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# How can the labor market accounts for the effectiveness of fiscal policy over the business cycle?

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## Abstract

We develop a new-Keynesian model with a two-sector search and matching labor market framework. We investigate the first and second order effects of fiscal policy on labor market and on output. The model includes four fiscal instruments: a labor income tax, a social protection tax paid by firms, public wage and public vacancies. First-order simulations of the model indicate that whatever instrument is used, fiscal expansion significantly increases total employment and reduce unemployment. We explicit the different transmission channels at work. The main contribution is to use a second-order approximation of the model to investigate the effects of fiscal shocks for two states of the economy: a low unemployment state (6%) and a high unemployment state (12%). For the four fiscal instruments, response of employment is greater when the steady-state unemployment rate is high. We also emphasize a new channel for explaining a larger output fiscal multiplier in periods of economic downturn: the wage channel that plays a crucial role for explaining the non-linear effects of fiscal policy.

***JEL classification:*** E62, J38.

***Keywords:*** Labor Market Search, Wage Bargaining, Public Wage, Business Cycle, Fiscal Policy, Second Order.

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# 1 Introduction

Recent literature indicates that the position over the business cycle greatly influences the size of the fiscal multiplier. Empirical studies show that the output fiscal multiplier is in periods of economic downturn <sup>1</sup>. However, the theoretical mechanisms behind this result still needs to be evidenced. If some intuitive mechanism could support this result <sup>2</sup>, explanations based on theoretical frameworks are still rare. The aim of our paper is to contribute to this theoretical literature. More precisely, our analysis focuses on the response of the labor market to fiscal policy assuming different values of the steady-state unemployment rate, thus to different positions of the economy over the business cycle. Especially, we attempt to show that the labor market dynamics, and especially the response of real wage, can explain different output fiscal multipliers according to the unemployment rate at the steady state.

Sims and Wolff (2013) and Michailat (2014) have attempted to investigate the non-linear effects of fiscal policy theoretically. Sims and Wolff (2013) investigate in a standard DSGE model the effects of fiscal policy on private consumption for different positions of the economy over the business cycle. Since a first version of their model does not include investment, the size of the multiplier (around 1) only depends on the response of private consumption. After the estimation of the model at the first-order, authors compute the output fiscal multiplier for each position over the business cycle. Their main finding is that the marginal utility of consumption is greater during economic downturn (of consumption is lower), thus the crowding-out effect of public spending on private consumption is reduced. This explanation of a greater fiscal multiplier in the trough of the economic cycle is an interesting first step. However, other aspects of an economy in a period of economic downturn must be taken into account, including the presence of non-Ricardian households <sup>3</sup> or the potential different dynamics of the labor market. Michailat (2014) focuses on the response of the labor market following a rise in public employment in a DSGE model with a search and matching labor market. The main

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<sup>1</sup>See Auerbach and Gorodnichenko (2012) and Creel et al. (2011) among others

<sup>2</sup>We could expect a higher price stickiness during economic downturn that produces greater real effects of spending expansion or a higher share of non-Ricardian households that diminish the crowding out effect of public consumption on private consumption.

<sup>3</sup>See for instance Coenen and Straub (2005) for discussions about the impact of the presence of non-Ricardian households on fiscal multipliers.

result is that when the unemployment rate is high (8% in the paper), a rise in public employment has a greater effect on total employment than in the case of a low unemployment rate (4%). By construction the model produces a crowding-out effect of public employment on private employment, as has been shown in empirical studies and notably in Ramey (2011). In Michailat (2014), the crowding-out effect is based on a lower pool of unemployed searching for a job in the private sector following the rise in public vacancies. When the pool of job seekers is high at the steady state, this crowding out effect is then lower. However, the author does not consider the role of the wage channel since the real wage law of motion is assumed as exogenous.

Our work follows Michailat (2014) by focusing on the effects of fiscal policy on the labor market according to the steady-state value of the unemployment rate. In comparison to this paper, we introduce a Nash efficient bargaining process that determines the law of motion of real wage. This is of first importance since our main result is based on the response of the real wage. Also, Michailat (2014) introduces only non-Ricardian households while we introduce both optimizing and *hand-to-mouth* households. Moreover, we show that the introduction of Ricardian households is necessary to produce higher output fiscal multipliers.

In this paper we construct a new-Keynesian model with a search and matching framework for the labor market. Workers can find a job in both the private and the public sector. Our modelling of the dual labor market is close to other papers like Afonso and Gomes (2014) and Brückner and Pappa (2012). Four fiscal instruments are introduced: a labor income tax, a social protection tax paid by firms, the public wage and finally the public vacancies. The first part of the paper is dedicated to analyzing the effects of these four fiscal instruments on the labor market. To achieve this investigation, we use a first-order approximation of the model. A main result is that the four fiscal expansions generate a drop in unemployment and a drop in private real wage, except for the cut in social protection tax. We disentangle the different transmission channels specific to each fiscal instrument.

The main contribution of the paper is to solve the model at the second-order in order to analyze the non-linear effects of fiscal policy according to two different steady-state levels of unemployment, in the spirit of Michailat (2014). A first result is that the four fiscal instruments triggers a larger rise

in employment in the case of a high steady-state unemployment rate (12% in our paper, in comparison with a lower unemployment rate of 6%). The explanation for this result is close to the one highlighted in Michailat (2014). A larger pool of job seekers at the steady state generates more job creations following the fiscal shocks.

This higher rise in private employment is the starting point for explaining a larger output fiscal multiplier during economic downturn. This stronger effect on employment when unemployment is high engenders a larger degradation of marginal productivity of labor and then a larger degradation of real wage. The greater drop in real wage implies a lower consumption for the non-Ricardians. However it also implies a lower marginal cost, inflation and a lower rise in interest rates. In this case, consumption of Ricardian households is less reduced than in the case of a low unemployment rate at the steady state. The total effect on aggregate demand depends on the relative strength of these two opposite effects on private consumption. Under our standard calibration, the higher consumption of Ricardian households prevails over the lower consumption of non-Ricardian households. It finally induces a larger output fiscal multiplier in the case of a high steady-state unemployment rate.

Thus, this paper attempts to offer a new theoretical explanation for variations of the output fiscal multiplier over the business cycle. Sims and Wolff (2013) argue for a larger output fiscal multiplier during economic downturn due to a larger marginal utility of consumption. It is important to note that our model does not include this transmission channel. On the contrary, the definition of steady-state values for both types of consumption implies a lower marginal utility of consumption for the Ricardian households during economic downturn. The wage channel we highlight in this paper is not in contradiction with the explanation found in Sims and Wolff (2013). However, the coexistence of these two effects could partly explain the sizeable difference found in the literature about the size of the output fiscal multiplier according to the position of the economy over the business cycle.

The rest of the paper is organized as follows: section 2 presents the model, section 3 presents the calibration, the steady-state calculations and the second-order solution of the model. Section 4 highlights the main results of the paper and section 5 concludes.

## 2 Model

The model used in this paper features nominal rigidity on prices and matching frictions on the labor market. Since the focus is on the effects of the public sector on the economy, two sectors coexist on the labor market, namely a public and a private sector. We make explicit the choice to work in the private sector or in the public sector. Also, we introduce an efficient Nash wage bargaining in which the public wage directly affects the determination of the private wage and thus employment in both sectors.

The model features also a rich fiscal side, with several types of expenditure/taxes in order to investigate the second-order effects of fiscal policy on the labor market according to the fiscal tool considered. On the expenditures side, we consider the effects of a rise of the public wage and of the public vacancies. On the taxes side, we investigate the effects of a labor revenue tax cut and a social protection tax cut.

### 2.1 Definitions and the matching process

Let us first define the non-employed pool  $1 - (1 - \rho)E_t^{tot}$  such as:

$$1 - (1 - \rho)E_t^{tot} = U_t + \rho E_t^{tot}, \quad (2.1)$$

where  $E_t^{tot}$  denotes the employed workers and  $U_t$  the pool of unemployed workers. The destruction rate  $\rho$  is assumed to be exogenous.

Moreover, the pool of job seekers  $S_t$  is expressed as

$$S_t = U_t + \rho E_t^{tot}. \quad (2.2)$$

Also, in the spirit of Trigari (2006), assuming that a new job becomes productive only in the following period and assuming that a match can be instantaneously broken, employment in a particular sector  $E_t^i$  can be expressed as:

$$E_t^i = (1 - \rho)E_{t-1}^i + p_{t-1}^i(1 - \rho)S_{t-1}, \quad (2.3)$$

with  $i = p, g$  that denotes both the public and private sectors. These definitions are common to both sectors. The job-finding probability in the sector

$i$ ,  $p_t^i$ , is defined later on. With these definitions, it is important to note that total employment is a predetermined variable.

Finally, the dynamic of job seekers is given by

$$S_t = (1 - p_{t-1}^p - p_{t-1}^g)S_{t-1} + \rho(p_{t-1}^p + p_{t-1}^g)S_{t-1} + \rho(E_{t-1}^p + E_{t-1}^g). \quad (2.4)$$

According to equation (2.4), the number of job seekers in the current period is equal to the number of job seekers who did not find a job either in the private sector or in the public sector in the previous period plus the number of job seekers is increased by the number of jobs which are destroyed in the previous period. Finally, we assume that there is a kind of trial period: a worker can match a firm in the beginning of the period but the relationship can be broken at the end of the period exogenously.

Let us now define the matching process occurring on a specific labor market, such as:

$$M_t^i = \kappa_e^i (S_t)^{\varphi^i} (V_t^i)^{(1-\varphi^i)}, \quad (2.5)$$

where  $\kappa_e^i$  denotes the matching technology in a particular sector while  $\varphi^i$  denotes the elasticity of employment for a supplementary unemployed worker.  $V_t^i$  is the number of vacancies in the sector  $i$ .

We can therefore set the following usual definitions:

$$p_t^i = \frac{M_t^i}{S_t}, \quad (2.6)$$

$$\text{and } q_t^i = \frac{M_t^i}{V_t^i} \quad (2.7)$$

with  $p_t^i$  the job finding probability in the sector  $i$  as previously introduced;  $q_t^i$  is the probability for a firm to fill the posted vacancy.

The labor market tightness (LMT thereafter) can be defined as

$$\theta_t^i = \frac{V_t^i}{S_t} = \frac{p_t^i}{q_t^i}. \quad (2.8)$$

## 2.2 Households' decisions

In this model two different types of agents are introduced. We assume a share  $\mu$  of non-Ricardian (*hand-to-mouth*) households and a share  $(1 - \mu)$  of Ricardian households. The difference between both types of households is their ability to participate in financial markets. Non-Ricardian can neither loan nor save so that they simply consume their disposable income in each period. On the contrary, Ricardian households can hold a riskless asset that allows them to optimize their consumption inter-temporally. Also, Ricardian households invest in physical capital that they then loan to firms. Both types of households formulate similar labor market decisions.

### 2.2.1 Ricardian households

A representative Ricardian household maximizes its lifetime utility and its utility function is defined as:

$$u(C_t^o, C_{t-1}^o, G_t, e_{jt}) = \frac{(C_t^o - HC_{t-1}^o)^{1-\sigma_c} - 1}{1 - \sigma_c} + M^o(e_{jt}) \quad (2.9)$$

where  $C_t^o$  denotes consumption of Ricardian households. Additively separable preferences of consumption and labor are introduced in an usual manner with  $\sigma_c$  the inter-temporal elasticity of substitution for consumption. The consumption decision is subject to a degree  $H$  of habit formation. The function  $M^o(e_{jt})$  represents the amount of leisure in terms of utility with regard to the presence of the household member on the labor market.

Following Ravn (2005, 2008),  $e_{jt}$  with  $j = n, u, l$  denoting the level of leisure according to the status of the household on the labor market *i.e.*  $e_{nt}$  for an employed worker,  $e_{ut}$  for an unemployed worker and  $e_{lt}$  for an inactive household such as:

$$e_{nt} = 1 - h - s, \quad (2.10)$$

$$e_{ut} = 1 - s, \quad (2.11)$$

$$e_{lt} = 1, \quad (2.12)$$

where  $h$  denotes hours worked that we assume as exogenous and  $s$  a fixed cost in the participate to labor market.



We consider the case of a representative worker in the spirit of Merz (1995), so that the function  $M^o(e_{it})$  contains the different possible statuses of a worker on the labor market, such as:

$$M^o(e_{it}) = \frac{[(E_t^{op} + E_t^{og})(1 - h - s)^{1-\zeta} + S_t^o(1 - s)^{1-\zeta} + (1 - (E_t^{op} + E_t^{og}) - S_t^o)]}{1 - \zeta} \quad (2.13)$$

where  $-1/\zeta$  defines the Frisch elasticity of labor supply and  $S_t^o$  is the share of Ricardians seeking employments in period  $t$ .  $E_t^{op}$  denotes employment of Ricardian households in the private sector while  $E_t^{og}$  denotes employment of Ricardian households in the public sector.

