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To seed, or not to seed

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Abstract

Within this paper, we develop a simple overlapping generations model (OLG) with a renewable resource (forest) in the spirit of Koskela et al. (2002). Seeding activities (more broadly, forestry) are introduced in the form of a domestic production as well as a joy-of-giving bequest motive regarding the resource. In this simple framework, we show that altruism always guarantee a positive resource level at the steady state. However, studying the dynamics, we point out that the stability of the steady state crucial depends upon both altruism and forestry productivity: under a low forestry productivity, the steady state is always stable while, under a high forestry productivity, two period-cycles (flip bifurcation) can emerge near the steady state if and only if altruism is sufficiently high which rises the question of resource preservation and leads to the conclusion that the road to hell is paved with good intentions.

Keywords: Renewable resource, OLG model, altruism, flip bifurcation.

JEL Classification: E32, O44.

1 Introduction

In a well-known note, reported and translated from the french by Baranzini and Allisson (2014), Léon Walras writes: "One must know what one is doing. If one wants to harvest promptly, one should plant carrots and salads ; if one has the ambition to plant oak trees, one must be wise enough to say: [posterity] will owe me this shade".

From this quote, we see three important features of a resource exploitation: seeding, harvesting and altruism. In the renewable resource literature, both harvesting and altruism have been studied extensively while, to the best of our knowledge, seeding has not yet been studied so far.

The first overlapping generations model (OLG) which incorporates a renewable resource dynamics dates back to Kemp and Van Long (1979). A representative household owns the resource and can decide to harvest and to sell it to a representative firm. We can observe two particular elements: (1) the resource is an inessential production factor in their model since the representative firm can produce without using the natural resource and (2) there is no capital accumulation. From this simple framework, Kemp and Van Long (1979) demonstrate that the competitive equilibrium could be inefficient. In contrast to Kemp and Van Long (1979), Koskela et al. (2002) consider that the resource is essential to produce and focus their analysis not only on the steady state efficiency but also on the dynamics. In particular, they point out that, under a concave reproduction function for the natural resource, two steady states appear: the first one is efficient but always unstable while the other is stable but can be inefficient. An important assumption made by the authors is the quasi-linear utility function which has been removed by Koskela et al. (2008). Using a logistic reproduction function for the natural resource, they prove the uniqueness of the steady state. Moreover, they observe that if the intertemporal elasticity of substitution in consumption is low enough, then periodic cycles can emerge near the steady state through a flip bifurcation.

The complex dynamics[1] pointed by Koskela et al. (2008) was also recovered recently by Amacher et al. (2018). In a framework close to Koskela et al. (2002, 2008), they assume that the natural resource has amenity benefits for the representative household: the resource affects directly the utility function. This extension always leads to a unique steady state which can be locally indeterminate if and only if the weight of the resource is high enough in the utility function, generating expectations driven fluctuations[2].

The existence of endogenous cycles is very interesting from an environmental perspective. Indeed, following Pezzey (1997)[3], a trajectory is said to satisfy the sustainable development criterion if the felicity never decreases. Typically, the flip bifurcation pointed out by Koskela et al. (2008) or the local indeterminacy stressed by Amacher et al. (2018) describe a violation of the sustainable development criterion because they imply fluctuations of the utility level. However, one can question the robustness of those results since both Koskela et al. (2008) or Amacher et al. (2018) consider an economy without capital accumulation and it is well-known that models are more volatile without capital[4].

As pointed out by the Walras' quote, resource exploitation refers also to altruism since the one who seed is not always the one who enjoy the benefits. Altruism has been usually study as in Barro (1974): parents take into account of their offspring's utility when they make their consumption/saving decisions. In contrast to this "pure altruism" à la Barro (1974), Andreoni (1989) proposes a "warm glow" or the so-called joy-of-giving bequest motive: the representative household bequests without considering her offspring's utility, she just makes her duty without other considerations. Such a joy-of-giving bequest motive has been introduced in the renewable resource literature by Bréchet and Lam-

[1] i.e. the flip bifurcation.

[2] See Azariadis (1981) among others.

[3] Among the others.

