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A Schumpeterian model of growth and inequality

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Abstract

This paper contributes to the analysis of the effects of demand structure on long-term growth. Introducing non-homothetic preferences in an otherwise standard quality-model, we first show that disparities in purchasing power generate positive R&D investment by quality leaders. This result is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers as well as without any concavity in the R&D cost function: in our framework, the incentive for a leader to invest in R&D stems from the possibility for an incumbent having innovated twice in a row to efficiently discriminate between rich and poor consumers displaying differences in their willingness to pay for quality. We hence exemplify a so far overlooked demand-driven rationale for innovation by incumbents. We then move to analyzing the impact of inequalities on long-term growth in our quality-ladder framework, and find that a lower level of wealth disparities always leads to an increase in the long-run growth rate. Finally, we show that beyond this negative impact on growth, inequalities also influence the allocation of the overall R&D effort between incumbents and challengers: a higher level of inequalities will in most cases lead to a bigger share of the overall R&D investment to be carried out by quality leaders.

Keywords: Growth, Innovation, Income inequalities.

JEL classification: O3, O4, F4.

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1 Introduction

Taking into account the impact of income distribution on the demand structure is a growing concern for the analysis of several macroeconomic phenomenons. Structural change models have recently shown how modifications in aggregate demand as households become richer can cause long-term alterations in the production and employment structure (Matsuyama, 2002; Foellmi and Zweimuller, 2008). In an international trade framework, income distribution within a country has lately been identified as a potential determinant of the quality of its exports, beyond supply considerations such as scale economies or factor endowments (Fajgelbaum et al., 2011). Finally, from a long-term growth perspective, the distribution of purchasing power across households and its effects on the profitability of a new product have been demonstrated to influence the intensity of innovation activities (Foellmi and Zweimuller, 2006).

The present paper contributes to this last strand of literature by showing that in the case of vertical innovation, the distribution of income within an economy decisively affects the incentives to invest in R&D activities of both challenger firms *and* incumbent patentholders. More precisely, while existing quality-ladder models featuring income inequality (Zweimuller and Brunner, 2005; Li, 2003) preclude any R&D investment by the current quality leader,[1] we claim that the existence of industry leaders investing in R&D to improve their own products can actually be *explained* by existing disparities in purchasing power among households. Indeed, leaders having at their disposal several quality-differentiated versions of their core product (obtained through successive innovations) are observed to simultaneously commercialize different price/quality bundles, aimed at different groups of consumers differing in income.[2] This is a well-explored microeconomic result (Mussa and Rosen, 1978): for a monopolist, serving costumers who do not care much for quality creates negative externalities, since it hinders the captation of costumer surplus from those who have a stronger taste for quality. In a microeconomic static set-up, the monopolist (who is assumed to have access to a whole product line) then internalizes the negative externalities by inducing less enthusiastic consumers to buy lower quality items charged at a lower price, opening the possibility of charging higher prices to more adamant buyers of high quality units. In our dynamic framework, the monopolist only has access to as many qualities as times he has innovated: internalization of the negative externality then leads to innovation by incumbent.

We hence extend and deepen the existing analysis of the impact of demand structure on innovation-based growth and provide a new, demand-based rationale for continued in-

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[1]Indeed, in standard quality-ladder models (Segerstrom et al., 1990; Grossman and Helpman, 1991b; Aghion and Howitt, 1992), quality leaders do not participate to the next innovation race: this absence of R&D investment by incumbents, known as the Arrow (1962) effect, is justified by the fact that innovating quality leaders would only cannibalize their own business.

[2]Intel currently sells three different families of microprocessors (Core, Pentium and Celeron), displaying different levels of speed and performance; Microsoft commercializes simultaneously Windows XP, Vista and 7; Nokia and Samsung sell numerous quality-differentiated mobile phones, displaying significant variations in prices and offered functionalities.

vestment in R&D by quality leaders.

We feature a traditional quality-ladder framework except for the use of non-homothetic preferences, hence allowing for more than one quality to be consumed at the equilibrium in the presence of differences in wealth endowment. This property is obtained by imposing unit consumption of the quality good, the rest of a consumer's income being spent on a standardized (composite) good: a given consumer will then buy the quality that, *given its price*, offers them the highest utility. The presence of heterogenous consumers in terms of income, combined with strategic pricing of firms producing different qualities, might then result in a situation of natural oligopoly (Shaked and Sutton, 1982) with more than one quality being produced and sold at the equilibrium. By contrast, in the standard quality-ladder models (Grossman and Helpman, 1991a; Segerstrom et al., 1990), the quality good is divisible and the preferences of the consumers are hence homothetic. Monopoly pricing by the current quality leader then ensures that only the highest price-adjusted quality is consumed at the equilibrium, even when differences in wealth endowments are allowed for: the poorest consumers only consume a lower share of the top quality good.

Using such a utility specification, we then model a two-class society in which the level of a consumer's income determines his willingness to pay for the highest quality. In such a framework, a *challenger* winning the latest innovation race and being the producer of the highest quality needs to decide between two alternatives: capturing the whole market by charging a price sufficiently low to appeal to the poorest households, or selling its product at a higher price only to the wealthiest consumers, at the cost of abandoning the rest of the market to its direct competitor (i.e. the previous quality leader). The profits of such an innovator are hence affected by income distribution through two effects, already identified by Foellmi and Zweimuller (2006) in an horizontal innovation setting: a price effect and a market size effect. Opting for a price that is acceptable for all consumers entails a positive market size effect but a negative price effect on the innovator's profits, since it prevents the latter from reaping maximum surplus from the higher willingness to pay of wealthier consumers. Charging the highest price acceptable for rich consumers has a positive price effect but a negative market size effect on profits of the current leader, since it means that the poor consumers will keep buying the second-best quality from the previous leader. For a successful challenger, the price and the market size effect can hence only work in opposite directions. On the other hand, an *incumbent* winning an innovation race is able to reconcile both effects, since he then has two successive qualities at its disposal: he can thus efficiently discriminate between rich and poor consumers by offering two distinct price/quality bundles, capturing the whole market *and* reaping the maximum surplus from the wealthy consumers at the same time.

In such a framework, we model R&D races with constant returns, and show that without any advantage of any kind in the R&D field, the incumbent still invests a strictly positive amount in R&D. Such a behavior directly stems from the existing increment between the profits realized when being a successful challenger and a successful incumbent. Hence,

while so far the incentives for innovation by quality leaders have essentially been modeled as stemming from the structure of the R&D process, this paper provides a *demand-driven* incentive for investment in R&D by incumbents. Also, by acknowledging the existence and assessing the importance of incumbent investment in R&D when facing purchasing power disparities, we are able to contribute further to the analysis of the effects of inequalities on innovation. We first show that in a majority of cases a lower level of inequalities is beneficial for long-term growth, demonstrating a negative relationship between wealth disparities and growth in a quality-ladder framework. We then also bring to light an impact of inequalities so far overlooked, showing that the distribution of income will also influence the allocation of the overall R&D effort between incumbents and challengers: most of the time, a higher level of inequalities will lead to a bigger share of the overall R&D investment to be carried out by quality leaders.

Relation to literature.

Our paper is part of the growing body of literature studying the occurrence and the impact on long-term growth of innovation by incumbents. Segerstrom and Zolnierek (1999) as well as Segerstrom (2007) have obtained positive investment in R&D by the incumbent by assuming that the expertise granted by quality leadership confers R&D cost advantages. Combined with diminishing returns to R&D efforts at the firm level, this assumption ensures that both incumbents and challengers participate to R&D races. Etro (2004, 2008) models sequential patent races with concave R&D costs where the incumbent, acting as a Stackelberg leader, is given the opportunity to make a strategic precommitment to a given level of R&D investment: the quality leader then has an incentive to invest in R&D in order to deter outsiders' entry. Acemoglu and Cao (2010) provide a model where incumbents and challengers participate to two different kinds of R&D races, differing in terms of costs and rewards: leaders invest in R&D to improve their products (incremental innovation), while challengers participate to R&D races in the hope of leapfrogging the existing incumbent (radical innovation). All those models have hence explored various possible incentives for innovation by incumbent stemming from the structure of the R&D process, i.e. from the *supply side*, while our paper on the other hand provides a *demand-based* rationale for leader R&D, stemming from the negative externalities generated by wealth inequalities on an innovator's profits.

Aghion et al. (2001) analyze the influence of the product market structure on innovation intensity, developing a framework in which goods of different quality are imperfect substitutes and can therefore coexist in the market. They show that the perspective to lessen the competition pressure (and broaden the market share) provides the incentive for the incumbent to carry out positive R&D investments in order to improve its own product, while the challenger invests in order to leapfrog the current leader. They however preclude free entry, by exogenously imposing that only the incumbent monopolist and a single outsider invest in R&D, while our paper on the other hand endogenizes both investment by

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incumbents and challenger entry.

Finally, all those papers assume concavity of the R&D cost function in order to ensure positive investment by both the incumbent and the challengers.[3] Our paper on the other hand provides a rationale generating positive investment by both R&D leaders and challengers even under the assumption of constant returns to R&D effort at the firm level.[4]

This paper is also closely related to the literature examining the relationship between long-term growth and inequalities operating through the demand side. Foellmi and Zweimuller (2006) model a similar two-class society, and demonstrate that a lower level of inequalities is systematically detrimental to long-term growth. They however obtain this result in an horizontal innovation framework, where the rewards for innovation are from a different nature than in Schumpeterian models. Zweimuller and Brunner (2005) on the other hand have studied the impact of disparities in purchasing power of households in a quality-ladder framework, showing that a reduction in the level of inequalities within the economy (whether it be through a decrease in the share of the population being poor or a redistribution from the rich to the poor) is beneficial for innovation intensity and hence for growth. While we rely on their modeling strategy and obtain results similar to theirs concerning the challenger innovation rate, their model however does not feature innovation by incumbent, hence only capturing part of the effects of the level of inequalities on the innovation rate.

The rest of the paper is organized as follows. Sections 2 to 4 present our model, while section 5 studies the steady state equilibrium. Section 6 then analyzes the effects of the extent of inequalities on the innovation intensity . Section 7 concludes.

2   Consumers

There is a fixed number L of consumers that live infinitely and supply one unit of labor each period, paid at a constant wage w. While all consumers have the same wage income, they are assumed to differ with respect to asset ownership $\omega_i(t)$: along Zweimuller and Brunner (2005), we assume a two-class society with rich (R) and poor (P) consumers, being distinguished by their wealth (respectively ω_R and ω_P).

The share of poor consumers within the population is denoted by β. The extent of inequalities within the economy is determined by this share, as well as by the repartition between rich and poor of the aggregate wealth Ω. $d \in (0, 1)$ is defined as the ratio of the value of assets owned by a poor consumer *relative* to the average per-capita wealth: $d = \frac{\omega_P}{\Omega/L}$. Given d, the wealth position of the rich can be computed as $d_R = \frac{1-\beta d}{1-\beta}$. We

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[3]A notable exception is Cozzi (2007), who shows that in the case of constant returns at the firm level, the incumbent is indifferent to its own R&D investment in the standard Schumpeterian environment. We however are able to clearly differentiate between incumbent and challenger R&D investment amounts, since the two kinds of actors do not face the same incentives in our model.

