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Multiple tortfeasors in high risk industries: how to share liability?

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Abstract

We develop a model in which two firms contribute to a risk of accident, each firm being financially unable to compensate for the entire damage. One firm directly operates the risky activity (and can make an effort in care to reduce the probability of an accident occurring), while the other firm provides an input technology whose quality has an impact on the likelihood an of accident occurring. We define a second-best rule of apportionment of liability between these two firms, and we show that this optimal sharing rule is sensitive to the market relationship on the technological market; thus calling for a collaboration between agencies in charge of risk regulation and those in charge of competition issues.

Keywords: multiple tortfeasors, sharing liability, insolvency, innovation, technical diffusion, market power

JEL Classification: K13, H23

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1 Introduction

Many human activities can inflict injuries on third parties. Energy supplying (power plants), chemicals production or processing, transportation (of hazardous substances) are examples of activities that can cause accidents to their external environment. Because of these negative externalities, these activities are subject to public regulations which aim both to provide incentives to control the level of risk and compensate the victims in case of damage. A public policy tool which responds to these two objectives is civil liability.

Civil liability is the legal provision whereby a tortfeasor has to repair the damage he causes, e.g. by paying financial compensation (called damages). The threat of having to pay damages in the event of an accident provides incentives to make efforts in terms of risk prevention measures (care). So, civil liability provides both compensation for victims (*ex post* justice) and *ex ante* incentives to control risk.

However, productive or industrial processes often require the involvement of several agents. Hence, accidents may be due to several contributors, among whom the liability has to be shared.

Many industrial processes can serve to illustrate this statement. As a first example we can cite the energy supply industry, and especially gas supply. In gas power plants, the level of risk of accident depends on the level of care provided by the operator (e.g. frequent control of the gas pressure). But for a given level of care provided, the level of risk also depends on the reliability of the production process, and so on elements which are often provided by other firms (e.g. the turbine is often provided by another firm). A second illustration is the agro-food industry. The level of health risk depends on the level of care taken by the operator, for example the quality of the sterilization process. However, for a given level of care, the level of health risk also depends on the quality of inputs provided by suppliers, for example the food containers (i.e. their ability not to oxidize in contact with food). Finally, transportation can also be cited as a case involving several firms, which have an impact on the overall risk of damage. Indeed, all carriers need the involvement of (at least) one other firm to run their activity, such as the aircraft or truck manufacturer for instance. Both the quality of the aircraft / truck (which is a manufacturer's decision) and the level of care during a journey (the carrier's decision) contribute to the determination of the level of risk of accident.

In all these examples, different contributors take part within a global process which can cause injuries to third parties. Because each contributor has an impact on the level of risk, each contributor has to be encouraged to provide a "sufficiently high" level of diligence to optimally mitigate the risk. And so the question arises of the optimal apportionment of liability, with the aim of providing optimal incentives to all contributors.

How can liability be shared? Two main sharing rules are currently used in common law countries: *joint and several* liability, and *non-joint* liability. Joint and several liability is characterized by two features: (i) if the global solvency of all contributors (taken together) exceeds the amount of damages to be paid, the victims are sure to be fully compensated; (ii) injured parties can recover the entire damages from any torfeasor: one torfeasor may be forced to pay for all the damages[1]. These two points imply that when a tortfeasor is financially unable to pay for his part of the judgement, the remaining part falls to the other solvent parties (which, as a result, pay more than their share). In the case of non-joint liability, any tortfeasor only has to pay for his share of liability (as long as he is financially able to do so): solvent parties do not pay for the insolvent ones.

In the USA, up to the 1980s joint and several liability was the default rule of apportionment. However, since the 1980s, this sharing rule has been disputed, especially in the case of environmental damage.

Joint and several liability, by allowing the victims to recover the full damages from only one defendant -the most solvent one-, provides the victims with a (relatively) high likelihood of success[2] in litigation (see Lee *et al.* (1994), p 298). This effect, combined

[1] And then he has to take action against the other tortfeasors, to be paid off for their share in liability.

[2] In case of injuries to workers, or to the environment, the increasing use of strict liability also explains the increase in the number of litigations (see Ringleb & Wiggins (1990) for the case of work exposures). Applying strict liability serves to establish the tortfeasor's liability without having to prove before the

with the (more than expected) difficulty in evaluating damage to the environment, led to an unexpected increase in debts for liabilities in some "risky" sectors (see Katzman (1998)). This "liability crisis" encouraged various partners of these "risky firms" to protect themselves, thus causing economic difficulties for these industries: liability insurers developed exclusions for damage resulting from pollution (see Anderson (1998)), and banks became more reluctant to grant loans[3].

To countervail these difficulties, in the 1980s some states instigated a "tort reform" (see Lee *et al.* (1994)), limiting the application of joint and several liability to economic damage: non-economic damage (such as damage to the environment) were subject to non-joint liability. But nowadays there is no consensus among the US states, and some federal laws -such as CERCLA[4]- still enforce joint and several liability: the debate is still open.

In the paper at hand, we question how to *efficiently* apportion liability between several tortfeasors, and we adopt a *normative* perspective. Efficiency refers here to the ability to provide optimal incentives, to all contributors, to make efforts in order to optimally mitigate the risk of accident. So we put aside other questions which arise when dealing with the apportionment of liability, such as the fairness of apportionment between contributors, or the fairness of the victims remedy. Our analysis is a fully normative one in the sense that we do not seek to compare different existing sharing rules, but we instead aim to propose an efficient rule which responds to a given and widespread situation. In particular, we only consider the application of strict liability (even if negligence remains the default liability rule for most cases, except the most dangerous activities (see Shavell (2004))). Nevertheless, we want the suggested rule to be enforceable without having to make far-reaching changes to the prevailing legal corpus

Court any deviation from a standard of due care. This reduces the victim's burden of proof, and so it increases the likelihood of success in litigation.

[3]Two reasons explain this reluctance: first, exclusions in liability insurances increase the financial burden on firms. This increases the likelihood of bankruptcy (and so of default) in case of accident. Moreover, some environmental laws -such as CERCLA- extends liabilities for environmental damage to the financial partners (see Pitchford (1995)).

[4]CERCLA means "Comprehensive Environmental Response, Compensation, and Liability Act". Among other provisions, CERCLA provides for liability of persons responsible for releases of hazardous waste on abandoned industrial sites.

(and, especially, without altering the limitation of liability principle).

The type of situation we are looking to regulate is the following: we consider two contributors, each one being potentially insolvent (i.e. financially unable to compensate, alone, for the entire damage), but taken together the two contributors are sufficiently solvent to pay for all the damage. We focus on the case of two firms already engaged in socially desirable activities. One of them can cause an accident during its activity. This firm, which is the "direct" author of the damage, needs a "technology" as an input to operate. This technology is provided by the second firm, which does not directly cause the damage. But the "quality" of the technology it provides has an impact on the *likelihood* of an accident occurring. The first firm, called the "operating firm", is necessary and sufficient for the damage to occur. The second one, called the "innovative firm", which provides the technology, is neither necessary nor sufficient to cause the damage. As outlined above, providers-operators relationships fit with this kind of situation.

Taking into account the possibility of insolvency in case of damage is a key feature of our analysis. No firm is able, alone, to compensate for the full damage. In such situations, since Shavell (1986) we know that civil liability fails to provide optimal incentives for risk mitigation. The intuition is the following: because of the legal provision of limited liability, firms' financial liabilities are capped to their wealth. So, for instance, a firm which has a financial wealth of USD 1 million and causes a damage of USD 10 millions has only to pay USD 1 million. As a consequence, part of the damage is externalized from the firm, and this leads the firm to make insufficient risk prevention efforts. In this paper, we propose to take this feature into account and to highlight its impact on the apportionment of liability.

