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Biodiversity, infectious diseases and the dilution effect

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Abstract

Biologists point out that biodiversity loss contributes to promote the transmission of diseases. In epidemiology, this phenomenon is known as dilution effect. Our paper aims to model this effect in an economic model where the spread of an infectious disease is considered. More precisely, we embed a SIS model into a Ramsey model (1928) where a pollution externality coming from production affects the evolution of biodiversity. Biodiversity is assimilated to a renewable resource and affects the infectivity of the disease (dilution effect). A green tax is levied on production at the firm level to finance depollution according to a balanced budget rule. In the long run, a disease-free and an endemic regime are possible. We focus only on the second case and we find that the magnitude of the dilution effect determines the number of steady states. When the dilution effect remains low, there are two steady states with high and low biodiversity respectively. Conversely, when the dilution effect becomes high, the steady state is always unique. Moreover, under a low dilution effect, a higher green-tax rate always impairs biodiversity at the low steady state, while this green paradox is over under a high dilution effect. In the short run, limit cycles can arise in both the cases even if only a low dilution effect can lead to the occurrence of Bogdanov-Takens and generalized Hopf bifurcations.

Keywords: dilution effect, pollution, SIS model, Ramsey model, local bifurcations of codimension one and two.

JEL Classification: C61, E32, O44.

1 Introduction

Paleontologists report that planet Earth has experienced five mass extinctions (also known as biotic crises) in the past 540 million years (Barnosky et al., 2011). A mass extinction is conventionally defined as a change where more than three-quarters of species disappear in a geologically short interval of time (Barnosky et al., 2011). MacLeod (2013) points out a number of causes of these past upheavals: tectonics, climate and sea-level changes, changes in ocean and atmosphere circulation patterns and large igneous province volcanism. According to the scientific community, a sixth mass extinction is under way. Nevertheless, it is not due to natural cycles as for the five other past biotic crisis but rather to human activities. As pointed out by Ceballos et al. (2015), deforestation and pollution are responsible of the global warming and the climate change at the origin of this mass extinction.

The heavy loss of biodiversity of the sixth biotic crisis is not just an environmental problem. It affects human well-being at a large extent. According to Keesing et al. (2010), biodiversity loss promotes the transmission of infectious diseases. For example, Allan et al. (2009) have reported that a low bird diversity increases the human transmission of West Nile encephalitis. This negative correlation is simply explained by Kessing et al. (2010): biodiversity loss reduces predation and competition on reservoir hosts and increases the pathogen's concentration in the remaining species (so-called dilution effect in epidemiology). The benefits of biodiversity in disease transmission are well-known in practice. According to Johnson and Thieltges (2010), in many African societies, cattle is placed near housing to distract malaria-carrying mosquitoes from people. Therefore, human pollution implies a biodiversity loss which promotes infectious diseases and weakens production and economic growth at the end.[1]

Biodiversity has not only a positive impact on physical health through the dilution effect, but also a positive influence on mental health. For instance, as reported by Dean et al. (2011), biodiversity in cities has some psychosocial benefits: recovery from stress, self-regulation of emotions, restoration of attention fatigue and enhanced sense of community. In their study, Dean et al. (2011) point out that these psychosocial benefits preserve mental health and prevent a depressive behavior. Even if there is, to the best of our knowledge, no empirical evidence about the effect of biodiversity on consumption demand, its benefits on mental health suggest also a role in consumption. Indeed, less biodiversity makes agents more depressive and can reduce their consumption demand. Conversely, they can compensate the loss of pleasure by increasing their consumption demand. The ambiguous effects of biodiversity on this demand remind us those of pollution on consumption highligted by Michel and Rotillon (1995). The novelty of our paper rests instead on the study of the dilution effect.

Thereby, human activities pollute entailing global warming and climate change which impair biodiversity. The loss of biodiversity affects physical health

[1] Illness is also recognized as one of the main causes of work absenteeism (Akazawa et al., 2003).

through the dilution effect. This effect lowers labor supply and worsens mental health affecting the consumption demand. Thus, a rise in economic activities today implies a drop in these activities tomorrow through the loss of biodiversity. To represent the interplay between economics and nature in terms of dilution effect, an interdisciplinary approach is needed: we focus on the effect of biodiversity on the immune system.

There is a panoply of epidemiological models to describe the spread of infectious diseases. In the spirit of Hethcote (2009), we apply the simplest model to represent the change of the share of healthy people through time: the SIS (Susceptible-Infective-Susceptible) dynamics are captured by two parameters: (1) the probability of a susceptible individual to become ill after a contact with an infected individual and (2) the recovery rate driving the lapse of time the infected individual spends to recover from the disease. Dynamics are straightforward. A disease-free steady state coexists with an endemic one. When (1) exceeds (2), the endemic steady state is stable while the other, unstable.

In our paper, we aim to bridge the gap between economy, ecology and epidemiology within a unified framework where the dilution effect is taken in account. More precisely, we embed the SIS model into a Ramsey model where the production activities pollute and impair the biodiversity. Biodiversity is assimilated to a renewable resource affecting the consumption demand and, in order to capture the dilution effect, the household's immune system. We introduce a two-sided dilution effect assuming the probability to become ill as a decreasing function of biodiversity and the recovery rate as an increasing function. As in Goenka et al. (2014), we consider that the labor force only consists of healthy people. Eventually, a green tax is levied on production at the firm level in order to finance depollution according to a balanced budget rule.

The integration of the SIS model into a Ramsey model is not new and dates back to Goenka and Liu (2012). They have considered a discrete time model in which healthy people tune their labor supply through a consumption-leisure arbitrage. In a continuous version of the model where labor supply is exclusively driven by the number of healthy people, Goenka et al. (2014) address the issue of optimal health expenditures. More recently, Bosi and Desmarchelier (2016a) have reconsidered the continuous time version developed by Goenka et al. (2014) to take in account the interplay between a flow of pollution and infectious diseases. Bosi and Desmarchelier (2016a) have pointed out that, when pollution becomes excessive, two limit cycles can appear (stable and unstable) near the endemic steady state through a Hopf bifurcation.

We characterize the equilibrium either in the long or the short run. In the long run, as in the standard SIS model, a disease-free regime coexists with the endemic one. In the endemic case, two steady state can coexist and display two different levels of biodiversity. A paradox emerges under a moderate dilution effect at the steady state with low biodiversity: a higher green-tax rate lowers the biodiversity. This counter-intuitive effect is similar to the static green paradox pointed out by Bosi and Desmarchelier (2017a).[2] Conversely, a green-tax rate

[2] In Bosi and Desmarchelier (2017a), a static green paradox is a positive relation between

always lowers the biodiversity level at the steady state with higher biodiversity. Interestingly, the static green paradox is ruled out by a strong dilution effect. In the short run, we show that both the dilution effects (low and high) are compatible with the existence of a Hopf bifurcation around the high-biodiversity steady state when preferences exhibit a complementarity between biodiversity and consumption. Focusing on codimension two, we show also that a Bogdanov-Takens bifurcation can occur only under a low dilution effect, as well as two generalized Hopf bifurcations.

The rest of the paper is organized as follows. Section 2 introduces the model. Sections 3 derives the equilibrium system. Sections 4 and 5 focus on the long and short-run dynamics. A numerical illustration with isoelastic fundamentals is provided in section 6. Section 7 concludes.

2 Model

2.1 Disease

Epidemiologists use the SIS model to study the spread of endemic diseases. Population (N) is divided in two classes: susceptible (S) and infective (I) with $S+I=N$. We consider a wide range of infectious diseases, not a specific one. So the infective class cover many different illnesses. The proportion of susceptible and infective are given by $s=S/N$ and $i=I/N$. $\beta>0$ denotes the average number of adequate contacts (sufficient to transmit the disease) of an infective per unit of time and S/N the probability to face a susceptible during a contact. β increases in the transmissibility due to the virulence and pathogenicity of microbes, which increases in turn in the loss of biodiversity. Thus, $\beta S/N$ is the average number of adequate contacts with susceptibles of one infective per unit of time, while the number of new infectives per unit of time is given by $\beta IS/N$. An infective is seek during a period of time after which he recovers and becomes a new susceptible ($\gamma=-\dot{I}/I$ is the recovery rate in absence of new contamination, a sort of exponential decay rate from infection). The recovery rate decreases with the virulence and, so, with the loss of biodiversity. Notice that the SIS model postulates that the infection does not confer immunity. In the following, for the sake of simplicity, we will omit the time argument t.