[4]As it has been demonstrated in a previous version of our paper (Latzer, 2010), our results are robust to the use of a concave R&D cost function.

hence have $\omega_P = d\frac{\Omega}{L}$ and $\omega_R = \frac{1-\beta d}{1-\beta}\frac{\Omega}{L}$.

Current income $y_i(t)$ of an individual belonging to the group i ($i = P, R$) is then of the form:

$$y_i(t) = w + r\omega_i(t) \tag{1}$$

with r being the interest rate.

Current income is then spent for the consumption of a single unit of a quality good with price $p_i(t)$ (depending on the quality $q_i(t)$ chosen by the consumer at time t), and of $c_i(t)$ units of a standardized good with price 1. Preferences are non-homothetic, with the instantaneous utility of a consumer of type i being described by the following utility function:

$$u_i(t) = \ln c_i(t) + \ln q_i(t) = \ln(y_i(t) - p_i(t)) + \ln q_i(t) \tag{2}$$

As shown by Zweimuller and Brunner (2005), at time τ the intertemporal decision problem of the consumer is then of the form:

$$\max_{c_i(t), q_i(t)} \int_\tau^\infty (lnc_i(t) + lnq_i(t))e^{-\rho(t-\tau)} \quad s.t. \quad \omega_i(\tau) + \int_\tau^\infty we^{-r(t)(t-\tau)}dt \geq$$
$$\int_\tau^\infty c_i(t)e^{-r(t)(t-\tau)}dt + \int_\tau^\infty p_i(t, q_i(t))e^{-r(t)(t-\tau)}dt$$

with ρ being the rate of time preference. Given an expected time path for both the interest rate $r(t)$ and the relation between quality and price $p_i(t, q_i(t))$, it is then possible to determine the optimal time path of $c_i(t)$, the consumption of the standardized good, and of $q_i(t)$, the chosen quality of the unit consumption good.

For any given time path of expenditures for the quality good $p_i(t, q_i(t))$ that does not exhaust life-time resources, the optimal path of consumption expenditures on the standardized good has to fulfill the standard first order condition of the maximization problem:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho \tag{3}$$

The optimal time path of $q_i(t)$, on the other hand, cannot be characterized by a differential equation, since the quality choice is discrete. We notice however that the choice in $q_i(t)$ is made simultaneously along with the decision on $p_i(t)$ by profit-maximizing firms. We hence set aside the choice of quality on the part of consumers until having defined the market and price structure for the quality good.

3   Market structure and pricing

There is a linear technology for the production of the standardized good, with labor as the unit input. We use the price of this standardized good as the numeraire, and since the market is assumed to be competitive, unit labor input is $1/w$.

The market for the quality good is non-competitive. At any date t, we assume that a continuum of qualities $q_j(t)$, $j = 0, -1, -2, ...$ exist and can be produced, with $q_0(t)$ being the best quality, $q_{-1}(t)$ the second-best, etc. Labor is the only input, with constant unit labor requirement $a < 1$.[5] Two successive quality levels differ by a fixed factor $k > 1$: $q_j(t) = k.q_{j-1}(t)$.

We will now define more precisely the structure of the quality good market. The quality good being characterized by unit consumption and fixed quality increments, firms use prices as strategic variables. We assume they know the shares of groups P and R in the population, the respective incomes y_R and y_P as well as the preferences of the consumers, but that they *cannot distinguish individuals by income*. In order to describe the strategic decisions operated by firms, we now define $p^T_{i,\{j,j-m\}}$, the "threshold" price for which a consumer belonging to group i is indifferent between quality j and quality $j - m$, i.e. $\ln(y_i - p^T_{i,\{j,j-m\}}) + \ln q_j = \ln(y_i - p_{j-m}) + \ln q_{j-m}$. Considering the fact that $q_j = k^{j-m} q_m$, solving for $p^T_{i,\{j,j-m\}}$ in the above equality yields:

$$p^T_{i,\{j,j-m\}} = y_i \left(\frac{k^m - 1}{k^m} \right) + \frac{p_{j-m}}{k^m} \tag{4}$$

The price $p^T_{i,\{j,j-m\}}$ is the maximum price that the firm selling the quality j can charge to a consumer belonging to the group i in order to have a positive market share, when facing the firm selling quality $j - m$. As one can see, this threshold price positively depends on the income y_i of consumer i, as well as on the price charged by the competitor p_{j-m}. Having defined this threshold price, and along with Zweimuller and Brunner (2005), we can state:

Lemma 1: *If $p_j \geq wa$ holds for the price of some quality q_j, $j = -1, -2, ...$, then for the producer of any higher quality q_{j+m}, $1 \leq m \leq -j$, there exists a price $p_{j+m} > wa$, such that any consumer prefers quality q_{j+m} to q_j.*

Proof: For a given group of consumers i, $p^T_{i,\{j+m,j\}} = y_i \left(\frac{k^m - 1}{k^m} \right) + \frac{p_j}{k^m}$ is a weighted average of y_i and p_j. Given the fact that only prices being below their income are taken into account by consumers i, we have that $p_j < y_i$, and we can hence conclude that $p^T_{j+m,j} > p_j$. Hence, it is always possible for the producer of the quality $j + m$ to set a price $p_{j+m} > p_j \geq wa$ such that $p_{j+m} \leq p^T_{i,\{j+m,j\}}$, i.e. such that quality q_{j+m} is preferred to quality q_j by the consumers of group i. This ends the proof. $\square$

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[5]Given the fact we have unit consumption of the quality good, a necessarily has to be inferior to 1.

Hence, if we take for granted that a producer never sells its quality at a price below the unit production cost wa, it is always possible for the producer of a higher quality to drive him out of the market, while still making strictly positive profits. Along this result, any firm entering the market with a new highest quality q_0 has to consider the following trade-off concerning the pricing of its product: setting the highest possible price for any given group of clients, vs. lowering its price in order to capture a further group of consumers.

It is then possible to show that in an economy characterized by two distinct groups of consumers (R and P), the equilibrium has the following properties:

Lemma 2: *At equilibrium,*
(1) The highest quality is produced,
(2) At most the two highest qualities q_0 and q_{-1} are actually produced,
(3) The equilibrium price p_{-1} fulfills $wa \leq p_{-1} \leq p^T_{P,\{-1,-2\}}$, with $p^T_{P,\{-1,-2\}}$ denoting the maximum price the producer of the q_{-1} quality can set in order to deter the producer of the q_{-2} quality from entry.

The proof is made in Zweimuller and Brunner (2005). The intuition is that since there are only two distinct groups of consumers, at most two distinct qualities can be sold, and at least one is always consumed, since it is assumed every individual buys one unit of the quality good. By Lemma 1, higher qualities drive out lower ones, hence the two qualities being still possibly active are q_0 and q_{-1}. At equilibrium, no firm can make a loss, hence the price p_j being charged for any quality q_j active on the market is necessarily superior or equal to the production cost wa. Finally, $p_{-1} \leq p^T_{P,\{-1,-2\}}$ follows from the fact that otherwise the producer of quality q_{-2} could enter the market.

As it can be seen from lemma 2, two different situations are possible for the equilibrium market structure and associated prices: either only the top quality good q_0 is sold to both groups of consumers (groups P and R), *or* the top quality good is sold only to the rich consumers (group R) while the second best quality good is sold to the poor consumers (group P). Lemma 1 shows that the decision regarding the market structure belongs to the producer having at its disposal the highest quality q_0, considering that he is always able to set a price that will drive its competitors out.

In Zweimuller and Brunner (2005) as well as in other quality ladder models studying the effects of inequality on growth through the product market (Zweimuller, 2000; Li, 2003), when two qualities are simultaneously sold on the market they are systematically designed and produced by two distinct firms. Indeed, in those models the incumbent does not engage in the next R&D race: when a new innovation occurs, the successful challenger becomes the quality leader, the previous quality leader becomes the producer of the second-best quality (whether he is still active or not depends on the pricing decision taken by the new quality leader), while the producer of the previous second-best quality is anyway driven out

of the market. These papers hence bear a close relationship with the static models of price competition in oligopoly markets with unit consumption of the quality good (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982, 1983). In this case, whether or not the second-best quality is being sold is solely determined by the distribution of income in the considered economy, since the latter determines the optimal pricing chosen by the quality leader. In other words, the equilibrium market structure in those models is *deterministic*, being set once and for all depending on the values assigned to the parameters β and d: it is not altered by the successive quality jumps.

In our model however, another possibility has to be taken into account: **a unique producer can have at its disposal the two highest qualities, since we leave the opportunity for the incumbent to still engage in R&D races**. If the incumbent wins the next innovation race, he then faces the monopoly pricing problem of a firm having at its disposal a spectrum of quality-differentiated goods (Mussa and Rosen, 1978).

Hence, in our model, the equilibrium market structure is stochastic, since it *does* depend on the outcome of the successive innovation races. In other words, the equilibrium market structure is a random process that we denote by $M(t)$. We define the state space of this stochastic process as $\{(SC), (SI)\}$, with the possible states (SC) and (SI) being characterized in the following way:[6]

- **"Successful Challenger" (SC) state**: a challenger is the winner of the last R&D race, i.e. the new quality leader is *different* from the former quality leader. The new quality leader then only has the highest quality at its disposal. One or two qualities can then be sold on the market, depending on the pricing strategy chosen by the new quality leader (which will itself depend on the wealth distribution in the economy). The market structure in this state can then either be a *monopoly* (only quality q_0 is sold), with the new quality leader charging a price that enables him to capture the whole market, or a *duopoly* (both qualities q_0 and q_{-1} are sold), with the new quality leader charging a higher price and serving only the upper part of the market, leaving the lower part to the second-best quality producer. One must however keep in mind that though either a monopoly or a duopoly, the market structure *inside* the (SC) case is *not* a stochastic process: for given values of the parameters β and d, it is *deterministically* determined.

- **"Successful Incumbent" (SI) state**: the former quality leader, still carrying out R&D, is the winner of the last R&D race, and hence has two successive qualities at its disposal. According to lemma 2, the market structure is then necessarily a *monopoly*. However, as we will show, the quality leader will then offer two different quality/price bundles in order to *discriminate* between the groups P and R of the population (Mussa and Rosen, 1978), and hence both qualities q_0 and q_{-1} are sold.

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[6]As it will be further demonstrated, the stochastic process $M(t)$ is a continuous time Markov process for which it will be possible to determine a stationary distribution.
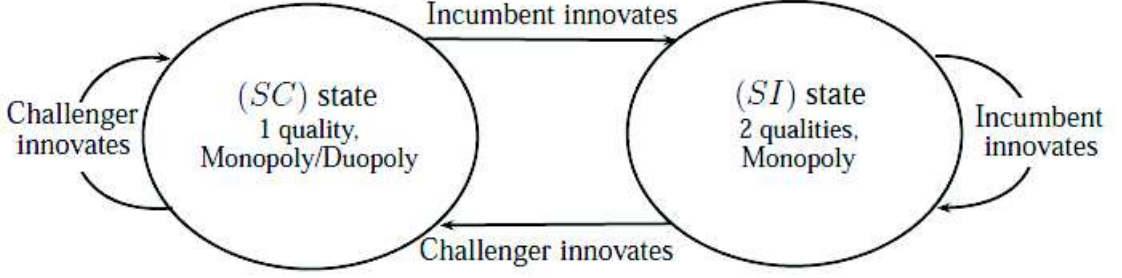
Figure 1: Two possible states

Figure 1 illustrates the fluctuations between the two possible states over time. We will now describe in more details the possible market structures, prices and associated profits in the two existing states.

