Documents de travail

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Auteurs

Patrice Loisel, Marielle Brunette, Stéphane Couture

Document de Travail n° 2022 – 26

Septembre 2022

Bureau d'Économie Théorique et Appliquée BETA

www.beta-economics.fr

@beta_economics

Contact :
jaoulgrammare@beta-cnrs.unistra.fr

Ambiguity, value of information and forest rotation decision under storm risk

Patrice Loisel [*], Marielle Brunette [†], Stéphane Couture [‡]

September 7, 2022

Abstract

Storm is a major risk in forestry. However, due to the more or less pessimistic scenarios of future climate change, storm frequency is now ambiguous and only partially known (i.e., scenario ambiguity). Furthermore, within each scenario, the quantification of storm frequency is also ambiguous due to the differences in risk quantification by experts, creating a second level of ambiguity (i.e., frequency ambiguity). In such an ambiguous context, knowledge of the future climate through accurate information about this risk is fundamental and can be of significant value. In this paper, we question how ambiguity and ambiguity aversion affect forest management, in particular, optimal cutting age. Using a classical Faustmann framework of forest rotation decisions, we compare three different situations: risk, scenario ambiguity and frequency ambiguity. We show that risk and risk aversion significantly reduce the optimal cutting age. We also show that both scenario and frequency ambiguities reinforce the effect of risk. Inversely, ambiguity aversion has no effect. The value of information that resolves scenario ambiguity is high, whereas it is null for frequency ambiguity.

Keywords: Rotation decision, Risk, Ambiguity, Ambiguity Aversion, Risk Aversion, Value of Information, Forests, Faustmann criterion.

JEL codes: D81, D90, Q23

[*]MISTEA, Université Montpellier, INRAE, Institut Agro, Montpellier, France. patrice.loisel@inrae.fr

[†]Université de Lorraine, Université de Strasbourg, AgroParisTech, CNRS, INRAE, BETA, 54000, Nancy, France. Phone: +33 (0)3 83 39 68 56. marielle.brunette@inrae.fr; Associate researcher at the Climate Economic Chair (CEC), Paris, France.

[‡]MIAT, INRAE, University of Toulouse, 31326 Castanet-Tolosan, France. stephane.couture@inrae.fr

1 Introduction

Natural events are the major threat facing forests worldwide today. At the global scale, van Lierop et al. (2015) estimated that over the period 2002-2013, 67 million hectares of forest burned annually worldwide, 85 million hectares were affected by insects, 38 million by severe weather conditions, and 12.5 million by disease. At the European scale, Schelhaas et al. (2003) indicated that over the period 1950-2000, an annual average of 35 million m^3 of wood was damaged by disturbances in Europe. Storms were responsible for 53% of the total damage, fire for 16%, snow for 3%, and biotic factors for 16%. Consequently, storms are the most damageable event for European forests. This is in line with Gardiner et al. (2013) who inventoried a total of 130 storms over the last 60 years affecting Europe, i.e., an average of two storms per year. In addition, Schelhaas et al. (2003) show that disturbances have increased over that period. This increase has continued in the first decade of the 21^{st} century (Seidl et al., 2014). More importantly, damage from these disturbances is likely to further increase in coming decades (Seidl et al., 2014). Climate change was identified as a main driver behind this increase (Seidl et al., 2011) and it impacts both the frequency and the intensity of disturbances (van Aalst, 2006), in particular, storms (Haarsma et al., 2013). The increase in the frequency and intensity of natural events is thus uncertain in that current scientific knowledge does not make it possible to precisely quantify these effects. Indeed, projecting past historical trends into the future in a context of climate change is not yet relevant. This imprecision translates into different climate scenarios such as the well-known IPCC scenarios (representative concentration pathways: RCP 2.6, 4.5, 6.0, 8.5) that make it possible to consider all the possible trajectories. However, among the possible scenarios, we cannot predict which future scenario will occur, thus creating a first level of ambiguity, referred to as scenario ambiguity.[1] In addition, each scenario provides an assessment of the storm risk. This quantification is based on past data or expert opinion, and is thus imprecise by nature, generating a second level of ambiguity, referred to as frequency ambiguity. Consequently, the relative knowledge of climate change and its impacts on forest ecosystems creates an environment that is ambiguous, difficult to apprehend and complex.

It is within this complex environment that forest owners have to manage their forests, and one of the most important decisions in forestry is the optimal cutting age. They have to decide when to harvest without precise knowledge about the future climate or the characteristics of natural

[1] The uncertainty of the probability of occurrence is referred to as ambiguity in decision theory. Camerer and Weber (1992) propose the following definition: "*Ambiguity is uncertainty about probability, created by missing information that is relevant and could be known*".

events that threaten their forests. Depending on the climate scenario, the frequency of the natural event will vary, as well as the management, meaning that the two levels of ambiguity will impact the optimal cutting age. Moreover, in this ambiguous context, the preferences of forest owners are fundamental, especially those related to their management behavior (Couture et al., 2016; Brunette et al., 2017). Indeed, the forest management decision strongly depends on the forest owner's preferences towards risk but also, more importantly in our context, towards ambiguity (Brunette et al., 2020). Forests are mainly privately owned in France and very heterogeneous in terms of forest management objectives and preferences towards risk and ambiguity. It is therefore essential to take this heterogeneity into account. In such a context, providing recommendations to private forest owners that take their preferences into account is a fundamental societal challenge in the face of climate change.

In addition, in such an ambiguous context, it is fundamental to improve our knowledge about the considered risks in order to ensure better decision-making. Consequently, a key challenge is to provide information that will make it possible to reduce or eliminate such ambiguities (Snow, 2010). We can easily imagine that knowing which climate scenario will occur (resolving the scenario ambiguity) or the precise probability of the occurrence of storm (resolving the frequency ambiguity) will affect the forest owner's management decision. This information has a value that is useful to know and to quantify in order to have a precise idea of what would be at stake before undertaking studies to obtain such information.

In this context, we question the impacts of the scenario and frequency ambiguities on the optimal cutting age. Does the optimal cutting age depend on the forest owner's preferences towards risk and ambiguity? What is the value of information that will make it possible to resolve each type of ambiguity? This article provides answers to these research questions.

The question of the optimal cutting age is classical in forest economics since the model proposed by Faustmann (1849). This approach allows the evaluation of the Land Expectation Value (LEV) over an infinite sequence of rotations. Initially, the model was deterministic, and Reed (1984) was the first to introduce risk into this classical framework. He showed that potential total destruction due to a fire risk acts as an increase in the discount rate and, consequently, reduces the optimal cutting age. This framework was used to analyze the impact of different types of risks in forestry, such as disease (Macpherson et al., 2018) and storm (Haight et al., 1995; Loisel, 2014; Rakotoarison and Loisel, 2017; Loisel et al., 2020). However, to our knowledge, ambiguity has never been considered in such a framework, whereas ambiguity under climate change better characterizes the forest owner's environment than risk.

