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Auteurs

**G. Dari-Mattiacci, B. Lovat, F. Parisi**

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**Faculté des sciences économiques et de gestion**
Pôle européen de gestion et d'économie (PEGE)
61 avenue de la Forêt Noire
F-67085 Strasbourg Cedex

Secrétariat du BETA
Géraldine Del Fabbro
Tél. : (33) 03 68 85 20 69
Fax : (33) 03 68 85 20 70
g.delfabbro @unistra.fr
www.beta-umr7522.fr

# Loss-sharing between Nonnegligent Parties*

Giuseppe Dari-Mattiacci,[†] Bruno Lovat,[‡] and Francesco Parisi[§]

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**Abstract**

Shavell (1980) established that all existing tort regimes fail to incentivize optimal activity levels. The bearer of residual loss adopts a socially optimal activity level, however the non-bearer of residual loss will adopt an excessive level of activity. In this paper, we explore alternative liability rules, which distribute the cost of accidents between nonnegligent parties, effectively rendering both parties (injurer and victim) partial residual bearers of loss. We introduce a bilateral accident model with care and activity levels, assuming risk neutrality. We determine conditions where loss-sharing for nonnegligent torts may be a desirable alternative for policymakers, and analyze the social cost of accidents under such shared-liability regimes. We also extend our analysis to account for role-uncertainty of the parties, as well as real-world implications for tort law.

*JEL classification*: K13, K32.

*Keywords*: tort, loss-sharing, negligence, strict liability, comparative fault, role-uncertainty.

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†Corresponding author: Amsterdam Center for Law and Economics, University of Amsterdam; email: gdarimat@uva.nl. The financial support provided by the NWO grant 016.075.332 is gratefully acknowledged.

‡Université de Lorraine, BETA-CNRS.

§University of Minnesota Law School and University of Bologna, Department of Economics.

# 1 Introduction

When an individual is negligent, the law prescribes that he must bear the cost of accidents that arise from his delinquency. A negligent victim must bear the harm done to him, while a negligent injurer must bear liability for harm done to others. However, it is not always the case that accidents arise from negligence. As the the military strategist John Muller (1755) eloquently observed: "But, notwithstanding all human precautions that can be taken, yet accidents will happen."

In a well-functioning liability system, parties are induced to adopt optimal care in equilibrium. The adoption of due care reduces the probability and severity of an accident loss, but the probability of an accident is not completely eliminated in such situations. Thus, a large proportion of accidents will likely arise from the activities where neither party is negligent. This begs the legal question how the cost of such "nonnegligent" accidents should be distributed. Although several legal rules exist to allocate or split an accident loss when both parties are negligent (e.g., through comparative negligence),[1] legal systems have not hitherto developed mechanisms for loss-sharing in cases where neither party is negligent.

The current literature devotes little attention to the study of accident losses that are not attributable to negligence. However, such accident losses are a significant portion of social cost of accidents, and their allocation is an important factor in incentivizing parties' activity levels. The bearer of residual liability (i.e., the party that bears the cost of accidents when both parties are nonnegligent) will adopt the socially optimal activity level,[2] while the non-bearer of residual liability will tend to adopt an excessive level. Thus, the conventional liability frameworks—negligence and strict liability—will fail to incentivize socially optimal activity levels for at least one of the parties (Shavell, 1980).

Under a negligence regime, potential injurers will adopt excessive activity levels, whereas under a strict liability regime, potential victims will adopt excessive activity levels. The choice of liability regime will therefore depend upon the value and riskiness of the parties' activities, among other factors. However, these two choices constrain policymakers to all-or-nothing alternatives, which in some cases may fail to create even second-best outcomes.

In this paper, we will consider a class of hitherto neglected middle-ground alternatives: shared liability in cases where neither party is negligent, analyzing the incentive properties of loss-sharing rules with respect to care and activity levels. We introduce a bilateral accident model of care levels and activity levels, assuming risk-neutrality. The traditional wisdom is that loss-sharing may be

[1]Both common law and civil law systems frequently, though not universally, address the issue of dual party negligence, through comparative negligence, according to which an accident loss should be shared between the parties. For a recent critical review of the literature on comparative negligence see Bar-Gill and Ben-Shahar (2003).

[2]This level of activity is socially optimal given the level of activity of the other party and the levels of due-care. Yet, it will typically differ from the first best level of activity. See further the analysis provided in Section 2.

desirable as a form of implicit insurance for risk-averse parties.[3] However, apart from such an insurance function, it was thought that there would be little motivation for such a policy device. Much of the interest for loss-sharing rules was further obfuscated by the belief that a loss-sharing rule was likely to undermine the parties' care incentives.[4]

We show that loss-sharing may be a valuable instrument for the reduction of the cost of accidents even under risk-neutrality. We specifically consider loss-sharing as a policy control variable independent of the parties' degree of negligence or causal contribution to the loss. Unlike the other rules that use all-or-nothing solutions in the apportionment of residual liability (thereby concentrating all activity level incentives on one party), rules that create loss-sharing in equilibrium spread both the threat of residual liability and the resulting activity level incentives between the parties. Both victims and injurers face some incentives to optimize their respective activity levels. Given these properties, we consider if and when it may be desirable to introduce a loss-sharing rule.

Although nonnegligent loss-sharing has not been systematically explored in the economic literature,[5] the desirability of such mechanisms was intuited by early scholars of the economic analysis of tort law. Calabresi (1965) noted that tort systems which apportion liability based on fault only deter accidents that are caused by negligent behavior and ignore the value of deterring accidents that are faultless. Calabresi explicitly suggested dividing the costs of an accident pro rata between the sub-activities involved, irrespective of legal notions of fault. In Calabresi's own example, if a walker, a bicyclist and an automobile are all involved in an accident without fault on any of these parties, the accident loss could be divided among these three activities.[6] Calabresi (1996) and Calabresi and Cooper (1996) revisited this issue, lamenting the lack of attention that contemporary scholars have paid to the apportionment of liability between nonnegligent parties.

In the next section, we will begin our analysis by addressing an unresolved question in the existing literature: whether the adoption of a loss-sharing rule between nonnegligent parties undermines the parties' incentives to adopt a socially optimal level of precautionary care.[7] Contrary to conventional wisdom, we

[3]See the discussion in Cooter (1991).

[4]For instance, early scholars, such as Brown (1973), were critical of sharing. While later literature, starting with Landes and Posner (1980) showed that sharing between negligent parties does not undermine incentives to take care, sharing between nonnegligent parties was left out of the analysis.

[5]The issue of loss-sharing has been considered by Calabresi (1996), Parisi and Fon (2004) and Parisi and Singh (2010) under the name of comparative causation. The latter two papers consider a comparative causation rule used in conjunction with negligence and strict liability rules. In their model, loss-sharing follows from a damage-apportionment rule based on causality. When neither party is at fault, the parties share damages based on their causal contribution to the loss. Dari-Mattiacci and De Geest (2005) discuss loss-sharing irrespective of fault and show that it might filter out the most harmful accidents.

[6]Calabresi suggested that if accidents of this sort occurred on a recurring basis, liability should be apportioned on the basis of the cumulative effect, assigning greater liability to those activities that result in more frequent and more severe accidents (Calabresi, 1965, pp. 740-741).

[7]The issue of incentives to abide by the due-care standards has been examined also by Singh

show that loss-sharing does not undermine care incentives. If a set of sufficient conditions are satisfied, parties will have incentives to comply with an optimally chosen standard of due care irrespective of the sharing rule implemented. This irrelevance result mirrors a similar result proven by Landes and Posner (1980, note 51) and Haddock and Curran (1985) with respect to sharing between negligent parties.[8] Building on this literature, our finding demonstrates that, in general, sharing accident losses between negligent or nonnegligent parties does not affect compliance with socially optimal negligence standards. If the conditions we identify are not satisfied, compliance with the negligence standards can be guaranteed by reducing the levels of due care below the socially optimal levels. Setting negligence standards as if both parties bore the residual loss guarantees compliance. Again, this result is valid under any sharing of the loss in equilibrium.

Having investigated the compatibility of loss-sharing rules with optimal care incentives, we turn in Section 3 to the related question: which sharing rule would most effectively promote a reduction of the social cost of accidents? Loss-sharing spreads the incentives to reduce activity levels for both parties. The desirability of spreading such incentives depends upon the relationship between the parties' efforts. We show that loss-sharing may induce greater overall reduction of the social costs of accidents than the traditional liability rules. The relative effectiveness of alternative liability rules is examined in a simplified example that provides a formal interpretation of Calabresi's (1965) recommendation to apportion residual losses according to the riskiness of the parties' activities.

In Section 4, we extend the analysis to consider the impact of parties' role-uncertainty. When parties are faced with role-uncertainty—uncertainty as to whether they will find themselves in the position of victims or injurers in a future accident—the law is incapable of affecting activity-level incentives. The limiting cases—strict liability and negligence—might have the same effects on parties' activity levels. Role-uncertainty leads to a distribution of expected losses, which may render loss-sharing rules superfluous. We suggest that our results may explain why loss-sharing rules, although desirable in principle, are seldom utilized by legal systems.

## 2 The model of negligence-based tort liability

In this section, we introduce a formal model of accident prevention. We assume that accidents may be prevented by taking two different types of precautionary measures. Following Shavell's (1980) terminology, we distinguish between "care levels" and "activity levels". Care levels are verifiable ex post in court and are

(2007), considering fixed levels of activity, and Parisi and Singh (2010), considering variable levels of activity and an endogenous loss-sharing rule based on causal contribution, which applies identically to two negligent parties and to two nonnegligent parties. As Proposition 2 shows, the latter feature simplifies the incentives problem. In our analysis, we are not concerned with causation and consider a more general loss function and two exogenous sharing rules, one for negligent parties and one for nonnegligent parties.