The evolution of S and I over time is simply given by:

$$\dot{S}=-\beta\frac{I}{N}S+\gamma I \tag{1}$$

$$\dot{I}=\beta\frac{I}{N}S-\gamma I \tag{2}$$

In an oversimplified world with no births, no deaths, no migrations, the population remains constant over time. Therefore, $N=S+I$ gives $\dot{S}+\dot{I}=0$

the green-tax rate and the pollution level at the steady state while, in the seminal contribution by Sinn (2008), this paradox is a positive relation along the transition path.

and equation (1) becomes:

$$\dot{s} = (1 - s)(\gamma - \beta s) \tag{3}$$

As in Goenka et al. (2014), we assume that the labor force (L) consists only of healthy people: $L = S$. Since $l = L/N \leq 1$, l inherits the dynamics of s:

$$\dot{l} = (1 - l)(\gamma - \beta l) \tag{4}$$

We can see that (4) exhibits two steady state: $l = 1$ and $l = \gamma/\beta$ with $\gamma < \beta$. The first one is called *disease-free* because the disease disappears while the other is called *endemic* because the disease persists.

Let B denotes biodiversity (species). Following Keesing et al. (2010) or Johnson and Thieltges (2010) among others, a biodiversity lost promotes infectious diseases (*dilution effect*). That is, $\beta = \beta(B)$ and $\gamma = \gamma(B)$ according to the following assumption.

Assumption 1 $\beta'(B) < 0$ *and* $\gamma'(B) > 0$ *with* $\lim_{B\to 0}\beta(B) = \infty$, $\lim_{B\to\infty}\beta(B) = 0$, $\lim_{B\to 0}\gamma(B) = 0$ *and* $\lim_{B\to\infty}\gamma(B) = \infty$.

Definition 1 (dilution effect) *The dilution effect is given by*

$$d(B) \equiv \varepsilon_\gamma(B) - \varepsilon_\beta(B) > 0$$

where

$$\varepsilon_\beta(B) \equiv \frac{B\beta'(B)}{\beta(B)} < 0 \text{ and } \varepsilon_\gamma(B) \equiv \frac{B\gamma'(B)}{\gamma(B)} > 0$$

are the first-order elasticities.

As we will see later, this effect has dramatic consequences on epidemiological and economic dynamics.

Isoelastic case

If $\beta(B) \equiv A_\beta B^{\varepsilon_\beta}$ and $\gamma(B) \equiv A_\gamma B^{\varepsilon_\gamma}$, the dilution effect is constant: $d \equiv \varepsilon_\gamma - \varepsilon_\beta > 0$.

2.2 Preferences

The household earns a capital income rh and a labor income ω, where r and h denote respectively the real interest rate and the individual wealth at time t. Income is consumed and saved/invested according to the budget constraint:

$$\dot{h} \leq (r - \delta)h + \bar{\omega} - c \tag{5}$$

In this model, healthy people work while sick people don't. However, for simplicity, we assume a perfect social security, that is a full unemployment insurance in the case of illness. Healthy and sick agents earn the same labor income $\bar{\omega}$. L healthy people supply one unit of labor at a wage w. Under a balanced-budget rule for social security, we obtain $\bar{\omega}N = wL$. Therefore, $\bar{\omega} = wl$.

Gross investments include the capital depreciation at the rate δ. For simplicity, the population of consumers-workers is normalized to unity: $N = 1$. Such a normalization implies $L = Nl = l$, $K = Nh = h$ and $h = K/N = kl$.

Let $u(c, B)$ be the utility function of the representative household. We assume that biodiversity affects marginal utility of consumption ($u_{cB} \neq 0$). If biodiversity increases the consumption demand, biodiversity and consumption are complement ($u_{cB} > 0$): it is the case when households like to consume in a pleasant environment, in presence of a large biodiversity. Conversely, if biodiversity lowers consumption demand, then biodiversity and consumption are substitutable: in this case, the household compensates the utility loss due to a loss of biodiversity by increasing her consumption demand ($u_{cB} < 0$). For now, we do not impose any restriction.

Assumption 2 *Preferences are rationalized by a non-separable utility function $u(c, B)$. First and second-order restrictions hold on the sign of derivatives: $u_c > 0$, $u_B > 0$ and $u_{cc} < 0$, jointly with the limit conditions: $\lim_{c \to 0^+} u_c = \infty$ and $\lim_{c \to +\infty} u_c = 0$.*

We introduce the second-order elasticities:

$$\begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cB} \\ \varepsilon_{Bc} & \varepsilon_{BB} \end{bmatrix} \equiv \begin{bmatrix} \frac{cu_{cc}}{u_c} & \frac{Bu_{cB}}{u_c} \\ \frac{cu_{Bc}}{u_B} & \frac{Bu_{BB}}{u_B} \end{bmatrix} \tag{6}$$

$-1/\varepsilon_{cc}$ represents the intertemporal elasticity of substitution in consumption while ε_{Bc} captures the effect of biodiversity on the marginal utility of consumption. Typically, if $\varepsilon_{cB} > 0$ (< 0), biodiversity and consumption are complement (substitute) for households.

The illness lowers labor supply and the individual income in turn. The agent maximizes the intertemporal utility function

$$\int_0^\infty e^{-\theta t} u(c, B)\, dt \tag{7}$$

under the budget constraint (5), where $\theta > 0$ is the rate of time preference.

Proposition 2 *The first-order conditions of the consumer's program are given by a static relation*

$$\mu = u_c(c, B) \tag{8}$$

a dynamic Euler equation and the budget constraint (1), now binding:

$$\dot{\mu} = \mu(\theta + \delta - r) \tag{9}$$

$$\dot{h} = (r - \delta) h + wl - c \tag{10}$$

jointly with the transversality condition $\lim_{t \to \infty} e^{-\theta t} \mu(t) h(t) = 0$. μ denotes the multiplier associated to the budget constraint.

Applying the Implicit Function Theorem to the static relation $\mu = u_c(c, B)$, we obtain the consumption function $c \equiv c(\mu, B)$ with elasticities

$$\frac{\mu}{c}\frac{dc}{d\mu} = \frac{1}{\varepsilon_{cc}} < 0 \text{ and } \frac{B}{c}\frac{dc}{dB} = -\frac{\varepsilon_{Bc}}{\varepsilon_{cc}} \tag{11}$$

2.3 Firms

The firm chooses the amount of capital and labor to maximize the profit taking as given the real interest rate r as well as the wage rate w. In addition, the government levies a proportional tax $\tau \in (0,1)$ on polluting production $F(k_j, l_j)$ of firm j to finance depollution expenditures.

Assumption 3 *The production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is C^2, homogeneous of degree one, strictly increasing and concave. Inada conditions hold.*

The profit maximization $\max_{K_j,N_j} [F(K_j, L_j) - rK_j - wL_j - \tau F(K_j, L_j)]$ entails the following first-order conditions:

$$r = (1-\tau) f'(k_j) \text{ and } w = (1-\tau)[f(k_j) - k_j f'(k_j)]$$

where $k_j \equiv K_j/L_j$ is the capital intensity and $f(k_j) \equiv F(k_j, 1)$ the average productivity of the firm j.

All the firms share the same technology and address the same demand for capital.

Corollary 3 *Let $k \equiv K/L$ with $K \equiv \sum_{j=1}^{J} K_j$ and $L \equiv \sum_{j=1}^{J} L_j$. In aggregate terms, $Y = F(K, L)$ and profit maximization yields*

$$r = (1-\tau)\rho(k) \text{ and } w = (1-\tau)\omega(k) \tag{12}$$

with $\rho(k) \equiv f'(k)$ and $\omega(k) \equiv f(k) - kf'(k)$.

We introduce the capital share in total disposable income and the elasticity of capital-labor substitution:

$$\alpha(k) \equiv \frac{rk}{(1-\tau) f(k)} = \frac{kf'(k)}{f(k)} \text{ and } \sigma(k) = \alpha(k)\frac{\omega(k)}{k\omega'(k)}$$

In addition, we determine the elasticities of factor prices:

$$\frac{k\rho'(k)}{\rho(k)} = -\frac{1-\alpha(k)}{\sigma(k)} \text{ and } \frac{k\omega'(k)}{\omega(k)} = \frac{\alpha(k)}{\sigma(k)}$$

2.4 Government

The government uses all the tax revenues to finance depollution expenditures (G) according to a balanced budget rule:

$$G = \tau F(K, L) \tag{13}$$

2.5 Biodiversity

The biodiversity is viewed as a renewable natural resource. Following Ayong Le Kama (2001) and Wirl (2004) the dynamics of natural resource is given by

$$\dot{B} = g(B) - P \tag{14}$$

where $g(B)$ and P represent the reproduction function and the pollution level respectively. In the following, we will refer to (14) as reproduction function in a broad sense.