The value of information in an ambiguity context has been used in the literature in different theoretical frameworks that incorporate ambiguity aversion. Some authors such as Nocetti (2018), Hoy et al. (2014) and Snow (2010) have shown the effect of ambiguity aversion on the value of information, with non-unanimous results. Snow (2010) shows that the value of information that resolves ambiguity increases with greater ambiguity and with greater ambiguity aversion. Hoy et al. (2014) propose an application of ambiguity theory to genetic tests. Contrary to Snow (2010), they show that such a test introduces ambiguity, which reduces the value of information to an ambiguity-averse decision-maker. Nocetti (2018) reconciles the two articles by indicating that the result of Hoy et al. (2014) is true for unconditional ambiguity aversion, whereas the one of Snow (2010) is validated in the event of a greater distaste for conditional ambiguity.[2] Such theoretical analyses are based on the decision criterion proposed by Klibanoff et al. (2005), called the smooth ambiguity model, which incorporates ambiguity aversion in the same way as risk aversion in a risky context. This literature is theoretical, and Peysakhovich and Karmarkar (2015) were the only ones to empirically study the impact of the arrival of partial information on decisions made under ambiguity. They found that the value of information increases or not depending on its favorable or unfavorable nature on the ambiguous perspective.

In this paper, we propose a theoretical model that extends the classical Faustmann framework under risk proposed by Reed (1984) to ambiguity. We consider that the forest is threatened by a storm risk. We assume the decision criterion proposed by Klibanoff et al. (2005), the smooth ambiguity model. In this context, we question if ambiguity reinforces the impact of risk, i.e., reduces the optimal cutting age more than under risk. We explicitly consider the forest owner's preferences towards risk and ambiguity. We also compute the value of information that makes it possible to be under risk rather than under ambiguity, i.e., favorable information fully resolving ambiguity. Finally, we solve the model numerically in a case study of a beech stand. We show that risk, risk aversion and scenario ambiguity reduce the optimal cutting age, whereas frequency ambiguity and ambiguity aversion have no impact. We obtain a positive value of information to resolve the two types of ambiguity. However, the value is high for scenario ambiguity, whereas it is close to zero for frequency ambiguity. Finally, the value of information that resolves scenario ambiguity increases with risk aversion, but ambiguity aversion has no impact. However, the value of information that

[2] According to Nocetti (2018), "*Conditional ambiguity refers to the uncertainty over the correct distribution of outcomes that remains after a message is received. Unconditional ambiguity refers to uncertainty over the message which will be received.*"

resolves frequency ambiguity increases with both risk and ambiguity aversion.

The rest of the paper is structured as follows. Section 2 presents the theoretical model. Section 3 solves the model for a case study. Finally, the results are discussed in Section 4, and Section 5 provides a conclusion.

2 Model of forest rotation and value of information

We first present some definitions and concepts, and then describe the model under risk and the model under ambiguity.

2.1 Definitions and mathematical formalization of risk and ambiguity

2.1.1 Storm risk: occurrence and impacts

We consider a private forest owner who manages a forest stand. The forest is exposed to a storm. We model the storm and its impact on the stand, like Loisel et al. (2020). The severity of the storm is given by the random variable A. Let τ be the period of time between the beginning of the stand and, either the storm occurrence or the final cutting. The storm is then described by the couple of random variables (τ,A). The impact of the storm is age-dependent and assumed to be low in the case of young stands. We assume a threshold in terms of height H_L that is reached at time t_L. Therefore, in the case of storm occurrence before t_L there is no damage, whereas above t_L, the proportion of damaged trees θ is dependent on A and on the characteristics of the stand at the time of the event (tree height, tree diameter).

Let τ be the time between the beginning of the stand and the first event after t_L, i.e., storm occurrence or cutting at time T. The distribution of the random variable τ is defined for $t_L < t \leq T$ as $F_\tau(t) = F(t - t_L) = 1 - e^{-\lambda(t-t_L)}$, where λ is the rate of return of the storm per unit of time.

Let $L(\theta, \tau)$ be the loss following the storm occurring at time τ for a proportion of damaged trees θ. This loss represents the non-harvesting of damaged trees: $L(\theta, \tau) = \theta V(\tau)$ where $V(\tau)$ is the potential final (without risk) income at time τ where $V'(.) > 0$.

Without risk, the forest stand grows from t (plantation or regeneration) to T, where T is the cutting age. The storm modifies the sequence of events as follows:

- If a storm occurs before the threshold time t_L, there is no impact, $L(.) = 0$ and stand growth continues.

- If a storm occurs after t_L but before T, the proportion of damaged trees is θ and the loss is $L(\theta, \tau) = \theta V(\tau)$; a clear-cutting and a regeneration (or plantation) of the stand take place.

- If no storm occurs before T, a clear-cutting and a regeneration (or plantation) of the stand take place at time T.

2.1.2 The ambiguity context: scenario and frequency ambiguities

In the ambiguous context, forest owners face the same consequences of the storm on their stand as in the risky case. Only two aspects are modified by ambiguity depending on the sources of ambiguity considered: ambiguity on the scenarios or ambiguity on the frequencies. There will then appear either a noise, a first source of ambiguity on the objective value of the rate of return, called the scenario ambiguity, or another noise, a second source of ambiguity around this objective value of the rate of return, called the frequency ambiguity.

Scenario ambiguity is characterized by different possible objective rates of return for different possible future climate scenarios, $(\lambda_0, \lambda_1, \lambda_2)$. An objective rate of return is in fact determined for each climate scenario. Forest owners do not know the future scenario in which they will evolve and have to make their decision in this ambiguous context. This is the source of ambiguity on the scenarios.

Frequency ambiguity is reflected in the fact that when the scenario is known, there is also a noise on the quantification of the objective rate of return. In this case, the objective rate of return belongs to a probability distribution known by the forest owners to which they associate a confidence parameter α_0. This parameter captures the forest owner's beliefs about the objective rate of return. It is then considered to be an additional source of ambiguity on the frequencies.

2.2 Model of forest rotation under risk

The decision problem of forest owners in a risky context is to choose the optimal cutting age that will maximize their expected utility from their forestry activity, as described by the Faustmann principle.

Since risk aversion has already been found to be decisive in the decision-making process of forest owners (Couture and Reynaud, 2008; Lobianco et al., 2016; Brunette et al., 2017), we assume a risk-averse private forest owner characterized by a von-Neumann Morgenstern (vNM) utility function u where $u'(.) > 0$ and $u''(.) < 0$. The forest owner is assumed to maximize the utility of the net economic return from forest management activity. We assume a risky context in which the probability of occurrence of the risk is known ($\lambda = \lambda_0$).