[8] See also Jain and Singh (2002).

taken into account in the determination of negligence, while activity levels are not. For example, a motorist's care level may include the speed at which he drives, whereas his activity level may include how often and how far he drives.[9]

We consider two parties: a prospective injurer ($U$) and his victim ($V$). We assume that they are strangers to each other, rational, and risk-neutral. We study the standard case of unilateral-risk accidents, such that the victim is the only party that suffers harm caused by the injurer if an accident occurs.[10] The expected accident loss is affected by both parties' activity and care levels (bilateral-precaution accidents). The parties' utilities $U$ and $V$ decrease in care $x$ and $y$ at a constant or increasing rate and increase[11] in their levels of activity $s$ and $t$ at a decreasing rate. We further assume that the expected accident loss increases in the level of activity and decreases in care. Therefore, let:

$U = U(x, s)$ be the injurer's utility, with $U_x < 0$, $U_{xx} \leq 0$, $U_s > 0$, and $U_{ss} < 0$

$V = V(y, t)$ be the victim's utility, with$V_y < 0$,$V_{yy} \leq 0$, $V_t > 0$, and $V_{tt} < 0$

$L = L(x, y, s, t)$ be the expected accident loss, with $L_x, L_y < 0$ and $L_{xx}$, $L_{yy}$, $L_s$, $L_t$, $L_{ss}$, $L_{tt} > 0$

Furthermore, all dependent and independent variables just listed are non-negative. Social welfare is assumed to be a simple sum of the parties utilities minus the expected accident loss:

$$W = U + V - L$$

The standard model used in Shavell (1980 and 1987) and subsequent literature is a special case of the general model described above. In this literature, the expected loss is defined as $L = stl(x, y)$ and the parties' utility functions are specified as $U = u(s) - sx$ and $V = v(t) - ty$, respectively.[12] Therefore, the social welfare function in Shavell's formulation is

$$w = u(s) + v(t) - stl(x, y) - sx - ty$$

[9] Activity levels are not utilized for the evaluation of negligence either because they yield prohibitively high verification costs (e.g., number of times that a pedestrian crosses the street in any given day), or because courts do not possess sufficient information (e.g., the private value of driving or other risk-creating activity) for establishing the socially optimal level of activity against which to compare the parties' behavior. On the optimal scope of the negligence inquiry, see Dari-Mattiacci (2005).

[10] See Arlen (1990) showing that results do not change when considering bilateral-risk accidents in a model without activity level.

[11] Note that activity level could be modeled as a normal care measure (which reduces the accident loss and also the party's utility) without changing the results of the analysis, as long all all other assumptions are satisfied. The only crucial difference between care and activity level is the inclusion in or exclusion from the negligence inquiry. On the interaction between care and activity level see also Nussim and Tabbach (2009).

[12] Our notation is consistent with Shavell (1987).

It easy to see that the parties' activities are substitutes[13] in Shavell's formulation of the problem, as $L_{st} = l(x, y) > 0$. The assumption of substitutability between the parties' activities is appealing because it describes most real life situations in which accidents occur as a result of encounters between an injurer and a victim, so that the likelihood of accidents goes down if either of them reduces his level of activity. We will therefore retain this assumption in our framework.

**Assumption 1.** The parties' levels of activity are substitutes in the reduction of the accident loss: $L_{st} \geq 0$.

The distinction between complement and substitute cases allows us to study the problem of accident prevention within the framework of supermodular games (Topkis, 1979 and 1998; Vives, 1990; Milgrom and Roberts, 1990). As we will demonstrate, if the parties' activities are complements, then the game played by the parties is supermodular; conversely, if the parties' activities are substitutes (the case on which we focus), then the game is submodular. Submodular games are an appealing theoretical framework for the problem at hand, in that they have at least one pure-strategy Nash equilibrium and convenient comparative-statics properties.

A similar distinction between complements and substitutes may be carried out with respect to the signs of $L_{xs}$ and $L_{yt}$ (determining the relationship between one party's care and his own activity level), $L_{ys}$ and $L_{xt}$ (determining the relationship between one party's care and the other party's activity level), and $L_{xy}$ (determining the relationship between the parties' care levels). Likewise, the relationship between care and activity level in the parties' utility functions, $U_{xs}$ and $V_{yt}$, determines whether increasing the level of activity increases or reduces the cost of care.[14]

## 2.1 First-best liability rules

The first-best socially optimal levels of care and activity $\left(\hat{x}, \hat{y}, \hat{s}, \hat{t}\right)$ solve:

$$\max_{x,y,s,t} [U + V - L]$$

In theory, a first-best liability rule should set the standards of care and activity to equal these first-best levels. However, as discussed above, activity

[13] Given the parties' objective to maximize the value of risk-creating activities at the net of accident costs, $L$ enters as a negative term. Thus, we look at the sign of $-L_{st}$. Therefore, the case $-L_{st} \geq 0$ (or $L_{st} \leq 0$) represents the situation where the parties' activities are complements in the reduction of the accident loss. The opposite holds true when the parties' activities are substitutes in the reduction of the accident loss, $-L_{st} \leq 0$ (or $L_{st} \geq 0$).

[14] Note that parties interact only in the production of the expected accident loss $L$, while a party's utility is independent from the other party's care and activity levels. This indicates that a party's cost of taking precautions (be it activity or care) does not depend on the behavior of the other party. While this assumption is standard in the literature, a notable exception is Dharmapala and Hoffmann (2005), studying the performance of negligence rules in a model in which the costs of precautions are interdependent.

levels are too costly to verify or to count in court as factors in determining liability and hence are not included in the determination of negligence. Thus, Shavell (1980) concludes that first-best accident prevention is not attainable under the set of liability rules considered here.

## 2.2 Second-best liability rules

Since activity levels are not included in the determination of negligence, courts have no direct means of manipulating parties' incentives within the traditional framework of tort law. The policy instruments are restricted to the choice of standards of due care for both parties $\left(x^d, y^d\right)$ and the choice of apportionment of liability between two nonnegligent parties (sharing rule $\sigma \in [0,1]$) and between two negligent parties (sharing rule $\vartheta \in [0,1]$), so that:

1. If both parties adopt due care, then the loss is shared according to $\sigma$;
2. If neither party adopts due care, then the loss is shared according to $\vartheta$;
3. If only one party adopts due care, then the unilaterally negligent party bears the entire loss.

This general framework is summarised in Figure 1 and encompasses all possible liability rules, which are further detailed in Figure 2. Care incentives are created by the negligence standards $\left(x^d, y^d\right)$, while the incentives to reduce activity levels depend both on the standards of due care $\left(x^d, y^d\right)$ and on the allocation of the residual loss in equilibrium, as determined by the sharing rule $\sigma$. More specifically, negligence rules shift the entire residual loss to the victim ($\sigma = 0$), while strict liability rules shift it entirely to the injurer ($\sigma = 1$). Unlike these standard rules, loss-sharing rules ($0 < \sigma < 1$) distribute the residual burden between the injurer and the victim, hence spreading the incentives to reduce activity levels between the parties.[15]

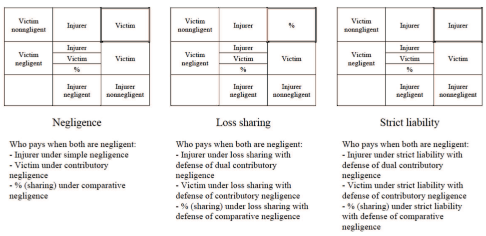


Figure 1: The structure of liability rules

[15] We will bracket off $\vartheta$ from the discussion, since the analysis of shared liability in cases of bilateral negligence is well-covered in the literature and irrelevant to the present discussion. We will, however, return to considering $\vartheta$ below, when examining the parties' compliance with the due-care standards.

| | | |
|---|---|---|
| Victim nonngligent | Injurer | $\sigma$ |
| Victim negligent | $\theta$ | Victim |
| | Injurer negligent | Injurer nonnegligent |

| | $\sigma = 0$ | $0 < \sigma < 1$ | $\sigma = 1$ |
|---|---|---|---|
| $\theta = 0$ | Contributory negligence | Loss sharing with defense of contributory negligence | Strict liability with defense of contributory negligence |
| $0 < \theta < 1$ | Comparative negligence | Loss sharing with defense of comparative negligence | Strict liability with defense of comparative negligence |
| $\theta = 1$ | Simple negligence | Loss sharing with defense of dual contributory negligence | Strict liability with defense of dual contributory negligence |

Figure 2: An overview of liability rules

Therefore, in the second-best scenario, the policymaker's problem can be expressed as follows:

$$\max_{x^d, y^d, \sigma} [U + V - L] \tag{1}$$
$$\text{s.t. } (i): \quad s = s\left(x^d, y^d, \sigma\right) \text{ and } t = t\left(x^d, y^d, \sigma\right)$$
$$\text{s.t. } (ii): \quad x = x^d \text{ and } y = y^d$$

The policy variables only indirectly influence the parties' choices of care and activity levels. The first restriction indicates that the parties' choice of activity levels depends in some way on all policy variables. We devote Section 2.2.1 to the study of how $s$ and $t$ are determined. The second restriction states that the desired policy outcome occurs only if the parties abide by the due-care standards, a problem we shall tackle in Section 2.2.3. If both restrictions are satisfactorily verified, the policymaker can set $\left(x^d, y^d, \sigma\right)$ to attain a second-best level of social welfare.

### 2.2.1 The parties' choice of activity levels

Let us assume that both parties exercise due care in equilibrium. We analyze the parties' choices of activity levels given the policymaker's choice of $x^d$, $y^d$ and $\sigma$. Thus, $x^d$, $y^d$ and $\sigma$ are parameters of the non-cooperative game $\Gamma(x^d, y^d, \sigma)$ played by the parties. The parties choose activity levels $s^*$ and $t^*$ in order to maximize their payoffs as follows:

$$\max_s \left[\Pi^U(s,t) = U\left(x^d, s\right) - \sigma L\left(x^d, y^d, s, t\right)\right] \tag{2}$$
$$\max_t \left[\Pi^V(s,t) = V\left(y^d, t\right) - (1-\sigma) L\left(x^d, y^d, s, t\right)\right] \tag{3}$$

Given the policymaker's choice of $x^d$, $y^d$ and $\sigma$, we assume that the activity levels $s$ and $t$ that solve (2) and (3) are interior values. Supermodular (and submodular) games provide the appropriate framework to study games where the best response of a player is a monotonic function of his rival, as is the case in our framework. We analyze the parties' choice of their activity levels given the policymaker's choice of $x^d$, $y^d$ and $\sigma$. Thus, $x^d$, $y^d$ and $\sigma$ are parameters of the non-cooperative game $\Gamma\left(x^d, y^d, \sigma\right)$ played by the parties. The parties choose activity levels $s, t$ in order to maximize their payoffs.

**Lemma 1.** Let $x^d$ and $y^d$ be two given numbers in $[0,+\infty[$ and $\sigma$ in $[0,1]$. If $L_{st} \geq 0$ for every $s,t \in D$, then $\Gamma\left(x^d, y^d, \sigma\right)$ is a submodular game. If $L_{st} \leq 0$ for every $s,t \in D$, then $\Gamma\left(x^d, y^d, \sigma\right)$ is a supermodular game.

