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Government expenditure, external and domestic public debts, and economic growth

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Abstract

This paper analyzes the relationship between government expenditure, tax on returns to asset, public debt, and economic growth. Public debt is composed of two components, domestic debt and external debt. We show that an increase in the tax rate on returns to asset leads to an increase in government expenditure, consumption, and domestic debt. However, the impact of tax rate on external debt is unclear. In some situation, in particular when the productivity of capital on production is low (high) and the tax rate is lower (higher) than a threshold, the relation between external debt and the tax rate has a bell-shaped form, i.e. external debt firstly rises then decreases with the tax rate.

Keywords: Taxation; public expenditure; domestic debt; external debt; growth.

JEL Classification: H50; H63; O40

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1 Introduction

Fiscal policy has been given an increasing part of economic reforms, especially in developing countries (see Tanzi 2004, Mundle 1999, Engen and Skinner 1992, and Camelia et al. 2013) with period of globalization. Most of existing studies on the relationship between public debt and growth considered domestic debt component of public debt (see, e.g., Bohn 1998, , Greiner 2007, and Checherita and Rother 2010). Bohn (1998) stated that the ratio of public debt to GDP has a positive relationship with primary surplus, similarly to Greiner (2007). In another research, Checherita and Rother (2010) stated that the public debt ratio is harmful for economic growth if this ratio is higher than 70%.

In the existing literature, the external debt also plays important role on economic growth. Clements et al. (2003) investigated the relationship between external debt, public investment, and growth in low-income countries and stated that high levels of public debt can depress economic growth in low-income countries and threshold levels of external debt was estimated around 50% of GDP.

In this paper, we investigate the interaction between government expenditure, taxation, economic growth, and two kinds of debt: domestic debt and foreign debt. In order to isolate the role of taxation on returns to asset, we assume this is the only kind of tax in the economy. The results show that the tax rate has a positive impact on government expenditure, consumption, and domestic debt, meanwhile its effect on external debt is unclear. However, if the productivity of capital on production is too small, the relationship between tax rate and external debt is negative. If the productivity of capital is large enough, the relationship between tax rate and external debt exhibits a bell shape, for example, external debt increase first and starts to decrease when tax rate exceeds a certain threshold.

The paper is organized as follows. The next section presents literature reviews on government expenditure, taxation, public debt and economic growth. In this section, we summarize the findings of previous papers and their arguments relate to subject. The theoretical model, based on Barro (1990) and Greiner (2007), is introduced in section 3. Section 5 characterizes the balanced-growth path (BGP) of the economy. The effects of tax on returns to asset on the economy is analyzed in section 6. Section 7 describes the dynamic behavior of macroeconomic variables around the BGP. The last section concludes the study and gives some perspectives for further research.

2 The role of government expenditure, public debt, and taxation on economic growth

According to Keynesian economists, economic growth in the short term depends on how expansionary public expenditure is. The authors suggest that if an economy has a depression or governments want to accelerate economic growth, they should expense more in the public sector, for instance, spending on education, health care, transportation and so on.

Following the neoclassical growth theory, the growth rate is determined by capital formulation and, consequently, fiscal policy has a major role (see Peacock and Shaw 1971, Peacock and Wiseman 1979). The neoclassical authors indicated that an increase of taxation will raise economic growth. They stated that a lower growth rate may imply a greater consumption net of external diseconomies if the latter (as a share of aggregate production) increases with growth. They also underlined that investment may cause more externalities than current expenditure, in particular, if the latter is related to personal services. These different schools converge on the same conclusion, i.e. public expenditure promotes economic growth in the short term. It is clear that governments can pursue two solutions together to enhance economic growth. On the one hand, they can implement an expansionary fiscal policy with the budget deficit financed by borrowing. On the other hand, they can employ an expansionary monetary policy which can increase the amount of private and public investments (through lowering interest rate) or the size of budget (through the monetization of public deficit).

Barro (1990) distinguished two types of government expenditure, productive and unproductive expenditure. He stated that the economy's growth is negatively correlated with the ratio of government spending to GDP and there is a positive relationship between public investment and output growth. In the same vein, Aschauer (1989) found that the government productive expenditure can stimulate output expansion. While Devarajan et al. (1996) agreed that government expenditure has a relationship with economic growth, each component of it has a different effects on growth. Particularly, current expenditure of government is associated with a higher growth whereas government productive expenditures in capital, transport, communication, health, and education have a negative impact on growth. In addition, Devarajan et al. (1996) and Angelopoulos et al. (2007) obtained that economic growth depends not only on the physical production of typical components of public spending, but also on the ratio of government expenditure allocated on them. On the contrary, Mundle (1999) and Glomm and Ravikumar (1997) stated that government's spending in infrastructure and social services have a significant impact on the long-run growth rate. Hence, these governments need to shift away from taxes on production and trade to taxes on income, consumption, and value added.

In their study about fiscal decentralization, government spending, and economic growth in China, Zhang and Zou (1998) showed that the central government's spending positively impacts economic growth. However, local government spending negatively affects growth. The same finding was also obtained by Xie et al. (1999) and Thornton (2007) when the authors studied about the decentralization and economic growth in the United States and in OECD countries, respectively. In contrast to previous studies, using cross-section data for the United States, Akai and Sakata (2002) got a different result following which fiscal decentralization contributes to economic growth.

In a research on growth effects of government expenditure and taxation in developed countries by using economic panel data of rich countries for the period 1970-1995, Folster and Henrekson (2001) found a negative relation between public expenditure and economic growth. Easterly and Rebelo (1993) stressed that international trade taxes have a strong association with economic growth in the poor countries whereas income taxes are a main factor of growth in industrial countries. Furthermore, Gupta et al. (2005) indicated that in low-income countries, the overall composition of public expenditure toward productive uses is particularly important for fostering growth, cutting current expenditures tend to trigger higher growth rates than adjustments based on revenue increases and cuts in more productive spending. Moreover, reductions in the public sector wage bill are not harmful for economic growth.