For a discount rate δ, this utility of the net economic return $\mathcal{Y}$, actualized at storm occurrence time τ or logging time T, is written as follows:

$$\mathcal{Y} = \begin{cases} \mathcal{H}_\delta(\underline{t}, \tau) + u(V_1(\theta_\tau, \tau) - c_1 - C_n(\theta_\tau, \tau)) & \text{if } t_L < \tau < T \\ \mathcal{H}_\delta(\underline{t}, T) + u(V(T) - c_1) & \text{if } \tau = T \end{cases}$$

where:

- $H_\delta(\underline{t}, \tau) = \int_{\underline{t}}^{\tau} u(h(s))e^{\delta(\tau - s)}ds$, the utility of the thinning incomes between time $\underline{t}$ and time τ actualized at time τ. Thinning makes it possible to smooth the forest owner's income over time.

- $V_1(\theta_\tau, \tau) = (1 - \theta_\tau)V(\tau)$, the final income in the event of a storm occurring at time τ.

- c_1, the regeneration (or plantation) cost.

- $C_n(\theta_\tau, \tau)$ the clearing costs for a storm occurring at time τ.

We deduce the following expression for the Faustmann value J_F:

$$J_F = \frac{E[e^{-\delta\tau}\mathcal{Y}]}{1 - E[e^{-\delta\tau}]} = \frac{1}{b(\delta, T)} \left[\frac{a_1(\delta, T)}{\delta} u(h) + W_F(0, T) \right]$$

where $a_0(\delta, T) = e^{(\delta+\lambda)T - \lambda t_L}(1 - e^{-\delta\underline{t}})$, $a_1(\delta, T) = e^{(\delta+\lambda)T - \lambda t_L - \delta\underline{t}} - \dfrac{\lambda e^{(\delta+\lambda)(T - t_L)} + \delta}{\delta + \lambda}$, $b(\delta, T) = a_0(\delta, T) + a_1(\delta, T)$ and $W_F(0, T)$ is a modified income such that:

$$W_F(0, T) = \lambda \int_{t_L}^{T} E[u(V_1(\theta, \tau) - c_1 - C_n(\theta_\tau, \tau))]e^{(\delta+\lambda)(T - \tau)}d\tau + u(V(0, T) - c_1)$$

In this risky context in which the probability of occurrence of the storm is known (λ), we look for the max of the Faustmann value as follows:

$$J(\lambda_i, T_i) = \max_T J(\lambda_i, T) \tag{1}$$

where T_i is the optimal cutting age obtained under risk for a known and unique storm occurrence probability level, λ_i.

2.3 Model of forest rotation under ambiguity

Under ambiguity, the forest owner's objective is to find the optimal cutting age that will maximize a functional form over the expected utility level that captures ambiguity preferences. We assume that the forest owner is characterized by an increasing and concave function $\phi(.)$, defined over the expectation of the utility function $u(.)$, representing the owner's ambiguity aversion. This way of implementing ambiguity follows the smooth ambiguity model of Klibanoff et al. (2005). One advantage of this approach is the separation of attitude towards risk (captured by the function $u(.)$) and the attitude towards ambiguity (captured by the function $\phi(.)$). This ambiguity reflects the fact that the forest owner has an imperfect knowledge of the rate of return of the storm.

2.3.1 Scenario ambiguity and the associated value of information

For scenario ambiguity, as described in Section 2.1.2, we assume that forest owners face three different rate of return $(\lambda_0, \lambda_1, \lambda_2)$ due to different possible future climate scenarios and that they do not know which one will occur. It is not objectively possible to know which climate scenario will occur in the future. Consequently, we assume a uniform distribution between the different possible scenarios. This means that the rates of return are associated with the following probabilities: $(p_0, p_1, p_2) = (1/3, 1/3, 1/3)$. Compared to the equation under risk (Eq. (1)), this leads to the following equation:

$$\sum_i p_i \phi(J(\lambda_i, T_*)) = \max_T E[\phi(J(\lambda, T))] = \sum_i p_i \phi(J(\lambda_i, T)) \tag{2}$$

where $E[.]$ denotes the expectation with respect to p. T_* is the optimal cutting age under scenario ambiguity. In an ambiguous context, forest owners can then modify their cutting decision compared to the decision taken in the risky situation. Indeed, the presence of ambiguity for ambiguity-averse forest owners will decrease their well-being and they may therefore modify their optimal cutting age and adjust their behavior in response to the level of ambiguity. Intuitively, an ambiguity-averse forest owner evaluates the ambiguous situation by giving more weight to states of nature that will provide a low level of expected utility compared to states of nature that generate higher levels of expected utility.

In this ambiguity context, we are interested in the value of information. We want to assess this value in view of favorable information that will allow us to go from ambiguity to risk. This favorable information allows us to fully resolve the ambiguity. Indeed, this value corresponds to the maximum amount of money the forest owner would be willing to pay to obtain information that would remove the ambiguity. The value of information is calculated as the value that makes the forest owner indifferent to the risky situation without ambiguity and the situation with ambiguity.

The value of information that eliminates the scenario ambiguity y_0^J associated with scenarios $J = J(\lambda, .)$ satisfies:

$$\phi(J(\lambda_0, T_0) - y_0^J) = \max_T \sum_i p_i \phi(J(\lambda_i, T)) \tag{3}$$

Hence, we deduce the value of information:

$$y_0^J = J(\lambda_0, T_0) - \phi^{-1}\left(\sum_i p_i \phi(J(\lambda_i, T_*))\right) \tag{4}$$

The monetary value of information may be computed as follows:

$$z_0^J = u^{-1}\left(J(\lambda_0, T_0)\right) - u^{-1}(EC) \text{ where } EC = \phi^{-1}\left(\sum_i p_i \phi(J(\lambda_i, T_*)\right)$$

The value of information that removes ambiguity is based on the comparison of an ambiguous situation with a risky one. Thus, an increase in ambiguity aversion decreases the right-hand side of the value-of-information equation (Eq. (3)) but leaves the left-hand side unchanged; hence the amount of value-of-information must increase to restore equality.

The resolution of ambiguity by information reveals the objective rate of return. Since the forest owner's optimal choice remains independent of the state after the ambiguity is removed, the value of the information that removes the ambiguity systematically depends on the forest owner's preferences towards ambiguity. In this case, since the forest owner is ambiguity-averse, the value of information that resolves scenario ambiguity should be positive, and should increase with greater ambiguity aversion. Indeed, an ambiguity-averse forest owner will be willing to pay for information that removes ambiguity (we focus on situations where the choice set is not affected by information acquisition). Forest owners are thus better off when the level of ambiguity is lower or even zero. They will then value this ambiguity-suppressing information and this value should increase with the degree of ambiguity aversion and the level of ambiguity.

2.3.2 Frequency ambiguity and the associated value of information

For frequency ambiguity, in addition, we assume, as described in Section 2.1.2, ambiguity on the distribution of the rate of return of the storm. This means that we consider an ambiguous distribution of the rate of return of the storm under the conditional distribution of probability $\pi(.|p)$. The equation is then given by:

$$E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right] = \max_T E\left[\phi\left(\sum_i q_i J(\lambda_i, T)\right)\right]$$
$$\text{where } q \sim \pi(.|p) \tag{5}$$

where $E[q|p] = p$ and T_* is the optimal cutting under the frequency ambiguity context. As in the scenario ambiguity context, the forest owner can then change the optimal cutting decision compared to the one chosen in the risky context. This change will depend on the owner's level of ambiguity aversion.

