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Cooperation in a differentiated duopoly when information is dispersed: A beauty contest game with endogenous concern for coordination*

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Abstract

The paper provides a micro-founded differentiated duopoly illustration of a beauty contest, in which the weight put on the strategic vs. the fundamental motive of the payoffs is not exogenous but may be manipulated by the players. We emphasize the role of the competition component of the strategic motive as a source of conflict with the fundamental motive. This conflict, already present in an oligopolistic setting under perfect information, is only exacerbated when information is imperfect and dispersed. We show how firm owners ease such conflict by opting for some cooperation, thus moderating the competitive toughness displayed by their managers. By doing so, they also influence the managers' strategic concern for coordination and consequently the weight put on public relative to private information.

Keywords: beauty contest, competition, cooperation, coordination, differentiated duopoly, dispersed information, public information.

JEL codes: D43, D82, L13, L21.

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1 Introduction

In a Keynesian 'beauty contest', agents make investment choices by referring to their expectations of some fundamental value and of the conventional value to be set by the market. In doing so, agents respond to fundamental and strategic motives, respectively. The strategic motive may itself be decomposed into coordination and competition motives: as far as they look for convention agents are willing to match the market, but as far as they take part in a contest they would be happy to beat the market. There is clearly some conflict involved in matching and beating the market at the same time. Now, the competition motive may be latent, when agents' actions have only an insignificant influence on the market, so that the market advantage occurs as an externality rather than as the consequence of strategic decisions. In this case and if information is perfect, the fundamental and the strategic motives are compatible, all the agents simply coordinating on the fundamental value. A conflict between the two motives will however emerge as soon as information is imperfect (blurring the fundamental) and dispersed (obstructing coordination), so that matching the fundamental value and matching the conventional value may then solely result from a trade-off.

The first point we want to emphasize is that when agents have significant market power, the competition motive may become active enough to create a conflict between the fundamental and the strategic motives even under perfect information. Imperfect and dispersed information will only exacerbate a conflict that was already present in this case, leading to a more adverse trade-off between the fundamental and the strategic motives. The second point we want to bring to the fore is that, as long as the relative weights on the fundamental and strategic motives are, as usual, part of the model structure, no space is left for the agents to improve the trade-off between those motives. We shall however admit that agents have some latitude in dealing with the conflict, by manipulating the relative weights on the two motives. By injecting some cooperation to tame the competition motive, such manipulation may ease the conflict, while enhancing the concern for coordination.

We illustrate these ideas with a simple IO example. We consider a two-firm, two-stage delegation game, in which firm owners control at the first stage the conduct to be adopted by firm managers at the second. Going one step upstream of an otherwise standard differentiated duopoly game allows for a strategic choice by each firm owner, committing on some degree of cooperation with the competitor so as to mitigate the competitive toughness at the second stage and thus to augment profits. The regime of competition is accordingly endogenous, strategically determined by the players as part of the equilibrium outcome, and not the consequence of the modeler's preferences for Cournot, Bertrand, or whatever.

Price setting at the second stage is performed on the basis of both public and private information on the random market size. Given demand linearity, we obtain a quadratic-payoff coordination game, which is reminiscent of a standard beauty contest, up to the addition of a first stage making the relative weights put by each firm on each type of information depend upon its concern for coordination, itself increasing with both degrees of cooperation. Each firm owner chooses the degree of cooperation which makes her marginal benefit from cooperation equal to her marginal information cost, which increases with the imprecision of private and principally public information. By moderating the competition motive, higher degrees of cooperation (prevailing under higher information quality) induce equilibrium prices that are closer to their fundamental, collusive, value. They also induce higher concerns for coordination, giving more weight to public information. Both cooperation and coordination increase at equilibrium as the two products become less and less differentiated and competition correspondingly more intense. In the limit case of the homogeneous duopoly, the degree of cooperation reaches for each firm its maximum, collusive, value, but the concern for coordination reaches its maximum value too, making public information all important and possibly disconnecting equilibrium prices from fundamentals.

Our paper contributes to two strands of the literature. First, our game fits into a large literature on beauty contest games initiated by Morris and Shin (2002) seminal contribution, distinguishing the roles of public and private information. In particular, we find in Angeletos and Pavan (2007) or Myatt and Wallace (2012, 2015, 2016) previous IO instances of beauty contest games. Our analysis differs however from theirs in at least four respects. First of all, we emphasize the role of the competition motive as a component of the strategic motive, whereas previous contributions focused on the opposition between the fundamental and coordination motives. Cornand and Heinemann (2008) already distinguished three components — the fundamental, the coordination and the competition motives — of a standard beauty contest payoff. Yet, the last motive is a mere externality when the set of agents is a continuum. Instead, by focusing on a duopoly, we confer a true strategic dimension to the competition motive.[1] Second, the previous IO applications, following Morris and Shin (2002), treat the relative weights put on the fundamental and strategic motives as pertaining to the model structure. Instead, we allow firm owners to manipulate these weights at a first stage of the game.[2] Third, contrary to the literature directly inspired by Morris and Shin (2002), we do not impose symmetry on the quality of private information, which allows us to distinguish the effects on a firm's behavior of changes in the precisions of each one of the two private signals. Fourth, by relying on the delegation game introduced under perfect information by Miller and Pazgal (2001),[3] and in contrast to previous applications of the beauty contest game to IO, our analysis is not based on the opposition between Cournot and Bertrand competition.[4]

[1]The competition motive may legitimately be ignored when market power is diluted within a large group of competitors. Myatt and Wallace (2016), in spite of addressing oligopolistic competition proper, merged in some sense the fundamental and competitive motives by taking the Bertrand price as the fundamental target, rather than a price, like the collusive price, which does not depend upon the intensity of competition.

[2]Martimort and Stole (2011) and Myatt and Wallace (2016) also consider a finite number of players in a beauty contest game *à la* Morris and Shin, in which the relative weights on the fundamental and coordination motives may vary across players, but are still fully exogenous.

[3]We borrow from Miller and Pazgal (2001) their second stage payoff. However, the significant point is here the ability to commit in the first stage to adopt some conduct in the second, not the specific second stage payoff. For alternative specifications, see d'Aspremont and Dos Santos Ferreira (2009, s.5) or d'Aspremont, Dos Santos Ferreira and Thépot (2016).

[4]As emphasized by the authors, the delegation game dilutes the opposition between price and quantity com-

The second strand of literature our work relates to is that on private information sharing and on the private and social value of information in oligopolies with uncertain demand (Vives, 1984, 1988, Raith, 1996), where again the opposition between Cournot and Bertrand competition appears crucial. More recently, the question has been reexamined under supply function competition – another way to go beyond that opposition – and the case of public information has been contemplated (Vives, 2011, 2013). Relative to this literature the present paper focuses on the comparative roles of public and private information, which is more in conformity with the first strand, and on the link between cooperation, coordination and these two types of information.

The remaining of the paper is structured as follows. Section 2 presents the two-firm, two-stage delegation game under perfect information. The full information benchmark allows us to identify the three motives of the beauty contest and to put light on the benefit from cooperation (that is maximized at the full information optimal degree of cooperation). Section 3 derives the subgame perfect equilibrium under imperfect and dispersed information. We analyze how changes in the quality of public and private information, as well as changes in the intensity of competition, affect the competitors' marginal information cost, and ultimately the extent of their cooperation. Section 4 concludes.

