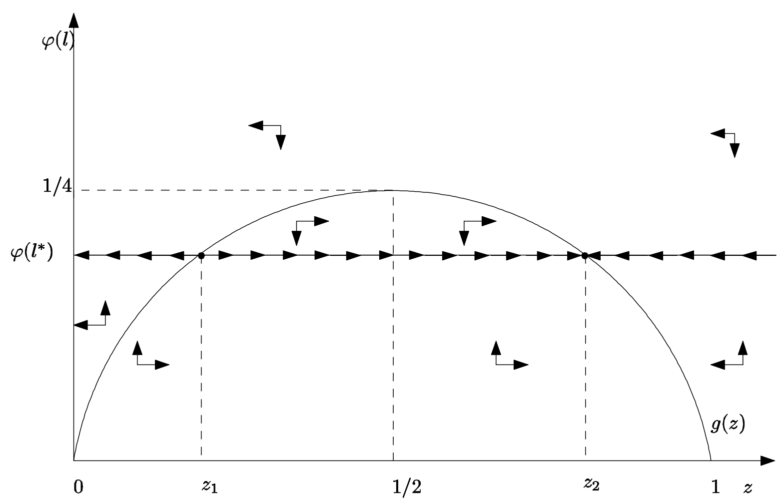

Fig.2. Phase diagram in cases (2.2)

In order to interpret the existence of both self-fulfilling prophecies and periodic cycles, we propose to proceed in two steps. First, let us reconsider the first order condition (7) with fixed prices. If the household born at date t increases her time devoted to harvest, this increases the relative marginal harvesting cost, namely $v'(l_t)/\varphi'(l_t)$, because of increasing marginal disutility of l_t (Assumption 2) as well as decreasing marginal productivity of l_t (Assumption 1). To fully compensate this increase of marginal cost, the household born at time t has to increase marginal benefits (i.e. $u'(c_{t+1})$), implying a decrease of c_{t+1} because of decreasing marginal utility (Assumption 2). Focusing on both production technology (8) and equilibrium conditions (11) and (12), it follows a lower resource demand at time $t+1$ inducing a lower time devoted to harvesting at $t+1$, namely a lower l_{t+1}. We then observe that a higher l_t is followed by a drop of l_{t+1} indicating the existence of endogenous cycles. This discussion, with fixed prices considered, gives the key elements to understand business cycles in this economy. The next step is to observe what happens with endogenous prices. That is, let us reconsider (15) with functional forms (21) and (22):

$$\Phi l_{t+1}^{\alpha\gamma(1-\varepsilon)} = l_t^{1+\eta} \tag{23}$$

With $\Phi \equiv \alpha\gamma B^{1-\varepsilon} A^{\alpha(1-\varepsilon)} > 0$. Since $\varepsilon > 1$ (Proposition 3, case 3):

$$\frac{\partial}{\partial l_t}\left[l_t^{1+\eta}\right] = (1+\eta)\, l_t^{\eta} > 0 \tag{24}$$

$$\frac{\partial}{\partial l_{t+1}}\left[\Phi l_{t+1}^{\alpha\gamma(1-\varepsilon)}\right] = \alpha\gamma(1-\varepsilon)\,\Phi l_{t+1}^{\alpha\gamma(1-\varepsilon)-1} < 0 \tag{25}$$

Now, assume that the young household born at date t increases her time devoted to harvest (l_t). Considering (24), this increases the RHS of (23). To keep equality holding, the LHS of (23) has also to increase. Focusing on (25), this is possible only if the young household born at date $t+1$ reduces her time devoted to harvest (i.e. l_{t+1}). As discussed before, this drop of l_{t+1} induces a drop of resource demand. That is, a higher l_t is followed by a lower l_{t+1}, inducing periodic cycles. Such a mechanism works also to understand the existence of self-fulfilling prophecies: if the household born at date t rationally expects a decrease of l_{t+1}, she has to increase l_t (see (23)). However, as discussed just before, an increase of l_t is followed by an effective decrease of l_{t+1}. That is, the expectation of a decrease of l_{t+1} is self-fulfilling.

