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## « Ambiguity and Optimal Technological Choice: Does the Liability Regime Matter? »

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# Ambiguity and Optimal Technological Choice: Does the Liability Regime Matter?\*

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## Abstract

We consider a firm, from a high-risk industry, facing two available technologies. One of the two technologies is *ambiguous* in the sense that its probability of accident lies in a interval of objective probabilities. The firm has the possibility to invest in seeking information in order to reduce the uncertainty inherent to the ambiguous technology. We apply a model inspired by Jaffray (1989) on imprecise probabilities to study the firm's behavior in such a context. Considering a strict liability rule, we compare the impact of two liability regimes, unlimited liability and limited liability, on the firm's technical choice. Whatever the firm's information seeking policy, which type of liability regime promotes which technology depends on the relative value of the marginal operating costs of the two technologies.

**Keywords:** Technical choice, technological risk, unlimited liability, limited liability, ambiguity.

**JEL Classification:** D21, D81, K32

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# 1 Introduction

Limited liability as the main legal liability regime for firms was adopted during the nineteenth century in order to “encourage capital investment in the massive aggregations that characterize modern corporate activity” (*dixit* [Alexander, 1992], p 390). This liability regime restricts the shareholders’ liability to the amount of their investment (whatever the magnitude of the firm’s liability). As a consequence, in the case of an accident, firms engaged in risky activities<sup>1</sup> have only to pay up to the net value of their assets. Considering this feature, some firms (or shareholders) may have an interest to undertake projects which have a negative net present value when taking into account all the technological risk that is transferred to Society. Trying to fight against these negative effects, some authors advocate for the introduction of unlimited liability<sup>2</sup>, thus holding shareholders personally liable<sup>3</sup> for damage. Other authors analyze the usefulness of extending the liability to the firm’s partners (e.g. banks). In the case of an accident, these partners will have to pay for damages if the concerned firm is not solvent enough to pay the damages in full<sup>4</sup>. Such a rule permits to increase the available funds for compensation, but it moves liability towards other operators. This may give less incentives to invest in prevention<sup>5</sup>.

Thus, since the seminal analyses of [Brown, 1973] and [Shavell, 1980], the field of *law and economics* analyzes the incentives provided by liability rules on agents (e.g. firms) to adopt adequate prevention measures to regulate their risky activities. Beyond organizing the compensation in the case of damage, liability rules are thus recognized as an indirect policy tool for risk regulation. This literature deals with incentives to technical change since increasing prevention measures to reduce the risk can be considered

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<sup>1</sup>e.g. the chemical or petrochemical industry, the energy industry, the explosives industry, . . .

<sup>2</sup>[Halpern *et al.*, 1980] advocate unlimited liability for small, closely-held corporations, [Leebron, 1991] and [Anonymous, 1986] for parent firms, [Hansmann & Kraakman, 1991] for all corporations. But these studies are lively discussed: see [Grundfest, 1992] and [Alexander, 1992] for arguments for retaining limited liability.

<sup>3</sup>Hence, shareholders’ personal assets can be confiscated for compensation if the firm is not sufficiently solvent to pay for damages.

<sup>4</sup>See [Klimek, 1990] for some practical cases involving US banks in the eighteens.

<sup>5</sup>See [Pitchford, 1995], [Boyer & Laffont, 1997], [Dionne & Spaeter, 2003], [Martimort & Hiriart, 2006].

to “small” technical change<sup>6</sup>. However, few papers deal with not perfectly known risks<sup>7</sup> and the possibility to invest in information acquisition to reduce the uncertainty. But nowadays technical innovations are more and more complex and a perfect knowledge about the risk they can impose on Society can require additional (and costly) research.

Hence technical risks, in particular when new technologies are introduced, can be imperfectly known. Several factors may be a source of ambiguity about the risk of accident, from purely technical (e.g. a lack of information about the reliability of the technology) to human and organizational factors (e.g. imperfect training to new operating systems, to new safety procedures, . . .).