The optimization problem for the representative Ricardian household is expressed as:

$$\max_{C_t^o, K_t^o, B_t, E_t^o, S_t^o, I_t^o} E_t \sum_{s=t}^{\infty} \beta^s u(C_{t+s}^o, C_{t-1+s}^o, G_{t+s}, e_{t+s}). \quad (2.14)$$

subject to

$$(1 + \tau^c)C_t^o + \frac{B_t}{P_t} + I_t^o \leq R_{t-1}^k K_{t-1} + \frac{R_{t-1} B_{t-1}}{P_t} + b(S_t^o) + (1 - \tau_t^w)[W_t^g h E_t^{og} + W_t^p h E_t^{op}] \quad (2.15)$$

$$K_t^o = (1 - \delta^k)K_{t-1}^o + [1 - A(I_t^o/I_{t-1}^o)]I_t^o \quad (2.16)$$

$$E_t^{op} = (1 - \rho)E_{t-1}^{op} + p_{t-1}^p(1 - \rho)S_{t-1}^o \quad (2.17)$$

$$E_t^{og} = (1 - \rho)E_{t-1}^{og} + p_{t-1}^g(1 - \rho)S_{t-1}^o \quad (2.18)$$

$$S_t^o = (1 - p_{t-1}^p - p_{t-1}^g)S_{t-1}^o + \rho(p_{t-1}^p + p_{t-1}^g)S_{t-1}^o + \rho(E_{t-1}^{op} + E_{t-1}^{og}) \quad (2.19)$$

that can be reduced to the following Bellman equation:

$$\begin{aligned} \Omega_t^o(K_t^o, E_t^o, B_t, I_t^o) = & \max_{C_t^o, K_t^o, S_t^o, n_t^o, B_t, I_t^o} \left\{ \frac{(C_t^o - HC_{t-1}^o)^{1-\sigma_c}}{1 - \sigma_c} + \frac{\zeta_g g_t^{1-\sigma_c} - 1}{1 - \sigma_c} \right. \\ & \left. + \frac{[(E_t^{op} + E_t^{og})(1 - h - s)^{1-\zeta} + S_t^o(1 - s)^{1-\zeta} + (1 - (E_t^{op} + E_t^{og}) - S_t^o)]}{1 - \zeta} \right\} \\ & + \beta \Omega_{t+1}^o(K_{t+1}^o, E_{t+1}^o, B_t, I_t^o), \end{aligned} \quad (2.20)$$

subject to the previous set of constraints with  $\beta$  the discount factor. Equation (2.15) is the budget constraint for the household. The optimizing household has access to perfect financial markets and can thus hold a riskless asset  $B_t$ . Furthermore, the household invests  $I_t^o$  in physical capital  $K_t^o$  and loan it to the firms at a rate  $R_t^k$ .  $\delta^k$  defines the depreciation rate of capital,  $R_t$  the nominal interest rate equals to  $\frac{1}{\beta}$  at the steady state and  $b$  the unemployment benefits.  $W_t^g$  and  $W_t^p$  are the real wages respectively in the public and the private sector.  $P_t$  defines the consumer price index (CPI thereafter). We note the appearance of two taxes, a constant VAT  $\tau^C$  and a labor revenue tax  $\tau_t^w$ . Equation (2.16) represents the law of motion of capital accumulation. We introduce an adjustment cost to investment changes with  $A(I_t^o/I_{t-1}^o) = \frac{\kappa}{2}(I_t^o/I_{t-1}^o - 1)^2$  similarly to Christiano et al. (2005) or Smets and Wouters (2007) among others with  $\kappa$  a constant cost associated to investment decisions.

First order conditions with respect to respectively  $C_t^o$ ,  $B_t$ ,  $I_t^o$ ,  $K_t^o$ ,  $E_t^{op}$ ,  $E_t^{og}$  and  $S_t^o$  yield:

$$\lambda_t^{rio} = \frac{[C_t^o - HC_{t-1}^o]^{-\sigma_c} - \beta HE_t\{[C_{t+1}^o - HC_t^o]^{-\sigma_c}\}}{1 + \tau^c} \quad (2.21)$$

$$\lambda_t^{rio} = r_t \beta E_t \left[ \frac{\lambda_{t+1}^{rio}}{\pi_{t+1}} \right], \quad (2.22)$$

with  $\pi_{t+1} = p_{t+1}/p_t$  defining the CPI inflation rate.

$$1 = Q_t [1 - A(I_t/I_{t-1})] \quad (2.23)$$

$$Q_t = \beta E_t \left[ \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} [(1 - \delta^k)Q_{t+1} + R_t^k] \right] \quad (2.24)$$

$$\begin{aligned} \lambda_t^{Eop} &= (1 - \tau_t^w) \lambda_t^{rio} W_t^p h - \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} \\ &\quad + \beta E_t [(1 - \rho)(\lambda_{t+1}^{Eop} - \lambda_{t+1}^{So}) + \lambda_{t+1}^{So}] \end{aligned} \quad (2.25)$$

$$\begin{aligned} \lambda_t^{Eog} &= (1 - \tau_t^w) \lambda_t^{rio} W_t^g h - \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} \\ &\quad + \beta E_t [(1 - \rho)(\lambda_{t+1}^{Eog} - \lambda_{t+1}^{So}) + \lambda_{t+1}^{So}] \end{aligned} \quad (2.26)$$

$$\begin{aligned}
\lambda_t^{S_o} &= b\lambda_t^{rio} - \frac{1 - (1 - s)^{1-\zeta}}{1 - \zeta} \\
&+ (1 - p_t^p - p_t^g)\beta E_t[\lambda_{t+1}^{S_o}] + \rho(p_t^p + p_t^g)\beta E_t[\lambda_{t+1}^{S_o}] \\
&+ (1 - \rho)\beta E_t[p_t^p \lambda_{t+1}^{E_{op}} + p_t^g \lambda_{t+1}^{E_{og}}]
\end{aligned} \tag{2.27}$$

with  $\lambda_t^{rio}$  the marginal utility of consumption for Ricardians,  $\lambda_t^{E_{op}}$  the marginal utility of working in the private sector,  $\lambda_t^{E_{og}}$  the marginal utility of working in the public sector and  $\lambda_t^{S_o}$  the marginal utility to be currently a job seeker.

Equation (2.25) defines the value of a job for a Ricardian household in the private sector while equation (2.26) determines the value of a job in the public sector. Also, equation (2.27) determines the decision for a Ricardian worker to participate in the labor market.

### 2.2.2 *Hand-to-mouth consumers*

Non-Ricardian households do not maximize consumption inter-temporally and then simply consume their disposable income in each period. For a representative non-Ricardian household, net VAT consumption is given by:

$$(1 + \tau^c)C_t^r = (1 - \tau_t^w)[W_t^g h E_t^g + W_t^p h E_t^p] + bS_t^r \tag{2.28}$$

with  $C_t^r$  the consumption level for non-Ricardians. The choices made by this class of households concerning the labor market is similar to the Ricardian case.

Similarly to Ricardian households, the utility function for this class of households is expressed as:

$$u(C_t^r, C_{t-1}^r, G_t, e_{it}) = \frac{(C_t^r - HC_{t-1}^r)^{1-\sigma_c} - 1}{1 - \sigma_c} + \frac{\zeta_g G_t^{1-\sigma_c}}{1 - \sigma_c} + M^r(e_{jt}) \tag{2.29}$$

with

$$M^r(e_{it}) = \frac{[(E_t^{rp} + E_t^{rg})(1 - h - s)^{1-\zeta} + S_t^r(1 - s)^{1-\zeta} + (1 - (E_t^{op} + E_t^{og}) - S_t^r)]}{1 - \zeta} \tag{2.30}$$

The corresponding Bellmann equation and constraints for this optimization program is therefore:

$$\Omega_t^r = \max_{S_t^r, E_t^r, E_t^g} \left\{ \frac{(C_t^r - HC_{t-1})^{1-\sigma_c}}{1-\sigma_c} + \frac{[(E_t^{rp} + E_t^{rg})(1-h-s)^{1-\zeta} + S_t^r(1-s)^{1-\zeta} + (1 - (E_t^{rp} + E_t^{rg}) - S_t^r)]}{1-\zeta} \right\} + \beta \Omega_{t+1}^r \quad (2.31)$$

s.t.

$$(1 + \tau^c)C_t^r \leq (1 - \tau_t^w)[W_t^g h E_t^{rg} + W_t^p h E_t^{rp}] + bS_t^r \quad (2.32)$$

$$E_t^{rp} = (1 - \rho)E_{t-1}^{rp} + p_{t-1}^p(1 - \rho)S_{t-1}^r \quad (2.33)$$

$$E_t^{rg} = (1 - \rho)E_{t-1}^{rg} + p_{t-1}^g(1 - \rho)S_{t-1}^r \quad (2.34)$$

$$S_t^r = (1 - p_{t-1}^p - p_{t-1}^g)S_{t-1}^r + \rho(p_{t-1}^p + p_{t-1}^g)S_{t-1}^r + \rho(E_{t-1}^{rp} + E_{t-1}^{rg}) \quad (2.35)$$

First order conditions with rapport to  $E_t^{rp}$ ,  $E_t^{rg}$  and  $S_t$  yield:

$$\lambda_t^{E_{rp}} = (1 - \tau_t^w)\lambda_t^{rir}W_t^p h - \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} + \beta E_t[(1 - \rho)(\lambda_{t+1}^{E_{rp}} - \lambda_{t+1}^{S_r}) + \lambda_{t+1}^{S_r}] \quad (2.36)$$

$$\lambda_t^{E_{rg}} = (1 - \tau_t^w)\lambda_t^{rir}W_t^g h - \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} + \beta E_t[(1 - \rho)(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r}) + \lambda_{t+1}^{S_r}] \quad (2.37)$$

$$\lambda_t^{S_r} = b\lambda_t^{rir} - \frac{1 - (1 - s)^{1-\zeta}}{1 - \zeta} + (1 - p_t^p - p_t^g)\beta E_t[\lambda_{t+1}^{S_r}] + \rho(p_t^p + p_t^g)\beta E_t[\lambda_{t+1}^{S_r}] + (1 - \rho)\beta E_t[p_t^p \lambda_{t+1}^{E_{rp}} + p_t^g \lambda_{t+1}^{E_{rg}}] \quad (2.38)$$

with  $\lambda_t^{E_{rp}}$  the marginal utility of working in the private sector for a non-Ricardian household,  $\lambda_t^{E_{rg}}$  similarly in the public sector and  $\lambda_t^{S_r}$  the marginal utility for a non-Ricardian to seek employment on the labor market.

Equation (2.36) defines the value of a job in the private sector for a non-Ricardian household while (2.37) defines the value of a job in the public sector. Also, equation (2.38) relates to the decision of a non-Ricardian worker to seek a job.

Even if non-Ricardian households do not maximize consumption intertemporally, maximization of (2.31) with rapport to  $C_t^r$  allows to obtain their marginal utility of consumption such as:

$$\lambda_t^{rir} = \frac{(C_t^r - HC_{t-1}^r)^{\sigma_c} - \beta E_t[H(C_{t+1}^r - HC_t^r)^{\sigma_c}]}{1 + \tau^c} \quad (2.39)$$

## 2.3 Firms

For the purposes of the model, we need to introduce three kinds of firms as in Trigari (2006). First, some firms we refer as "producers" produce goods with labor and private capital in a competitive environment. The producers then sell their aggregate goods to "intermediate firms", transforming the aggregate good on a continuum of differentiated goods in a monopolistic competition environment. The intermediate firms are the price-setters and set their optimal price subject to nominal rigidity as in Calvo (1983). Finally, a continuum of "final goods firms" in a competitive environment purchase the differentiated intermediate goods and package them to sell it to consumers. This dissociation between producers and intermediate firms is necessary because introducing the price-setting at the producers level would greatly complicate the decision of these firms on the labor market. However, this simplifying assumption has no important consequences either on the price dynamic or on the labor market dynamics<sup>4</sup>.

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<sup>4</sup>For more details, Christoffel et al. (2009a) made a survey on the implication of this assumption. In the spirit of Kuester (2010), Sveen and Weinke (2007) and Thomas (2011), Christoffel et al. (2009b) demonstrate that the dissociation assumption not only has no spurious consequences but also helps the standard Keynesian model to match stylized facts in terms of inflation reactions to monetary shocks.

### 2.3.1 The producers

Since the producers evolve in a competitive environment, they all behave similarly and we can consider the following optimization program with a representative firm, such as:

$$\max_{\tilde{K}_t, E_t^p, V_t} E_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \{Y_t - R_t^k \tilde{K}_t - (1 + \tau_t^{sp}) W_t^p E_t^p h - \kappa^v V_t\} \quad (2.40)$$

s.t.

$$Y_t = \epsilon_t^A (K_{t-1}^g)^{\alpha g} [\tilde{K}_t]^\alpha [E_t^p h]^{1-\alpha} \quad (2.41)$$

$$E_t^p = (1 - \rho) E_{t-1}^p + q_{t-1}^p V_{t-1}^p \quad (2.42)$$

where  $\tau_t^{sp}$  in equation (2.40) denotes a tax on labor paid by the firms for security protection purposes. The discount factor is  $\beta_{t,t+1} = \beta \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}}$ . Moreover, the producers take the probability of filling a vacancy  $q_t^p$  as given.  $V_t$  denotes the vacancies posted by the producers and  $\kappa^v$  the unitary cost of vacancy posting. We assume that the accumulated capital becomes effective for production after one quarter  $\tilde{K}_t = K_{t-1}$ . The Total-Factor Productivity (TFP thereafter)  $\epsilon_t^A$  is driven by the following AR(1) process

$$\left( \frac{\epsilon_t^A}{\epsilon_s^A} \right) = \left( \frac{\epsilon_{t-1}^A}{\epsilon_s^A} \right)^{\rho_\epsilon} \exp(\varepsilon_t^a),$$

where  $\epsilon_s^A$  stands for the TFP at the steady-state,  $\exp(\varepsilon_t^a)$  is a *iid* exogenous disturbance and  $\rho_\epsilon$  the duration of the shock.

Equation (2.41) represents the production function of the representative producer. Equation (2.42) represents the dynamic of employment in the producers' point of view.

The problem (2.40) can be represented as a Bellman equation such as:

$$V(\Omega_t) = \max_{k_t, E_t^p, V_t} \{Y_t - R_t^k k_t - (1 + \tau_t^{sp}) W_t^p E_t^p h - \kappa^v V_t + \beta \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} V(\Omega_{t+1})\} \quad (2.43)$$

Under the free entry condition, the first order conditions with respect to vacancy posting and employment yield:

$$\frac{\kappa^v}{q_t^p} = \beta_{t,t+1} \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} \lambda_{t+1}^{E_f} \quad (2.44)$$

$$\lambda_t^{E_f} = (1 - \alpha) \frac{Y_t}{E_t^p} - (1 + \tau_t^{sp}) W_t^p h + (1 - \rho) \beta_{t,t+1} \frac{\lambda_{t+1}^{rio}}{\lambda_t^{rio}} \lambda_{t+1}^{E_f} \quad (2.45)$$

Equation (2.44) defines the value of a posted vacancy and (2.45) the value of a job for a producer.

Cost minimization subject to equation (2.41) implies the following factor demand conditions,

$$R_t^k = \frac{\alpha Y_t}{K_t} m c_t \quad (2.46)$$

$$x_t = (1 - \alpha) m c_t \frac{Y_t}{E_t^p h} - (1 + \tau_t^{sp}) W_t^p h, \quad (2.47)$$

where  $m c_t$  represents the level of producers' marginal costs. Equation (2.46) characterizes the demand of capital by the producers and equation (2.47) defines the marginal cost of labor  $x_t$ .

### 2.3.2 Intermediate firms, final goods firms and Calvo price-setting

There is a continuum  $j$  of intermediate firms that purchase the homogeneous goods from the producers at their marginal cost since the producers are in a competitive environment. The intermediate firms then transform the homogeneous goods on a continuum  $j$  of differentiated goods and sell them at the final goods firms.