Moreover, in this context, we are also interested in the value of information. In keeping with Snow (2010), the value of information that eliminates the ambiguity y_π^J associated with scenarios $J = J(\lambda, .)$ and the conditional distribution of probability $\pi(.|p)$ satisfies:

$$\phi\left(\sum_i p_i J(\lambda_i, T_p) - y_\pi^J\right) = \max_T E\left[\phi(\sum_i q_i J(\lambda_i, T))\right]$$

where T_p is the optimal age of return obtained in the risky context for frequency ambiguity, with three different values for the probability of occurrence λ_0, λ_1 or λ_2, and defined by:

$$\sum_i p_i J(\lambda_i, T_p) = \max_T \sum_i p_i J(\lambda_i, T) \tag{6}$$

Hence, we deduce the value of information:

$$y_\pi^J = \sum_i p_i J(\lambda_i, T_p) - \phi^{-1}\left(E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]\right) \tag{7}$$

The monetary value of information may be computed as follows:

$$z_\pi^J = u^{-1}\left(\sum_i p_i J(\lambda_i, T_p)\right) - u^{-1}(EC) \text{ where } EC = \phi^{-1}\left(E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]\right) \quad (8)$$

For ambiguity-averse forest owners, the value of the information that cancels out the ambiguity should be positive and increase with their aversion.

We now consider the case where the vector q follows $\pi(.|p) = \mathrm{Dir}(K, \alpha)$ a Dirichlet distribution (Balakrishnan and Nevzorov, 2005), with vector $\alpha = \alpha_0$, where p is the vector of probabilities $p = (p_1, ..., p_K)$ and $\alpha_0 > 0$. The parameter α_0 represents a degree of confidence that characterizes the forest owner's beliefs about the objective rate of return: the higher α_0 is, the closer the random vector of probabilities q is to p. With this distribution, the expectation $E\left[\phi\left(\sum_i q_i J(\lambda_i, T)\right)\right]$ may be specifically expressed and we can obtain the following first-order approximation of the value of information (see Appendix A):

$$\widehat{y}_\pi^J = \frac{1}{2}\frac{V(J(\lambda, T_p))}{\alpha_0 + 1}\frac{-\phi''}{\phi'}(\mathcal{J}_1(T_p)) \quad (9)$$

where: $V(J(\lambda, T)) = E_\lambda\left[(J(\lambda, T) - \mathcal{J}_1(T))^2\right] = \mathcal{J}_2(T) - \mathcal{J}_1(T)^2$ and $\mathcal{J}_k(T) = E_\lambda[J(\lambda, T)^k] = \sum_i p_i J(\lambda_i, T)^k$.

This first-order approximation of the value of information is, up to a constant, the product of the variance of $J(\lambda, T_p)$ and the absolute ambiguity aversion coefficient. The coefficient $\frac{-\phi''}{\phi'}$ is the absolute ambiguity aversion coefficient associated with the ϕ function, exactly like the absolute risk aversion coefficient is associated with the utility function. A similar approximation result can be found for the risk premium, highlighting the fact that the value of information depends on two factors, one objective, linked to the risk under consideration, and the other psychological, specific to the forest owners, characterizing their aversion to ambiguity.

Moreover, the first-order approximation of the monetary value of information is given by:

$$\widehat{z}_\pi^J = u^{-1}\left(\mathcal{J}_1(T_p)\right) - u^{-1}(\widehat{EC}) \text{ where } \widehat{EC} = \phi^{-1}\left(\phi(\mathcal{J}_1(T_p)) + \frac{1}{2}\frac{V(J(\lambda, T_p))}{\alpha_0 + 1}\phi''(\mathcal{J}_1(T_p))\right) \quad (10)$$

The model cannot be solved analytically. We therefore propose to simulate it numerically through an illustration.

3 Illustration of a beech stand in France: results and discussion

First, we present the parameters and the values adopted to calibrate the model and to carry out the simulations. We then display the results of the simulations for both levels of ambiguity and, finally, we indicate the results of some sensitivity analyse.

3.1 Model calibration and solution method

We simulate the model for a beech stand of one hectare exposed to a storm risk. European beech (*Fagus sylvatica*) represents 9% of the total forest surface in France (IGN, 2016) and is the second most commonly found hardwood species after oak. Moreover, storms represent a major threat for French forests, and their frequency is expected to increase because of climate change (Haarsma et al., 2013). We consider three possible climate scenarios: the current scenario, the scenario based on the so-called optimistic RCP 4.5, and the scenario based on the so-called pessimistic RCP 8.5.

In this context, Brèteau-Amores et al. (2020) computed the probability of occurrence of a storm of the same intensity as Lothar in 1999, in the Grand-Est region of France.[3] Using historical data, they obtained a probability of 1/55 for the current period, of 1/47 under the RCP 4.5 and 1/23 under the RCP 8.5. These probabilities are rates of return, meaning that a storm occurs every 55 years, every 47 years and every 23 years, respectively. We used these estimates to solve our model. In addition, we considered that each scenario (current, RCP 4.5 and RCP 8.5) was equiprobable ($K = 3$, $p = 1/3, 1/3, 1/3$).

In order to represent the forest owner's behavior, we consider a power utility function $u(x) = \frac{x^{1-r}}{1-r}$ where r is the relative risk aversion coefficient as commonly assumed in decision theory (Gollier, 2001), and for forest owners as well (Brunette et al., 2020). As required and validated, it is a CRRA utility function. For ambiguity, we also assume a power function $\phi(x) = x^s$ where s is the coefficient of ambiguity aversion.[4] The relative risk aversion coefficient was initially fixed at $r = 0.59$ and the coefficient of ambiguity aversion at $s = 0.729$, as quantified by Brunette et al. (2020) for forest managers.

A discount rate of 2% is classical for beech stands in forest economics (Loisel, 2014), and the confidence parameter is arbitrarily chosen at 0.025.

[3] See Supplementary Materials A in Brèteau-Amores et al. (2020) for more information.

[4] As the functional form is less consensual for ambiguity than for risk, we also tested for an exponential function. See Appendix B for the results.

We summarized the values and functional forms considered for the simulations in Table 1.

Table 1: Functional forms and base values of parameters used in the illustration.

Type	**Function/parameter**	**Assumed form/value**
Number of scenarios	K	3
Discount rate	δ	0.02 $year^{-1}$
Confidence parameter	α_0	0.025
Risk rate	λ	1/55, 1/47, 1/23 $year^{-1}$
Utility function	u	$u(x) = \dfrac{x^{1-r}}{1-r}$
Risk aversion coef.	r	0.59
Ambiguity function	ϕ	$\phi(x) = x^s$
Ambiguity aversion coef.	s	0.729

The calculation of the Faustmann value is based on the average tree growth model proposed in Loisel (2014). The numerical model, based on the mathematical formalization described in Section 2, is directly programmed in the R language. The solution to the Faustmann criterion optimization problem is obtained numerically using the standard tools proposed by R software (R Core Team, 2020), due to the size of the problem spaces. The calculations are very inexpensive in terms of computing time.

