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Central bank's stabilization and communication policies when firms have motivated overconfidence in their own information accuracy or processing

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Abstract

Using a simple microfounded macroeconomic model with price making firms and a central bank maximizing the welfare of a representative household, it is shown that the presence of firms' *motivated beliefs* has stark consequences for the conduct of optimal communication and stabilization policies. Under pure communication (resp. communication and stabilization policies), motivated beliefs about own private information (resp. own ability to process information) reverse the bang-bang solution of transparency (resp. opacity with full stabilization) found in the literature under objective beliefs and lead to intermediate levels of communication (and stabilization).

Keywords: motivated beliefs, public and private information (accuracy), overconfidence, communication policy, stabilization policy.

JEL codes: D83, D84, E52, E58.

1 Introduction

Central banks' monetary policy mainly deals with the management of expectations. As Woodford (2003a, p.15) states it, "for [monetary policy to be most effective] not only do expectations about policy matter, but, at least under current conditions, very little else matters." Forward guidance policy is an emblematic example of instrument for managing expectations: by disclosing information on its future policy, the central bank aims to influence market expectations of future policy rates and thereby long-term interest rates and firms'

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inflation expectations. One characteristic of many expectations is that they are held in an overconfident manner,[1] possibly responding to motivational considerations (Bénabou and Tirole, 2016).[2] Firms may be tempted to interpret or process their information in order to increase their anticipatory utility, meaning that firms' managers experience pleasant emotions from thinking that they can reach high profits, for example by choosing to perceive their information in a more accurate manner or by considering their abilities to process information as better than they really are. Such motivated beliefs that maximize utility benefits of good outcomes characterize wishful thinking (or willfull blindness) in firms' information accuracy or processing. Firms' motivated beliefs in the accuracy of their information or in their ability to process information generate overconfidence in their own information or own abilitiy to process information. Motivated beliefs naturally affect the way firms' managers set their prices. Under these circumstances, how can central banks' communication and stabilization policies adjust to the induced distortions in firms' price setting? The aim of this paper is precisely to study how the central bank responds to the price setting of incompletely informed, overconfident firms.

Most economic models considering the role of heterogeneous and dispersed information derive policy recommendations on the assumption that private agents form objective beliefs. In the realm of monetary policy, for example, motivated beliefs about private agents' information knowledge or process have not yet received attention. This paper determines optimal stabilization and communication policies when firms exhibit motivated subjective beliefs about (i) the quality of their *own* private information on the fundamental (labor supply) shocks affecting the economy or (ii) their ability to process information in general, whether private or disclosed by the central bank. Using a simple model with price setting firms, shocks that do not affect market power and a policy maker maximizing the welfare of a representative household, it is shown that the presence of endogenously motivated subjective beliefs has stark consequences for the conduct of optimal stabilization and communication policies.

Two technical traits of our model help to keep it both tractable and accurate. First, following Maćkowiak and Wiederholt (2009), we have systematically resorted to second-order Taylor approximations of objective functions (rather than linear approximations) and applied Gaussian noises to deviations in the logarithms of the variables (thus avoiding violations of non-negativity constraints). Second, contrary to what is largely practiced in the literature, we have refrained from approximating by the arithmetic mean the generalized means issued from CES utility functions (an approximation losing information on dispersion).

In the model, each firm derives anticipatory utility from its profit prospects, and accordingly faces a trade-off: it can accept the grim implications of either

[1] A vast experimental and empirical literature shows that economic agents generally overestimate the accuracy of their expectations (Dunning et al., 1990). See Barrero (2021) for an application to firms' growth outcomes. The literature on the overconfidence bias also calls this 'overprecision' bias or 'miscalibration' (Ben-David et al., 2013).

[2] Bénabou and Tirole (2016, p. 145) point out that"individuals will overestimate or underestimate their own abilities depending upon which distortion is advantageous."

poorly accurate own information about the fundamental shock or poor capacity to process that information as well as central bank public disclosures and act in conformity, or else maintain hopeful beliefs by discounting and denying the fact that its information is poorly accurate or its abilities in information processing are low at the risk of making overoptimistic decisions.[3] The latter option is costly so that at the limit of an infinite cost, firms form objective beliefs. When such cost is limited however, in equilibrium firms exhibit overconfidence in the accuracy of their private signals (i.e. they overestimate the precision of their private information) or in their ability to process the information they receive (i.e. they believe they can extract the appropriate signal from their own private information or from the noisy public information sent by the central bank). In both cases, they may accordingly rely too much on private information to set their price, which can raise price dispersion and deteriorate welfare.

Because the trade-off between the supposed benefits of overconfidence and the cost of being objective is assumed to depend upon the relative quality of public and private informations, by influencing their relative precision central bank communication and stabilization policies have a role to play. We analyze the effect of endogenous subjective beliefs (motivated beliefs) on optimal policy when the central bank

(i) only discloses information to market participants (pure communication),

(ii) discloses information when it takes an action (communication and stabilization policies), and

(iii) takes an action that signals its economic assessment to market participants (signalling stabilization policy).

The case where the central bank only discloses information to market participants (pure communication) has been dealt with under objective beliefs by Morris and Shin (2002), who show in an abstract beauty contest game that disclosing public information when agents additionally receive private signals on the fundamentals can deteriorate welfare. Angeletos and Pavan (2007) have explained that transparency is always beneficial in a microfounded set-up, as the equilibrium degree of coordination is lower than the efficient degree of coordination, rationalizing the findings of e.g. Hellwig (2005). Under motivated beliefs about the accuracy of firms' information precision, we reverse the latter result. Under realistic assumptions, about the cost of motivated beliefs and the

[3] Note that this tension between holding 'accurate' beliefs, which lead to ex-post optimal actions and 'desirable' beliefs, which increase contemporaneous utility flows does not rely on firms exhibiting 'multiple selves'. Instead, following Banerjee et al. (2020), they consciously hold a single set of beliefs about the world. Indeed, as a modelling device, we assume that it is costly to deviate from the objective distribution. Banerjee et al. (2020, footnote 6, p.6) write that in doing so, their intention is "to capture the idea that individuals behave as though deviating too far from accurate beliefs is costly, perhaps due to previous experience. As in the literature on robust control, our use of the objective distribution in specifying the cost does not imply that the agent "knows" the true distribution. Caplin and Leahy (2019) discuss this distinction in more detail."

precisions of private and public signals, we obtain an optimal interior degree of transparency. Intuitively, by increasing the precision of its disclosure, the central bank makes firms overall more informed about the fundamental shocks and therefore reduces the cost of firms mistakenly believing that their private information is very precise. Firms thus tend to be even more overconfident in their private information and tend to overly rely on their private information when setting their prices. This raises price dispersion and deteriorates welfare. Under realistic conditions, the optimal communication policy is therefore an intermediate level of transparency.

The case where the central bank discloses information when it takes an action (communication and stabilization policies) has been tackled under objective beliefs by James and Lawler (2011), who show that full opacity is optimal. Indeed, by taking an action that is hidden from the public, the central bank succeeds in stabilizing the economy without creating overreaction to any disclosure. This result is robust to a microfounded set-up (Baeriswyl et al., 2020). We can reverse this result when firms hold motivated beliefs about their capacity to process their own private information as well as central bank public disclosures. The rationale for this result is that by being opaque, the central bank reduces the cost of firms mistakenly believing that they are able to process information. Firms thus tend to be more confident in their ability to process information correctly. They therefore rely more both on the public signal disclosed by the central bank *and their own private information*. More reliance on private information deteriorates welfare. In this case, it is optimal for the central bank to set an intermediate level of transparency, while implementing an intermediate level of stabilization policy (provided that the cost of being irrational is high).

Finally, the case where the central bank takes an action that signals its economic assessment to market participants (signalling stabilization policy) has been analyzed by Baeriswyl and Cornand (2010). They argue that taking an action inevitably provides public information because it signals the central bank belief to firms, qualifying the possibility of taking an action under opacity. Under objective beliefs, Baeriswyl et al. (2020) show that the optimal stabilization policy is indeterminate, but taking a signalling action can improve welfare compared to implementing no action. We show that this result is robust to both types of motivated beliefs.

Overall, motivated beliefs do not affect the central bank's policy under signalling stabilization. However, under pure communication (respectively communication and stabilization policies), motivated beliefs about own private information (respectively own abilities to process information) reverse the corner, bang-bang, solution of transparency (respectively opacity with full stabilization), found in the literature under objective beliefs and leads to intermediate levels of communication (and stabilization) policies.

The paper is organized as follows. Section 2 relates the paper to the literature. Section 3 describes the economy. Section 4 presents the information and belief structures, the timing of the game and solves for the equilibrium behavior of firms. Section 5 derives central bank stabilization and communication policies under objective beliefs, while section 6 considers the case of motivated beliefs.

Section 7 concludes the paper.

2 Related literature

Our paper shares three main ingredients: (i) coordination games (beauty contest framework) with heterogeneous and dispersed information (in the vein of Morris and Shin, 2002, and Angeletos and Pavan, 2007) applied to a macro-setting, (ii) central bank's monetary (stabilization) policy, and (iii) motivated beliefs or wishful thinking (Bénabou and Tirole, 2016), which translates in our framework into firms' overestimation of signals' precision or processing.

2.1 Beauty contest with heterogeneous and dispersed information

A growing literature has addressed the issue of central banks' communication in coordination games with heterogeneous and dispersed information. Morris and Shin (2002) presented a Keynesian beauty contest game where the equilibrium behavior of economic agents is driven by both a fundamental and a coordination motive. The focal role that public information exerts on higher-order beliefs of agents gives rise to an overreaction, which may be detrimental to welfare. If public information is not accurate, it distorts the market outcome away from the economic fundamental, challenging the presumed benefit of central bank transparency. Angeletos and Pavan (2007) questioned the pro-transparency result of Morris and Shin in microfounded beauty-contest set-ups, where the equilibrium degree of coordination is below the efficient degree of coordination, making public disclosure always beneficial. Hellwig (2005) and Angeletos, Iovino, and La'O (2016) studied the welfare consequences of public disclosures in microfounded business cycle models.

2.2 Central bank's monetary policy under heterogeneous and dispersed information

While Morris and Shin (2002) refer to the case where the provider of public information only considers the possibility of disclosing information, James and Lawler (2011) analyze the optimal disclosure strategy when the central bank takes an action and find that full opacity is optimal. Some microfounded macroeconomic models also give a role to central bank stabilization policy (possibly in addition to communication policy) under dispersed and heterogeneous information (e.g. Woodford, 2003b; Adam, 2007; Baeriswyl and Cornand, 2010; Lorenzoni, 2010; Paciello and Wiederholt, 2014; Angeletos and La'O, 2020; Baeriswyl et al., 2020; Benhima and Blengini, 2020; Chahrour and Ulbricht, 2021).

2.3 Motivated and subjective beliefs

Our paper contributes to this literature on communication and stabilization policies under dispersed and heterogeneous information but departs from it by relaxing the key assumption of rational expectations (i.e., the knowledge of the true joint distribution of signals and fundamentals) and instead allowing for subjective beliefs, so that firms may incorrectly interpret the information available to them. Following Bénabou and Tirole (2016), these subjective beliefs are not given in an exogenous manner but are instead endogenized through motivated reasoning. Banerjee et al. (2020) extend the generalized quadratic-Gaussian model of Angeletos and Pavan (2007) to allow for motivated belief choice about the precision of both private and public information about fundamentals.[4] We differ from them in various respects. First, while their set-up is abstract and does not allow to obtain analytical results, we derive the consequence of motivated beliefs in a fully microfounded model and obtain analytical results. Second, since our model is a microfounded one, we are able to study the role of central bank stabilization policy. Third, they apply motivated beliefs to both types of signals. Instead, we apply motivated beliefs to firms' own private information and to their ability to process central bank disclosure.