[4] For instance, the famous OLG model proposed by Grandmont (1985) and displaying endogenous cycles is a monetary economy without capital accumulation.

brecht (2011). In their paper the representative household owns the resource as in Koskela et al. (2002) and decides to harvest and to sell the resource to the production sector. During her old age, the household not only obtain utility from consumption but also from the amount of resource bequeathed. In addition to the introduction of this joy-of-giving bequest motive, Bréchet and Lambrecht (2011) complete Koskela et al. (2002) by considering capital accumulation and then, the production function possesses three inputs: labour, capital and resource. Within their paper, they show that altruism is a key element for resource preservation: there exists a threshold for the degree of altruism under which the resource level at the steady state is zero while this stock is positive above the threshold. Analyzing the dynamics, they point out the impossible occurrence of complex dynamics. This result clearly contrasts with the dynamical results obtained by Koskela et al. (2008) and by Amacher et al. (2018). As in Bréchet and Lambrecht (2011), the resource also enter into the utility function in Amacher et al. (2018) but the very difference between those two contributions, in addition to the capital accumulation, is that the resource is a separable argument of the household's utility function in Bréchet and Lambrecht (2011) while it is a non-separable argument in Amacher et al. (2018). A question arises: is the capital accumulation or the separable utility which avoid the possible existence of complex dynamics in Bréchet and Lambrecht (2011) comparing to Amacher et al. (2018)? One of the aim of the present paper is to answer this question by reconsidering the possibility of complex dynamics under capital accumulation.

Following Walras' quote, seeding activities appear also to be an important feature of resource exploitation. Seeding takes time and can be viewed as a domestic production. That is, for an household which owns a natural resource, such as a forest for instance, there is a trade-off between working in a firm or in her forest. To the best of our knowledge, this type of trade-off has not yet been studied in the renewable resource literature. To fill this gap, the following paper develops a simple OLG model with capital accumulation and a renewable natural resource. The representative household lives for two periods (youth and old age). During her youth, she inherits the forest from her parents and has to decide how much time she allocates to working in her forest (seeding activities or more broadly, forestry) or in the firm and the amount of timber she want to sell to the firm (harvesting). During her old age, she consumes and bequests the resource to her offsprings (joy-of-giving bequest motive). Moreover, there is a delay between seeding (forestry) and its effect and the forest size. To take into account of this element, we assume that this is not the household engaged in forestry who obtain the benefits but her offsprings: seeding (forestry) is made only for posterity in the spirit of Walras' quote. Moreover, this paper also refer to Amacher et al. (2018) in the sense that the amount of resource bequeathed (Joy-of-giving bequest motive) affects the marginal utility of consumption.

Within this simple framework, we prove the existence of a unique steady state. Interestingly, we show that, in contrast to Bréchet and Lambrecht (2011), there is no lower bond on the degree of altruism for resource preservation: the resource level at the steady state is always positive whatever the degree of altruism. We explain this result because of the non-separable utility function. After defining the optimal resource exploitation as the maximal level of the resource obtained by Nature without human activities, we show that there exists a particular level of altruism which allows to recover this optimum when harvesting and forestry are taking into account. Altruism below (above) this level leads to overexploitation (underexploitation) of the natural resource. This result is disturbing since it shows that altruism is not always a good thing for resource management. Finally, studying the resource dynamics, two configurations occur depending upon productivity of forestry: under a low forestry productivity, the steady state is always stable while, under a high forestry productivity, two period-cycles (flip bifurcation) can emerge near the steady state if and only if the resource is overexploited (that is, when altruism is too high regarding the maximum sustainable yield). This result is interesting because it completes both Koskela et al. (2008) and Amacher et al. (2018): complex dynamics can well arise in an OLG model with a renewable resource when capital accumulation is taking into account.

The paper is organized as follow: In section 2, we present the model, in section 3 the steady state existence is considered while section 4 studies both resource exploitation and resource dynamics. Section 5 concludes.

2 The model

2.1 The natural resource

Let z_t be the stock of the natural resource (forest) at time t. Following Koskela et al. (2002) (among others), without human activities, its evolution is given by the following equation:

$$z_{t+1} = z_t + g(z_t) \tag{1}$$

Where $g(z_t)$ represents the growth function (resource reproduction). As usual in the renewable resource literature[5], $g(z_t)$ has a bell shape. To simplify the exposition, we assume that $g(z_t) = z_t(1 - z_t)$. That is, if $z_t < 1\,(> 1)$, then $g(z_t) > 0\,(< 0)$: a congestion occur when $z_t > 1$. This situation can be simply explained, indeed, when $z_t > 1$, there is too much trees implying that the nutrients in the soil are not enough to allow the resource growth. Without human activities, considering the functional for $g(z_t)$, (1) writes simply:

$$z_{t+1} = z_t + z_t(1 - z_t) \tag{2}$$

Two steady state arises, $z = 0$ and $z = 1$: the zero steady state is unstable while the other one is stable. Now, let us introduce human activities and discuss how they affect resource dynamics (2).