3.1 Prices and profits in the (SC) state

As already stated, the market structure in the (SC) state is deterministically either a monopoly or a duopoly, depending on the extent of inequalities in wealth distribution in the economy.

3.1.1 Case 1: Monopoly price regime in the (SC) state

It corresponds to the case where the wealth structure makes it optimal for the quality leader to set a price enabling him to sell the unique quality he has at its disposal to the entire market, driving the former quality leader out of the market.

$p_{i,\{0,-1\}}^T$ is the maximum price the producer of quality q_0 can set in order to capture the consumers of the group i for a given price p_{-1} of quality q_{-1}. We first notice that setting a price that captures the consumers belonging to the group P automatically ensures that the rich consumers will consume the highest quality q_0 too, since $p_{i,\{0,-1\}}^T$ is increasing along y_i. Hence, the optimal price chosen by a quality leader willing to capture the whole market is $p_{P,\{0,-1\}}^T$, given that the producer of quality q_{-1} engages in marginal cost pricing (i.e. $q_{-1} = wa$).

We denote by p_M the price being then charged by the quality leader:

$$p_M = y_{SC}^P \frac{k-1}{k} + \frac{wa}{k}$$

with y_{SC}^P being the income of the poor consumers in the (SC) state, of the form $y_{SC}^P = w + rd\frac{\Omega_{SC}}{L}$. We also define the associated profits π_M:

$$\pi_M = L(p_M - wa)$$

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3.1.2 Case 2: Duopoly price regime in the (SC) state

It corresponds to the case where the wealth structure makes it optimal for the new quality leader to set a price capturing only the upper part of the market, abandoning the lower part to the producer of the second-best quality. The two highest qualities q_0 and q_{-1} are then sold at the equilibrium, being produced by two different firms.

Zweimuller and Brunner (2005) have defined a possible equilibrium in that case, under the condition on the punishment strategies of the infinitely repeated pricing game that no firm is punished if it changes its price without affecting the other firm's profit (Proof: cf Zweimuller and Brunner (2005), p. 242). At this equilibrium, the new quality leader optimally chooses to charge the highest possible price enabling him to capture the group of rich consumers $p_{R,\{0,-1\}}^T$, given the expected strategy of the producer of the second-best quality. The former quality leader charges the highest possible price enabling him to capture the poor group of consumers $p_{P,\{-1,-2\}}^T$, given that the producer of quality q_{-2} engages in marginal cost pricing.[7]

We call p_L the price being charged by the new quality leader for the highest quality, while p_F is the price charged by the follower for the second-best quality. They are of the following form:

$$p_F = y_{SC}^P \frac{k-1}{k} + \frac{wa}{k}, \qquad p_L = y_{SC}^R \frac{k-1}{k} + y_{SC}^P \frac{k-1}{k^2} + \frac{wa}{k^2}$$

with y_{SC}^R being the income of the rich consumers in the (SC) state, of the form $y_{SC}^R = w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_{SC}}{L}$. We also define the associated profits π_L for the quality leader and π_F for the producer of the second-best quality:

$$\pi_F = \beta(p_F - wa), \qquad \pi_L = (1-\beta)(p_L - wa)$$

Selection of the equilibrium price regime. Having described the prices and profits for both possible market structures, we still need to define under which parametric conditions on wealth distribution each price regime occurs. It can be however be seen from the expressions of π_M, π_L and π_F that they depend on the endogenous equilibrium values of overall wealth Ω_s ($s \in \{SC, SI\}$) in both possible cases. We will hence extensively comment the parametric conditions governing the occurence of each regime once we have fully defined the steady state equilibrium of our economy (section 5.3). For the time being however, it is sufficient to keep in mind that the choice of the market structure pertains to the new quality leader, who, considering the distribution parameters d and β, optimally decides to set a price capturing either the whole market or only the rich consumers.

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[7]We insist once more on the fact that the strategy chosen in this case by the producer of quality q_{-1} is only made possible because of the decision of the new quality leader to charge a higher price, capturing only the upper part of the market: had the new leader found optimal to charge $p_{P,\{0,-1\}}^T$ instead of $p_{R,\{0,-1\}}^T$ for quality q_0, the former leader would have been driven out of the market and we would be back to case 1 (monopoly price regime).

3.2 Prices and profits in the (SI) state

Two qualities are systematically sold in the (SI) state. Indeed, a leader having at its disposal two successive qualities and facing two groups of consumers having different levels of income will always find it optimal to offer two distinct price-quality bundles in order to maximize its profit (Mussa and Rosen, 1978). The market structure is then a **monopoly**. The price charged by the monopolist for its second-best quality will be the maximal price enabling him to capture the poor group of consumers $p_{P,\{-1,-2\}}^T$, given that the producer of quality q_{-2} engages in marginal cost pricing. The price charged for the highest quality will then be the maximal price $p_{R,\{0,-1\}}^T$, given $p_{P,\{-1,-2\}}^T$.

We call p_{SI}^R and p_{SI}^P the prices being charged respectively to the rich and poor consumers by the monopolist in the (SI) state:

$$p_{SI}^P = y_{SI}^P \frac{k-1}{k} + \frac{wa}{k}, \quad p_{SI}^R = y_{SI}^R \frac{k-1}{k} + y_{SI}^P \frac{k-1}{k^2} + \frac{wa}{k^2}$$

with the associated profits for the discriminating monopolist:

$$\pi_{SI} = \beta L (p_{SI}^P - wa) + (1-\beta) L (p_{SI}^R - wa)$$

and y_{SI}^P and y_{SI}^R being of the form:

$$y_{SI}^P = w + rd \frac{\Omega_{SI}}{L}, \quad y_{SI}^R = w + r \frac{1 - \beta d}{1 - \beta} \frac{\Omega_{SI}}{L}$$

We hence notice that the prices charged for the two qualities in the duopoly case of the (SC) state and in the (SI) state are strongly similar, even if the number of active firms are different. However, the overall wealth (Ω_s, $s \in (SC, SI)$) is different depending on the state the economy finds itself in, hence making it necessary to clearly differentiate the prices charged in the two possible cases in which 2 qualities are sold.

Having defined the possible market structure, prices and profits in every possible state, we can now move to the description of the R&D process, which is the engine of growth in our model.

4 R&D sector

Firms carry out R&D in order to discover the next quality level. Two types of firms engage in R&D races: the current quality leader (incumbent), and followers (challengers). We assume free entry, with every firm having access to the same R&D technology. Innovations are random, and occur for a given firm i according to a Poisson process of hazard rate ϕ_i. Labor is the only input, and we assume constant returns to R&D at the firm level: in order to have an immediate probability of innovating of ϕ_i, a firm needs to hire $F\phi_i$ labor units, F being a positive constant inversely related to the efficiency of the R&D technology.[8]

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[8]The condition of constant marginal costs of R&D can however be loosened, since a previous version of our model (Latzer, 2010) has demonstrated that our results are robust to the use of a concave R&D cost

We define v_C as the value of a challenger firm, v_{SC} as the expected present value of a quality leader having innovated once, and v_{SI} as the expected present value of a quality leader having innovated twice. Free entry and constant returns to scale imply that R&D challengers have no market value, whatever state the economy finds itself in: $v_C = 0$. Free entry of challengers in the successive R&D races also yields the traditional equality constraint between expected profits of innovating for the first time $\phi_C v_{SC}$ and engaged costs $\phi_C wF$ (free entry condition):

$$v_{SC} = wF \tag{5}$$

The incumbent on the other hand participates to the race with the advantage of having already innovated at least once, and hence being the current producer of the leading quality in case (SC)/of the two highest qualities in case (SI). It is hence not subject to the free entry constraint of equality between engaged costs and expected profits.[9]
In the (SC) state, he faces the following Hamilton-Jacobi-Bellman equation:

$$rv_{SC} = \max_{\phi_{I,SC} \geq 0} \{\pi_M - wF\phi_{I,SC} + \phi_{I,SC}(v_{SI} - v_{SC}) + \phi_C(v_F - v_{SC})\} \tag{6}$$

The incumbent in the (SC) state earns the profits π_{SC} (the precise form of π_{SC} depends on the equilibrium price regime and corresponding market structure in the (SC) state), and incurs the R&D costs $wF\phi_{I,SC}$. With instantaneous probability $\phi_{I,SC}$, the leader innovates once more, the economy jumps to the state (SI), and the value of the leader (now producing and selling two distinct qualities) climbs to v_{SI}. However, with overall instantaneous probability ϕ_C, some R&D challenger innovates, and the quality leader falls back to being a follower: its value drops to v_F (again, the precise form of v_F depends on the market structure in the (SC) case). The economy then remains in the state (SC), and only one quality is produced.

In the (SI) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$rv_{SI} = \max_{\phi_{I,SI} \geq 0} \{\pi_{SI} - wF\phi_{I,SI} + \phi_{I,SI}(v_{SI} - v_{SI}) + \phi_C(v_F - v_{SI})\} \tag{7}$$

The incumbent in the (SI) state earns the profits π_{SI} of a monopolist being able to discriminate between rich and poor consumers by offering two distinct price/quantity bundles. He incurs the R&D costs $wF\phi_{I,SI}$. With instantaneous probability $\phi_{I,SI}$, the incumbent innovates once more, in which case its value remains v_{SI}, since we have established with Lemma 2 that at most two successive quantities are sold at equilibrium. Hence, the incumbent will still be the producer of the two qualities being sold, but he will drive himself

function.

[9] The incumbent is however subject to exactly the same R&D costs and participates to the same race than the challengers, leading to the same size of innovation is successful: this is why we claim that they do not benefit from any advantage in the R&D field.

out of the market for the former quality q_{-1}, that has become quality q_{-2} with the latest quality jump. The economy then remains in state (SI). With instantaneous probability ϕ_C,[10] some R&D follower innovates, and the quality leader then falls back to being an R&D challenger: its value falls to v_F. The economy then jumps to the state (SC), and only the new highest quality is sold by the latest successful innovator.

In both states, the incumbent firm chooses its R&D effort so as to maximize the right-hand side of its Bellman equation. (6) and (7) then yield the following first order conditions:

$$v_{SI} - v_{SC} = wF \tag{8}$$
$$-wF = 0 \quad \Rightarrow \quad \phi_{I,SI} = 0 \tag{9}$$

Hence, we obtain a relation between the R&D costs and the incremental value that would result from innovating in both states. Given that the incremental value of a further innovation for an incumbent in the (SI) state is null in our economy with only two distinct population groups, we obtain that the optimal investment in R&D in that state is zero.[11] From then on, we hence refer to the investment in R&D of the incumbent firm in the (SC) state as simply ϕ_I.