Finally, our work relates to the literature that introduces overconfidence bias in models with dispersed and heterogeneous information. While there is a vast psychological literature on overconfidence, especially in finance, the overconfidence bias is rather novel in macro models. Benigno and Karantounias (2019) and Broer and Kohlhas (2019) represent two recent contributions aiming at refining models to account for empirical evidence. Our paper differs from these as, instead of imposing an exogenous bias, over-precision of private information results from an endogenous choice of firms that maximize their anticipatory profits and incur a cost from deviating from rational expectations. In equilibrium, we get overconfidence in information precision and processing. Moreover, we study the optimal communication and stabilization policies of the central bank.

3 The economy

We use a simple New Keynesian model, which is a variant of Adam (2007).

3.1 The household

The representative household derives utility from consuming a volume C of a composite good and disutility from supplying an amount L of homogeneous labour:

$$U(\Theta C) - \Theta L, \tag{1}$$

[4] Other works that generate endogenously optimistic biases in beliefs, by taking into account the utility benefits of good outcomes include Brunnermeier and Parker (2005), Brunnermeier et al. (2007), and Caplin and Leahy (2019).

where U is a twice-differentiable, increasing and strictly concave function, and where Θ is a random variable allowing to introduce labor supply shocks around 1 ($\mathbb{E}(\Theta) = 1$). The household maximizes its utility in (C, L) under the budget constraint $PC \leq WL + \Pi$, where W is the competitive money wage, P the price index of the components of the composite good and Π the nominal aggregate profit received by the household. The first order maximization condition requires the marginal rate of substitution of leisure for consumption to be equal to the real wage:

$$\frac{1}{U'(\Theta C)} = \frac{W}{P}. \tag{2}$$

This equation determines C, and L is then computed by inserting C in the budget equation.

The volume C of the composite good is a CES aggregate of a continuum of differentiated products:

$$C = \left(\int_0^1 C_i^{\frac{s-1}{s}} di \right)^{\frac{s}{s-1}}, \tag{3}$$

where C_i is the output of firm i and s is the constant elasticity of substitution between the differentiated goods, which we will assume to be larger than 1 (the case of substitutable goods). The consumer minimizes the expenditure $\int_0^1 P_i C_i di$ required to ensure a volume C of consumption, that is, under the constraint (3). The first order condition for this minimization gives the demand for each good i:

$$C_i = \left(\frac{P_i}{P} \right)^{-s} C, \text{ with } P = \left(\int_0^1 P_i^{1-s} di \right)^{\frac{1}{1-s}} \tag{4}$$

as the price index of all the differentiated goods, so that $\int_0^1 P_i C_i di = PC = Z$, the nominal expenditure that we will assume given to begin with, and then under the control of the central bank.

Each firm $i \in [0, 1]$ produces the quantity C_i of a single differentiated good with C_i units of labor. Hence, equilibrium in the labor market requires the condition $L = \int_0^1 C_i di$. In a symmetric equilibrium, $L = C$ and the first order utility maximization condition (2) implicitly defines a labor supply function. For simplicity, we shall assume this function to be iso-elastic by taking $U(\Theta C) = (\Theta C)^{1-\xi} / (1 - \xi)$, which gives:

$$\frac{W}{P} = \frac{1}{U'(\Theta C)} = (\Theta C)^{\xi}, \text{ with } \xi \in (0, 1). \tag{5}$$

The parameter ξ, which is the coefficient of relative risk aversion, has been restricted to be smaller than 1, resulting in prices being strategic complements, the case on which we want to focus.

Without symmetry across firms, the arithmetic mean $L = \int_0^1 C_i di$ of the employment (or the output) $L_i = C_i$ of all firms is higher than the mean

$C = \left(\int_0^1 C_i^{\frac{s-1}{s}} di\right)^{\frac{s}{s-1}}$ obtained by applying the CES aggregator, which is intermediate between the two limits of the arithmetic mean, when $s \to \infty$, and of the geometric mean, when $s \to 1$ (see Appendix A). As a consequence, asymmetry is welfare degrading: as dispersion of the output levels across firms increases, the same level of consumption requires more and more labor to be feasible.

3.2 The firms

The information structure assumed in this paper will be explicitized in the next section. Now it suffices to say that neither the firms (nor the central bank) observe the realizations of the fundamental Θ; instead, they receive some signals on Θ.

There is a continuum of identical firms represented by the interval $[0, 1]$, each firm i being assumed to set its price P_i so as to maximize its expected *real* profit (deflated by the price index P), conditional on its information set Γ_i (which allows to form expectations of C and P):

$$\mathbb{E}\left[\left(\left(\frac{P_i}{P}\right)^{1-s} - (\Theta C)^{\xi}\left(\frac{P_i}{P}\right)^{-s}\right) C \middle| \Gamma_i\right]. \tag{6}$$

Following Maćkowiak and Wiederholt (2009), we work with a log-quadratic approximation of the profit function around the nonstochastic solution of the model, obtained under certainty ($\Theta = 1$), perfect information and symmetry, namely

$$C^* = (1 - 1/s)^{1/\xi} \text{ and } P^* = W^*/(1 - 1/s) = \overline{Z}/C^*, \tag{7}$$

where $\overline{Z}$ is some nominal expenditure, which is given in the absence of any central bank manipulation. Notice that $1/s$ is the firms' degree of monopoly, which introduces a price distortion contracting consumption. If competition were perfect, we would indeed obtain the efficient equilibrium values $C^{**} = 1$ and $P^{**} = W^{**} = \overline{Z}$.

Using (2) and taking a lower case letter to denote the log-deviation of the variable from its value at the nonstochastic solution, the real profit can be rewritten as

$$\pi(p_i - p, c, \theta) = (1 - 1/s)^{1/\xi}\left[e^{c-(s-1)(p_i-p)} - (1 - 1/s)\, e^{\xi\theta+(1+\xi)c-s(p_i-p)}\right], \tag{8}$$

where $\theta = \ln\Theta - \ln 1 = \ln\Theta$, $p_i = \ln P_i - \ln P^*$ and $p = \ln P - \ln P^*$, so that $p_i - p = \ln P_i - \ln P$. We show in Appendix B that the second order Taylor approximation of this function at the origin is given (up to a constant with respect to $p_i - p$) by

$$\widetilde{\pi}(p_i - p, c + \theta) = (s - 1)(p_i - p)\left(\xi(c + \theta) - \frac{1}{2}(p_i - p)\right). \tag{9}$$

The first order condition for the maximization in $p_i - p$ of this function can be formulated as the log-linear equation:

$$\arg\max_{p_i} \tilde{\pi}\left(p_i - p, c + \theta\right) = p + \xi\left(c + \theta\right) \equiv \widehat{p}. \tag{10}$$

Given some realization of the fundamental θ, setting a price p_i that differs from $\widehat{p}$ leads to a profit loss which, by (9), writes:

$$\tilde{\pi}\left(\widehat{p} - p, c + \theta\right) - \tilde{\pi}\left(p_i - p, c + \theta\right) = \frac{s-1}{2}\left(p_i - \widehat{p}\right)^2. \tag{11}$$

We see that price dispersion increases the aggregate profit loss. As for the household, asymmetry is, for firms, welfare degrading.

Now, since p, c and θ are not known with certainty, firm i sets a price maximizing its *expected* profit, hence equal to its expectation of $\widehat{p}$, conditional on its information:

$$p_i = \mathbb{E}_i\left[\widehat{p}\right] = \mathbb{E}_i\left[p + \xi\left(c + \theta\right)\right], \text{ with } \mathbb{E}_i \equiv \mathbb{E}\left[\cdot \mid \Gamma_i\right]. \tag{12}$$

As $p + c = z$, where z is the log-deviation of the nominal aggregate expenditure Z (0 if $Z = \overline{Z}$, a constant), we can alternatively take as firm i's pricing rule

$$p_i = \left(1 - \xi\right)\mathbb{E}_i\left[p\right] + \xi\mathbb{E}_i\left[z + \theta\right]. \tag{13}$$

In other words, the expected profit maximizing price (deviation) is a convex combination of the expected mean price (deviation), reflecting a *coordination motive*, and of the expected sum of the fundamental and policy deviations, reflecting a *fundamental motive*. The weight $1-\xi$ put on coordination increases as the inverse labour supply elasticity ξ decreases, augmenting firms' market power. By this pricing rule, the price set by firm i responds positively to a demand pull (triggered by z), to a cost push (triggered by θ) and, through p, to others' price strategies (the assumption $\xi < 1$ entails, as stated above, strategic complementarity).

3.3 The central bank

The central bank seeks to maximize the expected welfare of households conditional on the information it receives about the fundamental θ. Indeed, the central bank receives a noisy signal $y = \theta + \eta$, with $\eta \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$. The welfare function is described in subsubsection 3.3.1. To maximize the expected welfare, the central bank can disclose to the firms information about the realization of the fundamental and/or take a policy action affecting z, depending on the considered operational framework (Baeriswyl *et al.* 2020). These operational frameworks are presented in subsubsection 3.3.2.

3.3.1 The welfare function

Central bank policy decisions are assumed to be taken so as to maximize a social welfare function. Since profits are entirely distributed to the representative household, welfare can be simply identified with household's utility

$$U(\Theta C) - \Theta L = U(\Theta C) - \Theta \int_0^1 \left(\frac{P_i}{P}\right)^{-s} C di. \tag{14}$$

As shown in Appendix C, after approximating at the second order and neglecting a constant term as well as the remainder term, we may refer to the transformed welfare function:

$$V\left(\sigma_{\mathbf{p}}^2\right) = -\frac{s\xi - 1}{\xi}\left(\frac{\sigma_{\mathbf{p}}^2}{2}\right) - \frac{1}{2}\left(\frac{(s-1)(3s\xi - 1)}{\xi^2} + s^2\right)\left(\frac{\sigma_{\mathbf{p}}^2}{2}\right)^2. \tag{15}$$

We assume that $s\xi \geq 1$, so that V is a decreasing function, the minimization of price variance $\sigma_{\mathbf{p}}^2$ being consequently the objective of the central bank.

3.3.2 Three operational frameworks

We will distinguish three operational frameworks: two with a single policy instrument and one with two policy instruments.

The first is *pure communication*, an example of which is forward guidance, whereby the central bank influences the behavior of economic agents by simply disclosing information. This situation corresponds to the framework of Morris and Shin (2002), examining how the central bank should optimally communicate when it takes no stabilizing action (implying $z = 0$ in our context). Following Baeriswyl and Cornand (2014), to allow for an intermediate level of disclosure, we assume that the central bank chooses the variance σ_ϕ^2 of the idiosyncratic noise affecting the signal $y_i = y + \phi_i$, with $\phi_i \sim N(0, \sigma_\phi^2)$, that it communicates. This noise captures the idea that each firm may interpret differently the same equivocal statement made by the central bank, rather than the idea that the central bank discloses a specific signal to each firm. It thus formalizes the notion that the central bank communicates its information y with more or less ambiguity. The signal y_i can be considered as a semi-public signal.[5] Under *transparency*, all firms interpret without ambiguity the same unequivocal signal ($\sigma_\phi^2 = 0$). The central bank disclosure y is then a public signal that is common knowledge among firms. Under *opacity*, each firm interprets differently its individual signal that contains an infinite idiosyncratic noise ($\sigma_\phi^2 \to \infty$), and the central bank disclosure does not contain any valuable information.

In the case of *signalling stabilization*, the central bank does not try to blur the public signal it receives, which is common knowledge among the firms, thanks to the full observation of the stabilization action z, taken by the central bank. This situation corresponds to the framework of Baeriswyl and Cornand (2010), examining how the central bank should optimally take its action when it is perfectly observable by the firms. Formally, $\sigma_\phi^2 = 0$, the central bank setting z aiming at the stabilization of the economy, by neutralizing the shocks on

[5]For other ways to model intermediate degrees of transparency in beauty contest games, see Cornand and Heinemann (2008) and Myatt and Wallace (2012, 2014).

the fundamental. Ideally, under perfect information, the central bank would choose $z = -\theta$. However, as the central bank has only the information y on the realization θ of the fundamental, it chooses instead $z = -(\rho y + (1-\rho)\,0) = -\rho y$, where 0 is the non-stochastic fundamental value and $\rho \in [0,1]$ is the value of the policy instrument, set by the central bank and known by the firms.

