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Can inflation contract discipline central bankers when agents are learning?

Marine Charlotte André* and Meixing Dai†

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Abstract

This paper studies how the government should design a linear inflation contract to deal with the time-inconsistency problem arising from incentives for the central bank to exploit the inflation-output tradeoff with an overambitious output-gap objective when private expectations are based on adaptive learning. An intertemporal trade-off due to learning leads the central bank to accommodate less the effect of inflation expectations and cost-push shocks on inflation. An optimal linear inflation contract is able to achieve many of the benefits, i.e., reducing inflation bias and stabilization bias, resulting from central bank conservatism and inflation targeting rules. The government can impose either a long-term or a short-term contract. The first is equivalent to appointing a hawkish central banker. The second implies that inflation penalty rate should be adjusted for inflation expectations in each period, and could be positive or negative, i.e., the central banker should shift between hawkish and dovish stances depending on inflation expectations and the speed of learning.

Keywords: adaptive learning, inflation bias, stabilization bias, inflation contract, monetary policy delegation, central bank conservatism, optimal monetary policy.

JEL Classification: C62, D83, D84, E52, E58.

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1 Introduction

In the theory of monetary policy, conventional wisdom holds that discretionary monetary policy could lead to an inflation bias, defined as an excess of inflation over the socially optimal level (Kydland and Prescott 1977, and Barro and Gordon 1983). The inflation bias and the associated time-inconsistency problem stem from the fact that central banks (CB) optimally respond to the wrong incentive structure while targeting a level of output that exceeds the natural rate. Furthermore, in New Keynesian models, the time-inconsistency problem could give rise to a dynamic phenomenon called stabilization bias (Svensson 1997, and Clarida, Galí and Gertler 1999).[1] It means that the inflation volatility is higher out of steady state under discretionary monetary policy than under commitment. The explanation is that monetary policy under commitment, by reducing the response of inflation to cost-push shocks, can subdue expected inflation, thus making stabilizing inflation less costly in terms of future output contraction. Both inflation bias and stabilization bias can be reduced or fully offset by delegating monetary policy to an independent and conservative central banker (Rogoff 1985, and Lohmann 1992) or adopting inflation targeting regimes where the government penalizes the central banker for deviating from targets (Svensson 1997, and Walsh 2003). Despite their effectiveness in reducing these biases, such approaches might not be socially optimal in models incorporating a trade-off between inflation prevention and stabilization policy (Chortareas and Miller 2007).

A strand of the literature treats monetary policy delegation as a principal-agent problem and proposes a CB contract, i.e., a linear inflation contract, to eliminate the trade-off between inflation reduction and stabilization policy and hence the inflation bias (Persson and Tabellini 1993, Walsh 1995).[2] The basic idea of this "contracting" approach is that the

[1]Dennis and Söderström (2006) have quantified this bias in various forward-looking New-Keynesian models with rational expectations.

[2]Frattianni, Von Hagen and Waller (1997) have compared various institutional arrangements designed to eliminate these biases and have found that CB independence and inflation contracts work best. Candel-Sánchez and Campoy-Miñarro (2004) and Chortareas and Miller (2007) have discussed the implications of the cost of inflation contract. Several studies have examined how linear inflation contracts are affected by uncertainty about the CB's output target (Muscatelli, 1999), about preference weights that the CB put on

government, as the principal, designs an optimal incentive scheme for the central banker with the intention of obtaining monetary policy outcomes equivalent to those under credible commitment. Generally, such scheme includes an efficient punishment (or transfer) mechanism that, by sufficiently raising the welfare costs of surprise inflation, counteracts the central banker's tendency to conduct a more accommodative monetary policy.

Insofar this principal–agent literature is built on the hypothesis of rational expectations (RE). The practicality of policy recommendations resulting from this literature might be limited since this hypothesis requires private agents to have a very good understanding of the economic structure and a great capacity of computation. As Woodford remarks (2013), familiar results in the theory of monetary policy obtained with this hypothesis could be challenged by alternative approaches to the specification of the expectations of economic decision makers. One of the most significant issues that have emerged in recent monetary policy literature is about how private expectations formed using learning algorithms would change the optimal design of monetary policy and institutions compared to RE. The mechanism that underlies the findings from studies examining this issue is that learning gives rise to an intertemporal trade-off, leading the CB to accommodate less the effect of inflation expectations and cost-push shocks on inflation than at the RE equilibrium (Molnár and Santoro 2014, Mele, Molnár and Santoro 2014, Airaudo, Nisticò and Zanna 2015). Introducing a non-linear inflation contract with negative inflation penalty helps the CB reduce the undesirable effect of distortions introduced by adaptive learning on the stabilization bias (André and Dai 2017). One might wonder if this result holds with a linear inflation contract.

The intention of our paper is to study how adaptive learning could change the well-known findings, in the literature on optimal inflation contracts, regarding the design of punishment or transfer mechanism and the dynamics of endogenous variables. As in this literature, we assume that both the government and the CB share the same preferences over inflation and

its inflation and output stabilization objectives (Beetsma and Jensen 1998, and Muscatelli 1998, 1999), or about the central banker's responsiveness to incentive schemes (Chortareas and Miller 2003). In addition to transparency issue, Dai and Spyromitros (2012) consider if model robustness affects the design of inflation contract and Ciccarone and Marchetti (2012) think about common agency.

output fluctuations. To achieve the inflation target with an inflation contract, incentives for the central banker must be strong enough. The government must thus implement an inflation penalty if the central banker does not respect the goal previously fixed. Moreover, such punishment mechanism has to be conceived to allow the CB to offset distortions introduced by the learning behavior of the private sector.

The main findings of our paper are as follows: (1) Under adaptive learning, the government can adopt either a long-term or a short-term approach to inflation contract. (2) Under long-term approach, the optimal inflation penalty rate is positively related to the output-gap target and its sensitiveness to the latter is substantially smaller under learning than under RE, particularly when the learning gain is high. (3) Under short-term approach, the optimal penalty rate depends on both the output-gap target and private agents' inflation expectations. The government should reset in each period the terms of the contract since optimal inflation penalty rate can be either positive or negative according to the learning gain and the sign of the deviation of inflation expectations from the inflation target.

Our paper contributes to a growing literature that focuses on the effects of learning in macroeconomic models, in particular the strands of literature investigating the consequences of adaptive learning for monetary policy decisions. These studies emphasize that learning has significant implications for monetary policy analysis and design. Marcet and Nicolini (2003) find that the learning process matches remarkably well some major stylized facts observed during the hyperinflations in the 1980's. Slobodyan and Wouters (2012) report that inflation expectations based on small forecasting models replicate well the survey evidence, and the model with an inertial Taylor rule fits the data better under adaptive learning than under RE. Several studies, e.g., Bullard and Mitra (2002), and Evans and Honkapohja (2003, 2006), assert that Taylor rules, while ensuring determinacy under RE, can generate instability if private expectations slightly deviate from rationality and are formed using adaptive learning algorithms. The fact that departures from RE increase the potential for economic instability has led some authors such as Ferrero (2007), Gaspar, Smets and Vestin (2010), Marzioni

(2014), Moore (2014), and Airaudo, Nisticò and Zanna (2015) to highlight the importance of anchoring inflation expectations and advocate for a more aggressive response of the interest rate policy to expected inflation under learning than under RE.

The rest of the paper is organized as follows. The next section sets up the model. Section 3 establishes the closed-form solution to the model under RE. Section 4 gives the equilibrium solution under constant-gain learning and analyzes how learning influences the equilibrium. Section 5 examines the design of optimal inflation contracts. Section 6 investigates the implications of decreasing-gain learning for the design of optimal inflation contracts. Section 7 discusses some possible extensions. The last section concludes.

2 The model

The economic environment is described following a standard micro-founded Keynesian model featuring optimizing private-sector behavior and nominal price rigidities that has been extensively used in the recent literature on monetary policy (Clarida, Galí and Gertler 1999). We formulate monetary policy in terms of control over the output gap, i.e., output relative to the flexible-price equilibrium level. This allows us to neglect the goods market equilibrium condition. Adopting an alternative modelling approach that takes account of this condition and explicitly treats the nominal interest rate as monetary policy instrument to achieve output and inflation targets, will not alter our main results.

The model is then completed by a principal-agent framework in which the government delegates monetary policymaking to the CB with the help of a linear inflation contract. We close the model by specifying the learning algorithm used by private agents to form expectations.

The supply side of the economy is characterized by a forward-looking Phillips curve:

$$\pi_t = \beta E_t^* \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is the inflation rate, $E_t^* \pi_{t+1}$ the expected rate of future inflation, x_t the output gap, $e_t \sim \mathcal{N}(0,\sigma_e)$ a serially uncorrelated supply or cost-push shock. The expectation operator E_t^* represents private expectations conditional on information set available at time t, with the asterisk denoting that private agents may form RE or not. The parameter $\beta \in (0,\ 1)$ is the discount factor. The parameter κ stands for the output-gap elasticity of inflation and captures the effects of the output gap on real marginal costs and hence on inflation. Underpinnings to (1) are that in an environment with monopolistically competition, each firm's price-setting decision is derived from an explicit optimization problem. When a firm has the opportunity, it sets the nominal price of its product to maximize profits subject to the constraint on the frequency of future price adjustments as defined in Calvo (1983).

The expected social loss function is specified in terms of variances of inflation and the output gap as follows:

$$L_t^s = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[\pi_{t+i}^2 + \alpha \left(x_{t+i} - \widetilde{x} \right)^2 \right], \tag{2}$$

where $\alpha > 0$ is the relative weight assigned to output stabilization. The public knows the true value of α as well as inflation and output gap targets. The latter are set to zero and $\widetilde{x} > 0$ respectively. The fact that the society has an output-gap target above its potential level, set to zero in this model, implies the presence of an inflation bias. In the absence of an inflation contract, a CB that shares the government's objective function would conduct discretionary monetary policy that would not avoid the time-inconsistency problem and the associated inflation bias. The government, as benevolent social planner, imposes on the central banker an inflation contract stipulating that she or he receives a monetary transfer payment from the government according to a rule, i.e.,

$$T = \tau_0 - \tau \pi_t \tag{3}$$

where τ_0 is fixed in a way to ensure the CB's participation and τ is the penalty rate associated with inflation (Walsh 1995). Such a payment can be either considered as the direct income of the central banker or as the budget of the CB.

To implement optimal discretionary monetary policy, the CB solves the following problem:

$$\min_{x_t} L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i [\alpha (x_{t+i} - \widetilde{x})^2 + \pi_{t+i}^2 - \xi (\tau_0 - \tau \pi_{t+i})], \tag{4}$$

subject to the constraint imposed by the structure of the economy, i.e., the Philips curve (1). The parameter ξ indicates the extent to which the central banker cares about the incentive scheme relative to the social welfare loss. The participation constraint is assumed to hold such that the CB accepts the contract. Inflation contract, by imposing a penalty rate linked to inflation, is designed such that the higher this penalty rate is, the costlier it is for the CB to adjust inflation to achieve the overambitious output-gap target.

Private agents are assumed to follow a learning algorithm as in Marcet and Nicolini (2003), and Molnár and Santoro (2014). This assumption relies on the idea of a limited rationality among private agents, which corresponds to a restrained knowledge of the process governing the evolution of endogenous variables. To improve their decisions, private agents may recursively estimate a Perceived Law of Motion (PLM) in the terminology of Evans and Honkapohja (2001), which is consistent with the law of motion that the CB follows under RE, by using the following deterministic learning algorithm:

$$E_t^* \pi_{t+1} \equiv a_t = a_{t-1} + \gamma_t (\pi_{t-1} - a_{t-1}), \tag{5}$$

where $\gamma_t \in (0, 1)$ is the learning gain that could be constant (sections 4 and 5) or decreasing (section 6).[3] The learning gain defines the speed of integration of new data into current expectations. The learning algorithm (5) establishes a positive relationship between inflation expectations and last period inflation. Given that past inflations depend on past shocks, inflation expectations based on learning contain a share of past inflation shocks.

[3] Compared to decreasing-gain learning, constant-gain learning has a better analytical tractability of the model. This explains why our paper mainly focuses on constant-gain learning.

Using (5), we rewrite (1) as:

$$\pi_t = \beta \left[a_{t-1} + \gamma_t(\pi_{t-1} - a_{t-1})\right] + \kappa x_t + e_t. \tag{6}$$

Equation (6) relates the current inflation to the current cost-push shock, the current output-gap, and the past values of inflation and of expected inflation.

3 Rational expectations equilibrium

The interest of this section resides in providing a benchmark equilibrium solution and optimal inflation contract based on the assumption that the private sector forms rational inflation expectations conditional on information available at time t. They serve as a reference point for the discussion about the equilibrium solution in section 4 and the optimal inflation contract obtained under learning in section 5.