Following Wirl (2004) and Bella (2010), we specify $g(B)$ as a Pearl-Verhulst logistic function: $g(B) \equiv B(1-B)$ with $0 < B < 1$.

Interestingly, since $g'(B) = 1 - 2B$, the maximal sustainable yield occurs at $B = 1/2$. Wirl (2004) has pointed out that limit cycles can occur if and only if $B < 1/2$ (the maximal sustainable yield) at the steady state.

To simplify the presentation, we assume as in Itaya (2008) or in Fernandez et al. (2012) that pollution is a flow coming from production activity**:**

$$P = aY - bG \tag{15}$$

where a and b capture respectively the environmental impact of production and the depollution efficacity.

Considering (13), (14) and (15), we find the natural resource accumulation law.

$$\dot{B} = B(1-B) - aF(K,L) + b\tau F(K,L) \tag{16}$$

A negative net pollution requires an additional assumption.

Assumption 4 *$a > b\tau$.*

3 Equilibrium

We normalize the population ($N = 1$) and we obtain the natural resource accumulation law:

$$\dot{B} = B(1-B) + (b\tau - a)\, lf(k)$$

At the equilibrium, all the markets clear. This leads to the following proposition.

Proposition 4 *Equilibrium dynamics are driven by a four-dimensional dynamic system:*

$$\dot{\mu} = \mu\left[\theta + \delta - (1-\tau)\rho(k)\right] \tag{17}$$

$$\dot{k} = \left[(1-\tau)\rho(k) - \delta\right]k + (1-\tau)\omega(k) - \frac{c(\mu,B)}{l} - kz(l,B) \tag{18}$$

$$\dot{l} = lz(l,B) \tag{19}$$

$$\dot{B} = B(1-B) + (b\tau - a)\, lf(k) \tag{20}$$

with

$$z(l,B) \equiv \left[\gamma(B) - \beta(B)\, l\right]\frac{1-l}{l}$$

jointly with the transversality condition.

We observe that the shadow price μ is a non-predetermined variable, while k, l and B are predetermined.

4 Steady state

At the steady state allthe variables remain constant: $\dot{\mu} = \dot{k} = \dot{l} = \dot{B} = 0$. From equation (17), we obtain the Modified Golden Rule (MGR):

$$\rho(k) = \frac{\theta + \delta}{1 - \tau} \tag{21}$$

Assumption 3 ensures the invertibility of ρ. The capital intensity at the steady state given by $k = \rho^{-1}((\theta + \delta)/(1 - \tau)) > 0$.

Focus now on equation (19). At the steady state, $z(l, B) = 0$, that is

$$(1 - l)[\gamma(B) - \beta(B) l] = 0$$

with solutions

$$l = 1 \text{ or } l = \frac{\gamma(B)}{\beta(B)}$$

We recover one of the main feature of the SIS model: two steady states coexist: $l = 1$ is the disease-free steady state, while $l = \gamma(B)/\beta(B)$ is the steady state with an endemic disease.

Since $z(l, B) = 0$ at the steady state, equation (18) gives simply the consumption level:

$$c = l[(1 - \tau) f(k) - \delta k] \tag{22}$$

We know that k is unique and positive at the steady state. Thus, given l, according to equation (22), there is a unique and positive value of c. Moreover, (8) implies that, given c and B, there is a unique and positive shadow price μ.

Finally, at the steady state, the natural resource accumulation law (20) becomes

$$B^2 - B + (a - b\tau) l f(k) = 0 \tag{23}$$

with roots

$$B_1 \equiv \frac{1}{2}\left[1 - \sqrt{1 - 4(a - b\tau) l f(k)}\right] \tag{24}$$

$$B_2 \equiv \frac{1}{2}\left[1 + \sqrt{1 - 4(a - b\tau) l f(k)}\right] \tag{25}$$

Given l, the existence and multiplicity of steady states depend on the existence of real roots B_1 and B_2. Compare now the disease-free and the endemic regime.

4.1 Disease-free steady state

At the disease-free steady state, the disease no longer exists and all the labor force is emplyed, that is $l = 1$.

Proposition 5 *Let* $l = 1$.

(1) If $1 - 4(a - b\tau) f(k) < 0$*, there are no steady states.*

(2) If $1 - 4(a - b\tau) f(k) > 0$*, there are two steady states with* $0 < B_1 < 1/2 < B_2 < 1$.

And if $1 - 4(a - b\tau) f(k) = 0$, $B_1 = B_2 = 1/2$.

In this case, the disease does not persist in the long run. In other words, the dilution effect has no effect on the steady state. The same case is considered in Bosi and Desmarchelier (2017b) and, for the sake of brevity, reader is referred to this paper for a dynamics analysis. The added value of the current paper concerns instead the other case: the endemic regime.

4.2 Endemic steady state

Focus on the endemic steady state:

$$l = \gamma(B)/\beta(B)$$

From (23), the stationary biodiversity level satisfies

$$\varphi(B) \equiv B(1-B)\frac{\beta(B)}{\gamma(B)} = (a - b\tau) f(k) > 0 \tag{26}$$

$\varphi(B) > 0$ requires $B \in (0, 1)$.

We notice that

$$\frac{B\varphi'(B)}{\varphi(B)} = \frac{1-2B}{1-B} - d(B) \tag{27}$$

where $d(B) \equiv \varepsilon_\gamma(B) - \varepsilon_\beta(B) > 0$ captures the dilution effect. The magnitude of dilution effect matters for the steady state multiplicity.

Isoelastic case

Let $\beta(B) \equiv A_\beta B^{\varepsilon_\beta}$ and $\gamma(B) \equiv A_\gamma B^{\varepsilon_\gamma}$ with a constant dilution effect $d \equiv \varepsilon_\gamma - \varepsilon_\beta > 0$ and

$$B^* \equiv \frac{1-d}{2-d} \text{ and } a^* \equiv b\tau + \frac{\varphi(B^*)}{f(k^*)} \tag{28}$$

where k^* is given by the MGR.

Proposition 6 (small dilution effect) *Focus on the isoelastic case with* $0 < d < 1$ *and consider the endemic steady state* $l = \gamma(B)/\beta(B)$.

(1) If $a < a^*$*, there are two steady states* B_3 *and* B_4 *such that* $0 < B_3 < B^* < B_4 < 1$.

(2) If $a = a^*$*, there is a unique steady state* B^*.

(3) If $a > a^*$*, there are no steady states.*

Proposition 7 (large dilution effect) *Focus on the isoelastic case with* $d > 1$ *and consider the endemic steady state* $l = \gamma(B)/\beta(B)$*. In this case, there exists a unique steady state.*

Propositions (6) and (7) highlight the role of dilution effect in the existence and the multiplicity of steady states through the reproduction function of biodiversity: under a low dilution effect ($0 < d < 1$), the maximal sustainable yield is given by $B^* \in (0, 1/2)$, while, under a large dilution effect ($d > 1$), $B^* \notin [0, 1]$ makes no longer sense.

The next section shows that the effect of the green tax on the endemic steady state precisely depends on the magnitude of the dilution effect.

4.3 Comparative statics

Focus on the endemic steady state.

Proposition 8 *Let Assumption 4 hold and $B \in (0, 1)$. Focus on the qualitative impact of τ on the endemic steady state ($l = \gamma/\beta$). We have*

$$\frac{\tau}{k}\frac{dk}{d\tau} < 0 \tag{29}$$

Moreover,
(1) if $0 < d < 1$,

$$\frac{\tau}{B}\frac{dB}{d\tau} > 0 \Leftrightarrow B > B^*$$
$$\frac{\tau}{l}\frac{dl}{d\tau} > 0 \Leftrightarrow B > B^*$$
$$\frac{\tau}{\mu}\frac{d\mu}{d\tau} > 0 \text{ if } (B < B^* \text{ and } d > -\frac{\varepsilon_{Bc}}{\varepsilon_{cc}}) \text{ or } (B > B^* \text{ and } d < -\frac{\varepsilon_{Bc}}{\varepsilon_{cc}})$$

(2) if $d > 1$,

$$\frac{\tau}{B}\frac{dB}{d\tau} > 0$$
$$\frac{\tau}{l}\frac{dl}{d\tau} > 0$$
$$\frac{\tau}{\mu}\frac{d\mu}{d\tau} > 0 \text{ if } d < -\frac{\varepsilon_{Bc}}{\varepsilon_{cc}}$$

Proposition 8 deserves some economic interpretations.

The critical value of the dilution effect remains 1 as in Propositions 6 and 7.

First, we observe that τ is levied on the production level. Thus, a higher green-tax rate reduces production and income, entailing a lower capital level at the end.