Taxation affects not only individuals and firms but also economic growth. Cebula (1995) highlighted that the higher maximum levels of federal government personal income tax rate and corporate income tax rate have a negative impact on economic growth as a results of empirical investigation for the period 1955-1972 in the United States. Angelopoulos et al. (2007) found that the average tax rate (as measured by tax revenue over GDP) and the associated fiscal size of the government (as measured by total expenditure over GDP) are significantly and negatively correlated with growth. By using disaggregated taxes, their results indicated (but this is not robust) that the growth effect of effective labor income tax is negative. Similarly, Easterly and Rebelo (1993) addressed that the ration of tax revenues to GDP is negative impact on growth as a results of cross-section regression in OECD countries for the period 1960-1988. Lastly, the growth effect of effective capital income tax is positive although not significant. However, there is an evidence that even through the mix of direct and indirect taxes is an important determinant of long-run growth and investment rates, but in practice, Mendoza and Asea (1997) underlined that plausible changes in tax rates seem to be unlikely to affect growth. The evidence of empirical research using Harberger model with panel data regression for the period 1965-1991 in 11 OECD countries, the authors found that the effects of 10 percentage point tax cuts on the investment rate are about 0.5 and 1.5 percentage points but growth effects are very small, approximately 0.1 to 0.2 of a percentage point. Mullen

and Williams (1994) obtained that higher marginal tax rates are associated with slower output growth and that lower marginal tax rates are able to have a positive impact on economic growth. The results of Mullen and Williams (1994) mean that changes in effective tax rates have an important effect on economic growth and that average tax rates and economic growth constitutes a significant relationship. In a non stochastic model, Lee et al. (1997) showed that taxation significantly affects economic growth and a tax reduction will increase the economy's growth rate. However, if consumers are risk averse enough, the growth rate might be decreased with a tax cut. Furthermore, Kim (1998) supposed that tax systems across countries have a significant relation with growth in which differences in taxes can explain growth discrepancy. The author also stated that tax reform may influence economic growth and that the hypothetical elimination of all taxes in the US raises approximately 0.85 percentage points of growth rate in the calibrated model. Lin and Russo (1999) found different figures with Kim (1998), for instance, there would be an increase in the growth rate by 0.63 percentage points if all the income taxes were eliminated and US debt-to-capital ratio was about 33%. When the corporate tax for innovative companies is eliminated, the growth rate will decrease by 0.20 percentage points.

By analyzing taxation and growth in an overlapping generations model, Yakita (2003) showed that the flat-rate wage tax elevates the growth rate and the flat-rate income tax does not stimulate economic growth. These results are different with Lucas' (1996) findings that labor income taxes stimulate economic growth while capital taxes do not. In their research, Lee and Gordon (2005) concluded that corporate tax rates have a negative impact on economic growth (i.e a cut in corporate tax rate by 10% will raise economic growth from 1% to 2%) whereas the personal tax rates have no clear evidence. Angelopoulos et al. (2007) recognized that some kinds of taxes such as labor income tax are negatively related to growth, meanwhile capital income and corporate income taxes are positively related to growth.

Regarding public debt, Greiner (2007) assumed that the ratio of primary surplus to gross domestic income is a positive linear function of the debt to gross domestic debt ratio. The author also stated that a sustainable balanced growth path exist if the government uses a certain part of the tax revenue for the debt services. In other researches, Reinhart and Rogoff (2010) and Herndon et al. (2014) showed that public debt has a positive impact on economic growth and there is a higher ratio of public debt to GDP leads to the lower GDP growth rate. For instance, if the ratio of public debt to GDP is lower than 30%, the average GDP growth rate is about 4.1%. On the contrary, the growth rate is reduced to 2.2% if the ratio of public debt to GDP becomes larger than 90%. In a study on the role of government debt on economic growth across twelve Euro-area countries, Checherita and Rother (2010) found that public debt and economic growth have a nonlinear relation and

that a higher public debt-to-GDP ratio is on average associated with a lower long-term growth rate when debt is above the range of 90-100% of GDP. In practice, the ratio of public debt to GDP in each country is different, for example, in European countries where it is regulated at the level of 60% of GDP following the Maastricht criteria. In the case of developing countries such as Vietnam, the figure is 65%. Clements et al. (2003) stressed that high levels of public debt can depress economic growth in low-income countries and the corresponding threshold level of external debt is estimated around 50% of GDP in their simulation exercise.

In the following section, we present a growth model to investigate the relation between growth, government expenditure, taxation, and two types of public debt (domestic debt and external debt).

3 Model

The growth model presented in this section is based on the models developed by Barro (1990) and Greiner (2007). Our economy comprises three sectors, namely government, firms, and consumers.

3.1 Government

We assume that at each period t the government can collect tax on returns to assets held by private agents. It can also borrow from the domestic and international financial markets, which correspond to two types of public debt, domestic debt D_t with interest rate r_t^D and external debt B_t with interest rate r_t^B . On the spending side, the government can share its resources between public expenditure devoted to production of final goods and reimbursement of interests and capital of domestic and external debts. The government budget constraint can be expressed as follows:¹

$$G_t + (r_t^B + 1)B_t + (r_t^D + 1)D_t = \tau_t r_t^A A_t + B_{t+1} + D_{t+1}. \quad (1)$$

where A_t is the stock of assets held by private agents, τ_t is the tax rate on returns to asset, r_t^A is the interest rate of asset, and G_t is the flow of government expenditure.

Following Greiner (2007), we assume that threshold of public debt is not over a certain proportion of total output in order to guarantee sustainability of public debt as below:

$$G_t + \eta(B_t + D_t) \leq \phi Y_t + \tau_t r_t^A A_t. \quad (2)$$

with ϕ and $\eta \in \mathbb{R}$ are constants. ϕ determines whether the level of the primary surplus rises or falls with an increase in gross domestic income, η determines how strong the

¹All variables are expressed in terms of real values.

primary surplus reacts to changes in domestic debt and external debt, η may be considered as a feedback parameter of domestic debt and external debt.

Inequality (2) means total government expenditure and government's borrowing are not exceeded government's revenue which comes from tax collection and a certain proportion of total output.