**Proof.** Let us now verify Topkis' (1998) requirements in our setting. Let us first assume that $L_{st} \leq 0$ for every $s,t$. Given our previous assumptions we know that the parties' strategy space is $[0,+\infty[$, which is a convex but not compact subset of $\mathbb{R}$. Nevertheless, we assumed that the activity levels $s$ and $t$ that solve (2) and (3) are bounded and we can say that the effective set of $s$ and $t$ is in fact a compact subset $D$ of $[0,+\infty[$. $D$ is a sublattice of $\mathbb{R}$, is one-dimensional and is partially ordered.[16] Moreover, $\Pi^U : D \times D \to \mathbb{R}$ and $\Pi^V : D \times D \to \mathbb{R}$ are twice-continuously-differentiable payoff functions, which verify the following inequalities for every $(s,t)$ in $D \times D$ and for every $\sigma$ in $[0,1]$:

$$\begin{aligned}\Pi^U_{st} &= -\sigma L_{st} \geq 0 \\ \Pi^V_{st} &= -(1-\sigma) L_{st} \geq 0\end{aligned}$$

Thus, according to Topkis (1998), the game $\Gamma(x^d,\ y^d,\ \sigma)$ is a (smooth) supermodular game. Likewise, if $L_{st} \geq 0$ for every $s,t$ in $D$ then $\Pi^U_{st} \leq 0$ and $\Pi^V_{st} \leq 0$ and the game $\Gamma\left(x^d, y^d, \sigma\right)$ is a (smooth) submodular game. ■

**Lemma 2.** The game $\Gamma(x^d,\ y^d,\ \sigma)$ has at least one pure strategy Nash equilibrium.

**Proof.** We know from Vives (1990) that if the game is supermodular ($L_{st} \leq 0$) then the set of pure strategy Nash equilibria is non-empty. Likewise, if the game is submodular ($L_{st} \geq 0$) then we have a similar result because it is well-known that we can reverse the natural order of the strategy set of player 2 and the transformed payoffs display increasing differences just as in the first case. If the game is strictly supermodular or strictly submodular ($L_{st} < 0$ or $L_{st} > 0$) then the set of pure strategy Nash equilibria is a non-empty complete sub-lattice which is ordered. ■

Thus, we can conclude that parties will choose unique levels of activity $s^* = s\left(x^d, y^d, \sigma\right)$ and $t^* = t\left(x^d, y^d, \sigma\right)$, which are functions of the parameters under which the game is played and which are generally different from the first best. Thanks to the property of supermodular games, we can use monotonicity arguments to prove comparative-statics results. Let us consider a supermodular game indexed by a parameter in a partially ordered set with a unique Nash equilibrium. If each player's payoff function has increasing differences in its own strategy and the parameter, then the Nash equilibrium is increasing in this parameter. For given standards of care $x^d$ and $y^d$ in $[0,+\infty[$, optimal levels of activity $s^*$ and $t^*$ exist and depend on the sharing rule $\sigma \in [0,1]$. If the parties'

[16] Its natural partial ordering is denoted by $\leq$ with $x \vee y = \max\{x,y\}$ and $x \wedge y = \min\{x,y\}$ for $x,y$ in $\mathbb{R}$.

levels of activity are substitutes in the reduction of the accident loss and the game $\Gamma(x^d, y^d, \sigma)$ has a unique Nash equilibrium, then we have the following result:

**Lemma 3 (monotonicity result).** Let $(s^*, t^*)$ be the unique Nash equilibrium of the game $\Gamma(x^d, y^d, \sigma)$. Under Assumption 1, if $(x^d, y^d)$ is given, $s^* = s(\sigma)$ is a decreasing function of $\sigma$ and $t^* = t(\sigma)$ is an increasing function of $\sigma$.

**Proof.** Let $\sigma$ in $[0.1]$. If $L_{st} \geq 0$ for every $s, t \in D$, then $\Gamma(x^d,\ y^d,\ \sigma)$ is a submodular game. For $s' = -s$ and $t' = t$ the game is supermodular in $(s', t')$ since:

$$\begin{aligned} \Pi^U_{s't'} &= -\sigma L_{s't'} = \sigma L_{st} \geq 0 \\ \Pi^V_{st} &= -(1-\sigma) L_{s't'} = (1-\sigma) L_{st} \geq 0. \end{aligned}$$

and for every $\sigma$ in $[0,1]$, we have:

$$\begin{aligned} \Pi^U_{s'\sigma} &= -\frac{\partial}{\partial s'}(L(s,t)) = \frac{\partial}{\partial s}(L(s,t)) = L_s \geq 0 \\ \Pi^V_{t'\sigma} &= L_{t'} = L_t \geq 0. \end{aligned}$$

Then the unique Nash equilibrium of the game $(s\prime, t\prime) = (-s^*, t^*)$ is increasing in $\sigma$. We conclude that $s^* = s(\sigma)$ is a decreasing function of $\sigma$ and $t^* = t(\sigma)$ is an increasing function of $\sigma$. ■

The next assumption captures a realistic scenario in which, at low levels of care, increasing care initially makes the activity safer without adding too much to the costs of precaution and hence encourages activity. At higher levels of care, increasing care bears more on the cost side and hence discourages activity. Thus, Assumption 2 implies that a party's care and activity level are complements at low levels of care and substitutes at high levels of care

**Assumption 2.** For any given $\sigma$, there exists an $x_1 > 0$ such that $s^* = s\left(x^d, y^d\right)$ increases in $x^d$ for $x^d < x_1$, decreases in $x^d$ for $x^d > x_1$, and reaches its maximum at $x^d = x_1$. Likewise, there exists a $y_1 > 0$ such that $t^* = t\left(x^d, y^d\right)$ decreases in $y^d$ for $y^d > y_1$, increases in $y^d$ for $y^d < y_1$, and $t^* = t\left(x^d, y^d\right)$ reaches its maximum at $y^d = y_1$.

From the analysis that follows it will be clear that $\frac{ds}{dx} = -\frac{U_{xs} - \sigma L_{xs}}{U_{ss} - \sigma L_{ss}}$, which is greater or less than zero depending on whether $U_{xs} - \sigma L_{xs}$ is greater or less than zero, given that the denominator is negative. Therefore, Assumption 2 is naturally verified when $U_{xs}$ is constant and $L_{xs}$ increases in $x$. This is the case in Shavell's model, where $U_{xs} = -1$ and $L_{xs} = \sigma t l_x$, which is negative and increases in $x$. Analogous considerations apply to the victim. Therefore, like Assumption 1, also Assumption 2 generalizes a fundamental feature of Shavell's model.

### 2.2.2 The policymaker's choice of due care and sharing

The policymaker defines the socially optimal levels of care $x^{d*}$ and $y^{d*}$ and the sharing $\sigma^*$ that maximize:

$$\begin{aligned} & \max_{x^d, y^d, \sigma} [U + V - L] \\ \text{s.t. } (i): \quad & s = s^* \text{ and } t = t^* \\ \text{s.t. } (ii): \quad & x = x^d \text{ and } y = y^d \end{aligned}$$

where the parties' activity levels are determined as in the previous section:

$$\begin{aligned} s^* &\in \arg\max_s \left[U(x, s) - \sigma L(x, y, s, t^*)\right] \\ t^* &\in \arg\max_t \left[V(y, t) - (1 - \sigma) L(x, y, s^*, t)\right] \end{aligned}$$

### 2.2.3 Parties' compliance with the negligence standards

In the preceding analysis, optimal standards of care and optimal sharing were identified under the working assumption that parties would comply with the chosen standards of due care in equilibrium. In this section, we verify whether this rather critical assumption holds in the case under examination, where $\sigma^*$, $x^{d*}$, $y^{d*}$, and $\vartheta$ are the parameters a non-cooperative game denoted as $\Gamma(x^d, y^d, \sigma, \vartheta)$. Note that the sharing rule between nonnegligent parties and the standards of care are set at the socially optimal level (second best), while we allow for any sharing rule between negligent parties, that is, for any $\vartheta$. Under these conditions, the injurer and the victim choose $(x, s)$ and $(y, t)$, respectively, in order to maximize the following payoffs:

$$\Pi^U(x, y) = \begin{cases} U - \sigma L & \text{if} \quad x \geq x^d \text{ and } y \geq y^d \quad \text{I (both nonnegligent)} \\ U - L & \text{if} \quad x < x^d \text{ and } y \geq y^d \quad \text{II (injurer negligent)} \\ U - \vartheta L & \text{if} \quad x < x^d \text{ and } y < y^d \quad \text{III (both negligent)} \\ U & \text{if} \quad x \geq x^d \text{ and } y < y^d \quad \text{IV (victim negligent)} \end{cases}$$

$$\Pi^V(x, y) = \begin{cases} V - (1 - \sigma) L & \text{if} \quad x \geq x^d \text{ and } y \geq y^d \quad \text{I (both nonnegligent)} \\ V & \text{if} \quad x < x^d \text{ and } y \geq y^d \quad \text{II (injurer negligent)} \\ V - (1 - \vartheta) L & \text{if} \quad x < x^d \text{ and } y < y^d \quad \text{III (both negligent)} \\ V - L & \text{if} \quad x \geq x^d \text{ and } y < y^d \quad \text{IV (victim negligent)} \end{cases}$$

The numbering (I through IV) to the right refers to the quadrants in Figure 1, where the numbering starts from the upper-right cell and continues counter-clockwise. We are interested in determining whether the parties choose $\left(x = x^d, y = y^d\right)$. In order for compliance to obtain in a Nash equilibrium in the second best, it must be impossible for the injurer to improve his utility by choosing levels of care and activity different from $x^{d*}$ and $s^*$, given that the victim is complying with the rule. We will analyze the parties' incentives to comply with the chosen standard of due care and examine whether this equilibrium is unique. We investigate whether sharing the loss between nonnegligent parties undermines the parties' care incentives in equilibrium. To do so, we introduce two technical assumptions which are relevant for the analysis of the parties' compliance behavior, given their choice of activity levels.

**Assumption 3.** There exist $x_0 \geq 0$, $y_0 \geq 0$ such that $(\max_s [U - L])_x \geq 0$ and $(\max_t [V - L])_y \geq 0$ for $x \leq x_0$ and $y \leq y_0$.