As seen above, a higher green tax rate always reduces the capital level in the long-run, that is, a higher τ always lowers the right-hand side of (26).

Let $0 < d < 1$ (low dilution effect).

If, at the steady state, the economy is located along the upward-sloping branch of φ (that is $B < B^*$), this entails a lower biodiversity level. Conversely, if the economy is located on the downward-sloping branch of φ (that is $B > B^*$), a higher tax level lowers the right-hand side of (26) and hence, increases the biodiversity level at the steady state. Therefore, the effect of τ on B depends upon the slope of the reproduction function. The fact that a higher green-tax rate impairs the biodiversity level (when $B < B^*$) is counter-intuitive and refers to the static Green Paradox introduced in Bosi and Desmarchelier (2017a and 2017b).

The impact of the green tax on the labor supply mimics that on the biodiversity level because of Assumption 1. Indeed, because of the dilution effect, a lower biodiversity level implies a more infective disease (a higher β jointly with a lower γ) which reduces the labor supply ($l = \gamma/\beta$).

Let $d > 1$ (high dilution effect).

In this case, φ is always a decreasing function of B (see the proof of Proposition 7). Hence, a higher tax rate lowers the right-hand side of (26) which always leads to a higher biodiversity level.

Summing up, we observe that the Green Paradox (that is the negative impact of the green tax on the biodiversity level) only occurs for low levels of biodiversity ($B < B^*$) and dilution effect ($0 < d < 1$).

It is worthy to notice that when the biodiversity becomes a prominent determinant of human health because of a sufficiently large dilution effect ($d > 1$), the unpleasant (static) Green Paradox is ruled out.

μ is a shadow price (marginal utility of consumption: $\mu = u_c(c, B)$). From an economic point of view, it is more interesting to characterize the impact of the green tax on consumption than on this unobservable variable.

We observe that

$$c(\tau) = \frac{\theta + [1 - \alpha(k(\tau))]\delta}{\alpha(k(\tau))} k(\tau) l(\tau) \tag{30}$$

In order to provide a clear-cut comparative statics, we focus on the Cobb-Douglas case. In this case, the capital share $\alpha(k)$ becomes a constant and (30) entails

$$\frac{\tau}{c}\frac{\partial c}{\partial \tau} = \frac{\tau}{k}\frac{\partial k}{\partial \tau} + \frac{\tau}{l}\frac{\partial l}{\partial \tau} \tag{31}$$

Therefore, the impact of the green tax on consumption can be disentangled in its effects on the production factors.

Proposition 9 *Consider the endemic steady state $l = \gamma/\beta$ with $a = b$ and $\sigma = 1$ (Cobb-Douglas technology).*

(1) If $0 < d < 1$ (low dilution effect),

$$\frac{\partial c}{\partial \tau} > 0 \text{ iff } B^* < B < \frac{1}{2}$$

(2) If $d > 1$ (high dilution effect),

$$\frac{\partial c}{\partial \tau} > 0 \text{ iff } B < 1/2$$

The last proposition shows that the effect of τ on the consumption demand is ambiguous in the long run. This deserves some economic interpretations.

Consider the case of a low dilution effect ($0 < d < 1$) and suppose, for simplicity, no capital depreciation. As seen before, a higher green-tax rate always lowers the capital level (see (29)). In addition, since the economy is located along the increasing branch of the reproduction function ($B < B^*$), this implies a drop in the biodiversity level, rendering the disease more infective which lowers the labor supply. Focus on expression (22): $c = l(1-\tau) f(k)$. Since τ increases and k and l decrease, c decreases. Now, assume that the economy is located along the decreasing branch of the reproduction function ($B > B^*$). In this case, a higher green-tax rate implies more biodiversity making the disease less infective (dilution effect) and raising the labor supply. In contrast, as above, the capital intensity lowers. Then, l increases, while $(1-\tau) f(k)$ decreases. The total impact of τ on $c = l(1-\tau) f(k)$ is ambiguous. In order to know whether the increase in the labor supply dominates the decrease of $(1-\tau) f(k)$, we have to focus on the elasticity (27) of the reproduction function φ. $B > B^*$ jointly with $0 < d < 1$ implies $\varphi'(B) < 0$. The slope of φ becomes flatter when $B < 1/2$ and steeper when $B > 1/2$. In other words, an increase in the green-tax rate has a larger effect on biodiversity and labor supply when $B < 1/2$. Therefore, since $B > B^*$, if $B < 1/2$, the increase of labor supply l dominates the drop of $(1-\tau) f(k)$ which implies a higher consumption level in the long run. Conversely, if $B > 1/2$, the increase in labor income is dominated and consumption lowers in the long run.

Similar interpretations hold in the case of a strong dilution effect ($d > 1$).

5 Local dynamics around the endemic steady state

The dilution effect has short and long-run consequences. In the long run, it affects the steady state as seen above. In the short run, the stability properties of equilibrium also depend on the dilution effect. Let us study how it affects the local dynamics around the steady state.

The disease-free regime is the same as the one considered by Bosi and Desmarchelier (2017b). For brevity's sake, the reader is referred to their paper also for the local dynamics.

The novelty of the current paper are the endemic steady state and the local dynamics around. To study the equilibrium transition, we linearize the dynamic system (17)-(20) around the endemic steady state $l = \gamma(B)/\beta(B)$.

Setting

$$\begin{aligned} q &\equiv \frac{c}{kl} = \frac{\theta + (1-\alpha)\delta}{\alpha} \\ m &\equiv \beta(1-l) \\ n &\equiv (\theta+\delta)\frac{1-\alpha}{\sigma} \end{aligned}$$

and noticing that $\omega(k)/[k\rho(k)] = [1-\alpha(k)]/\alpha(k)$, we get the Jacobian matrix

$$J = \begin{bmatrix} 0 & n\frac{\mu}{k} & 0 & 0 \\ -\frac{q}{\varepsilon_{cc}}\frac{k}{\mu} & \theta & (m+q)\frac{k}{l} & \frac{k}{B}\left(q\frac{\varepsilon_{Bc}}{\varepsilon_{cc}} - md\right) \\ 0 & 0 & -m & \frac{l}{B}md \\ 0 & \alpha\frac{B}{k}(B-1) & \frac{B}{l}(B-1) & 1-2B \end{bmatrix}$$

where $d > 0$ is the dilution effect.

To study the local dynamics of this four-dimensional system, we apply the methodology developped by Bosi and Desmarchelier (2017c) and based on the sums of principal minors of the Jacobian.

The characteristic polynomial is given by

$$\begin{aligned} P(\lambda) &\equiv (\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)(\lambda-\lambda_4) \\ &= \lambda^4 - T\lambda^3 + S_2\lambda^2 - S_3\lambda + D \end{aligned}$$

where

$$S_1 = \lambda_1+\lambda_2+\lambda_3+\lambda_4 = T \quad (32)$$

$$S_2 = \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4 \quad (33)$$

$$S_3 = \lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4+\lambda_1\lambda_2\lambda_3 \quad (34)$$

$$S_4 = \lambda_1\lambda_2\lambda_3\lambda_4 = D \quad (35)$$

T and D denote the trace and the determinant of J while S_2 and S_3 represent the sum of principal minors of order two and three.

In our model,

$$\begin{aligned} T &= \theta - m - 2B + 1 \\ S_2 &= [2B-1+(1-B)d]m + (1-2B)\theta - (1-B)dm\alpha + q\alpha\frac{\varepsilon_{Bc}}{\varepsilon_{cc}}(1-B) - m\theta + \frac{nq}{\varepsilon_{cc}} \\ S_3 &= \frac{1}{\varepsilon_{cc}}[(1-2B-m)nq + (B-1)mq\alpha\varepsilon_{Bc}] + (B-1)dmq\alpha + [2B-1+(1-B)d]m\theta \\ D &= \frac{mnq}{\varepsilon_{cc}}[2B-1+(1-B)d] \end{aligned} \quad (36)$$

Lemma 10 *If $0 < d < 1$ (low dilution effect), $D < 0$ if and only if $B > B^* \in (0,1)$.*

Lemma 11 *If $d > 1$ (large dilution effect), $D < 0$.*

Proposition 12 *There is no room for saddle-node, Bogdanov-Takens, Gavrilov-Guckenheimer and double-Hopf bifurcations if (1) $0 < d < 1$ (low dilution effect) and $B > B^*$ or (2) $d > 1$ (large dilution effect). In both these cases the equilibrium is locally unique (determinate).*

Proposition 13 *Focus on the case of a low dilution effect ($0 < d < 1$). A saddle-node bifurcation occurs if and only if $a = a^*$ (B_3 and B_4 coalesce with $B_3 = B_4 = B^*$).*

In the sequel, we will focus on the isoelastic case:

$$u(c,B) = \frac{(cB^{\eta})^{1-\varepsilon}}{1-\varepsilon} \tag{37}$$

with

$$\begin{bmatrix} \varepsilon_{cc} & \varepsilon_{Bc} \\ \varepsilon_{cB} & \varepsilon_{BB} \end{bmatrix} \equiv \begin{bmatrix} \frac{cu_{cc}}{u_c} & \frac{Bu_{Bc}}{u_c} \\ \frac{cu_{cB}}{u_B} & \frac{Bu_{BB}}{u_B} \end{bmatrix} = \begin{bmatrix} -\varepsilon & \eta(1-\varepsilon) \\ 1-\varepsilon & \eta(1-\varepsilon)-1 \end{bmatrix}$$

Let $\varepsilon_1 \equiv \varepsilon_{cc} = -\varepsilon < 0$ and $\varepsilon_2 \equiv \varepsilon_{Bc} = \eta(1-\varepsilon)$.