In the case of a two-instruments policy, covering *communication and stabilization*, the central bank takes an action and discloses information. This situation corresponds to the framework of James and Lawler (2011), examining how the central bank should optimally combine its action and disclosure.[6] In the present context, this means that the central bank chooses σ_ϕ^2 and ρ.

4 Information and beliefs, timing and equilibrium

We start by explicitizing the information structure and introducing the notion of firms' motivated beliefs concerning the quality of their private information or their ability to treat it. Then we describe the timing of the game. Finally, we will characterize symmetric equilibria when firms' price strategies are linear affine with respect to the (private and semi-public) signals they receive.

4.1 Information structure and motivated beliefs

We develop upon the information structure. The random variable θ taken as the fundamental follows a normal distribution with zero mean and a variance which is unknown to all the players. Taken to be infinite by the players, this variance does not play any role in our analysis. The central bank does not observe the realized value θ of the fundamental – a labor supply shock – but receives a signal $y = \theta + \eta$, where $\eta \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ is a white Gaussian noise. Each firm i does not observe the realized labor supply shock either, but receives a *private* signal $x_i = \theta + \varepsilon_i$, where $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is a white Gaussian idiosyncratic noise, and a *semi-public* signal $y_i = y + \phi_i = \theta + \eta + \phi_i$, where $\phi_i \sim \mathcal{N}\left(0, \sigma_\phi^2\right)$ is also a white Gaussian idiosyncratic noise, the variance of which may be under the control of the central bank. All the noises are independently distributed.

The firms may form two types of subjective beliefs. First, firm i may form subjective beliefs about the objective quality of its *private* information (following Banerjee et al. 2020, who extend however the subjectivity of beliefs to the quality of public information). In this case, firm i perceives the variance of its *own* private signal as $\sigma_\varepsilon^2/\delta$. The benchmark of rational expectations is captured by $\delta = 1$. When δ is larger than one, firm i overweights the private information when forming expectations, meaning that it believes the signal to contain less noise than it objectively contains (and conversely when δ is smaller than one).

[6] Note that it differs though from James and Lawler (2011) in allowing for an intermediate degree of transparency.

Second, firm i may form subjective beliefs about its ability to treat any kind of information, meaning that the coefficient δ will apply to both variances σ_ε^2 and σ_ϕ^2 (while it only applies to the variance σ_ε^2 in the case of subjective beliefs on the quality of private information). To cover both cases, it will be convenient to start by distinguishing two coefficients δ_ε and δ_ϕ, according to the specific noise to which each one applies, ε_i or ϕ_i. The coefficient δ is adopted by each particular firm i, but we will omit the index i for simplicity of notation. As we will keep our analysis restricted to symmetric equilibria, this practice should not be harmful, under some caveats when considering deviations from equilibrium.

Subjective beliefs may be motivated, or endogenized, by making firms choose their preferred value of δ. By (11), firm i will be assumed to minimize its expected profit loss, resulting from a price set on the basis of its subjective belief δ and of its information (x_i, y_i) yet to come. Ideally, it would like to make equal to zero the expectation of its profit loss, conditional on some potential realization θ of the labor supply shock, before any information on that realization is disclosed.

The choice of the preferred value of δ, characterizing *wishful thinking*, responds to what we may be tempted to view as an instance of the "pleasure principle". This principle must however be confronted with the "reality principle" commanding δ to be equal to 1, the rational expectations benchmark. So, firms will have to find a trade-off between the two principles, specifically by minimizing, say, a weighted sum of the expected profit loss function and of some other function representing the cost of being irrational: $\mathcal{L}(\delta) + \psi\mathcal{C}(\delta)$. The cost function $\mathcal{C}$ is assumed to have a strict global minimum equal to zero at $\delta = 1$, and to be increasing (resp. decreasing) for $\delta > 1$ (resp. $\delta < 1$).

As to the weight ψ, a fundamental assumption in our framework is that it is increasing in the relative precision of the relevant information, namely in the ratio of the variance of the objectively assessed information to the variance of the subjectively assessed information. Indeed, the higher the relative importance of the latter information (because of a lower relative variance) the higher the cost of being irrational and over- (or under-) weighting it. Conversely, when the quality of the subjectively assessed information is poor, there is a stronger incentive to irrationally overestimate that quality and so to put a lower weight on the cost of being irrational.

4.2 Timing

Before we proceed to the analysis of the game, we want to define thoroughly its timing by characterizing its successive stages:

1. On the basis of its knowledge of the laws of distribution of signals y and x_i, and anticipating the later stages of the game, the central bank chooses the value of one or both of the two policy instruments: (i) the variance σ_ϕ^2 of the idiosyncratic white Gaussian noise blurring the public signal y and/or (ii) the stabilization rule ρ governing the response $-\rho y$ to that signal.

2. Knowing the laws of distribution of public and private information, as well as the values of the policy instruments chosen by the central bank, each firm i adopts its subjective belief concerning either (i) the quality of the information to which it has a private access ($\delta_\varepsilon = \delta$, with $\delta_\phi = 1$) or (ii) its ability to treat information in general ($\delta_\varepsilon = \delta_\phi = \delta$). This belief is motivated conditionally to a potential value θ of the fundamental, which is in fact irrelevant for the firm's decision. The motivation involves linear affine pricing responses to private and semi-public information available in the future: $p_i = \kappa_0 + \kappa_1 y_i + \kappa_2 x_i$.

3. Nature chooses a realization θ of the fundamental and sends specific noisy signals of this realization $y = \theta + \eta$ to the central bank and $x_i = \theta + \varepsilon_i$ to each particular firm i. The central bank discloses its information, although possibly blurring it by a white Gaussian noise, each firm i eventually receiving a signal $y + \phi_i = y + \eta + \phi_i$.

4. Firms set the coefficients κ_0, κ_1 and κ_2 of their linear affine pricing responses to the signals (x_i, y_i) they receive, conditionally on their adopted subjective beliefs δ and on the central bank policy $\left(\sigma_\phi^2, \rho\right)$.

5. The representative household supplies labor and consumes products at the prices set by the firms.

As usual, we will consider these different stages backwards. Stage 5 has already been treated in subsection 3.1. Stages 4 and 2 will successively be examined in subsections 4.3 and 4.4, respectively. Stage 1 will be analyzed in sections 5 and 6.

4.3 Equilibrium in linear price strategies

We have assumed that, at stage 4, each firm i sets its price as a linear affine function of the two signals it receives:

$$p_i = \kappa_0 + \kappa_1 y_i + \kappa_2 x_i. \tag{16}$$

Of course, the values of the coefficients κ_0, κ_1 and κ_2 must be compatible with the pricing rule (13) established in the former section. To determine these values, we must formulate the expression for the expected price of the composite good, taking into account the linear price strategies of the other firms. Sticking to symmetry along the whole paper, we suppose that every other firm uses the same triple of coefficients $\overline{\kappa}_0$, $\overline{\kappa}_1$ and $\overline{\kappa}_2$. By equation (52) in Appendix A, we may write for the second order Taylor approximation of p (denoting by $\mathcal{P}_1(\mathbf{p})$ the arithmetic mean of the p_j's)

$$p \equiv p_{1-s} \simeq \mathcal{P}_1(\mathbf{p}) - \frac{s-1}{2}\sigma_{\mathbf{p}}^2 = \overline{\kappa}_0 + \overline{\kappa}_1 y + \overline{\kappa}_2 \theta - \frac{s-1}{2}\left(\overline{\kappa}_1^2 \sigma_\phi^2 + \overline{\kappa}_2^2 \sigma_\varepsilon^2\right). \tag{17}$$

Notice that dispersed information introduces a negative bias in the price index relative to the aritmetic mean of the p_j's, which vanishes only in the limit case

$s \to 1$, when the price index P tends to the geometric mean of individual prices P_j's.

The expectation $\mathbb{E}_i[p]$ of the price p of the composite good conditional on the information of firm i involves of course $\mathbb{E}_i[y]$ and $\mathbb{E}_i[\theta]$:

$$\mathbb{E}_i[p] = \mathbb{E}[p|\, x_i, y_i] \simeq \overline{\kappa}_0 + \overline{\kappa}_1 \mathbb{E}_i[y] + \overline{\kappa}_2 \mathbb{E}_i[\theta] - \frac{s-1}{2}\left(\overline{\kappa}_1^2 \sigma_\phi^2 + \overline{\kappa}_2^2 \sigma_\varepsilon^2\right). \quad (18)$$

Firm i's expectation of the fundamental shock θ when its information is reduced to the two signals x_i and y_i is

$$\mathbb{E}_i[\theta] = \mathbb{E}[\theta|x_i, y_i] = \underbrace{\frac{\sigma_\varepsilon^2/\delta_\varepsilon}{\sigma_\varepsilon^2/\delta_\varepsilon + \sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{\lambda} y_i + \underbrace{\frac{\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}{\sigma_\varepsilon^2/\delta_\varepsilon + \sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{1-\lambda} x_i. \quad (19)$$

Like Morris and Shin (2002), we are supposing that firms have no *ex ante* information about the distribution of the fundamental. Their information is only *ex post*, on the realized value θ. Similarly, firm i's expectation of the central bank information y is

$$\begin{aligned}
\mathbb{E}_i[y] &= \mathbb{E}[y|x_i, y_i] = \underbrace{\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{\mu} y_i + \underbrace{\frac{\sigma_\phi^2/\delta_\phi}{\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{1-\mu} \mathbb{E}_i[\theta] \\
&= (\mu + (1-\mu)\lambda) y_i + (1-\mu)(1-\lambda) x_i \\
&= \underbrace{\frac{\sigma_\eta^2 + \sigma_\varepsilon^2/\delta_\varepsilon}{\sigma_\varepsilon^2/\delta_\varepsilon + \sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{\nu} y_i + \underbrace{\frac{\sigma_\phi^2/\delta_\phi}{\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{1-\nu} x_i. \quad (20)
\end{aligned}$$

Referring to the pricing rule (13) which ensures that firm i sets a profit maximizing price p_i, and further referring to the expectation expressions (18), (19) and (20), we then obtain, for $z = -\rho y$,

$$\begin{aligned}
p_i &= (1-\xi)\,\mathbb{E}_i[p] + \xi \mathbb{E}_i[z+\theta] \quad (21)\\
&= (1-\xi)\left(\overline{\kappa}_0 - \frac{s-1}{2}\left(\overline{\kappa}_1^2\sigma_\phi^2 + \overline{\kappa}_2^2\sigma_\varepsilon^2\right)\right) + ((1-\xi)\,\overline{\kappa}_1 - \xi\rho)\,\mathbb{E}_i[y] \\
&\quad + ((1-\xi)\,\overline{\kappa}_2 + \xi)\,\mathbb{E}_i[\theta],
\end{aligned}$$

so that, at a *symmetric equilibrium* where $(\overline{\kappa}_0, \overline{\kappa}_1, \overline{\kappa}_2) = (\kappa_0, \kappa_1, \kappa_2)$, we obtain by identification:

$$\begin{aligned}
\kappa_1 &= \lambda - \rho\nu = \frac{(1-\rho)\,\sigma_\varepsilon^2/\delta_\varepsilon - \rho\xi\sigma_\eta^2}{\xi\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi + \sigma_\varepsilon^2/\delta_\varepsilon} \equiv \kappa_1(\boldsymbol{\delta}), \text{ with } \boldsymbol{\delta} = (\delta_\varepsilon, \delta_\phi), \\
\kappa_2 &= (1-\lambda) - \rho(1-\nu) = \frac{\xi\sigma_\eta^2 + (1-\rho)\,\sigma_\phi^2/\delta_\phi}{\xi\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi + \sigma_\varepsilon^2/\delta_\varepsilon} \equiv \kappa_2(\boldsymbol{\delta}), \\
\kappa_0 &= -\frac{s-1}{2}\frac{1-\xi}{\xi}\left(\kappa_1^2\sigma_\phi^2 + \kappa_2^2\sigma_\varepsilon^2\right) \equiv \kappa_0(\boldsymbol{\delta}). \quad (22)
\end{aligned}$$

Notice that $\kappa_1(\boldsymbol{\delta}) + \kappa_2(\boldsymbol{\delta}) = 1 - \rho$.