[5] See for instance Koskela et al. (2002), Wirl (2004), Bella (2010), Bréchet and Lambrecht (2011) or more recently Bosi and Desmarchelier (2018).

In this economy, the representative household lives for two periods: youth and old-age. During her youth, she owns the resource inherited from her parents and she is endowed with one unit of time that she shares between working in the firm (l_t) and working in her forest (i.e. seeding or more generally, forestry, $1-l_t$).

The production function of forestry activities is given by φ. To simplify the exposition, φ is assumed to be linear:

$$\varphi(1-l_t) = A(1-l_t)$$

Where $A > 0$ represents productivity of forestry.

Moreover, the representative household also harvests the forest during her youth (h_t). Taking into account of both forestry and harvesting, the evolution of the natural resource is now given by:

$$z_{t+1} = z_t + z_t(1-z_t) + A(1-l_t) - h_t \tag{3}$$

For the representative household born at time t, z_{t+1} represents the resource stock bequeathed to her children born at time $t+1$.

2.2 The households

As introduced in the previous section, the representative household is assumed to live for two periods: youth and old age. To keep things as simple as possible, we consider that there is no birth, no death and then, the population remains constant over time. During her youth, the representative household born at time t shares her working force between firm (l_t) and forestry ($1-l_t$). While working in the production sector allows to obtain a wage rate w_t, she also harvests the forest (h_t) and sells it to the firm at price p_t. All her income (coming from working in the firm and from selling the resource) is fully saved to finance consumption during her old-age (c_{t+1}). This leads to the following first and second period budget constraints:

$$w_t l_t + p_t h_t = s_t \tag{4}$$

$$c_{t+1} = (1+r_{t+1}) s_t \tag{5}$$

where r_{t+1} is the real interest rate. Differently from Koskela et al. (2002) but in the same spirit than Bréchet and Lambrecht (2011), we assume that the resource level affects the household's utility:

$$u(c_{t+1}, z_{t+1}) = \frac{\left(c_{t+1} z_{t+1}^{\eta}\right)^{1-\varepsilon}}{1-\varepsilon} \tag{6}$$

$\varepsilon > 0$ captures the elasticity of the utility function with respect to the composite good $c_{t+1} z_{t+1}^{\eta}$ while $\eta > 0$ represents the weight of the resource in the household's utility. As in Bréchet and Lambrecht (2011), the presence of z_{t+1} in the utility function is a joy of giving bequest motive: the representative household enjoys to bequest resource to her children. However, differently from Bréchet and Lambrecht (2011), the utility function (6) is not separable between consumption and resource, this implies that the resource affects the marginal utility of consumption as in Amacher et al. (2018). It follows also that η captures the magnitude of the joy-of-giving bequest motive: if $\eta = 0$, then the representative household will not try to bequest resource to her children while if $\eta \to +\infty$, a small increase of z_{t+1} dramatically increases her utility which means that she will try to bequest a strong resource level to her children. Let us characterize more precisely the optimal behavior of the representative household: she chooses both l_t and h_t to maximize her utility function (6) with respect to (4), (5) and (3), taking as given prices (w_t, p_t, r_{t+1}). First order conditions gives:

$$(1 + r_{t+1})\, w_t = \eta \frac{c_{t+1}}{z_{t+1}} A \tag{7}$$

$$p_t\, (1 + r_{t+1}) = \eta \frac{c_{t+1}}{z_{t+1}} \tag{8}$$

Which implies that:

$$w_t = A p_t \tag{9}$$

At the optimum, the household works and harvests such that (9) is verified: in this case, the household is indifferent between working and harvesting.

Considering (7) (or (8)) jointly with (9), (3), (4) and (5), it follows that:

$$l_t = \frac{z_t + z_t\,(1 - z_t) + A - (1 + \eta)\, h_t}{A\,(1 + \eta)} \tag{10}$$

Injecting (10) into (3), we obtain the forest dynamics:

$$z_{t+1} = \frac{\eta}{1 + \eta}\,(A + z_t\,(2 - z_t)) \tag{11}$$

We recover a configuration close to Koskela et al. (2002) or to Bréchet and Lambrecht (2011): the resource dynamics is a non-linear first order difference equation which is not affected by capital accumulation. Moreover, we observe that if the representative household has no joy-of-giving bequest motive (i.e. $\eta = 0$), then $z_{t+1} = 0$, that is, the representative household born at date t harvests and sells all her forest. In this case, the resource cannot be preserved in the long run: the forest is completely depleted by the first generation.