Solving the minimization problem of the CB under discretion gives the following targeting rule:

$$\pi_t = -\frac{\alpha}{\kappa}\left(x_t - \widetilde{x}\right) - \frac{1}{2}\xi\tau. \tag{7}$$

Condition (7) indicates that the equilibrium solution of inflation and the output gap now depends on inflation penalty rate. A positive inflation penalty rate, by making costlier for the CB to adjust inflation to achieve the output-gap target, implies a lower output and hence a decrease in the output gap.

For given inflation expectations, solving (1) and (7) yields the Actual Laws of Motion (ALMs) that govern the evolution of inflation and the output gap at the equilibrium:

$$\pi_t = \frac{\alpha\beta}{\alpha+\kappa^2}E_t^*\pi_{t+1} - \frac{1}{2}\frac{\kappa^2}{\alpha+\kappa^2}\xi\tau + \frac{\alpha\kappa}{\alpha+\kappa^2}\widetilde{x} + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{8}$$

$$x_t = -\frac{\beta\kappa}{\alpha+\kappa^2}E_t^*\pi_{t+1} - \frac{1}{2}\frac{\kappa\xi}{\alpha+\kappa^2}\tau + \frac{\alpha}{\alpha+\kappa^2}\widetilde{x} - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{9}$$

The system of equations (1) and (7) has a unique non-explosive RE equilibrium (REE) solution, called the "minimal state variable" solution (McCallum 1983), in terms of state variable e_t as well as τ and $\widetilde{x}$. Under RE, i.e., $E_t^* = E_t$, the solution of π_t takes the form: $\pi_t = \zeta_0 + \zeta_1 e_t$. Since cost-push shocks are serially uncorrelated, i.e., $E_t e_{t+1} = 0$, it follows that $E_t \pi_{t+1} = \zeta_0 + \zeta_1 E_t e_{t+1} = \zeta_0$. Using the method of undetermined coefficients yields $\zeta_0 = \frac{\alpha\kappa}{\alpha(1-\beta)+\kappa^2}\widetilde{x} - \frac{\kappa^2}{2[\alpha(1-\beta)+\kappa^2]}\xi\tau$, and $\zeta_1 = \frac{\alpha}{\alpha+\kappa^2}$.

Substituting the solution of $E_t\pi_{t+1}$ into (8)-(9), we obtain the REE solution corresponding to the optimal discretionary monetary policy:

$$\pi_t = \frac{\alpha\kappa}{\alpha(1-\beta)+\kappa^2}\widetilde{x} - \frac{1}{2}\frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}\xi\tau + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{10}$$

$$x_t = \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\kappa^2}\widetilde{x} - \frac{1}{2}\frac{(1-\beta)\kappa}{\alpha(1-\beta)+\kappa^2}\xi\tau - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{11}$$

The optimal inflation penalty rate is determined by minimizing (2) taking account of the solutions of π_t and x_t given by (10)-(11) as:

$$\tau = \frac{2\alpha\beta\kappa}{\xi\left[\alpha(1-\beta)^2+\kappa^2\right]}\widetilde{x}. \tag{12}$$

This result is obtained under the RE hypothesis and in the presence of inflation bias. The government should set a penalty rate that balances between mitigating inflation and stabilization biases. As a result, the optimal inflation penalty rate is positively related to the output-gap target. Thus the higher the target is, the higher the inflation penalty rate.

To observe if the linear optimal inflation contract offsets the inflation bias, we insert (12) into (10)-(11). It yields:[4]

[4]The solution under discretion differs from that under constrained commitment (commitment to a rule) or unconstrained commitment (with timeless perspective). Consider for example the equilibrium under constrained commitment, the equilibrium solution can be obtained using the method of undetermined coefficients as in Clarida, Galí and Gertler (1999). In this case, the government should set the optimal inflation penalty rate as $\tau = \frac{2\alpha(1-\beta)}{\xi\kappa}\widetilde{x}$, and the equilibrium solutions for inflation and the output gap are $\pi_t = \frac{\alpha}{\alpha+\kappa^2}e_t$. and $x_t = -\frac{\kappa}{\alpha+\kappa^2}e_t$. Thus, if the output-gap target is positive, it is obvious from comparing the previous results with (13)-(14) that inflation is inefficiently stabilized under discretion, i.e., the variance of inflation under discretion is greater than under commitment. This bias, which is known as the stabilization bias,

$$\pi_t = \frac{\alpha\kappa(1-\beta)}{\alpha(1-\beta)^2+\kappa^2}\widetilde{x} + \frac{\alpha}{\alpha+\kappa^2}e_t, \quad (13)$$

$$x_t = \frac{\alpha(1-\beta)^2}{\alpha(1-\beta)^2+\kappa^2}\widetilde{x} - \frac{\kappa}{\alpha+\kappa^2}e_t. \quad (14)$$

Thus under RE, even with an optimal linear inflation contract, the inflation bias cannot be fully neutralized, unless the output-gap target is null. This is due to the forward-looking inflation expectations present in the New Keynesian Phillips curve.[5]

4 Equilibrium with constant-gain learning

As generally recognized in the learning literature (Evans and Honkapohja 2009), private agents use more likely a constant-gain learning algorithm if they believe in possible structural changes in the near future.[6]

The focus of this section is twofold. We first analyze how constant-gain learning and inflation penalty rate interact with macroeconomic stabilization compared to the benchmark case where private agents form RE, and then examine how the government should design inflation contract to deal with inflation bias and stabilization bias. The CB sets policy under discretion.

becomes larger if shocks are persistent. If the over-ambitious output-gap target is absent, the stabilization bias disappears for white noise shocks even under discretion.

[5]Using a static Phillips curve such as $\pi_t = \pi_t^e + \kappa x_t + e_t$, with π_t^e representing the expected inflation rate for the current period based on information available in the previous period. It is easy to show that even under discretion, the optimal inflation contract fully eliminates the inflation bias due to an overambitious output-gap target. Comparing this result with that obtained in the standard New Keynesian model leads to the idea that the stabilization bias is a dynamic phenomenon.

[6]We relax the assumption that the learning gain is constant by assuming in section 6 that it decreases with time. The appeal for decreasing gain resides in the idea that as time goes by, private agents are more experienced with learning process and are hence more confident in their expectations. As the learning gain decreases from 1 to 0 over time, the economy behaves as if it jumps from one dynamic path with higher constant learning gain to another one with lower constant learning gain (Molnár and Santoro 2014, André and Dai 2017).

4.1 Optimal monetary policy rule

The Lagrangian of the CB's optimization problem is:

$$\begin{aligned}\mathscr{L}_t^{CB} = & E_t \sum_{i=0}^{+\infty} \beta^i \left\{ \tfrac{1}{2} \left[\alpha \left(x_{t+i} - \widetilde{x} \right)^2 + \pi_{t+i}^2 - \xi \left(\tau_0 - \tau \pi_{t+i} \right) \right] \right. \\ & - \lambda_{1,t+i} \left[\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} - e_{t+i} \right] \\ & \left. - \lambda_{2,t+i} \left[a_{t+i+1} - a_{t+i} - \gamma_{t+i+1} (\pi_{t+i} - a_{t+i}) \right] \right\},\end{aligned}$$

where $\lambda_{i,t}$, with i=1, 2, are Lagrange multipliers associated with (1) and (5), respectively. The first-order conditions of the CB's optimization problem are obtained by deriving the Lagrangian with respect to x_t, a_{t+1} and π_t:

$$\frac{\partial \mathscr{L}}{\partial x_t} = 0 \quad \Leftrightarrow \quad \lambda_{1,t} = -\frac{\alpha}{\kappa} \left(x_t - \widetilde{x} \right), \tag{15}$$

$$\frac{\partial \mathscr{L}}{\partial a_{t+1}} = 0 \quad \Leftrightarrow \quad \lambda_{2,t} = \beta \left[\beta \lambda_{1,t+1} + \left(1 - \gamma_{t+2} \right) \lambda_{2,t+1} \right], \tag{16}$$

$$\frac{\partial \mathscr{L}}{\partial \pi_t} = 0 \quad \Leftrightarrow \quad \lambda_{1,t} = \pi_t + \frac{1}{2} \xi \tau + \gamma_{t+1} \lambda_{2,t}. \tag{17}$$

Solving (15)-(17) for $\gamma_{t+1} = \gamma_t = \gamma$, we obtain the optimal rule for intratemporal and intertemporal trade-off between inflation and the output gap:

$$\pi_t = \beta \left(1 - \gamma \right) E_t \pi_{t+1} - \frac{\alpha}{\kappa} \left(x_t - \widetilde{x} \right) + \frac{\alpha \beta \left[1 - \gamma (1 - \beta) \right]}{\kappa} \left(E_t x_{t+1} - \widetilde{x} \right) - \frac{1}{2} \left[1 - \beta \left(1 - \gamma \right) \right] \xi \tau. \tag{18}$$

The CB follows this rule when conducting discretionary monetary policy.

4.2 The equilibrium and the effect of learning

There exists a unique non-explosive equilibrium solution corresponding to the CB's control problem under constant-gain learning (Appendix A.1). The ALM for inflation is given by:

$$\pi_t = c_\pi^{cg} a_t + d_\pi^{cg} e_t + \Theta_\pi^{cg} \xi \tau + \Omega_\pi^{cg} \widetilde{x}, \tag{19}$$

where

$$
\begin{aligned}
c_{\pi}^{cg} &= -\frac{p_0 + p_2 \left(c_{\pi}^{cg}\right)^2}{p_1} > 0, \\
d_{\pi}^{cg} &= \frac{\alpha}{\kappa^2 + \alpha + \alpha\gamma^2\beta^2 \left(\beta - c_{\pi}^{cg}\right) + \gamma\beta\left(1-\gamma\right)\left\{\alpha\beta - \left(\alpha+\kappa^2\right)c_{\pi}^{cg}\right\}} > 0, \\
\Theta_{\pi}^{cg} &= -\frac{\kappa^2\left[1-\beta\left(1-\gamma\right)\right]}{2\Phi} < 0, \\
\Omega_{\pi}^{cg} &= \frac{\alpha\kappa\left\{1-\beta\left[1-\gamma\left(1-\beta\right)\right]\right\}}{\Phi} > 0,
\end{aligned}
$$

with

$$
\begin{aligned}
\Phi &= \alpha + \kappa^2 + \alpha\gamma\beta^2\left[1-\gamma(1-\beta)\right] - \beta\left(1+c_{\pi}^{cg}\gamma\right)\left[\left(\alpha+\kappa^2\right)\left(1-\gamma\right)+\alpha\gamma\beta\right], \\
p_0 &= \alpha\beta\left\{1-\beta(1-\gamma)\left[1-\gamma(1-\beta)\right]\right\} > 0, \\
p_1 &= -\kappa^2\left[1-\beta(1-\gamma)\right] - \alpha(1-\beta)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\} - p_0 - p_2 < 0, \\
p_2 &= \gamma\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2\left(1-\gamma\right)\right\} > 0.
\end{aligned}
$$

The solution for c_{π}^{cg} ensuring a non-explosive evolution of inflation is given by:

$$
c_{\pi}^{cg} = \frac{-p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2}. \tag{20}
$$

Since $c_{\pi}^{cg} \in \left(0;\ \frac{\alpha\beta}{\alpha+\kappa^2}\right)$ and that $c_{\pi}^{cg} < 1$ (see appendix A.1), current inflation rises proportionately less than inflation expectations (a_t). Inflation expectations in (5) are related to past inflation, hence to past cost-push shocks through the Phillips curve (1). Consequently, current inflation is influenced by the CB's policy responses to past shocks. An increase in learning gain γ has two opposite effects on c_{π}^{cg}. The learning algorithm (5) indicates that, if γ increases, there exists a positive feedback from current inflation π_t to future inflation expectations a_{t+1}, and hence an incentive for the CB to lower c_{π}^{cg}, i.e., the positive feedback from a_t to π_t in (19). Nevertheless, higher γ dampens the effect of a_t on a_{t+1}, therefore increasing c_{π}^{cg} without reducing social welfare. The first effect always dominates the second such that $\frac{\partial c_\pi^{cg}}{\partial \gamma} < 0$ (Appendix A.2).

The effect of cost-push shocks on current inflation is negatively related to the learning gain because the denominator of d_π^{cg} is decreasing in γ given that $\frac{\partial c_\pi^{cg}}{\partial \gamma} < 0$. Inflation penalty rate is negatively correlated with inflation and this correlation, represented by the feedback coefficient $\Theta_\pi^{cg} < 0$, is strengthened as γ rises, i.e., $\frac{\partial \Theta_\pi^{cg}}{\partial \gamma} < 0$. This can be explained by the fact that higher learning gains enhance the integration of information about past inflations into private agents' expectations such that inflation penalty is more effective in reducing inflation. An increase in the output-gap target generates higher inflation with its effect being reinforced by an increase in learning gain, i.e., $\Omega_\pi^{cg} > 0$ and $\frac{\partial \Theta_\pi^{cg}}{\partial \gamma} > 0$.