Fluctuations of l generate also fluctuations of the utility level across generations. Moreover, this also implies fluctuations of the resource level in the long run. Indeed, remember that z_1 and z_2 are roots of equation (20) and then, fluctuations of l imply fluctuations of $\varphi(l)$ and hence, fluctuations of both z_1 and z_2. This situation is very exotic since periodic cycles through a flip bifurcation are known to occur in economies with a unique steady state when a renewable resource is considered (Koskela et al. 2008, Desmarchelier and Mayol 2022).

5 Simulations

This section proposes simulations using Python 3.9 to illustrate proposition 3. Considering functional forms (21) and (22), the dynamical system (15)-(16) becomes:

$$
\begin{aligned}
l_{t+1} &= \left[\frac{l_t^{1+\eta}}{\alpha\gamma B^{1-\varepsilon} A^{\alpha(1-\varepsilon)}}\right]^{\frac{1}{\alpha\gamma(1-\varepsilon)}} \\
z_{t+1} &= z_t + z_t\left(1-z_t\right) - A l_t^{\gamma}
\end{aligned}
$$

The (non-trivial) steady state value of l is simply given by:

$$l^* = \left[\alpha\gamma B^{1-\varepsilon} A^{\alpha(1-\varepsilon)}\right]^{\frac{1}{1+\eta+\alpha\gamma(\varepsilon-1)}}$$

To simplify the exposition, let us fix B to ensure that $l^* = 1$, namely:

$$B = \left(\alpha\gamma A^{\alpha(1-\varepsilon)}\right)^{\frac{1}{\varepsilon-1}}$$

In this case, z_1 and z_2 are simply given by:

$$
\begin{aligned}
z_1 &= \frac{1}{2} - \frac{1}{2}\sqrt{1-4A} \\
z_2 &= \frac{1}{2} + \frac{1}{2}\sqrt{1-4A}
\end{aligned}
$$

$A < 1/4$ ensures the existence of two distinct steady states for the resource level. This case is analogous to $\varphi(l^*) < 1/4$ (see propositions 2 and 3). Let us focus on the following calibration:

Parameter	A	α	γ	ε
Value	1/6	1/2	9/10	4

Table 1

One can remark that calibration in table 1 ensures that $\varepsilon > 1 + 1/(\alpha\gamma)$ (see Proposition 3). With $A = 1/6$, $z_1 \approx 0.211\,32$ and $z_2 \approx 0.788\,68$. Moreover,

$$\eta^f = 0.35$$

Focus on Proposition 3. Let us begin by illustrating the saddle-path stability of (l^*, z_2) in case (2.1). To do so, let $\eta = 0.36$ as well as[13] $l_0 = l^*$. Fig. 3 and Fig. 4 represent respectively the time trajectory of z_t when $z_0 = 0.3 \in (z_1, z_2)$ and when $z_0 = 0.9 > z_2$.

[13] $l_0 = l^*$ gives the stable arm of (l^*, z_2).