At this step of the reasoning, we have to introduce a restriction on ε to ensure the concavity of u with respect to(l_t, h_t).

Assumption 1 c_{t+1} and z_{t+1} are substitutable goods for the representative household ($\varepsilon > 1$)[6].

[6] See the appendix.

2.3 The representative firm

As in Bréchet and Lambrecht (2011)(among others), the representative firm uses three inputs to produce a composite good (Y_t) which can be consumed or saved/invested: (1) capital (K_t), (2) labor (L_t) and (3) resource (H_t). To keep things as simple as possible, we consider a constant return to scale Cobb-Douglas production function:

$$Y_t = F\left(K_t, L_t, H_t\right) = BK_t^{\alpha} L_t^{\beta} H_t^{1-\alpha-\beta} \tag{12}$$

Where $\alpha \in (0,1)$ and $\beta \in (0,1)$ capture respectively the sensitivity of the production function with respect to capital and labor while $1-\alpha-\beta$ captures the sensitivity with respect to the resource such that $1-\alpha-\beta > 0$. The representative firm chooses the amount of productive inputs K_t, L_t and H_t in order to maximize its profit:

$$\pi_t = BK_t^{\alpha} L_t^{\beta} H_t^{1-\alpha-\beta} - \left(r_t + \delta\right) K_t - w_t L_t - p_t H_t$$

taking as given prices (r_t, w_t and p_t). $\delta \in (0,1)$ denotes the capital depreciation rate while $B > 0$ is a scaling parameter which captures the total factor productivity. As usual, first order conditions imply that the representative firm chooses K_t, L_t and H_t such that marginal productivity are just equal to prices:

$$\frac{\partial F}{\partial K_t} = \alpha BK_t^{\alpha-1} L_t^{\beta} H_t^{1-\alpha-\beta} = r_t + \delta \tag{13}$$

$$\frac{\partial F}{\partial L_t} = \beta BK_t^{\alpha} L_t^{\beta-1} H_t^{1-\alpha-\beta} = w_t \tag{14}$$

$$\frac{\partial F}{\partial H_t} = \left(1-\alpha-\beta\right) BK_t^{\alpha} L_t^{\beta} H_t^{-\alpha-\beta} = p_t \tag{15}$$

3 Equilibrium and steady state

At the equilibrium, saving is just equal to investment:

$$s_t = K_{t+1} - \left(1-\delta\right) K_t$$

As usual in the OLG literature, we assume that capital fully depreciates in one period of time ($\delta = 1$) which means that $s_t = K_{t+1}$. Since the population remains constant forever, to simplify the presentation, we normalize it to the unity, that is, $N_{t+1} = N_t = 1$ and then, $K_{t+1} = N_{t+1} k_{t+1} = k_{t+1}$ which implies that:

$$s_t = k_{t+1}$$

Moreover, at the equilibrium, demand for labor and for the resource are just equal to their supply, namely, $L_t = l_t$ and $H_t = h_t$. That is, considering (9), at the equilibrium, (14) and (15) imply that:

$$h_t = \frac{A(1-\alpha-\beta)}{\beta} l_t \tag{16}$$

Considering jointly (10) and (16) give l_t and h_t at the equilibrium:

$$h_t = h(z_t) = \frac{1-(\alpha+\beta)}{(1+\eta)(1-\alpha)} (z_t + z_t(1-z_t) + A) \tag{17}$$

$$l_t = l(z_t) = \frac{\beta}{A(1+\eta)(1-\alpha)} (z_t + z_t(1-z_t) + A) \tag{18}$$

We observe that, at the equilibrium, l_t and h_t are two functions of the resource level z_t. It is not surprising to recover (11) by injecting (17) and (18) into (3). Moreover, considering jointly (4), (9), (16), (18) and (15), we obtain the evolution of capital:

$$k_{t+1} = \gamma \left(\frac{1}{1+\eta}\right)^{1-\alpha} (z_t + z_t(1-z_t) + A)^{1-\alpha} k_t^{\alpha}$$

With:

$$\gamma \equiv B(1-\alpha)\left(\frac{\beta}{(1-\alpha)A}\right)^{\beta}\left(\frac{1-(\alpha+\beta)}{1-\alpha}\right)^{1-\alpha-\beta} > 0$$

Previous discussion leads to the following proposition.