4.4 Motivation of beliefs

We have assumed that firms' beliefs are adopted at stage 2 by maximizing their expected profits (or, equivalently, by minimizing their expected profit loss, as given by (11)), conditionally on some potential realization of the fundamental, on which they have not yet received any information:

$$\min_{\boldsymbol{\delta}} \frac{s-1}{2} \underbrace{\mathbb{E}\left(\left(p_i\left(\boldsymbol{\delta}\right) - \widehat{p}\right)^2 \middle| \theta \right)}_{\mathcal{L}(\boldsymbol{\delta})}, \text{ with} \tag{23}$$

$$\begin{aligned} p_i\left(\boldsymbol{\delta}\right) &= \kappa_0\left(\boldsymbol{\delta}\right) + \kappa_1\left(\boldsymbol{\delta}\right)\left(\eta + \phi_i\right) + \kappa_2\left(\boldsymbol{\delta}\right)\varepsilon_i + \left(1-\rho\right)\theta \text{ and} \\ \widehat{p} &= \left(1-\xi\right)\left(\overline{\kappa}_0 + \overline{\kappa}_1\eta - \frac{s-1}{2}\left(\overline{\kappa}_1^2\sigma_\phi^2 + \overline{\kappa}_2^2\sigma_\varepsilon^2\right) + \left(1-\rho\right)\theta\right) \\ &\quad + \xi\left(-\rho\eta + \left(1-\rho\right)\theta\right), \end{aligned}$$

using (13) and (18). Also, using (22),

$$\begin{aligned} \mathcal{L}\left(\boldsymbol{\delta}\right) &= \left(\kappa_0\left(\boldsymbol{\delta}\right) - \left(1-\xi\right)\overline{\kappa}_0 + \left(1-\xi\right)\frac{s-1}{2}\left(\overline{\kappa}_1^2\sigma_\phi^2 + \overline{\kappa}_2^2\sigma_\varepsilon^2\right)\right)^2 \\ &\quad + \left(\kappa_1\left(\boldsymbol{\delta}\right) - \left(1-\xi\right)\overline{\kappa}_1 + \xi\rho\right)^2\sigma_\eta^2 + \left(\kappa_1\left(\boldsymbol{\delta}\right)\right)^2\sigma_\phi^2/\delta_\phi \\ &\quad + \left(\kappa_2\left(\boldsymbol{\delta}\right)\right)^2\sigma_\varepsilon^2/\delta_\varepsilon. \end{aligned} \tag{24}$$

The loss is directly decreasing in the parameters of subjective beliefs δ_ε and δ_ϕ and also indirectly dependent, through the coefficients κ_0, κ_1 and κ_2, on these parameters.

In order to investigate the conditions for firm i to optimize its belief $\boldsymbol{\delta}$, we consider the gradient of the loss function:

$$\begin{aligned} \nabla\mathcal{L}\left(\boldsymbol{\delta}\right) &= 2\left(\kappa_0\left(\boldsymbol{\delta}\right) - \left(1-\xi\right)\overline{\kappa}_0 + \left(1-\xi\right)\frac{s-1}{2}\left(\overline{\kappa}_1^2\sigma_\phi^2 + \overline{\kappa}_2^2\sigma_\varepsilon^2\right)\right)\nabla\kappa_0\left(\boldsymbol{\delta}\right) \\ &\quad + 2\left(\begin{array}{c} \left(\kappa_1\left(\boldsymbol{\delta}\right) - \left(1-\xi\right)\overline{\kappa}_1 + \xi\rho\right)\sigma_\eta^2 + \\ \kappa_1\left(\boldsymbol{\delta}\right)\sigma_\phi^2/\delta_\phi - \left(1-\rho-\kappa_1\left(\boldsymbol{\delta}\right)\right)\sigma_\varepsilon^2/\delta_\varepsilon \end{array}\right)\nabla\kappa_1\left(\boldsymbol{\delta}\right) \\ &\quad - \left(\left(\kappa_2\left(\boldsymbol{\delta}\right)\right)^2\sigma_\varepsilon^2/\delta_\varepsilon^2, \left(\kappa_1\left(\boldsymbol{\delta}\right)\right)^2\sigma_\phi^2/\delta_\phi^2\right), \end{aligned} \tag{25}$$

which, by (22), can be seen to reduce to

$$\nabla\mathcal{L}\left(\boldsymbol{\delta}\right) = -\left(\left(\kappa_2\left(\boldsymbol{\delta}\right)\right)^2\sigma_\varepsilon^2/\delta_\varepsilon^2, \left(\kappa_1\left(\boldsymbol{\delta}\right)\right)^2\sigma_\phi^2/\delta_\phi^2\right) \tag{26}$$

at a *symmetric profile* of belief choices by all the firms, where $\left(\overline{\kappa}_0, \overline{\kappa}_1, \overline{\kappa}_2\right) = \left(\kappa_0\left(\boldsymbol{\delta}\right), \kappa_1\left(\boldsymbol{\delta}\right), \kappa_2\left(\boldsymbol{\delta}\right)\right)$. A symmetric equilibrium in motivated beliefs can however not be obtained on the basis of the minimization of the sole loss function $\mathcal{L}$, since the incentive to increase δ_ε and δ_ϕ remains indefinitely present $\left(\nabla\mathcal{L}\left(\boldsymbol{\delta}\right) < 0\right)$. As already stated in subsection 4.1, firms must take into account the cost of being irrational and minimize instead a weighted sum of the expected profit loss and that cost.

Now, as announced in subsection 4.1, we are interested in two cases of subjective beliefs: (i) the firm's motivated belief in the quality of the information to which it has access, which may be characterized by $\delta_\varepsilon = \delta$ and $\delta_\phi = 1$, and (ii) the firm's motivated belief in its ability to treat any kind of information, which we may identify with $\delta_\varepsilon = \delta_\phi = \delta$. In both cases the minimization involves a sole variable, so that we have to refer in fact to the total derivative

$$\frac{d\mathcal{L}(\boldsymbol{\delta})}{d\delta} = \begin{cases} -\frac{(\kappa_2(\delta,1))^2\sigma_\varepsilon^2}{\delta^2} & \text{if } \boldsymbol{\delta} = (\delta, 1) \\ -\frac{(\kappa_2(\delta,\delta))^2\sigma_\varepsilon^2 + (\kappa_1(\delta,\delta))^2\sigma_\phi^2}{\delta^2} & \text{if } \boldsymbol{\delta} = (\delta, \delta) \end{cases}. \tag{27}$$

In this context, it is convenient to take as a cost function

$$\mathcal{C}(\boldsymbol{\delta}) = \begin{cases} \left|\int_1^\delta (\kappa_2(h,1))^2 \sigma_\varepsilon^2 dh\right| & \text{if } \boldsymbol{\delta} = (\delta, 1) \\ \left|\int_1^\delta \left((\kappa_2(h,h))^2 \sigma_\varepsilon^2 + (\kappa_1(h,h))^2 \sigma_\phi^2\right) dh\right| & \text{if } \boldsymbol{\delta} = (\delta, \delta) \end{cases}, \tag{28}$$

which has a strict global minimum equal to zero at $\delta = 1$ and is increasing (resp. decreasing) for $\delta > 1$ (resp. $\delta < 1$). Under this specification of the cost function, the sign of the derivative of $\mathcal{L}(\boldsymbol{\delta}) + \psi\mathcal{C}(\boldsymbol{\delta})$ with respect to δ is -1 if $\delta \leq 1$, otherwise

$$\operatorname{sgn}\left(\frac{d\mathcal{L}(\boldsymbol{\delta})}{d\delta} + \psi\frac{d\mathcal{C}(\boldsymbol{\delta})}{d\delta}\right) = \operatorname{sgn}\left(-\frac{1}{\delta^2} + \psi\right), \tag{29}$$

entailing a minimum $\delta = 1$ if $\psi \geq 1$, otherwise an interior minimum $\delta = 1/\sqrt{\psi} \in (1, \infty)$, tending to infinity as $\psi \to 0$.

As also announced in subsection 4.1, we shall assume that ψ is an increasing function of the ratio of the variance of the objectively assessed information to the variance of the subjectively assessed information, explicitly

$$\psi = \left(\beta\frac{\sigma_\eta^2 + \sigma_\phi^2}{\sigma_\varepsilon^2}\right)^{2\alpha} \text{ in case (i) and } \psi = \left(\beta\frac{\sigma_\eta^2}{\sigma_\phi^2 + \sigma_\varepsilon^2}\right)^{2\alpha} \text{ in case (ii),} \tag{30}$$

with α and β positive. Notice that β is an index of the level of ψ, whereas α is an index of the sensitivity of ψ to the relative precision of the subjectively assessed information. Notice also that switching from case (i) to case (ii) reverses the sense of dependence of ψ with respect to central bank's instrument σ_ϕ^2.

5 Central bank policy under the benchmark of objective beliefs

In this section, we analyze the first stage of the game, namely central bank's communication and stabilization policies under the benchmark of objective beliefs ($\delta_\varepsilon = \delta_\phi = 1$). Recall from subsection 3.3 that the central bank aims at maximizing welfare, which translates into minimizing price dispersion

$$\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2\sigma_\phi^2 + \kappa_2^2\sigma_\varepsilon^2, \tag{31}$$

which depends directly upon the central bank policy instrument σ_ϕ^2 and indirectly, through the coefficients κ_1 and κ_2, again on σ_ϕ^2 but also on ρ.

Under homogeneous information, $\sigma_\varepsilon^2 = 0$. By choosing not to implement a stabilization policy ($\rho = 0$), the central bank ensures that, whatever the value of σ_ϕ^2, firms set $\kappa_0 = \kappa_1 = 0$ and $\kappa_2 = 1$, which leads to the absence of price dispersion ($\sigma_\mathbf{p}^2 = 0$) and no welfare loss.

In what follows, we consider the case of heterogeneous information, where the variance σ_ε^2 of the private signal is positive. Two cases can then be distinguished: the case of transparency of the central bank policy, in which $\sigma_\phi^2 = 0$, and the case of opacity, in which $\sigma_\phi^2 \to \infty$. Intermediate cases are allowed for, but they are not optimal and will not be chosen by the central bank. As we are going to show, the optimal one-intrument policy is transparency, whereas the optimal two-instruments policy is opacity.

5.1 One-instrument policy: transparency

In what we have called the s*ignalling stabilization* regime, the central bank's stabilization policy is directly observed by firms, which allows them to infer the central bank's information about the fundamental shock.[7] This corresponds to the case where $\sigma_\phi^2 = 0$, implying price dispersion $\sigma_\mathbf{p}^2 = \kappa_2^2 \sigma_\varepsilon^2$, with $\kappa_2 = \xi\sigma_\eta^2 / \left(\xi\sigma_\eta^2 + \sigma_\varepsilon^2\right)$, so that welfare does not depend upon the stabilization policy anymore. Any policy coefficient ρ yields the same welfare, the stabilization policy being indeterminate.[8] Price dispersion is then given by

$$\sigma_\mathbf{p}^2|_{\sigma_\phi^2=0} = \left(\frac{\xi\sigma_\eta^2}{\xi\sigma_\eta^2 + \sigma_\varepsilon^2} \right)^2 \sigma_\varepsilon^2. \tag{32}$$

In the *pure communication* framework, the central bank discloses information but does not take any action, which corresponds to $\rho = 0$. By differentiating $\sigma_\mathbf{p}^2$ (which is to be minimized) with respect to σ_ϕ^2, we obtain

$$\begin{aligned} \frac{\partial \sigma_\mathbf{p}^2}{\partial \sigma_\phi^2} &= \frac{\partial \left(\kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2 \right)}{\partial \sigma_\phi^2} = \kappa_1^2 + 2 \left(\kappa_1 \sigma_\phi^2 \frac{\partial \kappa_1}{\partial \sigma_\phi^2} + \kappa_2 \sigma_\varepsilon^2 \frac{\partial \kappa_2}{\partial \sigma_\phi^2} \right) \\ &= \left(\frac{\sigma_\varepsilon^2}{\xi\sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2} \right)^2 \frac{3\xi\sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2}{\xi\sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2} > 0, \end{aligned} \tag{33}$$

so that the optimal communication policy, the one which minimizes $\sigma_\mathbf{p}^2$, is $\sigma_\phi^{*2} = 0$. Transparency is now the result of an optimizing choice of the central bank, but the price dispersion outcome is the same (see equation (32)). As underlined in the literature (e.g. Hellwig 2005 and Baeriswyl et al. 2020), in a microfounded

[7] Empirical evidence about the signalling role of monetary policy actions is provided by Romer and Romer (2000).