Let r_{t-1}^{BD} denote the interest rate which satisfies:

$$B_{t-1}(1 + r_{t-1}^B - \eta) + D_{t-1}(1 + r_{t-1}^D - \eta) = (B_{t-1} + D_{t-1})(1 + r_{t-1}^{BD} - \eta). \quad (3)$$

Equation (3) can be rewritten as follows:

$$B_{t-1}r_{t-1}^B + D_{t-1}r_{t-1}^D = (B_{t-1} + D_{t-1})r_{t-1}^{BD} \quad (4)$$

equivalently

$$r_{t-1}^{BD} = \frac{B_{t-1}}{B_{t-1} + D_{t-1}}r_{t-1}^B + \frac{D_{t-1}}{B_{t-1} + D_{t-1}}r_{t-1}^D. \quad (5)$$

Equation (5) indicates that r_{t-1}^{BD} is an average interest rate of r_{t-1}^B and r_{t-1}^D . There always exists an interest rate r_{t-1}^{BD} with given r_{t-1}^B , r_{t-1}^D , B_{t-1} and D_{t-1} . At equilibrium equality (2) must bind. Together (1), (2) and (4) lead to

$$B_t + D_t = (B_{t-1} + D_{t-1})(1 + r_{t-1}^{BD} - \eta) + \phi Y_{t-1} \quad (6)$$

Proposition 1 Define that γ_t is growth rate of gross domestic income Y_t , and r_t^{BD} is determined by equation (5). The sufficient condition for the sustainability of public debt is $\max\{\sup_t \gamma_t, 0\} < \inf_t r_t^{BD} - \eta$.

Proof. Equation (6) can be expressed as follows

$$B_t + D_t = (B_0 + D_0) \prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta) + \sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta). \quad (7)$$

which is equivalent to

$$B_0 + D_0 = \frac{B_t + D_t}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} - \frac{\sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)}. \quad (8)$$

Following Greiner's (2007) terms, sustainability of public debt states that the current value of public debt must equal the sum of discounted future non-interest surpluses. Hence, sustainability of public debt is characterized by

$$B_0 + D_0 = \lim_{t \rightarrow \infty} \left(\frac{B_t + D_t}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} \right). \quad (9)$$

Condition (9) is verified if

$$\lim_{t \rightarrow \infty} \frac{\sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} = 0. \quad (10)$$

Denote that γ_t is growth rate of total production income Y_t . Hence, $Y_{t-s} = \prod_{j=0}^{t-s} (1 + \gamma_j) Y_0$. We then get

$$\begin{aligned} \frac{\sum_{s=1}^t \phi Y_{t-s} \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} &= \phi Y_0 \frac{\sum_{s=1}^t \prod_{j=0}^{t-s} (1 + \gamma_j) \prod_{j=1}^{s-1} (1 + r_{t-j}^{BD} - \eta)}{\prod_{j=1}^t (1 + r_{t-j}^{BD} - \eta)} \\ &= \phi Y_0 \sum_{j=1}^t \prod_{s=j}^t \left(\frac{1 + \gamma_{t-s}}{1 + r_{t-s}^{BD} - \eta} \right) \end{aligned}$$

Hence, if $\max\{\sup_t \gamma_t, 0\} < \inf_t r_t^{BD} - \eta$ then Condition (9) is verified .

■

3.2 Firms

We assume that the production of the final good depends on the stock of private capital and government spending:

$$Y_t = F(K_t, G_t) = H K_t^\alpha G_t^{1-\alpha} \quad (11)$$

where $0 < \alpha < 1$ is output elasticity with respect to capital (and $1 - \alpha$ is the elasticity corresponding to public spending), H is total factor productivity or technological level. The production function F is strictly increasing in both variables, strictly concave in K . The production function also verifies (i) $F(0, G) = 0$ and (ii) $F(K, 0) > 0$ if $K > 0$. Here, G may be considered as a positive externality for the production. The profit is given by $\pi_t = F(K_t, G_t) - r_t^K K_t$ (r_t^K is the interest rate of capital). The first-order condition (FOC) for profit maximization is

$$F'_K(K_t, G_t) = r_t^K. \quad (12)$$

By substituting equation (11) into equation (12), the interest rate of capital can be written as

$$r_t^K = \alpha H K_t^{\alpha-1} G_t^{1-\alpha} = \alpha H \left(\frac{G_t}{K_t} \right)^{1-\alpha}, \quad (13)$$

or, equivalently,

$$r_t^K = \alpha H g_t^{1-\alpha}, \quad (14)$$

where $g_t \equiv G_t/K_t$. Equation (13) implies that interest rate of private capital is determined by total factor productivity, output elasticity with respect to public spending, and the ratio of government expenditure and private capital.

3.3 Consumers

The representative consumer's instantaneous utility function is assumed to have the iso-elastic form

$$U(C_t) = \begin{cases} \frac{C_t^{1-\rho}-1}{1-\rho} & \text{if } \rho \neq 1 \\ \ln C_t & \text{if } \rho = 1 \end{cases} \quad (15)$$

where $\rho > 0$ is the coefficient of relative risk aversion. The representative consumer chooses her consumption, her stock of assets, and her government bonds to maximize her inter-temporal utility $\sum_{t=0}^{+\infty} \beta^t U(C_t)$, where $\beta > 0$ is the discount rate, under the budget constraint

$$C_t + A_{t+1} + D_{t+1} \leq [r_t^A(1 - \tau_t) + 1] A_t + (r_t^D + 1)D_t + \pi_t \quad (16)$$

and positivity constraints $C_t \geq 0$ and $A_t \geq 0$, $\forall t$. Note that C_t , A_t , D_t , and π_t are respectively consumption, private assets, domestic debt hold by the consumer, and the profit she receives as the firm owner.²

The Lagrangian is

$$L = \sum_{t=0}^{\infty} \beta^t U(C_t) - \sum_{t=0}^{\infty} \lambda_t \{ [r_t^A(1 - \tau_t) + 1] A_t + (r_t^D + 1)D_t + \pi_t - C_t - A_{t+1} - D_{t+1} \} + \sum_{t=1}^{\infty} \mu_t A_t.$$

The FOCs are given as follows, $\forall t$,

$$\beta^t U'(C_t) + \lambda_t = 0, \quad (17)$$

$$\lambda_t [(1 - r_t^A(1 - \tau_t))] - \lambda_{t-1} + \mu_t = 0, \quad (18)$$

$$\lambda_t(1 + r_t^D) - \lambda_{t-1} = 0, \quad (19)$$

$$\mu_t A_t = 0. \quad (20)$$

The slackness condition in (20) means that $A_t > 0$, $\mu_t = 0$ or $A_t = 0$, $\mu_t > 0$. These FOCs and the budget constraint will provide a solution of the consumer's optimization program.