The second key feature we take into account is the market relationship between the two tortfeasors. Indeed, we assume that the "direct" author buys an input technology from a provider (who is an "indirect" author). The selling price results from a bargain between the two firms, and it has an impact on their respective decision making. Depending on the degree of competition on the technological market, the technology provider (or innovator) is able to extract a larger or lesser part of the operating firm's surplus from the new technology. This directly alters the incentives to innovate. As a consequence, the optimal sharing rule, which aims to provide optimal incentives, is also sensitive to the degree of competition on the technological market.

To our knowledge, such a setup has never been investigated by the literature.

When reviewing the different contributions, we see that there is a vast range of different situations involving several parties causing a damage; each situation having *its* optimal apportionment. To summarize, these situations can be differentiated according to three criteria: (i) the type of acts which have to be regulated, (ii) the chronology of acts, (iii) the way in which the different acts combine to lead to injury.

The type of acts refers to the distinction introduced by Miceli & Segerson (1991) between *infra marginal* and *marginal* issues. The question here is to distinguish whether acts are of binary order (to engage or not to engage in an activity. See Shavell (1985, 1987), Young *et al.* (2004, 2007)), or whether they refer to efforts the degree of which lies in continuous scales (efforts in care, safety, innovation, etc., knowing that activity is engaged. See Kornhauser & Revesz (1989)). The way in which acts combine refers to the distinction introduced by Young *et al.* (2007) between acts *in series* and acts *in parallel*: in the first case, the action of all parties is necessary to cause a damage, while in the other case each party is sufficient, in itself, to cause the damage.

In our case, we consider efforts in a continuous scale[5]. Moreover, our setup cannot fit with the distinction introduced by Young *et al.* (2007) between acts in series and acts in parallel: the operating firm which "directly" causes the damage is necessary and sufficient: it can cause the damage alone, and without its activity no damage can occur. However, a damage can occur without the innovative firm: it is not necessary. And it cannot cause a damage alone: it is not sufficient. Its activity only alters the probability of the operating firm causing a damage. Regarding the case of insolvency, we note that Kornhauser & Revesz (1990) analyze the apportionment of liability when actors are in-

[5]We do not consider the question of whether to engage or not in the activity (which is a question of binary order). We suppose the two activities to be socially desirable, and already started.

solvent, but they consider a different framework: it is not the probability of an accident occurring but the magnitude of damage which depends on the contributors' efforts.

To sum up, in the framework of a large-scale risk of accident (with high potential damage), we look at how to share liability between two firms, an operator and an innovator, providing the output technology, in such a way as to provide optimal incentives for care and R&D, both of them having an impact on the level of risk. Each firm benefits from limited liability and cannot pay alone for the entire damage, and the two firms have a market relationship which is affected by the degree of competition on the technological market. We adopt a normative perspective, aiming to maximize a social objective.

We define a second-best sharing rule, serving to compensate victims and to provide optimal incentives for care and R&D given the individual insolvency constraints. This sharing rule is a kind of non-joint liability, which depends on both individual wealth and on the degree of competition on the technological market. In a case of monopoly on this market, maximum liability has to be applied to the operating firm: the innovative firm is able to extract the entire social surplus from R&D, independently of the apportionment of liability. However, as soon as competition (even imperfect) takes place, we show that optimal apportionment progressively moves toward a higher degree of liability for the innovative firm. So, there is evidences that the market characteristics alter the incentives to provide diligence to mitigate the level of risk. This calls for a collaboration between agencies in charge of risk regulation and those in charge of competition issues, in the definition as well as in the enforcement of civil liability.

The paper is structured as follows. Section 2 presents the model. Section 3 provides the first-best situation, used as a benchmark, and then the second-best solution is derived in section 4. Section 5 provides a numerical illustration, and section 6 concludes.

2 Basic assumptions

We consider two sectors, production and innovation. For each sector we consider a representative firm: on the innovation market an innovator (denoted I) makes an effort in

R&D to produce an output technology which can be sold to the operative firm (denoted O) from the production sector. Firm O is engaged within an activity which can cause a damage H to Society. But it can undertake an effort in risk prevention to reduce the likelihood of an accident occurring.

We denote x the level of risk prevention, which is chosen by the firm O. Implementing x is costly: $c.x$, with $c > 0$ the unit cost of prevention. Risk prevention reduces the probability of causing a damage, at a decreasing rate. Moreover, the probability of an accident occurring also depends on the efficiency of the output technology used by the firm O.

The efficiency of the output technology is denoted by e, which reflects the degree of R&D that is embedded within this technology. The firm O is initially endowed with a "default" technology, which is characterized by a degree $e = 0$ of R&D[6]. But this firm has the possibility of buying a new technology, sold by firm I. This new technology is characterized by a degree $e \geq 0$ of R&D, which is a decision by firm I. R&D is costly (cost $k.e$, with $k > 0$), but it reduces the probability of an accident occurring, at a decreasing rate.

So we suppose the probability $p(x, e)$ of causing a damage to satisfy:

$$p(x, e) = f(x) + g(e)$$

where $f_x < 0$, $g_e < 0$, $f_{xx} > 0$, $g_{ee} > 0$ and so we finally obtain: $p_x < 0$, $p_{xx} > 0$, $p_e < 0$, $p_{ee} > 0$. We also assume $g\ (g\ ')^{-1}$ to be a convex function on $\mathbb{R}$. The risk of accident can be represented by the lottery:

$$\widetilde{H} \equiv [0, H; (1 - p(x, e)), p(x, e)]$$

As indicated in the introduction, a damage can occur during firm O's activity. But the probability of a damage occurring is affected both by firm O's and by firm I's decisions.

[6] We also tested the case in which firm O is initially not endowed with any technology. This does not modify the results of the paper. As mentioned later, the default technology can be the result of a preceding transaction between the two firms I and O: in that case, a new technology could be seen as an upgrade of the default technology.

A "defect" in the quality/reliability of the technology (firm I's decision) as well as a "lack" of care during the production process (firm O's decision) can explain a damage occurring during firm O's activity. The first element contributes to $g(e)$ in $p(x,e)$, the second one contributes to $f(x)$ in $p(x,e)$. Prevention (x) and R&D effort (e) both reduce the probability $p(x,e)$ of an accident occurring, but their effects are independent[7] from each other: $p_{xe} = 0$.

We suppose that entering into an activity allows a firm to perceive a revenue $R_i, i = O, I$, from which it can finance its $R\&D$ activities (firm I), its prevention activities or the purchase of a new technology (firm O). Moreover, each firm is endowed with a level of wealth, $W_i, i = O, I$, which can be confiscated for compensation in case of accident.

When an accident occurs, civil liability applies. We suppose that a strict liability[8] rule holds and that each firm benefits from limited liability: damages cannot exceed the firm's wealth W_i.

We consider that each firm, at its individual level, is unable to repair the whole damage: $W_i < H$. This is the property we call *individual insolvency*. However, when the wealth of the two firms is brought together, sufficient funds can be raised to compensate for all the damage: $\sum_i W_i > H, i = O, I$. From Shavell (1986), we know that individual insolvency, leading to partial risk internalization, provides suboptimal incentives to prevent a risk of accident. So we aim to determine how to share liability between the two firms in such a way, ideally, as to provide optimal incentives for prevention and innovation, and to fully compensate victims. D_i is the amount in damages a firm i has to pay

[7] This means that a variation in x has no impact on the e best-response, and, similarly, a variation in e has no impact on the x best-response.