The ALM of the output gap is obtained by substituting π_t given by (19) into (1):

$$x_t = c_x^{cg} a_t + d_x^{cg} e_t + \Theta_x^{cg} \xi \tau + \Omega_x^{cg} \widetilde{x}, \tag{21}$$

where $c_x^{cg} = -\frac{1}{\kappa}(\beta - c_\pi^{cg})$, $d_x^{cg} = -\frac{1}{\kappa}(1 - d_\pi^{cg})$, $\Theta_x^{cg} = \frac{1}{\kappa}\Theta_\pi^{cg}$ and $\Omega_x^{cg} = \frac{1}{\kappa}\Omega_\pi^{cg}$. An increase in inflation expectations calls the CB to set a monetary policy with the aim of reducing current inflation. Nevertheless, an increase in inflation expectations leads to higher inflation and lower output, i.e., $c_\pi^{cg} < \frac{\alpha\beta}{\alpha+\kappa^2}$ and $c_x^{cg} < -\frac{\beta\kappa}{\alpha+\kappa^2}$. Notice that (1) implies an increase in current inflation smaller than that of inflation expectations and hence lower future inflation expectations according to the learning algorithm (5). The feedback effect of cost-push shocks on the output gap is negative. It is positively correlated with that on inflation, i.e., $\frac{\partial d_x^{cg}}{\partial d_\pi^{cg}} > 0$, and hence decreases with learning gain γ, i.e., $\frac{\partial d_x^{cg}}{\partial \gamma} < 0$. Inflation penalty negatively affects the output gap, i.e., $\Theta_x^{cg} < 0$. As with the ALM for inflation, the effect increases with higher learning gain, i.e., $\frac{\partial \Theta_x^{cg}}{\partial \gamma} < 0$. The feedback effect of an increase in the output-gap target on the output gap is positive, i.e., $\Omega_x^{cg} > 0$, and decreases as the learning gain increases, $\frac{\partial \Omega_x^{cg}}{\partial \gamma} < 0$.

The learning gain is the critical determinant of the time horizon within which private agents' beliefs converge to RE. It thus affects the persistence of inflation and the stance of monetary policy. Setting $\gamma = 0$, i.e., inflation expectations are constant over time such that $a_t = a_{t-1}$, we obtain:

$$c_\pi^{cg} = \frac{\alpha\beta}{\alpha + \kappa^2}, \quad (22)$$

$$d_\pi^{cg} = \frac{\alpha}{\alpha + \kappa^2}, \quad (23)$$

$$\Theta_\pi^{cg} = -\frac{\kappa^2}{2\left(\alpha + \kappa^2\right)}, \quad (24)$$

$$\Omega_\pi^{cg} = \frac{\alpha\kappa}{\alpha + \kappa^2}. \quad (25)$$

For $\gamma = 0$, (19) is reduced to the form given by (8). This corresponds to the case of exogenously given inflation expectations. All feedback coefficients (i.e., (22)-(22)) are equal to those observed under RE in (8). It is straightforward to show that this is also true for the ALM for the output gap. Given that future expected inflation is set equal to the past one, i.e., $a_{t+1} = a_t$, past shocks cannot induce inflation persistence.

In the case where $\gamma = 1$, i.e., inflation expectations become static such that $a_{t+1} = \pi_t$, we get:

$$c_\pi^{cg} = \frac{\left\{\kappa^2 + \alpha + \alpha\beta^3\right\} - \sqrt{\left\{\kappa^2 + \alpha + \alpha\beta^3\right\}^2 - 4\alpha^2\beta^3}}{2\alpha\beta^2},$$

$$d_\pi^{cg} = \frac{\alpha}{\kappa^2 + \alpha + \alpha\beta^2\left(\beta - c_\pi^{cg}\right)},$$

$$\Theta_\pi^{cg} = -\frac{1}{2}\frac{\kappa^2}{\alpha + \kappa^2 - \alpha\beta^2\left(1 + c_\pi^{cg} - \beta\right)},$$

$$\Omega_\pi^{cg} = \frac{\alpha\kappa\left(1 - \beta^2\right)}{\alpha + \kappa^2 - \alpha\beta^2\left(1 + c_\pi^{cg} - \beta\right)}.$$

Private agents form expectations on the basis of a time horizon that increasingly shortens when γ approaches 1. For $\gamma = 1$, inflation is self-sustained by past inflation that becomes the only determinant of private agents' inflation expectations.

For standard parameter values, i.e., $\beta = 0.99$, $\kappa = 0.024$, $\alpha = 0.048$ and $\sigma = 0.157$, Figures 1 and 2 show how the feedback coefficients of the ALMs for inflation and the output gap change with γ, respectively.

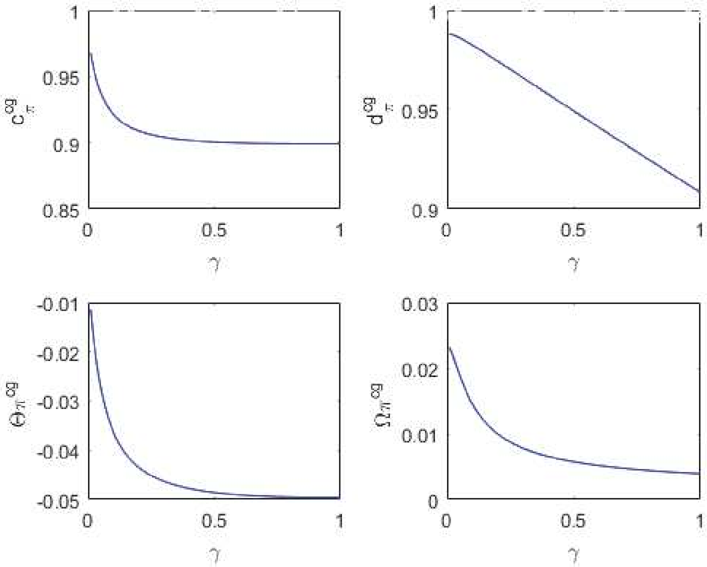

Figure 1: The feedback coefficients of the ALM for inflation.

Proposition 1. *An increase in learning gain reduces the feedback coefficients on inflation expectations and cost-push shocks in the ALMs for inflation and the output gap. The higher is the learning gain, the further away are these coefficients from their corresponding ones under RE. These effects are independent of inflation penalty rate.*

Proof. It follows from the definition of c_π^{cg}, d_π^{cg} c_x^{cg} and d_x^{cg} that the feedback coefficients on inflation expectations and cost-push shocks in the ALMs for inflation and the output gap are not function of inflation penalty rate. For the effect of an increase in γ on these feedback coefficients, see Appendix A.2.□

Linear inflation contract affects the dynamic effects of inflation expectations and cost-push shocks through shifting the dynamic path of inflation and the output gap. In comparison, non-linear inflation contract affects the dynamic effects of inflation expectations and

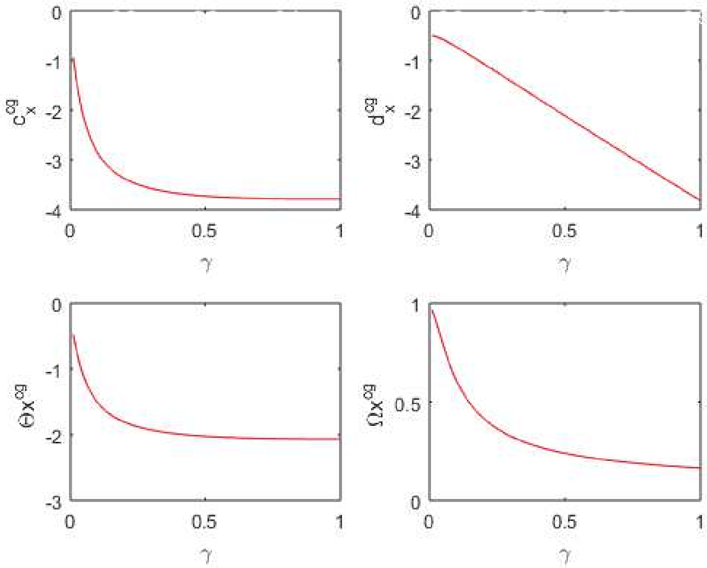

Figure 2: The feedback coefficients of the ALM for the output gap.

cost-push shocks by changing the slope of the dynamic path of inflation and the output gap (André and Dai 2017).

Proposition 2. *An inflation contract with positive inflation penalty rate allows offsetting to a certain extent the effect of an overambitious output-gap target on inflation and the output gap. An increase in learning gain strengthens this offsetting mechanism by lowering the negative feedback coefficients of inflation penalty rate and the positive feedback coefficients of the output-gap target in both ALMs for inflation and the output gap.*

Proof. It is straightforward to see from the definition of Θ_π^{cg}, Ω_π^{cg}, Θ_x^{cg} and Ω_x^{cg} that the feedback coefficients on inflation penalty rate and the output-gap target in the ALMs for inflation and the output gap are of opposite signs. Appendix A.2 shows the effect of an increase in γ on these feedback coefficients.□

It emerges from the comparison between the ALMs obtained under RE ((8)-(9)) and the ALMs obtained under learning ((19) and (21)) that learning alleviates (strengthens) the feedback effect of inflation expectations on the ALM for inflation (the output gap), i.e., $c_\pi^{cg} < \frac{\alpha\beta}{\alpha+\kappa^2}$ ($c_x^{cg} < -\frac{\beta\kappa}{\alpha+\kappa^2}$ respectively). As regards to the feedback coefficients on e_t, an increase in learning gain induces a weaker (stronger) response of inflation (the output gap) to current cost-push shocks than under RE since $d_{\pi}^{cg} < \frac{\alpha}{\alpha+\kappa^2}$ ($d_x^{cg} < -\frac{\kappa}{\alpha+\kappa^2}$).

Figure 3 shows that in the event of a positive inflation penalty shock, the higher the learning gain is, the larger the deviation of inflation expectations, inflation and the output gap from their steady state values, and the lower their persistence over time. This is explained by the fact that the higher is the learning gain, the less are past shocks taken into account in the formation of inflation expectations. Moreover, higher learning gain induces a stronger response of endogenous variables. This allows them to converge to their steady-state equilibrium at a higher rate in the first periods following the inflation penalty shock.

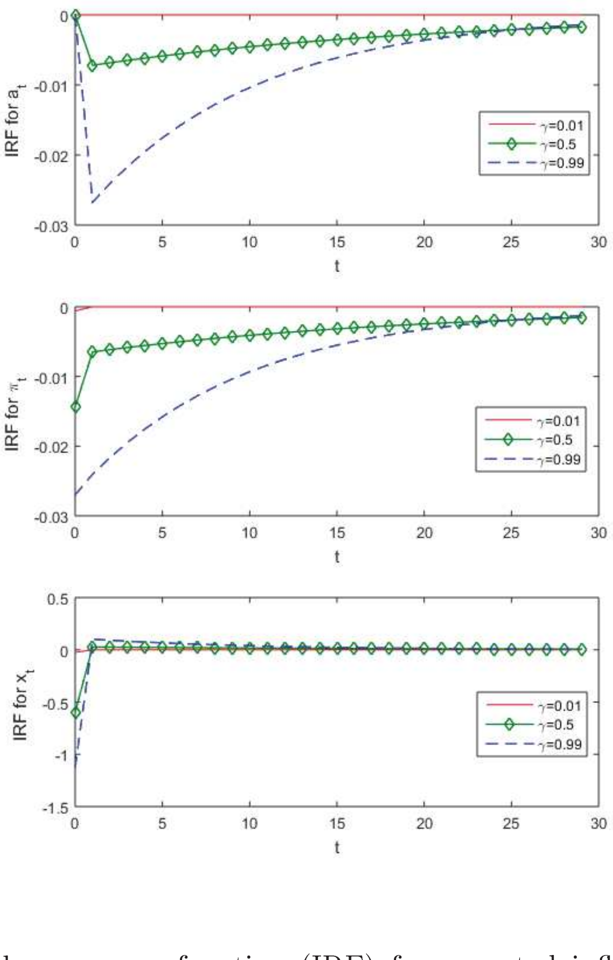

Figure 3: The impulse response function (IRF) for expected inflation, inflation and the output gap following an inflation penalty shock.

5 Optimal inflation penalty rate under constant-gain learning

In an economic environment where private agents are learning, the government can either sign a long-term contract with the central banker, which can be incorporated in the CB's statutes, or a short-term contract with the central banker, which can be periodically reviewed and adjusted. These two approaches are quite different since the first is independent of the evolution of private expectations formed with learning while the second is not. Moreover, the first approach implies a unique contract for every future central banker, while the second can be considered as an instrument that allows the government to closely monitor the central banker in each period. For each type of contract, we find closed-form solutions that may converge or not to the RE solution.