Solving for an interior solution ($A_t > 0$), conditions (18)-(20) give:

$$r_t^D = r_t^A(1 - \tau_t). \quad (21)$$

The equality between the interest rate of domestic debt and the net interest rate of private asset given in (21) represents the non-arbitrage condition between holding domestic debt

²We assume that there is no tax on government bond interest. Indeed, when such a tax exists, the consumer's budget constraint will include the term $r_t^D(1 - \tau_t^D)D_t$ instead of $r_t^D D_t$. In this case, the non-arbitrage condition between private assets and government bonds is $r_t^A(1 - \tau_t^A) = r_t^D(1 - \tau_t^D)$, which implies $r_t^A = r_t^D$ and $\tau_t^A = \tau_t^D$. For simplification purpose, we do not impose any tax on government bonds and consequently the implied non-arbitrage condition (see also below) will become equation (21).

and holding private capital. Furthermore, conditions (17) and (19) give

$$\frac{U'(C_{t-1})}{U'(C_t)} = \beta(1 + r_t^D), \quad (22)$$

which is the usual Keynes-Ramsey rule which states that the marginal utility of past consumption is equal to the discounted marginal utility of current consumption times the interest rate.

By using the utility function in (15), equation (22) becomes

$$\frac{C_t}{C_{t-1}} = [\beta(1 + r_t^D)]^{1/\rho}. \quad (23)$$

4 Equilibrium

Equilibrium of model is a solution of equations:

Balancedness of the government budget:

$$G_t + r_t^b B_t + r_t^d D_t + B_t + D_t = r_t^a \tau_t K_t + B_{t+1} + D_{t+1}$$

Sustainability of debt condition:

$$G_t + \eta(B_t + D_t) = \phi Y_t + \tau_t r_t^A A_t.$$

Balancedness of consumer budget:

$$C_t + A_{t+1} + D_{t+1} = [r_t^A(1 - \tau_t) + 1] A_t + (r_t^D + 1)D_t + \pi_t.$$

Keynes-Ramsey rule:

$$\frac{U'(C_{t-1})}{U'(C_t)} = \beta(1 + r_t^D).$$

Market clearing for the capital:

$$K_t = A_t.$$

Market clearing for the aggregate good :

$$C_t + K_{t+1} = F(K_t, G_t) + K_t.$$

Market clearing for the domestic debt :

$$D_{t+1} + \tau_t r_t^K K_t = (1 + r_t^D)D_t$$

and interest rates of capital, domestic debt:

$$\begin{aligned} r_t^K &= F'_K(K_t, G_t). \\ r_t^D &= r_t^A(1 - \tau_t). \end{aligned}$$

5 Balanced growth path

Let us define $g_t \equiv \frac{G_t}{K_t}$, $b_t \equiv \frac{B_t}{K_t}$, $d_t \equiv \frac{D_t}{K_t}$, $c_t \equiv \frac{C_t}{K_t}$, $\xi_c \equiv \frac{C_{t+1}}{C_t}$, $\xi_b \equiv \frac{B_{t+1}}{B_t}$, $\xi_d \equiv \frac{D_{t+1}}{D_t}$, and $\xi_k \equiv \frac{K_{t+1}}{K_t}$. The solution for the model with the variables G_t , C_t , B_t , D_t , and K_t is

equivalent to the solution with new variables g_t , c_t , b_t , and d_t .³ Equations (1), (2), (16), (23), and (??) become

$$g_t + (1 + r_t^B)b_t + (1 + r_t^D)d_t = r_t^K \tau_t + (b_{t+1} + d_{t+1})\xi_k, \quad (24)$$

$$\tau_t r_t^K - g_t = \phi H g_t^{1-\alpha} + \eta(b_t + d_t), \quad (25)$$

$$\frac{c_{t+1}}{c_t} \xi_k = [\beta(1 + r_t^D)]^{1/\rho}, \quad (26)$$

$$c_t + \xi_k + d_{t+1}\xi_k = (1 - \tau_t)r_t^K + 1 + (1 + r_t^D)d_t, \quad (27)$$

with $\xi_k = H g_t^{1-\alpha} + 1 - c_t$.

By substituting equation (13) into equations (24)-(27) and by using the non arbitrage condition (21), we get the following system

$$g_t + (1 + r_t^B)b_t + [1 + (1 - \tau_t)\alpha H g_t^{1-\alpha}] d_t = \alpha H g_t^{1-\alpha} \tau_t + (b_{t+1} + d_{t+1})(H g_t^{1-\alpha} + 1 - c_t), \quad (28)$$

$$\tau_t \alpha H g_t^{1-\alpha} - g_t = \phi H g_t^{1-\alpha} + \eta(b_t + d_t), \quad (29)$$

$$\frac{c_{t+1}}{c_t} (H g_t^{1-\alpha} + 1 - c_t) = [\beta(1 + (1 - \tau_t)\alpha H g_t^{1-\alpha})]^{1/\rho}, \quad (30)$$

$$c_t + (1 + d_{t+1})(H g_t^{1-\alpha} + 1 - c_t) = [1 + (1 - \tau_t)\alpha H g_t^{1-\alpha}] (1 + d_t). \quad (31)$$