2 Perfect information

Consider a market for two differentiated substitutes, where the demand for good i takes the form:

$$q_i = a - p_i + d\left(p_j - p_i\right), \text{ for } i, j = 1, 2, i \neq j, \text{ with } a > 0 \text{ and } 0 \leq d \leq \infty, \qquad (1)$$

q_i being the quantity of good i demanded at prices p_i and p_j of the two goods.[5] The parameter a measures the market size. The parameter d is the reciprocal of an index of product differentiation ($d = 0$ for independent goods, $d = \infty$ for perfect substitutes), and is consequently an indicator of the intensity of competition to which the firms are exposed.

2.1 Second stage: setting prices

Firms are price setters but, as in Miller and Pazgal (2001), we assume that the owner of each firm i has sufficient control over the respective manager's conduct to impose at a first stage a degree of cooperation $\gamma_i \in [0, 1]$ weighting the competitor's profit and turning firm i's payoff at the second stage into

$$\Pi_i\left(p_i, p_j; \gamma_i, a\right) = p_i\left(a - p_i + d\left(p_j - p_i\right)\right) + \gamma_i p_j\left(a - p_j + d\left(p_i - p_j\right)\right), \qquad (2)$$

petition, because "*if the owners have sufficient power to manipulate their managers' incentives, the equilibrium outcome is the same regardless of how the firms compete in the second stage*" (Miller and Pazgal, 2001, p. 284).

[5]Generality is not lost by assuming a demand curve with slope -1 instead of $-b < 0$.

to be maximized in p_i. By taking the extreme values of the two degrees of cooperation, we obtain Bertrand competition for a fully non-cooperative conduct ($\gamma_i = \gamma_j = 0$), and tacit collusion for a fully cooperative conduct ($\gamma_i = \gamma_j = 1$). A continuum of intermediate equilibrium outcomes, in particular the Cournot outcome (for $\gamma_i = \gamma_j = d/(1+d)$), is attainable between these extremes.

The first order condition for maximization of this payoff leads to the best reply price

$$p_i = \frac{\frac{a}{2} + d\frac{1+\gamma_i}{2}p_j}{1+d}, \tag{3}$$

a weighted mean of the monopoly price $a/2$ (which will be taken in the following as the fundamental θ) and of the discounted competitor's price $(1+\gamma_i)\,p_j/2$, the discount becoming heavier as cooperation weakens. The discount is an expression of the competition motive, in conflict with the fundamental motive, since it triggers a downward price deviation when the competitor sets the monopoly price. Notice also that the relative weight put on the fundamental decreases with the intensity of competition from 1 (when the products are independent) to 0 (when they are perfect substitutes).

A simple computation (using equations (3) for p_i and p_j) allows us to determine the equilibrium prices at the second stage of the game, when information is perfect:

$$p^*\left(\gamma_i, \gamma_j\right) = \frac{1+d+d\frac{1+\gamma_i}{2}}{(1+d)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}\frac{a}{2}, \tag{4}$$

for $i,j = 1,2$, $i \neq j$. These prices increase with both degrees of cooperation, from their fully non-cooperative Bertrand value $a/(2+d)$ to the collusive (or monopoly) price $a/2$, through the Cournot equilibrium price $a(1+d)/(2+3d)$. The price $p^*\left(\gamma_i, \gamma_j\right)$ is however more responsive to the degree of cooperation γ_i decided by the firm owner than to the degree of cooperation γ_j decided by the owner of the rival firm. Also, as the index of differentiation d increases from zero to infinity, the equilibrium prices decrease from their monopoly value to zero, except under fully cooperative conduct of the two firms. At the limit, when $d \to \infty$, the equilibrium price can be positive only if the degrees of cooperation tend both to 1.

2.2 First stage: choosing the extent of cooperation

Each firm owner, anticipating the second stage equilibrium prices, chooses at the first stage the degree of cooperation she wants to impose on her manager's conduct, in order to maximize her own profit, say $\Pi_i^*\left(\gamma_i, \gamma_j\right) = \Pi_i\left(p^*\left(\gamma_i, \gamma_j\right), p^*\left(\gamma_j, \gamma_i\right); 0, a\right)$ for firm i, that is,

$$\Pi_i^*\left(\gamma_i, \gamma_j\right) = p^*\left(\gamma_i, \gamma_j\right)\left(a - p^*\left(\gamma_i, \gamma_j\right) + d\left(p^*\left(\gamma_j, \gamma_i\right) - p^*\left(\gamma_i, \gamma_j\right)\right)\right). \tag{5}$$

Maximizing this profit in γ_i is clearly equivalent to minimizing in the same variable the loss with respect to the profit $\Pi_i^*(1,1)$ that would be obtained under tacit collusion, namely (recalling that $\theta \equiv a/2$ and denoting for shortness $p_i^* \equiv p^*(\gamma_i, \gamma_j)$ and $\overline{p}^* \equiv (p_i^* + p_j^*)/2$)

$$\begin{aligned} \Pi_i^*(1,1) - \Pi_i^*(\gamma_i, \gamma_j) &= \underbrace{(p_i^* - \theta)^2}_{\text{fundamental motive}} + 2d \underbrace{p_i^*(p_i^* - \overline{p}^*)}_{\text{strategic motive}} \\ &= \underbrace{(p_i^* - \theta)^2}_{\text{fundamental motive}} + 2d \underbrace{(p_i^* - \overline{p}^*)^2}_{\text{coordination motive}} + 2d \underbrace{\overline{p}^*(p_i^* - \overline{p}^*)}_{\text{competition motive}}, \end{aligned} \quad (6)$$

where we can identify three motives: the fundamental, the coordination and the competition motives. This loss function is reminiscent of a beauty contest game.[6] The competition motive adds to the loss of firm i only if its second stage equilibrium price is higher than the one of its competitor, and moderates the loss otherwise.[7]

Should the competition motive vanish, because of a genuine cooperative attitude of firm owners, instead of a cooperative conduct strategically imposed by them on firm managers, the subgame perfect equilibrium would be characterized under perfect information by maximum degrees of cooperation, with tacit collusion at the second stage. Of course, this outcome would crucially depend upon information being perfect, hence able to dissolve the potential conflict between the fundamental and the coordination motives. However, even keeping information perfect, the competition motive destroys in the present game the collusive outcome, as it leads at the first stage of the game to the best reply for firm i

$$\widehat{\gamma}_i(\gamma_j, d) = \frac{(2+3d)\, d\, (1+\gamma_j)}{4(1+d)^2 + (2+d)\, d\, (1+\gamma_j)}, \quad (7)$$

increasing in γ_j but always smaller than 1 as long as d is finite. Strategic complementarity at the first stage is not strong enough to translate into full cooperation. The (symmetric) subgame perfect equilibrium value of the degree of cooperation is indeed

$$\gamma^*(d) = \frac{d}{2+d}, \quad (8)$$

[6]It has the same structure as the standard payoff function of a beauty contest game (Morris and Shin, 2002): $u_i(\mathbf{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L})$, where $L_i = \frac{1}{n}\sum_j (a_j - a_i)^2$ and $\bar{L}$ is the arithmetic mean of the L_i's. This payoff is the sum of the fundamental and strategic motives or, developing L_i and $\bar{L}$, $u_i(\mathbf{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(a_i - \overline{a})^2 + r\frac{1}{n}\sum_j (a_j - \overline{a})^2$, the sum of the fundamental, coordination and competition motives. There is however an important difference as concerns aggregate payoffs: the competition motive counterbalances the coordination motive in the Morris and Shin specification, so that the fundamental motive stands alone as a component of social welfare. By contrast, the competition motive vanishes by aggregation of our loss functions, so that coordination contributes in our case to the players' welfare (identified here with expected total profits). Another important difference is that, becoming more and more insensitive to variations of a_i when n grows, the competition motive can be neglected in the individual optimization problem when n is large. This is indeed the case in Morris and Shin (2002), where a continuum of agents is assumed.