5.1 Long-term contracting

A long-term contract implies that the government will not change the terms of the contract over time, making the inflation penalty rate independent of short-run values of endogenous economic variables. Such a contract aims at maximizing the steady-state social welfare and allows the central banker to manage at her/his discretion the intertemporal trade-off induced by adaptive learning.

In the steady state characterized by $\pi_{t+i} = \pi_t = a_{t+i} = a_t$ and $e_{t+i} = e_{t+i} = 0$, the social welfare loss function can be rewritten using these steady-state conditions, $c_x^{cg} = -\dfrac{\beta - c_\pi^{cg}}{\kappa}$, $\Theta_x^{cg} = \frac{1}{\kappa}\Theta_\pi^{cg}$, $\Omega_x^{cg} = \frac{1}{\kappa}\Omega_\pi^{cg}$, (19) and (21) as:

$$L_t^s = \frac{1}{2(1-\beta)}\left[\left(\frac{\Theta_\pi^{cg}\xi\tau + \Omega_\pi^{cg}\widetilde{x}}{1-c_\pi^{cg}}\right)^2 + \alpha\left(\frac{(1-\beta)\left[\Theta_\pi^{cg}\xi\tau + \Omega_\pi^{cg}\widetilde{x}\right]}{\kappa(1-c_\pi^{cg})} - \widetilde{x}\right)^2\right]. \tag{26}$$

The government sets τ to minimize the social welfare loss (26). This leads to the optimal inflation penalty rate:

$$\tau = \frac{\alpha\kappa(1-\beta)(1-c_\pi^{cg}) - \left[\alpha(1-\beta)^2 + \kappa^2\right] \Omega_\pi^{cg}}{\left[\alpha(1-\beta)^2 + \kappa^2\right] \Theta_\pi^{cg} \xi} \widetilde{x}. \tag{27}$$

In the case where $\gamma = 0$ so that $c_\pi^{cg} = \frac{\alpha\beta}{\alpha+\kappa^2}$, $\Theta_\pi^{cg} = -\frac{\kappa^2}{2(\alpha+\kappa^2)}$ and $\Omega_\pi^{cg} = \frac{\alpha\kappa}{\alpha+\kappa^2}$, the optimal penalty rate is

$$\tau = \frac{2\alpha\beta\kappa}{\left[\alpha(1-\beta)^2 + \kappa^2\right] \xi} \widetilde{x}. \tag{28}$$

We notice that the optimal penalty rate given by (28) is identical to the one obtained under RE given by (12).

Using standard parameter values, the optimal inflation penalty rate (27) is drawn in Figure 4.

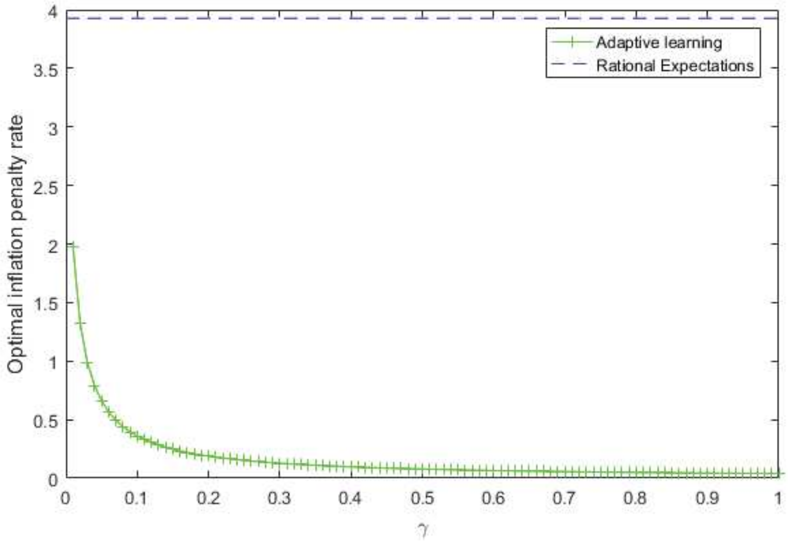

Figure 4: Optimal inflation penalty rate under long-term contracting approach.

It follows straightforwardly from Figure 4 that the optimal penalty rate decreases with learning gain. This observation leads to the following proposition:

Proposition 3. *The optimal inflation penalty rate in the long-term inflation contract is positively related to the output-gap target and decreases with learning gain. When the learning* *gain is zero, the optimal inflation penalty rate is equal to the one obtained under rational expectations, and as the learning gain approaches unity, the inflation penalty rate tends to zero.*

The fact that the optimal penalty rate decreases with learning gain can be explained by the reinforcing effect of an increase in learning gain on the integration of information about past inflations into private agents' expectations, making inflation penalty more effective in reducing inflation.

5.2 Short-term contracting

In the case of short-term contracting, the government takes private expectations as given and sets τ to minimize the social welfare loss during the lifetime of a one-period contract. The ALMs for inflation and the output gap are dependent on the output-gap target, inflation penalty rate and learning gain. This indicates that the contribution of their respective volatility to the social welfare loss is function of these parameters. Substituting (19) and (21) into the social welfare loss function (2) yields:

$$L_t^S = \frac{1}{2}\sum_{i=0}^{+\infty}\beta^i \left\{ \frac{\alpha}{\kappa^2}\left[\left(c_\pi^{cg} - \beta\right)a_{t+i} + \Theta_\pi^{cg}\xi\tau + \left(\Omega_\pi^{cg} - \kappa\right)\widetilde{x}\right]^2 + \left[c_\pi^{cg}a_{t+i} + \Theta_\pi^{cg}\xi\tau + \Omega_\pi^{cg}\widetilde{x}\right]^2 \right\} \tag{29}$$

The optimal inflation penalty rate results from minimizing (29):

$$\tau = -\frac{\alpha\left(\Omega_\pi^{cg} - \kappa\right) + \kappa^2\Omega_\pi^{cg}}{\Theta_\pi^{cg}\xi\left(\alpha + \kappa^2\right)}\widetilde{x} + \frac{\alpha\left(\beta - c_\pi^{cg}\right) - \kappa^2 c_\pi^{cg}}{\Theta_\pi^{cg}\xi\left(\alpha + \kappa^2\right)}a_t \tag{30}$$

When $\gamma \to 0$, the optimal inflation penalty rate is such that

$$\tau \to 0. \tag{31}$$

The optimal inflation penalty rate tends to zero when private agents ignore their expectations errors. This result can be explained by the fact that when $\gamma \to 0$, inflation expectations are entirely exogenous (equal to the inflation target) and will not be affected by the inflation penalty rate set by the government, thus making inflation penalty ineffective in affecting private agents behaviors. This is a distinguished feature of the short-term contracting approach under adaptive learning compared to the long-term one (28) and the inflation contract under RE (12).

For standard parameter values and for $a_t = \pm 1$, $\widetilde{x} = 1$, and $\xi = 1$, Figure 5 illustrates how the short-term optimal inflation penalty rate evolves with learning gain. We observe that the optimal inflation penalty rate is positive and constant under RE. However, under adaptive learning, depending on the deviation of private expectations from the steady-state inflation rate, the optimal inflation penalty rate can be either positive or negative. It is positive if private inflation expectations undershoot the steady-state inflation rate, and *vice versa*. For a given (positive or negative) deviation of inflation expectations and for standard parameter values, the optimal inflation penalty rate is sensitive to a change in the learning gain when the latter is small (i.e., $\gamma < 0.2$).

Both the composite coefficient associated with $\widetilde{x}$ and a_t in (30) are negative. The first component in (30) is negative given that the output-gap target is positive and the second component is either positive when $a_t < 0$ or negative when $a_t > 0$. This explains why the optimal inflation penalty rate is strictly decreasing in γ for $a_t > 0$ but is first increasing in γ and then decreasing in γ for $a_t < 0$ when γ is high enough (see Figure 5). Proposition 4 summarizes these results.

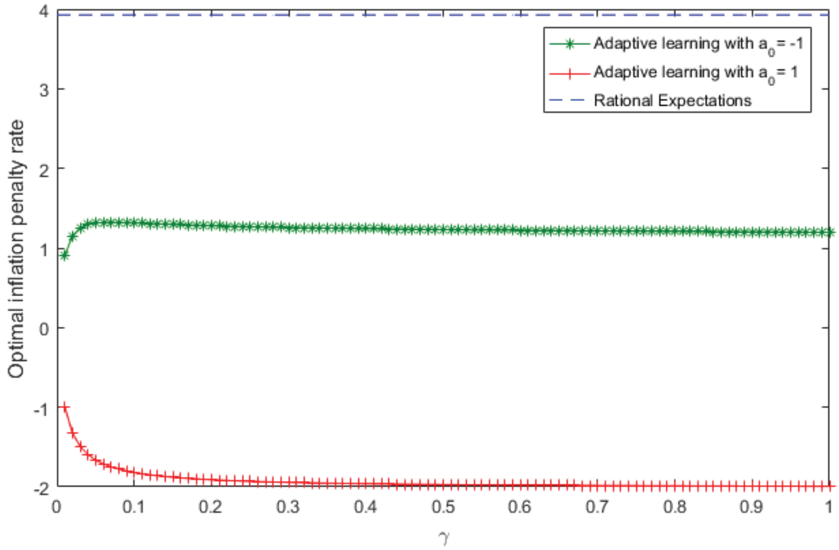

Figure 5: Optimal inflation penalty rate under short-term contracting approach.

Proposition 4. *When private agents are learning, the optimal inflation penalty rate of a short-term inflation contract is decomposed into two components. The component associated with the output-gap target is always negative. The component associated with expected inflation is positive for* $a_t < 0$*, and vice versa. The optimal inflation penalty rate is positive if private expected inflation undershoots enough the inflation target and positive otherwise.*

If the government sets the inflation penalty rate in each period, it also has to examine the sign of the deviation of private agents' expectations whereas under the pre-committed contract, it has only to take into account the output-gap target to fix the optimal inflation penalty rate. The level of optimal inflation penalty rate depends on the type of contract chosen in the first place, the learning gain, and finally, in the case of short-term contract, the deviation of private agents' inflation expectations from the inflation target. Indeed, under long-term contracting approach, the penalty rate remains positive, meaning that to deal with the inflation bias at the steady-state equilibrium, the learning behavior of private agents implies the appointment of a hawkish central banker but less than under RE. Whereas under short-term contracting approach, according to the sign of the deviation of private inflation expectations from the inflation target, the government can prefer either a hawkish or a dovish central banker.

Under adaptive learning, the linear inflation contract is quite different from the non-linear inflation contract studied in André and Dai (2017). When the overambitious output-gap target is absent, i.e., $\widetilde{x} = 0$, the optimal penalty rate of a linear inflation contract should be set to zero under both long-term and short-term contracting approaches at the steady state, and can be either positive or negative under short-term contracting approach out of steady state, while the optimal inflation penalty rate is always negative in the case of a non-linear contract. For comparison, the optimal penalty rate is always equal to zero under RE when $\widetilde{x} = 0$.

6 Equilibrium and inflation contract under decreasing-gain learning

The previous results are obtained under the assumption of constant-gain learning. In this section, we relax this assumption to show that our main results are still valid under the assumption of decreasing-gain learning. Indeed, constant-gain learning is more suitable in a time-varying environment because frequent structural changes rationalize an ongoing learning effort by private agents when forming expectations. However, if private agents confidently believe that the environment is stationary, modelling their learning behavior with a decreasing-gain rule is more appropriate since a steady state does not need continuing learning effort.[7] To model decreasing-gain learning, we substitute the parameter γ_t in the algorithm (5) by $\gamma_t = \frac{1}{t}$, which decreases over time. Indeed, $\gamma_t = 1$ if $t = 1$ and $\gamma_t \to 0$ as

[7] In practice, agents can shift from one approach to another. Most economic agents would begin to learn with a decreasing-gain learning as the first step in the expectations process before stabilizing the learning gain parameter. As shown by Milani (2014), private agents seem to have often switched to constant-gain learning, with a high gain, during most of the 1970s and until the early 1980s, before reverting to decreasing-gain learning.

$t \to +\infty$. More precisely, the decreasing-gain learning assigns increasingly lower weights to historical data as time goes by.

We solve the model using the same resolution method as in the case of constant-gain learning (Appendix A.3) and demonstrate that all feedback coefficients are bounded as $t \to +\infty$ (Appendix A.4).