[Ellsberg, 1961] defines the notion of *ambiguity* beyond the well-known distinction between “measurable uncertainty” or “risk” and “unmeasurable uncertainty”. According to him, three types of situations have to be distinguished: “risk” when probabilities are perfectly known, “complete ignorance” when nothing is known, and “ambiguity” when probabilities are not perfectly known but the decision maker still has an idea about them. Thanks to his urn experiments he shows that, in some situations, people might be affected by some *ambiguity aversion* which prevents them to build subjective probabilities, contrary to what was claimed by [Savage, 1954]. Jean-Yves Jaffray made a distinction between complete ignorance, risk, and “imprecise risk”. The two first situations are the same as the two first described by Ellsberg; these situations are very particular cases. The third one, which may be more often encountered, is a situation in which some information is available, but not sufficiently to define a unique probability distribution. In our paper, the firms face a new technology, the accident probability of

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<sup>6</sup>However, technical change is more deeply analyzed by *environmental economics* (see [Milliman & Prince, 1989], [Fischer *et al.*, 2003], [Magat, 1978], [Magat, 1979], [Parry, 1995]): different environmental policy tools are compared (but not liability rules) in terms of incentives to adopt a more efficient abatement technology. In the economic analysis of liability rules, technical changes are restricted to variations in risk prevention, say  $x$ , with *given* functions (prevention cost  $c(x)$ , probability of accident  $p(x)$ , with  $c'(x) > 0$ ,  $p'(x) < 0$ ,  $c(\cdot)$  and  $p(\cdot)$  given). No other technologies with different functions  $c_1(\cdot) \neq c(\cdot)$  and  $p_1(\cdot) \neq p(\cdot)$  are considered. To our knowledge, the first paper dealing with technical change (strictly speaking) and liability rules is [Endres & Bertram, 2006].

<sup>7</sup>To our knowledge, the only accident model under ambiguity has been developed by [Teitelbaum, 2007]. He compares strict liability and negligence on incentives to take risk prevention measures (under unlimited liability). The ambiguity results from a lack of confidence in a unique and objective probability of accident rather than an objective lack of precision about the probability of accident. So the degree of ambiguity is much more perceived as a psychological factor.

which lies within an interval of *objective* probabilities.

[Jaffray, 1989a], [Jaffray, 1989b] and [Jaffray, 1989c] develop a decision criterion under “imprecise risk”, i.e. under situations characterized by *imprecise probabilities*, lying in an interval of objective probabilities. His model consists in an extension of the Von Neumann - Morgenstern (VNM) utility function which includes a set of probability distributions allowing to take into account imprecise information about risk. Individuals’ preferences are represented by their VNM utility function, which represents their attitude towards risk, and by their Hurwicz pessimistic-optimistic perception index, which represents their attitude towards ambiguity<sup>8</sup>. An advantage of this model is that it is directly inspired by modeling methods adopted by engineers from risky industries (as power plants by instance)<sup>9</sup>. This kind of modeling, coming from the *evidence theory* initiated by [Dempster, 1967], is particularly used to analyze problems of *epistemic uncertainty*, i.e. uncertainty due to imprecise or incomplete information (e.g. in the risk assessment related to maintenance of industrial systems ([Fallet *et al.*, 2010])). Also, situations of imprecise risk and the application of Jaffray’s model go beyond “engineering contexts”: this model has been applied by [Eeckhoudt & Jeleva, 2004] to diagnostic risks and by [Jeleva, 1999] to insurance demand.

However, new information can sometimes enlighten the (technical) choice in ambiguous situations, particularly after technical tests for instance. To take into account the possibility to search for more information, and the consequences of additional information on the technical choice, is an important task notably because information acquisition in a context of imperfectly known risk could be required by a Regulator willing to apply the Precautionary Principle<sup>10</sup>. Contrary to some other papers<sup>11</sup>, the paper

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<sup>8</sup>Two particular cases of this model are expected utility (when there is no ambiguity, situation of risk) and the [Arrow & Hurwicz, 1972] criterion (when the interval of probabilities is  $[0, 1]$ ).

<sup>9</sup>To illustrate, see [Simon & Weber, 2008], [Fallet *et al.*, 2010] or [Magne & Vasseur, 2006] chapter 7 (about modeling methods developed by EDF).

<sup>10</sup>From the Rio Conference (1992), the Precautionary Principle states: “In order to protect the environment, the precautionary approach shall be widely applied by States according to their capabilities. Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation.” (Cf the *Report of the United Nations conference on environment and development*, <http://www.un.org/documents/ga/conf151/aconf15126-1annex1.htm>)

<sup>11</sup>To our knowledge, information seeking in a context of technical risk management was first stud-

at hand analyzes neither the investment decision, nor the different possible incentives to invest in information seeking. The question of whether a firm invests or does not invest in information seeking (and under what conditions) will be studied in a further paper (see [Jacob, 2011]). Here, we consider that the firms that have invested in information seeking receive an information signal about the risk of the new technology and choose to adopt or not to adopt the new technology in the light of this information; firms that have not invested keep their initial beliefs. Hence, we focus on the influence of the liability regime on the firms' technical choice strategy *given* their decision in terms of information seeking policy (and *given* the information available to them).