[7]Myatt and Wallace (2012, 2016) take the Bertrand price, instead of the collusive price, as the fundamental target. In our duopoly case, the Bertrand price is $\theta^B = 2\theta/(2+d)$ and firm i's loss function would then become (up to the multiplicative constant $1+d$) $(1-r)(p_i^* - \theta^B)^2 + r(p_i^* - p_j^*)^2 + h^*$, with $r = d/2(1+d)$ and $h^* = r(\theta\theta^B - p_j^{*2})$. If the game were reduced to its second stage, the term h^* would not depend upon the (price) strategy of player i, so that the fundamental and coordination motives would be the only relevant components of the loss function. However, in the two-stage game, h^* depends through p_j^* upon the degree of cooperation chosen by the owner of firm i, and cannot be treated as a constant. It conveys part of the competition motive, together with the very choice as the fundamental of the Bertrand price, depending upon the intensity of competition.

attaining 1 only in the limit case of perfect substitutability of the two products. Notice however that, although $\lim_{d\to\infty}\gamma^*(d) = 1$ (full cooperation), the collusive price is out of reach as the $\lim_{d\to\infty} p^*(\gamma^*(d), \gamma^*(d))$. Indeed, by (4) and (8),

$$p^*(\gamma^*(d), \gamma^*(d)) = \frac{1+d/2}{1+d}\theta, \tag{9}$$

which tends to $\theta/2$ (not to θ) as $d \to \infty$. This is of course a further consequence of the conflict between the fundamental and the competition motives.

Nevertheless, in the limit case where $d = \infty$, this symmetric subgame perfect equilibrium with $\gamma = 1$ and $p = \theta/2$ is just a limit point in a continuum of symmetric subgame perfect equilibria with $\gamma = 1$ and $p \in [0, a]$. Indeed, by (3), the best reply of each firm manager is to set the same price as his rival, whatever it is and independently of any reference to the fundamental. Also, by (4), each firm owner, by deviating to a lower degree of cooperation, would only generate zero equilibrium prices, and hence zero profits. There is in addition a continuum of trivial subgame perfect equilibria with arbitrary pairs of degrees of cooperation $(\gamma_1, \gamma_2) \neq (1, 1)$ and zero prices.

3 Imperfect and dispersed information

Going beyond Miller and Pazgal (2001), we shall now assume that the market size a is stochastic, with mean $\overline{a}$ and variance σ^2, so that $\mathbb{E}\left(\theta^2\right) = \left(\overline{a}^2 + \sigma^2\right)/4$, the sum of the mean square and of the variance of the fundamental.

3.1 Second stage: setting prices

The value $\theta = a/2$ of the fundamental is realized at the second stage of the game, but it is unknown by the managers, who have only access to imperfect public and private information on that value. Each manager i ($i = 1, 2$) receives unbiased public and private signals y and x_i, such that $y = \theta + \eta$ and $x_i = \theta + \varepsilon_i$, the random variables η and ε_i being independently and normally distributed, with mean 0 and finite variances $1/\alpha$ and $1/\beta_i$, respectively. Recall that, contrary to the literature in the vein of Morris and Shin (2002), we do not impose symmetry on the quality of private information, the precision β_i possibly differing from the precision β_j.[8]

According to the two signals received, and given Π_i as defined by (2), the problem of firm i's manager becomes: $\max_{p_i} \mathbb{E}\left(\Pi_i\left(p_i, p_j; \gamma_i, 2\theta\right) \middle| y, x_i\right)$. Referring to the first order

[8]As will become clear, this remains tractable in a two player game and allows us to distinguish the effects on a firm's behavior of changes in the precisions of each one of the two private signals. Distinguishing these effects is particularly important in a context where we have to disentangle the cooperative and coordinating roles played by the strategic variable γ_i.

condition (3), we may reformulate firm i's best reply as

$$p_i = \frac{\mathbb{E}_i\left(\theta\right) + d\frac{1+\gamma_i}{2}\mathbb{E}_i\left(p_j\right)}{1+d}, \tag{10}$$

where $\mathbb{E}_i\left(\cdot\right) \equiv \mathbb{E}(\cdot \,|\, y, x_i)$ is the expectation operator conditional on the signals received.

In particular, $\mathbb{E}_i(\theta) = (\alpha y + \beta_i x_i)/(\alpha + \beta_i)$. As to $\mathbb{E}_i\left(p_j\right)$, we follow the methodology developed by Morris and Shin (2002), and assume that each manager i follows a strategy which is linear in the received signals: $p_i = \kappa_i y + \kappa_i' x_i$.[9] As $\mathbb{E}_i\left(x_j\right) = \mathbb{E}_i\left(\theta\right)$, we thus obtain

$$\begin{aligned} p_i &= \frac{\frac{\alpha y+\beta_i x_i}{\alpha+\beta_i} + d\frac{1+\gamma_i}{2}\left(\kappa_j y + \kappa_j' \frac{\alpha y+\beta_i x_i}{\alpha+\beta_i}\right)}{1+d} \\ &= \underbrace{\frac{\left(1+d\frac{1+\gamma_i}{2}\kappa_j'\right)\alpha + d\frac{1+\gamma_i}{2}\left(\alpha+\beta_i\right)\kappa_j}{\left(1+d\right)\left(\alpha+\beta_i\right)}}_{\kappa_i} y + \underbrace{\frac{\left(1+d\frac{1+\gamma_i}{2}\kappa_j'\right)\beta_i}{\left(1+d\right)\left(\alpha+\beta_i\right)}}_{\kappa_i'} x_i, \end{aligned} \tag{11}$$

for $i, j = 1, 2$, $i \neq j$. Through identification of the coefficients (κ_i, κ_i') and use of the two equations, we can determine their equilibrium values. Rather than giving here the corresponding cumbersome expressions (to be found in the Appendix), let us equivalently establish two facts, which are more appropriate to interpretation.

First, the sum of the two coefficients applied to the public and private signals

$$\kappa_i + \kappa_i' = \frac{1+d+d\frac{1+\gamma_i}{2}}{\left(1+d\right)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}} \equiv K\left(\gamma_i, \gamma_j; d\right), \tag{12}$$

while variable, is independent of the information quality, as characterized by the precisions $\left(\alpha, \beta_i, \beta_j\right)$ of the three signals. It is equal to the coefficient applied to the fundamental in the expression of the equilibrium price under perfect information (see equation (4)).