**Assumption 4.** The negligence standard $x^{d*}$ maximizes social welfare when $y = y^{d*}$, $\sigma = \sigma^*$, and $t = t^*$ are given and $y^{d*}$ maximizes social welfare when $x = x^{d*}$, $\sigma = \sigma^*$, and $s = s^*$ are given.

Assumption 3 states that there exist $x_0$, $y_0$ such that the maximum in $s$ of the difference between utility and damage increases in the care of the parties when these levels are small enough. Note that this is a milder assumption than requiring concavity of the social welfare function. Therefore, the critical discriminant for compliance with the negligence standards is provided by Assumption 4, which in essence excludes that a party's level of activity is influenced by the other party's level of care. Assumption 4 will not be met if the level of activity cannot be taken as given but varies with care by the other party. However, cases in which the assumption is verified can be easily found, for instance, when damages are additive:

$$L(x, y, s, t) = L^I(x, s) + L^V(y, t)$$

In this case $s$ is independent of $y$ and $t$ is independent of $x$. Under these conditions, the inequality:

$$\begin{aligned} & U\left(x^{d*}, s(\sigma^*, x^{d*})\right) + V\left(y^{d*}, t(\sigma^*, y^{d*})\right) - L^I(x^{d*}, s(\sigma^*, x^{d*})) \\ & -L^V\left(y^{d*}, t(\sigma^*, y^{d*})\right) \\ > \quad & U\left(x, s(\sigma^*, x)\right) + V\left(y^{d*}, t(\sigma^*, y^{d*})\right) - L^I(x, s(\sigma^*, x)) - L^V\left(y^{d*}, t(\sigma^*, y^{d*})\right) \end{aligned}$$

shows that $x^{d*}$ maximizes social welfare when $y = y^{d*}$, $\sigma = \sigma^*$, and $t = t(\sigma^*, y^{d*}) = t^*$ are given. Likewise for the victim. In the following proposition we assume for the injurer that $x_0 \geq x^{d*} = x_1$ and we make a similar assumption for the victim. The first restriction can be read as meaning that the second-best level of due care is lower than the level of care that would be optimal if the injurer bore the entire loss (that is, under strict liability). This guarantees that the injurer cannot be better-off by choosing a lower level of care and paying damages. The second restriction guarantees that the optimal level of activity does not decrease in care below the due-care level and that it does not increase in care above the due-care level. This excludes the effects of adjustments in the level of activity and guarantees that the injurer will not prefer an even higher level of care than that imposed by tort law.

**Proposition 1.** Under Assumptions 1-4, if the standards of due care and the loss-sharing rule are set at the (second-best) socially optimal levels, $x^{d*}$, $y^{d*}$ and $\sigma^*$, and if the standards of due care are such that $x_0 \geq x^{d*} = x_1$; $y_0 \geq y^{d*} = y_1$, then both parties comply with the negligence standards in a unique Nash equilibrium, irrespective of the sharing rule $\vartheta$ applied when both parties are negligent.

**Proof.** We claim $(i)$ that deviations to higher care levels and then $(ii)$ deviations to lower care levels are excluded. We can now prove $(i)$ and $(ii)$ for the injurer. The proof for the victim is analogous.

$(i)$ We assume $x > x^{d*}$. The maximum of $(U - \sigma^* L)\left(x, y^{d*}, s, t^*\right)$ is reached for a value $s^{\sigma*} = s\left(\sigma^*, x\right)$, which depends continuously on $\sigma^*$ and $x$, and decreases in $x$ for $x > x^{d*}$ due to Assumption 2. In order for compliance to obtain in a Nash equilibrium, the following must hold true for the injurer:

$$(U - \sigma^* L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^*), t^*\right) > (U - \sigma^* L)\left(x, y^{d*}, s(\sigma^*, x), t^*\right)$$

for all $x > x^{d*}$. This condition shows that the injurer has no incentive to increase precautions above the required standard of care. By Assumption 4, $x^{d*}$ maximizes the social welfare when $y = y^{d*}$, $\sigma = \sigma^*$, and $t = t^*$ are given. Thus, we have:

$$(U + V - L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^{d*}), t^*\right) \geq (U + V - L)\left(x, y^{d*}, s(\sigma^*, x), t^*\right)$$

Since $V(y^{d*}, t^*)$ can be subtracted from both sides of the inequality, the former implies:

$$(U - L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^{d*}), t^*\right) \geq (U - L)\left(x, y^{d*}, s(\sigma^*, x), t^*\right)$$

Accordingly, since we have assumed $L_s > 0$, $L_x < 0$, and $x^{d*} \geq x_1$, then by Assumption 2 $s$ is a decreasing function of $x$ for $x > x^{d*}$ and we have:

$$\begin{aligned}
& (U - \sigma^* L)\left(x^{d*}, y^{d*}, s(\sigma^*, x), t^*\right) \\
= \; & (U - L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^{d*}), t^*\right) + (1 - \sigma^*) L(x^{d*}, y^{d*}, s(\sigma^*, x^{d*}), t^*) \\
> \; & (U - L)\left(x, y^{d*}, s(\sigma^*, x), t^*\right) + (1 - \sigma^*) L(x^{d*}, y^{d*}, s(\sigma^*, x), t^*) \\
> \; & (U - L)\left(x, y^{d*}, s(\sigma^*, x), t^*\right) + (1 - \sigma^*) L\left(x, y^{d*}, s(\sigma^*, x), t^*\right)
\end{aligned}$$

Therefore:

$$(U - \sigma^* L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^*), t^*\right) > (U - \sigma^* L)\left(x, y^{d*}, s(\sigma^*, x), t^*\right)$$

for all $x > x^{d*}$.

$(ii)$ We now assume $x < x^{d*}$ and continue to tackle the question of whether compliance can be obtained in a Nash equilibrium by studying whether the injurer has incentives to reduce precautions below the required standard of care. For all $x < x^{d*}$, when $\sigma^*$, $y^{d*}$ and $t^*$ are given, let us define $\gamma(x)$ as:

$$\gamma(x) = (U - \sigma^* L)\left(x, y^{d*}, s(1,x), t^*\right)$$

where $s = s(1,x)$ maximizes $U(x,s) - L\left(x, y^{d*}, s, t^*\right)$. That is, $\gamma(x)$ is the payoff of the injurer given that he chooses his level of activity as if he had to bear the full accident loss. We have:

$$\gamma(x) = (U - L)\left(x, y^{d*}, s(1,x), t^*\right) + (1-\sigma^*)L\left(x, y^{d*}, s(1,x), t^*\right)$$

Given $x^{d*} \leq x_0$, according to the Assumption 3, $(U - L)$ increases in $x$ for $x < x^{d*}$, and, since $s$ is an increasing function of $x$ for $x < x^{d*}$ due to Assumption 2, we can say that $\gamma$ increases in $x$ for $x < x^{d*}$. Playing the non-cooperative game $\Gamma(x^*, y^*, \sigma^*)$, the injurer chooses his level of activity $s^* = s(\sigma^*, x^{d*})$ in order to maximize his payoff $U(x^{d*}, s) - \sigma^* L\left(x^{d*}, y^{d*}, s, t^*\right)$ and hence:

$$\begin{aligned} & U(x^{d*}, s(\sigma^*, x^{d*})) - \sigma^* L\left(x^{d*}, y^{d*}, s(\sigma, x^{d*}), t^*\right) \\ > \quad & U(x^{d*}, s(1, x^{d*})) - \sigma^* L\left(x^{d*}, y^{d*}, s(1, x^{d*}), t^*\right) \end{aligned}$$

Therefore:

$$(U - \sigma^* L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^{d*}), t^\sigma\right) \geq \gamma(x^{d*}) > \gamma(x)$$

which follows from the fact that $\gamma$ increases in $x$ for $x < x^{d*}$. Since $\sigma^* \in [0,1]$, we have:

$$\begin{aligned} & (U - \sigma^* L)\left(x^{d*}, y^{d*}, s(\sigma^*, x^{d*}), t^*\right) \geq (U - \sigma^* L)\left(x, y^{d*}, s(1,x), t^*\right) \\ \geq \quad & (U - L)\left(x, y^{d*}, s(1,x), t^*\right) \end{aligned}$$

which proves the first part of Proposition 1. A similar analysis can be carried out for the victim's incentives to undertake optimal precautions in equilibrium. Note that this result holds for any $\vartheta$. That is, it holds regardless of the allocations of accident losses when both parties are negligent.

We should now verify whether the parties' compliance with the standards of due care represents a unique Nash equilibrium of the game. The results derived above show that unilateral negligence is not an equilibrium. In order to show uniqueness, we need to further prove that bilateral negligence is not an equilibrium.[17] In order to observe bilateral negligence in equilibrium, the following conditions should be simultaneously satisfied. These conditions state that, given that one party is negligent, the other party should also prefer to be negligent, rather than unilaterally nonnegligent:

$$\begin{aligned} U(x,s) - \vartheta L(x,y,s,t) &> U\left(x^{d*}, \bar{s}\right) \\ V(y,t) - (1-\vartheta) L(x,y,s,t) &> V\left(y^{d*}, \bar{t}\right) \\ &\text{s.t. (1): } s = s(x,y,\vartheta) \text{ and } t = t(x,y,\vartheta) \\ &\text{s.t. (2): } x < x^{d*} \text{ and } y < y^{d*} \end{aligned}$$

[17]Here we use the technique introduced by Landes and Posner (1980).

Summing these conditions, we obtain:

$$U(x,s)+V(y,t)-L(x,y,s,t)>U\left(x^{d*},\bar{s}\right)+V\left(y^{d*},\bar{t}\right) \tag{4}$$

Suppose that the loss-sharing rule is set at the socially optimal level, $\sigma=\sigma^*$. Notice that the left-hand side of (4) represents the second-best maximization problem, subject to analogous conditions on the parties' choices of activity level. It follows that the left-hand side of (4) is maximized by $\vartheta=\sigma^*$, levels of care equal to $x^{d*}$ and $y^{d*}$, and the resulting levels of activity equal to $s^*$ and $t^*$. Therefore, regardless of the sharing rule applied in the case of bilateral negligence, $\vartheta$, the parties' aggregate payoffs when bilaterally negligent cannot be larger than $U\left(x^{d*},s^*\right)+V\left(y^{d*},t^*\right)\leq U\left(x^{d*},\bar{s}\right)+V\left(y^{d*},\bar{t}\right)$, which proves that (4) cannot hold true and hence both parties' negligence cannot be an equilibrium if the loss-sharing rule and the negligence standards are optimally set. ■

### 2.2.4 Discussion

Proposition 1 shows that a second-best tort law system based on optimal negligence standards and optimal loss-sharing between nonnegligent parties is feasible. We have identified a set of conditions under which compliance results in equilibrium under any optimal loss-sharing rule. These results also confirm that compliance is not affected by the choice of sharing when both parties are negligent $\vartheta$. This finding extends a well established result in tort law and economics, proven by Landes and Posner (1980, note 51) in a model in which only care levels were considered. Here we show than the result holds also when activity level is taken into account. Our analysis departs from the conventional framework of Shavell (1980) in which only two corner sharing arrangements were allowed: $\sigma=0$ and $\sigma=1$, excluding the possibility of intermediate loss-sharing solutions even when optimal. Note, finally, that the compliance problem that underscores second-best liability rules is not specific to rules that share the loss in equilibrium, but applies more generally to any rule, including the traditional corner solutions $\sigma=0$ and $\sigma=1$.