3.2 Results of the simulations

We first present the results of the simulations for scenario ambiguity and we compare them with the results under risk. We then present the results for frequency ambiguity and compare them with the results for scenario ambiguity.

3.2.1 Scenario ambiguity vs. risk

Table 2 presents the results under risk and scenario ambiguity. In this table, we consider that $p = (1, 0, 0)$ under risk and that $p = (1/3, 1/3, 1/3)$ under ambiguity.

Looking at the first part of the table dedicated to risk, we can see the results of the simulations for the different rates of return. The benchmark ($\lambda = 1/55$) corresponds to the current situation in terms of storm occurrence. For this benchmark case under risk, the optimal cutting age is 79 years and the value of the Faustmann criterion is €1890/ha. This optimal cutting age is consistent with the reality of the study area since it is common to cut this species at around 80 years (ONF, 2007). We can see that by slightly increasing this level of risk ($\lambda = 1/47$), the forest owner's utility level

Table 2: Simulation results for scenario ambiguity.

RISK						
Risk rate λ (year^{-1})		Optimal criterion $J(\lambda_i, T_i)$	Opt. crit. without risk aversion $J_0(\lambda_i, T_i)$ (€)	Opt. cutting age T_i (year)		
1/55	(benchmark)	51.75	1890	79		
1/47		51.18	1794	78.5		
1/23		47.75	1335	76		
SCENARIO AMBIGUITY						
	Optimal criterion $\sum_i p_i \phi(J(\lambda_i, T_*))$	Certainty equivalent $\phi^{-1}\left(\sum_i p_i \phi(J(\lambda_i, T_*))\right)$	Opt. crit. without risk aversion $J_0(\lambda_1, T_*)$ (€)	Opt. cutting age T_* (year)	Value of information y_0^J	z_0^J (€)
ϕ	17.36	50.17	1807	78	1.57	125

(Optimal criterion), the Faustmann value without risk aversion and the optimal cutting age are lowered. Increasing the level of risk (i.e., doubling it from 1/55 to 1/23) exacerbates these effects. In a risky situation, the forest owner's behavior is guided by three opposing effects (Couture and Reynaud, 2008; Brunette et al., 2015). The first effect is a "wealth effect" representing an incentive to harvest in order to increase timber revenue. The second effect is a "risk effect" aimed at reducing exposure to risk. It corresponds to an incentive to harvest in order to reduce future potential damage. The last effect is a "continuation effect" since the forest owners try to smooth their utility over time and, as a consequence, they delay the harvest. These three effects then condition the behavior of the forest owners by encouraging them to increase or reduce the cutting age. When the risk increases, the "risk effect" incites the forest owner to reduce the cutting age.

Concerning scenario ambiguity, in the second part of the table, we observe that it reduces the optimal cutting age. Under ambiguity, forest owners modify their cutting age compared to the decision taken under risk; they reduce it because they adjust their behavior in response to the ambiguity. Risk reduces the cutting age and scenario ambiguity leads to the same trend. In this sense, scenario ambiguity reinforces risk. The "risk effect" outweighs the other two effects. Consequently, we extend the validity of the result of Reed (1984) to ambiguity. Ambiguity also reduces the Faustmann value without risk aversion, meaning that the value of the forest goes from €1890/ha to €1807/ha. The forest owner's utility level is also impacted by ambiguity, going from 51.75 under risk to around 50 under ambiguity. All in all, scenario ambiguity has a negative impact on all our indicators: optimal cutting age, Faustmann value without risk aversion, and optimal utility level.

The value of information is the difference in terms of utility level between the risk case and the ambiguous one. We observe that the value of information is 1.57. As assumed in the theoretical model, the value of information for an ambiguity-averse forest owner is positive. The monetary evaluation of this information, given by Eq. (5) is €125. Such a value corresponds to about 7% of the Faustmann value; such a percentage is a common order of magnitude (Bontems and Thomas, 2000; Amacher et al., 2008; Williams and Johnson, 2015; Couture et al., 2018).

We can summarize the main results as follows:

- ***Result 1.1: The higher the risk is, the lower the optimal cutting age will be.***
- ***Result 1.2: Scenario ambiguity reduces the optimal cutting age.***
- ***Result 1.3: The value of information to eliminate scenario ambiguity is positive and high.***

3.2.2 Frequency ambiguity vs. scenario ambiguity

In Table 3, we present the results of the simulations for frequency ambiguity, and we compare the results obtained with those of scenario ambiguity. In this table, we consider that $p = (1/3, 1/3, 1/3)$. The row "1^{st} order" refers to the first-order approximation provided by Eq. (9).

Table 3: Simulation results for frequency ambiguity.

SCENARIO AMBIGUITY WITHOUT AMBIGUITY AVERSION						
		Optimal criterion $\sum_i p_i J(\lambda_i, T_p)$	Opt. crit. without risk aversion $\sum_i p_i J_0(\lambda_i, T_p)$ (€)	Opt. cutting age T_p (year)		
		50.18	1665	78		
FREQUENCY AMBIGUITY						
	Optimal criterion $E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]$	Certainty equivalent $\phi^{-1}\left(E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]\right)$	Opt. crit. without risk aversion $\sum_i p_i J_0(\lambda_i, T_*)$ (€)	Opt. cutting age T_* (year)	Value of information y_π^J	Value of information z_π^J (€)
1^{st} order	17.36	50.17		78	0.009	0.71
Exact	17.36	50.17	1665	78	0.009	0.71

Table 3 reveals that frequency ambiguity has no impact on the optimal cutting age (always 78 years), on the Faustmann value without risk aversion (always €1665/ha), or on the forest owner's utility level (50.17 rather than 50.18) compared to scenario ambiguity. Indeed, if we compare the optimal decision under scenario ambiguity without ambiguity aversion (i.e., with a linear ϕ function) with the optimal decision under frequency ambiguity, there is no difference. This does not change the optimal criterion either and, in fact, the value of information is almost zero.

The monetary value of information (computed in Eq. (8) for "Exact" and Eq. (10) for "1^{st} order") is very close to zero, meaning that forest owners do not value information that would eliminate frequency ambiguity despite their aversion to ambiguity ($s = 0.729$ for the simulations). Note that in Table 3, the first-order approximation provides the value of information with excellent precision.

It is possible to assess the value of the overall information that eliminates the two sources of ambiguity by summing the two information values obtained, i.e., a total value of €125 + €0.71, which is not negligible. The forest owner would therefore be willing to pay this amount of money to obtain the information that would eliminate these two ambiguities.