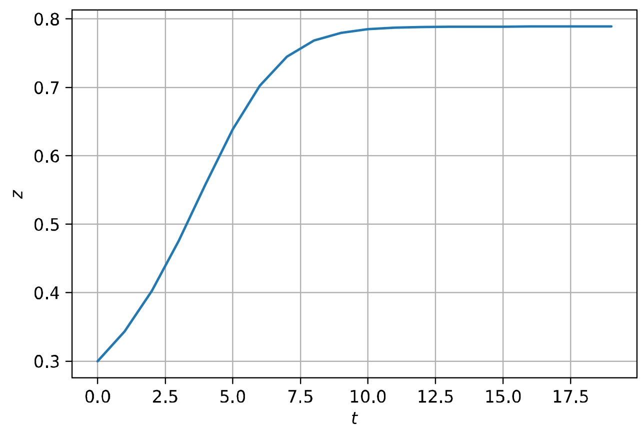

Fig. 3. z_t when $z_0 = 0.3$

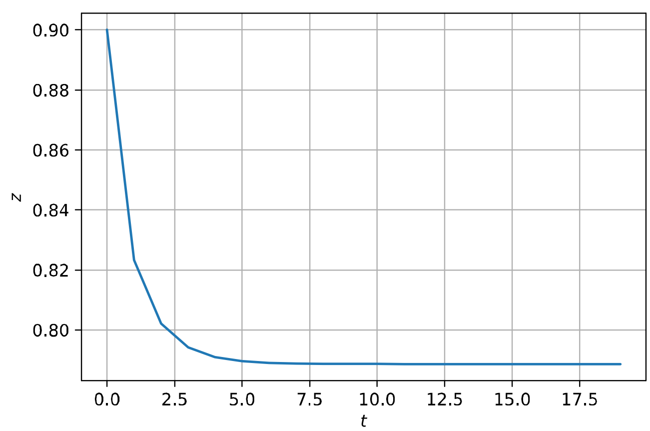

Fig. 4. z_t when $z_0 = 0.9$

To illustrate the occurrence of the flip bifurcation, Fig. 5 and Fig. 6 are done by taking initial values close (z_2, l^*), namely $l_0 = 0.99$ and $z_0 = 0.788$. Fig 5 depicts the stable cycle arising at the flip bifurcation (i.e. when $\eta = \eta^f = 0.35$) while Fig. 6 shows the converging oscillations when $\eta = 0.34 < \eta^f$.

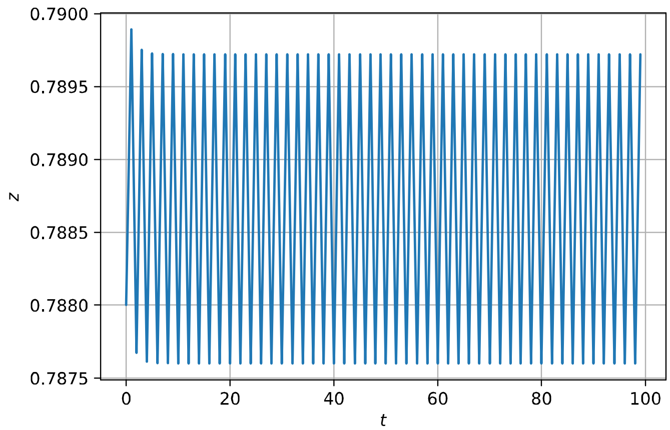

Fig. 5. Stable cycle when $\eta = 0.35$ (Flip bifurcation)

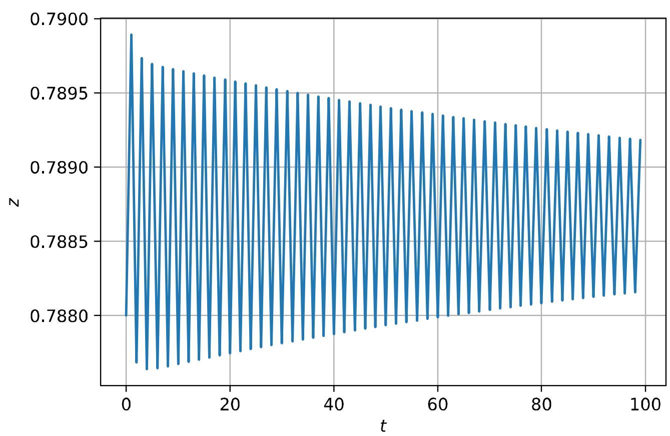

Fig. 6. Converging oscilations when $\eta = 0.34$

6 Conclusion

Within this paper, we have developed a simple monetary OLG model where a representative household has to arbitrate between time devoted to harvest a renewable resource and leisure. In this sense, harvesting is costly which contrasts with the existing literature[14]. Concerning the long run, it appears that two steady states can coexist: one with a low resource level (overexploitation), the other with a high resource level (underexploitation). That is, a costly harvesting is able to preserve the resource in the long run by avoiding a complete

[14] With the notable exception of Bednar-Friedl and Farmer (2013, 2014).

depletion. Such a property was already pointed out in the literature when the resource serves as a store of value or when households are altruistic. That is, a costly harvesting appears to be a third explanation for a potential long run resource preservation. Concerning the dynamics, a sufficiently productive harvesting technology is able to lead to local indeterminacy near the higher steady state, generating endogenous cycles due to self-fulfilling prophecies. Finally, a sufficiently productive harvesting technology is also able to generate periodic cycles around each steady state simultaneously which is very unusual. That is, costly harvesting appears to be both a source of resource preservation and a source of resource fluctuations.

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