The ALM for inflation takes the following form:

$$\pi_t = c^{dg}_{\pi,t} a_t + d^{dg}_{\pi,t} e_t + \Theta^{dg}_{\pi} \xi \tau + \Omega^{dg}_{\pi} \widetilde{x}. \tag{32}$$

where

$$c^{dg}_{\pi,t} = \frac{\beta \left\{ \alpha - (1-\gamma_{t+1}) \left[\alpha(1+\gamma_{t+1}\beta) \left(\beta - c^{dg}_{\pi,t+1} \right) - c^{dg}_{\pi,t+1}\kappa^2 \right] \right\}}{\alpha + \kappa^2 \left(1 - \beta\gamma_{t+1} c^{dg}_{\pi,t+1} \right) + \alpha\beta\gamma_{t+1} \left(1 + \beta\gamma_{t+1} \right) \left(\beta - c^{dg}_{\pi,t+1} \right)},$$

$$d^{dg}_{\pi,t} = -\frac{\alpha}{\alpha + \kappa^2 \left(1 - \beta\gamma_{t+1} c^{dg}_{\pi,t+1} \right) + \alpha\beta\gamma_{t+1} \left(1 + \beta\gamma_{t+1} \right) \left(\beta - c^{dg}_{\pi,t+1} \right)},$$

$$\Theta^{dg}_{\pi,t} = -\frac{1}{2} \frac{\kappa^2 (1-\beta)}{\alpha + \kappa^2 + \alpha\beta^2\gamma_{t+1} (1+\gamma_{t+1}\beta) - \beta \left(1 + c^{dg}_{\pi,t+1}\gamma_{t+1} \right) [\alpha + \kappa^2 + \alpha\gamma_{t+1}\beta]},$$

$$\Omega^{dg}_{\pi,t} = -\frac{\alpha\kappa \left[(1-\beta) - \beta^2\gamma_{t+1} \right]}{\alpha + \kappa^2 + \alpha\beta^2\gamma_{t+1} (1+\gamma_{t+1}\beta) - \beta \left(1 + c^{dg}_{\pi,t+1}\gamma_{t+1} \right) [\alpha + \kappa^2 + \alpha\gamma_{t+1}\beta]}.$$

The ALM for the output gap is:

$$x_t = c^{dg}_{x,t} a_t + d^{dg}_{x,t} e_t + \Theta^{dg}_{x,t} \xi \tau + \Omega^{dg}_{x,t} \widetilde{x}, \tag{33}$$

where $c^{dg}_{x,t} = -\frac{1}{\kappa}(\beta - c^{dg}_{\pi,t})$, $d^{dg}_{x,t} = -\frac{1}{\kappa}(1 - d^{dg}_{\pi,t})$, $\Theta^{dg}_{x,t} = \frac{1}{\kappa}\Theta^{dg}_{\pi,t}$ and $\Omega^{dg}_{x,t} = \frac{1}{\kappa}\Omega^{dg}_{\pi,t}$.

We remark that the feedback coefficients under decreasing-gain learning are similar to the ones obtained under constant-gain learning for $\gamma_t \in (0,1)$. This similarity allows us to discuss the implications of decreasing-gain learning by referring to those of constant-gain learning.

The steady state is reached as $t \to +\infty$ and $\gamma_{+\infty} \to 0$. As a result, inflation expectations become constant, i.e., there are no more expectations errors to be corrected through learning and private agents stop learning. The steady state is characterized by a constant value for inflation expectations. The feedback coefficients in the ALM for inflation, i.e., $c^{dg}_{\pi,t}$, $d^{dg}_{\pi,t}$, $\Theta^{dg}_{\pi,t}$ and $\Omega^{dg}_{\pi,t}$, obtained for $\gamma_{+\infty} \to 0$ are identical to c^{cg}_{π}, d^{cg}_{π}, Θ^{cg}_{π} and Ω^{cg}_{π} given by (22)-(25). In this particular case, since the learning gain is close to zero, the feedback effects of inflation expectations and current cost-push shocks on inflation are near to their maximum and the same is true for the feedback coefficients in the ALM for the output gap.

The assumption of decreasing-gain learning implies that, as long as the learning process is not terminated and the economy is not in the steady state, private agents will adjust their expectations by correcting past expectations errors. This makes possible for the CB to influence their future expectations in order to take advantage of distortions due to learning. To improve social welfare across the transitory learning equilibria compared to the REE, the government sets time-varying inflation penalty rate that depends on the type of contract and the evolution of learning-gain parameter in a way that the learning equilibrium converges as close as possible to the REE.

Under decreasing-gain learning, the effects of cost-push shocks, expected inflation, inflation penalty rate and the output-gap target on inflation and the output gap replicate more or less those observed under constant-gain learning. Due to the fact that the learning coefficient decreases as time goes by under decreasing-gain learning, the impact made by learning on the equilibrium decreases over time. Hence, inflation penalty rate evolves with the learning gain that decreases from unity to zero as time goes by. In the limit case where $\gamma_t = 0$, the CB cannot anymore manipulate private expectations and the possibility for the government to improve social welfare by appointing a liberal or dovish central banker disappears.

Before the learning gain coefficient tends to zero, decreasing-gain learning induces a deviation of the equilibrium solution from a more efficient REE and thus suggests the possibility for the government to improve social welfare by setting a long-term inflation contract (section 5.1) or a short-term inflation contract (section 5.2). Since the learning gain is decreasing over time, we conclude that the government cannot fix an invariable inflation penalty rate for all periods. As a result, the government has to change the inflation penalty rate in each period. However, the government can either set the path of inflation penalty rate under long-term contracting approach or the one under short-term contracting approach. Notice that inflation penalty rate under both approaches varies with decreasing learning gain. The difference is that under a long-term inflation contract, the government accounts for the change in learning gain but ignores the short-run issues of discretionary monetary policy, notably raised by managing the intertemporal trade-off and manipulating private expectations to improve the short-run macroeconomic stabilization, while under a short-term inflation contract, these short-run issues are taken into account by the government.

The above discussion allows us to formulate the following proposition by the similarity with the setting of inflation penalty rate under contracting approaches examined in section 5:

Proposition 5. *As time goes by, the learning coefficient γ_t decreases from unity to zero, implying that the feedback coefficients on inflation expectations, cost-push shocks, inflation penalty rate and the output-gap target in both ALMs increase over time. For a given volatility of inflation expectations and cost-push shocks, the optimal inflation penalty rate will increase from zero to the level that the government would set at the REE if it desires to replicate the long-term contract. Under the short-term approach, the optimal inflation penalty rate evolves from a positive or negative value according to past expected inflation rates to a value not far away from zero over time.*

7 Discussion

Our results account for the way and the extent to which an optimal linear inflation contract can correct the stabilization bias and the inflation bias under adaptive learning. Learning gives the CB the possibility to manipulate private expectations, and implies that the government has to design inflation contracts that are quite different from these under RE. These contracts are conceived with the aim of attaining the level of stabilization bias achieved under discretionary monetary policy with RE. The model can be promisingly extended in several directions.

We have only considered discretionary monetary policy when agents are learning. An alternative approach is that the CB conducts policy under full commitment, with RE and perfect knowledge about the economy, and the private sector is learning about the values of the parameters prevailing in equilibrium and knows that the CB is fully committed. According to Mele, Molnár and Santoro (2014), the optimal monetary policy under commitment drives the economy far from the RE commitment equilibrium, and to the RE discretionary equilibrium. It would be interesting to study the implications of this result for the design of optimal inflation contract.

A limitation that can rise in the present model is that learning agents cannot rationally infer about the future actions of the CB even if information about such actions are available. One extension of our model could be to introduce heterogeneous beliefs including both rational and learning expectations as Honkapohja and Mitra (2006). It is likely that, according to the proportion of agents who are learning, the inflation contract could be more or less similar to the one we obtain when all agents are learning.

Another promising extension is to consider the persistence of shocks, which is an important source of stabilization bias, and the CB's fear for model misspecification as in Tillmann (2009).[8] The latter has shown that the optimal degree of conservatism increases with the

[8] Alternative approaches to model misspecification could follow Leitemo and Söderström (2008) who consider a misspecification term in each equation of the structural model of a small open economy or as Giannoni (2007) who consider parameter uncertainty.

degree of uncertainty about such persistence. Indeed, introducing model uncertainty reinforces the argument in favor of the adoption of learning algorithm by private agents to compute expectations since such an approach is more appropriate for forming expectations in uncertain environments. One central research issue here is to understand how optimal monetary policy delegation is affected by learning and fear for model misspecification. It is likely that the learning algorithm amplifies the effects of the persistence in cost-push shocks, encouraging the government to choose a CB that is less conservative or impose a smaller inflation penalty rate than under RE.

It is documented that the first difference of inflation negatively depends on its own lag, and the sticky-price New Keynesian models emphasizing the role of firms' forward-looking pricing behavior cannot match with the stylized fact (Rudd and Whelan 2006). This finding justifies the incorporation of an inertial term into the New Keynesian Phillips curve as in Fuhrer and Moore (1995). The implications of learning for monetary policy in terms of E-stability in this kind of model have been studied by Evans and McGough (2005). It would be fruitful to examine the optimal design of monetary delegation and its implication for the dynamics of endogenous variables when agents are learning in such a model. As shown by Steinsson (2003), with the backward-looking term, it is optimal to endure a much larger contraction of the output gap to avoid getting too much inflation into the system while gradually bringing inflation back to its target instead of the overshooting that characterizes the purely forward-looking case. This implies a higher degree of interest rate inertia as illustrated by policies of both the Bundesbank and the Federal Reserve in the early 1990s and an attenuation of the aggressiveness in monetary policy implied by learning, and could thus dampen the effects of learning on inflation penalty rate.

High degree of interest rate inertia evidenced by empirical observations could be alternatively attributed to the optimal choice of a CB optimizing with an objective of interest rate smoothing. Woodford (2003) advocates that it is optimal to delegate monetary policy to a discretionary CB with this type of objective. Considering the issue of learnability, Bullard and Mitra (2007) have shown that interest-rate smoothing help alleviating problems of indeterminacy of stationary RE equilibria. By attenuating the aggressiveness in monetary policy caused by learning, optimal interest-rate smoothing will induce similar effect on optimal monetary delegation as the above-discussed hybrid New Keynesian Phillips curve.

8 Conclusion

The fact that private agents form inflation expectations using adaptive learning algorithm dramatically changes the logic in the design of linear inflation contracts. This paper reveals the new logic through analyzing the implications of adaptive learning for the delegation of monetary policy decisions by the government to a central banker with the help of a linear inflation contract.

Compared to the rational expectations equilibrium, learning dampens (strengthens) the responses of inflation (the output gap) to a change in inflation expectations and cost-push shocks. The higher the learning gain is, the larger such effects will be. An inflation penalty rate that is linearly indexed on the deviation of inflation from its target does not modify the feedback effects of inflation expectations and cost-push shocks on inflation and the output gap. However, without accounting for its indirect effect, a positive inflation penalty rate is able, as under rational expectations, to offset to some extent the inflation bias and the stabilization bias introduced by the presence of an overambitious output-gap target imposed by the society on the central banker.

An optimal inflation contract fully taking into account the equilibrium effects of private agents' learning behavior is quite different from that based on rational expectations. The government can choose between a long-term inflation contract, aiming at the long-term social optimum, and a short-term contract that gives– the flexibility to adjust the terms of the contract to private agents' inflation expectations to improve social welfare in the short-run. In the case of long-term contract, the optimal inflation penalty rate is positive and increases from zero to the level set under rational expectations, as the learning gain decreases from unity to zero.

If the short-term contract is adopted, the optimal inflation penalty rate is function of both the output-gap target and private agents' inflation expectations. Imposing an optimal short-term inflation contract when private agents are learning means that, given that inflation expectations are evolving over time, the government should face the challenge to reset in each period the terms of the contract to maximize social welfare. The inflation penalty rate can be either positive or negative depending on the learning gain and the sign of the deviation of inflation expectations from the inflation target.

Furthermore, an optimal linear inflation contract is quite different from an optimal non-linear inflation contract under adaptive learning. In the absence of an output-gap target different from its potential level, an optimal non-linear inflation contract implies that the central bank should be dovish, meaning that the government has to impose a negative inflation penalty rate intending to correct the distortions introduced by learning on the stabilization bias. In contrast, an optimal linear inflation contract is able to correct such distortions by discretionarily changing the sign of inflation penalty rate. The main results obtained under constant-gain learning remain valid under decreasing-gain learning.

Acknowledgements

We are grateful to Dmitry Matveev and Josef Hollmayr for very helpful remarks and suggestions.