We observe that ε_2 captures the impact of biodiversity on consumption demand and does not affect T and D but only S_2 and S_3. For these reasons, we choose ε_2 as bifurcation parameter. ε_2 can be positive or negative. According to (11), consumption and biodiversity are complements ($\varepsilon_2 > 0$) if and only if $0 < \varepsilon < 1$. Conversely, they are substitutable in the household's preferences ($\varepsilon_2 < 0$) if and only if $\varepsilon > 1$.

Let

$$\varepsilon_H \equiv \varepsilon_1 \frac{z_3 - T\frac{Z+\sqrt{Z^2-4mD(m+T)}}{2(m+T)}}{m\alpha q(1-B)} \tag{38}$$

with

$$z_3 \equiv qm\alpha d(B-1) + (T-\theta)\frac{nq}{\varepsilon_1} + \theta D\frac{\varepsilon_1}{nq}$$
$$Z \equiv m\left[m\alpha d(B-1) + \frac{nq}{\varepsilon_1} + D\frac{\varepsilon_1}{nq} + (T-\theta)\theta\right] + z_3$$

Notice that ε_H is independent of ε_2.

Proposition 14 *Let $B < (1+\theta)/2$. If*
(1) $0 < d < 1$ (low dilution effect) and $B > B^*$ (large biodiversity) or
(2) $d > 1$ (large dilution effect),
then a limit cycle occur near the endemic steady state if and only if $\varepsilon_2 = \varepsilon_H$.

The existence of limit cycles depends on the sign of ε_H. According to (38), this information is not analytically available and we need to perform a computer simulation to have an economic intuition of such fluctuations.

Proposition 15 *Let $0 < d < 1$ (low dilution effect). If $a = a^*$ jointly with*

$$\varepsilon_2 = \varepsilon_{BT} \equiv \varepsilon_1 \frac{z_3}{mq\alpha(1-B)}$$

then a Bogdanov-Takens bifurcation occurs.

Few economic models display a Bogdanov-Takens bifurcation. This bifurcation arises when the conditions for the Hopf and the saddle-node bifurcation are jointly satisfied. Kuznetsov et al. (2014) show that, in the case of a Bogdanov-Takens bifurcation, the limit cycle disappears. More precisely, they show that the limit cycle and the saddle point collide giving rise to a parasitic loop at the bifurcation point.

Proposition 16 *There is no room for a Gavrilov-Guckenheimer bifurcation.*

Proposition 17 *There is no room for a double-Hopf bifurcation.*

6 Simulations

We have characterized the occurrence of local bifurcations of codimension one and two. Now, we provide a numerical illustration of the analytical results provided in the previous section. Proposition 14 shows that a Hopf bifurcation may occur under low and high dilution effect. Since we are especially interested in the Bogdanov-Takens bifurcation, which arises only under a low dilution effect, for brevity's sake, we will focus only on the case of low dilution effect.

We reconsider the isoelastic utility (37) and the isoelastic functions $\beta(B) \equiv A_\beta B^{\varepsilon_\beta}$ and $\gamma(B) \equiv A_\gamma B^{\varepsilon_\gamma}$ with $\varepsilon_\beta < 0$ and $\varepsilon_\gamma > 0$. For simplicity, we consider also a Cobb-Douglas production function: $f(k) = Ak^\alpha$. According to the MGR (21), we find the stationary capital level:

$$k^* = \left[\frac{\alpha A(1-\tau)}{\theta+\delta}\right]^{\frac{1}{1-\alpha}}$$

Replacing k^* in (41) (see the Appendix), we obtain the biodiversity level as solution of

$$(1-B)B^{1-d} = A(a-b\tau)\frac{A_\gamma}{A_\beta}\left[\frac{\alpha A(1-\tau)}{\theta+\delta}\right]^{\frac{\alpha}{1-\alpha}} \tag{39}$$

According to Proposition 6, equation (39) possesses two solutions if and only if

$$a < a^* = b\tau + \frac{A_\beta}{A_\gamma}\frac{\left(1-\frac{1-d}{2-d}\right)\left(\frac{1-d}{2-d}\right)^{1-d}}{A\left[\frac{\alpha A(1-\tau)}{\theta+\delta}\right]^{\frac{\alpha}{1-\alpha}}}$$

where $d \equiv \varepsilon_\gamma - \varepsilon_\beta > 0$.

Parameters	A	A_γ	A_β	ε	ε_γ	ε_β	b	α	δ	θ	τ
Values	1	1	1	0.5	0.25	-0.25	0.0015	0.33	0.025	0.01	0.002

(40)

α, δ and θ are set at their usual quarterly values while b and τ[3] are fixed as in Bosi and Desmarchelier (2016b). We obtain also $d \equiv \varepsilon_\gamma - \varepsilon_\beta = 1/2 \in (0,1)$.

Calibration (40) yields $a^* = 0.1276$. We fix $a = 0.127 < a^*$ and we solve (39) for B. We obtain two roots: $B = 0.2968$ and $B = 0.3713$. We observe that the

[3] In this economy, τ captures the public air protection expenditures. Indeed $G/Y = \tau Y/Y = \tau = 0.2\%$. According to the OECD Environmental Performance reviews for France (2016) (p. 149), the public air protection expenditures amount to less than 5 billion Euros (2013 prices), which represents less than 0.25% of France GDP. Our calibration for τ is then in accordance with data.

necessary (but not sufficient) condition for the occurrence of a Hopf bifurcation in Proposition 14 is satisfied: $B^{*}=0.333<B=0.3713<(1+\theta)/2=0.505$.

Focusing on $B=0.3713$, we compute η such that $\varepsilon_2=\eta(1-\varepsilon)=\varepsilon_H$. We obtain $\eta_H=0.27569>0$. Therefore, under calibration (40), when $a=0.127$, the system undergoes a Hopf bifurcation at η_H and experiences a limit cycle near $B=0.3713$.

Summing up, under calibration (40), we get a Hopf bifurcation at $a=0.127$ and a saddle-node bifurcation at $a=0.1276$ near the higher endemic steady state ($B>B^{*}$ with $l=\gamma/\beta$ and $0<d<1$).

After having seen how to calibrate the model to find a Hopf bifurcation, we deepen our approach considering an equilibrium continuation.[4] We aim to plot the Hopf bifurcation curve and the saddle-node bifurcation curve in the (a,η)-plane and to show the occurrence of the Bogdanov-Takens bifurcation (according to Proposition 15) when these bifurcation curves meet each others. We will refer to Figure 1 where LP, H, BT and GH stand for Limit Point (elementary saddle-node), Hopf, Bogdanov-Takens and Generalized Hopf. These points are computed and represented by MATCONT when the corresponding bifurcations occur near the steady state.

To perform the equilibrium continuation using MATCONT, we consider first the bifurcation of codimension one (Hopf and the saddle-node). We fix $\eta=0.27569$ as above and we set an arbitrary value for a at which no bifurcation occurs near the endemic steady state: $a=0.1268\equiv a_0$. In this case, under calibration (40), the endemic steady state becomes

$$(\mu,k,l,B)=(0.7354,28.385671,0.61423845,0.37728887)$$

Let MATCONT raise a from $a_0\equiv 0.1268$ to $a^{*}=0.1276$ keeping $\eta=0.27569$ as constant. In Figure1, we are moving to the right along the horizontal line HLP. MATCONT detects a Hopf bifurcation (H) at $a=a_H=0.127$ and a saddle-node bifurcation (LP) at $a=a^{*}$.

[4] To this purpose, we use the MATCONT package for MATLAB.