We can summarize the main results as follows:

- ***Result 2.1: Frequency ambiguity has no impact on the optimal cutting age.***
- ***Result 2.2: The value of information to eliminate frequency ambiguity is positive but close to zero.***

3.3 Sensitivity analysis

We carried out a sensitivity analysis on the risk aversion coefficient with three values (other than the benchmark of $r = 0.59$): $r = -0.5$ (risk loving), $r = 0$ (risk neutrality), $r = 0.9$ (high risk aversion). In the same way, we carried out a sensitivity analysis on the ambiguity aversion coefficient with three values (other than the benchmark of $s = 0.729$): $s = 0.1$ (high ambiguity aversion), $s = 0.9$ (ambiguity neutrality) and $s = 1.5$ (ambiguity loving). We observe the impact of the variation of r when s is fixed, and inversely for scenario ambiguity (Table 4) and frequency ambiguity (Table 5). We consider here the benchmark case where $\lambda = 1/55$. As previously done, we compare the risk with scenario ambiguity in Table 4 and scenario ambiguity with frequency ambiguity in Table 5. In order to ease the comparisons, we recall the benchmarks from Tables 2 and 3, highlighted in gray.

3.3.1 Risk vs. scenario ambiguity

In the following table, we present the results of sensitivity analysis under risk and we compare the results obtained with those for scenario ambiguity.

Table 4: Sensitivity analysis results for scenario ambiguity and for $\lambda = 1/55$.

RISK							
r	Risk rate λ (year^{-1})		Optimal criterion $J(\lambda_i, T_i)$	Opt. crit. without risk aversion $J_0(\lambda_i, T_i)$ (€)	Opt. cutting age T_0 (year)		
-0.5	1/55		257233	2460	114		
0	1/55		2655	2655	100		
0.59	1/55		51.75	1890	79		
0.9	1/55		33.61	627	67*		
SCENARIO AMBIGUITY							
r	s	Optimal criterion $\sum_i p_i \phi(J(\lambda_i, T_*))$	Certainty equivalent $\phi^{-1}\left(\sum_i p_i \phi(J(\lambda_i, T_*))\right)$	Opt. crit. without risk aversion $J_0(\lambda_1, T_*)$ (€)	Opt. cutting age T_* (year)	Value of information y_0^J	z_0^J (€)
-0.5	0.729	7008	188440	2575	108.5	68793	989
0	0.729	274.9	2218	2634	96	437	437
0.59	0.729	17.36	50.17	1807	78	1.57	125
0.9	0.729	12.87	33.27	616	67*	0.34	17782**
0.59	0.1	1.479	50.15	1762	77.5	1.60	127
0.59	0.729	17.36	50.17	1807	78	1.57	125
0.59	0.9	33.92	50.18	1806	78	1.57	124
0.59	1.5	355.7	50.2	1806	78	1.54	123

* A tree diameter of 45 cm is considered to be a harvestable diameter for the timber industry. Consequently, this diameter is used in the optimization, resulting in a cutting age of 67 years.
** This outlier is due to our calculation of the inverse utility, which becomes problematic and invalid for strong aversions.

Looking at the first part of Table 4 dedicated to risk, we can observe that the optimal cutting age for a risk-neutral forest owner is 100 years, whereas it is 114 years for a risk lover and 67 years for a highly risk-averse owner. The higher the risk aversion is, the lower the optimal cutting age will be. This trend is also true for risk rate λ of 1/47 and 1/23. This conclusion is common for a forest owner whose behavior is described by a CRRA utility function, as in our case. Indeed, when the forest owner's risk aversion increases, the "risk effect" that tends to reduce the cutting age becomes stronger than the other two effects, thus justifying the reduction of the rotation period.

When looking at the second part of Table 4 dedicated to scenario ambiguity, we can observe: (i) the impact of risk attitudes on an ambiguity-averse owner, and (ii) the impact of ambiguity attitude on a risk-averse owner. First, when the forest owner is ambiguity-averse, the higher the risk aversion is, the lower the optimal cutting age will be. Second, when the owner is risk averse, we find almost no effect of ambiguity aversion on the optimal cutting age (from 78 to 77.5 years). Ambiguity aversion weakly modifies the level of the optimized criterion. The stronger the ambiguity aversion is, the lower the level will be. Ambiguity aversion modifies the "wealth effect". However, this effect is weak, justifying that the value of information is not impacted by ambiguity aversion.

We see that ambiguity attenuates the effects of risk aversion: even a risk-lover owner (optimal age of 114) decreases the optimal cutting age under ambiguity (108.5). In this sense, ambiguity modifies the behavior for the different levels of risk aversion and tends to make the agent more cautious and therefore more risk-averse and to thus decrease the optimal cutting age. Consequently, ambiguity has the same effect as risk aversion.

We can summarize the main results as follows:

- ***Result 3.1: Under risk, the higher the risk aversion is, the lower the optimal cutting age will be.***
- ***Result 3.2: Under scenario ambiguity, risk aversion reduces the optimal cutting age, whereas ambiguity aversion has no impact.***
- ***Result 3.3: The value of information that resolves scenario ambiguity increases with risk aversion, but ambiguity aversion has no impact.***

3.3.2 Scenario ambiguity vs. frequency ambiguity

In the following table, we present the results of the sensitivity analysis for frequency ambiguity and we compare the results obtained with those for scenario ambiguity.

The impact of risk preferences and ambiguity preferences on the optimal cutting age is identical under scenario and frequency ambiguity. An original aspect of this table is the negative value of information when the forest owner is risk averse and strongly ambiguity loving. This means that the owner really does not want to know the information that would make it possible to reduce the frequency ambiguity. For such a risk-averse forest owner who loves ambiguity, the value of the total information is then lower than the value of the information to remove only scenario ambiguity. The owner is not willing to pay for the information that would remove the frequency ambiguity and would even like to receive money if this ambiguity is removed.

Note that in Table 5, the first-order approximation gives results of varying precision: if the forest owner is risk averse the approximation provides the value of information with an error of

Table 5: Sensitivity analysis results for frequency ambiguity.