Using the optimality constraints (8) and (9) in (6) and (7), we obtain the following expressions for the expected values v_{SC} and v_{SI}:

$$v_{SC} = \frac{\pi_{SC} + \phi_C v_F}{r + \phi_C} \tag{10}$$
$$v_{SI} = \frac{\pi_{SI} + \phi_C v_F}{r + \phi_C} \tag{11}$$

We are now left to detail the possible values taken by v_F and π_{SC}, which depend on whether the equilibrium market structure in the (SC) state (depending on the income distribution within our economy) is a monopoly (Case 1) or a duopoly (Case 2).

- *Case 1 - Monopoly price regime in the (SC) state.* We then have $\pi_{SC} = \pi_M$, and the value v_F of a firm that has been leap-frogged by a challenger is null: indeed, the new leader charges a price that captures the whole market, leaving no room for the producer of the second-best quality. The previous leader then falls back to a challenger status, and we have $v_F = v_C = 0$. Using (5), (8), (10) and (11), we hence get the two following equalities between incurred costs and expected profits when we

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[10]The challengers invest the same amount in the R&D sector ϕ_C in both states (SC) and (SI), since they face the same expected reward v_{SC} in both cases: a successful innovation by a challenger indeed always brings the economy back to state (SC).

[11]We believe it would be possible to generalize our model to more than two groups of population, or a continuum of quality valuations as in Mussa and Rosen (1978). Intuitively, the incumbent would then keep investing in R&D beyond the second innovation in a row.

have a monopoly market structure in the (SC) state:

$$wF = \frac{\pi_M}{r + \phi_C} \tag{12}$$

$$2wF = \frac{\pi_{SI}}{r + \phi_C} \tag{13}$$

- *Case 2 - Duopoly price regime in the (SC) state.* We then have $\pi_{SC} = \pi_L$, and the value v_F of the previous leader (now producer of the second-best quality) is strictly positive, since in Case 2 the new leader has optimally chosen to charge a price capturing only the upper part of the market. This "follower" faces the following Hamilton-Jabobi-Bellman equation:

$$rv_F = \max_{\phi_F \geq 0} \{\pi_F - wF\phi_F + \phi_F(v_{SC} - v_F) + (\phi_C + \phi_I)(v_C - v_F)\} \tag{14}$$

The follower sells the second-best quality to the lower part of the market, earning the profits π_F. He incurs the R&D costs $wF\phi_F$. With instantaneous probability ϕ_F, he is successful in innovating once more, and its value jumps back to v_{SC}. With instantaneous probability $\phi_C + \phi_I$, either some R&D follower or the current quality leader innovates, and the follower is definitively driven out of the market: its value falls to $v_C = 0$. Solving for an interior solution to this maximization problem yields the condition $v_{SC} - v_F = wF$, which, combined with condition (5), would imply $v_F = 0$. This is however not the case, since the follower's profits π_F when the market structure in the (SC) state is a duopoly are strictly positive. We then necessarily have $\phi_F = 0$. Plugging this value back into (14), we obtain that $v_F = \frac{\pi_F}{r + \phi_C + \phi_I}$. Using (5), (8), (10) and (11), we finally get the two following equalities between incurred costs and expected profits when we have a duopoly market structure in the (SC) state:

$$wF = \frac{\pi_L + \phi_C \left(\frac{\pi_F}{r + \phi_C + \phi_I}\right)}{r + \phi_C} \tag{15}$$

$$2wF = \frac{\pi_{SI} + \phi_C \left(\frac{\pi_F}{r + \phi_C + \phi_I}\right)}{r + \phi_C} \tag{16}$$

5 Steady state equilibrium

5.1 Labor market equilibrium

We have two possible equations describing the equilibrium on the labor market, whether we are in the (SC) or the (SI) state. The equilibrium on the labor market in the (SC) state is of the form:

$$L = F(\phi_I + \phi_C) + aL + (L/w)(\beta(y_{SC}^P - p_{SC}^P) + (1 - \beta)(y_{SC}^R - p_M)) \tag{17}$$

with $F(\phi_I + \phi_C)$ being the number of people hired in the R&D sector, aL being the number of people hired for the production of the quality good, and finally $L/w(\beta(y_{SC}^P - p_{SC}^P) + (1-\beta)(y_{SC}^R - p_{SC}^R))$ being the number of people devoted to the production of the standardized good. p_{SC}^P and p_{SC}^R are the prices paid by consumers belonging respectively to the P and the R group in the (SC) state. Again, the values taken by those two variables depend on the equilibrium market structure in the (SC) state: $p_{SC}^P = p_{SC}^R = p_M$ when the market structure is a monopoly; $p_{SC}^P = p_F$ and $p_{SC}^R = p_L$ when the market structure is a duopoly.

The equilibrium on the labor market in the (SI) state is of the form:

$$L = F\phi_C + aL + (L/w)(\beta(y_{SI}^P - p_{SI}^P) + (1-\beta)(y_{SI}^R - p_{SI}^R)) \tag{18}$$

with $F\phi_C$ being the number of people hired in the R&D sector (the incumbent does not invest in R&D in the (SI) state), aL being the number of people hired for the production of the quality good, and finally $L/w(\beta(y_{SI}^P - p_{SI}^P) + (1-\beta)(y_{SI}^R - p_{SI}^R))$ being the number of people devoted to the production of the standardized good.

It will prove convenient to express (17) and (18) in terms of profit flows. Multiplying both sides by w and replacing y_{SC}^P, y_{SC}^R, y_{SI}^P and y_{SI}^R by their values expressed in Section 3, (17) and (18) respectively yield:

$$wL = wF\phi_C + wF\phi_I + waL + L(\beta(w + rd\frac{\Omega_{SC}}{L} - p_{SC}^P) + (1-\beta)(w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_{SC}}{L} - p_{SC}^R))$$

$$wL = wF\phi_C + waL + L(\beta(w + rd\frac{\Omega_{SI}}{L} - p_{SI}^P) + (1-\beta)(w + r\frac{1-\beta d}{1-\beta}\frac{\Omega_{SI}}{L} - p_{SI}^R))$$

Splitting waL into $\beta waL + (1-\beta)waL$ and rearranging terms, as well as distinguishing between the two possible cases in the (SC) state, we finally get:

- *Case 1 - Monopoly price regime in the (SC) state.* We have $p_{SC}^P = p_{SC}^R = p_M$, and obtain the two following equations defining labor equilibrium in both states:

$$wF\phi_I + wF\phi_C = \pi_M - r\Omega_{SC} \tag{19}$$
$$wF\phi_C = \pi_{SI} - r\Omega_{SI} \tag{20}$$

- *Case 2 - Duopoly price regime in the (SC) state.* We have $p_{SC}^P = p_F$ and $p_{SC}^R = p_L$, and obtain the two following equations defining labor equilibrium in both states:

$$wF\phi_I + wF\phi_C = \pi_F + \pi_L - r\Omega_{SC} \tag{21}$$
$$wF\phi_C = \pi_{SI} - r\Omega_{SI} \tag{22}$$

5.2 Steady state analysis

In order to be able to proceed to a steady state analysis, we first need to prove the existence of a stationary distribution for the stochastic equilibrium market structure $M(t)$.

Proposition 1: *The market structure $M(t)$ is a Markov process with state space $\{(SC), (SI)\}$, transition rate matrix $Q = \begin{pmatrix} -\phi_I & \phi_I \\ \phi_C & -\phi_C \end{pmatrix}$, and transition probability matrix $P(\Delta t) = I + Q\Delta t$.*

Proof: The continuous stochastic process $M(t)$ satisfies the Markov property:

$$P(M(t + \Delta t) = k | M(t) = j, M(t_i) = x_i \forall i) = P(M(t + \Delta t) = k | M(t) = j)$$

with $k, j \in \{(SC), (SI)\}$, $t_0 < t_1 < ... < t_n < t$ and $x_0, ..., x_n \in \{(SC), (SI)\}$. Indeed, the current state of the market structure $M(t)$ contains all the information that is needed to characterize the future stochastic behavior of the process: at a given time t, we only need to know the realization of the random variable $M(t)$ to be able to compute the probabilities associated to the possible realizations of $M(t + \Delta t)$. We define as $q_{i,j}$ the probability per time unit that the system makes a transition from state i to state j:

$$q_{i,j} = \lim_{\Delta t \to 0} \frac{P(M(t + \Delta t) = j | M(t) = i)}{\Delta t}$$

Considering the R&D races described in our model, we have $q_{SC,SI} = \phi_I$ and $q_{SI,SC} = \phi_C$. Indeed, ϕ_I corresponds to the immediate probability for the incumbent to innovate when in the (SC) state, while ϕ_C corresponds to the immediate aggregate probability for a challenger to innovate, whether it be in the (SC) or the (SI) state.

We define as q_i the total transition rate out of state i, and $q_{i,i} = -q_i$.

The transition rate matrix Q of such a Markov process is:

$$Q = \begin{pmatrix} q_{SC,SC} & q_{SC,SI} \\ q_{SI,SC} & q_{SI,SI} \end{pmatrix} = \begin{pmatrix} -\phi_I & \phi_I \\ \phi_C & -\phi_C \end{pmatrix}$$

and the transition probability matrix over time interval Δt is $P(\Delta t) = I + Q\Delta t$. Finally, the embedded (discrete time) Markov chain of the continuous time Markov process $M(t)$ can be represented in the following way:
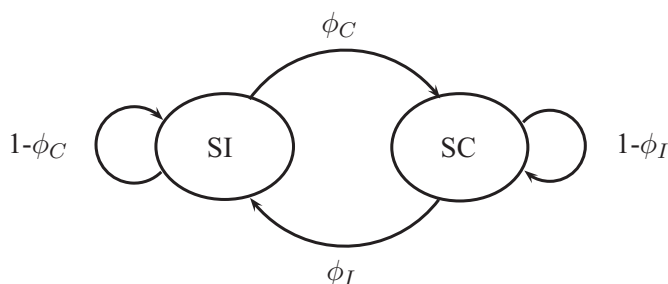
This ends the proof. □

Now that we have determined that $M(t)$ is a Markov process, we still need to prove that it admits a stationary distribution in order to be able to characterize a steady state for our economy.

Proposition 2: *The Markov process M(t), describing the market structure, admits a stationary distribution with stationary state probability vector* $\pi = \left(\frac{\phi_C}{\phi_C + \phi_I} \quad \frac{\phi_I}{\phi_C + \phi_I} \right)$.

Proof: We define the state probability vector $\pi(t)$, being a function of time and evolving as follows: $\frac{d}{dt}\pi(t) = \pi(t) \cdot Q$. The stationary solution $\pi = \lim_{t \to \infty} \pi(t)$ is independant of time, and thus satisfies $\pi \cdot Q = 0$. Being a probability distribution vector, it also satisfies $\pi \cdot e^T = 1$ with e being a row vector with all elements equal to 1. Defining $E = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, we then have that $\pi \cdot E = e$.