Proposition 1 *An intertemporal equilibrium for this economy is a sequence* $(k_t, z_t)_{t=0}^{+\infty}$ *such that the following system is verified:*

$$k_{t+1} = \gamma \left(\frac{1}{1+\eta}\right)^{1-\alpha} (z_t + z_t(1-z_t) + A)^{1-\alpha} k_t^{\alpha} \tag{19}$$

$$z_{t+1} = \frac{\eta}{1+\eta} (z_t + z_t(1-z_t) + A) \tag{20}$$

Where k_0 *and* z_0 *are given.*

At the steady state, $k_{t+1} = k_t = k$ and $z_{t+1} = z_t = z$. Considering (20), it follows that, at the steady state:

$$z = \frac{\eta}{1+\eta} (z + z(1-z) + A) \tag{21}$$

Two different values of z satisfy (21), namely z_1 and z_2 with:

$$z_1 = \frac{1}{2\eta}\left(\eta - 1 + \sqrt{(1+4A)\eta^2 - 2\eta + 1}\right) \tag{22}$$

$$z_2 = \frac{1}{2\eta}\left(\eta - 1 - \sqrt{(1+4A)\eta^2 - 2\eta + 1}\right)$$

We observe that $A > 0$ ensures that $(1+4A)\,\eta^2 - 2\eta + 1 > 0$. That is, since $A > 0$, both z_1 and z_2 exist. Moreover, $\forall\,(\eta, A) \in \mathbb{R}_+^2$, $z_2 < 0$ and $z_1 > 0$ which implies that z_1 is the only admissible steady state. Considering (21) and (22), equation (19) gives the capital level at the steady state:

$$k^* = \gamma^{\frac{1}{1-\alpha}} \frac{z_1}{\eta} \tag{23}$$

That is, the unique steady state of this economy is given by (k^*, z_1). The following proposition study how altruism affect the steady state.

Proposition 2 *The qualitative impact of η on the steady state is given by:*

$$\begin{aligned} \frac{\eta}{z_1}\frac{\partial z_1}{\partial \eta} &= \frac{1}{\sqrt{(1+4A)\,\eta^2 - 2\eta + 1}} > 0 \\ \frac{\eta}{k^*}\frac{\partial k^*}{\partial \eta} &= \frac{\eta}{z_1}\frac{\partial z_1}{\partial \eta} - 1 > (<)\, 0 \textit{ iff } \eta < (>)\, \frac{2}{1+4A} \end{aligned}$$

Proof. Simply differentiate (23) and (22) with respect to η by considering that, at the steady state, (21) holds. ■

The positive effect of η on the resource level can be simply explained. Indeed, a higher value of η means that the representative household has a higher incentive to bequest a more important level of natural resource to her offspring and then, to higher forestry (seeding) and to lower harvesting, increasing in turn the resource level in the long run. The effect of η on the capital level is more ambiguous. Indeed, we have just observed that a higher η implies a higher resource level because it implies a higher incentive to increase forestry (or to lower working time in the firm) and to lower harvesting. Considering (4), this means a lower saving level which implies a lower capital level in the long run. However, consider now a situation where the initial altruism level is low ($\eta < 2/(1+4A)$). In this case, the resource level at the steady state is also low which implies in turn a low production level since the resource is an essential production factor (see (12)). In this case, a higher η increases the resource level in the long run, implying a higher production and then a higher capital level at the steady state. That is, the only situation where η has a positive effect on both natural resource and capital in the long run is when altruism is low ($\eta < 2/(1+4A)$).

4 Resource dynamics and resource exploitation

As observed before, the dynamics of the natural resource is not affected by the capital accumulation. Since the purpose of the following paper is to observe how altruism affects the resource preservation, we focus on the resource dynamics. Let:

$$z_{t+1} \equiv \phi\,(z_t) = \frac{\eta}{1+\eta}\,(z_t + z_t\,(1 - z_t) + A)$$

We can remark simply that:

$$\phi(0) = \frac{\eta A}{1+\eta}, \lim_{z_t \to +\infty} \phi(z_t) = -\infty \text{ and } \phi\left(1+\sqrt{1+A}\right) = 0$$