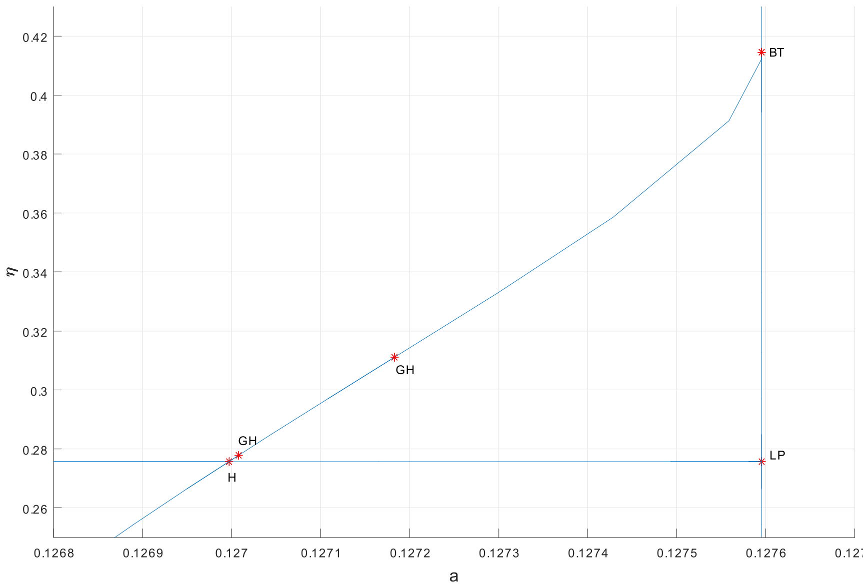

Fig 1. The equilibrium continuation

Now, let us move from H to BT along the locus of all the Hopf bifurcations: for any a, there is a Hopf critical value

$$\eta_H(a) \equiv -\frac{\varepsilon}{1-\varepsilon}\frac{z_3(a) - T(a)\frac{Z(a)+\sqrt{Z(a)^2-4m(a)D(a)[m(a)+T(a)]}}{2[m(a)+T(a)]}}{m(a)\,\alpha q\,[1-B(a)]}$$

The Hopf-bifurcation curve $\{(a, \eta_H(a))\}$ is precisely represented in Figure 1 by the curve HBT.

For any η, the elementary saddle-node bifurcation value for a is $a^* = 0.1276$ (the line $LPBT$ is vertical because a^* does not depend on η). In particular, the Limit Point corresponding to $\eta = 0.275\,69$ is $LP = (0.1276, 0.275\,69)$. The vertical line $LPBT$ represents a third equilibrium continuation, the set of all the pairs $(a, \eta) = (a^*, \eta)$ for which a saddle-node bifurcation occurs.

To sum up, increasing a from $a_0 = 0.1268$ to $a^* = 0.1276$, we obtain all the Hopf bifurcations along the curve $HBT \equiv \{(a, \eta_H(a))\}_{a\in[a_0,a^*]}$ going from H to BT. Moreover, in the range $[a_0, a^*) \ni a$, we find two distinct steady states. When a attains the maximal value a^* these two steady states coalesce and the Hopf bifurcation point $(a, \eta_H(a))$ reaches the ending point BT along the curve HBT while the economy experiences a Bogdanov-Takens bifurcation. Indeed, a Bogdanov-Takens bifurcation generically arises when a Hopf bifurcation curve crosses a locus of saddle-node bifurcations.

Along the locus of Hopf bifurcations, two Generalized Hopf (Bautin) bifurcations also appear. A Hopf bifurcation can be subcritical or supercritical, leading respectively to an unstable or stable limit cycle. The Generalized Hopf bifurcation point implies a change in the stability of the limit cycle arising near the steady state, that is, the bifurcation from subcritical becomes supercritical or viceversa. If the first Lyapunov coefficient (l_1) is negative (positive), the bifurcation is said to be supercritical (subcritical), leading to a stable (unstable) limit cycle near the steady state. At the Generalized Hopf bifurcation point, l_1 vanishes.

Let us explain the relation between a double-Hopf and a generalized Hopf bifurcation.[5] At the double-Hopf bifurcation, two limit cycles emerge simultaneously. The interaction between these two limit cycles can produce a wide range of dynamics depending upon higher-order terms of the Taylor series, such as a torus or local chaos. Concerning the Jacobian matrix, a double pair of purely imaginary eigenvalues appear at the double-Hopf bifurcation. In the case of a generalized Hopf bifurcation, the arising limit cycle is unique, as for a standard Hopf bifurcation: the Jacobian possesses a single pair of purely imaginary eigenvalues. The distinction between a standard or a generalized Hopf bifurcation rests on the value of the first-order Lyapunov coefficient and, thus, on higher-order terms of the Taylor representation of the dynamical system. Indeed, at the generalized Hopf bifurcation, the first Lyapunov coefficient is equal to zero which means a change of stability for the limit cycle.

At the Hopf bifurcation point (H), the steady state is given by:

$$(\mu, k, l, B) = (0.736723, 28.38567, 0.609338, 0.371292)$$

with eigenvalues:

$$\lambda_1 = -0.34187,\ \lambda_2 = 0.108813 \text{ and } \lambda_3 = 0.0539646i = -\lambda_4$$

In order to visualize the limit cycle arising at the Hopf bifurcation point, we project the four-dimensional dynamics on a three-dimensional space. Since the shadow price μ is not directly observable variable, we prefer to represent the trajectory in the (k, l, B)-space.

The corresponding first Lyapunov coefficient is given by $l_1 = 6.182045 * 10^{-5} > 0$. Its positivity means that the Hopf bifurcation is subcritical, that is the limit cycle arising near the steady state is unstable (Fig. 2).

[5] The reader is referred to pages 307 and 349 in Kuznetsov (1998) for the generalized Hopf bifurcation and the double-Hopf bifurcation respectively.

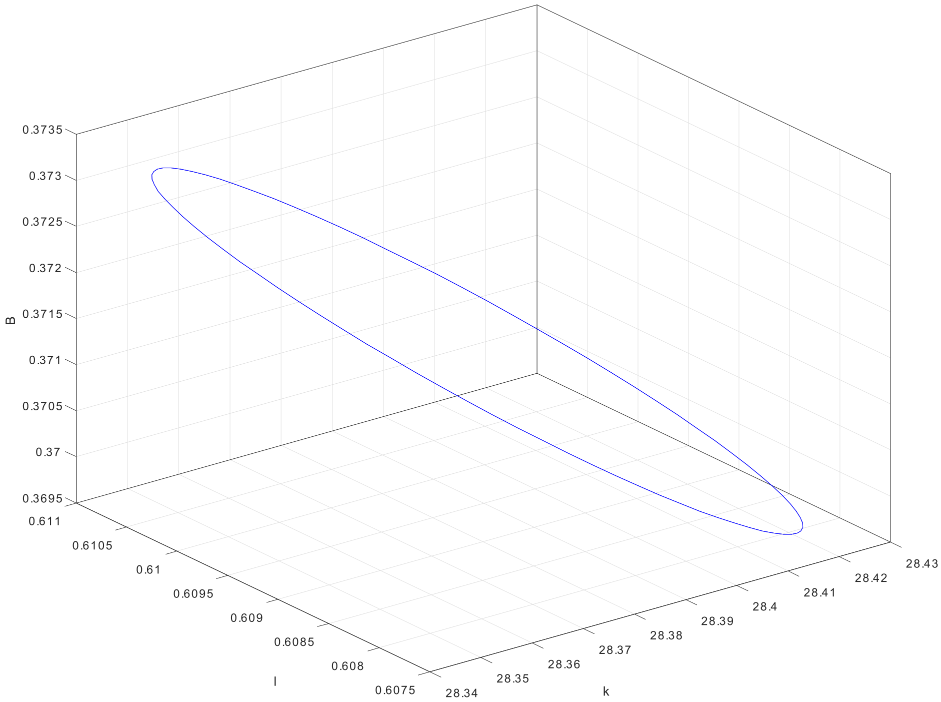

Fig. 2. The unstable limit cycle

According to (40), we obtain a positive value of η_H with $0 < \varepsilon < 1$ or, equivalently, $\varepsilon_2 > 0$ which means that complementarity between consumption and biodiversity is needed to generate a limit cycle around the endemic steady state. We can interpret the role of complementarity as follows.