Using all those results together, we have that $\pi(Q + E) = e$, and hence the stationary distribution is obtained by solving for $\pi = e \cdot (Q + E)^{-1}$, provided $Q + E$ is an invertible matrix. We have $Q + E = \begin{pmatrix} 1 - \phi_I & 1 + \phi_I \\ 1 + \phi_C & 1 - \phi_C \end{pmatrix}$. This matrix is indeed invertible, with $(Q + E)^{-1} = \frac{1}{-2\phi_C - 2\phi_I} \begin{pmatrix} 1 - \phi_C & -1 - \phi_I \\ -1 - \phi_C & 1 - \phi_I \end{pmatrix}$. Finally, we obtain $\pi = e \cdot (Q + E)^{-1} = \left(\frac{\phi_C}{\phi_C + \phi_I} \quad \frac{\phi_I}{\phi_C + \phi_I} \right)$. This ends the proof. □

The steady state equilibrium is hence defined by the overall wealths in both states Ω_{SC} and Ω_{SI}, as well as the stationary distribution of M, determined by the endogenous transition rates ϕ_I and ϕ_C. As already stated, those transition rates are being determined by the R&D investment decisions of the incumbent (transition from the (SC) to the (SI) state) and the challengers (transition from the (SI) to the (SC) state), taken according to the rewards accruing to successful innovators, π_{SC} and π_{SI}. Since those rewards differ whether we have a monopoly or a duopoly in the (SC) state, we need to distinguish both cases in the definition of the steady state:

- *Case 1 - Monopoly price regime in the (SC) state.* The 4 equations defining the economy steady state are the equality constraints between the incurred R&D costs and the expected value of an innovation in both states (12) and (13), as well as the

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labor market equilibrium conditions in both states (19) and (20).

- *Case 2 - Duopoly price regime in the* (SC) *state.* The 4 equations defining the economy steady state are the equality constraints between the incurred R&D costs and the expected value of an innovation in both states (15) and (16), as well as the labor market equilibrium conditions in both states (21) and (22).

Proposition 3 (Existence and uniqueness of a steady state equilibrium):

- (1) *Monopoly price regime in the* (SC) *state: for* $\beta > 0.5$, *and under the parametric conditions (i)-(ii) (cf Appendix A) on* $\frac{L}{F}$ *and* k, *the system formed by equations* (12), (13), (19) *and* (20) *has a unique solution in* $(\phi_C, \phi_I, \Omega_{SC}, \Omega_{SI})$, *all strictly positive.*

- (2) *Duopoly price regime in the* (SC) *state: for* r *sufficiently small, the system formed by equations* (15), (16), (21) *and* (22) *has a unique solution in* $(\phi_C, \phi_I, \Omega_{SC}, \Omega_{SI})$, *all strictly positive.*

Proof: cf Appendix A.

Several comments are in order. First, since the equilibrium price regime in the (SC) case depends on the distribution parameters β and d (cf section 3.1 and section 5.3 below), it might seem problematic that a condition is made on β for an equilibrium to exist when we have a monopoly price regime in the (SC) state. However, such a lower bound on β seems acceptable, since monopoly is anyway only likely to emerge as an equilibrium price regime for high values of β. Indeed, it is intuitively clear (cf section 5.3) that for the new leader to accept to charge a lower price in order to capture the poor consumers, those consumers must not be too poor (low value of d) or too few (low value of β).[12]

Second and most importantly, we have demonstrated that **in an economy where disparities in purchasing power exist, incumbents have an incentive to keep investing in R&D beyond their first successful innovation**. The intuition behind this result is that leaders participate to the next R&D race because of the positive increment in profits that exists when innovating for a second time in an economy with wealth inequalities. Indeed, in our framework, a *challenger* that has just won the latest innovation race needs to decide between two alternatives: selling its product to the whole market at a price being sufficiently low to attract the poorest households, or extract the maximum surplus from wealthy consumers by charging a higher price, but at the cost of abandoning the poor consumers to its direct competitor (i.e. the previous quality leader). Those two alternatives respectively correspond to Cases 1 and 2 for the (SC) state market structure (monopoly or duopoly), and the equilibrium price regime is optimally chosen by the successful challenger according to the wealth distribution within the economy (β and d). However, since he only has one quality at its disposal, such an innovator cannot efficiently exploit the differences

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[12]It should furthermore be noted that such a condition is sufficient but not necessary (cf Appendix A), and simulations carried out in section 5.3 to determine the equilibrium price regime in the (SC) case also show that a unique equilibrium exists in Case 1 for $\beta < 0.5$ under a wide array of parametric cases.

in the willingness to pay existing between rich and poor consumers: he always needs to sacrifice either a portion of the maximum price he could charge (Case 1) or a chunk of the market (Case 2). On the other hand, an *incumbent* winning an innovation race has two successive qualities at its disposal: he can thus efficiently discriminate between rich and poor consumers by offering two distinct price/quality bundles, capturing the whole market *and* reaping the maximum surplus from the wealthy consumers at the same time.

This result can be commented in the light of the microeconomic literature analyzing price discrimination by a monopolist having at its disposal a product range including different quality levels. In such a context, Mussa and Rosen (1978) have demonstrated that serving costumers who place smaller valuations on quality creates **negative externalities** for the monopolist, preventing him from capturing the maximum costumer surplus from those who have a stronger taste for quality. In their static framework, the multi-quality monopolist then internalizes the existing negative externalities by inducing less enthusiastic consumers to buy lower quality items charged at a lower price, opening the possibility of charging higher prices to more adamant buyers of high quality units. In our dynamic model with endogenous innovation, the monopolist only has access to as many qualities as R&D races he has won: the negative externalities stemming from having to serve two distinct groups of consumers having different quality valuation is then internalized by expanding the line of product towards *higher* (and not lower) qualities, i.e. through R&D investment.

Finally, another implication of this results is that in the case there exist wealth disparities within an economy, positive investment in R&D by quality leaders is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers, as well as without any concavity in the R&D cost function. We are indeed modeling constant returns to R&D investments, and not allowing for any R&D cost advantage of the incumbent over the followers (Segerstrom and Zolnierek, 1999; Segerstrom, 2007) or any sequentiality in the patent races (Etro, 2008). Our model hence exemplifies the existence of so far overlooked incentives for innovation by incumbent stemming from the demand structure rather than from the supply side (R&D sector characteristics and R&D capabilities of challenger and incumbent firms).

Such a result, beyond its novelty, enables us to furthermore extend and deepen the study of the impact of income inequality on long term economic growth, by taking into account investment of *both* challengers and incumbents. This will be the aim of the next section. Since the impact of inequalities on growth depends on the price regime in the (SC) state, we however first comment the parametric conditions governing the choice of the equilibrium price regime when a challenger has won the latest innovation race.

5.3   Selection of the equilibrium price regime in the (SC) state

As noted in section 3.1, the selection of the (deterministic) equilibrium market structure in the (SC) state is up to the winner of the latest innovation race (i.e. the quality winner), that chooses the optimal price regime considering the distribution of wealth in the economy.

Intuitively, a duopoly price regime should arise at equilibrium for low values of β and d: indeed, for the leader to optimally abandon the poorer consumers to its direct competitor, the wealthy group of consumers has to be significantly richer (d sufficiently small) or sufficiently big (β sufficiently small). Hence, we expect the equilibrium market structure to be a monopoly for high values of β and/or high values of d, and to be a duopoly for low values of β and/or low values of d. More formally, the decision will be taken by the new leader comparing the expected profits in both cases, assuming the latter anticipates correctly the overall wealth as well as the R&D investment rates in both cases. The condition for the leader to choose a monopoly rather than a duopoly market structure is hence of the form:

$$\frac{\pi_M(\Omega^M_{SC})}{r+\phi^M_C} + \phi^M_I \frac{\pi_{SI}(\Omega^M_{SI})}{r+\phi^M_C} > \frac{\pi_L(\Omega^D_{SC}) + \phi^D_C \frac{\pi_F(\Omega^D_{SC})}{r+\phi^D_C+\phi^D_I}}{r+\phi^D_C} + \phi^D_I \frac{\pi_{SI}(\Omega^D_{SI}) + \phi^D_C \frac{\pi_F(\Omega^D_{SC})}{r+\phi^D_C+\phi^D_I}}{r+\phi^D_C}$$

with the superscript M (respectively D) describing the value taken by the endogenous variables in the case of a monopoly (resp. duopoly) price regime in the (SC) state. Although this condition might seem complex, constant returns to R&D effort at the firm level enable us to simplify this expression using (10) and (11). Indeed, whatever the equilibrium price regime in the (SC) state, expected incremental values $\phi^i_C v_{SC}$ and $\phi^i_I(v_{SI} - v_{SC})$ of a first and a second innovation have to be equal to the incurred costs, i.e. $\phi^i_C wF$ and $\phi^i_I wF$. The above condition then simplifies to $wF + 2wF\phi^M_I > wF + 2wF\phi^D_I$, i.e. $\phi^M_I > \phi^D_I$. Determining the equilibrium market structure in the (SC) state hence amounts to comparing the investment by incumbent in both possible price regimes. Such a condition implies that the leader systematically chooses the market structure ensuring him the greatest probability of reaching the status of "discriminating" monopolist, i.e. of endogenizing the negative externalities stemming from wealth inequalities. It is important to keep in mind that this however does not amount to choosing the case displaying the highest long-term growth rate, since part of the overall R&D effort is carried out by challengers, and this conditions gives no information on the respective size of ϕ^M_C and ϕ^D_C.

We are hence now able to determine the conditions on the wealth distribution parameters β and d governing the occurrence of a given case at the equilibrium. Given the absence of explicit analytical expression of the different endogenous variables in the case of a duopoly price regime (cf. Appendix A), we are left to carry out numerical simulations. We hence proceed to simulations for β varying from $0,3$ to 1, and for d varying from $0,1$ to 1, carrying out a sensitivity analysis along different values of F, k, a and r. Severel salient numerical regularities emerge:

- *Numerical finding 1*: Under a wide array of parametric cases, a unique steady state equilibrium with positive $(\phi_C, \phi_I, \Omega_{SC}, \Omega_{SI})$ exists under a monopoly price regime in the (SC) state for values of β inferior to 0.5.

- *Numerical finding 2*: For parameter values for which a unique steady state equilibrium exists in both cases, the dominant market structure is always a duopoly for $\beta < 0.5$ and a monopoly for $\beta > 0.6$ (the market structure for $0.5 < \beta < 0.6$ varies

along values of F, k, a and d).

- *Numerical finding 3*: Except for very high values of the productivity parameter in the quality good sector a ($a > 0.8$), varying values of the parameter d do not influence the equilibrium price regime for a given β.

- *Numerical finding 4*: Whatever the equilibrium price regime, a majority of the R&D investment in the (SC) state is carried out by the incumbent for low values of the quality increment k.

Numerical findings 1 and 2 confirm that the condition imposed on the share of the population being poor within the economy ($\beta > 0.5$) in order to have a unique and positive steady state equilibrium in Case 1 (monopoly price regime in the (SC) state) is not problematic. Indeed, even if a steady state equilibrium exists for lower values of β (Numerical finding 1), the monopoly market structure only becomes dominant for values of β superior to 0.5 (Numerical finding 2).

Numerical findings 2 and 3 state that it is the *share* of the poor group within the economy rather than the wealth repartition between the two groups that will determine the equilibrium price regime in the (SC) state. Hence, it seems that for a successful challenger having only one quality at its disposal and seeking to maximize its expected profits, the size of the market matters more than the extent of the immediate price surplus he can reap. Indeed, even if the rich group concentrates a significant part of the overall wealth (low values of d), the leader will not be ready to abandon the poor group's consumption to its competitor if the size of the latter is big enough (high values of β).