Moreover,

$$\frac{dz_{t+1}}{dz_t} = \phi'(z_t) = 2\frac{\eta}{1+\eta}(1-z_t) \tag{24}$$

That is, ϕ increases (decreases) if and only if $z_t < 1\,(> 1)$. It follows that the household born at date t maximizes the resource bequeathed to her offspring if and only if $z_t = 1$. It is interesting to observe the relation with the reproduction function of the natural resource[7]: $g(z_t) > 0\,(< 0)$ if and only if $z_t < 1\,(> 1)$. As discussed in section 2, without human interventions, the maximal resource level in the long run is given by $z = 1$. This level is known as the Maximum Sustainable Yield (MSY). We can characterize the resource level at the steady state regarding the MSY: if $z_1 < 1\,(> 1)$ the resource is overexploited (underexploited) while $z_1 = 1$ appears to be optimal from an ecological point of view. The following proposition summarizes all possible configurations with respect to both A and η.

Proposition 3 *Let $A \neq 0$:*
-If $\eta < 1/A$, then $z_1 < 1$, the resource is overexploited.
-If $\eta > 1/A$, then $z_1 > 1$, the resource is underexploited.
And if $\eta = 1/A$, the resource exploitation is optimal ($z_1 = 1$).

Proof. Simply consider (22). ■

The last proposition shows that altruism is important to preserve the resource level in the long run. In particular, $\eta = 1/A$ allows to recover the MSY. However, altruism can become too high ($\eta > 1/A$) which generates a congestion. Figure 1 depicts all possible configurations.

[7] Remember that the reproduction function of the natural resource is given by $g(z_t) = z_t(1-z_t)$.

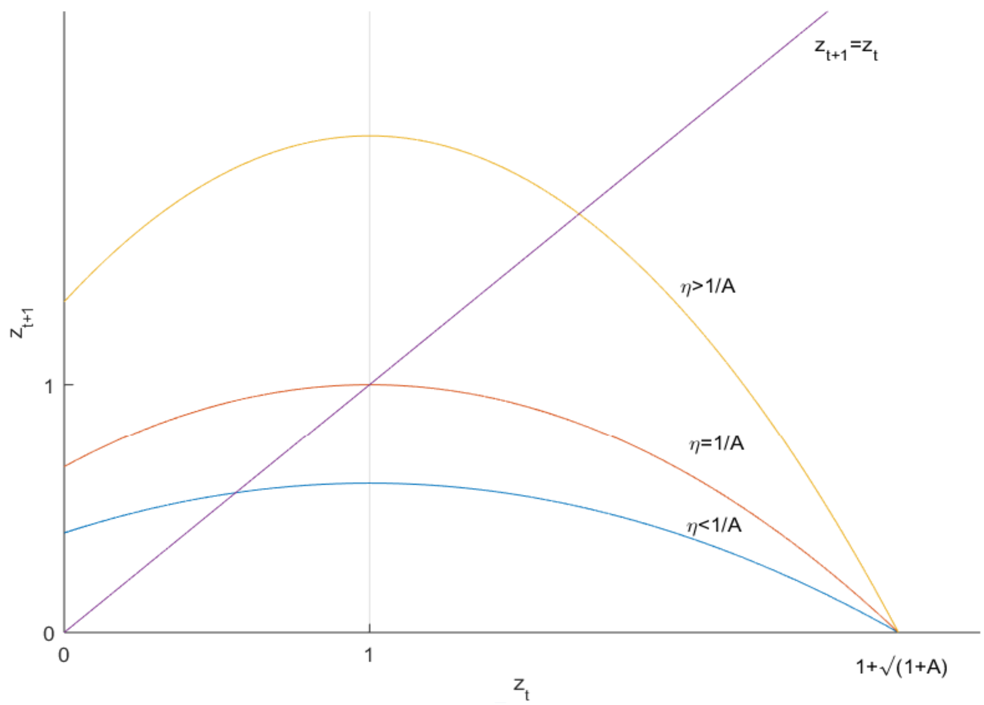

Fig.1. Steady state w.r.t η (A is fixed).

Despite the steady state level, the resource is preserved in the long run if and only if z_1 is stable.

Let:

$$\eta_1 = \frac{5 - 2\sqrt{3A+4}}{4A-3}$$
$$\eta_2 = \frac{2\sqrt{3A+4}+5}{4A-3}$$

Proposition 4 *Assume that $\eta > 0$:*

-If $A < 3/4$, then z_1 is always locally stable.

-If $A > 3/4$, z_1 is locally stable if and only if $\eta < \eta_2$ while z_1 is locally unstable if and only if $\eta > \eta_2$. When $\eta = \eta_2$ a cycle of period two arises near z_1 through a flip bifurcation.