Final goods firms produce a package of the intermediate differentiated goods according to:

$$Y_t = \left[ \int_0^1 Y_{jt}^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \quad (2.48)$$

where  $\varepsilon$  is the elasticity of substitution across intermediate goods. Demand for each intermediate good is of the form:

$$Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t, \quad (2.49)$$

with the following definition for the CPI  $P_t$ :

$$P_t = \left[ \int_0^1 P_{jt}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, \quad (2.50)$$

and with  $P_{jt}$  the price of good  $j$  in the period  $t$ .

Following Calvo (1983), intermediate firms are allowed to re-optimize their price only with a probability  $\theta_p \in [0, 1)$  in each period. This probability is assumed to be independent from the re-optimization decision taken in the last period.

Intermediate firms seek to maximize their lifetime profit according to their own price level such as:

$$E_t \sum_{k=0}^{\infty} (\beta \theta_p)^s \frac{\lambda_{t+s}^{rio}}{\lambda_t^{rio}} \left[ \frac{P_{j,t}}{P_{t+s}} - mc_{t+s} \right] Y_{j,t+s}, \quad (2.51)$$

subject to the demand function expressed in the equation (2.49). The first order condition yields:

$$P_{jt}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \frac{\lambda_{t+s}^{rio}}{\lambda_t^{rio}} [mc_{t+s} P_{t+s}^{\varepsilon} Y_{t+s}]}{E_t \sum_{s=0}^{\infty} (\beta \theta_p)^s \frac{\lambda_{t+s}^{rio}}{\lambda_t^{rio}} [P_{t+s}^{\varepsilon-1} Y_{t+s}]} \quad (2.52)$$

where  $P_{jt}^*$  is the optimal price of the intermediate firm  $j$  and  $\frac{\varepsilon}{\varepsilon - 1}$  the desired (natural) mark-up. The law of motion for aggregate prices is given by

$$P_t = [(1 - \theta_p) P_t^{*1-\varepsilon} + \theta_p P_{t-1}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \quad (2.53)$$

Equations (2.52) and (2.53) yield the New-Keynesian Phillips Curve once log-linearized and after some mathematical rearrangements.



## 2.4 Wage bargaining

Following Stähler and Thomas (2012) the union utility corresponds to the mean of the surplus on employment of all its members. With  $\mu$  the share of non-Ricardian households, let us express the union utility  $\Upsilon_t$  as:

$$\Upsilon_t = (1 - \mu)[\lambda_t^{E_{op}} - \lambda_t^{S_o}] + \mu[\lambda_t^{E_{rp}} - \lambda_t^{S_r}] \quad (2.54)$$

Let us now describe the surplus for both sorts of households. The surplus for a Ricardian household to stay employed and accept the wage level agreed during the wage bargaining rather than seek for a new job in both sectors is, after some re-arrangements and calculations:

$$\begin{aligned} \lambda_t^{E_{op}} - \lambda_t^{S_o} &= (1 - \tau_t^w)\lambda_t^{rio}W_t^p h - \lambda_t^{rio}b + \frac{(1 - h - s)^{1-\zeta} - (1 - s)^{1-\zeta}}{1 - \zeta} \\ &\quad + \beta E_t[(1 - p_t)(1 - \rho)(\lambda_{t+1}^{E_{op}} - \lambda_{t+1}^{S_o}) - p_t^g(1 - \rho)(\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o})] \end{aligned} \quad (2.55)$$

and for the non-Ricardian workers:

$$\begin{aligned} \lambda_t^{E_{rp}} - \lambda_t^{S_r} &= (1 - \tau_t^w)\lambda_t^{rir}W_t^p h - \lambda_t^{rir}b + \frac{(1 - h - s)^{1-\zeta} - (1 - s)^{1-\zeta}}{1 - \zeta} \\ &\quad + \beta E_t[(1 - p_t)(1 - \rho)(\lambda_{t+1}^{E_{rp}} - \lambda_{t+1}^{S_o}) - p_t^g(1 - \rho)(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r})] \end{aligned} \quad (2.56)$$

### 2.4.1 Nash product and efficient bargaining

Under the free entry condition, the Nash product can be expressed as:

$$\mathcal{N}_t = \Upsilon_t^\eta [\lambda_t^{E_f}]^{1-\eta}, \quad (2.57)$$

with  $\eta$  the union bargaining power.

In the case of efficient bargaining, firms and union jointly determine the real wage but not the hours worked in our model since we assume them as exogenously fixed.

Maximization of the Nash product with respect to the private real wage leads to the following optimal rule for the surplus allocation:

$$\eta \frac{\partial \Upsilon_t}{\partial W_t^p} \lambda_t^{E_f} = (1 - \eta) \frac{-\partial \lambda_t^{E_f}}{\partial W_t^p} \Upsilon_t \quad (2.58)$$

After several calculation steps (fully described in appendix C.4.1), we finally obtain this rule for the private real hourly wage (net of the income tax):

$$\begin{aligned}
(1 - \tau_t^w)W_t^p h &= \eta \frac{(1 - \alpha)(1 - \tau_t^w) Y_t}{(1 + \tau_t^{sp}) E_t^p} \\
&+ (1 - \eta) \left[ b + \frac{(1 - s)^{1-\zeta} - (1 - h - s)^{1-\zeta}}{(1 - \zeta)(\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio})} \right] \\
&+ \eta(1 - \rho) E_t \left\{ \beta_{t,t+1} \left[ 1 - (1 - p_t^p) \frac{(1 - \tau_{t+1}^w)}{(1 + \tau_{t+1}^{sp})} \tilde{\Lambda}_{t+1} \right] \lambda_{t+1}^{E_f} \right\} \\
&+ (1 - \eta)(1 - \rho) p_t^g \beta E_t [\Lambda_t (\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r}) + (1 - \Lambda_t) (\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o})],
\end{aligned} \tag{2.59}$$

with  $\Lambda_t = \frac{\mu\lambda_t^{rir}}{\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio}}$  the relative part of non-Ricardian consumers in the consumer pool and  $\tilde{\Lambda}_t = \frac{\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio}}{\mu\lambda_{t-1}^{rir} + (1 - \mu)\lambda_{t-1}^{rio}}$ .

## 2.5 Monetary and fiscal policies

In each period, the monetary authority set the nominal interest rate according to the following standard Taylor rule:

$$\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\alpha^r} \left( \frac{Y_t}{\bar{Y}} \right)^{\alpha^y} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\alpha^\pi} \tag{2.60}$$

with  $\alpha^r$  the degree of inertia of the nominal interest rate and  $\alpha^y$  and  $\alpha^\pi$  the relative weights given by the monetary authority to the stabilization of output and inflation.

The budget constraint in each period for the government equals to:

$$\tau_t^c C_t + (\tau_t^w + \tau_t^{sp})(W_t^p E_t^p h) + D_t = W_t^g E_t^g h + C^g + I^g + bS_t \tag{2.61}$$

The government is allowed to create a deficit  $D_t$  to finance supplementary expenditure or deterioration of the tax bases.  $I^g$  denotes public investment.

Public wage and public vacancies are considered as an AR(1) process such as:

$$\frac{W_t^g}{\bar{W}^g} = \left( \frac{W_{t-1}^g}{\bar{W}^g} \right)^{\rho^g} \exp(\xi_t^{Wg}) \quad (2.62)$$

$$\frac{V_t^g}{\bar{V}^g} = \left( \frac{V_{t-1}^g}{\bar{V}^g} \right)^{\rho^g} \exp(\xi_t^{Vg}) \quad (2.63)$$

where  $\rho^g$  denotes the duration of the shock. The term  $\xi_t$  is the white noises associated to the shock.

Each tax is also considered as an AR(1) process such as:

$$\frac{\tau_t^w}{\tau_s^w} = \left( \frac{\tau_{t-1}^w}{\tau_s^w} \right)^{\rho^g} \exp(\xi^{\tau^w}) \quad (2.64)$$

$$\frac{\tau_t^{sp}}{\tau_s^{sp}} = \left( \frac{\tau_{t-1}^{sp}}{\tau_s^{sp}} \right)^{\rho^g} \exp(\xi^{\tau^{sp}}) \quad (2.65)$$

## 2.6 Aggregation and market clearing

In order to clear the model, total demand addressed by both government and households to firms is expressed as:

$$Y_t = C_t + I_t + C^g + I^g \quad (2.66)$$

Given the previous description, aggregation yields

$$E_t^{tot} = E_t^p + E_t^g, \quad (2.67)$$

$$E_t^g = (1 - \mu)E_t^{og} + \mu E_t^{rg}, \quad (2.68)$$

$$E_t^p = (1 - \mu)E_t^{op} + \mu E_t^{rp}, \quad (2.69)$$

$$S_t = S_t^o + S_t^r \quad (2.70)$$

$$\theta_t = \theta_t^p + \theta_t^g \quad (2.71)$$

## 3 Calibration, steady-state calculations and second-order solution

We calibrate our model to a quarterly frequency. Some parameters are chosen so that long-run targeted values are reproduced. Table (1) presents the

baseline calibration for the households' preferences and for the firms' production function. Table 2 presents the baseline calibration for the fiscal and monetary policies and for the labor market.

The time-discount factor  $\beta$  is set to 0.997 in order to match an average annual real rate of 3%. According to Chetty et al. (2013) and to Peterman (2012), we set  $-\zeta$  to 1/3 in order to match the macro estimates of the Frisch elasticity. This parameter is slightly higher than the calibration chosen by Smets and Wouters (2007) which is around 2. Following Smets and Wouters (2003) and Stähler and Thomas (2012), we set the value of the risk aversion coefficient to  $\sigma_c = 2$ . Knowing that we set  $h = 0.33$ , we set the value of the fixed cost of participating in the labor market to  $s = 7.5\%$  of the time endowment. This value is halfway between Burnside and Eichenbaum (1996)'s value and Ravn (2005)'s value which are respectively equal to 5% and 9.9% of the time endowment. Following Smets and Wouters (2003) and Stähler and Thomas (2012), we set  $H = 0.85$ . Finally, we set  $\mu = 0.3$  which is quite similar to the choice made by Coenen and Straub (2005).

Regarding the monetary policy's parameters, we set the coefficient response to the output gap and to inflation to the respective values  $\alpha^y = 0.5$  and  $\alpha^\pi = 1.5$  as in Clarida et al. (2000) and Trigari (2006). The nominal interest rate smoothing coefficient is set to  $\alpha^r = 0.8$  as in Christoffel et al. (2009a).

Following Stähler and Thomas (2012), we set the public sector capital influence in the private production  $\alpha^g = 0.015$ , the adjustment cost parameter  $\kappa = 2.48$ . The share of the public sector in the whole economy is equal to  $fracpub = 0.19$ . Following Afonso and Gomes (2014) and Stähler and Thomas (2012), we set the elasticity of matches to unemployment in the public sector  $\varphi^g = 0.3$  in order to give greater importance to vacancies in the public sector. However, the elasticity of matches to unemployment in the private sector is equal to  $\varphi^p = 0.5$ . Finally, in order to satisfy the Hosios (1990) condition, we set the bargaining power as equal to the elasticity of matches to unemployment in the private sector.

Regarding the production side, we set the elasticity of substitution between differentiated goods at  $\varepsilon = 7$  in order to obtain an optimal markup of around 17%. The depreciation rate of capital is set to  $\delta^k = 0.025$  just as

in Moyen and Stähler (2010) and Stähler and Thomas (2012). The private sector capital influence coefficient follows the choice of Moyen and Stähler (2010) and it is set to  $\alpha = 0.3$ .

Table 1: Parameters and their calibrated values I

<u>Preferences</u>		
$\beta$	0.997	Time-discount factor
$-\zeta$	1/3	Reverse of Frisch elasticity
$\sigma^c$	2	Risk aversion
$h$	0.33	Worked hours
$s$	0.075h	Fixed cost of participating in the labor market
$H$	0.85	Degree of Consumption habits
$\mu$	0.3	Share of non-Ricardian workers in the economy
<u>Production</u>		
$\varepsilon$	7	Elasticity of substitution of goods
$\delta^k$	0.025	Depreciation rate of capital
$\alpha$	0.3	Private sector capital influence
$\kappa^v$	0.2	Vacancies posting costs

Regarding the long-run targeted values table 3 presents the different choices.

Table 2: Parameters and their calibrated values II

<u>Monetary Policy</u>		
$\alpha^r$	0.8	Interest rate smoothing
$\alpha^y$	0.5	Response coefficient to the output gap
$\alpha^\pi$	1.5	Response coefficient to inflation
<u>Fiscal Policy</u>		
$\rho^g$	0.6	Duration of the fiscal policy shock
<u>Labor market and wage bargaining</u>		
$\kappa$	2.48	Adjustment cost parameter
$\eta$	0.5	Workers' bargaining power
$\rho$	0.06	Job destruction
$\varphi^p$	0.5	Elasticity of matches to unemployment in the private sector
$\varphi^g$	0.3	Elasticity of matches to unemployment in the public sector
$fracpub$	0.19	Share of the public sector in the whole economy

Table 3: Targeted Values

$\pi_s$	1	Inflation
$p_s$	1	Prices
$Y_s$	1	Output
$C^g$	0.2	Public Consumption
$I^g$	0.03	Public Investment
$b$	$0.3Y_s$	Unemployment benefit
$\tau^c$	0.20	VAT
$\tau_s^w$	0.16	Income tax
$\tau_s^{sp}$	0.16	Social Protection tax
$U_s$	0.08	Unemployment
$q_s^p$	0.7	Job filling probability in private sector
$q_s^g$	0.8	Job filling probability in public sector

## 4 The effects of fiscal policy on the labor market and output

We simulate the model with all fiscal shocks in turn. We begin by using a first-order approximation of the model in order to emphasize the transmission

channels of the different fiscal instruments. Also, in the case of the public wage, we compare our results with those of Afonso and Gomes (2014). Then, the model is solved at the second-order in order to analyze the effects of the different fiscal shocks according to two steady-states for the unemployment rate. The low unemployment rate state consists in  $U_s = 6\%$  while the labor market in bad times is represented by  $U_s = 12\%$ .

## 4.1 Effects in normal time

The IRFs of the first order are presented in appendix A and B.

### 4.1.1 Public wage expansion financed by debt

A rise in  $W_t^g$  has a direct positive impact on consumption of non-Ricardian households. This effect is amplified by a rise in employment. On the other hand, we observe a drop in private real wage that produces downward pressures on non-Ricardian consumption. However, total response is unambiguously positive.

Output thus increases right at the moment of the shock. However, the rise in prices generates higher interest rates that progressively crowds out Ricardian consumption and investment. This crowding-out effect on private activity produces a negative response of output in the mid-term, as shown in the IRFs.