Numerical finding 4, although not related to the conditions for a given price regime to occur in the (SC) state, is still worth to mention. Indeed, it demonstrates that our model is able to replicate the stylized fact emphasized by Etro (2008) and Acemoglu and Cao (2010) that a major bulk of the overall R&D investment is carried out by current incumbent patentholders. The intuition linking the fraction of overall R&D carried out by the incumbent and the size of the innovation k is found considering the increment in profits $\pi_{SI} - \pi_{SC}$ when innovating for the second time. For a given level of wealth in both states, this increment is non-monotonous along k, first increasing for $k < 2$ and then decreasing.

Having established that both price regimes occur at the equilibrium as wealth distribution progresses along the β and d dimensions, we now proceed to studying the impact on overall growth of the level of inequalities.

6 Distribution of income and long-term growth

Does an increase in the level of inequalities, whether it be through an increasing concentration of wealth among a small group of people (increasing β) or a more unequal distribution of overall wealth between rich and poor (decreasing d) have a positive or negative impact on the long-run growth rate in a quality-ladder framework? Our models allows

us to answer that question by studying the impact of the level of inequalities on both the incumbent's and the challengers' investment in R&D, and by extension on long-run growth. Indeed, the economy growth rate is directly linked to the innovation intensity of both challengers and leaders, since consumers become better off due to the successive improvements of the quality consumption good. More precisely, we have the following relationship:

Proposition 4: *The steady state utility growth rate of our economy is* $\gamma = (\ln k)\phi_C(1 + \frac{\phi_I}{\phi_I + \phi_C})$.

Proof: Considering equation (2) and the fact that in a given state, the consumption of the standard good c remains constant when at the steady state equilibrium, we have $\gamma_i = \frac{\dot{u}_i(t)}{u_i(t)} = \frac{\dot{q}_i(t)}{q_i(t)}$. In state (SC), we hence have that $\gamma_R = \frac{\dot{q}_R(t)}{q_R(t)} = (\ln k)(\phi_C + \phi_I)$ and $\gamma_P = \frac{\dot{q}_P(t)}{q_P(t)} = (\ln k)(\phi_C)$. Indeed, if the next innovation race is won by a challenger, the latter will sell to both population groups the *unique* quality he has at its disposal, having a quality increment k with respect to the previous quality being consumed by both groups. However, if the next innovation race is won by the incumbent, the latter will sell the highest quality he has at its disposal to the rich consumers, whose utility will indeed increase. He will however keep selling the second-best quality to the poor consumers, whose utility will hence not increase following this quality jump. In state (SI), only challengers carry out R&D, and in the case they win the next innovation race, they will again sell the highest quality to the two groups of consumers. Hence we have that $\gamma_R = (\ln k)\phi_C$, while $\gamma_P = 2(\ln k)\phi_C$. Indeed, the poor consumers were consuming quality q_{-1} before the quality jump. Hence, considering the stationary probability distribution of the market structure, we have that the average utility growth rate of rich consumers is $\gamma_R = (\ln k)((\phi_C + \phi_I)(\frac{\phi_C}{\phi_C + \phi_I}) + \phi_C(\frac{\phi_I}{\phi_C + \phi_I})) = (\ln k)\phi_C(1 + \frac{\phi_I}{\phi_I + \phi_C})$, while the average utility growth rate of poor consumers is $\gamma_P = (\ln k)\phi_C(\frac{\phi_C}{\phi_C + \phi_I}) + 2(\ln k)\phi_C(\frac{\phi_I}{\phi_C + \phi_I}) = (\ln k)\phi_C(1 + \frac{\phi_I}{\phi_I + \phi_C})$. This ends the proof. $\square$

We consider two types of variations in the extent of wealth disparities: (a) an increase in β for a given d, and (b) an increase in d for a given β. We obtain analytical results in the case we have a monopoly price regime in the (SC) state:

Proposition 5 (Wealth distribution and long-term growth):
When the equilibrium market structure is a monopoly in the (SC) state, we have the following comparative statics for varying values of β and d:

- *(a) Effect of an increase in the population share of the poor β: the incumbent's R&D intensity ϕ_I increases along β while the challengers' innovation rate ϕ_C as well as the overall wealth in both states Ω_{SC} and Ω_{SI} decrease along β.*

- *(b) Effect of an increase in the relative wealth of poor consumers d: the challengers' innovation rate ϕ_C and the overall wealth in the (SI) state Ω_{SI} increase along d,*

while the directions of variation of the incumbent R&D intensity ϕ_I as well as the overall wealth in the (SC) state Ω_{SC} are ambiguous.

Proof: Full analytical expressions for the above comparative statics can be obtained from the expressions (23), (24), (25) and (26):

$$\frac{\partial \phi_C}{\partial \beta} = -\frac{(k-1)k((1-a)(k+1+d(k-1))L+2dFkr)}{F(k(k+1)-d(k-1)(1-\beta))^2} < 0$$

$$\frac{\partial \phi_I}{\partial \beta} = \frac{k^2((1-a)(k+1+d(k-1))L+2dFkr)}{dF(k(k+1)-d(k-1)(1-\beta))^2} > 0$$

$$\frac{\partial \Omega_{SC}}{\partial \beta} = -\frac{k^2((1-a)(k+1+d(k-1))L+2dFkr)}{dr(k(k+1)-d(k-1)(1-\beta))^2} < 0$$

$$\frac{\partial \Omega_{SI}}{\partial \beta} = -\frac{(k-1)k((1-a)(k+1+d(k-1))L+2dFkr)}{r(k(k+1)-d(k-1)(1-\beta))^2} < 0$$

$$\frac{\partial \phi_C}{\partial d} = \frac{(k-1)(1-\beta)(2Fk^2r+(1-a)(k-1)L(k+1-\beta))}{F(k(k+1)-d(k-1)(1-\beta))^2} > 0$$

$$\frac{\partial \Omega_{SI}}{\partial d} = \frac{(k-1)w(2Fk^2r+(1-a)(k-1)L(1+k-\beta))(1-\beta)}{r(k(k+1)-d(k-1)(1-\beta))^2} > 0$$

This ends the proof.□

(a) Let us first comment the effects of an increase in β when we have a monopoly price regime in the (SC) state. A rise in the share of the population being poor β corresponds to a growing level of inequalities, since it leads to a higher concentration of wealth through an increase in the relative income of a rich consumer ($\frac{\partial d_R}{\partial \beta} = \frac{1-d}{(1-\beta)^2} > 0$): there are more poor with the same income, and fewer rich with more income. Intuition for the variations of ϕ_C and ϕ_I in that case can be gained from considering the resource constraints (19) and (20), keeping in mind that for given levels of wealth Ω_{SC} and Ω_{SI}, an increase in β leaves π_M unchanged and leads to a decrease in π_{SI}.[13] In the (SI) state, a lower π_{SI} means that a smaller part of the overall wealth of the consumers has been spent on the consumption of the quality good, leading to a mechanic increase in the consumption of the homogenous good. Keeping in mind that the labor demand for the production of the quality good is fixed at aL (unit consumption of the quality good), such an increase in consumption of the standard good necessarily leads to a labor reallocation from the R&D sector to the production of the homogenous good sector. Hence, according to the labor constraint (20), ϕ_C drops for an increasing β. Since the right-hand side of the labor constraint in the (SC) state (19) is left unchanged for a given Ω_{SC}, ϕ_I then necessarily needs to rise in order to compensate the fall in ϕ_C.

Beyond labor market considerations, variations in ϕ_C and ϕ_I following a shock on β can also be explained considering the variations in expected gains of successfully innovating for the first and the second time. Figure 2 illustrates in both possible cases (monopoly or duopoly market structure in the (SC) state) the profits of a successful challenger (resp.

―――――――――
[13]Indeed, we have $\frac{\partial \pi_{SI}}{\partial \beta} = -\frac{(k-1)(rd\Omega_{SI}+(1-a)Lw)}{k^2} < 0$
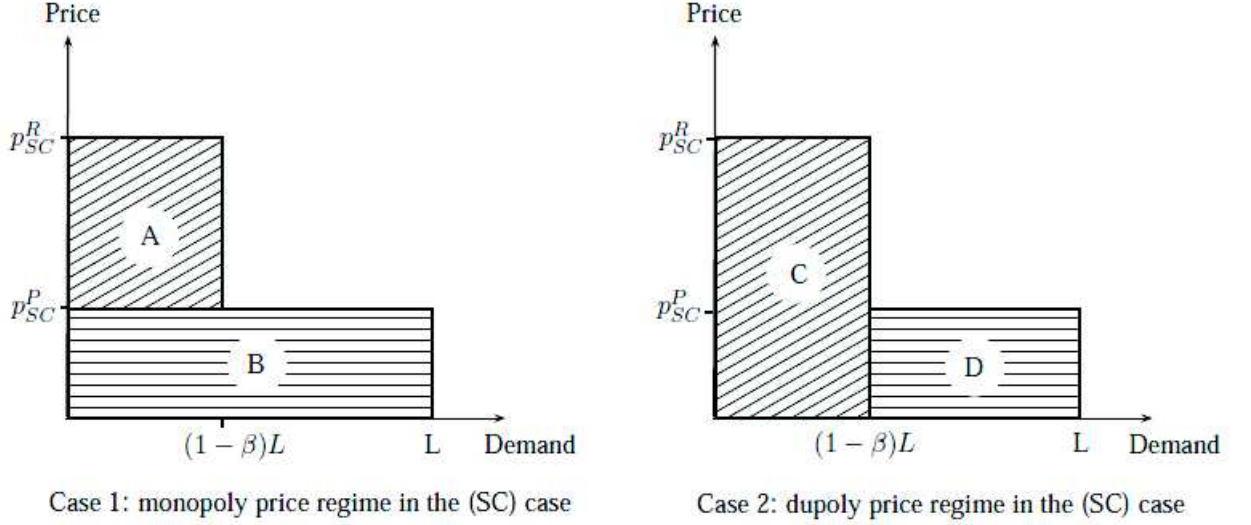
Figure 2: Profits in the (SC) and (SI) states in cases 1 and 2

areas B and C) and the increment in profits of a successful incumbent (resp. areas A and D). In the case of a monopoly price regime in the (SC) state, variations in the R&D investment of challengers will depend on variations of the overall area A+B. Indeed, innovating for the first time not only yields immediate profits (B), but also offers the opportunity to enter the next race as an incumbent, with the expected incremental reward (A). On the other hand, variations in the R&D investment of the incumbent will only depend on variations of the area A, corresponding to the increment in profits when innovating a second time.

For given values of Ω_{SC} and Ω_{SI}, an increase in β in Case 1 leaves area B unchanged and impacts area A through both a *positive price effect* (the price a successful incumbent can charge for its highest quality has increased) and a *negative market size effect* (the number of people for which he can extract that extra surplus has decreased). Taking into account the negative variations of Ω_{SC} and Ω_{SI} following an increase in β however changes the picture: area B shrinks (negative price effect), and the positive price effect on the incremental profits of an incumbent is mitigated. The two negative effects (negative price effect on B and negative market size effect on A) on the expected profits of a successful challenger dominate the positive price effect on A: ϕ_C drops. On the other hand, the incumbent invests in R&D only taking into account the expected incremental gain A: the positive price effect dominates the negative market size effect, and ϕ_I rises.