A Appendix

A.1 Equilibrium solution of inflation under constant-gain learning

Using (1) and (5), we obtain:

$$x_t = \frac{1}{\kappa}\pi_t - \beta\frac{1}{\kappa}a_t - \frac{1}{\kappa}e_t, \tag{A.1}$$

$$x_{t+1} = \frac{1}{\kappa}\pi_{t+1} - \frac{\beta}{\kappa}\left[a_t + \gamma(\pi_t - a_t)\right] - \frac{1}{\kappa}e_{t+1}. \tag{A.2}$$

Substituting x_t and x_{t+1} respectively given by (A.1) and (A.2) into (18) and arranging terms lead to

$$E_t\pi_{t+1} = A_{11,t}\pi_t + A_{12,t}a_t + A_{13}\xi\tau + A_{14}\widetilde{x} + P_{1,t}e_t, \tag{A.3}$$

with

$$A_{11} \equiv \frac{\alpha + \kappa^2 + \alpha\gamma\beta^2\left[1-\gamma(1-\beta)\right]}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2(1-\gamma)\right\}}, \tag{A.4}$$

$$A_{12} \equiv -\frac{\alpha\beta\left\{1-\beta(1-\gamma)\left[1-\gamma(1-\beta)\right]\right\}}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2(1-\gamma)\right\}}, \tag{A.5}$$

$$A_{13} \equiv \frac{\kappa^2\left[1-\beta(1-\gamma)\right]}{2\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2(1-\gamma)\right\}}, \tag{A.6}$$

$$A_{14} \equiv -\frac{\alpha\kappa\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\}}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2(1-\gamma)\right\}}, \tag{A.7}$$

$$P_1 \equiv -\frac{\alpha}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2(1-\gamma)\right\}}, \tag{A.8}$$

where A_{13} and P_1 are coefficients related to exogenous variables in this equation.

According to the proposition 1 from Blanchard and Kahn (1980), the solution of the ALM for inflation takes the following form:

$$\pi_t = c_\pi^{cg}a_t + d_\pi^{cg}e_t + \Theta_\pi^{cg}\xi\tau + \Omega_\pi^{cg}\widetilde{x}. \tag{A.9}$$

We obtain with the help of (5) and (A.9):

$$E_t\pi_{t+1} = c_\pi^{cg}\left[(1-\gamma)a_t + \gamma\pi_t\right] + \Theta_\pi^{cg}\xi\tau + \varOmega_\pi^{cg}\widetilde{x}. \tag{A.10}$$

Using (A.10) to eliminate $E_t\pi_{t+1}$ in (A.3) and arranging terms yield:

$$\pi_t = \frac{\left[A_{12} - c_\pi^{cg}(1-\gamma)\right]}{c_\pi^{cg}\gamma - A_{11}}a_t + \frac{P_1}{c_\pi^{cg}\gamma - A_{11}}e_t + \frac{A_{13} - \Theta_\pi^{cg}}{c_\pi^{cg}\gamma - A_{11}}\xi\tau + \frac{A_{14} - \varOmega_\pi^{cg}}{\gamma c_\pi^{cg} - A_{11}}\widetilde{x}. \tag{A.11}$$

This implies that:

$$c_\pi^{cg} = \frac{A_{12} - c_\pi^{cg}(1-\gamma)}{c_\pi^{cg}\gamma - A_{11}}, \tag{A.12}$$

$$d_\pi^{cg} = \frac{P_1}{c_\pi^{cg}\gamma - A_{11}}, \tag{A.13}$$

$$\Theta_\pi^{cg} = \frac{A_{13}}{1 + c_\pi^{cg}\gamma - A_{11}}, \tag{A.14}$$

$$\varOmega_\pi^{cg} = \frac{A_{14}}{1 + \gamma c_\pi^{cg} - A_{11}}. \tag{A.15}$$

Gathering (5) and (A.3), while using (A.1) to substitute x_t, we obtain:

$$E_t y_{t+1} = Z + A_t y_t + P_t e_t,$$

where

$$y_t \equiv \left[\pi_t,\, a_t\right],\; Z \equiv \begin{bmatrix} A_{13}\xi\tau + A_{14}\widetilde{x} \\ 0 \end{bmatrix},\; A \equiv \begin{bmatrix} A_{11} & A_{12} \\ \gamma & 1-\gamma \end{bmatrix},\; \text{and } P \equiv \begin{bmatrix} P_1 \\ 0 \end{bmatrix}.$$

The above system is subject to two boundary conditions: a_0 and $\lim_{s\to\infty}|E_t\pi_{t+s}| < \infty$. The eigenvalues of A are $(1-\gamma)$ and the two eigenvalues of A_1:

$$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ \gamma & 1-\gamma \end{bmatrix}. \tag{A.16}$$

We can show that A_1 has an eigenvalue inside and one outside the unit circle. Among infinite stochastic sequences of c_π^{cg} satisfying (A.12), we focus on a non-explosive solution, i.e., the solution corresponding to the eigenvalue of A_1 inside the unit circle.

It is straightforward to show that the trace and determinant of A_1 are both positive. Thus, for A_1 to have two real eigenvalues (μ_1, μ_2), one inside and one outside the unit circle, it is sufficient to show that $(1-\mu_1)(1-\mu_2) < 0$. This is equivalent to:

$$\mu_1 + \mu_2 > 1 + \mu_1\mu_2. \tag{A.17}$$

Knowing that $\mu_1 + \mu_2$ is equal to the trace of A_1 and $\mu_1\mu_2$ equal to its determinant, we rewrite (A.17) as:

$$\frac{\alpha + \kappa^2 + \alpha\gamma\beta^2\left[1-\gamma(1-\beta)\right]}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2\left(1-\gamma\right)\right\}} + 1 - \gamma \;\; > \;\; 1 + \frac{\alpha + \kappa^2 + \alpha\gamma\beta^2\left[1-\gamma(1-\beta)\right]}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2\left(1-\gamma\right)\right\}}(1-\gamma) + \frac{\alpha\beta\left\{1-\beta\left(1-\gamma\right)\left[1-\gamma(1-\beta)\right]\right\}}{\beta\left\{\alpha\left[1-\gamma(1-\beta)\right] + \kappa^2\left(1-\gamma\right)\right\}}\gamma.$$

After simplification, we get:

$$\alpha\left(1-\beta\right)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\} + \kappa^2\left[1-\beta\left(1-\gamma\right)\right] > 0,$$

which is always verified given that $\beta \in (0,1)$ and $\gamma \in (0,1)$.

Rewriting (A.12) as $c_\pi^{cg}c_\pi^{cg}\gamma - c_\pi^{cg}A_{11} - A_{12} + c_\pi^{cg}(1-\gamma) = 0$ and substituting A_{11} and A_{12} by their expression, we obtain:

$$p_2(c_\pi^{cg})^2 + p_1 c_\pi^{cg} + p_0 = 0 \tag{A.18}$$

with

$$
\begin{aligned}
p_0 &= \alpha\beta\left\{1-\beta(1-\gamma)\left[1-\gamma(1-\beta)\right]\right\} > 0, \\
p_1 &= \beta\left(1-\gamma\right)\left\{\kappa^2(1-\gamma)+\alpha\left[1-\gamma(1-\beta)\right]\right\} - \alpha\gamma\beta^2\left[1-\gamma(1-\beta)\right] - \alpha - \kappa^2, \\
p_2 &= \gamma\beta\left\{\alpha\left[1-\gamma(1-\beta)\right]+\kappa^2\left(1-\gamma\right)\right\} > 0.
\end{aligned}
$$

We rewrite p_1 as $p_1 = -\kappa^2\left[1-\beta(1-\gamma)\right] - \alpha(1-\beta)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\} - p_0 - p_2$, it follows immediately that $p_1 < 0$. Then, it is straightforward to show that the discriminant of the polynomial (A.18) is positive.

To characterize the two solutions of c_π^{cg}, we rewrite (A.18) as:

$$
c_\pi^{cg} = -\frac{p_0 + p_2\left(c_\pi^{cg}\right)^2}{p_1} \equiv f(c_\pi^{cg}) \tag{A.19}
$$

As $f(c_\pi^{cg})$ is strictly increasing for $c_\pi^{cg} \in [0,1]$ with $f'(c_\pi^{cg}) = -\dfrac{2p_2}{p_1}c_\pi^{cg} > 0$ for $c_\pi^{cg} \in [0,1]$. To prove $f(c_\pi^{cg}) : [0,1] \to (0,1)$, it is sufficient to show that $f(0) > 0$ and that $f(1) < 1$. It is straightforward to see that $f(0) = -\dfrac{p_0}{p_1} > 0$ and

$$
f(1) = \frac{p_0 + p_2}{\kappa^2\left[1-\beta(1-\gamma)\right] + \alpha(1-\beta)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\} + p_0 + p_2} < 1.
$$

Since $f(c_\pi^{cg}) : [0,1] \to (0,1)$ and $f(c_\pi^{cg})$ is strictly increasing, it follows from the theorem of Brouwer that there exists one unique solution of c_π^{cg} in the interval (0,1). This solution corresponds to

$$
c_\pi^{cg} = \frac{-p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2} \tag{A.20}
$$

The other possible solution $c_\pi^{cg} = \frac{-p_1+\sqrt{p_1^2-4p_2p_0}}{2p_2}$ is larger than unity, which is excluded to avoid an explosive evolution of inflation.

Substituting A_{11}, A_{13}, A_{14} and P_1 into (A.13)-(A.15) leads to:

$$d_\pi^{cg} = \frac{\alpha}{\kappa^2 + \alpha + \alpha\gamma^2\beta^2\left(\beta - c_\pi^{cg}\right) + \gamma\beta\left(1-\gamma\right)\left\{\alpha\beta - \left(\alpha+\kappa^2\right)c_\pi^{cg}\right\}}. \tag{A.21}$$

$$\Theta_\pi^{cg} = -\frac{1}{2}\frac{\kappa^2\left[1-\beta\left(1-\gamma\right)\right]}{\Phi(\gamma)} < 0, \tag{A.22}$$

$$\Omega_\pi^{cg} = \frac{\alpha\kappa\left\{1-\beta\left[1-\gamma\left(1-\beta\right)\right]\right\}}{\Phi(\gamma)} > 0, \tag{A.23}$$

where $\Phi(\gamma) \equiv \alpha + \kappa^2 + \alpha\gamma\beta^2\left[1-\gamma(1-\beta)\right] - \beta\left(1+c_\pi^{cg}\gamma\right)\left[\left(\alpha+\kappa^2\right)\left(1-\gamma\right)+\alpha\gamma\beta\right]$.

The common denominator of Θ_π^{cg} and Ω_π^{cg}, i.e., $\Phi(\gamma)$, is positive for $\gamma = 0$ and $\gamma = 1$. We have $\Phi(1) > \Phi(0)$ and

$$\begin{aligned}\frac{\partial\Phi}{\partial\gamma} &= \beta\left\{\alpha(1-\beta\gamma)(1-\beta) + \kappa^2\left[1-\beta\left(1-\gamma\right)\right]\right\} + \beta\gamma c_\pi^{cg}\left[\left(\alpha+\kappa^2\right) - \alpha\beta\right] \\ &\quad + \beta\left[\left(\beta - c_\pi^{cg}\right) - \gamma\frac{\partial c_\pi^{cg}}{\partial\gamma}\right]\left[\left(\alpha+\kappa^2\right)\left(1-\gamma\right)+\alpha\gamma\beta\right] > 0.\end{aligned}$$

It follows that $\Phi(\gamma) > 0$, for $0 < \gamma < 1$.

We now show that $f(c_\pi^{cg}) : [0; \frac{\alpha\beta}{\alpha+\kappa^2}] \to (0; \frac{\alpha\beta}{\alpha+\kappa^2})$. Knowing that $f(0) > 0$ and substituting c_π^{cg} by $\frac{\alpha\beta}{\alpha+\kappa^2}$ into the function $f(c_\pi^{cg})$ defined by (A.19), we find

$$f\left(\frac{\alpha\beta}{\alpha+\kappa^2}\right) = -\frac{p_0 + p_2\left[\frac{\alpha\beta}{\alpha+\kappa^2}\right]^2}{p_1} = \frac{\frac{\alpha\beta}{\alpha+\kappa^2}\left\{\frac{\alpha+\kappa^2}{\alpha\beta}p_0 + \frac{\alpha\beta}{\alpha+\kappa^2}p_2\right\}}{-p_1}. \tag{A.24}$$

Using $p_2 = \frac{\alpha(1-\beta)+\kappa^2}{\alpha+\kappa^2}p_2 + \frac{\alpha\beta}{\alpha+\kappa^2}p_2$, $p_0 = -\frac{\alpha(1-\beta)+\kappa^2}{\alpha\beta}p_0 + \frac{\alpha+\kappa^2}{\alpha\beta}p_0$ and the definition of p_0, p_1, and p_2 given above, we rewrite the denominator in (A.24) as

$$\begin{aligned} -p_1 &= \kappa^2\left[1-\beta(1-\gamma)\right]+\alpha(1-\beta)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\}+p_0+p_2 \\ &= \kappa^2\left[1-\beta(1-\gamma)\right]+\alpha(1-\beta)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\} \\ &\quad -\tfrac{\alpha(1-\beta)+\kappa^2}{\alpha\beta}p_0+\tfrac{\alpha+\kappa^2}{\alpha\beta}p_0+\tfrac{\alpha(1-\beta)+\kappa^2}{\alpha+\kappa^2}p_2+\tfrac{\alpha\beta}{\alpha+\kappa^2}p_2 \\ &= -(1-\beta)p_2+\tfrac{\alpha+\kappa^2}{\alpha\beta}p_0+\tfrac{\alpha(1-\beta)+\kappa^2}{\alpha+\kappa^2}p_2+\tfrac{\alpha\beta}{\alpha+\kappa^2}p_2 \\ &= \beta p_2+\tfrac{\alpha+\kappa^2}{\alpha\beta}p_0 \qquad \text{(A.25)} \end{aligned}$$

Substituting the above expression of $-p_1$ into (A.24), we obtain:

$$f\left(\frac{\alpha\beta}{\alpha+\kappa^2(1+\tau)}\right) = \frac{\frac{\alpha\beta}{\alpha+\kappa^2}\left\{\frac{\alpha+\kappa^2}{\alpha\beta}p_0+\frac{\alpha\beta}{\alpha+\kappa^2}p_2\right\}}{\frac{\beta\kappa^2}{\alpha+\kappa^2}p_2+\frac{\alpha+\kappa^2}{\alpha\beta}p_0+\frac{\alpha\beta}{\alpha+\kappa^2}p_2} < \frac{\alpha\beta}{\alpha+\kappa^2}.$$

Given that $f'(c_\pi^{cg}) = -\dfrac{2p_2}{p_1}c_\pi^{cg} > 0$ for $c_\pi^{cg} \in [0,1]$, $f(c_\pi^{cg})$ is strictly increasing in the interval $\left[0;\ \frac{\alpha\beta}{\alpha+\kappa^2}\right]$. This property and the fact that $f(c_\pi^{cg}):\ [0;\ \frac{\alpha\beta}{\alpha+\kappa^2}] \to (0;\ \frac{\alpha\beta}{\alpha+\kappa^2})$ imply that there is a unique solution for c_π^{cg} so that $0 < c_\pi^{cg} < \frac{\alpha\beta}{\alpha+\kappa^2}$.