[8] One way to solve this indeterminacy is to introduce frictions to make welfare depend on the price level. This would however not affect welfare.

macroeconomic model, transparency always improves welfare by reducing price dispersion across firms. According to Angeletos and Pavan (2007), the reason is that the equilibrium degree of coordination is lower than the optimal degree of coordination.

5.2 Two-instruments policy: opacity

In practice, central banks do not only communicate, they also implement a stabilization policy. In the communication and stabilization policies regime, the central bank chooses jointly its optimal stabilization policy ρ and its communication policy σ_ϕ^2 in order to minimize $\sigma_{\mathbf{p}}^2$. A simple inspection of (22) shows that, by choosing opacity ($\sigma_\phi^2 \to \infty$), the central bank leads firms to set $\kappa_1 = 0$ and actually obtains $\kappa_1^2 \sigma_\phi^2 = 0$. The coefficient $\kappa_2 = 1 - \rho$ can then be also made equal to zero by setting $\rho = 1$. Welfare loss is thus minimized at zero, with no price dispersion, an outcome that can be achieved thanks to the use of two instruments.

Taking an action is thus more efficient for maximizing welfare than disclosing information. If the central bank were not fully opaque, firms would overreact to public disclosure due to strategic complementarities in price setting: they would make an inefficient use of information. As emphasized by Baeriswyl et al. (2020), reducing price dispersion does not require public information *per se* but a weaker response to private information, which the central bank can achieve through its stabilization policy, by setting $\rho = 1$. In conformity with Lucas' critique, the manipulation of the nominal expenditure ($z = -\rho y$) does not stabilize the economy by compensating the labor supply shocks, it does so by moderating the firms' response to the information on those shocks ($\kappa_1 + \kappa_2 = 1 - \rho$).

6 Central bank policy under motivated beliefs

In this section, we analyze the first stage of the game under motivated beliefs. We successively consider firms' motivated beliefs about the quality of their own private information and about their ability to process information in general. These two cases are however indistinguishable under *signalling stabilization* ($\sigma_\phi^2 = 0$). Indeed, by equations (22) and (30), price variance, given by (31), which is the objective function the central bank wants to minimize, is equal in both cases to

$$\sigma_{\mathbf{p}}^2 = \kappa_2^2 \sigma_\varepsilon^2 = \left(\frac{\xi \sigma_\eta^2 / \sigma_\varepsilon^2}{\xi \sigma_\eta^2 / \sigma_\varepsilon^2 + \min\left(\left(\beta \sigma_\eta^2 / \sigma_\varepsilon^2 \right)^\alpha, 1 \right)} \right)^2 \sigma_\varepsilon^2. \tag{34}$$

Price variance may thus be higher than in the case of objective beliefs (if $\beta \sigma_\eta^2 / \sigma_\varepsilon^2 < 1$), but it cannot be manipulated by the stabilization instrument ρ. We obtain the same result of stabilization policy indeterminateness as in the benchmark case of objective beliefs. In the following, we will consequently restrict our analysis to the policies of pure communication and communication with stabilization.

6.1 Motivated beliefs about the quality of private information

We consider how motivated beliefs about the quality of firms' private information ($\delta_\varepsilon = \delta$ and $\delta_\phi = 1$) affect central bank's policy in the two operational regimes involving communication by the central bank (compared to the benchmark of objective beliefs as described in section 5). Recall that, in this case, the weight ψ on the cost of being irrational is an *increasing* function of central bank's communication instrument σ_ϕ^2.

6.1.1 Robustness of objective belief outcomes under active stabilization

From subsection 4.4, we know that the equilibrium subjective belief is given by $\delta^* = 1/\min\left(\sqrt{\psi}, 1\right)$, with $\sqrt{\psi} = \left(\beta\left(\sigma_\eta^2 + \sigma_\phi^2\right)/\sigma_\varepsilon^2\right)^\alpha$. The weights put on semi-public and private information in firms' pricing rule are then, respectively,

$$\begin{aligned}\kappa_1\left(\delta^*, 1\right) &= \frac{\left(1-\rho\right)\sigma_\varepsilon^2 \min\left(\sqrt{\psi}, 1\right) - \rho\xi\sigma_\eta^2}{\xi\sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2 \min\left(\sqrt{\psi}, 1\right)}, \\ \kappa_2\left(\delta^*, 1\right) &= \frac{\xi\sigma_\eta^2 + \left(1-\rho\right)\sigma_\phi^2}{\xi\sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2 \min\left(\sqrt{\psi}, 1\right)}.\end{aligned} \tag{35}$$

In the case of a two-instruments policy, nothing is changed compared to the benchmark of objective beliefs. Opacity still ensures that $\kappa_1\left(\delta^*, 1\right) \to 0$ (more significantly, that $\left(\kappa_1\left(\delta^*, 1\right)\right)^2 \sigma_\phi^2 \to 0$) and also, with $\rho = 1$, that $\kappa_2\left(\delta^*, 1\right) \to 0$, leading to price uniformity. Indefinitely increasing σ_ϕ^2 can only make $\sqrt{\psi}$ become larger than 1, so that any difference with the case of objective beliefs ceases to be effective.

6.1.2 Reversal of the case for transparency under pure communication

Contrary to the regime with two policy instruments, which leads to opacity under both objective and motivated beliefs, the pure communication regime ($\rho = 0$) leads to transparency in the former case, but not necessarily in the latter. Full opacity is still excluded, since $\lim_{\sigma_\phi^2 \to \infty} \delta^* = 1$, entailing the situation of objective beliefs with $\sigma_{\mathbf{p}}^2$ increasing in σ_ϕ^2. The full transparency outcome can however be reversed provided an indefinite decrease of σ_ϕ^2 makes $\sqrt{\psi}$ become smaller than 1, that is, provided $\lim_{\sigma_\phi^2 \to 0} \psi^{1/2\alpha} = \beta\sigma_\eta^2/\sigma_\varepsilon^2 < 1$ (a situation of a low weight on the cost for a firm of parting with objectivity). Price dispersion

$\sigma^2_{\mathbf{p}} = (1-\kappa_2)^2\,\sigma^2_\phi + \kappa_2^2\sigma^2_\varepsilon$ has then, for a small enough value of σ^2_ϕ, a derivative

$$\frac{\partial\sigma^2_{\mathbf{p}}}{\partial\sigma^2_\phi} = (1-\kappa_2)^2 - 2\left((1-\kappa_2)\,\sigma^2_\phi - \kappa_2\sigma^2_\varepsilon\right)\frac{\partial\kappa_2}{\partial\sigma^2_\phi}, \tag{36}$$

with sign

$$\operatorname{sgn}\left(\frac{\partial\sigma^2_{\mathbf{p}}}{\partial\sigma^2_\phi}\right) = \operatorname{sgn}\left(\sqrt{\psi} - 2\frac{\xi\sigma^2_\eta + \left(1-\sqrt{\psi}\right)\sigma^2_\phi}{\xi\sigma^2_\eta + \sigma^2_\phi + \sigma^2_\varepsilon\sqrt{\psi}}\left(\alpha\frac{\xi\sigma^2_\eta + \sigma^2_\phi}{\sigma^2_\eta + \sigma^2_\phi} - 1\right)\right). \tag{37}$$

If α is small, in any case smaller than 1, the derivative $\partial\sigma^2_{\mathbf{p}}/\partial\sigma^2_\phi$ is positive and transparency is again the optimal communication policy. However, if $\alpha\xi > 1$, the derivative is negative for σ^2_ϕ and ψ both small enough. Formally,

$$\lim_{\sigma^2_\phi\to 0}\operatorname{sgn}\left(\frac{\partial\sigma^2_{\mathbf{p}}}{\partial\sigma^2_\phi}\right) = \operatorname{sgn}\left(\left(\beta\sigma^2_\eta/\sigma^2_\varepsilon\right)^\alpha - \frac{2\xi\left(\alpha\xi-1\right)}{\xi + \beta^\alpha\left(\sigma^2_\eta/\sigma^2_\varepsilon\right)^{\alpha-1}}\right) = -1 \tag{38}$$

if β and/or $\sigma^2_\eta/\sigma^2_\varepsilon$ are low enough. A combination of a high value of α and a small value of β (a low but sensitive weight on the cost of being irrational) destroys the optimality of a fully transparent communication policy. High values of ξ (a small degree of strategic complementarity) and $\sigma^2_\varepsilon/\sigma^2_\eta$ (a low precision of private relative to public information)[9] will reinforce this result.

If $\sigma^2_{\mathbf{p}}$ is decreasing in σ^2_ϕ when this variance is close to zero, we get an interior solution to the minimization of $\sigma^2_{\mathbf{p}}$ in terms of σ^2_ϕ. Now, there are two cases depending on whether this solution is smaller or larger than the value $\sigma^2_\varepsilon/\beta - \sigma^2_\eta$ that makes ψ equal to one. Formally,

$$\operatorname{sgn}\left(\left.\frac{\partial\sigma^2_{\mathbf{p}}}{\partial\sigma^2_\phi}\right|_{\sigma^2_\phi=\sigma^2_\varepsilon/\beta-\sigma^2_\eta}\right) = \operatorname{sgn}\left(1 - 2\alpha\xi\beta\frac{\sigma^2_\eta}{\sigma^2_\varepsilon}\frac{1-1/\alpha-(1-\xi)\,\beta\left(\sigma^2_\eta/\sigma^2_\varepsilon\right)}{1+\beta-(1-\xi)\,\beta\left(\sigma^2_\eta/\sigma^2_\varepsilon\right)}\right), \tag{39}$$

positive in particular for a small enough value of the weight $\beta\sigma^2_\eta/\sigma^2_\varepsilon$ on the cost (under transparency) of being irrational, that is, for a low precision $\sigma^2_\eta/\sigma^2_\varepsilon$ of private relative to public information and/or a small value of the parameter β. The value of σ^2_ϕ which minimizes the welfare loss is then interior to the interval $\left(0, \sigma^2_\varepsilon/\beta - \sigma^2_\eta\right)$, resulting in a motivated overconfidence in private information ($\delta^* > 1$). A negative sign is however not excluded, leading to $\delta^* = 1$: the possibility of irrationality, rightly conjectured by the central bank, drives the outcome but is not observed at equilibrium.

Thus, transparency is not always optimal under firms' subjective beliefs about the accuracy of their private information, contrasting with the case of objective beliefs. By reducing the variance σ^2_ϕ, the central bank makes firms

[9] The case where private information is less precise than public information is the more realistic one.

overall more informed about the fundamental shocks and therefore reduces the importance they give to the cost of mistakenly believing that their private information is very precise. Firms become then even more confident in their private information (since $\delta^* = \left(\beta\left(\sigma_\eta^2 + \sigma_\phi^2\right)/\sigma_\varepsilon^2\right)^{-\alpha}$ is decreasing in σ_ϕ^2) and may overcompensate the direct impact of the reduction of σ_ϕ^2 on the weight $\kappa_2(\delta, 1)$ put on private information. Hence, increasing the precision of semi-public information ends up in increasing price dispersion. Conversely, opacity cannot be optimal: by making more important the cost for firms of being irrational, the central bank makes them eventually rely on objective beliefs and turn entirely to their more informative private signals ($\lim_{\sigma_\phi^2 \to \infty} \kappa_2(\delta^*, 1) = 1$). Again, full reliance on private information enhances price dispersion and deteriorates welfare.