Second, the ratio of the weight put on the private signal to the weight put on the public signal is

$$\frac{\kappa_i'}{\kappa_i} = \left(1 - r_i\right)\frac{\beta_i}{\alpha}, \tag{13}$$

with r_i given by

$$r\left(\gamma_i, \gamma_j; \frac{\beta_i}{\alpha}, \frac{\beta_j}{\alpha}, d\right) \equiv \frac{d\frac{1+\gamma_i}{2}}{1+d}\frac{\left(1+d+d\frac{1+\gamma_j}{2}\right)\left(1+d\right)\left(1+\frac{\beta_i}{\alpha}\right) + \left(1+d+d\frac{1+\gamma_i}{2}\right)d\frac{1+\gamma_j}{2}\frac{\beta_j}{\alpha}}{\left(1+d+d\frac{1+\gamma_i}{2}\right)\left(1+d\right)\left(1+\frac{\beta_j}{\alpha}\right) + \left(1+d+d\frac{1+\gamma_j}{2}\right)d\frac{1+\gamma_i}{2}\frac{\beta_i}{\alpha}}. \tag{14}$$

The coefficient $r_i = r\left(\gamma_i, \gamma_j; \frac{\beta_i}{\alpha}, \frac{\beta_j}{\alpha}, d\right)$, depending upon the degrees of cooperation γ_i and γ_j decided by both firm owners, upon the relative quality of private (with respect to public) information available to both managers, as measured by β_i/α and β_j/α, and upon the

[9] When assuming normal distributions of the public and private signals, we are ignoring the consequences of obtaining values of $\mathbb{E}_i\left(\theta\right)$ and $\mathbb{E}_i\left(p_j\right)$ that are either negative or too high to ensure a positive demand.

competitive intensity d resulting from the extent of product substitutability,[10] can be seen as an index of the *strategic concern for coordination* of the manager of firm i. The ratio κ_i'/κ_i of the weights put on the two signals would just be equal to the ratio of the corresponding precisions β_i/α if there were no competitive interaction between the two firms, that is, if the goods were independent ($d = 0$, entailing $r_i = 0$). At the other extreme, all the weight would be put on the public signal ($\kappa_i'/\kappa_i = 0$) under perfect substitutability of the two goods ($d = \infty$) and fully cooperative conduct of the two firms ($\gamma_i = \gamma_j = 1$), entailing a maximum strategic concern for coordination $r_i = 1$.

At this stage, the quality of information steps in only relatively, so that the impact of, say, an increase in the precision α of the public signal is indistinguishable from a simultaneous proportionate decrease in the precisions β_i and β_j of both private signals. Also, the quality of information can only have an impact on the strategic concern for coordination through an asymmetry effect given by the second ratio in the expression of r, which is equal to 1 when information and cooperation are both symmetric ($\beta_i = \beta_j$ and $\gamma_i = \gamma_j$).

On the basis of the two facts above, we can formulate two propositions. The first asserts existence and uniqueness of a second stage equilibrium (outside the limit case where $d = \infty$ and $\gamma_i = \gamma_j = 1$, left to subsection 3.3).

Proposition 1 *Take $d < \infty$ or $(\gamma_1, \gamma_2) \neq (1, 1)$. Then, conditionally on the realization of the public signal y and the private signals x_i and x_j, the second stage equilibrium price of good i is uniquely determined by $p_i^*\left(y, x_i, \gamma_i, \gamma_j\right) = \kappa_i y + \kappa_i' x_i$ ($i, j = 1, 2$, $i \neq j$), with*

$$\kappa_i = \frac{1}{1 + (1 - r_i)\, \beta_i/\alpha} K_i \text{ and } \kappa_i' = \frac{(1 - r_i)\, \beta_i/\alpha}{1 + (1 - r_i)\, \beta_i/\alpha} K_i, \tag{15}$$

$K_i = K\left(\gamma_i, \gamma_j; d\right)$ and $r_i = r\left(\gamma_i, \gamma_j; \frac{\beta_i}{\alpha}, \frac{\beta_j}{\alpha}, d\right)$ being given by equations (12) and (14), respectively.

Proof. The determination of the expressions of κ_i and κ_i' in (15) from the two equations (12) and (13), for $i = 1, 2$, is straightforward. □

The second proposition concerns the effects of the changes in the relative quality of private and public information on the weight put on private relative to public information. There is a first evident effect of an increase in relative quality of private vs. public information, as measured by β_i/α, on its relative weight κ_i'/κ at equilibrium (see equation (13)). This effect is however modified and completed by the already mentioned asymmetry effects working through the strategic concern for coordination r_i, the resultant of which is stated as follows.

Proposition 2 *The ratio κ_i'/κ_i of the weight on the private signal received by firm i to that on the public signal is increasing in the relative precisions β_i/α and β_j/α of both private signals with*

[10] In the Morris and Shin (2002) standard beauty contest game, the ratio of the weights put on the private and public signals has the same specification but is exogenous and uniform across agents. Here r_i is manipulable by the owner of firm i through the degree of cooperation γ_i she decides to set.

respect to that of the public one.

Proof. As shown by (14), the index of firm i's strategic concern for coordination is a homographic function of the ratios of precisions β_j/α and β_i/α. Thus, a simple computation allows to establish that $r\left(\gamma_i, \gamma_j; \beta_i/\alpha, \cdot, d\right)$ is decreasing, so that κ_i'/κ_i is, by (13), increasing in β_j/α. It is also easy to check that

$$\frac{\kappa_i'}{\kappa_i} = \left(1 - \frac{b + h\left(\beta_i/\alpha\right)}{l + h\left(\beta_i/\alpha\right)}\right)\left(\beta_i/\alpha\right) = \frac{\left(l - b\right)\left(\beta_i/\alpha\right)}{l + h\left(\beta_i/\alpha\right)},$$

with positive constants b, h and l. Hence, as $l > b$, κ_i'/κ_i is increasing in β_i/α. □

So, an increase in the precision of one of the private signals relative to the public one, say β_i/α, induces an increase in the relative weight put by *both* firms on their respective private signals, namely κ_i'/κ_i and κ_j'/κ_j, but for different reasons. By (13), the increase in κ_i'/κ_i is explained by the direct effect of the increase in the quality of firm i private information β_i/α, an effect only attenuated by its contrary indirect effect through the strategic concern for coordination r_i. By contrast, the increase in κ_j'/κ_j is due to the decrease in r_j: firm j's manager, whose rival is better privately informed, becomes less strategically concerned.

To conclude, let us briefly consider how the extent of cooperation influences the concern for coordination. First, there is a direct positive influence of γ_i on r_i expressed by the first ratio on the RHS of (14), $d\left(1 + \gamma_i\right)/2\left(1 + d\right)$, which is the derivative of the own price with respect to the competitor's price in the best reply of firm i (see (3) or (10)). Second, there are asymmetry effects, through the second ratio on the RHS of (14), of the same type of those stated in Proposition 2. Since this ratio is a homographic function of γ_i and γ_j, it is easy to check that it is decreasing in γ_i (which attenuates the just mentioned direct positive influence) and increasing in γ_j. Ultimately, the strategic concern for coordination responds positively (and the ratio κ_i'/κ_i negatively) to a higher degree of cooperation by anyone of the two firms: more willingness to cooperate induces more concern for coordination.

3.2 First stage: choosing the extent of cooperation

At the first stage, the owner of firm i maximizes in γ_i her expected profit $\mathbb{E}\left(\Pi_i^*\left(\gamma_i, \gamma_j\right)\right)$, with Π_i^* defined by (5), before the uncertainty on the fundamental is resolved. In the expression of Π_i^* we take $a = 2\theta$ and $p_i^*\left(\gamma_i, \gamma_j\right) = \kappa_i\left(\gamma_i, \gamma_j\right) y + \kappa_i'\left(\gamma_i, \gamma_j\right) x_i$, with weights $\kappa_i\left(\gamma_i, \gamma_j\right)$ and $\kappa_i'\left(\gamma_i, \gamma_j\right)$ given by (15) and depending upon γ_i and γ_j through r_i as given by (14). These weights do not depend upon the random values θ, y, x_i and x_j, all of which have the same expected value $\overline{a}/2$. Firm i's expected profit can accordingly be expressed as

$$\mathbb{E}\left(\Pi_i^*\right) = \mathbb{E}\left(\theta^2\right)\left[\begin{array}{c} \underbrace{\left(\kappa_i + \kappa_i'\right)\left(2 - \left(1 + d\right)\left(\kappa_i + \kappa_i'\right) + d\left(\kappa_j + \kappa_j'\right)\right)}_{F_i\left(\gamma_i, \gamma_j\right)} \\ \underbrace{-\frac{\kappa_i}{\mathbb{E}\left(\theta^2\right)\alpha}\left(\kappa_i + d\left(\kappa_i - \kappa_j\right) + \left(1 - r_i\right)\left(1 + d\right)\kappa_i'\right)}_{G_i\left(\gamma_i, \gamma_j\right)} \end{array}\right]. \qquad (16)$$