In order to study the firms' technical choice strategy in a situation of imprecise risk, we develop a model inspired by Jaffray [[Jaffray, 1989a],[Jaffray, 1989c]]. We consider that the firms were given the possibility to invest in information seeking; this information permits them to update their beliefs about the risk (and the profitability) of the “*ambiguous*” technology in a manner similar to Orset [[Orset, 2010],[Chemarin & Orset, 2010]]. In order to contribute to the debate on the relevance of the limited liability regime to regulate risky activities, the firms' choices are analyzed under limited and unlimited liability. Both regimes are used with a strict liability rule, i.e. the firm is liable for damage whenever an accident occurs, whatever her behavior. This rule is the easiest to apply and it is one of the most used to regulate activities that are dangerous to the environment<sup>12</sup>. We find conditions under which limited liability can provide less incentives to adopt the “ambiguous” technology than unlimited liability, given the available information.

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ied in the 1980's, notably by [Schwartz, 1985] and [McCardle, 1985]. [Shavell, 1992] studies investment in information seeking about an imperfectly known technical risk under different liability rules. [Chemarin & Orset, 2010] and [Orset, 2010] also analyze investment on information acquisition about a project the risk of which is uncertain.

<sup>12</sup>Strict liability was adopted in USA notably in the legislation CERCLA (Comprehensive Environmental Response, Compensation and Liability Act, 1980-1986), in response to concern about the release of hazardous substances from abandoned waste sites. The Council of Europe (Convention on Civil Liability for Damages Resulting from Activities Dangerous to the Environment (ETS No. 150).) and the European Union (Directive 2004/35/CE on environmental liability with regard to the prevention and remedying of environmental damage) have also adopted such a rule, even if the Directive 2004/35/CE allows the Member States to use the negligence rule in some particular cases (e.g. concerning the burden of remedial costs of environmental damages; or the damage to protected species and natural habitats).

We organize this paper as follows. Section 2 introduces the model and presents the firm’s choices under each liability regime. In section 3 we conduct a comparison between both liability regimes. Section 4 draw conclusions and points out further research.

## 2 Theoretical analysis

In this section, we first define the assumptions of the basic model. Then we present the two liability regimes and we determine the firm’s choices under each regime.

### 2.1 The basic model

We consider a firm in a competitive market selling a product at a price  $q$ . Demand is infinitely elastic. Consider the firm as being risk-neutral in order to focus on effects related to ambiguity. Her activity induces a large-scale risk of accident, the probability of occurrence of which depends on the chosen technology. The firm initially uses the technology  $A$  (the default technology) with a probability of accident,  $p_A$ , being perfectly known. A new technology, the technology  $B$ , is available in the industry but its probability of accident,  $p_B$ , is not perfectly known by Society: the state of the art is such that  $p_B$  lies in an interval of *objective* probabilities<sup>13</sup>  $[p_B^L; p_B^H]$ . So, the risk inherent to the technology  $B$  is imprecise in the sense of [Jaffray, 1989a]; the width of the interval  $[p_B^L; p_B^H]$  gives a measure of the degree of ambiguity<sup>14</sup>. The level of the damage is  $D$  whatever the technology and the level of activity. Hence, the random variable defining the risk of accident for the technology  $i$  can be denoted as  $\tilde{D}_i \equiv (1 - p_i, p_i; 0, D)$ ,  $i = A, B$ . The cost function for technology  $i$  is  $Z_i y_i^2$ , with  $y_i$  the level of activity and  $Z_i$  a cost parameter with  $Z_A < Z_B$  because of adoption costs inherent to a new technology<sup>15</sup>.

To avoid trivial cases, we assume  $0 < p_B^L < p_A < p_B^H < 1$  in a way to obtain  $E[\tilde{\Pi}_B]_{p_B=p_B^H} < E[\tilde{\Pi}_A] < E[\tilde{\Pi}_B]_{p_B=p_B^L}$ , i.e. the expected profit with the technology

<sup>13</sup>*Objective* probabilities means probabilities known and accepted by everybody.

<sup>14</sup>Smaller the interval, the less ambiguity about the value of  $p_B$ . Contrary to [Teitelbaum, 2007], the ambiguity is here a “physical” factor, and not a “psychological” one.