Proof. Consider (24). We observe that $\phi'(z_1) < 1$ whatever $A > 0$ and $\eta > 0$.

Moreover, $\phi'(z_1) > -1$ is equivalent to:

$$(4A-3)(\eta - \eta_1)(\eta - \eta_2) < 0$$

and remark that, since $A < 3/4$, $\eta_1 < 0$ and $\eta_2 < 0$ while since $A > 3/4$, $\eta_1 < 0$ and $\eta_2 > 0$. Finally, $\phi'(z^*) = -1$ when $\eta = \eta_2$. The last proposition follows. ■

In Bréchet and Lambrecht (2011), resource preservation is defined as a non-zero steady state level for the resource. In this context, they have pointed out that resource preservation requires a sufficiently high altruism. In our paper, we have observed earlier that since $\eta > 0$, then $z_1 > 0$. That is, following the definition proposed by Bréchet and Lambrecht (2011), in the present paper, the natural resource is always preserved in the long run. This difference comes from the fact that resource and consumption are two non-separable arguments in the utility function in our model. Indeed, in case of a separable utility function (i.e. the Bréchet and Lambrecht (2011) case), the utility could be non-zero under a positive consumption despite a zero resource level. Conversely, under a non-separable utility function (see (6)), a zero resource level leads to a zero utility level (since $\eta > 0$).

However, we think that the definition of resource preservation followed by Bréchet and Lambrecht (2011) is not sufficient. Indeed, a non-zero resource level at the steady state does not mean that the economy will converge to this steady state. In particular, if z_1 is locally unstable, the resource preservation is not ensured in the long run. We believe that resource preservation requires two dimensions: (1) a non-zero resource level at the steady state and (2) a locally stable steady state. Following proposition 4, it appears that resource preservation is not guarantee under an excessive altruism ($\eta > \eta_2$) which contrasts with Bréchet and Lambrecht (2011): in their paper, there is a lower bound in altruism to ensure resource preservation while this is an upper bound in our framework.

Now, let us explain simply the occurrence of endogenous (periodic) cycles (flip bifurcation) pointed out in proposition 4. First of all, it is important to observe that those cycles could only occur when the steady state is located on the downward sloping branch of resource evolution function (ϕ, see Fig.1). Assume that the economy is at the steady state at date t and assume an exogenous increase of the resource level z_t. Since the steady state is located on the downward sloping branch of ϕ, it follows a drop a z_{t+1}. That is, a higher z_t is followed by a lower z_{t+1} which seems to be related to the existence of endogenous cycles. Moreover, remember that a strong value of η implies a strong substitutability between consumption (c_{t+1}) and resource (z_{t+1})[8], that is, the drop of z_{t+1} implies an increase of c_{t+1}. To be able to achieve this goal, the representative household, born at date t, has to increase her saving. That is, she has to increase her labor supply (to reduce forestry) as well as to increase harvesting: following (3), this reinforces the drop of z_{t+1}, rendering possible sustained fluctuations. Moreover, following (3), we observe that the effect of a reduction in the forestry effort (a lower $(1 - l_t)$) on z_{t+1} is magnified under a strong level of A which is coherent with proposition 4 ($A > 3/4$). The existence of endogenous cycles is very interesting because previous papers pointing out such complex dynamics in an OLG context with a renewable resource havealways ignored capital accumulation from their analysis (Koskela et al. (2008), Amacher et al. (2018)).

[8] Indeed, remember that $\varepsilon > 1$.

5 Conclusion

Through this paper, we have reconsidered the OLG model with a renewable resource by introducing seeding activities (forestry). In this model, we consider also capital accumulation as well as a joy-of-giving bequest motive regarding the natural resource as in Bréchet and Lambrecht (2011). In the long run, we observe that altruism always implies a positive resource level at the steady state. Moreover, we point out that there exists a unique relation between forestry productivity and altruism for which the resource level at the steady state coincides with the maximum sustainable yield. Concerning the local dynamics, we prove that the interplay between altruism and forestry productivity is crucial. Indeed, under a low forestry productivity, the steady state is always stable while, under a high forestry productivity, two period-cycles can emerge near the steady state through a flip bifurcation if and only if altruism is sufficiently high. This result completes Koskela et al. (2008) and Amacher et al. (2018) by showing that endogenous cycles can well arise in a simple OLG model with a renewable resource when capital accumulation is taking into account.