## 2.3 Third-best liability rules

The previous analysis in which both negligence standards and sharing were set at the socially optimal level can now be extended in two ways: sharing could be suboptimal or the negligence standards could be suboptimal. We first consider cases in which sharing is not possible, or is otherwise set at a socially suboptimal level.[18] This setting, which can be referred to as third best, is especially important if one considers that traditional negligence rules implement corner solutions ($\sigma=1$ or $\sigma=0$) and hence do not necessarily apply the optimal sharing. We will demonstrate that the results stated in the first part of Proposition 1 about compliance with the optimal standards apply to

[18] See further Section 4 on this point.

the third best. However, the proof of the uniqueness of the equilibrium is based on the fact that the sharing is optimal. Hence, we identify a different sufficient condition which unveils an important interdependence between the loss-sharing rule $\sigma$ (applicable when both parties are nonnegligent) and the loss-sharing rule $\vartheta$ (applicable when both parties are instead negligent).

A second way in which the second-best results may fail to apply concerns compliance with the due-care levels of care. Since Propositions 1 and 2 only identify sufficient condition for compliance with the negligence standards, they leave open the possibility that, under a different set of assumption, parties will not abide by the second-best levels of care. A solution to this potential problem consists in lowering the due-care standards to some feasible third-best levels and we identify levels that are feasible under any sharing of the loss in equilibrium.

The Appendix contains a simpler model studying an additive loss function, which allows us to compute the optimal sharing rule explicitly and to check compliance. The example shows that compliance can be guaranteed in equilibrium irrespective of whether the sharing rule or the negligence standards are optimally set—hence, the example relates both to the third best and to the second best—and illustrates a case in which the optimal sharing rule is an interior value.

### 2.3.1 Suboptimal sharing

In this case, we take $\sigma$ as given. The policymaker only defines the socially optimal levels of care $x^{d\sigma}$ and $y^{d\sigma}$ that maximize:

$$\begin{aligned} & \max_{x^d, y^d} [U + V - L] \\ \text{s.t. } (i)\text{:} \quad & s = s^* \text{ and } t = t^* \\ \text{s.t. } (ii)\text{:} \quad & x = x^d \text{ and } y = y^d \end{aligned}$$

where the parties' activity levels are determined as in the previous section. Note that the levels of due care are set at the socially optimal levels, $x^{d\sigma}$ and $y^{d\sigma}$, but the loss-sharing rule is not optimally set, $\sigma \neq \sigma^*$. We need to verify again whether compliance with the standards of due care results in a Nash equilibrium and whether the resulting equilibrium is unique.

**Proposition 2.** Under Assumptions 1-4, if the levels of due care are set at the socially optimal levels, $x^{d\sigma}$ and $y^{d\sigma}$, but the loss-sharing rule is not optimally set $\sigma \neq \sigma^*$ and if the standards of due care are such that $x_0 \geq x_1 = x^{d\sigma}$; $y_0 \geq y_1 = y^{d\sigma}$, then both parties comply with the negligence standards in equilibrium irrespective of the sharing rule $\vartheta$. This equilibrium is unique if $\vartheta$ is close to $\sigma$.

**Proof.** The proof of the first part of Proposition 1 is still valid when $\sigma \neq \sigma^*$. The second part of Proposition 2 requires verifying whether the parties' compliance with the standard of due care represents a unique Nash equilibrium of the game. Knowing that unilateral negligence is not an equilibrium, we should verify if bilateral negligence can be excluded as a possible equilibrium of the game.

It is easy to see that if the loss is shared according to the same loss-sharing criterion used when both parties are negligent (that is, if $\vartheta = \sigma$), the left-hand side of (4) represents the policymaker's maximization problem. Given $\vartheta = \sigma$, when both parties are negligent their aggregate net utilities are maximized when parties adopt levels of care equal to $x^{d\sigma}$ and $y^{d\sigma}$ and levels of activity equal to $s^{\sigma}$ and $t^{\sigma}$. We can hence conclude that, if $\vartheta = \sigma$, the condition in (4) necessary for bilateral negligence cannot be satisfied: at least one party would have an incentive to deviate. This proves that when $\vartheta = \sigma$, parties cannot both be negligent in equilibrium. By continuity, the result holds true for $\vartheta$ in a neighborhood of $\sigma$. The equilibrium where both parties comply with the due standard of care will thus be unique in this case, which proves the second part of Proposition 2. ■

### 2.3.2 Suboptimal negligence standards

Let us now turn to cases in which the sufficient conditions identified in Proposition 1 (and Proposition 2) are not verified. This eventuality opens the door to parties' noncompliance with the due-care standards set by the policymaker at the second-best level.

If this is the case, to facilitate compliance, the due-care standards can be lowered to some less demanding levels. In particular, if the injurer's negligence standards is set at the "strict liability level", that is at the level that, given the behavior of the victim, would be optimal if the injurer bore the entire accident loss, than the injurer will abide by due care under any sharing of the loss. The intuition is straightforward: the threshold level of the negligence standard proposed above is the level of care that the injurer would choose if he chose to be negligent. Likewise, compliance by the victim can be guaranteed by setting the victim's due-care level at the "no liability level", that is at the level of care that, given the behavior of the injurer, would be optimal if the victim bore the entire accident loss.

**Proposition 3.** Under Assumptions 1-2, given $y^d$ and $t^* = \left(x^d, y^d, \sigma\right)$, if the injurer's due-care level is such that $x^d \leq \arg\max_x \left[(U - L)\left(x, s\left(x, 1\right), y^d, t^*\right)\right]$, then the injurer complies with due-care; likewise, given $x^d$ and $s^* = \left(x^d, y^d, \sigma\right)$, if the victim's due-care level is such that $y^d \leq \arg\max_y \left[(V - L)\left(x^d, s^*, y, t\left(y, 1\right)\right)\right]$, then the victim complies with due-care.

**Proof.** Let us show that, given $\left(x^d, y^d\right)$ as in Proposition 3, if the victim chooses $y^d$ and $t^*$, then the injurer has no incentive to choose $x < x^d$. Assume that the injurer chooses $x < x^d$. Given the victim's compliance choice, a negligent injurer bears full liability and, hence, chooses $x$ and $s$ as to maximize $(U - L)\left(x, s, y^d, t^*\right)$. The chosen levels of $x$ and $s$ simultaneously satisfy the following two first order conditions, calculated at $y = y^d$ and $t = t^*$:

$$\begin{aligned}(U-L)_x &= 0 \\ (U-L)_s &= 0\end{aligned}$$

Then, $s = s(x,1)$ is the level of activity that satisfies the latter first order condition. Substituting into the injurer's maximization problem, we have that a negligent injurer chooses $x$ in order to maximize $(U-L)\left(x, s(x,1), y^d, t^*\right)$. Thus, by construction, the level of care that maximizes a negligent injurer's payoff is greater than or equal to $x^d$. Therefore, the injurer does not have an incentive to reduce his level of care below $x^d$. An analogous argument applies to the victim. ■

### 2.3.3 Discussion

Proposition 2 shows that, under some sufficient conditions, compliance with the standard of due care can be achieved in equilibrium for any $\sigma$ and any $\vartheta$. This irrelevance result mirrors and further extends the work of Landes and Posner (1980). We show that the parties' incentives to comply with the standard of due care are present not only under any sharing rule for bilateral negligence (Landes and Posner's result), but also under any sharing rule for bilateral non-negligence. Landes and Posner (1980) used their irrelevance result with respect to $\vartheta$ to demonstrate the incentive-equivalence of different negligence rules (simple negligence, comparative negligence and contributory negligence), all of which were shown to lead to the adoption of due levels of care and identical levels of social welfare. The same equivalence result holds when alternative negligence defenses are applied to strict liability (strict liability with defense of contributory negligence, strict liability with defense of dual contributory negligence, and strict liability with defense of comparative negligence). However, the internal equivalence of these rules within each set of regimes cannot be used to compare the different allocations of the residual loss induced by the alternative negligence or strict-liability regimes—a task we have taken on in this paper.

We have proven a higher-level irrelevance among all combinations of sharing $\sigma$ (when parties are nonnegligent) and $\vartheta$ (when parties are negligent), showing that all liability rules[19] are equivalent with respect to care incentives, in the sense that they all induce both parties to comply with the negligence standards. However, unlike the sharing rule applied in the case of bilateral negligence $\vartheta$, which will rarely affect social welfare, because it occurs out of equilibrium, the sharing rule $\sigma$ applies to nonnegligent parties and is implemented in equilibrium, hence more likely impacting social welfare.

The sufficient conditions identified in Proposition 2 guarantee that compliance with the negligence standards is the only equilibrium if the sharing $\sigma$ applied when parties are negligent is close to the sharing $\vartheta$ applied when parties are nonnegligent. This, in turn, is the case in three broad sets of situations:

[19] Recall that no-liability and strict liability without negligence defenses are excluded from this count.

1) under comparative negligence when a loss-sharing rule for residual losses applies in a similar proportion; 2) under contributory negligence when the victim pays the whole accident loss if the parties are both negligent or if they are both nonnegligent; and 3) under strict liability with defense of dual contributory negligence when the injurer pays—as it is easy to verify in Figure 2.

A second reason why the second best cannot be achieved is that parties might choose not to comply with the negligence standards, a possibility that cannot be excluded by Propositions 1 and 2. In this case, Proposition 3 shows that that the negligence standards can be reduced to levels that actually induce compliance. A level of due-care that induces compliance under all circumstances is the level of care that maximizes a party's payoff when that party bears the whole loss, given the behavior of the other party. If both negligence standards are set at such levels, compliance is guaranteed for any sharing of the loss.