SCENARIO AMBIGUITY WITHOUT AMBIGUITY AVERSION								
r				Optimal criterion $\sum_i p_i J(\lambda_i, T_p)$	Opt. crit. without risk aversion $\sum_i p_i J_0(\lambda_i, T_p)$ (€)	Opt. cutting age T_p (year)		
-0.5				192229	2086	109		
0				2231	2231	96		
0.59				50.18	1665	78		
0.9				33.27	617	67*		
FREQUENCY AMBIGUITY								
r	s		Optimal criterion $E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]$	Certainty equivalent $\phi^{-1}\left(E\left[\phi\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]\right)$	Opt. crit. without risk aversion $\sum_i p_i J_0(\lambda_i, T_*)$ (€)	Opt. cutting age T_* (year)	Value of information y_π^J	Value of information z_π^J (€)
-0.5	0.729	1^{st} order	7024	189000		108.5	3229	48
		Exact	7011	188558	2095	108.5	3671	55
0	0.729	1^{st} order	275	2219		96	12.3	12.3
		Exact	275	2218.4	2231	96	13	13
0.59	0.729	1^{st} order	17.36	50.17		78	0.009	0.71
		Exact	17.36	50.17	1665	78	0.009	0.71
0.9	0.729	1^{st} order	12.87	33.27		67*	0.001	31
		Exact	12.87	33.27	617	67*	0.001	31
0.59	0.1	1^{st} order	1.479	50.15		77.5	0.027	2.13
		Exact	1.479	50.15	1673	77.5	0.027	2.13
0.59	0.729	1^{st} order	17.36	50.17		78	0.009	0.71
		Exact	17.36	50.17	1665	78	0.009	0.71
0.59	0.9	1^{st} order	33.92	50.18		78	0.0032	0.246
		Exact	33.92	50.18	1665	78	0.0034	0.261
0.59	1.5	1^{st} order	355.7	50.20		78	-0.0159	-1.23
		Exact	355.7	50.20	1666	78	-0.0161	-1.25

* A tree diameter of 45 cm is considered to be a harvestable diameter for the timber industry. Consequently, this value is used in the optimization, resulting in a cutting age of 67 years.

1%, whereas if the forest owner is risk loving, then the value of information is provided only with an accuracy of 10%.

We can summarize the main results as follows:

- ***Result 4.1: For the frequency ambiguity level, risk aversion reduces the optimal cutting age, whereas ambiguity aversion has no impact. This result is identical to the one obtained for scenario ambiguity.***
- ***Result 4.2: The value of information that resolves frequency ambiguity increases with risk aversion and ambiguity aversion.***

4 Discussion

The results of our numerical simulations allow us to evaluate the effect of risk and ambiguity on the optimal cutting decision of a risk-averse and ambiguity-averse forest owner. The simulations make it possible to confirm the result of Reed (1984) that risk reduces the optimal cutting age. We complement this result by showing that the higher the risk level is, the lower the optimal cutting age will be (Result 1.1). In addition, we show that the higher the forest owner's risk aversion is, the lower the optimal cutting age will be (Results 3.1 and 3.2). We go further and demonstrate that scenario ambiguity also reduces the optimal cutting age (Result 1.2), whereas frequency ambiguity has no effect (Result 2.1). Moreover, ambiguity preferences have no effect on the optimal cutting age for both ambiguities (Results 3.2 and 4.1). Indeed, the optimal cutting age is approximately the same for ambiguity-averse, ambiguity-loving and ambiguity-neutral owners both for the scenario and frequency ambiguities. This result is in line with Brunette et al. (2020) who show that ambiguity aversion has no effect on the forest manager's adaptation decisions towards climate change, whereas risk aversion has a significant one. It seems that risk preferences drive the results and that ambiguity preferences do not.

All in all, scenario ambiguity reduces the optimal cutting age, whereas frequency ambiguity and ambiguity aversion have no effect. This means that only scenario ambiguity is relevant in the end. A potential explanation is that the effect of ambiguity has been captured by risk aversion and by the higher levels of risk that ambiguity generates, as well as by the noise that disturbs the probability, which has little impact because the probability level of the storm risk is low. On the other hand, due to the specification of the criterion considered, ambiguity aversion will weakly modify the level of the optimized criterion.

Concerning the value of information, we show that it is positive and significant for scenario ambiguity (Result 1.2), whereas it is close to zero for frequency ambiguity (Result 2.1). In addition, we also observe that the value of information that resolves both ambiguities increases with risk aversion (Results 3.3 and 4.2). However, only the value of information to remove the frequency ambiguity increases with ambiguity aversion (Results 3.3 and 4.2). This last result is in line with Snow (2010) who shows that the value of information that resolves ambiguity increases with greater ambiguity aversion. These results mean that forest owners are willing to pay to have precise information that will allow them to resolve the scenario ambiguity (and to then make decisions in a risky environment), and this willingness to pay increases with the forest owner's risk aversion.

Such information may be helpful for public policy issues. Indeed, the orientation of public efforts to try to reduce ambiguity is an increasingly frequent question. More precisely, our results reveal that forest owners would like to remove the ambiguity when it is characterized by low imprecision on the probabilities (three possible values in this article: λ_0, λ_1, λ_2, i.e., scenario ambiguity) rather than when imprecision is high (uncertainty on the distribution of the rate of return, i.e., frequency ambiguity). This means that public authorities should invest in the reduction of the ambiguity on scenarios in order to allow forest decision-makers to take relevant management decisions.

In this article, we assume that probability is ambiguous. It may be interesting to consider another type of ambiguity, that of the outcome. Indeed, we classically assume that ambiguity concerns the probability of the occurrence of the event. However, ambiguity may also characterize the amount of the damage (outcome level). We can imagine that ambiguity on the outcome plays a role too. The literature in experimental economics has already addressed the question of the individual's behavior towards various sources of ambiguity. This literature shows that individuals prefer facing the ambiguity of the probability rather than the ambiguity of the outcome. For example, Brunette et al. (2022) already showed that for a low probability level, subjects prefer facing uncertainty as to the probability rather than uncertainty as to the outcome, whereas the opposite is true for medium and high probability levels. Du and Budescu (2005) reported that individuals prefer to increase the precision of the outcomes rather than the precision of the probabilities. This result corresponds to Result 2.1. Indeed, we show that the value of information is very low, meaning that individuals are not ready to pay to increase the precision of the probabilities. Perhaps they are more willing to pay to reduce the imprecision of the outcome? Further research in this direction is necessary.

5 Conclusion

In this article, we propose to introduce ambiguity within a classical forest economics approach. For that purpose, we extend the model of Reed (1984) by considering two different types of ambiguity. We then solve the model numerically through simulations on a beech stand exposed to a storm risk that increases due to climate change. The main conclusions are that risk and risk aversion significantly reduce the optimal cutting age. In addition, we show that both types of ambiguity reinforce the effect of risk. Inversely, ambiguity aversion has no effect. The other main message is that the information to resolve scenario ambiguity has a positive and high value, whereas the value is close to zero for the information to resolve frequency ambiguity.

An interesting extension of this article would be to experimentally test our theoretical and simulation results. Indeed, we can easily imagine placing forest owners in the different scenarios considered in this paper and then estimating the value of information. An assessment of their preferences towards risk and ambiguity could also be carried out.

Acknowledgements

This work was supported by a grant overseen by the French National Research Agency (ANR) as part of the "Investissements d'Avenir" program (ANR-11-LABX-0002-01, Lab of Excellence ARBRE).