In contrast with Afonso and Gomes (2014), a rise in public wage produces a drop in private real wage and a rise in employment. In Afonso and Gomes (2014), authors explain higher private real wage through three different channels. First, a higher public wage increases the value of being unemployed, and we also share this channel. Secondly, their model generates a rise in marginal productivity of labor which creates upward pressures on private real wage. In our model, marginal productivity of labor clearly decreases due to a negative total effect on output and a clear rise in employment. This important aspect partly explains the different dynamics of private real wage produced by our model following a rise in public wage. Thirdly, Afonso and Gomes (2014) assume that the wage bill is entirely financed by a rise in labor income tax. The authors argue that this rise in the labor income tax has contradictory effects on real wage. In this simulation, we assume that the

supplementary spending is financed by debt. Indeed, in our model, all things being equal, an increase in the labor income tax triggers a raise in private real wage. Thus, introducing the labor income tax as financing the wage bill puts upward pressures on private real wage. For comparison purposes, we simulate a scenario similar to the one in Afonso and Gomes (2014) by replacing equation (2.64) by:

$$\tau^c(C_t^o + C_t^r) + \tau_t^w W_t^p E_t^p h + \tau^{sp} E_t^p W_t^p h = C^g + I^g + (1 - \tau_t^w)(W_t^g E_t^g h) + bS_t \quad (4.1)$$

In equation (4.1), we assume that  $\tau^c$  and  $\tau^{sp}$  are constant. In this case, the degradation of the public deficit is totally counterbalanced by a raise in the labor income tax.

As shown in appendix A, our model reproduces similar results in this case. Employment falls, unemployment rises and private real wage increases. We conclude that the rise in private real wage following a rise in public real wage strongly depends on the assumption made about the financing. As shown before, in case of a debt-based public wage expansion, our model produces a clear decrease in private real wage.

#### 4.1.2 Public vacancies expansion

Following Michailat (2014), a rise in public vacancies triggers a positive effect on total employment despite a crowding-out effect on private employment. The hiring of job seekers by the public sector increases the labor market tightness and thus triggers less job creation in the private sector. Since in our model a rise in public vacancies is wasteful (the public sector is unproductive), the effect on output is clearly negative because of a crowding-out effect on Ricardian households' consumption since real interest rate increases. Consumption of non-Ricardian households increases with the rise in total employment despite the decrease in real wage. For the first few periods, the response of output is positive, thanks to a rise in private investment. This rise in aggregate demand triggers a rise in private employment. However, after few periods the crowding-out effect of public employment on private employment prevails over the positive effect induced by the aggregate demand.



### 4.1.3 Labor income tax cut

First, the cut in the labor income tax yields a drop in private real wage. This drop can be explained thanks to a direct impact of the labor income tax on the wage dynamic. Indeed, the drop in the labor income tax increases the match surplus going to the worker. In the bargaining process, it puts a downward pressure on private sector wage. It induces a raise in private sector employment. Also, marginal productivity of labor is reduced, which causes additional downward pressures on private sector wage.

Following the increase in private employment and despite the drop in private sector wage, consumption of non-Ricardian households increases. With a rise in inflation and interest rates, Ricardian consumption drops and this crowding-out effect triggers a drop in output at the mid-term.

### 4.1.4 Social protection tax cut

Following the cut in social protection tax, the match surplus going to the firm hikes which induces an upward pressure on the private sector wage. As a consequence, consumption of non-Ricardians rises. There is a limited crowding-out effect on Ricardians consumption. On the labor market, the decrease in  $\tau_t^{sp}$  rises directly the present and future value of a job for firms. The marginal productivity of labor decreases slightly but the response of private real wages remains unambiguously positive. Employment in the private sector increases while in the public sector the rise in private real wages and the drop in unemployment reduce employment. However, total employment increases strongly.

## 4.2 What impact over the business cycle?

For all simulations in this paper we use the Dynare program created by the CEPREMAP team. The algorithm used by Dynare for the second order approximation of our model is very close to the one developed in Schmitt-Grohé and Uribe (2004). In addition, the simulations are done by using the pruning method<sup>5</sup>, in order to avoid triggering polynomials of increasing degrees when simulating the model. The IRFs of the second order simulations are

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<sup>5</sup>See for instance Lombardo and Uhlig (2014) for a presentation of the pruning method.

presented in appendix C.

For all the fiscal shocks considered, we find a similar result: fiscal policies have a greater effect on employment, unemployment and output in the case of the high steady-state value for the unemployment rate. As we will see throughout this section, these results are driven by two main elements: a wider pool of job seekers and the crucial role of the wage channel.

**The wage channel:** The starting point is that with a higher unemployment rate ( $U_s = 12\%$ ), the pool of job seekers is wider at the steady state. In the case of expansionary fiscal shocks, the rise in private vacancies generates more matches when the initial pool of job seekers is wider. This channel is very close to the result expounded by Michailat (2014).

From then on, since employment increases more when  $U_s = 12\%$ , all things being equal, marginal productivity of labor also decreases more sharply. Indeed, even if the better response of output when  $U_s = 12\%$  eases this channel, the response of marginal productivity of labor remains stronger when unemployment is high. It causes larger downward pressures on real wage, as shown in the IRFs.

**Effects on output fiscal multipliers:** Moreover, the wage channel is a crucial element for understanding and comparing the response of output according to different steady-state unemployment rates. We now explore the conditions under which we obtain a better response of output in the case of a high unemployment rate, thanks to a higher degradation of real wage .

First, except for the social protection tax shock, the greater degradation of real wage when  $U_s = 12\%$ , principally driven by the decrease in productivity, has a direct negative effect on consumption of the non-Ricardians. Indeed, non-Ricardian households' consumption increases more when  $U_s = 6\%$  than when  $U_s = 12\%$ . The case for the social protection tax shock is different in the sense that a decrease in the social protection tax produces a positive response of private wage. However, this positive response is larger when unemployment is low than when unemployment is high so the non-Ricardian

households' consumption reacts in the same way as previously.

The consequence of the previous result is the following: if our economy were composed only of non-Ricardian households, like in Michailat (2014) for instance, our model would produce higher output fiscal multipliers with the low steady-state unemployment rate. In that sense, we need to introduce Ricardian households to produce higher output multipliers at the bad state of the economy. As observed in the IRFs, consumption of the Ricardians is higher when the unemployment rate is high, which produces better output fiscal multipliers. This is due to the greater degradation of real wage, causing lower inflation pressures for the firms and thus, a lower rise of the interest rate in the medium and long term.

Thus, when  $U_s = 12\%$  the larger negative response of real wage produces a higher response of Ricardians' consumption but a lower non-Ricardians' consumption, in comparison with the simulations when  $U_s = 6\%$ . Total response of aggregate consumption and output depends on the strength of these two opposite effects and of the relative share of both types of households in the economy. With a share of non-Ricardians in line with previous estimates<sup>6</sup>, that is  $\mu = 0.3$ , the response of aggregate demand is better when the unemployment rate is high. With this model calibration, the positive effect of a lower inflation on Ricardians' consumption when the unemployment rate is high prevails over the weaker response of non-Ricardian's consumption due to a greater degradation of real wage.

It is important to notice that the more positive response of consumption of the Ricardians is not due in our model to a higher marginal utility of consumption in economic downturns, as this is the case in Sims and Wolff (2013). The authors highlight this transmission channel for explaining different output fiscal multipliers over the business cycle. This is not the case in our model according to the definition of the steady-states. The value of Ricardian consumption at the steady state is obtained residually with the steady-state value of non-Ricardian consumption such as:

$$C_s^o = \frac{C_s - \mu C_s^r}{1 - \mu}, \quad (4.2)$$

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<sup>6</sup>See for instance Coenen and Straub (2005)

whith  $C_s^o$ ,  $C_s^r$  and  $C_s$  respectively the steady-state value of  $C_t^o$ ,  $C_t^r$  and  $C_t$ .

The steady-state value of non-Ricardian consumption is larger with  $U_s = 12\%$  since real wage is larger than unemployment benefits at the steady state. It triggers a higher marginal utility of consumption for this class of households but it has no impact on their consumption behavior since they simply consume their disposable income. However, a lower level of consumption at the steady state for the non-Ricardian households implies a higher consumption for the Ricardians.

## 5 Conclusion

This paper attempts to investigate the non-linear effects of fiscal policy over the business cycle with a focus on the labor market. A first part of the results section is dedicated to the analysis of the effects of different fiscal instruments on the labor market and on output. We use a first-order approximation of the model in order to disentangle the main transmission channels at work. The main result is that all fiscal instruments increase employment and decrease unemployment. Also, response of output is positive in the short term but negative in the medium term because of a strong and permanent crowding-out effect on Ricardian consumption.

Using a second-order approximation of the model, we show that all fiscal shocks are more effective when the steady-state unemployment rate is high: both employment and output increase more. Following Michailat (2014), the stronger effect on employment is due to a larger pool of job seekers when the shocks occur. We then investigate the assumptions needed to produce a better response of output. In our model, if we introduced only non-Ricardian households, the output fiscal multiplier would be lower when the unemployment rate is at 12%.

The introduction of Ricardian households is necessary to produce a higher output fiscal multipliers as explained in the results section. However, the transmission channel is very different from the one in Sims and Wolff (2013). In our model, it is the wage channel and a lower rise in interest rates that produce the larger output fiscal multiplier during economic downturn while it

is a higher marginal utility of consumption during bad times that mitigates the degradation of consumption of the Ricardian households in Sims and Wolff (2013). On the contrary, our definition of the steady states triggers a lower marginal utility of consumption for the Ricardians when the unemployment rate is high. We can expect that when introducing a higher marginal utility of consumption for the Ricardians during economic downturn at the steady-state, this result would be amplified.

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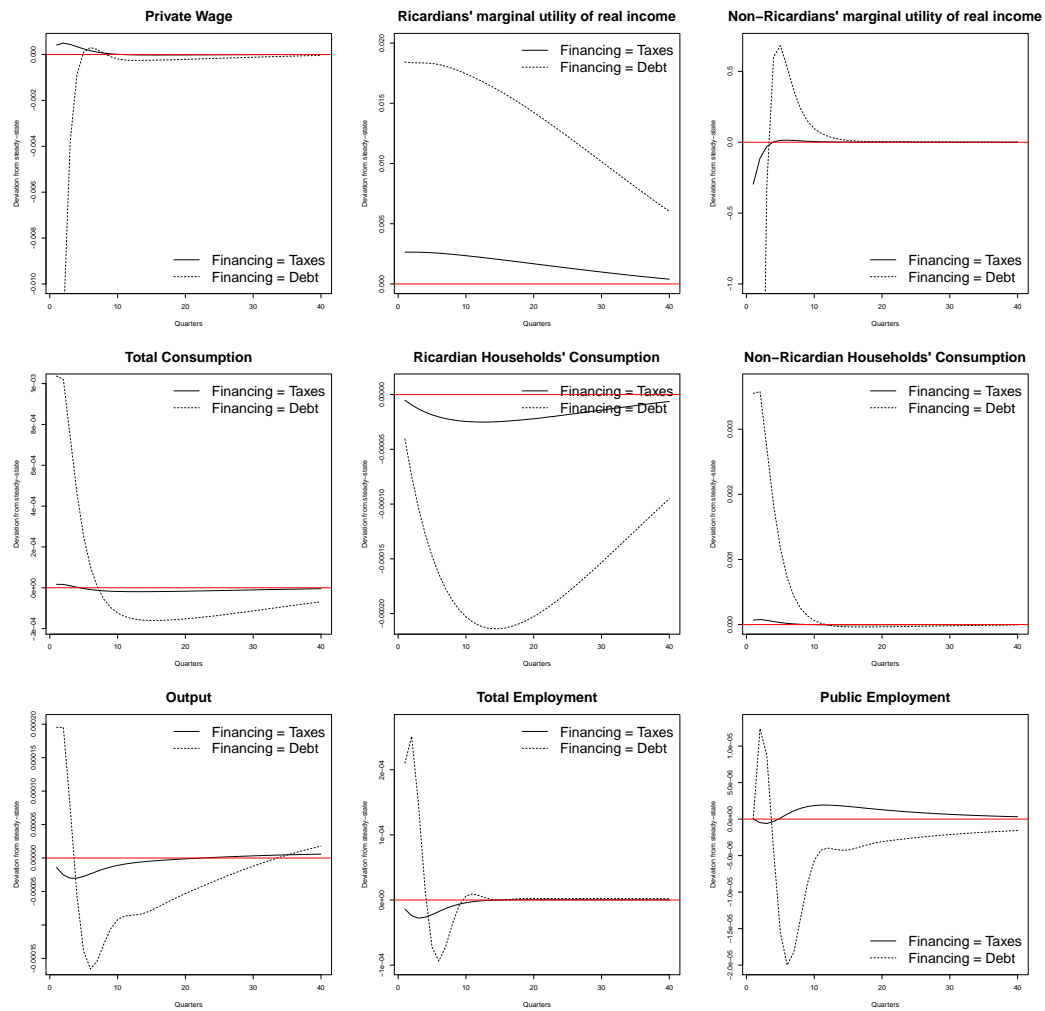
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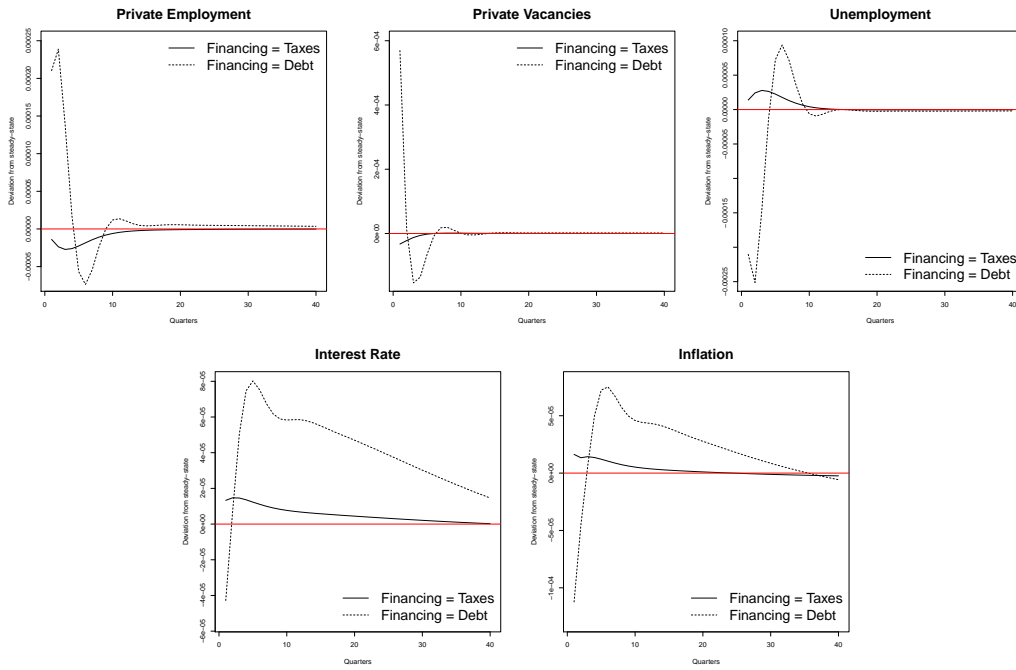
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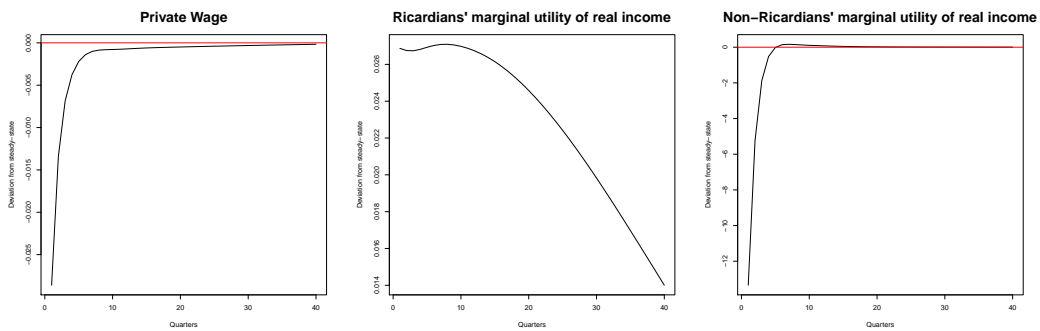
# A A comparison with Afonso and Gomes (2014)