[8] Strict liability rule is opposed to negligence rule, according to which liability is subject to the deviation from a standard of due care. Under strict liability, the liability applies whenever an accident occurs (whatever the firm's behavior). There are two reasons for supposing strict liability: first, strict liability is increasingly used in the framework of damage to the environment and to regulate large-scale risks (see CERCLA, or Shavell (2004), chapter 2, quoting Fleming (1983) and Keeton *et al.* (1984) on the use of strict liability in the case of harms arising from "abnormally dangerous" or "ultrahazardous" activities); second, strict liability serves to put aside the question of the contribution to negligence (and so the question of the fairness of apportionment), thus focusing on the efficiency issue.

in case of accident, and D is the overall amount of penalties paid by the two firms in case of accident (i.e. $D = \sum_i D_i$, $i = O, I$). This overall amount can be different from H .

Now we have first to determine the first-best situation, towards which we should strive.

3 First-best

First-best decisions are derived by maximizing a social welfare function. Taking the point of view of a benevolent, omniscient and omnipotent dictator, this social welfare function includes all social costs and benefits from activity. Considering that both innovative and productive activities are socially desirable, the social welfare function to maximize can be written as:

$$SW(x,e) = R_I + R_O + W_I + W_O - k.e - c.x - p(x,e)H \tag{1}$$

with $p(x,e) = f(x) + g(e)$. This social welfare is the sum of the firms' revenues and wealth minus the cost of R&D, the cost of risk prevention, and the total expected cost of damage[9].

First-best values for x and e can be derived in the following manner. The first-best level of care, x^{**}, satisfies:

$$\begin{aligned} &SW_x(x,e) = 0 \Leftrightarrow -c - Hf'(x) = 0 \\ &\Leftrightarrow x^{**} = (f\,')^{-1}\left[\frac{-c}{H}\right] \end{aligned} \tag{2}$$

[9]The purchase of a new technology (by firm O, from firm I) is a transfer between innovative and productive activities. It is not relevant to take it into account from a global viewpoint, which is a "fusion" of all private interests.

and the first-best effort in R&D, e^{**}, satisfies:

$$SW_e(x,e) = 0 \Leftrightarrow -k - Hg'(e) = 0$$
$$\Leftrightarrow e^{**} = (g\,')^{-1}\left[\frac{-k}{H}\right] \tag{3}$$

Below, we check that a higher level of damage H requires a higher level of care and R&D, and so it requires a lower probability of an accident occurring. Indeed, at the first-best equilibrium, the probability of causing a damage:

$$p(x^{**}, e^{**}) = p(H) = f\left((f\,')^{-1}\left[\frac{-c}{H}\right]\right) + g\left((g\,')^{-1}\left[\frac{-k}{H}\right]\right)$$

is decreasing in H.

We underline the "double dimension" of this problem: in "classical" studies from economic analysis of liability (accident models) there is only one variable that defines the risk prevention strategy. In our context both prevention *and* R&D take part in a global risk prevention strategy. Both of these two variables have to be set at their first-best values (x^{**}, e^{**}) to reach the first-best situation.

In an "ideal" world, where the firms have no solvency constraint, the first-best situation could be reached by forcing each firm to pay H in case of accident. So there would be overcompensation of victims. An alternative mechanism could be to use (x^{**}, e^{**}) as policy variables: if neither firm adopts (x^{**}, e^{**}), penalties could be imposed on the firms. When only one firm adopts the first-best value, then the unilaterally deviant party bears the entire damage.

But here these first-best mechanisms are not enforceable because we assume that each firm does not have sufficient wealth to pay for the full damage, and they benefit from limited liability[10] ($W_i < H$). Under such an assumption, in case of a very low level of wealth it may be easy for a firm to deviate from the first-best solution, if it is (privately) too costly to comply with it. As a consequence, it seems impossible

[10]No extension of liability to partners, or shareholders, is considered.

to implement these first-best mechanisms[11] to provide socially optimal incentives for prevention and innovation. A second-best policy has to be designed.

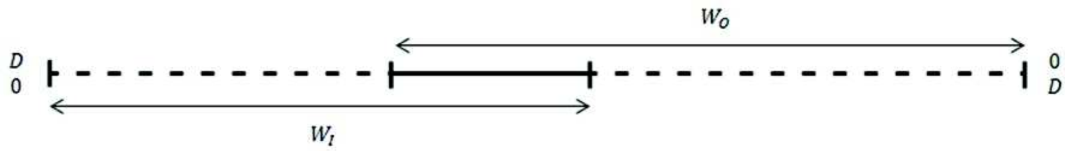
4 Sharing liability: the second-best solution

We aim to investigate how to share liability between the two firms in order to provide, if possible, optimal incentives for prevention and innovation. In a first step, we have to introduce the scheme of liability in this particular context. For the sake of simplicity, we assume that $D = H$ (complete compensation).

4.1 Introducing liability sharing

Given the individual insolvency constraints, the apportionment of liability can be described in the following manner.

Figure 1. Liability with multiple actors and individual insolvency



Bear in mind that we set $D = H$, and we suppose that perfect information takes place. Each firm is subject to an individual insolvency constraint ($W_i < H$) but, brought together, the two firms are able to compensate for the damage in full (i.e. $W_O + W_I > H$). In that case, taking into account the individual insolvency constraints, we know that the maximal individual payment in damages is capped to individual wealth: $\max\{D_i\} = W_i$, $i = I, O$. As a consequence, only the sum of individual wealth ($W_O + W_I$) in excess of the amount of global damages (D) can be subject to sharing between the two firms ($W_O + W_I - D$, i.e. the full black segment in figure 1). Proposition 1 shows that there is always a share $[W_O + W_I - D]\,\theta$, with $\theta \in [0, 1]$ of this amount that is allocated to firm I and a share $[W_O + W_I - D]\,(1 - \theta)$ that is allocated to firm O.

[11] Moreover, using (x^{**}, e^{**}) as policy variables suppose a perfect observation of the public authorities on these variables. Informational costs should also be taken into account.

It is important to note that θ in this proposition only applies to the "sharing zone", $W_O + W_I - D$, and not on the global value of D. Indeed, because of individual insolvency constraints, individual payments are bounded: $D_i \in [D - W_j, W_i]$, $i = O, I$, $j = O, I$, $i \neq j$. So, $\theta = 0.5$ does not mean that 50 percent of the global damages D have to be paid by each firm. It means that 50 percent of the amount to be shared, $W_O + W_I - D$, has to be paid by each firm[12].

Proposition 1 ***The sharing proposition***

Two firms O and I, with initial endowments W_O and W_I, have to collectively pay a debt D. Each one is individually unable to pay the entire debt, but the sum of their wealth allows them to pay D, i.e.:

$$\begin{aligned} W_O &< D \\ W_I &< D \\ W_O + W_I &> D \end{aligned}$$

In that case, there is always a unique value $\theta \in [0, 1]$ for which firm O's payment is necessarily:

$$D_O = W_O - [W_O + W_I - D]\,\theta$$

and firm I's payment is:

$$D_I = W_I - [W_O + W_I - D]\,(1 - \theta)$$

Proof.: see the appendix.

We remark that θ is a "kind of" non-joint liability, in the sense that each firm only pays for its share of liability (this share applying only to the sharing zone, $W_O + W_I - D$).