The case where $\gamma = 0$. Substituting $\gamma = 0$ into (A.4)-(A.8) and using the results in (A.12)-(A.15), we obtain:

$$\begin{aligned} c_\pi^{cg} &= \frac{\alpha\beta}{\alpha+\kappa^2}, \\ d_\pi^{cg} &= \frac{\alpha}{\alpha+\kappa^2}, \\ \Theta_\pi^{cg} &= -\frac{\kappa^2}{2\left(\alpha+\kappa^2\right)}, \\ \Omega_\pi^{cg} &= \frac{\alpha\kappa}{\alpha+\kappa^2}. \end{aligned}$$

The case where $\gamma = 1$. Inserting $\gamma = 1$ into (A.4)-(A.8) and using the results in (A.12)-(A.15) lead to:

$$
\begin{aligned}
c_{\pi}^{cg} &= \frac{\{\kappa^2+\alpha+\alpha\beta^3\} - \sqrt{\{\kappa^2+\alpha+\alpha\beta^3\}^2 - 4\alpha^2\beta^3}}{2\alpha\beta^2}, \\
d_{\pi}^{cg} &= \frac{\alpha}{\kappa^2+\alpha+\alpha\beta^2\left(\beta - c_{\pi}^{cg}\right)}, \\
\Theta_{\pi}^{cg} &= -\frac{1}{2}\frac{\kappa^2}{\alpha+\kappa^2-\alpha\beta^2\left(1+c_{\pi}^{cg}-\beta\right)}, \\
\Omega_{\pi}^{cg} &= \frac{\alpha\kappa\left(1-\beta^2\right)}{\alpha+\kappa^2-\alpha\beta^2\left(1+c_{\pi}^{cg}-\beta\right)}.
\end{aligned}
$$

A.2 The effect of an increase in learning gain

Deriving p_0, p_1 and p_2 and using (A.25) yield:

$$
\begin{aligned}
\frac{\partial p_2}{\partial \gamma} &= \beta\left\{\alpha\left[1-2\gamma(1-\beta)\right]+\kappa^2\left(1-2\gamma\right)\right\}. \\
\frac{\partial p_0}{\partial \gamma} &= \alpha\beta^2\left[(2-\beta)(1-\gamma)+\gamma\beta\right] > 0, \\
\frac{\partial p_1}{\partial \gamma} &= -\ \beta\frac{\partial p_2}{\partial \gamma} - \tfrac{\alpha+\kappa^2}{\alpha\beta}\frac{\partial p_0}{\partial \gamma}.
\end{aligned}
$$

Deriving the solution of c_{π}^{cg} given by (A.20), we obtain:

$$
\frac{\partial c_{\pi}^{cg}}{\partial \gamma} = \frac{\left(-p_2 - \frac{p_1 p_2}{\sqrt{p_1^2-4p_2p_0}}\right)\frac{\partial p_1}{\partial \gamma} + \frac{2p_2p_2}{\sqrt{p_1^2-4p_2p_0}}\frac{\partial p_0}{\partial \gamma} + \left(p_1 + \frac{p_1^2-2p_2p_0}{\sqrt{p_1^2-4p_2p_0}}\right)\frac{\partial p_2}{\partial \gamma}}{2p_2^2}.
$$

Using $\frac{\partial p_2}{\partial \gamma} = -\frac{1}{\beta}\frac{\partial p_1}{\partial \gamma} - \frac{\alpha+\kappa^2}{\alpha\beta^2}\frac{\partial p_0}{\partial \gamma}$ deduced from (A.25), we get:

$$
\frac{\partial c_{\pi}^{cg}}{\partial \gamma} = \frac{1}{2p_2^2}\left(F\frac{\partial p_1}{\partial \gamma} + G\frac{\partial p_2}{\partial \gamma}\right),
$$

where

$$\begin{aligned} F &= -p_2 + \frac{-p_1 p_2}{\sqrt{p_1^2 - 4p_2 p_0}} - \frac{1}{\beta} p_1 - \frac{1}{\beta} \frac{p_1^2 - 2p_2 p_0}{\sqrt{p_1^2 - 4p_2 p_0}}, \\ G &= \frac{2p_2 p_2}{\sqrt{p_1^2 - 4p_2 p_0}} - \left(p_1 + \frac{p_1^2 - 2p_2 p_0}{\sqrt{p_1^2 - 4p_2 p_0}} \right) \tfrac{\alpha+\kappa^2}{\alpha\beta^2}. \end{aligned}$$

Using (A.25), i.e., $p_1 = -\beta p_2 - \frac{\alpha+\kappa^2}{\alpha\beta} p_0$, after fastidious arrangements of terms, we finally obtain:

$$\frac{\partial c_\pi^{cg}}{\partial \gamma} = \frac{1 - \frac{\alpha+\kappa^2}{\alpha\beta} c_\pi^{cg}}{\beta p_2 \sqrt{p_1^2 - 4p_2 p_0}} \left(p_0 \frac{\partial p_1}{\partial \gamma} - p_1 \frac{\partial p_0}{\partial \gamma} \right).$$

Using $c_\pi^{cg} < \frac{\alpha\beta}{\alpha+\kappa^2}$, we find: $1 - \frac{\alpha+\kappa^2}{\alpha\beta} c_\pi^{cg} > 1 - \frac{\alpha+\kappa^2}{\alpha\beta} \frac{\alpha\beta}{\alpha+\kappa^2} = 0$. To determine the sign of $H \equiv p_0 \frac{\partial p_1}{\partial \gamma} - p_1 \frac{\partial p_0}{\partial \gamma}$, we first check its value for $\gamma = 1$ and then the sign of its derivative with respect to γ. For $\gamma = 1$, we have : $\frac{\partial p_0}{\partial \gamma} = \alpha\beta^3 > 0$, $\frac{\partial p_1}{\partial \gamma} = -\beta^2 \alpha (1+\beta) < 0$, $p_1 = -\left(\kappa^2 + \alpha + \alpha\beta^3\right)$ and $p_0 = \alpha\beta$. It is straightforward to show that for $\gamma = 1$:

$$H \equiv -\alpha\beta^3 \left[1 - (\kappa^2 + \alpha) + \beta(1 - \alpha\beta^2)\right]$$

Given that κ and α are very small and $\beta < 1$ in the New-Keynesian literature, we have $H < 0$. Deriving H with respect to γ yields

$$\frac{\partial H}{\partial \gamma} = \frac{\partial p_0}{\partial \gamma} \frac{\partial p_1}{\partial \gamma} + p_0 \frac{\partial^2 p_1}{\partial^2 \gamma} - \frac{\partial p_1}{\partial \gamma} \frac{\partial p_0}{\partial \gamma} - p_1 \frac{\partial^2 p_0}{\partial^2 \gamma} = p_0 \frac{\partial^2 p_1}{\partial^2 \gamma} - p_1 \frac{\partial^2 p_0}{\partial^2 \gamma}.$$

Deriving twice p_0 and p_1 with respect to γ for $\gamma \in (0,1)$ leads to

$$\begin{aligned} \frac{\partial^2 p_0}{\partial^2 \gamma} &= -2\alpha\beta^2(1-\beta) < 0, \\ \frac{\partial^2 p_1}{\partial^2 \gamma} &= 2\beta \left[\alpha(1-\beta^2) + \kappa^2\right] > 0. \end{aligned}$$

Using these second-order derivatives, we get

$$\frac{\partial H}{\partial \gamma} \;=\; 2\alpha^2\beta^3(1-\beta)\left\{1-\beta\left[1-\gamma(1-\beta)\right]\right\}+2\alpha\beta^3\left[1-\beta(1-\gamma)\right]\kappa^2>0.$$

Consequently, given that $H<0$ for $\gamma=1$ and $\frac{\partial H}{\partial \gamma}>0$ for $\gamma\in[0,1]$, we conclude that

$$\frac{\partial c_\pi^{cg}}{\partial \gamma}<0.$$

Using d_π^{cg} with respect to γ yields:

$$\frac{\partial d_\pi^{cg}}{\partial \gamma}=\frac{-\alpha\beta\left\{\Upsilon-\gamma\left[\alpha\beta\gamma+(1-\gamma)\left(\alpha+\kappa^2\right)\right]\frac{\partial c_\pi^{cg}}{\partial \gamma}\right\}}{\left\{\kappa^2+\alpha+\alpha\beta^2\gamma^2(\beta-c_\pi^{cg})+\beta\gamma(1-\gamma)\left[\alpha\beta-\left(\alpha+\kappa^2\right)c_\pi^{cg}\right]\right\}^2},$$

Using $c_\pi^{cg}<\frac{\alpha\beta}{\alpha+\kappa^2}$, we find that $\Upsilon\equiv 2\alpha\beta\gamma(\beta-c_\pi^{cg})+(1-2\gamma)\left[\alpha\beta-(\alpha+\kappa^2)c_\pi^{cg}\right]>2\alpha\beta\gamma(\beta-c_\pi^{cg})>0$, it follows that:

$$\frac{\partial d_\pi^{cg}}{\partial \gamma}<0.$$

Using the definition of c_x^{cg}, d_π^{cg} and d_x^{cg}, it is straightforward to show the sign of their partial derivative with respect to γ.

Deriving (A.14) with respect to γ gives :

$$\begin{aligned}\frac{\partial \Theta_\pi^{cg}}{\partial \gamma} \;=\; & -\frac{\kappa^2\beta}{2\Phi^2}\left\{\left\{\alpha\beta\left(1-\beta\right)+\left[\kappa^2+\alpha(1-\beta)\right]\left[(1-\gamma)\left[1-\beta(1-\gamma)\right]-\gamma\right]\right\}c_\pi^{cg}\right.\\ & \left.+\alpha\gamma\beta(1-\beta)\left[2(1-\beta)+\beta\gamma\right]+\gamma\left[1-\beta\left(1-\gamma\right)\right]\left\{(1-\gamma)\left[\alpha(1-\beta)+\kappa^2\right]+\alpha\beta\right\}\frac{\partial c_\pi^{cg}}{\partial \gamma}\right\}.\end{aligned}$$

Given that it is impossible to analytically determine the sign of $\frac{\partial \Theta_\pi^{cg}}{\partial \gamma}$, we have checked that $\frac{\partial \Theta_\pi^{cg}}{\partial \gamma}<0$ using standard parameter values, i.e., $\beta=0.99$, $\kappa=0.024$ and $\alpha=0.048$.