6 Appendix

Concavity of u

Considering (9), the Hessian matrix H of u is given by:

$$H \equiv \begin{bmatrix} \frac{\partial^2 u}{\partial l_t^2} & \frac{\partial^2 u}{\partial l_t \partial h_t} \\ \frac{\partial^2 u}{\partial h_t \partial l_t} & \frac{\partial^2 u}{\partial h_t^2} \end{bmatrix}$$

Such that:

$$\frac{\partial^2 u}{\partial l_t^2} = \omega \left[(1-\varepsilon) \left(\Omega - \frac{\eta A}{z_{t+1}} \right)^2 - \left(\Omega^2 + \eta \left(\frac{A}{z_{t+1}} \right)^2 \right) \right]$$

$$\frac{\partial^2 u}{\partial h_t^2} = \omega \left[(1-\varepsilon) \left(\Phi - \frac{\eta}{z_{t+1}} \right)^2 - \left(\Phi^2 + \frac{\eta}{z_{t+1}^2} \right) \right]$$

$$\frac{\partial^2 u}{\partial l_t \partial h_t} = \omega \left[(1-\varepsilon) \left(\Omega - \frac{\eta A}{z_{t+1}} \right) \left(\Phi - \frac{\eta}{z_{t+1}} \right) - \left(\left(\frac{1+r_{t+1}}{c_{t+1}} \right)^2 w_t p_t + \frac{\eta A}{z_{t+1}^2} \right) \right]$$

$$\frac{\partial^2 u}{\partial l_t \partial h_t} = \frac{\partial^2 u}{\partial h_t \partial l_t} \text{ (Young's theorem)}$$

With $\omega \equiv \left(c_{t+1} z_{t+1}^{\eta} \right)^{1-\varepsilon} > 0$ and:

$$\Omega \equiv \frac{(1+r_{t+1})\, w_t}{c_{t+1}} > 0 \text{ and } \Phi \equiv \frac{(1+r_{t+1})\, p_t}{c_{t+1}} > 0$$

u is strictly concave if and only if

$$\frac{\partial^2 u}{\partial l_t^2} < 0 \text{ and } \det H = \eta\omega^2 \left(Ap_t - w_t\right)^2 \left(1 + r_{t+1}\right)^2 \frac{\varepsilon + \eta\left(\varepsilon - 1\right)}{c_{t+1}^2 z_{t+1}^2} > 0 \qquad (25)$$

Assumption 1 ensures that (25) is verified. ■

References

[1] Amacher G., Ollikainen M. and M. Puhakka. (2018). Renewable Resource Use and Nonseparable Amenity Benefits. *Environmental & Resource Economics* 69, 637-659.

[2] Andreoni J. (1989). Giving with impure altruism: applications to charity and Ricardian equivalence. *Journal of Political Economy* 96, 1447-1458.

[3] Baranzini R. and F. Allisson. (2014). *Economics and Other Branches - In the Shade of the Oak Tree*. Routledge, New York.

[4] Barro, R.J. (1974). Are government bonds net wealth? *Journal of Political Economy* 82, 1095–1117.

[5] Bella G. (2010). Periodic solutions in the dynamics of an optimal resource extraction model. *Environmental Economics* 1, 49-58.

[6] Bosi S. and D. Desmarchelier. (2018). Natural cycles and pollution. *Mathematical Social Sciences* 96, 10-20.

[7] Bréchet T. and S. Lambrecht. (2011). Renewable resource and capital with a joy-of-giving resource bequest motive. *Resource and Energy Economics* 33, 981-994.

[8] Kemp M. and N. Van Long. (1979). The under-exploitation of natural resources: a model with overlapping generations. *Economic Record* 55, 214-221.

[9] Koskela E., Ollikainen M. and M. Puhakka. (2008). Saddles and Bifurcations in an Overlapping Generations Economy with a Renewable Resource. *Finnish Economic Papers* 21, 3-21.

[10] Koskela E., Ollikainen M. and M. Puhakka. (2008). Renewable resources in an overlapping generations economy without capital. *Journal of Environmental Economics and Management* 43, 497-517.

[11] Pezzey J. C. V. (1997). Sustainability constraints versus "optimality" versus intertemporal concern, and axioms versus data. *Land Economics* 73, 448-466.

[12] Wirl F. (2004). Sustainable growth, renewable resources and pollution: thresholds and cycles. *Journal of Economic Dynamics & Control* 28, 1149-1157.