By contrast, an intermediate level of transparency ($0 < \sigma_\phi^{*2} < \sigma_\varepsilon^2/\beta - \sigma_\eta^2$) balances the benefit of increasing firms' overall information on the fundamental shocks (making them rely objectively less on their private information) and the detrimental effect of firms' subjective overconfidence (making them rely more on that information). By not being fully opaque, the central bank makes firms more informed and less dependent on private information, and by not being fully transparent, the central bank imposes a larger cost on overconfidence in the precision of private information.

The following proposition summarizes these results.

Proposition 1 *When firms hold motivated beliefs about the quality of their private information, (i) opacity is, as under objective beliefs, the optimal communication policy of the central bank when the latter also stabilizes shocks; (ii) for a weight on the cost of being irrational low enough ($\beta\sigma_\eta^2/\sigma_\varepsilon^2$ small) but sufficiently sensitive to the relative precision of private information ($\alpha\xi > 1$), an intermediate level of transparency is the optimal communication policy of a central bank that can only communicate, contrasting with the case of objective beliefs under which transparency is always optimal.*

6.2 Motivated beliefs about the ability to process information

We now consider how motivated beliefs about firms' ability to process their information ($\delta_\varepsilon = \delta_\phi = \delta$) affect central bank's policy in the regimes of pure communication and communication coupled with stabilization (compared to the benchmark of objective beliefs). Recall that the weight ψ on the cost of being irrational is now a *decreasing* function of central bank's communication instrument σ_ϕ^2.

6.2.1 Robustness of objective belief outcomes under pure communication

In subsection 4.4, we have established that equilibrium motivated beliefs are given by $\delta^* = \max\left(1/\sqrt{\psi}, 1\right)$, with $\sqrt{\psi} = \left(\beta\sigma_\eta^2/\left(\sigma_\phi^2 + \sigma_\varepsilon^2\right)\right)^\alpha$, and that the corresponding weight on private information is

$$\kappa_2\left(\delta, \delta\right) = \frac{\xi\sigma_\eta^2 + (1-\rho)\,\sigma_\phi^2 \min\left(\sqrt{\psi}, 1\right)}{\xi\sigma_\eta^2 + \left(\sigma_\phi^2 + \sigma_\varepsilon^2\right)\min\left(\sqrt{\psi}, 1\right)}. \tag{40}$$

Price variance, to be minimized by the central bank is

$$\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2\sigma_\phi^2 + \kappa_2^2\sigma_\varepsilon^2 = \left(1-\rho-\kappa_2\right)^2\sigma_\phi^2 + \kappa_2^2\sigma_\varepsilon^2, \tag{41}$$

with derivatives with respect to the policy instruments

$$\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\sigma_\phi^2} = \left(1-\rho-\kappa_2\right)^2 + 2\left(\kappa_2\sigma_\varepsilon^2 - \left(1-\rho-\kappa_2\right)\sigma_\phi^2\right)\frac{\partial\kappa_2}{\partial\sigma_\phi^2} \tag{42}$$

and

$$\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\rho} = -2\left(1-\rho-\kappa_2\right)\sigma_\phi^2 + 2\left(\kappa_2\left(\sigma_\phi^2+\sigma_\varepsilon^2\right) - (1-\rho)\,\sigma_\phi^2\right)\frac{\partial\kappa_2}{\partial\rho}. \tag{43}$$

Under *pure communication* ($\rho = 0$), the sign of

$$\begin{aligned}\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\sigma_\phi^2} \quad = \quad & \left(\frac{\sigma_\varepsilon^2\min\left(\sqrt{\psi},1\right)}{\xi\sigma_\eta^2 + \left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\min\left(\sqrt{\psi},1\right)}\right)^2 \\ & \left[1 + \frac{2\xi\sigma_\eta^2}{\xi\sigma_\eta^2 + \left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\min\left(\sqrt{\psi},1\right)}\left(1 - \frac{\xi\sigma_\eta^2\partial\min\left(\sqrt{\psi},1\right)/\partial\sigma_\phi^2}{\left(\min\left(\sqrt{\psi},1\right)\right)^2}\right)\right]\end{aligned} \tag{44}$$

is positive since $\partial\min\left(\sqrt{\psi},1\right)/\partial\sigma_\phi^2 \leq 0$, so that we retrieve the situation of objective beliefs, any reduction of σ_ϕ^2 being always beneficial for the central bank. We thus end up with transparency as the optimal pure communication policy.

6.2.2 Reversal of the case for opacity under active stabilization

If the weight ψ on the cost of being irrational is not too sensitive to the relative precision of the subjectively assessed information, more precisely if $\alpha < 1$, so that $\lim_{\sigma_\phi^2\to\infty}\left(\sigma_\phi^2\sqrt{\psi}\right) = \infty$, the central bank can still obtain $\kappa_2\left(\delta,\delta\right) \to 0$ (and $\kappa_1\left(\delta,\delta\right) = \left(1-\rho-\kappa_2\left(\delta,\delta\right)\right) \to 0$) by choosing $\rho = 1$ and letting $\sigma_\phi^2 \to \infty$

(opacity), as in the case of objective beliefs. However, $\lim_{\sigma_\phi^2\to\infty}\left(\sigma_\phi^2\sqrt{\psi}\right)=0$ if $\alpha>1$ and $\lim_{\sigma_\phi^2\to\infty}\left(\sigma_\phi^2\sqrt{\psi}\right)=\beta\sigma_\eta^2$ if $\alpha=1$. So,

$$\lim_{\sigma_\phi^2\to\infty}\kappa_2=1 \text{ if } \alpha>1 \text{ and } \lim_{\sigma_\phi^2\to\infty}\kappa_2=\frac{\xi+(1-\rho)\beta}{\xi+\beta} \text{ if } \alpha=1, \tag{45}$$

destroying the favorable outcome obtained by opacity under objective beliefs. If $\alpha\geq 1$, which we shall assume from now on, increasing the variance σ_ϕ^2 strongly diminishes the weight on the cost of being irrational, leading to overconfidence on the quality of private information, hence to higher price dispersion.

The exclusion of opacity ($\sigma_\phi^2=\infty$) as an optimal communication policy can actually be more generally established. Indeed, by equation (42) and taking ρ as given, we obtain, for $\alpha>1$,

$$\begin{aligned}\lim_{\sigma_\phi^2\to\infty}\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\sigma_\phi^2} &= \rho^2+2\lim_{\sigma_\phi^2\to\infty}\left(\left(\rho\sigma_\phi^2+\sigma_\varepsilon^2\right)\frac{\partial\kappa_2}{\partial\sigma_\phi^2}\right) \qquad (46)\\ &= \rho^2+2\alpha\frac{\beta^\alpha}{\xi}\left(\sigma_\eta^2\right)^{\alpha-1}\lim_{\sigma_\phi^2\to\infty}\left(\frac{\left(\rho\sigma_\phi^2+\sigma_\varepsilon^2\right)^2}{\left(\sigma_\phi^2+\sigma_\varepsilon^2\right)^{\alpha+1}}\right)=\rho^2\end{aligned}$$

and, for $\alpha=1$,

$$\begin{aligned}&\lim_{\sigma_\phi^2\to\infty}\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\sigma_\phi^2}\\ &= \left(\frac{\rho\xi}{\xi+\beta}\right)^2+\frac{2\xi\beta}{(\xi+\beta)^3}\lim_{\sigma_\phi^2\to\infty}\left(\frac{\left(\left(\xi+(1-\rho)\beta\right)\sigma_\varepsilon^2-\rho\xi\sigma_\phi^2\right)\left(\rho\sigma_\phi^2+\sigma_\varepsilon^2\right)}{\left(\sigma_\phi^2+\sigma_\varepsilon^2\right)^2}\right)\\ &= \rho^2\frac{\xi^2(\xi-\beta)}{(\xi+\beta)^3}, \qquad (47)\end{aligned}$$

so that $\lim_{\sigma_\phi^2\to\infty}\partial\sigma_{\mathbf{p}}^2/\partial\sigma_\phi^2>0$ for $\rho>0$ if either $\alpha>1$ or $\alpha=1$ and $\xi>\beta$.

Is the reversal of outcomes complete, with transparency as the new optimal communication policy? We show in Appendix D that, for any $\sigma_\phi^2/\sigma_\varepsilon^2\in(0,\infty)$, the optimal value of ρ is

$$\rho^*\left(\sigma_\phi^2/\sigma_\varepsilon^2\right)=1-\frac{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2\right)^2}{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2+\min\left(\sqrt{\psi},1\right)\right)^2+\left(\sigma_\phi^2/\sigma_\varepsilon^2\right)\left(\min\left(\sqrt{\psi},1\right)\right)^2}, \tag{48}$$

belonging to the interior of $(0,1)$. We further show that $\lim_{\sigma_\phi^2\to 0}\partial\sigma_{\mathbf{p}}^2/\partial\sigma_\phi^2<0$ for $\rho=\rho^*\left(\sigma_\phi^2/\sigma_\varepsilon^2\right)$ if $\beta\sigma_\eta^2/\sigma_\varepsilon^2>1$ (a high cost of being irrational), implying that $\min\left(\sqrt{\psi},1\right)=1$ as σ_ϕ^2 becomes close enough to zero. Full transparency

($\sigma_\phi^2 = 0$) is then equally excluded as the optimal communication policy. In this case both communication and stabilization policy instruments must take intermediate values: $\sigma_\phi^{*2} \in (0, \infty)$ and $\rho^* \left(\sigma_\phi^{*2} / \sigma_\varepsilon^2 \right)$.

This result can be interpreted in the terms applied to the case (i) where motivated beliefs concern the accuracy of their private information, by just reversing the effects of a manipulation of the instrument σ_ϕ^2. Opacity under a two-instruments policy is not always optimal when firms' subjective beliefs concern their ability to process information, contrasting with the case of objective beliefs. Indeed, by increasing the variance σ_ϕ^2, hence diminishing the relative precision of the privately assessed information, the central bank actually reduces the cost for firms of mistakenly believing that this information is more precise than it really is. Firms thus tend to be even more confident in their private information (as $\delta^* = \left(\beta\sigma_\eta^2 / \left(\sigma_\phi^2 + \sigma_\varepsilon^2 \right) \right)^{-\alpha}$ is increasing in σ_ϕ^2) and may consequently overcompensate the impact of the increase in σ_ϕ^2 on the weight $\kappa_2(\delta, \delta)$ put on private information. Hence, decreasing the precision of semi-public information ends up in increasing price dispersion.

Conversely, as we have seen, transparency may not be optimal since, by excessively augmenting the cost for firms of being irrational, the central bank makes them eventually rely on objective beliefs. However, in the case of a low cost of being irrational, if $\beta\sigma_\eta^2 / \sigma_\varepsilon^2 < 1$, implying that $\min \left(\sqrt{\psi}, 1 \right) = \left(\beta\sigma_\eta^2 / \sigma_\varepsilon^2 \right)^\alpha$ for σ_ϕ^2 close enough to zero, $\lim_{\sigma_\phi^2 \to 0} \partial\sigma_{\mathbf{p}}^2 / \partial\sigma_\phi^2 > 0$ when taking $\rho = \rho^* \left(\sigma_\phi^2 / \sigma_\varepsilon^2 \right)$, ensuring that full transparency is at least locally optimal.

The following proposition summarizes these results.

Proposition 2 *When firms hold motivated beliefs about their ability to process information, (i) transparency is, as under objective beliefs, optimal for a central bank that can only communicate and opacity, combined with full stabilization, is optimal under a two-instruments policy, provided in this case that the weight on the cost of being irrational is not too sensitive to the relative precision of the subjectively assessed information; (ii) otherwise and contrary to what happens under objective beliefs, the optimal two-instruments policy is characterized by an intermediate stabilization level coupled with partial (full) transparency if the cost of being irrational is high (low).*

7 Conclusion

We have shown how firms' motivated beliefs about the precision of their private information or about their ability to process the information that underlies their pricing decisions can impact the optimal policy of the central bank. First, motivated beliefs may make monetary policy less efficient without however commanding any change in the optimal behavior of the central bank. This is what happens in the case of signalling stabilization. The absence of idiosyncratic noise affecting the public information displayed by the central bank deprives the latter, in this case, of the capacity to modulate motivated beliefs.