Let the economy be at the steady state today and assume an exogenous rise in the pollution level. (14) implies a drop in biodiversity with two consequences: (1) a lower labor supply ($l = \gamma(B)/\beta(B)$ decreases) and (2) a lower consumption demand due to complementarity ($\mu = u_c(c, B)$ decreases). The Euler equation (intertemporal consumption smoothing) implies $\dot{\mu}/\mu = \theta + \delta - (1-\tau)\rho(k) < 0$. Thus, $\rho(k)$ increases from the Modified Golden Rule $(\theta + \delta)/(1-\tau)$ today to the new transition value tomorrow and, since ρ is a decreasing function, the capital intensity k lowers and the average productivity $f(k)$ as well. Pollution is given by $P = (a - b\tau) l f(k)$. Hence, under Assumption 4, the drops in labor supply l and in productivity $f(k)$ entail a lower pollution level. Thus, a higher pollution today entails a weaker pollution tomorrow giving rise to an endogenous fluctuation.

At the saddle-node bifurcation (LP), the steady state becomes:

$$(\mu, k, l, B) = (0.745\,69, 28.38567, 0.577347, 0.333333)$$

with eigenvalues:

$$\lambda_1 = -0.401245,\ \lambda_2 = 0,\ \lambda_3 = 0.0204607 \text{ and } \lambda_4 = 0.16789$$

At the Bogdanov-Takens bifurcation (BT) when $a = a^* = 0.1276$ jointly with $\eta = \eta_{BT} = 0.414576$, the steady state becomes:

$$(\mu, k, l, B) = (0.690915, 28.385671, 0.577350, 0.333333)$$

with eigenvalues:

$$\lambda_1 = -0.399\,61,\ \lambda_2 = \lambda_3 = 0, \text{ and } \lambda_4 = 0.186\,71$$

The Bogdanov-Takens bifurcation occurs when conditions for the elementary saddle-node bifurcation and for the Hopf bifurcation meet each other.

As in Kuznetsov et al. (2014), at the Bogdanov-Takens point, the orbit describes a parasitic loop near the saddle-point (Fig. 3). The parasitic loop typically arises when the limit cycle and the saddle-point coalesce.

As above, to represent the trajectory, we project the four-dimensional dynamics on the three-dimensional (μ, l, B)-space, where the parasitic loop appears (Fig. 3).

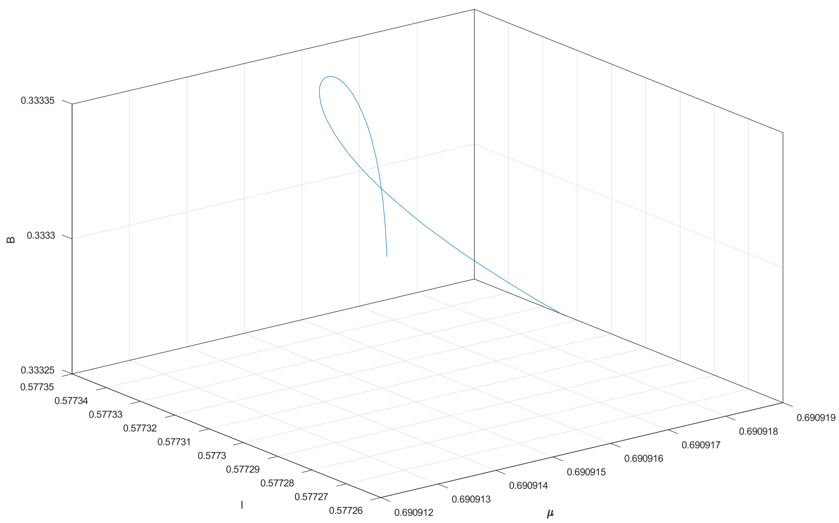

Fig. 3. The parasitic loop

At the Generalized-Hopf bifurcations (GH), we obtain:

parameters	steady state	eigenvalues	l_2
$a = 0.12700794$ $\eta = 0.2778149$	$\mu = 0.73602393$ $k = 28.38567$ $l = 0.60905296$ $B = 0.37094551$	$\lambda_1 = -0.342374$ $\lambda_2 = 0.109537$ $\lambda_3 = -0.0535261i$ $\lambda_4 = 0.0535261i$	$2.61837 * 10^{-3}$
$a = 0.12718298$ $\eta = 0.31106998$	$\mu = 0.7251378$ $k = 28.38567$ $l = 0.60395327$ $B = 0.36475955$	$\lambda_1 = -0.351468$ $\lambda_2 = 0.122331$ $\lambda_3 = -0.0460886i$ $\lambda_4 = 0.0460886i$	$1.744669 * 10^{-2}$

A Generalized-Hopf bifurcation implies a change in the stability of the limit cycle arising through the Hopf bifurcation. Typically, such a bifurcation occurs when the first Lyapunov coefficient vanishes. This phenomenon cannot be detected through a simple analysis of the eigenvalues. According to Kuznetsov (1998), a GH bifurcation is non-degenerated bifurcation if the second-order Lyapunov coefficient is different from zero ($l_2 \neq 0$). It is the case under our calibration for both the GH bifurcations.

7 Conclusion

We have provided a unified framework at the crossroad of economics, ecology and epidemiology, and studied how the negative relation between biodiversity and disease transmission (the so-called dilution effect) affects the economy in the long and the short run. More precisely, we have embedded a SIS model into a Ramsey model where a pollution externality coming from production impairs a biodiversity measure. For the sake of simplicity, we have assimilated biodiversity to a renewable resource and introduced a two-sided dilution effect assuming that both the probability to become ill and the recovery rate from the infectious disease depend on the biodiversity level. To complete the model, we have considered a proportional tax levied on production at firm level to finance depollution.

In long run, we have recovered a standard feature of the SIS model: a disease-free regime coexists with an endemic one. In the endemic case, the number of steady states depends on the magnitude of the dilution effect. Indeed, two steady states can coexist under a low dilution effect (with high and low biodiversity respectively). Conversely, under a large dilution effect, the steady state is always unique. Moreover, we have highlighted a kind of green paradox in the endemic regime: under a low dilution effect, a higher green-tax rate always impairs biodiversity at the low steady state. This counter-intuitive result is comparable to the static green paradox considered in Bosi and Desmarchelier (2017a). Conversely, the green paradox is over under a large dilution effect.

In the short run, limit cycles can arise under both the low and the high dilution effect through a Hopf bifurcation near the steady state. This happens in the endemic case when preferences exhibit a complementarity between biodiversity and consumption. Conversely, both Bogdanov-Takens and generalized Hopf bifurcations require a low dilution effect as a necessary condition.

8 Appendix

Proof of Proposition 2

The agent maximizes the intertemporal utility function (7) under the budget constraint (5). Setting the Hamiltonian $H = e^{-\theta t} u(c, B) + \lambda [(r - \delta) h + \bar{\omega} - c]$, deriving the first-order conditions $\partial H / \partial c = 0$, $\partial H / \partial h = -\dot{\lambda}$ and $\partial H / \partial \mu = \dot{h}$, and defining $\mu \equiv \lambda e^{\theta t}$, we get (8), (9) and (10). ■

Proof of Proposition 4

Consider (4), (12), (16) and Proposition 2. ■

Proof of Proposition 5

Consider (24) and (25) with $l = 1$. ■

Proof of Proposition 6

The steady state equation (26) becomes

$$\varphi(B) = \frac{A_\beta}{A_\gamma} (1 - B) B^{1-d} = (a - b\tau) f(k^*) \tag{41}$$

We observe that φ is a concave function with $B^* = \arg\max \varphi$ and $\varphi(0) = \varphi(1) = 0$. Then, $\varphi(B^*) > (a - b\tau) f(k^*)$ or, equivalently, $a < a^*$ implies (1). $\varphi(B^*) = (a - b\tau) f(k^*)$ or, equivalently, $a = a^*$ implies (2). Finally, $\varphi(B^*) < (a - b\tau) f(k^*)$ or, equivalently, $a > a^*$ implies (3). ■

Proof of Proposition 7

Reconsider (41). If $d > 1$, $\varphi'(B) < 0$ for any $B \in (0, 1)$. Moreover, $\lim_{B \to 0} \varphi(B) = +\infty$ and $\lim_{B \to 1} \varphi(B) = 0$. The continuity of φ implies that a steady state exists and it is unique. ■