Shavell (1987, pp. 42-43) employs a model in which the standards of due care are set at the first-best levels $\hat{x}$ and $\hat{y}$ and shows that compliance results in equilibrium. He proceeds to show that social welfare can be improved by increasing the negligence standards above those first-best levels, as long as the increase is not high enough to induce parties to violate them. In this paper, we provide conditions under which the negligence standards can be increased up to the second-best standards without compromising parties' compliance and a strategy to induce third-best compliance when this is not possible.

# 3 Optimal loss-sharing between nonnegligent parties

In the previous section, we showed that the adoption of loss-sharing rules between nonnegligent parties does not undermine the parties' care incentives and that this result holds true (with some qualifications) even when the loss-sharing rule is not optimally chosen. The allocation of liability between nonnegligent parties can take one of three possible forms, namely $\sigma = 0$ (i.e., the allocation of traditional negligence rules), $\sigma = 1$ (i.e., the allocation of strict liability rules), or $0 < \sigma < 1$ (i.e., loss-sharing between nonnegligent parties). The choice of the optimal sharing rule depends on the characteristics of the relevant accident functions.

In this section, we study the optimal setting of $\sigma$ in a simplified case, where the loss function is given an additive form. Thus, we have $L_{st} = 0$; that is, the parties activity levels are independent of each other in the production of the expected accident loss. This formulation allows us to discuss the conditions under which sharing the loss between nonnegligent parties is preferable to allocating it entirely to either of them and has the advantage that the optimal level of $\sigma$ can be made explicit without having to calculate the due-care standards. Here we provide a simple example with quadratic utility functions and identify the conditions under which it is optimal to share the residual loss between the parties. Let:

$$\begin{aligned} U &= s(2a - s) - sx \\ V &= t(2b - t) - ty \\ L &= sl_{\mathrm{I}}(x, y) + tl_{\mathrm{V}}(x, y) \end{aligned}$$

Note that the parties' utility functions have two components as in Shavell (1980). The first component indicates that a party's utility increases at a decreasing rate with his level of activity. The second component indicates that care has a linear cost that increases with the party's activity, because the greater the activity the more often care has to be taken. The variables $a$ and $b$ are positive parameters and we assume that they have the values required to support the analysis below (that is, to ensure that activity levels are nonnegative), since these parameters do not play any crucial role in the result. The loss function is additive and linear in the parties' levels of activity. Its two components $l_{\mathrm{I}}$ and $l_{\mathrm{V}}$ are convex nonnegative functions of the parties' care levels as in the standard model of torts.

In the Appendix, we fully study a model with additive loss function, where we also address the question of compliance. Here we assume that the parties will abide by the due-care standards, so that we can focus on the parties' choice of activity levels:

$$\begin{aligned} &\max_{s} \left[s(2a - s) - s\left[\sigma l_{\mathrm{I}}(x^d, y^d) + x^d\right] - \sigma t l_{\mathrm{V}}(x^d, y^d)\right] \\ &\max_{t} \left[t(2b - t) - t\left[(1-\sigma) l_{\mathrm{V}}(x^d, y^d) + y^d\right] - (1-\sigma)\, s l_{\mathrm{I}}(x^d, y^d)\right] \end{aligned}$$

Differentiating these objective functions with respect to $s$ and $t$ and setting the derivatives equal to zero yields the parties' chosen levels of activities:

$$\begin{aligned} s &= a - \frac{1}{2}\sigma l_{\mathrm{I}}(x^d, y^d) - \frac{1}{2}x^d \\ t &= b - \frac{1}{2}(1-\sigma) l_{\mathrm{V}}(x^d, y^d) - \frac{1}{2}y^d \end{aligned}$$

The social welfare function is

$$\begin{aligned} W(\sigma, x^d, y^d) &= s(\sigma, x^d, y^d)(2a - s(\sigma, x^d, y^d)) + t(\sigma, x^d, y^d)(2b - t(\sigma, x^d, y^d)) \\ &\quad - s(\sigma, x^d, y^d) l_{\mathrm{I}}(x^d, y^d) - t(\sigma, x^d, y^d) l_{\mathrm{V}}(x^d, y^d) \\ &\quad - s(\sigma, x^d, y^d) x^d - t(\sigma, x^d, y^d) y^d \end{aligned}$$

Differentiating the social welfare function with respect to $\sigma$, substituting the parties' chosen levels of $s$ and $t$, and setting the derivative equal to zero gives:

$$\frac{1-\sigma}{2} l_{\mathrm{I}}^2(x^d, y^d) - \frac{\sigma}{2} l_{\mathrm{V}}^2(x^d, y^d) = 0$$

This indicates that the socially optimal sharing rule is:

$$\sigma^* = \frac{l_{\mathrm{I}}^2(x^d, y^d)}{l_{\mathrm{I}}^2(x^d, y^d) + l_{\mathrm{V}}^2(x^d, y^d)} \in [0, 1]$$

In this example, without computing $x^d, y^d$, we can see that the socially optimal allocation of the residual loss is $\sigma^* = 0$ (the allocations of residual loss induced by traditional negligence rules), when $l_{\mathrm{I}}(x^d, y^d) = 0$ (the damage attributable to the injurer at the due level of care is equal to zero); it is $\sigma^* = 1$ (the allocation of residual loss induced by strict liability rules) when $l_{\mathrm{V}}(x^d, y^d) = 0$ (the damage attributable to the victim at the due level of care is equal to zero); it is $0 < \sigma^* < 1$, when both $l_{\mathrm{I}}(x^d, y^d) > 0$ and $l_{\mathrm{V}}(x^d, y^d) > 0$. In this case, loss-sharing is necessary to advance social welfare. In particular, the socially optimal loss-sharing rule requires an equal splitting of the loss between the two nonnegligent parties, $\sigma^* = \frac{1}{2}$, when $l_{\mathrm{I}}(x^d, y^d) = l_{\mathrm{V}}(x^d, y^d)$.

This result is consistent with Calabresi (1965), who put forth the idea of a non-fault liability system apportioning liability according to the riskiness of the activity at the optimal level of care, irrespective of legal notions of fault. Calabresi suggested that this criterion of (partial) non-fault liability could be implemented dividing the costs of an accident pro rata according to the cumulative effect of the nonnegligent activity on accident losses.[20]

# 4 Role-uncertainty and loss-sharing

The study of the incentive effects of liability rules carried out in the literature implicitly or explicitly assumes that parties adjust their care and activity levels as potential injurers and victims, and thus that they know with certainty whether they will play the role of injurers or victims in a potential accident. However, in many real life situations, parties face uncertainty as to whether they will be victims or injurers.[21] In traffic accidents, for instance, it is normally difficult for two motorists to know ex ante which of their vehicles involved in a collision will be (more seriously) damaged. By applying the results of the previous sections, it is possible to study the effect of role-uncertainty on the parties' incentives.[22]

Uncertainty with respect to the parties' roles—as injurers or victims—in the event of a future accident creates a *de facto* sharing of the expected residual loss. With respect to incentives, this ex ante sharing of the expected residual loss occasions effects that are analogous to those produced by the ex post loss-sharing considered in this paper. The findings of this paper thus illuminate the workings of liability rules in real-life cases where parties face role-uncertainty.

Role-uncertainty can be interpreted as a form of implicit loss-sharing as

[20] Calabresi, (1965, pp. 740-741).

[21] Furthermore, as pointed out by Coase (1960), causation is often reciprocal and ambiguous. It is often by means of conventional legal constructs that the ambiguities are resolved with the labeling of one party as victim and the other as tortfeasor. But it is not until after the harmful event has occurred that such ambiguity is resolved. Note also that the notions of victim and injurer are defined in Section 2 with reference to where the loss initially falls and not in relation to causation, which we do not examine in this paper. Role-uncertainty has also been studied by Feldman and Kim (2006) in a different setting from ours.

[22] For simplicity we assume that role-uncertainty does not affect due-care standards or else that they are the same for both parties.

follows: Given a legal sharing rule $\sigma$ and an ex ante probability $\pi$ that a given party be the injurer in the event of an accident, the expected sharing for that party is given by

$$\zeta = \pi\sigma + (1 - \pi)(1 - \sigma) \tag{5}$$

Conversely, the expected share of residual loss borne by the other party would be given by $1 - \zeta = (1 - \pi)\sigma + \pi(1 - \sigma)$. The expected sharing $\zeta$ coincides with the legally chosen sharing rule $\sigma$ only when no uncertainty exists, $\pi = 1$. At the limit, when role-uncertainty is maximal and the parties have equal probabilities of finding themselves as victims or injurers of an accident, the legal sharing rule $\sigma$ becomes irrelevant. It is in fact easy to show that if $\pi = \frac{1}{2}$ then also $\zeta = \frac{1}{2}$, irrespective of the value of $\sigma$: the legal allocation of residual liability between nonnegligent parties is thus irrelevant when parties face full role-uncertainty. Because of role-uncertainty, potential injurers will internalize some of the costs associated with their activities even under a negligence rule, as they may find themselves as uncompensated victims in the event of an accident.

This yields to the interesting insight that role-uncertainty may in fact distort or even completely hamper any policy aimed at controlling the parties' activity levels. When role-uncertainty is maximal, risk-neutral parties will behave as if the loss were to be equally shared, regardless of the chosen allocation of residual liability $\sigma$. In this case, negligence ($\sigma = 0$) and strict liability ($\sigma = 1$) produce identical incentives with respect to activity levels, undermining the most important rationale for choosing between one or the other liability regime. Under complete role-uncertainty the allocation of residual liability is thus irrelevant. The choice of a loss-sharing regime, although having no effect on the parties' expected liability, would however reduce the variance of actual outcomes—because the sharing is actual, rather than a mere expectation—and may lead to a more desirable allocation of residual liability when risk-averse parties are involved.