A balanced-growth path equilibrium is defined by $x_{t+1} = x_t = x^*$, $x = c, b, d, g$. The BGP is hence given by the following quantities:

$$g^* = \left(\frac{1 - \beta}{\beta(1 - \tau)\alpha H} \right)^{\frac{1}{1-\alpha}}, \quad (32)$$

$$c^* = \frac{1 - \beta}{\beta(1 - \tau)\alpha}, \quad (33)$$

$$d^* = \frac{1 - \alpha(1 - \tau)}{\alpha(1 - \tau)}, \quad (34)$$

$$b^* = \frac{1}{\eta} \left[\frac{(1 - \beta)\tau}{\beta(1 - \tau)} - \left(\frac{1 - \beta}{\beta(1 - \tau)\alpha H} \right)^{\frac{1}{1-\alpha}} - \frac{(1 - \beta)\phi}{\beta(1 - \tau)\alpha} - \eta \frac{1 - \alpha(1 - \tau)}{\alpha(1 - \tau)} \right]. \quad (35)$$

and the following interest rates

$$r^{D*} = \frac{1 - \beta}{\beta}, \quad (36)$$

$$r^{K*} = \frac{1 - \beta}{\beta(1 - \tau)}. \quad (37)$$

The results show that the ratios of government expenditure, consumption, and domestic debt over private capital at the BGP depend on parameters such as tax rate (τ), discount rate (β), output elasticities (α and $1 - \alpha$), and technological level (H). In addition to these parameters, the BGP value of the ratio of external debt to capital also

³More computational details can be found in Appendix .

depend on the slopes of the budget surplus function with respect to output (ϕ) and total public debt (η). Furthermore, we observe that the interest rates of domestic debt and private capital at the BGP are determined only by the consumer's discount rate and the tax rate on returns to asset. At the BGP, tax rate impacts on almost of all macroeconomic variables while it does not impact on interest rate of domestic debt which is determined only by discount rate (β). It is easy to find that the relationship between interest rate of domestic debt (r^{D*}) and discounted rate (β) is negative because derivative of the interest rate of domestic debt by discount rate is negative. In the next section, we will investigate the impact of tax rate on the rest of macroeconomic variables.

6 Impact of taxation on economy

The impacts of the tax rate on returns to assets on the macroeconomic variables of the model (government expenditure, consumption, domestic debt, external debt, and interest rate of capital) can be summarized in the following proposition.

Proposition 2 *Other things being equal,*

(i) g^* , c^* , d^* , and r^{K*} increase with τ ,

(ii) a) If $\alpha \leq \frac{\eta\beta}{1-\beta} + \phi$, b^* decreases with τ ,

b) If $\alpha > \frac{\eta\beta}{1-\beta} + \phi$, b^* increases with τ if $\tau < \bar{\tau}$ and decreases with τ if $\tau \geq \bar{\tau}$, where

$$\bar{\tau} \equiv 1 - \frac{\left(\frac{1-\beta}{\alpha\beta H}\right)^{\frac{1}{\alpha}}}{\left[\left(\frac{1-\beta}{\beta}(\alpha - \phi) - \eta\right) \frac{1-\alpha}{\alpha}\right]^{\frac{1-\alpha}{\alpha}}}. \quad (38)$$

Proof. The derivatives of g^* , c^* , d^* , b^* with respect to τ can be obtained from equations (32)- (35) and (37):

$$\frac{\partial g^*}{\partial \tau} = \frac{1}{(1-\alpha)(1-\tau)^2} \left(\frac{1-\beta}{\alpha\beta H}\right)^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\tau}\right)^{\frac{\alpha}{1-\alpha}} \geq 0, \quad (39)$$

$$\frac{\partial c^*}{\partial \tau} = \frac{1-\beta}{\alpha\beta(1-\tau)^2} \geq 0, \quad (40)$$

$$\frac{\partial d^*}{\partial \tau} = \frac{1}{\alpha(1-\tau)^2} \geq 0, \quad (41)$$

$$\frac{\partial b^*}{\partial \tau} = \frac{1}{\eta(1-\tau)^2} \left[\frac{(1-\beta)(\alpha - \phi) - \eta\beta}{\alpha\beta} - \frac{1}{1-\alpha} \left(\frac{1-\beta}{\alpha\beta H}\right)^{\frac{1}{1-\alpha}} \left(\frac{1}{1-\tau}\right)^{\frac{\alpha}{1-\alpha}} \right] \begin{matrix} \leq \\ \geq \end{matrix} 0 \quad (42)$$

$$\frac{\partial r^{K*}}{\partial \tau} = \frac{1-\beta}{\beta(1-\tau)^2} \geq 0. \quad (43)$$

We observe that the derivative of g^* , c^* , d^* , and r^{K^*} with respect to τ as given in equations (39), (40) and (41) are positive because $0 < \alpha, \beta, \tau < 1$, which verify points (i) of the proposition.

Finally, concerning the derivative of b^* with respect to τ , the result is unclear. Indeed, from equation (42), we can easily check that condition $\alpha \leq \frac{\eta\beta}{1-\beta} + \phi$ sufficiently implies that $\frac{\partial b^*}{\partial \tau} \leq 0$. However, when $\alpha > \frac{\eta\beta}{1-\beta} + \phi$, b^* can either increase or decrease depending on a threshold value of the tax rate. The latter is obtained after some arithmetic manipulation as

$$\bar{\tau} \equiv 1 - \frac{\left(\frac{1-\beta}{\alpha\beta H}\right)^{\frac{1}{\alpha}}}{\left[\left(\frac{1-\beta}{\beta}(\alpha - \phi) - \eta\right) \frac{1-\alpha}{\alpha}\right]^{\frac{1-\alpha}{\alpha}}}.$$

As a result, in the case of $\alpha > \frac{\eta\beta}{1-\beta} + \phi$, external debt increases with τ if τ is lower than this threshold and decreases if τ is higher. This verifies point (ii.b) of the proposition. ■

This result means that at the steady state if tax rate τ increases, government expenditure g^* , consumption c^* , and domestic debt d^* increase. In other words, an increase in tax rate on returns to asset leads to increase in government expenditure, consumption, domestic debt, and interest rate of capital. On the one hand, when the tax rate increases that leads to increase in total tax revenue, consequently government expenditure increases. On the consumers side, if the tax rate increases that leads to an decrease in total available income and consumers will prefer consumption and hold domestic bonds instead of assets. Following the increase of tax rate, consumers revise their trade-off between consumption and government bonds. Moreover, when the tax rate on returns to assets (capital) increases that leads to decrease in assets (capital), subsequently one demands for assets (capital) have to pay the higher interest rate of assets (capital) in the market.