[12] Another illustration could be the following case: if $\theta = 0$, firm I does not contribute to the amount to be shared. A maximum payment is allocated to firm O, which pays all its wealth W_O, and firm I only pays the remainders $D - W_O$.

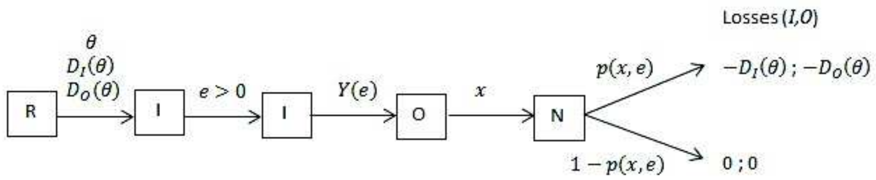
But, as we will see later, this apportionment depends on parameters related to the characteristics of the two defendants, and on parameters related to the relationship between them.

Now we have to provide more details about how the different parties interact with each other.

4.2 Timing

The schedule can be described by the following figure.

Figure 2. Decision tree



In a first step, the regulator (player R in figure 2) announces the sharing rule $(\theta, 1-\theta)$. This rule is common knowledge to the two firms I and O. Recall that from θ, we can deduce the amount in damages each firm has to pay in case of accident: $D_I = D_I(\theta)$, $D_O = D_O(\theta)$.

Next, firm I has to choose its effort e in R&D, which allows it to offer a new technology to firm O for a price $Y(e)$; this price results from a bargain between them.

Finally, firm O chooses its level of care and, in case of accident the regulator enforces liability.

Now we turn to determine private equilibria for care and innovation, depending on the liability sharing rule θ. The model is solved backward.

4.3 Private equilibria

In a first step we determine the private level of prevention, for a technology with a given level e in R&D. Firm O's objective is:

$$\max_{x} \widetilde{\Pi}^{O}(x,e,\theta) = R_O + W_O - Y(e) - c.x - p(x,e).D_O$$

with $p(x,e) = f(x) + g(e)$ and $D_O = W_O - [W_O + W_I - D]\,\theta$

We find one private level of prevention, x^*, which satisfies:

$$\begin{aligned}
&\widetilde{\Pi}^{O}_{x}(x,e,\theta,D) = 0\\
&-c - D_O f'(x) = 0\\
&x^* = (f\,')^{-1}\left[\frac{-c}{D_O}\right]\\
&x^* = (f\,')^{-1}\left[\frac{c}{[W_O + W_I - D]\,\theta - W_O}\right] \qquad (4)
\end{aligned}$$

Remark 1: the second-order condition is satisfied :

$$\widetilde{\Pi}^{O}_{xx}(x,e,\theta,D) = -D_O f''(x) < 0$$

and since$(f\,')^{-1}$ is an increasing function, x^* is a decreasing function of θ. Moreover, we have :

$$x^* = (f\,')^{-1}\left[\frac{-c}{D_O}\right] \leq x^{**} = (f\,')^{-1}\left[\frac{-c}{D}\right]$$

In a second step, before determining the private level e^* in R&D, it is necessary to determine at what price a new technology can be sold.

This price results from a bargain between the two firms. We suppose the bargaining power to be correlated with the degree of competition on the R&D market. The two extreme cases are the following:

(i) when perfect competition holds on the R&D market, firm I is a price taker so that the selling price of the new technology is independent from the effort in R&D. So we have:

$$Y(e) = Y_0$$

(ii) in case of monopoly on the R&D market, firm I is a price maker so that it is able to extract all firm O's surplus from R&D. So we have:

$$Y_I(e) = D_O(g(0) - g(e))$$

which corresponds to firm O's net benefit from R&D (relatively to using its own default technology, which has no R&D embedded).

In other words, in case of perfect competition on the R&D market (case (i)), firm I is under the threat of other competitors, which allows firm O to negotiate the price. In case of monopoly, firm I fixes the maximum selling price, which makes firm O indifferent between buying or not buying the new technology. In case of an imperfectly competitive market, there is a mixed situation where the selling price is:

$$Y(e) = \lambda Y_0 + (1 - \lambda)Y_I(e)$$

$$Y(e) = \lambda Y_0 + (1 - \lambda)\left[D_O(g(0) - g(e))\right] \text{ with } 0 \leq \lambda \leq 1$$

where λ is an index of firm O's bargaining power: when $\lambda = 0$ the R&D market is a monopoly and firm I can charge the maximum selling price (no bargaining power for firm O); when $\lambda = 1$ the R&D market is perfectly competitive and the price is an exogenous variable (maximum bargaining power for firm O).

As a consequence, in a last step, we can deduce the private effort in R&D, e^*, which is undertaken by firm I. This effort responds to the following problem:

$$\max_{e} \widetilde{\Pi}^I(x^*, e, \theta) = R_I + W_I + Y(e) - ke - p(x^*, e)D_I$$

with: $Y(e) = \lambda Y_0 + (1-\lambda)\left[D_O(g(0) - g(e))\right]$, $p(x^*, e) = f(x^*) + g(e)$

And so, e^* satisfies $\widetilde{\Pi}^I_e(x^*, e, \theta) = 0$, which can be written as:

$$\widetilde{\Pi}^I_e(x^*, e, \theta) = (1-\lambda)\left[D_O g'(e)\right] - k - D_I g'(e) = 0$$

$$g'(e^*) = \frac{k}{-D_O(1-\lambda) - D_I}$$

$$e^* = (g\ ')^{-1}\left[\frac{k}{\lambda D_O - D}\right] \tag{5}$$

Remark 2:

(i) We know $g'(e^*) \leq 0$ and the second-order condition is satisfied : $\widetilde{\Pi}^I_{ee}(x^*, e, \theta) < 0$ since $g^{"}(e) > 0$. Moreover $g'(e^*) = \frac{k}{\lambda D_O - D} \leq g\ '(e^{**}) = \left[\frac{k}{D}\right]$. So we can deduce: $e^* \leq e^{**}$

(ii) e^* depends on θ. Starting from (5) we have:

$$e^* = (g\ ')^{-1}\left[\frac{k}{\lambda D_O - D}\right]$$

$$e^* = (g\ ')^{-1}\left[\frac{k}{\lambda(W_O - [W_O + W_I - D]\,\theta) - D}\right]$$

And so e^* is an increasing function of θ since $k > 0$.

(iii) When $\lambda = 0$ (monopoly of firm I on the R&D market), the equilibrium effort e^* is equal to the first-best one, e^{**}.

Point (iii) of Remark 2 is easily interpretable: in case of monopoly, firm I has the ability to set its own selling price. Its only constraint is firm O having an interest in buying the new technology. So the maximum selling price can be fixed, so that firm O is indifferent between buying or not buying the new technology. Through this maximum selling price, firm I internalizes the whole social benefit from its R&D activity; encouraging it to provide an effort equal to e^{**}.

From Remark 1 and 2, we can see that for any sharing rule $(\theta, 1-\theta)$, we have : $x^* < x^{**}$, $e^* \leq e^{**}$. It follows:

$$p(x^*, e^*) > p(x^{**}, e^{**})$$

Proposition 2

For any sharing rule $(\theta, 1-\theta)$, the probability of an accident occurring with private equilibria $p(x^*, e^*)$ is higher than probability $p(x^{**}, e^{**})$ of causing damage when both prevention and R&D are set at their first-best values.