The first term inside the brackets, $F_i(\gamma_i, \gamma_j) \equiv \Pi_i(\kappa_i + \kappa'_i, \kappa_j + \kappa'_j; 0, 2)$, is the profit that firm i would obtain at prices $\kappa_i + \kappa'_i \equiv K_i = K(\gamma_i, \gamma_j; d)$ and $\kappa_j + \kappa'_j \equiv K_j = K(\gamma_j, \gamma_i; d)$ if θ were expected to be equal to 1 with certainty.[11] It only depends upon the sums of the weights on the private and public signals, which are independent of the precisions α, β_i and β_j of the three signals. Increasing the degree of cooperation γ_i exerts a strategic cooperation effect through the expected price on the expected profit, described by the derivative

$$\frac{\partial F_i}{\partial \gamma_i} = \frac{\partial \Pi_i}{\partial K_i}\frac{\partial K_i}{\partial \gamma_i} + \frac{\partial \Pi_i}{\partial K_j}\frac{\partial K_j}{\partial \gamma_i}, \tag{17}$$

which can be seen as the *marginal cooperation benefit*, fully independent of the information quality.

The second term inside the brackets, $G_i(\gamma_i, \gamma_j)$, is the information cost that arises because the second stage equilibrium price is expected to differ under imperfect information both from the fundamental and from the competitor's price. By referring to equation (6) and using (13), it can be checked that, abstracting from the terms in $\kappa_i + \kappa'_i$ and $\kappa_j + \kappa'_j$ which are taken into account in $F_i(\gamma_i, \gamma_j)$ and concentrating on $\mathbb{E}(\theta^2) G_i(\gamma_i, \gamma_j)$, the term $\kappa_i^2/\alpha + \kappa_i'^2/\beta_i$ corresponds to the fundamental motive and its complementary term $d(\kappa_i(\kappa_i - \kappa_j)/\alpha + \kappa_i'^2/\beta_i)$ to the (weighted) strategic motive, combining its coordination and competition components.[12] The derivative $\partial G_i/\partial \gamma_i$ is the *marginal information cost* of an increase in the degree of cooperation.

In equation (16), the term $\mathbb{E}(\theta^2)\alpha = \mathbb{E}(\theta^2)/\mathbb{E}(\eta^2)$ appears in the denominator of the information cost $G_i(\gamma_i, \gamma_j)$. This term is the ratio of the expected mean square $\mathbb{E}(\theta^2) = (\overline{a}^2 + \sigma^2)/4$ of the fundamental to the mean square (also the variance) of the noise affecting the public signal. Clearly, the expected information cost decreases as public information becomes more precise *relative to* the mean and variance of the fundamental. It vanishes when information reaches perfection, either because the precision α of the public signal tends to infinity, or because *both* precisions β_i and β_j tend to infinity, leading (by (13)) to the vanishing of the factor κ_i which multiplies the information cost $G_i(\gamma_i, \gamma_j)$. Notice that, in the latter case, perfect private information shared by the two players spontaneously entails free coordination, in addition to the knowledge of the fundamental. In the limit case of a vanishing information cost, expected profit maximization requires the marginal benefit of cooperation $\partial F_i/\partial \gamma_i$ to be zero, and we are back to the optimal degree of cooperation given by (7) in the preceding section. By contrast, a high information cost due to the low precision of the public signal (a small $\mathbb{E}(\theta^2)\alpha$) may be an obstacle to the existence of equilibrium by

[11] At the first stage, under uncertainty on θ, the owner of firm i expects, conditionally to the choices γ_i and γ_j, the equilibrium price $\mathbb{E}(p_i^*) = \kappa_i \mathbb{E}(y) + \kappa'_i \mathbb{E}(x_i) = (\kappa_i + \kappa'_i)\mathbb{E}(\theta) = K(\gamma_i, \gamma_j; d)(\overline{a}/2)$. The term $F_i(\gamma_i, \gamma_j)$ thus corresponds to a non-stochastic normalized market size $\overline{a} = 2$. In the expression of the expected profit $\mathbb{E}(\Pi_i^*)$, this term is multiplied by $\mathbb{E}(\theta^2) = (\overline{a}^2 + \sigma^2)/4$, taking into account variability and uncertainty of the market size.

[12] The term multiplied by the weight $2d$ in $\mathbb{E}(\theta^2) G_i(\gamma_i, \gamma_j)$, namely (using (13)) $(\kappa_i(\kappa_i - \kappa_j)/\alpha + \kappa_i'^2/\beta_i)/2$, is the sum of $\left((\kappa_i - \kappa_j)^2/\alpha + \kappa_i'^2/\beta_i + \kappa_j'^2/\beta_j\right)/4$ and $\left(\left(\kappa_i^2 - \kappa_j^2\right)/\alpha + \kappa_i'^2/\beta_i - \kappa_j'^2/\beta_j\right)/4$, representing the coordination and competition motives, respectively (see (6)).

preventing expected profits from reaching positive values.

Our aim is to analyze the impact of changes in the precision of the different signals on the equilibrium value of the degrees of cooperation of the two firms. To do that, we shall refer to the marginal cooperation benefit $\partial F_i/\partial\gamma_i \equiv f_i\left(\gamma_i, \gamma_j; d\right)$ and to the marginal information cost $\partial G_i/\partial\gamma_i \equiv g_i\left(\gamma_i, \gamma_j; \mathbb{E}\left(\theta^2\right)\alpha, \beta_i/\alpha, \beta_j/\alpha, d\right)$. The first order condition for the maximization of the payoff defined by (16) is $f_i\left(\gamma_i^*, \gamma_j^*\right) \leq g_i\left(\gamma_i^*, \gamma_j^*\right)$, with equality if $\gamma_i^* > 0$.[13] In order to appreciate the impact of changes in the precisions $\left(\alpha, \beta_i, \beta_j\right)$, it is convenient to start from a symmetric equilibrium, associated to the case $\beta_i = \beta_j = \beta$. We first state existence of such a symmetric equilibrium in the following proposition, and then proceed by simulation.

Proposition 3 *Assume symmetry in the precision of both private signals ($\beta_i = \beta_j = \beta$). Also assume finite variances σ^2, $1/\alpha$, $1/\beta$ of the fundamental and of the public and private signals, as well as a finite index of product differentiation d. Then there exists a symmetric subgame perfect equilibrium, with a degree of cooperation $\gamma^* \in [0, d/(2+d)]$, provided the quality of information is high enough to ensure non-negativity of the expected profit: $F_i(\gamma^*, \gamma^*) \geq G_i(\gamma^*, \gamma^*)$. The degree of cooperation tends to its maximum value as information becomes perfect ($\alpha \to \infty$ or $\beta \to \infty$).*

Proof. See Appendix. □

A symmetric equilibrium is illustrated in Figure 1 at the intersection of the two thick solid curves, representing the graphs of the marginal cooperation benefit $f(\cdot, \gamma^*; 10)$ (the hump shaped curve) and of the marginal information cost $g(\cdot, \gamma^*; 10, 1, 1, 10)$ (the increasing curve), with $\gamma^* = 0.775$ in this example.