In terms of impact of tax rate on external debt, if the productivity of physical capital (α) is too small or the ratio of debt to GDP and constant ratio of GDP (ϕ and η) are large, external debt decreases with the tax rate. However, if the productivity of capital is large, the external debt increases if the tax rate is lower than a certain threshold and when tax rate is larger than this threshold, external debt decreases. This is a bell-shaped form relation between external debt and the tax rate in the case of high productivity of capital. This explains that when tax rate increases while the productivity of capital is small, the government has difficulties to borrow money from international financial markets. Obviously, if the tax rate is at high level, it becomes harmful for the economy because consumers reduce their capital, subsequently government expenditure is declined. Moreover, when total tax revenue reduces that may leads to the lower payment's ability of the government and it is too hard to borrow from international financial markets.

Our findings show that in a particular case when the productivity of capital is too small that usually happens in poor countries where governments control almost of economic activities, an increase in tax rate leads to decrease in external debt (similarly to findings of Greiner, 2007).⁴ However, if the productivity of capital is high, an increase in tax rate can boost external debt. Furthermore, our results indicate that increasing tax rate can lead to increased total output as government expenditure, domestic debt, and, in some cases, external debt increase. This result is similar to Greiner (2007).

7 Dynamic analysis

In this section, we analyze the behavior of the economy, characterized by the system of equations (28)-(31), around the BGP equilibrium. Firstly, equation (29) can be rewritten as

$$\tau_{t+1}\alpha Hg_{t+1}^{1-\alpha} - g_{t+1} = \phi Hg_{t+1}^{1-\alpha} + \eta(b_{t+1} + d_{t+1}). \quad (44)$$

By substituting equation (44) into equation (28) we obtain

$$g_t + (1 + r_t^B)b_t + [1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha}] d_t = \alpha Hg_t^{1-\alpha}\tau_t + \frac{Hg_t^{1-\alpha} + 1 - c_t}{\eta} (\tau_{t+1}\alpha Hg_{t+1}^{1-\alpha} - g_{t+1} - \phi Hg_{t+1}^{1-\alpha}), \quad (45)$$

or, equivalently,

$$g_{t+1} \equiv \Sigma(g_t, b_t, d_t, c_t). \quad (46)$$

Similarly, equations (30) and (31) can be rewritten as follows

$$c_{t+1} = \frac{c_t}{Hg_t^{1-\alpha} + 1 - c_t} [\beta[1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha}]]^{1/\rho} \equiv \Gamma(g_t, c_t), \quad (47)$$

$$d_{t+1} = \frac{[1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha}](1 + d_t)}{Hg_t^{1-\alpha} + 1 - c_t} - \frac{c_t}{Hg_t^{1-\alpha} + 1 - c_t} - 1 \equiv \Theta(g_t, d_t, c_t). \quad (48)$$

Finally, by substituting the expression for d_{t+1} given by equation (31) into equation (44) we derive the equation for b_{t+1} as

$$b_{t+1} = \Xi(g_t, b_t, d_t, c_t). \quad (49)$$

With the system of equations (46)-(49) at hands we can formally compute the Jacobian matrix at the BGP equilibrium (computational details are reported in Appendix). The analytical computation of the eigenvalues and the eigenvectors of this 4×4 matrix is cumbersome. For simplicity's sake, we provide numerical calculations by varying α , and β in interval $(0, 1)$ and by assuming $H = 1$, $\phi = 0.6$, $\eta = 0.2$, $\tau = 0.3$, and $\rho = 1$.

⁴Greiner (2007) used income tax.

Figure 1 reports the eigenvalues obtained from this numerical exercise. Remark that any eigenvalue whose real part is comprised between -1 and 1 corresponds to a stable convergence path. We observe in Figure 1 that many eigenvalues are located inside the circle which has the modulus 1. Table 1 reports all the four eigenvalues corresponding to each of the combinations between specific values of α and β , other parameters being fixed as previously, i.e. $H = 1$, $\phi = 0.6$, $\eta = 0.3$, $\tau = 0.2$, $r^B = 0.03$ and $\rho = 1$. For all the combinations of α and β considered, there is at least one eigenvalue whose real part is included in interval $(-1,1)$ implying the presence of a convergence path. This numerical exercise shows that there are many situations where the BGP of the model correspond to a saddle point.

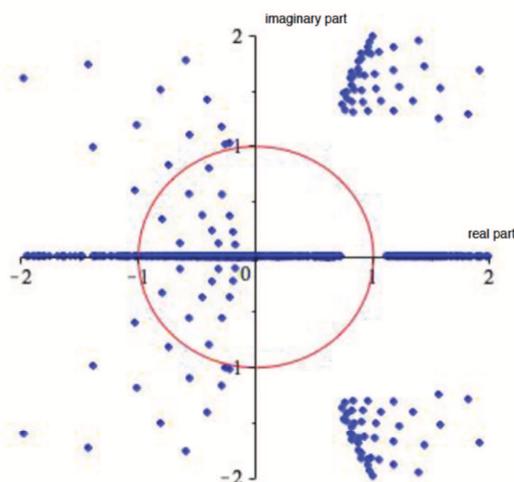


Figure 1: Eigenvalues of the Jacobian matrix with α and β varying in interval $(0, 1)$ Other parameters are $H = 1$, $\phi = 0.6$, $\eta = 0.2$, $\tau = 0.3$, and $\rho = 1$. The abscissa corresponds to the real part and the ordinate corresponds to the imaginary part.

Table 1: Eigenvalues corresponding to specific values of α and β and types of stability

α	β	Eigenvalues	Type of stability
0.3	0.9	-1.65, 1.25, 0.17, 0.46	a saddle point
0.6	0.9	1.18, -0.55, 0.43, -0.09	a saddle point
0.7	0.4	0.003+0.34i, 0.003-0.34i, -0.002, 1.49	a saddle point
0.7	0.8	0.36, -0.03, -0.31, 1.15	a saddle point
0.7	0.9	1.13, -0.39, 0.4, -0.06	a saddle point

8 Conclusion

In this paper, we investigate the relationship between government expenditure, taxation, economic growth, and public debt. With assumptions that public debt consists of two types, domestic debt and external debt, and that the ratio of the primary surplus to gross domestic income is a linear function of the debt income ratio which assures that public debt is sustainable. The results show that government expenditure, consumption, and domestic debt increase with tax rate. However, if the productivity of physical capital is small or the ratio of debt is large, the effect of tax rate is negative. In case of high productivity of capital, the impact of tax rate on external debt is positive if tax rate does not exceed a certain threshold, otherwise, the relation is decreasing.