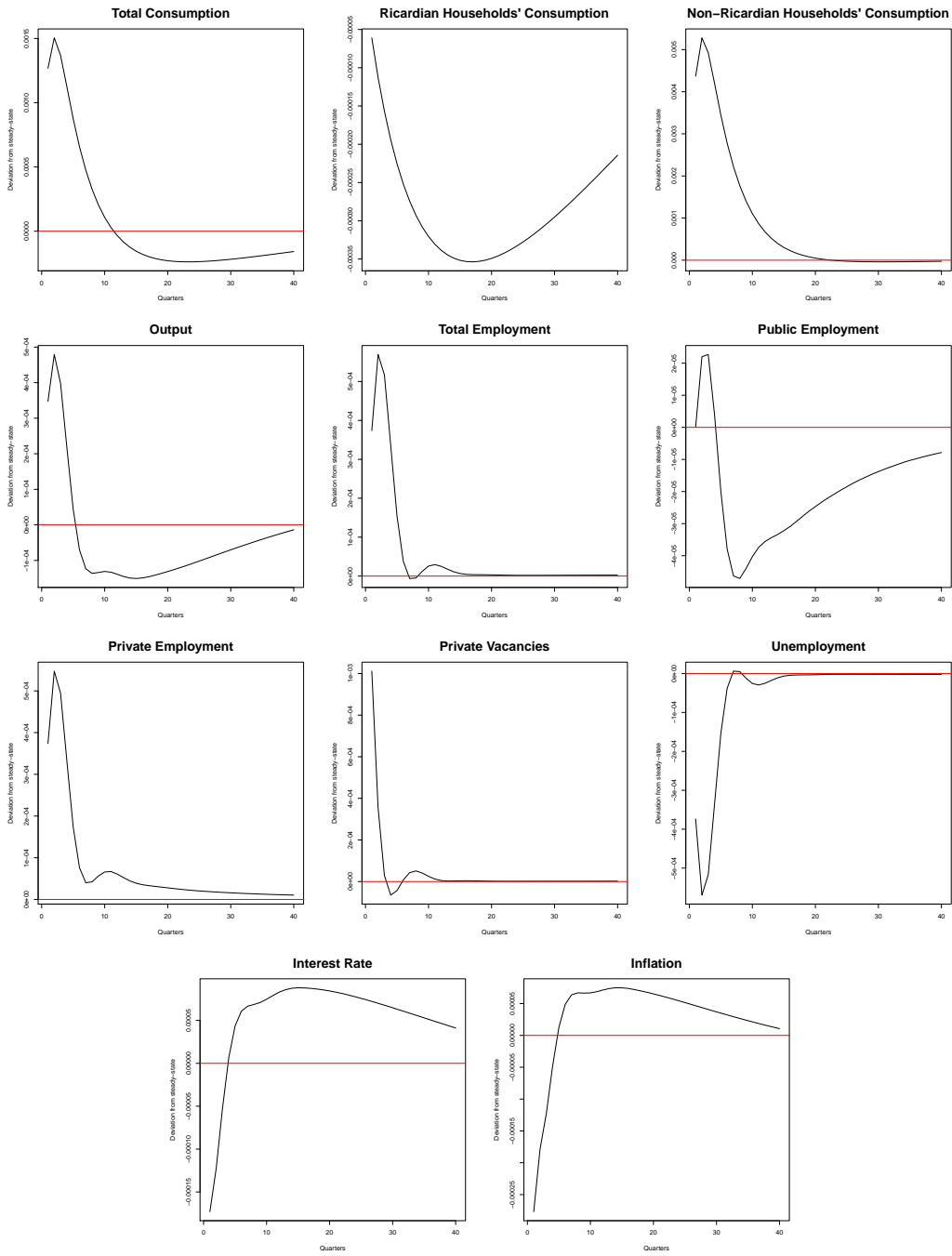




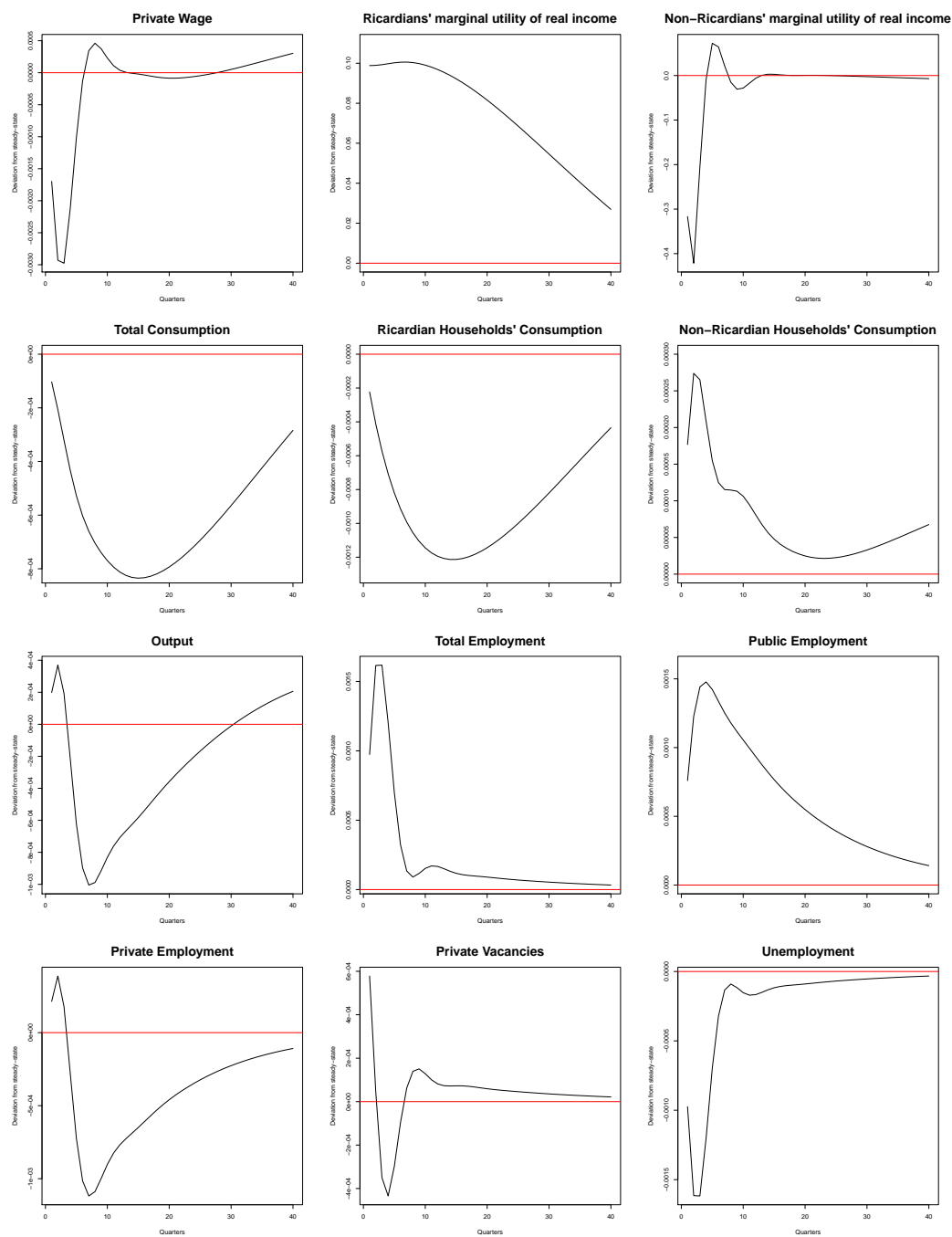
## B The IRFs for the different shocks in the middle of the business cycle