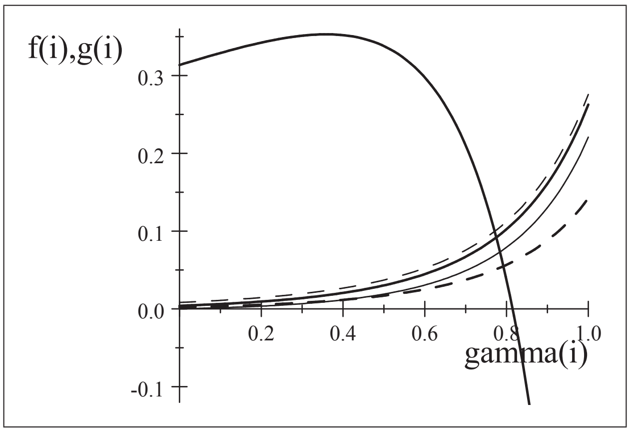

Figure 1 - The equilibrium value of γ_i as determined by the intersection of f_i and g_i

An increase in the precision of the signals received by the manager of firm i diminishes his marginal information cost, shifting the corresponding curve downwards and creating an incentive to more cooperation (increasing γ_i), hence easing the conflict between the competition and the fundamental motives. The thick dashed curve below the solid one

[13]The case $f_i\left(1, \gamma_j^*\right) > g_i\left(1, \gamma_j^*\right)$ for the first order condition is excluded if $d < \infty$, since γ_i^* is upper-bounded by its perfect information value, given by (7), which is always smaller than one under a finite d.

illustrates a doubling of α, and the intermediate thin solid curve a doubling of β_i. The shift of the marginal information cost is larger when induced by a higher precision of the public signal, because of the direct first stage effect via $\mathbb{E}\left(\theta^2\right)\alpha$ (see equation (16)).[14] The variation in β_i, by contrast, has only indirect effects via the changes induced at the second stage through the β_i/α ratio. Finally, a higher precision of the private signal received by rival firm j *increases* the marginal information cost of firm i, by decreasing its strategic concern for coordination r_i. The marginal information cost curve shifts upwards: the thin dashed curve above the thick solid one illustrates a doubling of the precision β_j. We end up with an incentive for firm i to cooperate less (a smaller γ_i) and also to coordinate less (a larger κ_i'/κ_i, by Proposition 2).[15]

3.3 The influence of competitive intensity

We finally consider the consequences of higher competitive intensity, associated with a larger value of the differentiation parameter d. As well known, an indefinite increase in product substitutability augments dramatically the need for *cooperation*, so as to avoid the Bertrand outcome, with the eventual vanishing of profits. Indeed, by equation (12), $\lim_{d\to\infty} K\left(\gamma_i,\gamma_j;d\right)=0$ unless $\gamma_i=\gamma_j=1$: equilibrium prices will tend to zero as the products become perfectly substitutable unless both firms decide to collude. In addition however, the increase in d also augments the need for *coordination*: as d becomes higher and higher, total profits are more and more eroded by the divergence between the competitors' prices rather than by their distance from the fundamental. This is clear if we refer to the total loss with respect to the maximum total profit, namely (by (6)) the sum of the fundamental motive $\left(p_i^*-\theta\right)^2+\left(p_j^*-\theta\right)^2$ and the coordination motive $d\left(p_i^*-p_j^*\right)^2$, with a weight d on the latter increasing indefinitely with respect to the unit weight on the former.

As to the first point, the influence of competitive intensity on cooperation, Figure 2 illustrates, under symmetry, the increase in the equilibrium degree of cooperation $\gamma=\gamma_i=\gamma_j$ induced by an increase in the differentiation parameter d. The thin curves represent, with the parameter values of Figure 1 ($\alpha=\beta_i=\beta_j=10$ and $d=10$), the graphs of the marginal cooperation benefit $f\left(\cdot,\cdot;10\right)$ (the hump shaped curve) and of the marginal information cost $g\left(\cdot,\cdot;10,1,1,10\right)$ (the increasing curve). The thick curves, which intersect at a higher value of γ,[16] result from $d=100$ with the same values of the information parameters, thus corresponding to the graphs of $f\left(\cdot,\cdot,100\right)$ and $g\left(\cdot,\cdot,10,1,1,100\right)$.

[14] An increase in the expected mean square $\mathbb{E}\left(\theta^2\right)$ of the fundamental would have an even stronger effect than the one of a proportionate increase in α, since it would not be partially compensated by an increase in the marginal information cost due to the decrease in β_i/α.

[15] The new (asymmetric) equilibria, after perturbation of each one of the three precisions α, β_i and β_j may be straightforwardly computed. Figure 1 does not immediately allow us to visualize them, since the adjustment by the owner of firm j of her strategy determines a further shift of firm i's marginal information cost curve, together with a shift of its marginal cooperation benefit curve.

[16] The equilibrium degree of cooperation is $\gamma^*=0.775$ for $d=10$, and $\gamma^*=0.972$ for $d=100$.

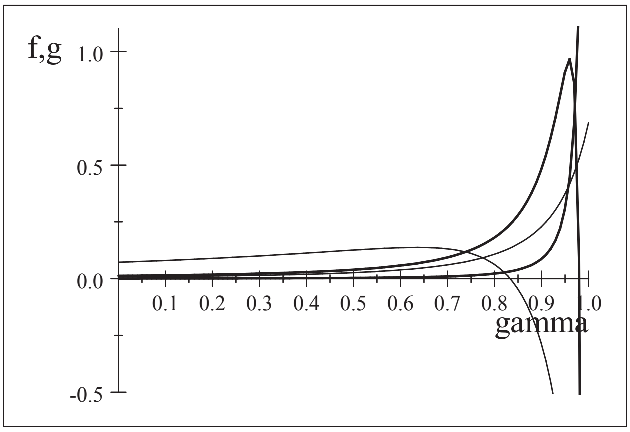

Figure 2 - Stronger cooperation induced by higher competitive intensity

As to the second point, the influence of competitive intensity on coordination, consider the expression, again under symmetry, of each firm's concern for coordination, namely $r = (1+\gamma)/2(1+1/d)$, increasing in both γ and d. Higher competitive intensity induces directly and indirectly, through stronger cooperation, a higher strategic concern for coordination, which translates into a weaker relative weight κ'/κ on private information. In the limit case of perfect product substitutability and full cooperation, this strategic concern for coordination reaches its maximum value ($r = 1$), the issue ceasing to be the reference to the fundamental and becoming instead the coordination between competitors through the public signal.[17]

To conclude, the following proposition applies to the limit case of perfect substitutability between the two products, with its strong incentive to full cooperation and full coordination.

Proposition 4 *If $d = \infty$, there is a continuum of subgame perfect equilibria with full cooperation ($\gamma_i = \gamma_j = 1$), conditional on the sole (random) public signal y, such that the second stage equilibrium price of both goods is equal to κy, for any $\kappa \in \left[0, 2\mathbb{E}\left(\theta^2\right)/\mathbb{E}\left(y^2\right)\right]$. There is in addition a continuum of trivial subgame perfect equilibria with arbitrary pairs of degrees of cooperation $\left(\gamma_i, \gamma_j\right) \neq (1,1)$ and zero prices.*

Proof. Through identification of the coefficients κ_i and κ'_i in equation (11), we see that we generically obtain, if $d = \infty$ and $\gamma_i = \gamma_j = 1$, $\kappa'_i = \kappa'_j = 0$ and $\kappa_i = \kappa_j = \kappa \geq 0$. Hence, by (16), $F_i(1,1;\infty) = (2-\kappa)\kappa$ and $G_i(1,1;\infty) = \kappa^2/\mathbb{E}\left(\theta^2\right)\alpha$, so that firm i's expected profit is non-negative if

$$\kappa \leq 2\frac{\mathbb{E}\left(\theta^2\right)}{\mathbb{E}\left(\theta^2\right)+\mathbb{E}\left(\eta^2\right)} = 2\frac{\mathbb{E}\left(\theta^2\right)}{\mathbb{E}\left(y^2\right)}.$$

As to the trivial equilibria, observe that for $\gamma_i\gamma_j < 1$, by equations (12), (15) and (16), $\kappa = K\left(\gamma_i, \gamma_j; \infty\right) = 0$ and $F_i\left(\gamma_i, \gamma_j; \infty\right) = G_i\left(\gamma_i, \gamma_j; \infty\right) = 0$. □

[17] Coordination might even be achieved in this case through a biased public signal (a sunspot disconnected from the fundamental).