We have hence demonstrated that an increase in the level of inequalities through a rise in β unambiguously leads to an increase in R&D intensity of the incumbent ϕ_I and a decrease in challengers' R&D investment ϕ_C. It is however important to emphasize that the effect on long-run growth is ambiguous, since the latter depends on both the challengers' and the incumbent's investment in R&D: $\gamma = (\ln k)\phi_C(1 + \frac{\phi_I}{\phi_I + \phi_C})$. Clear analytical predictions are not possible to obtain, and we hence carry out simulations for β varying from 0.5 to 0.9 (lower values of β lead to a duopoly market structure in the (SC) case) and for a wide array of values of F, k, a and r. We find the growth rate to be strictly decreasing
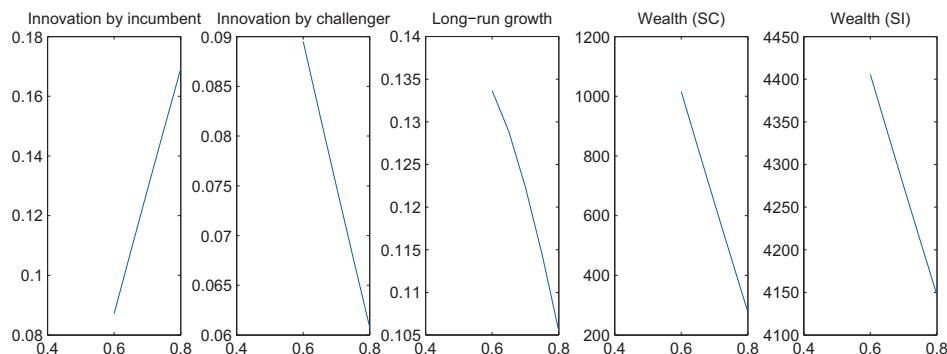
Figure 3: Effects of an increase in β under a **monopoly** price regime in the (SC) case $(r = 0.2,\ L = 200,\ F = 120,\ a = 0.7,\ d = 0.7,\ k = 2)$

along β, implying a negative relationship between long-run growth and the level of inequalities (as measured by β) when we have a monopoly price regime in the (SC) state (Figure 3).

(b) We now move to comment the effects of an increase in d when we have a monopoly price regime in the (SC) state. We first note that such a rise in the ratio of the wealth of a poor consumer *relative* to the average per-capita wealth leads to a *decrease* in the level of inequalities. A simple intuition for the positive variation of ϕ_C in the case of an increase in d can then be found by considering the variations in expected gains of successfully innovating for the first and the second time. For given levels of wealth, an increase in d has a *positive price effect* on the profits of a successful challenger, since he can charge a higher price and still capture the whole market (the critical income in the (SC) state is the income of poor households): area B increases. On the other hand, such an increase in d had a *negative price effect* on the incremental profits of a successful incumbent, shrinking area A. One would then expect an unambiguous decrease in the R&D investment of the current leader (smaller incremental profits), and an ambiguous variation in the R&D investment of challengers, who base their decisions on variations in the overall area A+B. Taking into account the positive variation in Ω_{SI} however modifies the picture, since such an increase in overall wealth in the (SI) state will at least mitigate and might reverse the negative impact on area A of a higher d. Indeed, analytical results confirm that the positive price effect on B always dominates the ambiguous price effect on A, leading to an unambiguous increase in ϕ_C. On the other hand, simulations carried out for a wide array of values of F, k, a and r show that the leader's R&D investment ϕ_I might decrease or increase along d, illustrating the ambiguous effect of a variation of d on the incremental profits described by area A.

We carry out simulations for d varying from 0.1 to 0.9 and for a wide array of values of F, k, a and r in order to determine the overall impact of an increase in d on long-term growth. We systematically obtain a positive relationship between the long-run growth rate and the relative wealth of a poor consumer d (Figure 4 depicts a parametric case where the growth rate of the economy increases along d, even though the leader's R&D investment
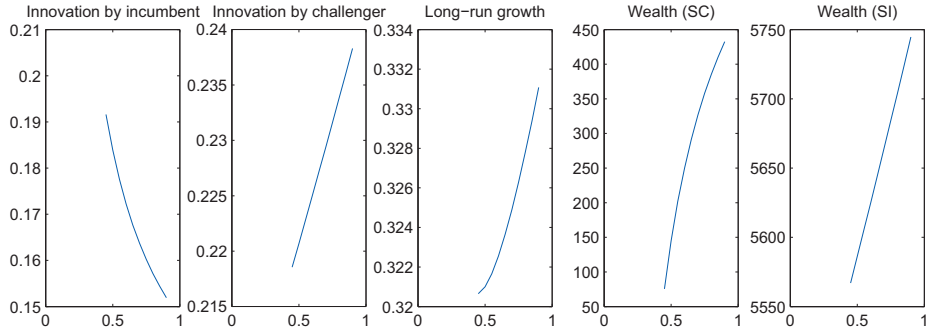
Figure 4: Effects of an increase in d under a **monopoly** price regime in the (SC) case $(r = 0.2, L = 200, F = 120, a = 0.7, \beta = 0.6, k = 2)$
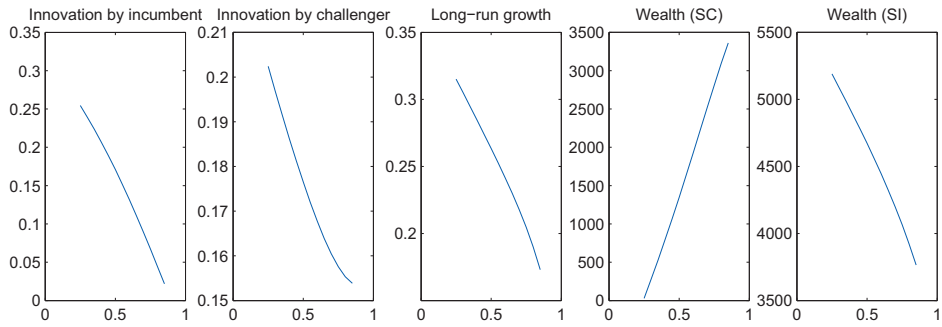

Figure 5: Effects of an increase in β under a **duopoly** price regime in the (SC) case $(r = 0.2, L = 200, F = 120, a = 0.7, d = 0.3, k = 2)$

ϕ_I decreases along d). In other words, we find a decreasing level of inequalities (modeled through variations in d) to have a positive impact on long-term growth when we have a monopoly market structure in the (SC) state.

In the case of a duopoly price regime in the (SC) state, the absence of explicit analytical expression of the different endogenous variables (cf. Appendix A) leads us to resort to numerical simulations. We carry out a sensitivity analysis along a wide array of values for parameters F, r, a and k, and the following numerical regularities emerge:

- *Numerical finding 5*: When the equilibrium market structure is a duopoly in the (SC) state, an increase in β leads to a decrease in both the incumbent's and the challengers' investment in R&D under a wide array of parametric cases. The long-run growth rate then unambiguously decreases (Figure 5).

- *Numerical finding 6*: When the equilibrium market structure is a duopoly in the (SC) state, an increase in d decreases the incumbent's but increases the challengers' investment in R&D *and* the long-run growth rate under a wide array of parametric cases (Figure 6).

Two main conclusions can be derived from the results presented in this section. First, an increase in the level of inequalities, whether it be through an increasing concentration
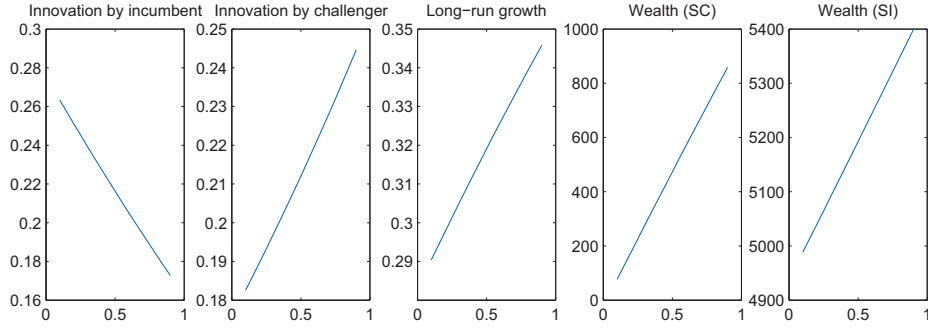
Figure 6: Effects of an increase in d under a **duopoly** price regime in the (SC) case ($r = 0.2$, $L = 200$, $F = 120$, $a = 0.7$, $\beta = 0.4$, $k = 2$)

of wealth among a small group of people (increasing β) or a more unequal distribution of overall wealth between rich and poor (decreasing d) is systematically detrimental for long-term growth. Indeed, whether we have a monopoly or a duopoly market structure in the (SC) state, our analytical results and our simulations show that a rise in β or a decrease in d lead to a decrease in the long-run growth rate of the economy. Second, reactions to a variation in the level of inequalities differ greatly between incumbents and challengers. A decrease in the level of inequalities through a rise in d for example systematically triggers opposite variations in the leader's and the challengers' R&D intensity, whether we have a monopoly or a duopoly price regime in the (SC) state: ϕ_I decreases, while ϕ_C rises. Hence, beyond the evolution of long-run growth, the level of inequalities also influences the *allocation* of the overall R&D effort among challengers and leaders: in most cases, greater disparities in wealth distribution imply that a bigger share of the overall R&D investment will be carried out by the incumbent. Indeed, a greater level of inequalities yields stronger negative externalities on the profits of the quality leader having innovated only once, and will increase its incentive to invest in R&D in order to be able to efficiently discriminate between rich and poor consumers.

We hence contribute to the analysis of the influence of wealth disparities on long-run growth operating through the demand side. Our results confirm the predictions obtained by Zweimuller and Brunner (2005) in a similar quality-ladder framework: a reduction in the level of inequalities leads to an increase in long-run growth. Furthermore, by being able to differentiate the impact of variations in the level of inequalities on the incumbent's and the challengers' investment in R&D, we exemplify a so far overlooked influence of wealth distribution on the allocation of R&D spending between the leader and the challengers. Finally, it is interesting to notice that while we obtain a negative relationship between inequalities and growth in a Schumpeterian creative destruction context, Foellmi and Zweimuller (2006) had exemplified a positive one in an horizontal innovation framework. Such opposite results can be explained by the fundamental differences existing in the nature of innovation between the two frameworks, and its influence on the market structure.

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7   Conclusion

In this paper we provided two major contributions to the analysis of the impact of inequalities on long-term growth operating through the demand side. We first show that disparities in purchasing power justify investment in R&D by both leaders and challengers, providing a demand-driven rationale for innovation by incumbents. By introducing non-homothetic preferences in an otherwise standard quality-model, we show that the perspective to discriminate efficiently between consumers differing in their willingness to pay for quality is sufficient for the industry leader to overcome the Arrow (1962) effect and keep investing in R&D. The strictly positive innovation rate of the incumbent is here obtained with constant returns to R&D efforts and without any advantage of the incumbent in the R&D field (supply side), by allowing for income inequalities to generate different quality valuation of poor and rich consumers (demand side). Second, we then study the impact of a variation in the level of inequalities on long-run growth, and obtain a negative relationship between inequalities and growth. Finally, we show that the level of inequalities impacts not only the long-term growth rate, but also the allocation of the R&D effort between challengers and leaders.