Deriving (A.15) with respect to γ and using $c_\pi^{cg}<\frac{\alpha\beta}{\alpha+\kappa^2}$ allow to show that:

$$\frac{\partial \Omega_\pi^{cg}}{\partial \gamma}<-\frac{\alpha\kappa\beta\left(1-\beta\right)}{\Phi^2}\left\{\beta\kappa^2+\frac{\alpha\beta^2\gamma\kappa^2(2+\beta\gamma)}{\alpha+\kappa^2}-\gamma(1+\beta\gamma)\left[\left(\alpha+\kappa^2\right)(1-\gamma)+\alpha\gamma\beta\right]\frac{\partial c_\pi^{cg}}{\partial \gamma}\right\}<0.$$

A.3 Equilibrium solution under decreasing learning gain

Using (15) and (17) lead to

$$\lambda_{2,t} = -\frac{1}{\gamma_{t+1}}\left[\pi_t + \frac{1}{2}\xi\tau + \frac{\alpha}{\kappa}\left(x_t - \widetilde{x}\right)\right]. \tag{A.26}$$

Advancing in time (15) and the previous equation, and taking account of the fact that the CB minimizes the expected welfare function, we get:

$$\lambda_{1,t+1} = -\frac{\alpha}{\kappa}\left(E_t x_{t+1} - \widetilde{x}\right) \tag{A.27}$$

$$\lambda_{2,t+1} = -\frac{1}{\gamma_{t+2}}\left[E_t\pi_{t+1} + \frac{1}{2}\xi\tau + \frac{\alpha}{\kappa}\left(E_t x_{t+1} - \widetilde{x}\right)\right] \tag{A.28}$$

Substituting $\lambda_{2,t}$, $\lambda_{1,t+1}$ and $\lambda_{2,t+1}$ given by (A.26)-(A.28) into (16), we obtain

$$\begin{aligned}\pi_t \;=\; & -\frac{\alpha}{\kappa}\left(x_t - \widetilde{x}\right) + \frac{\alpha\beta\gamma_{t+1}}{\kappa}\left(\frac{1-\gamma_{t+2}\left(1-\beta\right)}{\gamma_{t+2}}\right)\left(E_t x_{t+1} - \widetilde{x}\right) + \gamma_{t+1}\frac{\left(1-\gamma_{t+2}\right)}{\gamma_{t+2}}\beta E_t\pi_{t+1} \\ & + \frac{1}{2}\left[\frac{\beta\gamma_{t+1}\left(1-\gamma_{t+2}\right) - \gamma_{t+2}}{\gamma_{t+2}}\right]\xi\tau\end{aligned} \tag{A.29}$$

Using (1) and (5), we obtain:

$$x_t = \frac{1}{\kappa}\pi_t - \frac{\beta}{\kappa}a_t - \frac{1}{\kappa}e_t \tag{A.30}$$

$$E_t x_{t+1} = \frac{1}{\kappa}E_t\pi_{t+1} - \frac{\beta}{\kappa}\left[a_t + \gamma_{t+1}(\pi_t - a_t)\right]. \tag{A.31}$$

Using (A.30) and (A.31) and the expression of decreasing gain learning parameters, i.e., $\gamma_{t+1} = \frac{1}{t+1}$, $\gamma_{t+2} = \frac{1}{t+2}$, and $1 - \gamma_{t+1} = \frac{t}{t+1}$, in (A.29), and arranging terms yield:

$$E_t\pi_{t+1} = A_{11,t}\pi_t + A_{12,t}a_t + A_{13}\xi\tau + A_{14}\widetilde{x} + P_{1,t}e_t, \tag{A.32}$$

with

$$A_{11,t} \equiv \frac{(\alpha+\kappa^2)+\alpha\beta^2\left(\frac{1}{t+1}\right)\left[1+\left(\frac{1}{t+1}\right)\beta\right]}{\alpha\beta\left(1+\beta\frac{1}{t+1}\right)+\beta\kappa^2}, \tag{A.33}$$

$$A_{12,t} \equiv -\frac{\alpha\beta-\alpha\beta^2(1-\frac{1}{t+1})(1+\frac{1}{t+1}\beta)}{\alpha\beta\left(1+\beta\frac{1}{t+1}\right)+\beta\kappa^2}, \tag{A.34}$$

$$A_{13,t} \equiv \frac{1}{2}\frac{\kappa^2(1-\beta)}{\alpha\beta\left(1+\beta\frac{1}{t+1}\right)+\beta\kappa^2}, \tag{A.35}$$

$$A_{14,t} \equiv -\frac{\alpha\kappa\left[(1-\beta)-\frac{\beta^2}{t+1}\right]}{\alpha\beta\left(1+\beta\frac{1}{t+1}\right)+\beta\kappa^2}, \tag{A.36}$$

$$P_{1,t} \equiv -\frac{\alpha}{\alpha\beta\left(1+\beta\frac{1}{t+1}\right)+\beta\kappa^2}. \tag{A.37}$$

The solution of the ALM of inflation takes the following form:

$$\pi_t = c^{dg}_{\pi,t}a_t + d^{dg}_{\pi,t}e_t + \Theta^{dg}_{\pi}\xi\tau + \Omega^{dg}_{\pi}\widetilde{x}.. \tag{A.38}$$

Using (5) and (A.38), we obtain:

$$E_t\pi_{t+1} = c^{dg}_{\pi,t+1}\left[(1-\gamma_{t+1})a_t + \gamma_{t+1}\pi_t\right] + \Theta^{dg}_{\pi}\xi\tau + \Omega^{dg}_{\pi}\widetilde{x}. \tag{A.39}$$

Using (A.39) and (A.32) to eliminate $E_t\pi_{t+1}$ and arranging terms yield the equilibrium solution of inflation. Comparing this solution with (A.38) gives the feedback coefficients for the ALM of inflation:

$$c^{dg}_{\pi,t} = \frac{A_{12,t}-(1-\frac{1}{t+1})c^{dg}_{\pi,t+1}}{\frac{1}{t+1}c^{dg}_{\pi,t+1}-A_{11,t}}, \tag{A.40}$$

$$d^{dg}_{\pi,t} = \frac{P_{1,t}}{\frac{1}{t+1}c^{dg}_{\pi,t+1}-A_{11,t}}, \tag{A.41}$$

$$\Theta^{dg}_{\pi,t} = \frac{A_{13,t}}{\frac{1}{t+1}c^{dg}_{\pi,t+1}+1-A_{11,t}}, \tag{A.42}$$

$$\Omega^{dg}_{\pi,t} = \frac{A_{14}}{\frac{1}{t+1}c^{dg}_{\pi,t+1}+1-A_{11,t}}. \tag{A.43}$$

Inserting (A.33)-(A.37) into (A.40)-(A.43) yields:

$$c^{dg}_{\pi,t} = \frac{\beta\left\{\alpha - (1-\gamma_{t+1})\left[\alpha(1+\gamma_{t+1}\beta)\left(\beta - c^{dg}_{\pi,t+1}\right) - c^{dg}_{\pi,t+1}\kappa^2\right]\right\}}{\alpha + \kappa^2\left(1-\beta\gamma_{t+1}c^{dg}_{\pi,t+1}\right) + \alpha\beta\gamma_{t+1}\left(1+\beta\gamma_{t+1}\right)\left(\beta - c^{dg}_{\pi,t+1}\right)}, \tag{A.44}$$

$$d^{dg}_{\pi,t} = -\frac{\alpha}{\alpha + \kappa^2\left(1-\beta\gamma_{t+1}c^{dg}_{\pi,t+1}\right) + \alpha\beta\gamma_{t+1}\left(1+\beta\gamma_{t+1}\right)\left(\beta - c^{dg}_{\pi,t+1}\right)}, \tag{A.45}$$

$$\Theta^{dg}_{\pi,t} = -\frac{1}{2}\frac{\kappa^2(1-\beta)}{\alpha + \kappa^2 + \alpha\beta^2\gamma_{t+1}\left(1+\gamma_{t+1}\beta\right) - \beta\left(1 + c^{dg}_{\pi,t+1}\gamma_{t+1}\right)\left[\alpha + \kappa^2 + \alpha\gamma_{t+1}\beta\right]}, \tag{A.46}$$

$$\Omega^{dg}_{\pi,t} = -\frac{\alpha\kappa\left[(1-\beta) - \beta^2\gamma_{t+1}\right]}{\alpha + \kappa^2 + \alpha\beta^2\gamma_{t+1}\left(1+\gamma_{t+1}\beta\right) - \beta\left(1 + c^{dg}_{\pi,t+1}\gamma_{t+1}\right)\left[\alpha + \kappa^2 + \alpha\gamma_{t+1}\beta\right]}. \tag{A.47}$$

Gathering (5) and (A.32), while using (A.30) to substitute x_t, we obtain:

$$E_t y_{t+1} = Z + A_t y_t + P_t e_t,$$

where

$$y_t \equiv [\pi_t,\, a_t]\,,\; Z \equiv \begin{bmatrix} A_{13}\xi\tau + A_{14}\widetilde{x} \\ 0 \end{bmatrix},\; A \equiv \begin{bmatrix} A_{11} & A_{12} \\ \frac{1}{t+1} & \frac{t}{t+1} \end{bmatrix},\text{ and } P \equiv \begin{bmatrix} P_1 \\ 0 \end{bmatrix}.$$

The above system is subject to two boundary conditions: a_0 and $\lim_{s\to\infty}|E_t\pi_{t+s}| < \infty$. The eigenvalues of A_t are $1-\gamma_{t+1} = \frac{t}{t+1}$ and the two eigenvalues of $A_{1,t}$:

$$A_{1,t} = \begin{bmatrix} A_{11,t} & A_{12,t} \\ \frac{1}{t+1} & \frac{t}{t+1} \end{bmatrix}. \tag{A.48}$$

We can show that $A_{1,t}$ has a real eigenvalue inside and one outside the unit circle.

A.4 The single stable solution under decreasing-gain learning

Our attention focuses on a non-explosive solution among infinite stochastic sequences satisfying equation (A.44). To put into evidence the properties of this solution, we consider the value of $c^{dg}_{\pi,t}$ when $t \to +\infty$. With the help of the boundary conditions $\lim_{t\to+\infty} A_{11,t} = \frac{1}{\beta}$ and $\lim_{t\to+\infty} A_{12,t} = \frac{-\alpha(1-\beta)}{\alpha+\kappa^2}$, we find that in the limit, $c^{dg}_{\pi,t}$ evolves according to:

$$\lim_{t\to+\infty} c^{dg}_{\pi,t} = \beta\left[\lim_{t\to+\infty} c^{dg}_{\pi,t+1} + (1-\beta)\frac{\alpha}{\alpha+\kappa^2}\right]. \tag{A.49}$$

The boundary condition imposed on inflation $\lim_{n\to\infty}|\pi_{t+n}| < \infty$ yields that $\lim_{n\to+\infty}\beta^n c^{dg}_{\pi,t+n} = 0$. Using this condition and solving (A.49) forward yield one and only one bounded solution for $c^{dg}_{\pi,t}$:

$$\lim_{t\to+\infty} c^{dg}_{\pi,t} = \frac{\alpha\beta}{\alpha+\kappa^2}.$$

Equation (A.49) yields

$$\lim_{t\to+\infty} c^{dg}_{\pi,t+1} > \lim_{t\to+\infty} c^{dg}_{\pi,t}, \tag{A.50}$$

meaning that when $t \to +\infty$, we get $c^{dg}_{\pi,t} < \dfrac{\alpha\beta}{\alpha+\kappa^2}$.

We assume that $c^{dg}_{\pi,n+1} < \dfrac{\alpha\beta}{\alpha+\kappa^2}$ for $t = n+1$. It follows straightforwardly from (A.40) that

$$c^{dg}_{\pi,n} = \frac{A_{12,n} - \frac{n}{n+1}c^{dg}_{\pi,n+1}}{\frac{1}{n+1}c^{dg}_{\pi,n+1} - A_{11,n}} \Rightarrow$$

$$c^{dg}_{\pi,n+1} = \frac{A_{12,n} + c^{dg}_{\pi,n}A_{11,n}}{c^{dg}_{\pi,n}\frac{1}{n+1} + \frac{n}{n+1}} < \frac{\alpha\beta}{\alpha+\kappa^2} \Rightarrow$$

$$\left[A_{11,n} - \frac{1}{n+1}\frac{\alpha\beta}{\alpha+\kappa^2}\right] c^{dg}_{\pi,n} < \frac{\alpha\beta}{\alpha+\kappa^2}\frac{n}{n+1} - A_{12,n}$$

Substituting $A_{11,n}$ and $A_{12,n}$ given by (A.33)-(A.34), we obtain after some fastidious computations:

$$c_{\pi,n}^{dg} < \frac{\alpha\beta}{\alpha+\kappa^2} \frac{(\alpha+\kappa^2) - \beta^2\kappa^2 \frac{1}{n+1}(1-\frac{1}{n+1})}{(\alpha+\kappa^2) + \left(\frac{1}{n+1}\right)^2 \frac{\alpha\beta^3\kappa^2}{\alpha+\kappa^2}} \ll \frac{\alpha\beta}{(\alpha+\kappa^2)}.$$

By recurrence, we conclude that $c_{\pi,t}^{dg}$ is increasing with time and is bounded:

$$c_{\pi,t}^{dg} < \frac{\alpha\beta}{\alpha+\kappa^2}, \text{ for } t \in [1, +\infty).$$

Given the definition of $c_{x,t}^{dg}$, $d_{\pi,t}^{dg}$ $d_{x,t}^{dg}$, it is straightforward to find their limit and their evolution over time.

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