Proof of Proposition 8

We differentiate system (20) and we obtain

$$\begin{bmatrix} \frac{\tau}{\mu}\frac{d\mu}{d\tau} \\ \frac{\tau}{k}\frac{dk}{d\tau} \\ \frac{\tau}{l}\frac{dl}{d\tau} \\ \frac{\tau}{B}\frac{dB}{d\tau} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1-\alpha}{\sigma} & 0 & 0 \\ -\frac{\phi}{\varepsilon_{cc}} & \theta & \phi + \beta(1-l) & \phi\frac{\varepsilon_{Bc}}{\varepsilon_{cc}} - \gamma d \frac{1-l}{l} \\ 0 & 0 & 1 & -d \\ 0 & \alpha & 1 & -\frac{1-2B}{1-B} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\tau}{1-\tau} \\ \frac{\theta+\delta}{\alpha}\frac{\tau}{1-\tau} \\ 0 \\ \frac{b\tau}{a-b\tau} \end{bmatrix}$$

where

$$\phi \equiv \frac{c}{kl} = \frac{\theta + (1-\alpha)\delta}{\alpha}$$

that is

$$\left[\begin{array}{c} \frac{\tau}{\mu}\frac{d\mu}{d\tau} \\ \frac{\tau}{k}\frac{dk}{d\tau} \\ \frac{\tau}{l}\frac{dl}{d\tau} \\ \frac{\tau}{B}\frac{dB}{d\tau} \end{array}\right] = \left[\begin{array}{c} \varepsilon_{cc}\left[\left(\frac{b\tau}{a-b\tau}+\frac{\alpha\sigma}{1-\alpha}\frac{\tau}{1-\tau}\right)\frac{1-B}{B^*-B}\frac{d+\frac{\varepsilon_{Bc}}{\varepsilon_{cc}}}{d-2}-\left[1+\frac{\alpha}{1-\alpha}\frac{\sigma\theta+(1-\alpha)\delta}{\theta+(1-\alpha)\delta}\right]\frac{\tau}{1-\tau}\right] \\ -\frac{\sigma}{1-\alpha}\frac{\tau}{1-\tau} \\ \left(\frac{\alpha\sigma}{1-\alpha}\frac{\tau}{1-\tau}+\frac{b\tau}{a-b\tau}\right)\frac{1-B}{B^*-B}\frac{d}{d-2} \\ \left(\frac{\alpha\sigma}{1-\alpha}\frac{\tau}{1-\tau}+\frac{b\tau}{a-b\tau}\right)\frac{1-B}{B^*-B}\frac{1}{d-2} \end{array}\right] \tag{42}$$

Under Assumption 4 and $B \in (0,1)$, we obtain easily Proposition **8**. ■

Proof of Proposition 9

In the Cobb-Douglas case, $\sigma = 1$ and, according to expressions (42), (31) yields

$$\frac{\tau}{c}\frac{\partial c}{\partial \tau} = -\frac{1}{1-\alpha}\frac{\tau}{1-\tau} + \left(\frac{\alpha}{1-\alpha}\frac{\tau}{1-\tau} + \frac{b\tau}{a-b\tau}\right)\frac{1-B}{B^*-B}\frac{d}{d-2}$$

In the case $a = b$, we obtain

$$\frac{\tau}{c}\frac{\partial c}{\partial \tau} = \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\left(\frac{1-B}{B^*-B}\frac{d}{d-2} - 1\right) = \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\frac{1-2B}{(d-2)(B^*-B)}$$

Proposition 9 immediately follows. ■

Proof of Lemma 10

Simply, consider expression (36). ■

Proof of Lemma 11

Consider again (36). If $1 < d < 2$, then $D < 0$ is equivalent to $B > B^*$ which is always satisfied because $B > 0 > B^*$. If $d > 2$, then $D < 0$ is equivalent to $B < B^*$ which is always satisfied because $B < 1 < B^*$. ■

Proof of Proposition 12

Lemmas 11 and 11 imply $D < 0$. Propositions 13, 16, 17 and 19 in Bosi and Desmarchelier (2017c) apply.

$D < 0$ implies that at least one eigenvalue is real and positive. Indeed, if all the eigenvalues are nonreal, the determinant is positive, a contradition. If two eigenvalues are real and negative, and two nonreal, the determinant is positive, a contradiction. If all the eigenvalues are real and negative, then the determinant is positive, a contradiction. Therefore, at least one eigenvalue is positive. Local indeterminacy requires a dimension of the stable manifold strictly greater than the number of predetermined variables. In our case, the dynamic system is four-dimensional and there are three predetermined variables. Therefore, local indeterminacy requires a full-dimensional (four-dimensional) stable manifold. But, the dimension of the stable manifold is equal to the number of negative real eigenvalues plus the number of negative real parts of nonreal eigenvalues. In our case, this number cannot exceed three, because one eigenvalue is positive. Therefore, the stable manifold is not full-dimensional and the equilibrium cannot be indeterminate. ■

Proof of Proposition 13

When $a = a^*$, $B_3 = B_4 = B^*$ and $D = 0$. We apply Proposition 13 in Bosi and Desmarchelier (2017c). ■

Proof of Proposition 14

According to Corollary 15 of Bosi and Desmarchelier (2017c), a Hopf bifurcation arises iff $S_2 = S_3/T + DT/S_3$ and T and S_3 have the same sign.

Let us rewrite S_2 and S_3 as follows:

$$S_2 = \frac{Z}{m} - \frac{T}{m}\frac{S_3}{T} \tag{43}$$

$$S_3 = z_3 - m\alpha q\,(1 - B)\,\frac{\varepsilon_2}{\varepsilon_1} \tag{44}$$

Replacing (43), equation

$$S_2 = \frac{S_3}{T} + D\frac{T}{S_3}$$

becomes

$$\frac{S_3}{T} = \frac{Z \pm \sqrt{Z^2 - 4mD\,(m+T)}}{2\,(m+T)} \tag{45}$$

We observe that, if $0 < d < 1$, $D < 0 < m + T$ iff $B^* < B < (1+\theta)/2$. In this case, $m + T > 0$ and $4mD\,(m+T) < 0$. If $d > 1$, then $D < 0$ and, thus, $D < 0 < m + T$ iff $B < (1+\theta)/2$. Even in this case, $m + T > 0$ and $4mD\,(m+T) < 0$.

Then, in both the cases,

$$\left(\frac{S_3}{T}\right)_- < 0 < \left(\frac{S_3}{T}\right)_+$$

Clearly, the solution ε_2 of

$$\frac{S_3\,(\varepsilon_2)}{T} = \left(\frac{S_3}{T}\right)_- < 0$$

is not acceptable as Hopf bifurcation value because T and S_3 have opposite sign. Let ε_H be solution of

$$\frac{S_3\,(\varepsilon_2)}{T} = \left(\frac{S_3}{T}\right)_+$$

Replacing (44) in the LHS and (45) in the RHS, we obtain (38). ■

Proof of Proposition 15

Consider Proposition 16 in Bosi and Desmarchelier (2017c). Since $a = a^*$, $B = B^*$ and then $D = 0$. In addition, $S_3 = 0$ if and only if $\varepsilon_2 = \varepsilon_{BT}$. ■

Proof of Proposition 16

According to Proposition 16 in Bosi and Desmarchelier (2017c), a Gavrilov-Guckenheimer bifurcation arises if and only if $D = 0$ jointly with $S_3 = TS_2$ such that $S_2 > 0$.

We notice that $S_3 = TS_2$ iff

$$\varepsilon_2 = \varepsilon_{GG} \equiv \varepsilon_1 \frac{mz_3 - T(Z - z_3)}{m\alpha q(1 - B)(m + T)}$$

In addition, $S_2(\varepsilon_{GG}) = Z/(\theta + 1 - 2B)$. We observe that $D = 0$ and $a = a^*$, that is $B_3 = B_4 = B^*$, imply $1 - 2B = (1 - B)d > 0$ and

$$\begin{aligned} Z &= m\theta(T - \theta) - d(1 - B)\left[\alpha m^2 + qm\alpha - nq/\varepsilon_1\right] \\ &= -d(1 - B)(m[\alpha m + (1 - \alpha)\delta] - nq/\varepsilon_1) - \theta m^2 < 0 \end{aligned}$$

Thus, $S_2(\varepsilon_{GG}) < 0$ and any Gavrilov-Guckenheimer bifurcation is ruled out. ■

Proof of Proposition 17

Proposition 18 in Bosi and Desmarchelier (2017c) states that a double-Hopf bifurcation occurs if and only if $T = S_3 = 0$ with $D > 0$, $S_2 > 0$ and $S_2^2 \geq 4D$.

According to Lemmas 10 and 11, $D > 0$ if and only if $0 < d < 1$ jointly $B < B^*$, and, according to (28), $B^* < 1/2$.

If $T = S_3 = 0$, then $S_2 = Z/m$ with

$$Z = m\left[m\alpha d(B - 1) + D\frac{\varepsilon_1}{nq} - \theta^2\right] + qm\alpha d(B - 1) + (m - \theta)\frac{nq}{\varepsilon_1} + \theta D\frac{\varepsilon_1}{nq}$$

Since $T = 0$ jointly with $B < B^* < 1/2$, it follows that $m - \theta = 1 - 2B > 0$. This implies $Z < 0$ and, therefore, $S_2 < 0$ which rules out any double-Hopf bifurcation. ■

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