<sup>15</sup>Later we also discuss the case where  $Z_B < Z_A$ , the new technology being thus more efficient in terms of operating cost.

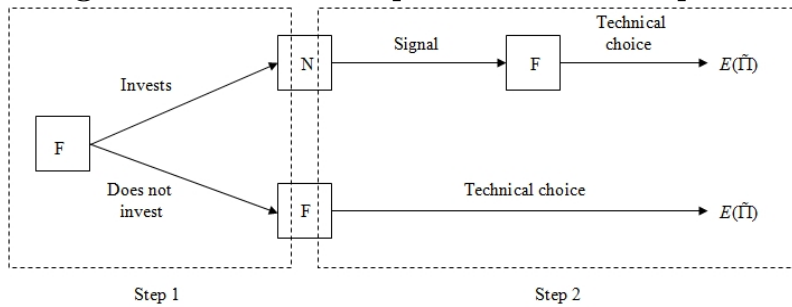
$B$  when  $p_B = p_B^H$  is smaller than the one with the technology  $A$  (the expected profit with  $B$  when  $p_B = p_B^L$  is higher than the one with  $A$ ). Thus, it is possible to define a “threshold probability” of accident for technology  $B$ , say  $p_B^T$  for instance, which induces the same expected profit with technology  $A$  and technology  $B$ . From there, two states of Nature about the true  $p_B$  can be defined:

- State  $l$ : the true  $p_B$  is smaller than the *threshold probability*  $p_B^T$
- State  $h$ : the true  $p_B$  is higher than the *threshold probability*  $p_B^T$

Thus, if the economy is in state  $l$ , we have  $E[\tilde{\Pi}_B] > E[\tilde{\Pi}_A]$  and the firm would choose the technology  $B$  if she had the information about the true state (but without knowing  $p_B$  perfectly). If the economy is in state  $h$ , we have  $E[\tilde{\Pi}_B] < E[\tilde{\Pi}_A]$  and the firm would choose the technology  $A$ .

Initially, the firm does not know which of  $l$  or  $h$  is the true state of Nature. The firm has *prior* beliefs about the likelihood of these states. We assume the firm is given the possibility to undertake information acquisition to obtain more information about the true state (see below Diagram 1, step 1).

**Diagram 1: The two steps of the decision process**



with  $F$  denoting the firm,  $N$  denoting Nature.

As noted above, in the paper at hand we consider the decision whether to invest or not to invest in information seeking (step 1) as *given*. So we focus on the technical choice *given* the decision concerning the information seeking policy and, therefore, *given* the



information available to the firm (step 2).<sup>16</sup>

We suppose that information seeking has a monetary cost,  $\bar{I}$ . The information seeking process consists, for instance, in technical tests, or reliability tests. Hence, we suppose that the time elapsing before receiving information is relatively short, so that we consider a unique horizon and no discount factor. Information consists in an imperfect signal  $\theta^j$  meaning “the true state is  $j$ ”, with  $j = l, h$ .

Let  $f$  be an exogenous variable that takes values in  $[\frac{1}{2}, 1]$ . When receiving a signal, the firm observes  $f$  and knows the signal is true at  $f\%$ ;  $f$  represents the degree of confidence of the signal. Hence if  $f = \frac{1}{2}$ , the signal is not informative. If  $f = 1$ , it is perfectly informative.

Hence, if the firm has decided not to invest in information seeking, her beliefs about states  $h$  and  $l$  remain unchanged (the firm keeps her *prior* beliefs). If she has decided to invest, the firm updates her beliefs of being in a given state ( $l$  or  $h$ ) thanks to Bayes’ rule.

In the presence of ambiguity about the risk of accident, we assume that the firm builds beliefs on the occurrence of an accident in a manner similar to [Jaffray, 1989a] model. In this model, the set of probability distributions consistent with the available information about the concerned variable (here  $\tilde{D}$ , when technology  $B$  is used) is characterized by its lower envelope (the minimum probability of each event consistent with the available information). This lower envelope is associated with a function, its Möbius inverse, which permits to determine the *belief assignment* about the different possible events<sup>17</sup>. When there are only two possible events (0 and  $D$ ), the interpretation is easy.

If, for instance, the firm were able to know with certainty that state  $h$  is the true one, then information about the true  $p_B$  would be:

- Event “accident” ( $\tilde{D} = D$ ) occurs at least with probability  $p_B^T$  (the *threshold probability*),

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<sup>16</sup>The decision of whether to invest or not to invest in information seeking (Step 1), *given* the technical choice strategy we define in this paper, is treated in [Jacob, 2011].