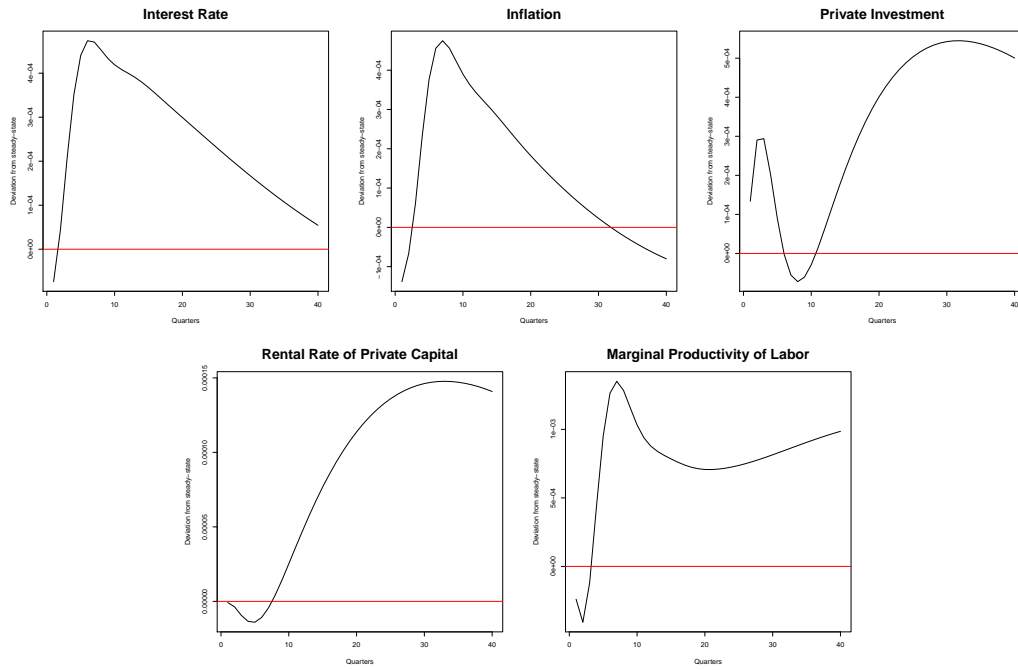
### B.1 The Wage Tax Shock



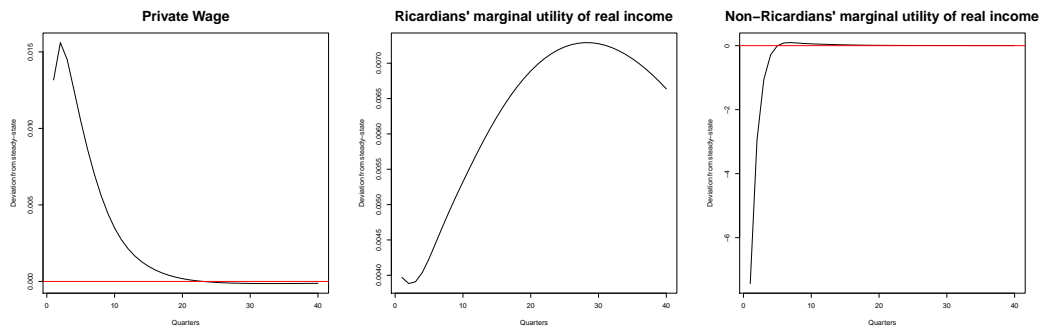


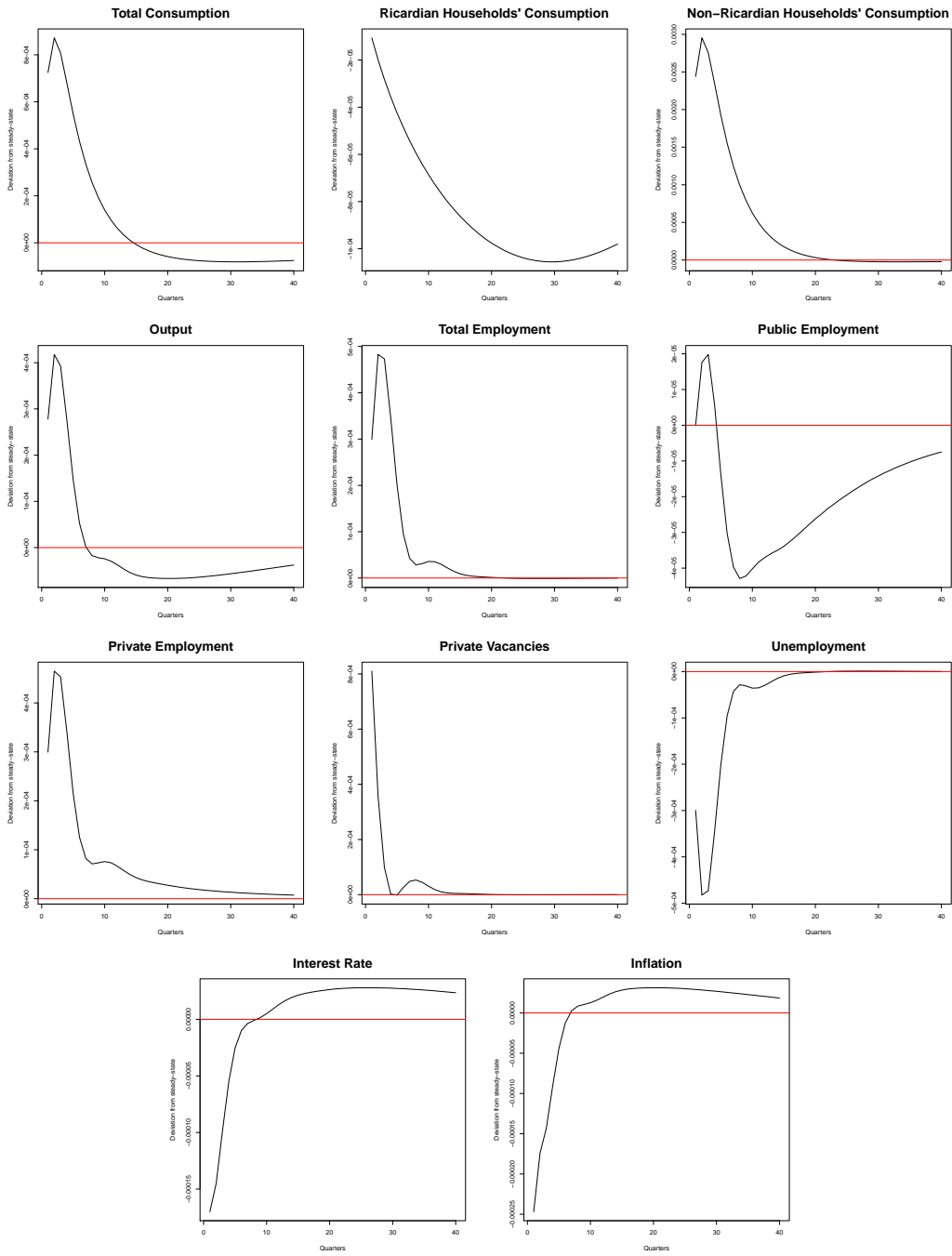
## B.2 The Public Vacancies Shock





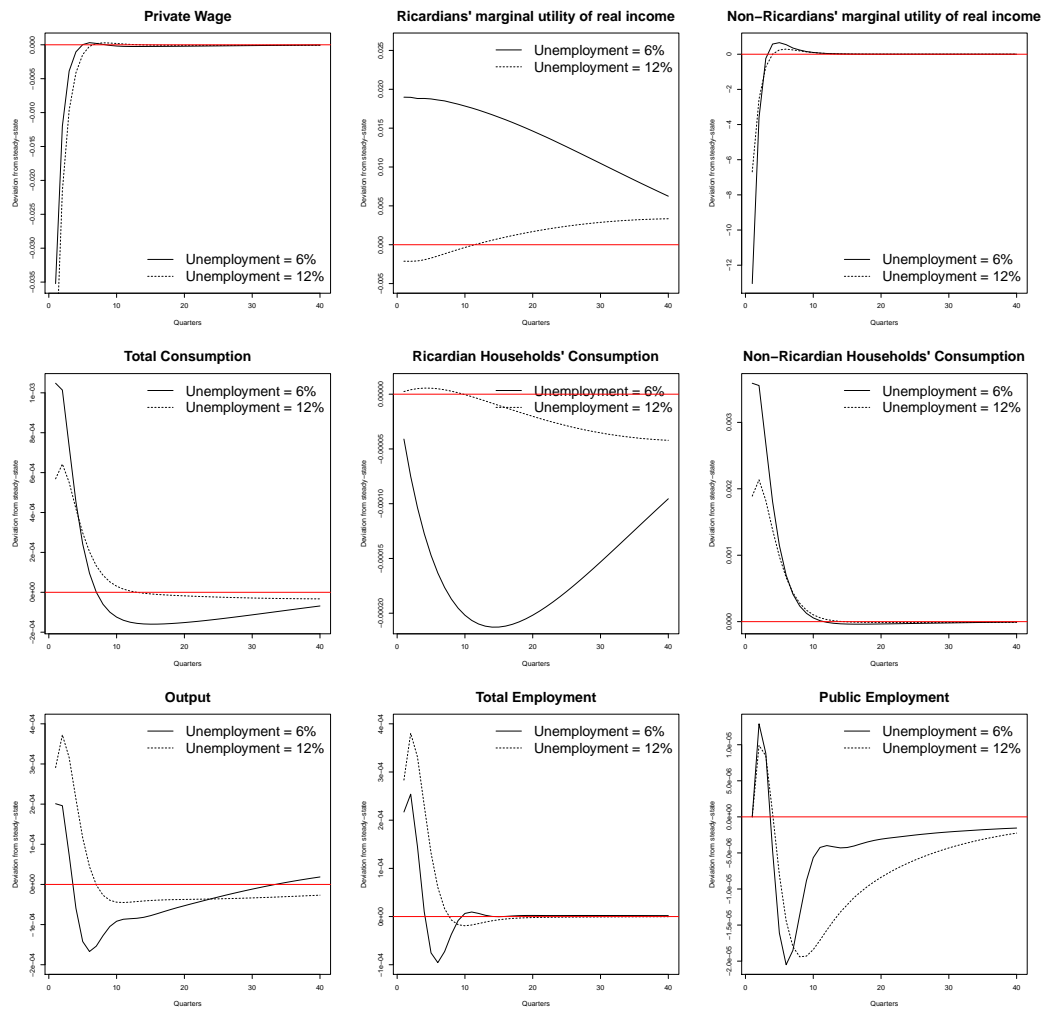
### B.3 The Social Protection Tax Shock

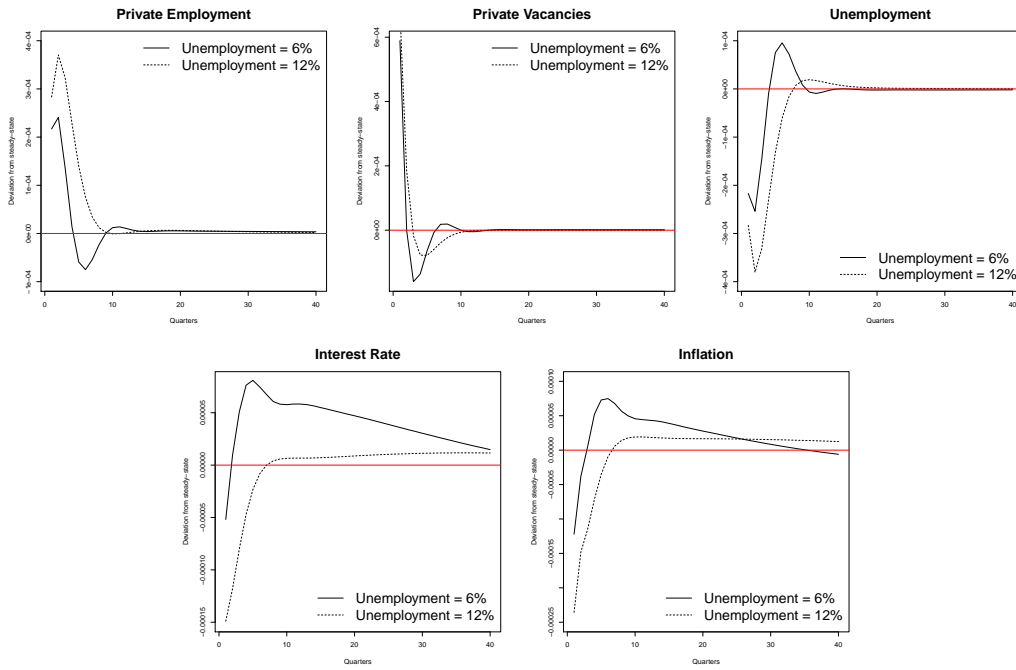




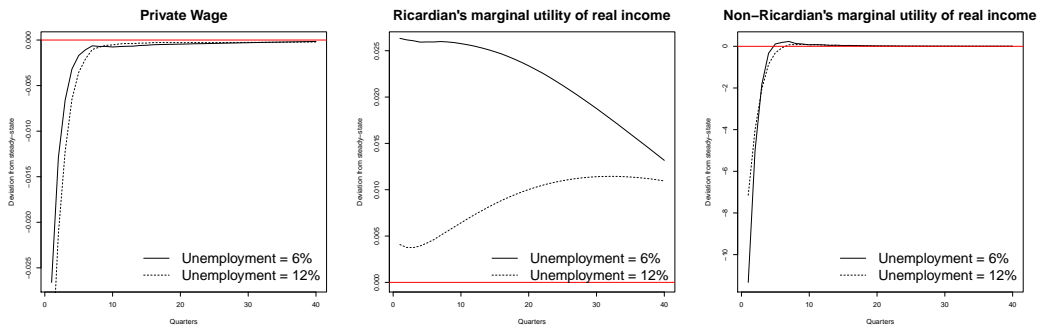
# C The IRFs for the different shocks over the business cycle