4.4 The second-best sharing rule

Now we turn to determining the (second-best) optimal sharing rule θ^*. This consists of finding the value of θ which maximizes the social welfare function $SW(x, e)$(as defined by (1)), by taking into account the private equilibria x^* and e^* (respectively defined by (4) and (5)). So we have:

$$\max_{\theta} SW(x^*, e^*, \theta) = R_O + R_I + W_O + W_I - c.x^* - k.e^* - p(x^*, e^*).D$$

In the following Proposition, we have determined the second-best sharing rule θ^* when f'' and g'' are bounded, and when W_O and W_I are sufficiently close to (but still lower than) D.

Proposition 3 ***The second-best rule proposition***

(i) The second best policymaker's problem:

$$\begin{aligned} & \max_{\theta} \left[SW(x^*, e^*, \theta)\right] \\ e^* \quad \in & \quad \arg\max_{e} \widetilde{\Pi}^I(x^*, e, \theta) \\ x^* \quad \in & \quad \arg\max_{x} \widetilde{\Pi}^O(x, e, \theta) \end{aligned}$$

has an optimal sharing rule $\theta^* \in [0,1[$ and optimal equilibria x^* and e^* such that

$$g'(e^*) = \frac{k}{\lambda(W_O - [W_O + W_I - D]\,\theta^*) - D} \text{ and } f'(x^*) = \frac{c}{W_O - [W_O + W_I - D]\,\theta^*}$$

when the values of W_O and W_I are sufficiently close and high.

(ii) In the specific case of monopoly on the R&D market ($\lambda = 0$), the optimal sharing is $\theta^* = 0$ (i.e. maximum liability on firm O).

Proof.: see the appendix.

Point (ii) of Proposition 3 directly follows from Remark 2 point (iii): in case of monopoly, whatever the sharing rule θ, firm I is able to internalize the whole social benefit from its R&D activity, so that $e^* = e^{**}$. So the only problem to overcome is the lack of incentives for risk prevention provided to firm O. This is a well-known problem of partial risk internalization (e.g. Shavell (1986)), which is fixed through a higher payment in damages in case of accident: x^* decreases with θ. So, in case of monopoly, the second-best situation requires a maximum liability on firm O.[13]

Note that we check the stability of these equilibria: for a given θ^*, the firms chooses the corresponding x^* and e^* and have no interest in deviating, by adopting a lower effort in prevention or in R&D[14].

4.5 The role of the R&D market

Now we analyze how the second-best rule of apportionment is sensitive to the degree of competition on the R&D market.

Proposition 4 *The market power proposition*

The second-best sharing rule is sensitive to the market relationship which links the two firms.

[13]But x^* remains lower than x^{**} because of firm O's insolvency: $W_I < D = H$

[14]The payment of a fine in case of deviation - which could be observed *ex post*, when parties are before the Court - ensures a strict stability of equilibria. Demonstration is available upon request.

The higher the degree of competition on the R&D market, the higher the share of liability to apportion to the innovative firm (firm I).

Proof.: see the appendix.

Starting from the monopoly case developed in Proposition 3 point (ii), the intuition is the following: as soon as the degree of competition on the R&D market increases, the ability of firm I to set its own selling price decreases. Its capacity to absorb the whole social benefit from its R&D activity decreases, so that a problem of partial internalization appears. This lessens the incentives to do $R\&D$, and so it is socially desirable to (partially) offset this lack of incentives through an increase in firm I's share of liability.

We have identified second-best sharing rules, depending on the degree of competition on the R&D market (or, similarly, depending on the relative bargaining power between the two parties). We now analyze the existence of such situations more deeply by developing a numerical example.

4.6 A numerical example

In order to see to what extent our sharing rule can be applied (or, similarly, to see to what extent the conditions of application of our sharing rule are restrictive or not), we provide a numerical illustration.

For this, we suppose the probability of an accident occurring to be as: $p(x,e) = \frac{exp(-\alpha x)+exp(-\beta e)}{\gamma}$, $\gamma \geq 2$. This specification satisfies the derivatives properties which we posed in our assumptions, including the separability between x and e. We redevelop our theoretical analysis for this special case. We find the following specific result.

Result 1 *Consider the specific case where the probability of an accident occurring satisfies:* $p(x,e) = \frac{exp(-\alpha x)+exp(-\beta e)}{\gamma}$.

Except for the case of monopoly (i.e. $\lambda \neq 0$*), there is an optimal apportionment* $\theta \in]0,1[$ *if the following conditions are met:*

(i) $\frac{\alpha/c}{\beta/k}$ *is higher than* $1/3\frac{(\lambda-1)^2}{2\lambda+1}$

(ii) The level of individual wealth W_O *and* W_I *are sufficiently high.*

Proof.: see the appendix.

Point (i) of Result 1 teaches us that for an interior solution to exist, in this particular case, it is necessary for the efficiency - cost ratio of prevention to be sufficiently higher than that of innovation. Because $\lambda \in [0,1]$, $1/3\frac{(\lambda-1)^2}{2\lambda+1} \in [0,1/3]$. And so, for instance, if the efficiency - cost ratio of prevention is equal to the efficiency - cost ratio of innovation, this necessary condition is satisfied. Note that point (ii) confirms one of the results of the (more general) theoretical analysis: an interior solution requires W_O and W_I to be sufficiently close to D. Two reasons explains this feature: first, the higher the sum of the individual wealth, the higher the "sharing zone" $W_O + W_I - D$. Secondly, $\theta\ in]0,1[$ means that the situation requires "balanced incentives" to be provided: there is no need to provide maximum incentives to one agent (and minimum incentives to the other one). In a way, the lack of incentives is similar for the two agents. All else being equal, incentives are provided through the size of the wealth: the more an agent can lose in case of accident, the higher the incentives to avoid the accident. So, all else being equal, the more similar the agents' wealth, the more balanced (between the two agents) the incentives to be provided.

Then we numerically test for the impact of the degree of competition on the R&D market, λ, on the optimal apportionment θ^*. For this we calculate, for a given set of parameters, the private efforts in care and R&D, depending on the sharing rule θ. By including these private reactions in a function of social welfare $SW(\theta)$, we deduce the optimal value of θ (which maximizes $SW(\theta)$, given the functions $x^*(\theta)$ and $e^*(\theta)$). We repeat this methodology for different values of λ. We pose the following values as our basic scenario[15].

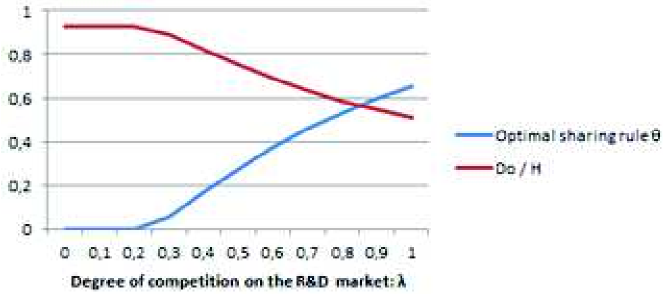
[15]For that specific case, the optimal apportionment of liability is: $\theta^* = 0,653$. This means that it is optimal to allocate 65.3% of the sharing zone to the innovator, and 34.7% to the operator. Recall that the sharing zone is: $W_I + W_O - H = 1000 + 1300 - 1400 = 900$. It follows that: $D_O = 400 + 0.347 * 900 = 712.3$, which represents 50.88% of the total damage H, and $D_I = 100 + 0.653 * 900 = 687.7$, i.e. 49.12% of the total damage H.