The existence of these two sets of subgame perfect equilibria does not depend upon the imperfection of information and was already found in section 2. The novelty arises here from the form taken by the influence of the extreme competitive intensity on the strategic concern for coordination, ending up in the zero weight put on the private signals. It arises further from the ceiling on κ, hence on the expected price $\kappa\mathbb{E}(y) = \kappa\theta$, not $a = 2\theta$ as under perfect information (when $\mathbb{E}(\eta^2) = 0$), but a lower and lower ceiling as the mean square $\mathbb{E}(y^2)$ of the public signal is less and less attributable to the mean square $\mathbb{E}(\theta^2)$ of the fundamental itself, in other words as public information becomes more and more noisy.

4 Conclusion

The main contribution of this paper is to provide a micro-founded differentiated duopoly illustration of a beauty contest, in which the weight put on the strategic vs. the fundamental motive of the payoffs is not given by the model structure but may be strategically manipulated by the players. The analysis of beauty contest games, of which the IO illustrations are no exception, has up to now emphasized the conflict between the fundamental and coordination motives, a conflict which is the consequence of dispersed information. By contrast, we emphasize the role of the other component of the strategic motive, the competition motive, as a source of conflict with the fundamental motive. This conflict is already present in an oligopolistic setting under perfect information, and is only exacerbated when information is imperfect and dispersed. We have shown how firm owners ease such conflict by opting for some cooperation, thus moderating the competitive toughness displayed by their managers. By doing so, they also influence the strategic concern for coordination of these managers and consequently the weight they put on public relative to private information. While the paper provides only an illustration of how an endogenous concern for coordination may arise in a beauty contest under dispersed information, the conflicts at stake and the way the agents deal with them may well carry over to other instances of beauty contest games in which a trade-off between the motives present in the payoffs is allowed for.

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Appendix

Computation of the marginal cooperation benefit and of the marginal information cost

Using (12) and (16), we obtain for the marginal cooperation benefit:

$$\frac{\partial F_i\left(\gamma_i,\gamma_j\right)}{\partial \gamma_i} = \left(2\left(1-K_i\right) - d\left(K_i - K_j\right)\right)\frac{\partial K_i}{\partial \gamma_i} - dK_i\left(\frac{\partial K_i}{\partial \gamma_i} - \frac{\partial K_j}{\partial \gamma_i}\right), \tag{18}$$

with

$$K_i = \frac{c\left(z_i\right)}{C\left(z_i,z_j\right)},\; z_i \equiv \frac{1+\gamma_i}{2},\; c\left(z_i\right) \equiv 1+d+dz_i,\; C\left(z_i,z_j\right) \equiv \left(1+d\right)^2 - d^2 z_i z_j, \tag{19}$$

and

$$\frac{\partial K_i}{\partial \gamma_i} = \frac{d\left(1+d\right)c\left(z_j\right)}{2\left(C\left(z_i,z_j\right)\right)^2},\; \frac{\partial K_j}{\partial \gamma_i} = \frac{d^2 z_j c\left(z_j\right)}{2\left(C\left(z_i,z_j\right)\right)^2}. \tag{20}$$

Hence,

$$\frac{\partial F_i\left(\gamma_i,\gamma_j\right)}{\partial \gamma_i} = \frac{dc\left(z_j\right)\left(\left(1+d/2\right)C\left(z_i,z_j\right) - \left(1+d\right)\left(c\left(z_i\right) + d^2\left(z_i - z_j\right)\right)\right)}{\left(C\left(z_i,z_j\right)\right)^3}. \tag{21}$$

For $d > 0$, the function $\partial F_i\left(\cdot, \gamma_j\right)/\partial \gamma_i$ is always positive at $z_i = 1/2$ and negative at $z_i = 1$, changing signs only once, at $z_i = \left(1 + \widehat{\gamma}_i\left(\gamma_j, d\right)\right)/2$ where $\widehat{\gamma}_i\left(\gamma_j, d\right)$ is the best response to γ_j under perfect information given by (7). Thus, $F_i\left(\cdot, \gamma_j\right)$ is strictly quasi-concave.

Using again (16), we obtain for the marginal information cost:

$$\frac{\partial G_i\left(\gamma_i, \gamma_j\right)}{\partial \gamma_i} = \frac{1}{\mathbb{E}\left(\theta^2\right)\alpha}\left(2\left(\kappa_i \frac{\partial \kappa_i}{\partial \gamma_i} + \frac{1+d}{\beta_i/\alpha}\kappa_i' \frac{\partial \kappa_i'}{\partial \gamma_i}\right) + d\left(\left(\kappa_i - \kappa_j\right)\frac{\partial \kappa_i}{\partial \gamma_i} + \kappa_i\left(\frac{\partial \kappa_i}{\partial \gamma_i} - \frac{\partial \kappa_j}{\partial \gamma_i}\right)\right)\right), \tag{22}$$

with

$$\kappa_i = \frac{1+d}{A\left(z_i, z_j\right)}\left(\left(1+d\right)\left(1 + \beta_i/\alpha + \beta_j/\alpha\right)K_i - \beta_i/\alpha\right), \tag{23}$$

$$A\left(z_i, z_j\right) \equiv \left(1+d\right)^2\left(1 + \beta_i/\alpha\right)\left(1 + \beta_j/\alpha\right) - d^2\left(\beta_i/\alpha\right)\left(\beta_j/\alpha\right) z_i z_j, \tag{24}$$

$$\kappa_i' = \frac{\left(1+d\right)\left(1 + \beta_j/\alpha\right) + d\left(\beta_j/\alpha\right) z_i}{A\left(z_i, z_j\right)}\beta_i/\alpha, \tag{25}$$

$$\begin{aligned}\frac{\partial \kappa_i}{\partial \gamma_i} &= \frac{\left(1+d\right) d^2\left(\beta_i/\alpha\right)\left(\beta_j/\alpha\right) z_j}{2\left(A\left(z_i, z_j\right)\right)^2}\left(\left(1+d\right)\left(1 + \beta_i/\alpha + \beta_j/\alpha\right)K_i - \beta_i/\alpha\right) \\ &\quad + \frac{\left(1+d\right)^2\left(1 + \beta_i/\alpha + \beta_j/\alpha\right)}{2A\left(z_i, z_j\right)}\frac{\left(1+d\right) dK_j}{C\left(z_i, z_j\right)},\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\partial \kappa_j}{\partial \gamma_i} &= \frac{\left(1+d\right) d^2\left(\beta_i/\alpha\right)\left(\beta_j/\alpha\right) z_j}{2\left(A\left(z_i, z_j\right)\right)^2}\left(\left(1+d\right)\left(1 + \beta_i/\alpha + \beta_j/\alpha\right)K_j - \beta_j/\alpha\right) \\ &\quad + \frac{\left(1+d\right)\left(1 + \beta_i/\alpha + \beta_j/\alpha\right)}{2A\left(z_i, z_j\right)}\frac{\left(1+d\right) d^2 K_j z_j}{C\left(z_i, z_j\right)},\end{aligned} \tag{27}$$

$$\frac{\partial \kappa_i'}{\partial \gamma_i} = \frac{\left(1+d\right) d\left(\beta_j/\alpha\right)\left(1 + \beta_j/\alpha\right)\left(1 + d + c\left(z_j\right)\left(\beta_i/\alpha\right)\right)}{2\left(A\left(z_i, z_j\right)\right)^2}\beta_i/\alpha. \tag{28}$$

It is easy to check that κ_i, κ_i', $\partial \kappa_i/\partial \gamma_i$, $\partial \kappa_j/\partial \gamma_i$ and $\partial \kappa_i'/\partial \gamma_i$ are all positive increasing functions of z_i.