When role-uncertainty is present, but less than complete, our findings suggest that legal sharing $\sigma$ could be instrumentally adapted in order to achieve the desired expected sharing $\zeta$, although not perfectly. An example will better illustrate this point. Assume that an accident might occur between two parties, A and B, and that A is the injurer in $\frac{1}{3}$ of the cases. If the desired apportionment of liability for A is $\frac{2}{3}$, setting $\sigma = \frac{2}{3}$ will not reach this outcome; in fact, in this case the expected share of party A as calculated from (5) would be $\zeta = \frac{4}{9}$, far less than the goal. Instead, lowering the ex post sharing to $\sigma = 0$ (the simple negligence rule) would result in an increase of the expected sharing to the desired level, $\zeta = \frac{2}{3}$. The reason for this apparently contradictory outcome is that, since A is more often a victim than an injurer, his expected exposure to residual liability increases if his exposure as victim increases. From (5) we can derive a general rule for the setting of $\sigma$ in the case of role-uncertainty:

$$\sigma = \frac{\zeta + \pi - 1}{2\pi - 1}$$

From this formulation, we can note that, if $\zeta = \pi$, that is, if the desired expected sharing is equal to the role-uncertainty value, strict liability ($\sigma = 1$)

is optimal, while if the desired sharing is the opposite of the role-uncertainty, $\zeta = 1 - \pi$, then the negligence rule ($\sigma = 0$) is optimal. It is also important to note that not all distortions created by role-uncertainty can be corrected by an appropriate setting of $\sigma$. In general, the range of achievable ex ante sharing can be derived from (5) and is $\zeta \in [\pi, 1 - \pi]$. Role-uncertanty is largest when $\pi$ is close to $\frac{1}{2}$, which makes the constraint more binding as it restricts the set of sharing policies that can be implemented. In our example, no party can be made the residual bearer in expectation. In fact, $\zeta = 1 > 1 - \pi$. To make A the residual bearer, A should bear $\sigma = \frac{3}{2}$ of the loss when he is the injurer and $1 - \sigma = \frac{1}{2}$ when he is the victim. This implies a decoupling of liability that cannot be implemented by ordinary liability rules.[23]

A second important result of our analysis is that, as previously shown for loss-sharing $\sigma$, role-uncertainty $\zeta$ does not undermine the parties' care incentives. Far from being a problem, both loss-sharing and role-uncertainty may indeed be desirable, given the fact that the all-or-nothing allocations of the residual loss are often suboptimal.

# 5 Conclusion

It is a well-established result in the economic analysis of torts that negligence rules (and indeed, all existing liability rules) are incapable of incentivizing potential injurers and potential victims to exercise optimal activity levels. The residual bearer of liability (the party bearing the cost of the accident when neither party behaved negligently) will have incentives to take the socially optimal activity level; however, the non-bearer of residual liability will not be so constrained, and will tend to exercise excessive (i.e., socially suboptimal) activity levels.

Historically, courts have engaged two all-or-nothing options. In cases where both parties in an accident were nonnegligent, courts have either allocated all the residual liability to the injurer, or all the residual liability to the victim. In this paper, we explored the benefits of shared residual liability regimes—an appealing intermediate policy, which spreads partial activity-level incentives between the parties in a more fine-grained way.

In our analysis of shared residual liability, we first showed that such tort regimes would not interfere with the prior established results in the economic analysis of torts literature. We then characterized in general terms the optimal sharing of nonnegligent losses. Finally, we introduced the element of role-uncertainty as a possible explanation for the absence of shared residual liability in real-world adjudication.

This leads us to a somewhat comforting reconciliation of our results with real-life observations, in line with a positive efficiency hypothesis. The interesting intuition is that loss-sharing, although in principle desirable, may often

[23] In ordinary liability rules, damages are perfectly compensatory or, if punitive, are a trasfer from the victim to the injurer. Decoupling could be implemented by subsidies or fines. In a different framework, decoupling is studied in Polinsky and Che (1991).

be unnecessary—it might be for this reason that it is rarely observed in the adjudication of tort cases. Role-uncertainty *de facto* creates an expected loss-sharing, even absent a legal loss-sharing rule. Role-uncertainty might create incentives that are similar to those of a loss-sharing rule. Role-uncertainty, however, requires that injurers and victims be similarly situated, as in the case of automobile torts. When parties are very differently situated, as in the case of industrial polluters and the general population, role-uncertainty cannot effectively be an alternative for loss-sharing through tort law. Our results open the door to the important policy question on the desirability of loss-sharing solutions when parties' roles (as prospective victims or tortfearors) are known ex ante in the multifarious taxonomy of tort cases.

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# Appendix: Additive loss function

Here we consider a special case of the general model developed in the text, where the parties' behavior can be easily computed. The parties' utility functions are specified as in Shavell (1987). Let:

$$\begin{aligned} U &= u(s) - sx \\ V &= v(t) - ty \end{aligned}$$

with $u_s > 0$, and $u_{ss}$ an increasing and nonpositive function satisfying $u_{ss} < 0$; likewise, $v_t > 0$, and $v_{tt}$ an increasing and nonpositive function satisfying $v_{tt} < 0$. Note that the functions $u_s(s)$ and $v_t(t)$ are continuous and strictly decreasing functions and their inverses $g = (u_s)^{-1}$ and $h = (v_t)^{-1}$ exist and are also continuous and strictly decreasing functions. Here let us use an additive loss function:

$$L = sl_{\mathrm{I}}(x) + tl_{\mathrm{V}}(y)$$

where $l_{\mathrm{I}}$ and $l_{\mathrm{V}}$ are positive and decreasing functions of $x$ and $y$. It is easy to show that the activity level chosen by the injurer, is now a function which depends only on $x$ and $\sigma$ and likewise the activity level chosen by the victim is a function of $y$ and $\sigma$ only. Since $\sigma$ is taken as given, we can write:

$$\begin{aligned} s^*(x) &= \arg\max_s \left[u(s) - sx - \sigma(sl_{\mathrm{I}}(x) + tl_{\mathrm{V}}(y))\right] \\ t^*(y) &= \arg\max_t \left[v(t) - ty - (1-\sigma)(sl_{\mathrm{I}}(x) + tl_{\mathrm{V}}(y))\right] \end{aligned}$$

It is easy to see that this model is compatible with Assumptions 1-4. First we consider the case where $\sigma$ is given and then the case where $(x^d, y^d)$ is given. We do so because this allows us to consider both second- and third-best levels of sharing and due care in the same model. We will show compliance and the existence of an internal value for the sharing rule. This shows that our results are not dependent on the policymaker setting both sharing and due care at the optimal levels.

## Optimal due-care standards for a given sharing rule

Let $(x^d, y^d)$ be the socially-optimal standards of due care when $\sigma$ is given but not necessarily optimally set. We assume that $s(x)$ is a decreasing function for $x > x^d$ and $t(y)$ is a decreasing function for $y > y^d$. This simply means that pushing the due-care standards to (inefficiently) high levels increases the parties' costs per unit of activity and hence reduces their activity levels, which is clearly plausible. Due to the additive form of the loss function, with this unique assumption we can verify directly that compliance with the standards of due care results in a Nash equilibrium. If the injurer chooses $\overline{x} > x^d$ his payoff is:

$$\max_s \left[u(s) - sx - s\sigma l_{\mathrm{I}}(x) - \sigma t^* l_{\mathrm{V}}(y^d)\right] = u(s(\overline{x}) - s(\overline{x})\overline{x} - \sigma s(\overline{x}) l_{\mathrm{I}}(\overline{x}) - \sigma t^* l_{\mathrm{V}}(y^d)$$

The injurer chooses $\overline{x} > x^d$ if:

$$\begin{aligned}
& u(s(\overline{x})) - s(\overline{x})\overline{x} - \sigma s(\overline{x})l_{\mathrm{I}}(\overline{x}) - \sigma t^* l_{\mathrm{V}}(y^d) \\
\geq \;& u(s^*(x^d)) - s^*(x^d)x^d - \sigma s^*(x^d)l_{\mathrm{I}}(x^d) - \sigma t^* l_{\mathrm{V}}(y^d) \\
\geq \;& u(s^*(x^d)) - s^*(x^d)x^d - s^*(x^d)l_{\mathrm{I}}(x^d) - t^* l_{\mathrm{V}}(y^d) \qquad (6)
\end{aligned}$$

Furthermore, $\overline{x} > x^d$ implies $s(\overline{x})l_{\mathrm{I}}(\overline{x}) < s(x^d)l_{\mathrm{I}}(x^d)$ since $s(x)l_{\mathrm{I}}(x)$ is a decreasing function for $x > x^d$ and hence we have:

$$-(1-\sigma)s(\overline{x})l_{\mathrm{I}}(\overline{x}) > -(1-\sigma)s(x^d)l_{\mathrm{I}}(x^d) \qquad (7)$$

By computing (6)+(7) we obtain:

$$\begin{aligned}
& u(s(\overline{x})) - s(\overline{x})\overline{x} - s(\overline{x})l_{\mathrm{I}}(\overline{x}) + v(t^*) - t^* l_{\mathrm{V}}(y^d) \\
> \;& u(s^*(x^d)) - s^*(x^d)x^d - s^*(x^d)l_{\mathrm{I}}(x^d) + v(t^*) - t^* l_{\mathrm{V}}(y^d)
\end{aligned}$$

However, the latter inequality cannot be true because, by definition, $x^d$ maximizes social welfare. Hence, the injurer does not have incentives to deviate upwards from the due-level of care. Likewise, if the injurer chooses $\overline{x} < x^d$, his payoff is:

$$\max_s \left[u(s) - sx - sl_{\mathrm{I}}(x) - t^* l_{\mathrm{V}}(y^d)\right] = u(s(\overline{x}) - s(\overline{x})\overline{x} - s(\overline{x})l_{\mathrm{I}}(\overline{x}) - t^* l_{\mathrm{V}}(y^d)$$

and he chooses $\overline{x} < x^d$ if:

$$\begin{aligned}
& u(s(\overline{x}) - s(\overline{x})\overline{x} - s(\overline{x})l_{\mathrm{I}}(\overline{x}) - t^* l_{\mathrm{V}}(y^d) \\
> \;& u(s(x^d) - s(x^d)x^d - \sigma s(x^d)l_{\mathrm{I}}(x^d) - \sigma t^* l_{\mathrm{V}}(y^d) \\
> \;& u(s(x^d) - s(x^d,)x^d - s(x^d)l_{\mathrm{I}}(x^d) - t^* l_{\mathrm{V}}(y^d)
\end{aligned}$$

However, this inequality cannot be true because, by definition, $x^d$ maximizes social welfare. The same results hold for the victim. Thus, we can conclude that both parties will comply with the optimally set due-care standards, irrespective of whether the sharing rule is optimally set or not.