Table 1: Parameters values for the basic scenario

$H = 1400$	$R_O = 100$	$c = 2$	$\alpha = 0,5$
$W_O = 1300$	$R_I = 100$	$k = 1,98$	$\beta = 0,55$
$W_I = 1000$		$\lambda = 1$	$\gamma = 2$

The figure below depicts the evolution of the optimal sharing rule (and, as a consequence, the evolution of the firm O's relative contribution on the reparation of H) depending on the degree of competition θ.

Figure 3. Optimal sharing rule θ^* and share of global liability ($\frac{D_O}{H}$) depending on the degree of competition λ



This numerical example confirms the theoretical prediction that in case of monopoly on the R&D market, it is socially desirable to put a maximum liability on the operating firm. So $\theta = 0$ and firm O pays the maximum amount of damages which can be assigned to it: $W_O = 1300$. H being equal to 1400, the share of damages assigned to firm O, $\frac{D_O}{H}$ equals $92,86\%$ (1300/1400). Then, in this example we find that a maximum liability on the operator remains optimal for some (low but) positive degrees of competition and then, as the degree of competition increases, the allocation of liability progressively switches towards the innovator. However, in case of perfect competition ($\lambda = 1$), the optimal value of θ is not equal to 1: imperfect risk internalization requires maintaining incentives for the operator, all the more so as prevention and innovation have relatively similar efficiency - cost ratios: if prevention were less efficient in risk mitigation (relatively to R&D, because of a high marginal cost c for instance), then it would be socially desirable to allocate more liability to the innovator: the value of θ would be higher.

5 Conclusion

In this paper we analyze how to apportion liability between two firms which have an impact on the level of a large-scale risk of accident. More precisely, one firm provides the other one with an output technology, and this latter firm can cause damage to society. We adopt a normative perspective, and question how to share liability in order to provide second-best incentives to each contributor, knowing that each one is subject to potential insolvency (but they can, together, repair the full damage).

The second-best sharing rule we find, based on a strict liability system, is a “kind of” non-joint liability with an apportionment which depends (i) on individual wealth and (ii) on the market relationship between the two tortfeasors.

Our sharing rule only applies to the excess of global wealth (of the two contributors) with respect to the amount to be paid for the victims’ compensation. Moreover, the degree of competition on the technological market (on which the two firms interact) is of paramount importance in the definition of the second-best sharing rule.

In case of monopoly of the innovative firm, a maximum liability has to be apportioned to the operative firm. Because of its high market power, the innovative firm is able to extract all the social benefit from R&D: its incentives for R&D are optimal, whatever the apportionment of liability. Inefficiency is only due to a suboptimal level of care, and so a maximum liability has to be assigned to the operative firm to provide maximum incentives for care. In case of a higher degree of competition (but not perfect competition), a second-best sharing rule exists (and it is not necessarily a corner-solution).

This relationship between the market characteristics and the optimal definition of the liability system leads to (at least) two remarks.

First, considering the specific case of monopoly on the R&D market (which calls for a minimum liability on the innovator), we could consider this result in combination with the rationale of the patenting system (which gives an innovator a monopolistic position), in such a way as to foster the incentives provided by this system.

A second consideration calls for a greater cooperation between agencies in charge of risk regulation and those in charge of competition issues: having knowledge about the degree of competition on the provider's market (λ in our paper) is a necessary condition to properly configuring the liability system. Here, the fundamental idea is to build a common-knowledge "liability formula" which gives the amount to be paid for liability by each defendant, depending on the degree of competition on the provider's market (among other parameters). The expertise of the competition authority should be necessary *ex ante* and *ex post*, both *ex ante* to determine the general formula and then, *ex post*, to help the Court to determine the amounts to be paid by each defendant (depending on the degree of competition which prevailed on the market). The value of λ could be deduced from the relative distance of the (observed) market price from perfect competition price (i.e. the minimum cost of production) and monopoly price (maximum selling price, including all social benefits from the innovation). Remark that collaboration is also extended to regulation agencies, since the optimal apportionment also depends on characteristics of the production technologies.

Whatever the degree of competition on the market, we define a strict-based liability system which provides incentives for the two parties to adopt a second best equilibrium (in terms of care and R&D). So we adopt a normative perspective, since no such system is applied anywhere. But such a system could be applied without any "dramatic" change in the legal corpus, especially by maintaining the limited liability principle.

This paper is a first contribution, which calls for further research. We suppose only one "innovator" (or provider), while production processes often involve several input providers. A first natural extension could be to extend our model to a sharing rule with N contributors. Also, we focus on the classical objective of maximizing social welfare. However, alternative objectives could be imagined, such as minimizing the level of risk (in the perspective of the Precautionary Principle): it would be interesting to see how our liability rule is sensitive to a change in the social objective.

Appendix

Proof of Proposition 1

Let X_i to be the amount of i's remaining wealth (after its contribution D_i to the payment of D), and let θ_j to be the share of j's contribution to the amount to be shared $(W_O + W_I - D)$. We have:

$$\theta_j = \frac{X_j}{W_O + W_I - D}.$$

which leads to: $X_j = \theta_j(W_O + W_I - D)$, and:

$$X_O + X_I = (\theta_O + \theta_I)(W_O + W_I - D) \tag{6}$$

Finally we have:

$$D = D_O + D_I = (W_O - X_O) + (W_I - X_I) => X_O + X_I = W_O + W_I - D \tag{7}$$

Comparing (6) and (7) we find: $\theta_O + \theta_I = 1$. Let us note:

$$\theta = \theta_I = 1 - \theta_O$$

Firm O's contribution can be written as:

$$D_O = W_O - X_O = W_O - [W_O + W_I - D]\,\theta$$

and firm I's contribution is:

$$D_I = W_I - X_I = W_I - (W_O + W_I - D)(1 - \theta)$$

Q.E.D

Proof of Proposition 3

Point (i):

$$SW(\theta) = R_I + R_O + W_I + W_O - cx^* - ke^* - g(e^*)D - f(x^*)D$$

The second-best sharing rule θ^* responds to:

$$\begin{aligned}\frac{dSW(\theta)}{d\theta} &= (-k - Dg'(e^*))\frac{de^*}{d\theta} - \frac{dx^*}{d\theta}\left[c + Df'(x^*)\right] \\ &= (-k - D(\frac{k}{\lambda D_O - D}))\frac{de^*}{d\theta} + \frac{dx^*}{d\theta}\left[c(\frac{D - D_O}{D_O})\right]\end{aligned}$$

Note that: $-k - D\frac{k}{\lambda D_0 - D} = \lambda\frac{kD_0}{D - \lambda D_0} > 0$.

So, for $\lambda \neq 0$, the two expressions which are a part of $\frac{dSW(\theta)}{d\theta}$ are of opposite signs

$$(-k - D(\frac{k}{\lambda D_O - D}))\frac{de^*}{d\theta} \geq 0 \quad (8)$$

$$\frac{dx^*}{d\theta}\left[c(\frac{D - D_O}{D_O})\right] \leq 0 \quad (9)$$

We remark:

. When $\theta = 0$ and $W_O = D$, the expression (9) equal zero since:

$$c + D\frac{c}{[W_O + W_I - D]\,\theta - W_O} = c + D(\frac{c}{-D}) = 0$$

while the expression (8) remains positive. So we have: $\frac{dSW(\theta)}{d\theta}\,|_{\theta=0} > 0$ when D and W_O are closed.