Some lines of further work can be quickly sketched. An obvious extension to our model would be to treat the more general case of more than two types of consumers, in order for the incumbent to keep investing in R&D after the second successful race.[14] A model such as ours can also be extended to a two-country framework, in order to contribute to the developing literature studying the role of multi-product firms in international trade (Fajgelbaum et al., 2011): while the impact on growth of inter-industrial quality trade has already been extensively studied (product life cycle), we believe our framework would be a good starting point for the elaboration of a dynamic model of intra-industrial quality trade (quality life cycle).

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[14]Indeed, as already pointed out, the null investment in R&D by the incumbent in the (SI) state solely stems from the fact that we have only two distinct groups of consumers: once having offered two distinct price-quality bundles, the incumbent does not have any incentive to keep carrying out R&D.

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Appendix A

Monopoly in the (SC) state

We first notice that π_M and π_{DL} are linear functions of respectively Ω_{SC} and Ω_{SI}. The stationarized steady state Markovian equilibrium defined by equations (12), (13), (19) and (20) is hence a linear system of 4 equations with 4 unknowns which can be solved analytically:

$$\phi_C = \frac{(1-a)(k-1)L(k+1-\beta) - 2Fr(k-(k-1)d(1-\beta))}{F(k(k+1) - d(k-1)(1-\beta))} \tag{23}$$

$$\phi_I = \frac{(1-a)L(\beta - (k-1)(1-\beta)) + Fr(k\beta - d^2(k-1)(1-\beta))}{F(k(k+1) - d(k-1)(1-\beta))} \tag{24}$$

$$\Omega_{SC} = \frac{w(d(1-\beta)((1-a)(k-1)L + Fkr) + k(Fkr - (1-a)\beta L))}{F(k(k+1) - d(k-1)(1-\beta))} \tag{25}$$

$$\Omega_{SI} = \frac{w(2Fk^2r + (1-a)(k-1)L(k+1-\beta))}{F(k(k+1) - d(k-1)(1-\beta))} > 0 \tag{26}$$

Since we have $a < 1$, $\beta < 1$ and $k > 1$, Ω_{SI} is positive without any further condition on the parameters. It can be seen from (23) and (25) that for both ϕ_C and Ω_{SC} to be simultaneously positive, we need for the ratio $\frac{L}{F}$ to respect the following condition:

$$\underbrace{\frac{2r(k-(k-1)d(1-\beta))}{(1-a)(k-1)(k+1-\beta)}}_{=LB} < \frac{L}{F} < \underbrace{\frac{kr(k+d(1-\beta))}{(1-a)(k\beta - d(1-\beta)(k-1))}}_{=UB} \tag{i}$$

Imposing $k \geq 2$ guarantees that the numerator and the denominator of the upper bound UB are respectively greater and smaller than the numerator and the denominator of the lower bound LB, hence ensuring that the interval $[LB, UB]$ is not empty. Finally, it can be seen from (24) that a sufficient (but not necessary) condition for ϕ_I to be positive is $k \leq \frac{1}{1-\beta}$, since it guarantees for the two terms of the numerator to be positive. We then have two conditions on k, that can be summarized as:

$$2 \leq k \leq \frac{1}{1-\beta} \tag{ii}$$

The interval $[2, \frac{1}{1-\beta}]$ is non empty provided $\beta > 0.5$. Hence, for $\beta > 0.5$ and under the parametric conditions (i)-(ii), the system $\{(12), (13), (19), (20)\}$ has a unique solution $(\phi_C, \phi_I, \Omega_{SC}, \Omega_{SI})$, all strictly positive.

Duopoly in the (SC) state

We first notice that π_L, π_F and π_{DL} can be re-expressed as $\pi_L = A_l + B_l\Omega_{SC}$, $\pi_F = A_f + B_f\Omega_{SC}$ and $\pi_{DL} = A_{dl} + B_{dl}\Omega_{SC}$, with:

$$
\begin{aligned}
A_l &= \left(\frac{k-1}{k^2}\right)(1-a)wL(k-1)(1-\beta), \quad B_l = \left(\frac{k-1}{k^2}\right)r(k(1-\beta d) + d(1-\beta)) \\
A_f &= \beta\left(\frac{k-1}{k}\right)(1-a)wL, \quad B_f = \beta\left(\frac{k-1}{k}\right)dr \\
A_{dl} &= \left(\frac{k-1}{k^2}\right)(1-a)wL(k+1-\beta), \quad B_l = \left(\frac{k-1}{k^2}\right)r(k+(1-\beta)d)
\end{aligned}
$$

We also have $A_l + A_f = A_{dl}$ and $B_l + B_f = B_{dl}$.

Considering the stationarized steady state Markovian equilibrium defined by the system of equations $\{(15),(16),(21),(22)\}$, it is possible to obtain the following expressions for ϕ_C and Ω_{SI}:

$$
\Omega_{SI} = \Omega_{SC} + \frac{wF\phi_I}{r - B_{dl}} \tag{27}
$$

$$
\phi_C = \frac{A_f}{wF} - r + \frac{B_f}{wF}\Omega_{SC} + \frac{B_{dl}}{r - B_{dl}}\phi_I \tag{28}
$$

Since $r - B_{dl} > 0$, we have $\Omega_{SI} > 0$ provided there exists an equilibrium with Ω_{SC} and ϕ_I positive. The sign of ϕ_C is on the other hand ambiguous, but a sufficient condition to ensure that $\phi_C > 0$ for Ω_{SC}, $\phi_I > 0$ is to impose $r < \beta\left(\frac{k-1}{k}\right)(1-a)\frac{L}{F}$.

Substituting for Ω_{SI} and ϕ_C using (27) and (28), the R&D free-entry condition in the (SC) state (15) and the labor equilibrium condition in the (SC) state (21) yield two implicit functions $\Omega_{SC} = \psi^R(\phi_I)$ and $\Omega_{SC} = \psi^L(\phi_I)$. ψ^R and ψ^L are implicitly defined by writing (15) and (21) respectively as $R(\phi_I, \Omega_{SC}) = 0$ and $L(\phi_I, \Omega_{SC}) = 0$, with:

$$
R(.) = A_f - A_l + (B_f - B_l)\Omega_{SC} + \frac{B_{dl}wF}{r - B_{dl}}\phi_I - \left(\frac{A_f - wFr + B_f\Omega_{SC} + \frac{B_{dl}wF\phi_I}{r - B_{dl}}}{A_f + B_f\Omega_{SC} + \frac{rwF\phi_I}{r - B_{dl}}}\right)(A_f + B_f\Omega_{SC})
$$

$$
L(.) = -A_l - rwF + (r - B_l)\Omega_{SC} + \frac{rwF}{r - B_{dl}}\phi_I
$$

Using implicit differentiation, we easily obtain $\frac{\partial\psi^L}{\partial\phi_I} = -\frac{(r-B_l)(r-B_{dl})}{wFr} < 0$. We then study the sign of $\frac{\partial\psi^L}{\partial\phi_I} = -\frac{\partial R/\partial\Omega_{SC}}{\partial R/\partial\phi_I}$. We have:

$$
\frac{\partial R}{\partial\Omega_{SC}} = \underbrace{B_f - B_l}_{<0} - B_f\underbrace{\left(\frac{A_f - wFr + B_f\Omega_{SC} + \frac{B_{dl}wF\phi_I}{r - B_{dl}}}{A_f + B_f\Omega_{SC} + \frac{rwF\phi_I}{r - B_{dl}}}\right)}_{>0 \text{ under the conditions ensuring } \phi_C > 0} - B_f(A_f + B_f\Omega_{SC})\underbrace{\left(\frac{wF(r + \phi_I)}{(A_f + B_f\Omega_{SC} + \frac{rwF\phi_I}{r - B_{dl}})}\right)}_{>0 \text{ for } \Omega_{SC},\ \phi_I > 0} < 0
$$

$$
\frac{\partial R}{\partial\phi_I} = \frac{B_{dl}wF}{r - B_{dl}} + \underbrace{(A_f + B_f\Omega_{SC})\left(\frac{wF(A_f + B_f\Omega_{SC} + r)}{(A_f + B_f + \frac{rwF\phi_I}{r - B_{dl}})^2}\right)}_{>0 \text{ for } \Omega_{SC},\ \phi_I > 0} > 0
$$

We can then conclude that $\frac{\partial\psi^L}{\partial\phi_I} > 0$ for Ω_{SC}, $\phi_I > 0$. Hence, in the plane (Ω_{SC}, ϕ_I)

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with $\Omega_{SC} > 0$ and $\phi_I > 0$, the function $\Omega_{SC} = \psi^R(\phi_I)$ (R-line) has a positive slope, and the function $\Omega_{SC} = \psi^L(\phi_I)$ (L-line) has a negative slope. We then only need to prove that those two lines indeed intersect for Ω_{SC}, $\phi_I > 0$. We consider the roots ϕ_{I0}^R and ψ_{I0}^L defined as $\psi^R(\phi_{I0}^R) = 0$ and $\psi^L(\phi_{I0}^L) = 0$. Computing those two roots amounts to solving for $R(\phi_{I0}^R, 0) = 0$ and $L(\phi_{I0}^L, 0) = 0$:

$$R(\phi_{I0}^R, 0) = 0 \quad \Leftrightarrow \quad \phi_{I0}^R = \frac{(A_l - A_f)(r - B_{dl})r \pm (r - B_{dl})wF\sqrt{r^2(A_l - A_f)^2 + 4B_{dl}rA_f(A_l - wFr)}}{2rB_{dl}(wF)^2}$$

$$L(\phi_{I0}^L, 0) = 0 \quad \Leftrightarrow \quad \phi_{I0}^L = \frac{(A_l + rwF)(r - B_{dl})}{rwF} > 0$$

Since we have demonstrated that ψ^R is monotonically decreasing in ϕ_I for ϕ_I, $\Omega_{SC} > 0$, the smaller of its two roots is necessarily negative. Comparing the positive root $\phi_{I0}^R = \frac{(A_l - A_f)(r - B_{dl})r + (r - B_{dl})wF\sqrt{r^2(A_l - A_f)^2 + 4B_{dl}rA_f(A_l - wFr)}}{2rB_{dl}(wF)^2}$ and the unique root of ψ^L that can be re-expressed as $\phi_{I0}^L = \frac{2wFB_{dl}(A_l + rwF)(r - B_{dl})}{2rB_{dl}(wF)^2}$, we then see that given the conditions on the parameters of the model, we necessarily have that $\phi_{I0}^R < \psi_{I0}^L$. This ensures that the R-line and the L-line have a unique intersection for $\Omega_{SC} > 0$ and $\phi_I > 0$.

We have hence demonstrated that for r small enough there exists a unique, positive equilibrium to the system $\{(15), (16), (21), (22)\}$ in $(\phi_C, \phi_I, \Omega_{SC}, \Omega_{SI})$, all strictly positive.

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