In addition,

$$\kappa_i - \kappa_j = \frac{1+d}{A\left(z_i, z_j\right)}\left(\left(1+d\right)\left(1 + \beta_i/\alpha + \beta_j/\alpha\right) d\frac{z_i - z_j}{C\left(z_i, z_j\right)} - \frac{\beta_i - \beta_j}{\alpha}\right) \tag{29}$$

and

$$\begin{aligned}\frac{\partial \kappa_i}{\partial \gamma_i} - \frac{\partial \kappa_j}{\partial \gamma_i} &= \frac{\left(1+d\right) d^2\left(\beta_i/\alpha\right)\left(\beta_j/\alpha\right) z_j}{2\left(A\left(z_i, z_j\right)\right)^2}\left(\left(1+d\right)\left(1 + \beta_i/\alpha + \beta_j/\alpha\right) d\frac{z_i - z_j}{C\left(z_i, z_j\right)} - \frac{\beta_i - \beta_j}{\alpha}\right) \\ &\quad + \frac{\left(1+d\right)^3 d\left(1 + \beta_i/\alpha + \beta_j/\alpha\right)}{2A\left(z_i, z_j\right)}\frac{\left(1+d\right)^2 - d^2 z_j^2}{\left(C\left(z_i, z_j\right)\right)^2}\end{aligned} \tag{30}$$

are also both positive and increasing in z_i if $z_i \geq z_j$ and $\beta_i \leq \beta_j$.

Proof of Proposition 3

As each firm owner has a non-empty, compact, convex strategy space $[0,1]$ and a continuous payoff $F_i(\gamma_i,\gamma_j) - G_i(\gamma_i,\gamma_j)$, we need only to show that this payoff is quasi-concave in γ_i in order to prove existence of a pure Nash equilibrium of the first stage game. Strict quasi-concavity results from $\partial F_i(\cdot,\gamma_j)/\partial\gamma_i - \partial G_i(\cdot,\gamma_j)/\partial\gamma_i$ being decreasing at any positive solution γ_i to the first order condition $\partial F_i(\gamma_i,\gamma_j)/\partial\gamma_i = \partial G_i(\gamma_i,\gamma_j)/\partial\gamma_i$. By (18) and (22), this first order condition can be written as

$$\underbrace{d\left(K_j\frac{\partial K_i}{\partial\gamma_i} + K_i\frac{\partial K_j}{\partial\gamma_i}\right)}_{f^1(\gamma_i,\gamma_j)} + \underbrace{2\frac{\partial K_i}{\partial\gamma_i}}_{f^2(\gamma_i,\gamma_j)} + \underbrace{\frac{d}{\mathbb{E}(\theta^2)\alpha}\left(\kappa_j\frac{\partial\kappa_i}{\partial\gamma_i} + \kappa_i\frac{\partial\kappa_j}{\partial\gamma_i}\right)}_{g^1(\gamma_i,\gamma_j)} \tag{31}$$

$$= \underbrace{d\left(2K_i\frac{\partial K_i}{\partial\gamma_i}\right)}_{f^3(\gamma_i,\gamma_j)} + \underbrace{2K_i\frac{\partial K_i}{\partial\gamma_i}}_{f^4(\gamma_i,\gamma_j)} + \underbrace{\frac{d}{\mathbb{E}(\theta^2)\alpha}\left(2\kappa_i\frac{\partial\kappa_i}{\partial\gamma_i}\right)}_{g^2(\gamma_i,\gamma_j)} + \underbrace{\frac{2}{\mathbb{E}(\theta^2)\alpha}\left(\kappa_i\frac{\partial\kappa_i}{\partial\gamma_i} + \frac{1+d}{\beta_i/\alpha}\kappa_i'\frac{\partial\kappa_i'}{\partial\gamma_i}\right)}_{g^3(\gamma_i,\gamma_j)}.$$

Each handside of this equation is a sum of positive increasing functions of γ_i. By (19) and (20), we see that the elasticity of $K_i(\cdot,\gamma^*)$ (resp. $\partial K_i(\cdot,\gamma^*)/\partial\gamma_i$) is larger than (resp. equal to) the elasticity of $K_j(\cdot,\gamma^*)$ (resp. $\partial K_j(\cdot,\gamma^*)/\partial\gamma_i$), so that the elasticity of $f^3(\cdot,\gamma_j)$ is larger than the elasticity of $f^1(\cdot,\gamma_j)$. Clearly, the elasticity of $f^4(\cdot,\gamma_j)$ is also larger than the elasticity of $f^2(\cdot,\gamma_j)$. By (19), (23), (26) and (27), the elasticities of $\kappa_i(\cdot,\gamma^*)$ and $\partial\kappa_i(\cdot,\gamma^*)/\partial\gamma_i$ are larger than the elasticities of $\kappa_j(\cdot,\gamma^*)$ and $\partial\kappa_j(\cdot,\gamma^*)/\partial\gamma_i$, respectively, so that the elasticity of $g^2(\cdot,\gamma_j)$ exceeds the elasticity of $g^1(\cdot,\gamma_j)$. We may add the unpaired positive elasticity of $g^3(\gamma_i,\gamma_j)$. As a result, the elasticity with respect to γ_i of the LHS is smaller than the corresponding elasticity of the RHS, so that the elasticity of $\partial F_i(\cdot,\gamma_j)/\partial\gamma_i - \partial G_i(\cdot,\gamma_j)/\partial\gamma_i$ is negative. Existence of a Nash equilibrium of the first stage game, hence existence of a subgame perfect equilibrium of the two-stage game is thus proved.

To conclude the proof of Proposition 3, recall that, by (21), (22) and (30), $\partial F_i(\gamma,\gamma)/\partial\gamma_i < 0$ if $\gamma > d/(2+d)$, whereas $\partial G_i(\gamma,\gamma)/\partial\gamma_i > 0$. By continuity of both functions, either there exists a symmetric solution $\gamma^* \in (0, d/(2+d))$ to the equation $\partial F_i(\gamma,\gamma)/\partial\gamma_i = \partial G_i(\gamma,\gamma)/\partial\gamma_i$, or the first order condition takes the form: $\partial F_i(0,0)/\partial\gamma_i \leq \partial G_i(0,0)/\partial\gamma_i$ (that is, $\gamma^* = 0$). Finally, for the symmetric solution to the first order condition to be the first stage component of a subgame perfect equilibrium, firm i's expected profit must be non-negative: $F_i(\gamma^*,\gamma^*) \geq G_i(\gamma^*,\gamma^*)$, implying that $\mathbb{E}(\theta^2)\alpha$ should not be too small. $\square$