. When $\theta = 1$ and $W_I \to D$, the expression (9) tends to $-\infty$ since :

$$\begin{aligned}\frac{dx^*}{d\theta} &= \frac{d}{d\theta}(\frac{c}{[W_O + W_I - D]\,\theta - W_O})\frac{1}{f\,"\left[(f\,')^{-1}(\frac{1}{[W_O+W_I-D]\theta - W_O})\right]} \\ &= -\frac{[W_O + W_I - D]}{(-[W_O + W_I - D]\,\theta + W_O)^2}\frac{c}{f\,"\left[(f\,')^{-1}(\frac{c}{[W_O+W_I-D]\theta - W_O})\right]}\end{aligned}$$ tends to $-\infty$ too.

while the expression (8) remains positive and bounded. So we have: $\frac{dSW(\theta)}{d\theta}\mid_{\theta=1} < 0$ when D and W_I are closed.

As a consequence, if we suppose f'', g'' to be bounded, W_O and W_I sufficiently close (but still lower than) D, we can state that, by continuity on $[0,1]$, $\max_\theta [SW(x^*, e^*, \theta)]$ exists and is met. Moreover, for $\lambda \neq 0$,the inequalities $\frac{dSW}{d\theta}(0) > 0$ and $\frac{dSW}{d\theta}(1) < 0$ show that it is an interior solution.

Point (ii): from Remark 2 point (iii) we know that when $\lambda = 0$, we have $e^* = (g\prime)^{-1}(\frac{-k}{D}) = e^{**}$. The R&D effort is equal to the first-best one, and it is independent from the sharing rule θ. About prevention, we have $x^* = (f\prime)^{-1}(\frac{c}{D_O})$ which is lower than x^{**} but increases with D_O. D_O is maximum for $\theta = 0$ (maximum liability on the firm O, for a maximum risk internalization). To sum up, we have $e^* = e^{**}, \forall\theta$ but x^* decreases with θ: the second-best sharing rule is $\theta^* = 0$. This is also verified by the equalization of equation (8) to 0 when $\lambda = 0$.

Q.E.D

Proof of Proposition 4

We aim to demonstrate that: $\theta^*(\lambda) = \arg\max_\theta [SW(\theta, \lambda)]$ is an increasing function of λ. For this, it is sufficient to show that: $\frac{\partial^2 SW(\theta,\lambda)}{\partial\theta\partial\lambda} > 0$ for all $(\theta, \lambda) \in [0,1]^2$.

Recall that we assume $g\ (g\ ')^{-1}$ to be a convex function on $\mathbb{R}$.

Recall:

$$SW(\theta, \lambda) = R_I + R_O + W_I + W_O - ke^* - g(e^*)D - f(x^*)D - cx^*$$

We have:

$$\begin{aligned}\frac{dSW(\theta,\lambda)}{d\theta} &= (-k - Dg'(e^*))\frac{de^*}{d\theta} - \frac{dx^*}{d\theta}\left[c + Df'(x^*)\right] \\ &= \frac{-k\lambda D_0}{\lambda D_0 - D}\frac{de^*}{d\theta} + \frac{dx^*}{d\theta}\left[c(\frac{D - D_O}{D_O})\right]\end{aligned}$$

with $g'(e^*) = \frac{k}{\lambda D_O - D}$, and $D_O = W_O - \theta(W_O + W_I - D)$
For a given θ, $x^* = (f\ ')^{-1}\left[\frac{c}{D_0}\right]$ does not depend on λ. However, $e^* = (g\ ')^{-1}\left[\frac{k}{\lambda D_O - D}\right]$ is a function of λ. So we have:

$$\frac{\partial^2 SW(\theta,\lambda)}{\partial\theta\partial\lambda} = \frac{d}{d\lambda}\left((-k - Dg'(e^*))\frac{de^*}{d\theta} - \frac{dx^*}{d\theta}\left[c + Df'(x^*)\right]\right)$$

with $\frac{d}{d\lambda}\left(\frac{dx^*}{d\theta}\left[c + Df'(x^*)\right]\right) = 0$ and:

$$\frac{de^*}{d\theta} = \frac{d}{d\theta}\left((g\ ')^{-1}\left[\frac{k}{\lambda D_O - D}\right]\right)$$
$$= \frac{(W_O + W_I - D)}{(\lambda D_O - D)^2}\frac{\lambda k}{g\ "\left[(g\ ')^{-1}(\frac{-k}{D - \lambda D_0})\right]}$$

It follows:

$$\frac{\partial^2 SW(\theta,\lambda)}{\partial\theta\partial\lambda} = \frac{d}{d\lambda}\left[\frac{-k\lambda D_0}{\lambda D_0 - D}\frac{de^*}{d\theta}\right]$$
$$= k^2(W_O + W_I - D)D_0\frac{d}{d\lambda}\left(\frac{\lambda^2}{(D - \lambda D_0)^3}\frac{1}{g\ "\left[(g\ ')^{-1}(\frac{-k}{D - \lambda D_O})\right]}\right)$$

Showing that $\frac{\lambda^2}{(D-\lambda D_0)^3}\frac{1}{g\ "\left[(g\ ')^{-1}(\frac{-k}{D-\lambda D_O})\right]}$ is an increasing function of λ is a sufficient condition to demonstrate that $\frac{\partial^2 SW(\theta,\lambda)}{\partial\theta\partial\lambda} > 0$ for all $(\theta,\lambda) \in [0,1]^2$.

Consider: $z = \frac{-k}{D-\lambda D_O} < 0$. For λ lying in $[0,1]$, z lies in $[\frac{-k}{D-D_O}, \frac{-k}{D}]$. Remark that $\lambda = \frac{D}{D_O} + \frac{k}{zD_O}$ is decreasing with z, so that saying $\frac{\lambda^2}{(D-\lambda D_0)^3}\frac{1}{g\ "\left[(g\ ')^{-1}(\frac{-k}{D-\lambda D_O})\right]}$ to be an increasing function of λ (with $\lambda \in [0,1]$) is equivalent to say that $\frac{-z(zD+k)^2}{g\ "[(g\ ')^{-1}(z)]}$ is a decreasing function of z (with $z \in [\frac{-k}{D-D_O}, \frac{-k}{D}]$).
We know: $(g\ [(g\ ')^{-1}(z)])' = \frac{z}{g\ "[(g\ ')^{-1}(z)]}$. Because of the assumption of convexity of $g[(g')^{-1}(z)]$, we have: $(g\,[(g\ ')^{-1}(z)])'' = \left(\frac{z}{g\ "[(g\ ')^{-1}(z)]}\right)' \geq 0$. Knowing $z < 0$, $(g\ ')^{-1}(z) > 0$ and $g(.)$ to be decreasing and convex, we deduce that $\frac{z}{g\ "[(g\ ')^{-1}(z)]}$ is negative and increasing with z, and so $\frac{-z}{g\ "[(g\ ')^{-1}(z)]}$ is positive and decreasing with z (with $z \in [\frac{-k}{D-D_O}, \frac{-k}{D}]$). It is easy to check that $(zD+k)^2$ is positive and decreasing with z (with $z \in [\frac{-k}{D-D_O}, \frac{-k}{D}]$). As a result, we obtain $\frac{-z(zD+k)^2}{g\ "[(g\ ')^{-1}(z)]}$ to be a positive and decreasing

function of z. And so we obtain:

$$\frac{\partial^2 SW(\theta,\lambda)}{\partial\theta\partial\lambda} \geq 0.$$

meaning θ^* to be an increasing function of λ.

Q.E.D

Proof of Result 1

To lighten the calculations, we directly search for an optimal value of D_O. Then, from D_O we can easily deduce the value of θ: $\theta = \frac{W_O - D_O}{W_I + W_O - D}$. Keep in mind that D_O takes value in $[D - W_I, W_O]$: if the optimal value of D_O should be higher than W_O, limited liability constraints us to cap its value at W_O.