Appendix. Derivatives of $g_{t+1}, b_{t+1}, c_{t+1}, d_{t+1}$ with respect to g_t, b_t, c_t, d_t .

From equation (28) and equation (29) we obtain:

$$(b_{t+1} + d_{t+1})(Hg_t^{1-\alpha} + 1 - c_t) = g_t + (1 + r_t^B)b_t + \{1 + (1 - \tau_t)\alpha Hg_t^{1-\alpha}\}d_t - \alpha Hg_t^{1-\alpha}\tau_t. \quad (50)$$

$$b_{t+1} + d_{t+1} = \frac{Hg_{t+1}^{1-\alpha}(\tau_{t+1}\alpha - \phi) - g_{t+1}}{\eta}. \quad (51)$$

Derivative both sides of equation (50) and equation (51) by g_t we have:

$$\begin{aligned} \left(\frac{\partial b_{t+1}}{\partial g_t} + \frac{\partial d_{t+1}}{\partial g_t}\right)(Hg_t^{1-\alpha} + 1 - c_t) + (b_{t+1} + d_{t+1})H(1 - \alpha)g_t^{-\alpha} &= 1 + Hd_t(1 - \tau_t)\alpha(1 - \alpha)g_t^{-\alpha} - \alpha(1 - \alpha)H\tau_tg_t^{-\alpha}. \\ \frac{\partial b_{t+1}}{\partial g_t} + \frac{\partial d_{t+1}}{\partial g_t} &= \left\{\frac{H(1 - \alpha)(\tau_{t+1}\alpha - \phi)g_{t+1}^{-\alpha} - 1}{\eta}\right\}\frac{\partial g_{t+1}}{\partial g_t}. \end{aligned}$$

Consequently

$$\frac{\partial g_{t+1}}{\partial g_t} = \frac{\eta\{1 + Hd_t(1 - \tau_t)\alpha(1 - \alpha)g_t^{-\alpha} - \alpha(1 - \alpha)H\tau_tg_t^{-\alpha} - (b_{t+1} + d_{t+1})H(1 - \alpha)g_t^{-\alpha}\}}{(Hg_t^{1-\alpha} + 1 - c_t)[H(1 - \alpha)(\tau_{t+1}\alpha - \phi)g_{t+1}^{-\alpha} - 1]}.$$

Taking derivatives of both sides of equation (50) and equation (51) with respect to b_t , we have:

$$\begin{aligned} \left(\frac{\partial b_{t+1}}{\partial b_t} + \frac{\partial d_{t+1}}{\partial b_t}\right)(Hg_t^{1-\alpha} + 1 - c_t) &= 1 + r_t^B \\ \frac{\partial b_{t+1}}{\partial b_t} + \frac{\partial d_{t+1}}{\partial b_t} &= \left\{\frac{H(1 - \alpha)(\alpha\tau_{t+1} - \phi)g_{t+1}^{-\alpha} - 1}{\eta}\right\}\frac{\partial g_{t+1}}{\partial b_t}. \end{aligned}$$

Consequently,

$$\frac{\partial g_{t+1}}{\partial b_t} = \frac{\eta(1 + r_t^B)}{(Hg_t^{1-\alpha} + 1 - c_t)[H(1 - \alpha)(\alpha\tau_{t+1} - \phi)g_{t+1}^{-\alpha} - 1]}.$$

Derivative both sides of equation (50) and equation (51) by c_t we obtain:

$$\begin{aligned} \left(\frac{\partial b_{t+1}}{\partial c_t} + \frac{\partial d_{t+1}}{\partial c_t}\right)(Hg_t^{1-\alpha} + 1 - c_t) - (b_{t+1} + d_{t+1}) &= 0. \\ \frac{\partial b_{t+1}}{\partial c_t} + \frac{\partial d_{t+1}}{\partial c_t} &= \left\{\frac{H(1 - \alpha)(\alpha\tau_{t+1} - \phi)g_{t+1}^{-\alpha} - 1}{\eta}\right\}\frac{\partial g_{t+1}}{\partial c_t} \end{aligned}$$

It results that

$$\frac{\partial g_{t+1}}{\partial c_t} = \frac{\eta(b_{t+1} + d_{t+1})}{(Hg_t^{1-\alpha} + 1 - c_t)[H(1 - \alpha)(\alpha\tau_{t+1} - \phi)g_{t+1}^{-\alpha} - 1]}.$$

Derivative both sides of equation (50) and equation (51) by d_t we have:

$$\begin{aligned} \left(\frac{\partial b_{t+1}}{\partial d_t} + \frac{\partial d_{t+1}}{\partial d_t}\right)(Hg_t^{1-\alpha} + 1 - c_t) &= 1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha}. \\ \frac{\partial b_{t+1}}{\partial d_t} + \frac{\partial d_{t+1}}{\partial d_t} &= \left\{\frac{H(1 - \alpha)(\alpha\tau_{t+1} - \phi)g_{t+1}^{-\alpha} - 1}{\eta}\right\} \frac{\partial g_{t+1}}{\partial d_t}. \end{aligned}$$

Consequently,

$$\frac{\partial g_{t+1}}{\partial d_t} = \frac{\eta(1 + \alpha Hg_t^{1-\alpha})}{(Hg_t^{1-\alpha} + 1 - c_t)[H(1 - \alpha)(\alpha\tau_{t+1} - \phi)g_{t+1}^{-\alpha} - 1]}.$$

From equation (31) we obtain:

$$81 \quad (Hg_t^{1-\alpha} + 1 - c_t)d_{t+1} = \alpha(1 - \tau_t)Hg_t^{1-\alpha} + \{1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha}\}d_t - Hg_t^{1-\alpha}. \quad (52)$$