## C.1 The Public Wage Shock

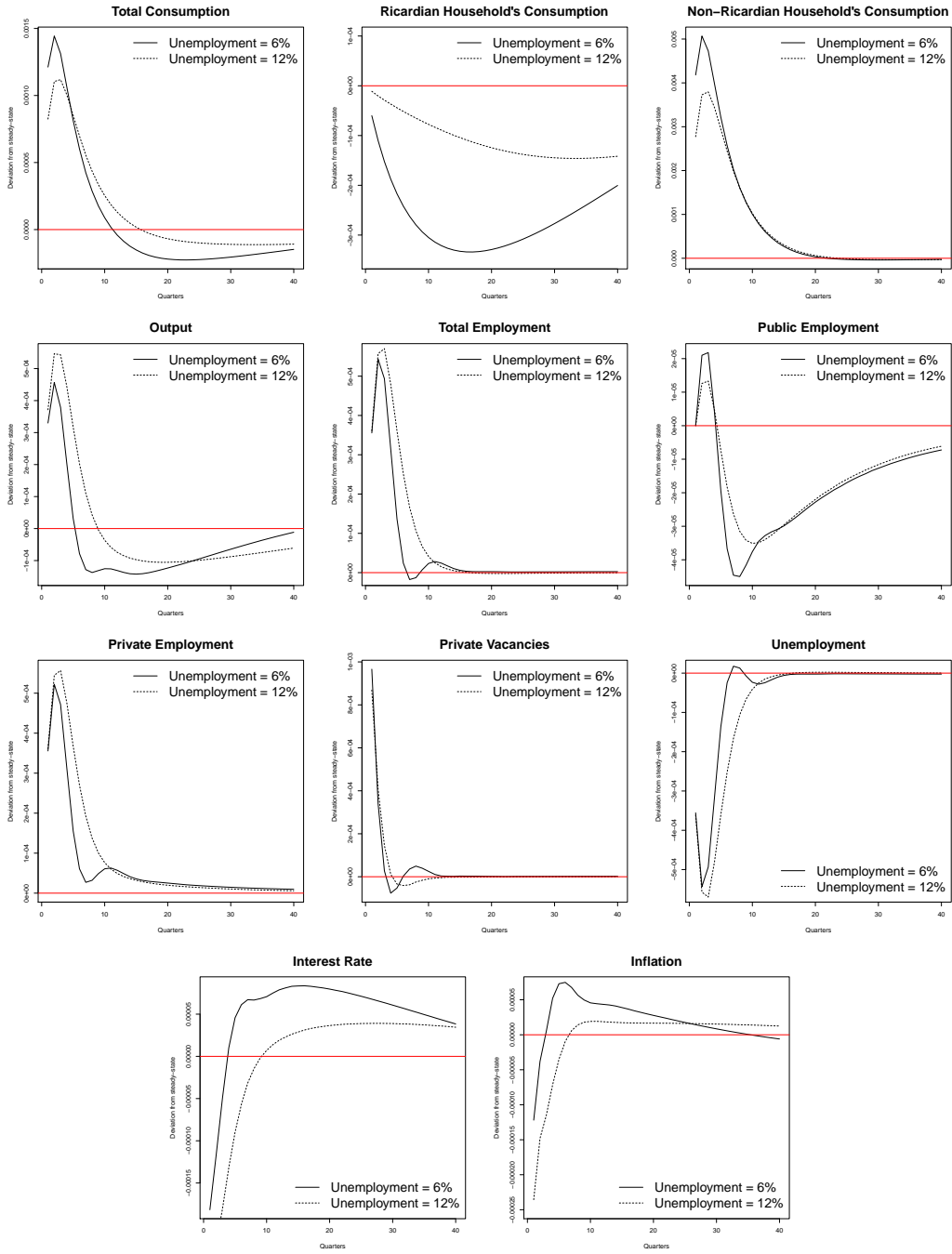




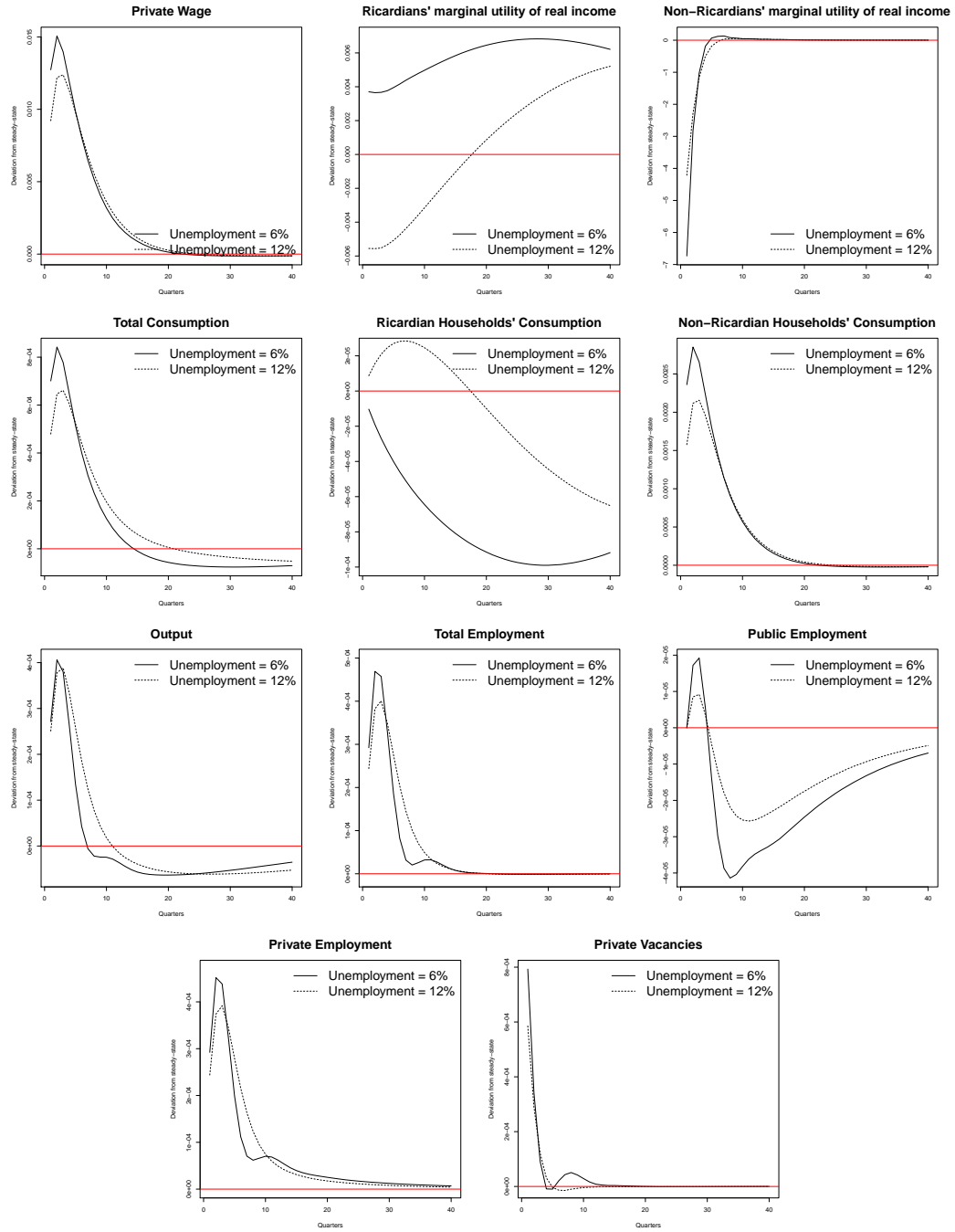
## C.2 The Wage Tax Shock

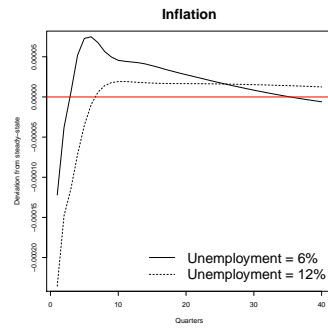
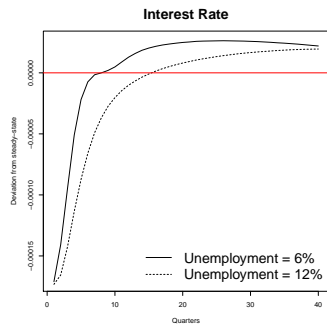
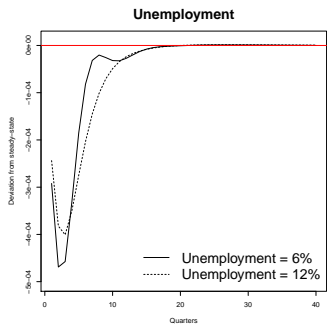




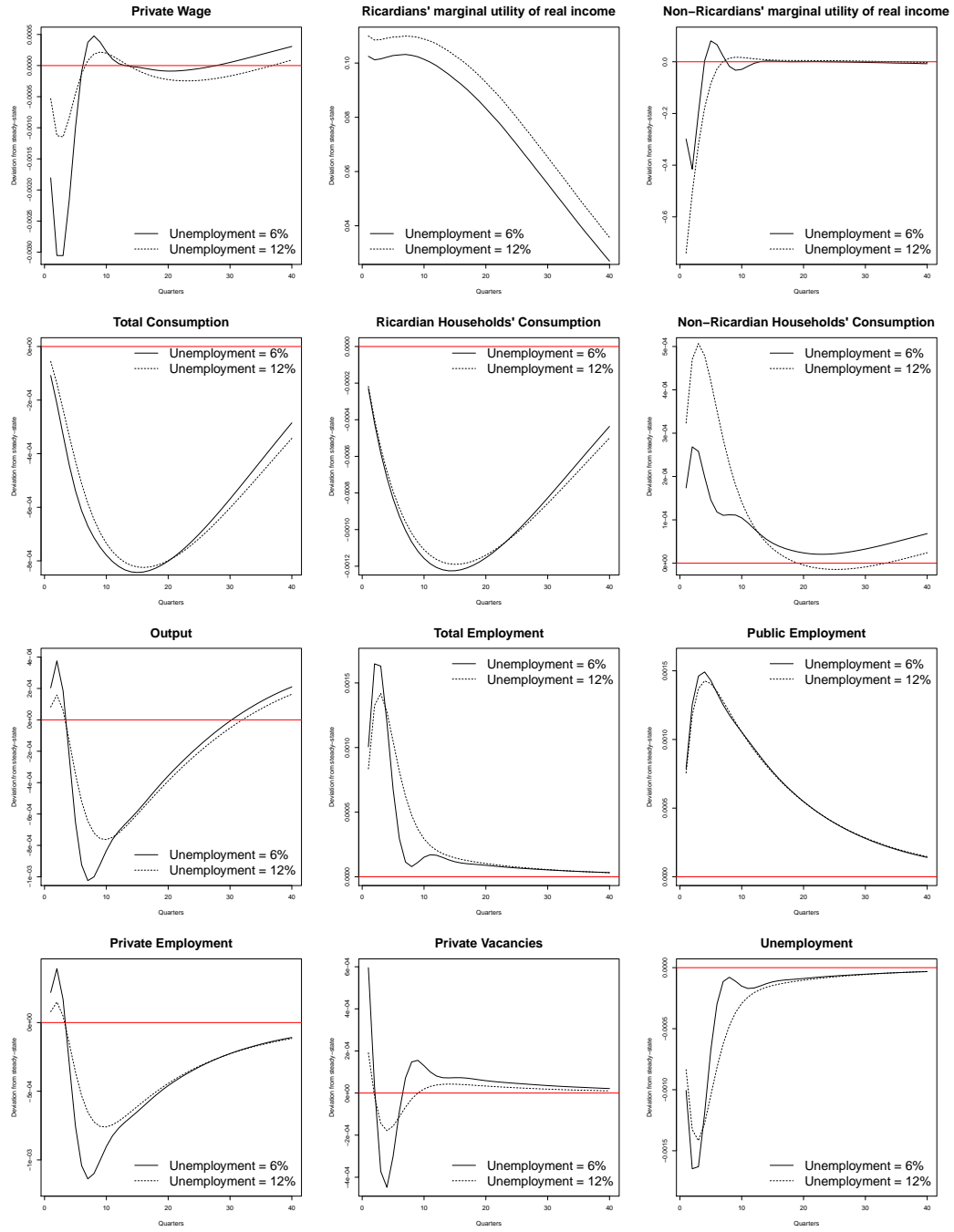


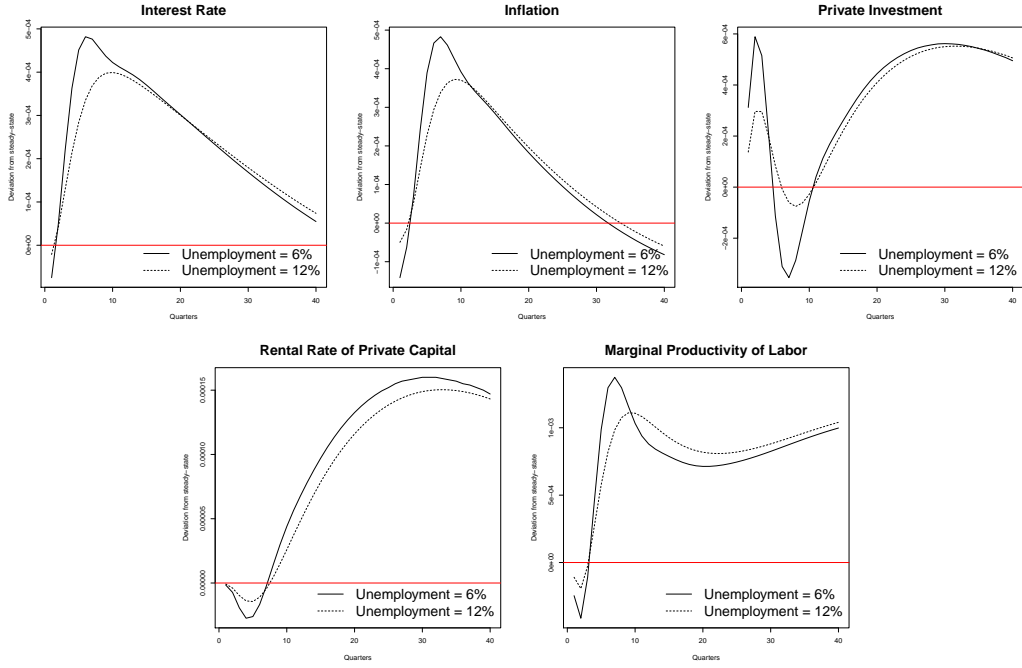
## C.3 The Social Protection Tax Shock





## C.4 The Public Vacancies Shock





#### C.4.1 Wage equation calculation

We start from the surplus' optimal sharing rule given by the equation (2.58). Given that

$$\frac{\partial \Upsilon_t}{\partial W_t^p} = (1 - \mu)(1 - \tau_t^w)\lambda_t^{rio}h + \mu(1 - \tau_t^w)\lambda_t^{rir}h, \quad (C.1)$$

and

$$\frac{\partial \Upsilon_t^{Ef}}{\partial W_t^p} = -(1 + \tau_t^{sp})h, \quad (C.2)$$

and after giving to  $\Upsilon_t$  and  $\lambda_t^{E_f}$  their respective value described by equations(2.54) and (2.45), (2.58) yields

$$\begin{aligned}
& \eta [(1 - \mu)(1 - \tau_t^w)\lambda_t^{rio} + \mu(1 - \tau_t^w)\lambda_t^{rir}] \\
& \times \left[ (1 - \alpha)\frac{Y_t}{E_t^p} - (1 + \tau_t^{sp})W_t^p h + (1 - \rho)\beta_{t,t+1}\lambda_{t+1}^{E_f} \right] \\
& = (1 - \eta)(1 + \tau_t^{sp}) \left\{ \mu [(1 - \tau_t^w)\lambda_t^{rir}W_t^p h - \lambda_t^{rir}b \right. \\
& \quad \left. + \frac{(1 - h - s)^{1-\zeta} - (1 - s)^{1-\zeta}}{1 - \zeta} \right. \\
& \quad \left. + \beta E_t [(1 - \rho)(1 - p_t^p)(\lambda_{t+1}^{E_{rp}} - \lambda_{t+1}^{S_r}) - p_t^g(1 - \rho)(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r})] \right] \\
& + (1 - \mu) \left[ (1 - \tau_t^w)\lambda_t^{rio}W_t^p h - \lambda_t^{rio}b + \frac{(1 - h - s)^{1-\zeta} - (1 - s)^{1-\zeta}}{1 - \zeta} \right. \\
& \quad \left. + \beta E_t [(1 - \rho)(1 - p_t^p)(\lambda_{t+1}^{E_{op}} - \lambda_{t+1}^{S_o}) - p_t^g(1 - \rho)(\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o})] \right] \left. \right\} \\
& \Leftrightarrow (1 + \tau_t^{sp})(1 - \tau_w)(\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio})W_t^p h \\
& = \eta(1 - \tau_t^w)(\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio}) \left[ \frac{(1 - \alpha)Y_t}{E_t^p} + (1 - \rho)\beta_{t,t+1}\lambda_{t+1}^{E_f} \right] \\
& + (1 - \eta)(1 + \tau_t^{sp}) \left[ [\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio}]b + \frac{(1 - s)^{1-\zeta} - (1 - h - s)^{1-\zeta}}{1 - \zeta} \right] \\
& \quad - (1 - \eta)(1 + \tau_t^{sp})(1 - \rho)(1 - p_t^p)\beta E_t[\Upsilon_{t+1}] \\
& + (1 - \eta)(1 + \tau_t^{sp})(1 - \rho)p_t^g\beta E_t[\mu(\lambda_{t+1}^{E_{rg}} - \lambda_{t+1}^{S_r}) + (1 - \mu)(\lambda_{t+1}^{E_{og}} - \lambda_{t+1}^{S_o})]
\end{aligned}$$

Moreover, since equation (2.58) yields

$$\beta E_t[\Upsilon_{t+1}] = \frac{\eta}{(1 - \eta)} E_t \left[ \beta_{t,t+1} \frac{(1 - \tau_{t+1}^w)(\mu\lambda_{t+1}^{rir} + (1 - \mu)\lambda_{t+1}^{rio})}{(1 + \tau_{t+1}^{sp})} \lambda_{t+1}^{E_f} \right],$$

we finally obtain

$$\begin{aligned}
& \Leftrightarrow (1 + \tau_t^{sp})(1 - \tau_w)(\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio})W_t^p h \\
& = \eta(1 - \tau_t^w)(\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio}) \left[ \frac{(1 - \alpha)Y_t}{E_t^p} + \frac{1 - \rho}{1 + \tau_t^{sp}} E_t[\beta_{t,t+1}\lambda_{t+1}^{Ef}] \right] \\
& + (1 - \eta)(1 + \tau_t^{sp}) \left[ (\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio})b + \frac{(1 - s)^{1-\zeta} - (1 - h - s)^{1-\zeta}}{1 - \zeta} \right] \\
& - \eta(1 + \tau_t^{sp})(1 - p_t^p)(1 - \rho)E_t \left[ \beta_{t,t+1} \frac{(1 - \tau_{t+1}^w)}{(1 + \tau_{t+1}^{sp})} (\mu\lambda_{t+1}^{rir} + (1 - \mu)\lambda_{t+1}^{rio})\lambda_{t+1}^{Ef} \right] \\
& + (1 - \eta)(1 + \tau_t^{sp})(1 - \rho)p_t^g \beta E_t [\mu(\lambda_{t+1}^{Erg} - \lambda_{t+1}^{Sr}) + (1 - \mu)(\lambda_{t+1}^{Eog} - \lambda_{t+1}^{So})] \\
& \\
& (1 - \tau_t^w)W_t^p h = \eta \frac{(1 - \alpha)(1 - \tau_t^w)}{(1 + \tau_t^{sp})} \frac{Y_t}{E_t^p} \\
& + (1 - \eta) \left[ b + \frac{(1 - s)^{1-\zeta} - (1 - h - s)^{1-\zeta}}{(1 - \zeta)\mu\lambda_t^{rir} + (1 - \mu)\lambda_t^{rio}} \right] \\
& + \eta(1 - \rho)E_t \left\{ \beta_{t,t+1} \left[ 1 - (1 - p_t^p) \frac{(1 - \tau_{t+1}^w)}{(1 + \tau_{t+1}^{sp})} \tilde{\Lambda}_{t+1} \right] \lambda_{t+1}^{Ef} \right\} \\
& + (1 - \eta)(1 - \rho)p_t^g \beta E_t [\Lambda_t(\lambda_{t+1}^{Erg} - \lambda_{t+1}^{Sr}) + (1 - \Lambda_t)(\lambda_{t+1}^{Eog} - \lambda_{t+1}^{So})] \quad (C.3)
\end{aligned}$$

#### C.4.2 Steady-State calculations

Starting from the long-run targeted values described in table 3, we now describe the steady-state calculations. We first assume that  $W_s^g = W_s^p$ .