First of all, we determine, for this example (with $p(x,e) = \frac{exp(-\alpha x)+exp(-\beta e)}{\gamma}$), the private efforts in prevention and R&D. x^* satisfies:

$$\max_x \widetilde{\Pi}^O(x,e,D_O) = W_O + R_O - cx - Y(e) - p(x,e^*)D_O$$

and we find: $x^* = \frac{1}{\alpha} ln\left(\frac{\alpha D_O}{\gamma c}\right)$

e^* satisfies:

$$\max_e \widetilde{\Pi}^I(x,e,D_I) = W_I + R_I + Y(e) - ke^* - p(x,e^*)D_I$$

with: $Y(e) = \lambda Y_0 + (1-\lambda)[D_O(p(x,0) - p(x,e))]$. We find: $e^* = \frac{1}{\beta} ln\left(\frac{\beta(D-\lambda D_O)}{\gamma k}\right)$

Considering these private responses, the second-best problem is:

$$\max_{D_O} SW(x,e,D_O) = W_I + W_O + R_I + R_O - cx^* - ke^* - p(x^*,e^*)D$$

The first order condition is:

$$\frac{-c}{\alpha D_O}+\frac{cD}{\alpha(D_O)^2}-\frac{\lambda k D}{\beta(D-\lambda D_O)^2}+\frac{\lambda k}{\beta(D-\lambda D_O)}=0$$
$$\Rightarrow \frac{-(D_O)^3\lambda^2(c\beta+k\alpha)+(D_O)^2 c\beta D(2\lambda+\lambda^2)-D_O c\beta D^2(2\lambda+1)+c\beta D^3}{\alpha\beta(D_O)^2(D-\lambda D_O)^2}=0$$

So the optimal value of D_O satisfies:

$$(D_O)^3\lambda^2(c\beta+k\alpha)-(D_O)^2 c\beta D(2\lambda+\lambda^2)+D_O c\beta D^2(2\lambda+1)-c\beta D^3=0$$

Dividing by $c\beta$ we have:

$$F(D_O)=(D_O)^3\lambda^2(1+\tau)-(D_O)^2 D(2\lambda+\lambda^2)+D_O D^2(2\lambda+1)-D^3=0$$

with: $\tau=\frac{\alpha/c}{\beta/k}$.
This first order condition is a third-degree equation in D_O, which we denote $F(D_O)$. Differentiating $F(D_O)$ leads to a second degree polynomial:

$$F'(D_O)=3(D_O)^2\lambda^2(1+\tau)-2D_O D(2\lambda+\lambda^2)+D^2(2\lambda+1)$$

whose discriminant is: $\Delta=4D^2[(2\lambda+\lambda^2)^2-3\lambda^2(\tau+1)(2\lambda+1)]$, which is of the sign of: $(2\lambda+\lambda^2)^2-3\lambda^2(\tau+1)(2\lambda+1)$. We can deduce that for $\tau>1/3\frac{(\lambda-1)^2}{2\lambda+1}$, $F'(D_O)$ is always non negative, and so $F(D_O)$ is an increasing function with a unique root. So before D_O the function SW increases, and decreases after D_O: D_O is the unique maximize of SW. Now we have to ensure that the optimal value of D_O lies in $[D-W_I, W_O]$. For this, let us define:

- $\tau_O=\frac{W_O}{D}$, the maximum payment in damages the operator can bear (i.e. its wealth W_O), expressed as a percentage of the overall payment D. This allows us to rewrite: $max\,\{D_O\}=W_O=\tau_O D$.

- $\tau_I=\frac{D-W_I}{D}$, the remaining amount of damage (which remains to be remedied) when a maximum liability is assigned to the innovator ($D_O=D-W_I$ when $D_I=W_I$), expressed as a percentage of the overall payment D. This allows us to rewrite: $min\{D_O\} = D - W_I = \tau_I D$.

$D_O \in]D - W_I, W_O[$ requires:

$$F(\tau_I H) \leq 0 \leq F(\tau_O H)$$
$$\Rightarrow \frac{(\tau_O)^3(-\lambda^2) + (\tau_O)^2(\lambda^2 + 2\lambda) + \tau_O(-2\lambda - 1) + 1}{(\tau_O)^3\lambda^2} \leq \tau$$
$$\leq \frac{(\tau_I)^3(-\lambda^2) + (\tau_I)^2(\lambda^2 + 2\lambda) + \tau_I(-2\lambda - 1) + 1}{(\tau_I)^3\lambda^2}$$

We observe that: $\frac{(\tau_O)^3(-\lambda^2)+(\tau_O)^2(\lambda^2+2\lambda)+\tau_O(-2\lambda-1)+1}{(\tau_O)^3\lambda^2}$ is decreasing in W_O, and $\frac{(\tau_I)^3(-\lambda^2)+(\tau_I)^2(\lambda^2+2\lambda)+\tau_I(-2\lambda-1)+1}{(\tau_I)^3\lambda^2}$ is increasing in W_I. So the condition is more easily satisfied when both W_O and W_I are sufficiently high.

Q.E.D

References

ANDERSON, D.R. 1998. Development of environmental liability risk management and insurance in the United States: lessons and opportunities. *Risk Management and Insurance Review*, **2**, 1–23.

FLEMING, J. 1983. *The Law of Torts (6th ed.)*. Law Book Company, Perth.

KATZMAN, M.T. 1998. Pollution Liability Insurance and Catastrophic Environmental Risk. *The Journal of Risk and Insurance*, **55**, 75–100.

KEETON, W.P., DOBBS, D., KEETON, R., & OWEN, D. 1984. *Prosser and Keeton on Torts (5th ed.)*. West, St. Paul.

KORNHAUSER, L. A., & REVESZ, R. L. 1989. Sharing damages among multiple tortfeasors. *The Yale Law Journal*, **98**, 831–884.

KORNHAUSER, L. A., & REVESZ, R. L. 1990. Apportioning Damages among Potentially Insolvent Actors. *The Journal of Legal Studies*, **19**, 617–651.

Lee, H-D., Browne, M.J., & Schmit, J.T. 1994. How Does Joint and Several Tort Reform Affect the Rate of Tort Filings? Evidence from the State Courts. *The Journal of Risk and Insurance*, **61**, 295–316.

Miceli, T.J., & Segerson, K. 1991. Joint liability in torts: marginal and infra-marginal efficiency. *International Review of Law and Economics*, **11**, 235–249.

Pitchford, R. 1995. How liable should a lender be? The case of judgment-proof firms and environmental risk. *The American Economic Review*, **85**, 1171–1186.

Ringleb, A.H., & Wiggins, S.N. 1990. Liability and Large Scale, Long-Term Hazards. *Journal of Political Economics*, **98**, 574–595.

Shavell, S. 1985. Uncertainty over causation and the determinants of civil liability. *Journal of Law and Economics*, **28**, 587–609.

Shavell, S. 1986. The Judgment Proof Problem. *International Review of Law and Economics*, **6**, 45–58.

Shavell, S. 1987. Economic Analysis of Accident Law. *Harvard University Press, Cambridge, MA*.

Shavell, S. 2004. *Foundations of Economic Analysis of Law*. Harvard University Press.

Young, R., Faure, M., & Fenn, P. 2004. Causality and causation in tort law. *International Review of Law and Economics*, **24**, 507–523.

Young, R., Faure, M., Fenn, P., & Willis, J. 2007. Multiple tortfeasors: An economic analysis. *Review of Law and Economics*, **3**, 111–132.