Derivative both sides of equation (52) by g_t we have:

$$\frac{\partial d_{t+1}}{\partial g_t}(Hg_t^{1-\alpha} + 1 - c_t) + d_{t+1}H(1 - \alpha)g_t^{-\alpha} = \alpha(1 - \alpha)(1 - \tau_t)Hg_t^{-\alpha} + d_tH\alpha(1 - \alpha)(1 - \tau_t)g_t^{-\alpha} - H(1 - \alpha)g_t^{-\alpha},$$

or

$$\frac{\partial d_{t+1}}{\partial g_t} = \frac{H(1 - \alpha)g_t^{-\alpha}\{\alpha(1 - \tau_t) + \alpha(1 - \tau_t)d_t - 1 - d_{t+1}\}}{Hg_t^{1-\alpha} + 1 - c_t}.$$

Similar to b_t , c_t , and d_t , derivative both sides of equation (52), we obtain:

$$\begin{aligned} \frac{\partial d_{t+1}}{\partial b_t} &= 0. \\ \frac{\partial d_{t+1}}{\partial c_t} &= \frac{d_{t+1}}{Hg_t^{1-\alpha} + 1 - c_t}. \\ \frac{\partial d_{t+1}}{\partial d_t} &= \frac{1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha}}{Hg_t^{1-\alpha} + 1 - c_t}. \end{aligned}$$

From equation (30) we can rewrite as following:

$$c_{t+1}(Hg_t^{1-\alpha} + 1 - c_t) = c_t\{\beta(1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha})\}^{1/\rho}. \quad (53)$$

Derivative both sides of equation (53) by g_t we have:

$$\frac{\partial c_{t+1}}{\partial g_t}(Hg_t^{1-\alpha} + 1 - c_t) + c_{t+1}H(1 - \alpha)g_t^{-\alpha} = \frac{c_t\beta^{\frac{1}{\rho}}\alpha(1 - \alpha)(1 - \tau_t)Hg_t^{-\alpha}(1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha})^{\frac{1-\rho}{\rho}}}{\rho}.$$

Consequently,

$$\frac{\partial c_{t+1}}{\partial g_t} = \frac{H(1 - \alpha)g_t^{-\alpha}\{c_t\beta^{\frac{1}{\rho}}\alpha(1 - \tau_t)(1 + H\alpha g_t^{1-\alpha})^{\frac{\rho}{1-\rho}} - c_{t+1}\}}{\rho(Hg_t^{1-\alpha} + 1 - c_t)}.$$

Derivative equation (53) by b_t and d_t we have:

$$\begin{aligned} \frac{\partial c_{t+1}}{\partial b_t} &= 0. \\ \frac{\partial c_{t+1}}{\partial d_t} &= 0. \end{aligned}$$

Derivative equation (53) by c_t we obtain:

$$\left(\frac{\partial c_{t+1}}{\partial c_t}\right)(Hg_t^{1-\alpha} + 1 - c_t) - c_{t+1} = \{\beta(1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha})\}^{1/\rho},$$

or

$$\frac{\partial c_{t+1}}{\partial c_t} = \frac{\{\beta(1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha})\}^{\frac{1}{\rho}} + c_{t+1}}{Hg_t^{1-\alpha} + 1 - c_t}.$$

From equation (28) we can rewrite as following:

$$b_{t+1}(Hg_t^{1-\alpha} + 1 - c_t) = g_t + (1 + r_t^B)b_t + \{1 + \alpha(1 - \tau_t)Hg_t^{1-\alpha}\}d_t - \alpha\tau_tHg_t^{1-\alpha} - d_{t+1}(Hg_t^{1-\alpha} + 1 - c_t). \quad (54)$$

Derivative equation (54) by g_t , we obtain:

$$\frac{\partial b_{t+1}}{\partial g_t}(Hg_t^{1-\alpha} + 1 - c_t) + b_{t+1}H(1-\alpha)g_t^{-\alpha} = 1 + d_t\alpha(1-\alpha)(1-\tau_t)Hg_t^{-\alpha} - \alpha(1-\alpha)\tau_tHg_t^{-\alpha} - \frac{\partial d_{t+1}}{\partial g_t}(Hg_t^{1-\alpha} + 1 - c_t) - d_{t+1}H(1-\alpha)g_t^{-\alpha},$$

with

$$\frac{\partial d_{t+1}}{\partial g_t} = \frac{H(1-\alpha)g_t^{-\alpha}\{\alpha(1-\tau_t) + \alpha(1-\tau_t)d_t - 1 - d_{t+1}\}}{Hg_t^{1-\alpha} + 1 - c_t},$$

or

$$\begin{aligned} \frac{\partial b_{t+1}}{\partial g_t}(Hg_t^{1-\alpha} + 1 - c_t) &= 1 + H(1-\alpha)g_t^{-\alpha}(2\alpha\tau_t d_t - \alpha - 1 - b_{t+1}). \\ \frac{\partial b_{t+1}}{\partial g_t} &= \frac{1 + H(1-\alpha)g_t^{-\alpha}(2\alpha\tau_t d_t - \alpha - 1 - b_{t+1})}{Hg_t^{1-\alpha} + 1 - c_t}. \end{aligned}$$

Derivative equation (54) by b_t , we obtain:

$$\frac{\partial b_{t+1}}{\partial b_t}(Hg_t^{1-\alpha} + 1 - c_t) = 1 + r_t^B,$$

or

$$\frac{\partial b_{t+1}}{\partial b_t} = \frac{1 + r_t^B}{Hg_t^{1-\alpha} + 1 - c_t}.$$

Similarly, derivative equation (54) by c_t we have:

$$\frac{\partial b_{t+1}}{\partial c_t}(Hg_t^{1-\alpha} + 1 - c_t) - b_{t+1} = -\frac{\partial d_{t+1}}{\partial c_t}(Hg_t^{1-\alpha} + 1 - c_t) + d_{t+1},$$

with

$$\frac{\partial d_{t+1}}{\partial c_t} = \frac{d_{t+1}}{Hg_t^{1-\alpha} + 1 - c_t}.$$

Consequently,

$$\frac{\partial b_{t+1}}{\partial c_t} = \frac{b_{t+1}}{Hg_t^{1-\alpha} + 1 - c_t}.$$

Derivative equation (54) by d_t we obtain:

$$\frac{\partial b_{t+1}}{\partial d_t} = 0.$$

At the steady-state point, we compute all the components of the Jacobian matrix by using Maple software.

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