Second, motivated beliefs may by contrast require the optimizing behavior of the central bank to be adjusted. Outcome prospects are enhanced by overconfidence in the quality of firms' own information, which ends up in more price dispersion and sub-optimal coordination of individual decisions. These decisions result however from a trade-off between the supposed benefit created by overconfidence and the cost of parting with objectivity. If this trade-off is modulated by the signal to noise ratio of central bank communication, the manipulation of this ratio can induce effects distorted enough to temper or even to reverse the direction of optimal monetary policy.

The way the trade-off is modulated depends upon the object of motivated beliefs. If they address the quality of firm's own information relative to that of the semi-public information provided by the central bank, firms' overconfidence is tamed when the latter is noisier, hence less decisive as a basis for pricing decisions. As a consequence, the central bank may weaken price dispersion by abandoning full transparency, optimal under objective beliefs, and introducing some idiosyncratic noise. By contrast, if motivated beliefs address the ability for each firm to extract a signal from noisy information, whatever its source, by providing noisier information the central bank can only reinforce firms' overconfidence, making them wishfully perceive a signal even under full opacity. Then opacity, combined with full stabilization (in order to weaken firms' reactivity), may cease to be optimal, some (or even full) transparency being welcome to reduce price dispersion.

Overall, motivated beliefs tend to give rise to intermediate solutions for the central bank (instead of corner, bang-bang, solutions under objective beliefs), due to effects that go in two opposite directions and that compensate each other: an intermediate level of transparency balances the benefits of increasing firms' information on the fundamental shocks and the detrimental effect of firms' subjective overconfidence.

References

Adam, K. (2007). Optimal Monetary Policy with Imperfect Common Knowledge. *Journal of Monetary Economics*, 54(2): 267-301.

Angeletos, G.-M., and La'O, J. (2020). Optimal Monetary Policy with Informational Frictions. *Journal of Political Economy*, 128(3): 1027-64.

Angeletos, G.-M., and Pavan A. (2007). Efficient Use of Information and Social Value of Information. *Econometrica*, 75(4): 1103-42.

Baeriswyl, R., and Cornand, C. (2010). The Signaling Role of Policy Actions. *Journal of Monetary Economics*, 57(6): 682-95.

Baeriswyl, R., Cornand, C., and Ziliotto, B. (2020). Observing and shaping the market: the dilemma of central banks. *Journal of Money, Credit and Banking*, 52(8): 1973-2005

Banerjee, S., Davis, J., and Gondhi, N. (2020). Motivated beliefs in Coordination Games. unpublished manuscript, July 28.

Barrero, J. M. (2021). The Micro and Macro of Managerial Beliefs. *Journal of Financial Economics*, forthcoming.

Benigno, P. and Karantounias, A. (2019). Overconfidence, subjective perception and pricing behavior. *Journal of Economic Behavior and Organization*, 164: 107-32.

Ben-David, I., Graham, J. R., and Harvey, C. R. (2013). Managerial miscalibration. *Quarterly Journal of Economics*, 128(4): 1547-84.

Benhima K., and Blengini, I. (2020). Optimal Monetary Policy when Information is Market-Generated. *Economic Journal*, 130(628): 956-975.

Brunnermeier, M. K., and Parker, J. A. (2005). Optimal expectations. *American Economic Review*, 95(4):1092-118.

Brunnermeier, M. K., Gollier, C., and Parker, J. A. (2007). Optimal beliefs, asset prices, and the preference for skewed returns. *American Economic Review*, 97(2):159-65.

Caplin, A., and Leahy, J. (2019). Wishful thinking. NBER Working Paper 25707.

Chahrour, R. A., and Ulbricht R. (2021), Robust Predictions for DSGE Models with Incomplete Information. *American Economic journal: Macroeconomics*, conditionally accepted.

Cornand, C., and Heinemann, F. (2008), Optimal Degree of Public Information Dissemination, *Economic Journal*, 118: 718-42.

Dunning, D., Griffin, D., Milojkovic, J., and Ross, L. (1990). The Overconfidence Effect in Social Prediction. *Journal of personality and social psychology*, 58: 568-81. 10.1037/0022-3514.58.4.568.

Hellwig, C. (2005), Heterogeneous Information and the Welfare Effects of Public Information Disclosures.

James, J. G., and Lawler, P. (2011). Optimal Policy Intervention and the Social Value of Public Information. *American Economic Review*, 101(4): 1561-74.

Lorenzoni, G. (2010). Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information. *Review of Economic Studies*, 77(1): 305-38.

Maćkowiak, B., and Wiederholt, M. (2009). Optimal Sticky Prices under Rational Inattention. *American Economic Review*, 99(3): 769-803.

Morris, S., and Shin, H.S. (2002). Social Value of Public Information. *American Economic Review*, 92(5): 1521-34.

Myatt, D., and Wallace, C. (2012). Endogenous Information Acquisition in Coordination Games. *Review of Economic Studies*, 79: 340-74.

Myatt, D., and Wallace, C. (2014). Central Bank Communication Design in a Lucas-Phelps Economy. *Journal of Monetary Economics*, 63: 64-79.

Paciello, L., and Wiederholt, M. (2014). Exogenous Information, Endogenous Information, and Optimal Monetary Policy. *Review of Economic Studies*, 81(1): 356-88.

Woodford, M. (2003a). Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

Woodford, M. (2003b). Imperfect Common Knowledge and the Effects of Monetary Policy, In Knowledge, Information, and Expectations in Modern

Macroeconomics: In Honor of Edmund S. Phelps, edited by P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, 25-58. Princeton, NJ: Princeton Univ. Press.

Appendix

A Generalized means

A *generalized* (or *power*) *mean* of a continuum of non-negative values $\mathbf{P} = (P_i)_{i \in [0,1]} \in \mathbb{R}_+^{[0,1]}$ is defined, for some non-zero real number a, by the equality

$$\mathcal{P}_a(\mathbf{P}) = \left(\int_0^1 P_i^a di \right)^{1/a}. \tag{49}$$

Limit cases are $\mathcal{P}_0(\mathbf{P}) = \prod_{i=0}^{i=1} P_i$ (the geometric mean), $\mathcal{P}_{-\infty}(\mathbf{P}) = \min(P_i)$ and $\mathcal{P}_{+\infty}(\mathbf{P}) = \max(P_i)$. Two currently used special cases are $\mathcal{P}_1(\mathbf{P}) = \int_0^1 P_i di$ (the arithmetic mean) and $\mathcal{P}_{-1}(\mathbf{P}) = \left(\int_0^1 P_i^{-1} di \right)^{-1}$ (the harmonic mean). The CES quantity and price aggregators are generalized means with a restricted to $(-\infty, 1)$ in the case where goods are substitutes ($s > 1$). An important property of the family of generalized means is that

$$a < b \Longrightarrow \mathcal{P}_a(\mathbf{P}) \leq \mathcal{P}_b(\mathbf{P}) \tag{50}$$

with equality if and only if $P_i = P_j$ for any i and j in $[0,1]$.

A transformation of the variables into their log-deviations from a reference fixed value P^*, with $p_i \equiv \ln P_i - \ln P^*$, gives:

$$p_a \equiv \ln \mathcal{P}_a(\mathbf{P}) - \ln P^* = \frac{1}{a} \ln \int_0^1 e^{a p_i} di \tag{51}$$

and, taking a second order Taylor approximation of p_a around $p^* = 0$,

$$p_a \simeq \int_0^1 p_i di + \frac{a}{2} \left(\int_0^1 p_i^2 di - \int_0^1 \int_0^1 p_i p_j di dj \right) = \mathcal{P}_1(\mathbf{p}) + \frac{a}{2} \sigma_{\mathbf{p}}^2, \tag{52}$$

where $\mathcal{P}_1(\mathbf{p})$ and $\sigma_{\mathbf{p}}^2$ are the arithmetic mean and the variance of $\mathbf{p}$, respectively. As $\mathcal{P}_a(\mathbf{P}) = P^* e^{\mathcal{P}_1(\mathbf{p}) + (a/2)\sigma_{\mathbf{p}}^2}$, we thus retrieve the property that $\mathcal{P}_a(\mathbf{P})$ is increasing in a, except when $\sigma_{\mathbf{p}}^2 = 0$.

An application of formula (52) concerns the CES price aggregator

$$p \equiv p_{1-s} \simeq \mathcal{P}_1(\mathbf{p}) - \frac{s-1}{2} \sigma_{\mathbf{p}}^2 \tag{53}$$

which, in the case of the linear pricing rule $p_j = \overline{\kappa}_0 + \overline{\kappa}_1 y_j + \overline{\kappa}_2 x_j$ for any firm j, becomes

$$\begin{aligned} p &= \overline{\kappa}_0 + \overline{\kappa}_1 \left(y + \int_0^1 \phi_j dj \right) + \overline{\kappa}_2 \left(\theta + \int_0^1 \varepsilon_j dj \right) \\ & -\frac{s-1}{2} \int_0^1 \left(\overline{\kappa}_1 \left(\phi_j - \int_0^1 \phi_k dk \right) + \overline{\kappa}_2 \left(\varepsilon_j - \int_0^1 \varepsilon_k dk \right) \right)^2 dj, \end{aligned}$$

B The profit function

Take the expression of the real profit in terms of log deviations from the non-stochasting solution:

$$\pi \left(p_i - p, c, \theta \right) = \left(1 - 1/s\right)^{1/\xi} \left[e^{c-(s-1)(p_i - p)} - \left(1 - 1/s\right) e^{\xi\theta + (1+\xi)c - s(p_i - p)} \right]. \tag{54}$$

The second order Taylor approximation of this function at the origin is given by

$$\begin{aligned} \frac{\pi \left(p_i - p, c, \theta \right)}{\left(1 - 1/s\right)^{1/\xi}} &\simeq 1/s + \frac{\partial \pi}{\partial \left(p_i - p \right)} \left(p_i - p \right) + \frac{\partial \pi}{\partial c} c + \frac{\partial \pi}{\partial \theta} \theta \\ & + \left(p_i - p \right) \left(\frac{1}{2} \frac{\partial^2 \pi}{\partial \left(p_i - p \right)^2} \left(p_i - p \right) + \frac{\partial^2 \pi}{\partial \left(p_i - p \right) \partial c} c + \frac{\partial^2 \pi}{\partial \left(p_i - p \right) \partial \theta} \theta \right) \\ & + c \left(\frac{1}{2} \frac{\partial^2 \pi}{\partial c^2} c + \frac{\partial^2 \pi}{\partial c \partial \theta} \theta \right) + \frac{1}{2} \frac{\partial^2 \pi}{\partial \theta^2} \theta^2 + R_2 \left(p_i - p, c, \theta \right), \end{aligned} \tag{55}$$

where R_2 is the remainder term at the second order. By computing the second order derivatives at the origin, we obtain the log-quadratic approximation of the profit function

$$\begin{aligned} \frac{\pi \left(p_i - p, c, \theta \right)}{\left(1 - 1/s\right)^{1/\xi}} &\simeq \underbrace{\left(s-1 \right) \left(p_i - p \right) \left(\xi \left(c + \theta \right) - \frac{1}{2} \left(p_i - p \right) \right)}_{\widetilde{\pi}(p_i - p, c+\theta)} \\ & \frac{1}{s} + \frac{1 - (s-1)\xi}{s} c - \frac{(s-1)\xi}{s} \theta + \frac{s - (s-1)(1+\xi)^2}{2s} c^2 \\ & - \frac{(s-1)(1+\xi)\xi}{s} c\theta - \frac{(s-1)\xi^2}{2s} \theta^2 + R_2 \left(p_i - p, c, \theta \right), \end{aligned} \tag{56}$$

where all but the first term do not depend upon $p_i - p$ or involve a higher order of approximation. Hence, we may take

$$\widetilde{\pi} \left(p_i - p, c + \theta \right) = \left(s-1 \right) \left(p_i - p \right) \left(\xi \left(c + \theta \right) - \frac{1}{2} \left(p_i - p \right) \right) \tag{57}$$

as the approximated profit function to be maximized in $p_i - p$.