From equation (2.1), one can easily define the value of total employment at the steady-state such as

$$E_s^{tot} = 1 - U_s. \quad (C.4)$$

From equation (2.2), the number of job seekers in the economy as a whole is equal to

$$S_s = U_s + \rho E_s^{tot}. \quad (C.5)$$

By definition, assuming that *fracpub* is the size of the public sector on the labor market, we can define the value of public employment as

$$E_s^g = E_s^{tot} \times \text{fracpub}. \quad (C.6)$$

Then, from equations (C.6) and (2.67), we define the value of private employment at the steady state as

$$E_s^p = E_s^{tot} - E_s^g. \quad (C.7)$$

By definition we have

$$E_s^r = \mu E_s^{tot} \quad (C.8)$$

$$\text{and } E_s^o = (1 - \mu) E_s^{tot} \quad (C.9)$$

Thanks to equation (2.40), we can define

$$V_s^p = \rho \frac{E_s^p}{q_s^p} \quad (C.10)$$

and we assume similarly that

$$V_s^g = \rho \frac{E_s^g}{q_s^g}. \quad (C.11)$$

Joining the matching functions and the definition of the probability for a firm to fill its job, described by the equations (2.5) and (2.7) we are able to define the matching technology in each sector as

$$\kappa_e^p = \frac{V_s^p q_s^p}{S_s^{\varphi^p} (V_s^p)^{1-\varphi^p}} \quad (C.12)$$

$$\kappa_e^g = \frac{V_s^g q_s^g}{S_s^{\varphi^g} (V_s^g)^{1-\varphi^g}} \quad (C.13)$$

Thanks to the previous equations and to the equations (2.5), we can define the number of matches in each sector at the steady state as

$$M_s^p = \kappa_e^p S_s^{\varphi^p} (V_s^p)^{1-\varphi^p} \quad (C.14)$$

$$\text{and } M_s^g = \kappa_e^g S_s^{\varphi^g} (V_s^g)^{1-\varphi^g}. \quad (C.15)$$

Thanks to equations (C.5), (C.14) and (C.15), we can define the probability for a worker to find a job in each sector at the steady state as

$$p_s^p = \frac{M_s^p}{S_s} \quad (C.16)$$

$$\text{and } p_s^g = \frac{M_s^g}{S_s}. \quad (C.17)$$



According to equation (2.24) we have

$$R_s^k = r_s + \delta^k - 1. \quad (\text{C.18})$$

We assume that at the steady-state, marginal cost is equal to the desired (flexible prices) markup such as

$$mc_s = \frac{\varepsilon}{\varepsilon - 1}. \quad (\text{C.19})$$

Thanks to the previous equations and using equation (2.47), we can define the marginal cost of labor at the steady state such as

$$x_s = (1 - \alpha)mc_s \left( \frac{Y_s}{E_s^p h} \right) - (1 + \tau_s^{sp} W_s^p h). \quad (\text{C.20})$$

From equation 2.25 and the definition of  $S \left( \frac{I_t^o}{I_{t-1}^o} \right)$ , the steady-state of Tobin's Q is:

$$Q_s = 1. \quad (\text{C.21})$$

According to equation (2.46), we have

$$k_s = \alpha mc_s \frac{Y_s}{R_s^k}, \quad (\text{C.22})$$

while from aggregation we have

$$k_s^o = \frac{k_s}{(1 - \mu)} \quad (\text{C.23})$$

$$\text{and } I_s^o = \frac{I_s}{(1 - \mu)}. \quad (\text{C.24})$$

Thanks to the equation (2.41), we can define the TPF at the steady-state as

$$\epsilon_s^a = \frac{Y_s}{(K_s^g)^{\alpha g} k_s^\alpha (E_s^p h)^{1-\alpha}}. \quad (\text{C.25})$$

According to the market clearing condition defines by equation (2.66), we have

$$C_s = Y_s - C_g - I_g - I_s. \quad (\text{C.26})$$

The definition of the LMT given by equation (2.8) yields

$$\theta_s^p = \frac{V_s^p}{S_s} \quad (\text{C.27})$$

$$\text{and } \theta_s^g = \frac{V_s^g}{S_s}. \quad (\text{C.28})$$

Aggregation yields

$$\theta_s = \theta_s^p + \theta_s^g. \quad (\text{C.29})$$

By construction, we have

$$q_s^1 = \frac{\lambda_s^{rio} Y_s m c_s}{1 - \beta \theta^p \pi_s^{\epsilon-1}} \quad (\text{C.30})$$

$$q_s^2 = \frac{\lambda_s^{rio} Y_s}{1 - \beta \theta^p \pi_s^{\epsilon-1}}, \quad (\text{C.31})$$

and thanks to equation (2.52)

$$p_s^{opt} = \frac{\varepsilon}{\varepsilon - 1} \frac{q_s^1}{q_s^2}. \quad (\text{C.32})$$

The value of a job at the steady-state for a firm is equal to

$$\lambda_s^{Ef} = \frac{1 - \alpha}{1 - (1 - \rho)\beta} \frac{Y_s}{E_s^p} - \frac{1 + \tau_s^{sp}}{1 - (1 - \rho)\beta} W_s^p h. \quad (\text{C.33})$$

Thanks to the previous equations we can now define the value of posting a vacancy

$$\kappa^v = \beta \left( \frac{(1 - \alpha) Y_s}{E_s^p} - (1 + \tau_s^{sp} W_s^p h + (1 - \rho)\beta \lambda_s^{Ef}) q_s^p \right) \quad (\text{C.34})$$

The utility function of the union at the steady state can be defined as

$$\Upsilon_s = (1 - \mu)(\lambda_s^{Eop} - \lambda_s^{So}) + \mu(\lambda_s^{Erp} - \lambda_s^{Sr}). \quad (\text{C.35})$$

Finally, by definition,

$$mpl_s = \frac{(1 - \alpha) Y_s}{E_s^p h}. \quad (\text{C.36})$$

**Marginal utility of real income in terms of non-Ricardian consumption** If we admit that  $W_s^g = W_s^p$ , the non-Ricardian consumption at the steady state can be expressed as

$$C_s^r = \{(1 - \tau_s^w)[E_s^r W_s^p h + (1 - E_s^r)b]\}(1 + \tau_s^c) \quad (\text{C.37})$$

We express the Ricardians' consumption at the steady state in terms of wage as

$$C_s^o = \frac{C_s - mu C_s^r}{1 - mu} \quad (\text{C.38})$$

Then, the marginal utility of real income for Ricardian and non-Ricardian households can be expressed as

$$\begin{aligned} \lambda_s^{rio} &= \frac{1 - \beta H}{1 + \tau_s^c} [(1 - H)C_s^o]^{-\sigma_c} \\ \Leftrightarrow \lambda_s^{rio} &= \frac{1 - \beta h}{1 + \tau_s^c} \left\{ (1 - H) \frac{1}{1 - \mu} \left\{ C_s \right. \right. \\ &\quad \left. \left. - \frac{\mu}{1 + \tau_s^c} [(1 - \tau_s^w)E_s^r W_s^p h + (1 - E_s^r)b] \right\} \right\}^{-\sigma_c} \end{aligned} \quad (\text{C.39})$$

$$\begin{aligned} \lambda_s^{rir} &= (1 - \beta H)[(1 - H)C_s^r]^{-\sigma_c} \\ \Leftrightarrow \lambda_s^{rir} &= \frac{1 - \beta H}{1 + \tau_s^c} \{(1 - H)\{(1 - \tau_s^w)[E_s^r W_s^p h + (1 - E_s^r)b]\}\}^{-\sigma_c} \end{aligned} \quad (\text{C.40})$$

**Workers' marginal utilities in terms of unemployment marginal utility**

- For Ricardian workers

$$\begin{aligned} \lambda_s^{E_{op}} &= (1 - \tau_s^w)\lambda_s^{rio}W_s^p h \\ &= \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} + (1 - \rho)\beta\lambda_s^{E_{op}} + \rho\beta\lambda_s^{S_o} \\ \Leftrightarrow [1 - (1 - \rho)\beta]\lambda_s^{E_{op}} &= (1 - \tau_s^w)\lambda_s^{rio}W_s^p h \end{aligned}$$

$$\begin{aligned}
& -\frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} + \rho\beta\lambda_s^{S_o} \\
\Leftrightarrow \lambda_s^{E_{op}} &= \frac{1}{1 - (1 - \rho)\beta} \left[ (1 - \tau_s^w)W_s^p h \lambda_s^{rio} \right. \\
& \left. - \frac{1 - (1 - h - s)^{1-\zeta}}{(1 - \zeta)} + \beta\rho\lambda_s^{S_{os}} \right] \tag{C.41}
\end{aligned}$$

$$\begin{aligned}
\lambda_s^{E_{og}} &= (1 - \tau_s^w)\lambda_s^{rio}W_s^g h - \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} \\
& \quad + (1 - \rho)\beta\lambda_s^{E_{og}} + \rho\beta\lambda_s^{S_o} \\
\Leftrightarrow [1 - (1 - \rho)\beta]\lambda_s^{E_{og}} &= (1 - \tau_s^w)\lambda_s^{rio}W_s^g h \\
& \quad - \frac{1 - (1 - h - s)^{1-\zeta}}{1 - \zeta} + \rho\beta\lambda_s^{S_o} \\
\Leftrightarrow \lambda_s^{E_{og}} &= \frac{1}{1 - (1 - \rho)\beta} \left[ (1 - \tau_s^w)W_s^g h \lambda_s^{rio} \right. \\
& \quad \left. - \frac{1 - (1 - h - s)^{1-\zeta}}{(1 - \zeta)} + \beta\rho\lambda_s^{S_{os}} \right] \\
\Leftrightarrow \lambda_s^{E_{og}} &= \frac{1}{1 - (1 - \rho)\beta} \left[ (1 - \tau_s^w)W_s^p h \lambda_s^{rio} \right. \\
& \quad \left. - \frac{1 - (1 - h - s)^{1-\zeta}}{(1 - \zeta)} + \beta\rho\lambda_s^{S_{os}} \right] \tag{C.42}
\end{aligned}$$

$$\begin{aligned}
\lambda_s^{S_o} &= b\lambda_s^{rio} - \frac{1 - (1-s)^{1-\zeta}}{1-\zeta} + (1-p_s^p - p_s^g)\beta\lambda_s^{S_o} \\
&\quad + \rho(p_s^p + p_s^g)\beta\lambda_s^{S_o} + (1-\rho)\beta[p_s^p\lambda_s^{E_{op}} + p_s^g\lambda_s^{E_{og}}] \\
\Leftrightarrow \lambda_s^{S_o}[1 - \beta + \beta(1-\rho)(p_s^p + p_s^g)] &= b\lambda_s^{rio} - \frac{1 - (1-s)^{1-\zeta}}{1-\zeta} \\
&\quad + \frac{\beta(1-\rho)(p_s^p + p_s^g)}{1-\beta(1-\rho)} \left[ (1-\tau_s^w)\lambda_s^{rio}W_s^ph \right. \\
&\quad \quad \left. - \frac{1 - (1-h-s)^{1-\zeta}}{1-\zeta} + \beta\rho\lambda_s^{rio} \right] \\
\Leftrightarrow \lambda_s^{S_o} \left[ 1 - \beta + \beta(1-\rho)(p_s^p + p_s^g) \left( 1 - \frac{\beta\rho}{1-\beta(1-\rho)} \right) \right] & \\
&= b\lambda_s^{rio} - \frac{1 - (1-s)^{1-\zeta}}{1-\zeta} \\
+ \frac{\beta(1-\rho)(p_s^p + p_s^g)}{1-\beta(1-\rho)} \left[ (1-\tau_s^w)\lambda_s^{rio}W_s^ph - \frac{1 - (1-h-s)^{1-\zeta}}{1-\zeta} \right] & \quad (C.43)
\end{aligned}$$

$$\Leftrightarrow \lambda_s^{S_o} = \frac{b\lambda_s^{rio} - B_1^S + B_2^S W_s^p h \lambda_s^{rio}}{B_3^S} \quad (C.44)$$

with

$$B_1^S = \frac{1 - (1-s)^{1-\zeta}}{1-\zeta} + \frac{\beta(1-\rho)(p_s^p + p_s^g)}{1 - (1-\rho)\beta} \frac{1 - (1-h-s)^{1-\zeta}}{(1-\zeta)}$$

$$B_2^S = \frac{\beta(1-\rho)(p_s^p + p_s^g)}{1-\beta(1-\rho)} (1-\tau_s^w)$$

$$B_3^S = 1 - \beta + \beta(1-\rho)(p_s^p + p_s^g) \left( 1 - \frac{\beta\rho}{1-\beta(1-\rho)} \right)$$

- For non-Ricardian workers

In a similar way, we obtain

$$\begin{aligned}
\lambda_s^{E_{rp}} &= \frac{1}{1 - (1-\rho)\beta} \left[ (1-\tau_s^w)W_s^ph\lambda_s^{rir} \right. \\
&\quad \left. - \frac{1 - (1-h-s)^{1-\zeta}}{(1-\zeta)} + \beta\rho\lambda_s^{S_{rs}} \right] \quad (C.45)
\end{aligned}$$

$$\lambda_s^{Erg} = \frac{1}{1 - (1 - \rho)\beta} \left[ (1 - \tau_s^w) W_s^p h \lambda_s^{rir} - \frac{1 - (1 - h - s)^{1-\zeta}}{(1 - \zeta)} + \beta \rho \lambda_s^{Srs} \right] \quad (\text{C.46})$$

$$\lambda_s^{S_r} = \frac{b \lambda_s^{rir} - B_1^S + B_2^S W_s^p h \lambda_s^{rir}}{B_3^S} \quad (\text{C.47})$$