## Optimal sharing rule for given due-care standards

Let $(x^d, y^d)$ be the given standards of due care, which are not necessarily set at the socially optimal levels. In order to identify the socially optimal loss-sharing rule $\sigma$ for nonnegligent parties, we must find again the maximum of the social welfare function. As in the main analysis, let us proceed by backward induction and first analyze the parties' choices of activity levels, $s$ and $t$, given the due-care standards, $x^d$ and $y^d$, and the loss-sharing rule, $\sigma$. The relevant maximisation problems for the injurer and the victim are, respectively:

$$\begin{aligned}
&\max_s \left[u(s) - \sigma(sl_{\mathrm{I}}(x) + tl_{\mathrm{V}}(y)) - sx\right] \\
&\max_t \left[v(t) - (1-\sigma)(sl_{\mathrm{I}}(x) + tl_{\mathrm{V}}(y)) - ty\right]
\end{aligned}$$

Let us calculate the first-order conditions for the first strictly concave function with respect to $s$ and for the second with respect to $t$:

$$u_s(s) - \sigma l_{\mathrm{I}}(x) - x = 0 \tag{8}$$
$$v_t(t) - (1-\sigma) l_{\mathrm{V}}(y)) - y = 0 \tag{9}$$

Note that $g = (u_s)^{-1}$ and $h = (v_t)^{-1}$ exist and are continuous, strictly decreasing functions so that, for given due-care standards $(x^d, y^d)$, we have:

$$s^*(\sigma) = g(\sigma l_{\mathrm{I}}(x^d) + x^d)$$
$$t^*(\sigma) = h((1-\sigma)\, l_{\mathrm{V}}(y^d) + y^d))$$

are the parties' chosen levels of activity, so that we can evaluate social welfare at the equilibrium levels of activity. For each $(x^d, y^d)$, we need to show the existence and the uniqueness of $\sigma$, which solves the following maximization problem:

$$\max_{\sigma\in[0,1]} W(\sigma) = \sup_{\sigma\in[0,1]} \{u(s^*(\sigma)) + v(t^*(\sigma)) - s^*(\sigma) l_{\mathrm{I}}(x^d) - t^*(\sigma) l_{\mathrm{V}}(y^d)$$
$$-s^*(\sigma)x^d - t^*(\sigma)y^d\}$$

The existence of $\sigma$ is evident by noting that $W$ is a continuous function on $[0,1]$. If the optimal sharing rule $\sigma$ is an internal critical point, the first-order condition is:

$$\frac{ds^*(\sigma)}{d\sigma}(u_s(s^*(\sigma) - l_{\mathrm{I}}(x^d) - x^d) + \frac{dt^*(\sigma)}{d\sigma}(v_t(t^*(\sigma)) - l_{\mathrm{V}}(y^d) - y^d) = 0$$

Now we can use (8) and (9). Differentiating with respect to $\sigma$ gives:

$$\frac{ds^*(\sigma)}{d\sigma} = \frac{l_{\mathrm{I}}(x^d)}{u_{ss}(s^*(\sigma))}$$
$$\frac{dt^*(\sigma)}{d\sigma} = -\frac{l_{\mathrm{V}}(y^d)}{v_{tt}(t^*(\sigma))}$$

Substituting gives:

$$\frac{ds^*(\sigma)}{d\sigma}(\sigma l_{\mathrm{I}}(x^d) - l_{\mathrm{I}}(x^d)) + \frac{dt^*(\sigma)}{d\sigma}((1-\sigma) l_{\mathrm{V}}(y^d) - l_{\mathrm{V}}(y^d)) = 0$$

which can be rewritten as

$$u_{ss}(s^*(\sigma)) l_{\mathrm{V}}^2(y^d)\sigma - v_{tt}(t^*(\sigma)) l_{\mathrm{I}}^2(x^d)(1-\sigma) = 0$$

One can remark that

$$| \, u_{ss}(s^*(\sigma)) l_{\mathrm{V}}^2(y^d)\sigma - v_{tt}(t^*(\sigma)) l_{\mathrm{I}}^2(x^d)(1-\sigma) \, |_{\sigma=0} = -v_{tt}(t^*(0)) l_{\mathrm{I}}^2(x^d) > 0$$
$$| \, u_{ss}(s^*(\sigma)) l_{\mathrm{V}}^2(y^d)\sigma - v_{tt}(t^*(\sigma)) l_{\mathrm{I}}^2(x^d)(1-\sigma) \, |_{\sigma=1} = u_{ss}(s^*(1)) l_{\mathrm{V}}^2(y^d) < 0$$

Given that $s^*(\sigma) = g(\sigma l_{\mathrm{I}}(x^d)+x^d)$ is a decreasing function of $\sigma$ and given the assumptions on $u$, we have that $u_{ss}(s^*(\sigma))$ is a decreasing function of $\sigma$. Likewise, $t^*(\sigma)$ is an increasing function of $\sigma$ and, hence, $v_{tt}(t^*(\sigma)$ is an increasing function of $\sigma$. Then $u_{ss}(s^*(\sigma))l_{\mathrm{V}}^2(y^d)\sigma - v_{tt}(t^*(\sigma))l_{\mathrm{I}}^2(x^d)(1-\sigma)$ is a continuous and monotonic function of $\sigma$ and the equation has a unique solution $\sigma^*$on $[0,1]$.

Since the internal critical point is unique, it is easy to note that corner solutions are ruled out when the maximum of the social welfare function is not attained for $\sigma = 0$ and $\sigma = 1$. Let

$$\begin{aligned}
A_\sigma(x^d, y^d) &= u(g(\sigma l_{\mathrm{I}}(x^d) + x^d)) + v(h((1-\sigma)\, l_{\mathrm{V}}(y^d) + y^d))) \\
&\quad - g(\sigma l_{\mathrm{I}}(x^d) + x^d)(l_{\mathrm{I}}(x^d) - x^d) \\
&\quad - h((1-\sigma)\, l_{\mathrm{V}}(y^d) + y^d))(l_{\mathrm{V}}(y^d) - y^d) \\
B(x^d, y^d) &= u(g(x^d)) + v(h(l_{\mathrm{V}}(y^d) + y^d))) - g(x^d)(l_{\mathrm{I}}(x^d) - x^d) \\
&\quad - h(l_{\mathrm{V}}(y^d) + y^d))(l_{\mathrm{V}}(y^d) - y^d) \\
C(x^d, y^d) &= u(g(l_{\mathrm{I}}(x^d) + x^d)) + v(h(y^d))) \\
&\quad - g(l_{\mathrm{I}}(x^d) + x^d)(l_{\mathrm{I}}(x^d) - x^d) - h(y^d))(l_{\mathrm{V}}(y^d) - y^d)
\end{aligned}$$

Therefore, an interior level of $\sigma$ is optimal if $(x^d, y^d)$ satisfy the inequality

$$A_\sigma(x^d, y^d) > \max\left\{B(x^d, y^d); C(x^d, y^d)\right\} \tag{10}$$

The following proposition summarises these results on the uniqueness of the optimal loss-sharing rule $\sigma^*$ and gives a condition on $(x^d, y^d)$ to rule out corner solutions.

**Proposition A (additive loss function).** In the additive version of our model, sharing the loss between nonnegligent parties maximizes social welfare if and only if $(x^d, y^d)$ satisfy (10). Then, for each $(x^d, y^d)$, $0 < \sigma^* < 1$ is the unique optimal loss-sharing rule maximising social welfare.

**Proof.** See text above. ■

Now we need to verify whether compliance with the standards of due care results in a Nash equilibrium. Here is a sufficient condition.

**Proposition B (additive loss function).** Let $x + l_{\mathrm{I}}(x)$ and $y + l_{\mathrm{V}}(y)$ be decreasing functions of $x$ and $y$, respectively. If $(x^d, y^d)$ is chosen such that $x + \sigma(x^d, y^d)l_{\mathrm{I}}(x)$ and $y + (1 - \sigma(x^d, y^d))l_{\mathrm{V}}(y)$ are increasing functions of $x$ and $y$, respectively, then both parties comply with the negligence standards in equilibrium.

**Proof.** If the injurer chooses $x > x^d$ his payoff is:

$$\max_s \left[u(s) - s\left[\sigma(x^d, y^d)l_{\mathrm{I}}(x) + x\right] - \sigma(x^d, y^d)t^* l_{\mathrm{V}}(y^d)\right]$$

but $\sigma(x^d, y^d)l_{\mathrm{I}}(x)+x$ is an increasing function of $x$ and $\max_s \left[u(s) - s\left[\sigma(x^d, y^d)l_{\mathrm{I}}(x) + x\right]\right]$ is a decreasing function of $x$, so that:

$$\sigma(x^d, y^d)l_{\mathrm{I}}(x^d) + x^d \leq \sigma(x^d, y^d)l_{\mathrm{I}}(x) + x$$

and:

$$\begin{aligned} & \max_s \left[u(s) - s\left[\sigma(x^d, y^d)l_{\mathrm{I}}(x) + x\right] - \sigma(x^d, y^d)t^* l_{\mathrm{V}}(y^d)\right] \\ \leq \quad & \max_s \left[u(s) - s\left[\sigma(x^d, y^d)l_{\mathrm{I}}(x^d) + x^d\right] - \sigma(x^d, y^d)t^* l_{\mathrm{V}}(y^d)\right] \end{aligned}$$

which shows that the injurer will not choose $x > x^d$.

If the injurer chooses $x < x^d$ his payoff is:

$$\max_{s\geq 0} \left[u(s) - s\left[l_{\mathrm{I}}(x) + x\right] - t^* l_{\mathrm{V}}(y^d)\right]$$

Since $l_{\mathrm{I}}(x) + x$ is a decreasing function of $x$, we have:

$$l_{\mathrm{I}}(x^d) + x^d < l_{\mathrm{I}}(x) + x$$

and since $\max_{s\geq 0}\left[u(s) - s\left[l_{\mathrm{I}}(x) + x\right]\right]$ is now an increasing function of $x$, we have:

$$\begin{aligned} & \max_s \left[u(s) - s\left[l_{\mathrm{I}}(x) + x\right] - t^* l_{\mathrm{V}}(y^d)\right] \\ \leq \quad & \max_s \left[u(s) - s\left[l_{\mathrm{I}}(x^d) + x^d\right] - t^* l_{\mathrm{V}}(y^d)\right] \\ \leq \quad & \max_s \left[u(s) - s\left[\sigma(x^d, y^d)l_{\mathrm{I}}(x^d) + x\right] - \sigma(x^d, y^d)t^* l_{\mathrm{V}}(y^d)\right] \end{aligned}$$

which shows that the injurer will not choose $x < x^d$. The same results hold for the victim. Therefore, both parties will comply with the due-care standards. ■