Appendix A: The first-order approximation

The expectation $E\left[\phi\left(\sum_i q_i J(\lambda_i, T)\right)\right]$ may be expressed as follows:

$$\phi\left(\sum_i p_i J(\lambda_i, T)\right) + \frac{1}{2}E\left[\left(\sum_i (q_i - p_i)J(\lambda_i, T)\right)^2\right]\phi''\left(\sum_i p_i J(\lambda_i, T)\right)$$
$$+\frac{1}{6}E\left[\left(\sum_i (q_i - p_i)J(\lambda_i, T)\right)^3 \phi^{(3)}\left(\sum_i (p_i + t(q_i - p_i))J(\lambda_i, T)\right),\ 0 < t < 1\right]$$

where: $E\left[\left(\sum_i (q_i - p_i)J(\lambda_i, T)\right)^2\right] = \sum_i Var(q_i)J(\lambda_i, T)^2 + 2\sum_{j>i} Cov(q_i, q_j)J(\lambda_i, T)J(\lambda_j, T))$.

In the particular case of a Dirichlet distribution (Balakrishnan and Nevzorov, 2005), the associated covariance matrix is composed of $Var(q_i) = \frac{p_i(1-p_i)}{\alpha_0 + 1}$ and $Cov(q_i, q_j) = -\frac{p_i p_j}{\alpha_0 + 1}$. After reordering the terms, we obtain:

$$\begin{aligned}
E\left[\left(\sum_i (q_i - p_i)J(\lambda_i, T)\right)^2\right] &= \frac{1}{\alpha_0 + 1}\left[\sum_i p_i(1-p_i)J(\lambda_i, T)^2 - \sum_{j\neq i} p_i p_j J(\lambda_i, T)J(\lambda_j, T)\right] \\
&= \frac{1}{\alpha_0 + 1}\left[\sum_i p_i J(\lambda_i, T)^2 - \sum_{i,j} p_i p_j J(\lambda_i, T)J(\lambda_j, T)\right] \\
&= \frac{1}{\alpha_0 + 1}\left[\mathcal{J}_2(T) - \mathcal{J}_1(T)^2\right]
\end{aligned}$$

where $\mathcal{J}_k(T) = E_\lambda[J(\lambda, T)^k] = \sum_i p_i J(\lambda_i, T)^k$.

Hence $E\left[\left(\sum_i (q_i - p_i)J(\lambda_i, T)\right)^2\right] = \frac{V(J(\lambda, T))}{\alpha_0 + 1}$

where: $V(J(\lambda, T)) = E_\lambda\left[(J(\lambda, T) - \mathcal{J}_1(T))^2\right] = \mathcal{J}_2(T) - \mathcal{J}_1(T)^2$

$$E\left[\phi\left(\sum_i q_i J(\lambda_i, T)\right)\right] = \phi\left(\mathcal{J}_1(T)\right) + \frac{1}{2}\frac{V(J(\lambda, T))}{\alpha_0 + 1}\phi''(\mathcal{J}_1(T)) + O\left(\frac{1}{(\alpha_0 + 1)^2}\right)$$

The value of information is defined by:

$$\phi(\mathcal{J}_1(T_p) - y_\pi^J) = \max_T \left[\phi(\mathcal{J}_1(T)) + \frac{1}{2}\frac{V(J(\lambda, T))}{\alpha_0 + 1}\phi''(\mathcal{J}_1(T)) + O\left(\frac{1}{(\alpha_0 + 1)^2}\right)\right]$$

Let the parameter $\epsilon = \frac{1}{\alpha_0 + 1}$. Applying the envelop theorem to differentiate with respect to ϵ, we obtain the expression of the derivative of y_π^J with respect to ϵ at $\epsilon = 0$:

$$(y_\pi^J)'_\epsilon = \frac{1}{2} V(J(\lambda, T_p) \frac{-\phi''}{\phi'}(\mathcal{J}_1(T_p))$$

Hence $y_\pi^J = \widehat{y}_\pi^J + O\left(\frac{1}{(\alpha_0+1)^2}\right)$ and $E\left[\phi\left(\sum_i q_i J(\lambda_i, T_p)\right)\right] = \widehat{\phi}_\pi^J + O\left(\frac{1}{(\alpha_0+1)^2}\right)$ where:

$$\widehat{y}_\pi^J = \frac{1}{2}\frac{V(J(\lambda, T_p)}{\alpha_0 + 1}\frac{-\phi''}{\phi'}(\mathcal{J}_1(T_p))$$

and $\widehat{\phi}_\pi^J = \phi(\mathcal{J}_1(T_p)) + \frac{1}{2}\frac{V(J(\lambda, T_p))}{\alpha_0 + 1}\phi''(\mathcal{J}_1(T_p))$.

Moreover, the first-order approximation of the monetary value of information is given by:

$$\widehat{z}_\pi^J = u^{-1}\left(\sum_i p_i J(\lambda_i, T_p)\right) - u^{-1}(\widehat{EC}) \text{ where } \widehat{EC} = \phi^{-1}\left(\widehat{\phi}_\pi^J\right)$$

Appendix B: Exponential function for ambiguity

In addition to the power utility function considered in the simulations, we test another functional form, an exponential negative: $\phi_1(x) = \frac{1-e^{-sx}}{s}$. The value of the ambiguity aversion coefficient s is arbitrarily set at 0.001 because, to our knowledge, there is no empirical study that has quantified this coefficient with such a specification.

The simulation results associated with this functional form are presented in this section.

Table 6: Simulation results for scenario ambiguity with ϕ_1.

RISK						
Risk rate λ (year^{-1})		Optimal criterion $J(\lambda_i, T_0)$	Opt. crit. without risk aversion $J_0(\lambda_i, T_0)$ (€)	Opt. cutting age age T_0 (year)		
1/55	(benchmark)	51.75	1890	79		
1/47		51.18	1794	78.5		
1/23		47.75	1335	76		
SCENARIO AMBIGUITY						
	Optimal criterion $\sum_i p_i \phi_1(J(\lambda_i, T_*))$	Certainty equivalent $\phi_1^{-1}\left(\sum_i p_i \phi_1(J(\lambda_i, T_*))\right)$	Opt. crit. without risk aversion $J_0(\lambda_1, T_*)$ (€)	Opt. cutting age T_* (year)	Value of information y_0^J	z_0^J (€)
ϕ_1	48.94	50.18	1807	78	1.54	122

Table 7: Simulation results for frequency ambiguity with ϕ_1.

SCENARIO AMBIGUITY WITHOUT AMBIGUITY AVERSION						
		Optimal criterion $\sum_i p_i J(\lambda_i, T_p)$	Opt. crit. without risk aversion $\sum_i p_i J_0(\lambda_i, T_p)$ (€)	Opt. cutting age T_p (year)		
		50.18	1665	78		
FREQUENCY AMBIGUITY						
	Optimal criterion $E\left[\phi_1\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]$	Certainty equivalent $\phi_1^{-1}\left(E\left[\phi_1\left(\sum_i q_i J(\lambda_i, T_*)\right)\right]\right)$	Opt. crit. without risk aversion $\sum_i p_i J_0(\lambda_i, T_*)$ (€)	Opt. cutting age T_* (year)	Value of information y_π^J	z_π^J (€)
1^{st} order	48.94	50.18		78	0.0016	0.124
Exact	48.94	50.18	1665	78	0.0016	0.124

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