C The welfare function

Welfare is identified with the household's utility

$$U(\Theta C) - \Theta L = U(\Theta C) - \Theta \int_0^1 \left(\frac{P_i}{P}\right)^{-s} C di = \frac{(\Theta C)^{1-\xi}}{1-\xi} - \Theta C \mathcal{P}_{1-s}(\mathbf{P})^s \mathcal{P}_{-s}(\mathbf{P})^{-s}, \tag{58}$$

using the generalized mean $\mathcal{P}_a(\mathbf{P}) \equiv \left(\int_0^1 P_i^a di\right)^{1/a}$ (see Appendix A). Equivalently, in log-deviations from the nonstochastic solution of the model,

$$v(p_{-s} - p_{1-s}, c+\theta) = (1 - 1/s)^{1/\xi} \left(\frac{e^{(1-\xi)(c+\theta)}}{(1-\xi)(1-1/s)} - e^{c+\theta - s(p_{-s} - p_{1-s})}\right). \tag{59}$$

The second order Taylor approximation of this function, at the origin, is

$$\begin{aligned} \frac{\widetilde{v}(p_{-s} - p_{1-s}, c+\theta)}{(1-1/s)^{1/\xi}} &= \underbrace{\frac{1+\xi(s-1)}{(1-\xi)(s-1)}}_{\widetilde{v}(\mathbf{0})/(1-1/s)^{1/\xi}} + \frac{c+\theta}{s-1} + s(p_{-s} - p_{1-s}) \\ &+ \frac{1}{2}\left(\frac{1-s\xi}{s-1}(c+\theta)^2 - s^2(p_{-s} - p_{1-s})^2 + 2s(p_{-s} - p_{1-s})(c+\theta)\right) \\ &+ R_2(p_{-s} - p_{1-s}, c+\theta). \end{aligned} \tag{60}$$

where R_2 is the remainder term at the second order.

This welfare function has two arguments, which may be seen as expressing coordination and stabilization intermediate objectives of the central bank policy. By equation (52), its first argument, $p_{-s} - p_{1-s}$, is indeed always non-positive, attaining its maximum at zero, when all the prices are identical:

$$p_{-s} - p_{1-s} = -\frac{1}{2}\sigma_{\mathbf{p}}^2. \tag{61}$$

As to the second argument of the approximated welfare function, we can use the equality $c + \theta = z + \theta - p$, with z possibly under the control of the central bank. By (52) and then (13), we have at equilibrium:

$$p = \mathcal{P}_1(\mathbf{p}) - \frac{s-1}{2}\sigma_{\mathbf{p}}^2 = (1-\xi)p + \xi(z+\theta) - \frac{s-1}{2}\sigma_{\mathbf{p}}^2, \tag{62}$$

so that

$$p = z + \theta - \frac{s-1}{2\xi}\sigma_{\mathbf{p}}^2, \tag{63}$$

and finally

$$c + \theta = \frac{s-1}{2\xi}\sigma_{\mathbf{p}}^2. \tag{64}$$

The second potential intermediary objective of the central bank, stabilization, is out of reach, leaving us with coordination. This is of course in line with Lucas' critique.

By (60) and neglecting the constant term as well as the remainder term at the second order, we may refer to the transformed welfare function

$$V\left(\sigma_{\mathbf{p}}^2\right) = -\frac{s\xi - 1}{\xi}\left(\frac{\sigma_{\mathbf{p}}^2}{2}\right) - \frac{1}{2}\left(\frac{(s-1)(3s\xi - 1)}{\xi^2} + s^2\right)\left(\frac{\sigma_{\mathbf{p}}^2}{2}\right)^2, \tag{65}$$

which is clearly a decreasing function under the assumption $s\xi \geq 1$.

D Optimal policy under motivated beliefs about the ability to process information

We recall that the loss function to be minimized by the central bank is

$$\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2\sigma_\phi^2 + \kappa_2^2\sigma_\varepsilon^2 = (1-\rho-\kappa_2)^2\sigma_\phi^2 + \kappa_2^2\sigma_\varepsilon^2, \tag{66}$$

with derivatives with respect to the policy instruments

$$\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\sigma_\phi^2} = (1-\rho-\kappa_2)^2 + 2\left(\kappa_2\sigma_\varepsilon^2 - (1-\rho-\kappa_2)\sigma_\phi^2\right)\frac{\partial\kappa_2}{\partial\sigma_\phi^2} \tag{67}$$

and

$$\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\rho} = -2(1-\rho-\kappa_2)\sigma_\phi^2 + 2\left(\kappa_2\left(\sigma_\phi^2+\sigma_\varepsilon^2\right) - (1-\rho)\sigma_\phi^2\right)\frac{\partial\kappa_2}{\partial\rho}. \tag{68}$$

The coefficient κ_2 can be expressed as

$$\kappa_2 = \frac{\xi\sigma_\eta^2 + (1-\rho)\sigma_\phi^2\min\left(\sqrt{\psi},1\right)}{\xi\sigma_\eta^2 + \left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\min\left(\sqrt{\psi},1\right)}, \text{ where } \sqrt{\psi} = \left(\beta\sigma_\eta^2/\left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\right)^\alpha, \tag{69}$$

with derivatives

$$\begin{aligned}\frac{\partial\kappa_2}{\partial\sigma_\phi^2} &= \frac{1}{\left(\xi\sigma_\eta^2 + \left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\min\left(\sqrt{\psi},1\right)\right)^2} \\ &\quad \left[\begin{array}{c}\min\left(\sqrt{\psi},1\right)\left(-\rho\xi\sigma_\eta^2 + (1-\rho)\sigma_\varepsilon^2\min\left(\sqrt{\psi},1\right)\right) \\ -\xi\sigma_\eta^2\left(\rho\sigma_\phi^2+\sigma_\varepsilon^2\right)\partial\min\left(\sqrt{\psi},1\right)/\partial\sigma_\phi^2\end{array}\right]\end{aligned} \tag{70}$$

and

$$\frac{\partial\kappa_2}{\partial\rho} = -\frac{\sigma_\phi^2\min\left(\sqrt{\psi},1\right)}{\xi\sigma_\eta^2 + \left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\min\left(\sqrt{\psi},1\right)}. \tag{71}$$

Given $\sigma_\phi^2 \in (0,\infty)$, the sign of $\partial\sigma_{\mathbf{p}}^2/\partial\rho$ is

$$\begin{aligned}\operatorname{sgn}\left(\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\rho}\right) &= \operatorname{sgn}\left(\rho - \left(1 - \frac{\xi\sigma_\eta^2\left(\sigma_\phi^2 + \left(\sigma_\phi^2+\sigma_\varepsilon^2\right)\frac{\partial\kappa_2}{\partial\rho}\right)}{\sigma_\phi^2\left[\xi\sigma_\eta^2\left(1+\frac{\partial\kappa_2}{\partial\rho}\right) + \sigma_\varepsilon^2\min\left(\sqrt{\psi},1\right)\right]}\right)\right) \\ &\operatorname{sgn}\left(\rho - \left(1 - \frac{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2\right)^2}{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2 + \min\left(\sqrt{\psi},1\right)\right)^2 + \left(\sigma_\phi^2/\sigma_\varepsilon^2\right)\left(\min\left(\sqrt{\psi},1\right)\right)^2}\right)\right).\end{aligned} \tag{72}$$

This ensures, for any $\sigma_\phi^2/\sigma_\varepsilon^2 \in (0,\infty)$, an optimal value of ρ, namely $\rho^*\left(\sigma_\phi^2/\sigma_\varepsilon^2\right)$, which belongs to the interior of $(0,1)$ if $\sigma_\eta^2/\sigma_\varepsilon^2 \in (0,\infty)$. Notice that, as soon as $\sigma_\phi^2 = 0$, $\partial\sigma_{\mathbf{p}}^2/\partial\rho = 0$, since this derivative is the product of σ_ϕ^2 and a decreasing function of $\sigma_\phi^2/\sigma_\varepsilon^2$. The consequence is that the optimal value of ρ is then indeterminate.

Let us now consider the behavior of $\partial\kappa_2/\partial\sigma_\phi^2$ as $\sigma_\phi^2 \to 0$. First, notice that $\lim_{\sigma_\phi^2\to 0}\sqrt{\psi} = \left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha$, leading to two cases.

- $\beta\sigma_\eta^2/\sigma_\varepsilon^2 > 1$

  In this case, $\min\left(\sqrt{\psi},1\right) = 1$ and $\partial\min\left(\sqrt{\psi},1\right)/\partial\sigma_\phi^2 = 0$ for σ_ϕ^2 close enough to zero. Also,

$$\lim_{\sigma_\phi^2\to 0}\rho^*\left(\sigma_\phi^2/\sigma_\varepsilon^2\right) = 1 - \left(\frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+1}\right)^2. \tag{73}$$

  Then,

$$\lim_{\sigma_\phi^2\to 0}\kappa_2 = \frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+1} \text{ and } \lim_{\sigma_\phi^2\to 0}\frac{\partial\kappa_2}{\partial\sigma_\phi^2} = -\frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\sigma_\varepsilon^2\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2+1\right)^3}, \tag{74}$$

  so that, by introducing these values in the expression for $\partial\sigma_{\mathbf{p}}^2/\partial\sigma_\phi^2$, we obtain:

$$\operatorname{sgn}\left(\lim_{\sigma_\phi^2\to 0}\frac{\partial\sigma_{\mathbf{p}}^2}{\partial\sigma_\phi^2}\right) = -1. \tag{75}$$

- $\beta\sigma_\eta^2/\sigma_\varepsilon^2 < 1$

  Now, $\min\left(\sqrt{\psi},1\right) \to \left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha$ and $\partial\min\left(\sqrt{\psi},1\right)/\partial\sigma_\phi^2 \to -\alpha\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha/\sigma_\varepsilon^2$ as $\sigma_\phi^2 \to 0$. Also,

$$\lim_{\sigma_\phi^2\to 0}\rho^*\left(\sigma_\phi^2/\sigma_\varepsilon^2\right) = 1 - \left(\frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha}\right)^2. \tag{76}$$

  Then,

$$\begin{aligned}\lim_{\sigma_\phi^2\to 0}\kappa_2 &= \frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha} \text{ and} \\ \lim_{\sigma_\phi^2\to 0}\frac{\partial\kappa_2}{\partial\sigma_\phi^2} &= \frac{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2\right)\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\left[\alpha\xi\sigma_\eta^2/\sigma_\varepsilon^2+(\alpha-1)\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\right]}{\sigma_\varepsilon^2\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\right)^3},\end{aligned} \tag{77}$$

  leading to

$$2\alpha\xi\sigma_\eta^2/\sigma_\varepsilon^2+(2\alpha-1)\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha$$

$$
\begin{aligned}
& \operatorname{sgn}\left(\lim_{\sigma_\phi^2\to 0}\frac{\partial\sigma_{\mathbf{P}}^2}{\partial\sigma_\phi^2}\right) \\
= & \operatorname{sgn}\left(\begin{array}{c} \left(\frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha}\right)^2\left(\frac{-\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha}\right)^2 \\ -2\frac{\xi\sigma_\eta^2/\sigma_\varepsilon^2}{\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha}\frac{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2\right)\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\left[\alpha\xi\sigma_\eta^2/\sigma_\varepsilon^2+(\alpha-1)\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\right]}{\left(\xi\sigma_\eta^2/\sigma_\varepsilon^2+\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\right)^3} \end{array}\right) \\
= & \operatorname{sgn}\left(2\alpha\xi\sigma_\eta^2/\sigma_\varepsilon^2+(2\alpha-1)\left(\beta\sigma_\eta^2/\sigma_\varepsilon^2\right)^\alpha\right)=+1. \qquad (78)
\end{aligned}
$$