<sup>17</sup>For a clear explanation, see [Lefèvre, 2001], [Simon & Weber, 2008] and [Fallet *et al.*, 2010].

- Event “no accident” ( $\tilde{D} = 0$ ) occurs at least with probability  $(1 - p_B^H)$ ,
- but the remaining probability mass  $(1 - (1 - p_B^H) - p_B^T = p_B^H - p_B^T)$  cannot objectively be assigned to one or to the other event. So, considering the Möbius inverse, it is assigned to the union<sup>18</sup> of the two events depending on an “optimistic-pessimistic” Hurwicz’s index [Hurwicz, 1951]. Formally we have:  $(p_B^H - p_B^T)[\alpha D + (1 - \alpha)0]$ ,  $\alpha \in ]0, 1[$

Therefore, the firm’s belief about the true  $p_B$  knowing that state  $h$  occurs would be:  $\alpha p_B^H + (1 - \alpha)p_B^T$ .

Following the same reasoning, if the state  $l$  turned out to be the true one, the firm’s belief about the true  $p_B$  would be:  $\alpha p_B^T + (1 - \alpha)p_B^L$ .

As noted by [Giraud & Tallon, 2009], by construction the set of priors coincides with the available information provided by the objective probabilities. Interpretation is easy, the coefficient  $\alpha$  weighting the minimum expected profit<sup>19</sup> is thus a pessimism index: the higher  $\alpha$ , the higher the firm’s belief about the occurrence of the event “accident”.  $\alpha$  corresponds to the attitude towards ambiguity, which is clearly distinct from attitude towards risk (the characteristics of the VNM utility function). It is a psychological characteristic of the firm, helping to define her preferences. We suppose it to be constant.

Recall that we want to focus on the behavior of firms in risky industries. We can suppose that these firms analyze information in a way that is “more rational” than individuals: we can think that such firms have engineers team(s) specially trained to process information in the most “objective way”<sup>20,21</sup>. In this regard, this model seems to be particularly appropriate because, as noted before, it is directly inspired from modeling methods developed by engineers from risky industries.

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<sup>18</sup>See [Jaffray, 1989a] for theoretical foundations. See [Eeckhoudt & Jeleva, 2004] for an application.

<sup>19</sup>In a more general way, we would say “the coefficient weighting the minimum expected utility”. But here firms are risk neutral, so their utility is linear: expected utility is equivalent to expected profit.

<sup>20</sup>We can think of a team of engineers who submit various scenarios to the steering committee, which choose ones of them according to its pessimism ( $\alpha$ ).

<sup>21</sup>This also justifies risk neutrality: compared to individuals, firms have more (financial) resources and knowledge to put up with risk (or even to eliminate it, *via* diversification). However, ambiguity can not be eliminated; thus justifying some degree of optimism/pessimism.

Next, we turn to determine the firm's behavior depending on the liability regime that is enforced, namely an *unlimited* liability or a *limited* liability regime.

## 2.2 Behavior under unlimited liability

We first determine the implications of unlimited liability on the model. Then we study the firm's behavior when this regime is enforced.

### 2.2.1 Introducing the unlimited liability regime

In this part, we present the unlimited liability regime in our specific context. Recall that in this paper, we only consider a *strict* liability rule. So, the firm is liable for accident whenever a damage occurs, regardless of her *ex ante* behavior. But in this subsection we consider that liability is *unlimited*. This implies that the firm bears all the risk, even if it leads to a negative profit<sup>22</sup> in the case of an accident. As a result, for a given technology  $i = A, B$ , the firm's expected profit is:

$$E[\tilde{\Pi}_i^U(\alpha)] = qy_i - Z_i y_i^2 - p_i D - I^* \quad (1)$$

with  $I^*$  the optimal decision concerning the information seeking policy ( $I^* = 0, \bar{I}$ ) defined in Step 1.<sup>23</sup>

The private level of activity  $y_i^{U*}$  that is solution to (1) satisfies:  $y_i^{U*} = \frac{q}{2Z_i}$ . The optimal expected profit can be rewritten as:  $\frac{q^2}{4Z_i} - p_i D - I^*$ .

**Definition 1:** Let  $p_B^{TU}$  be the *threshold probability* of an accident occurring for technology  $B$ , under unlimited liability, that induces the same expected profit with technology  $A$  the technology  $B$ , i.e. for which:

$$\frac{q^2}{4Z_A} - p_A D = \frac{q^2}{4Z_B} - p_B^{TU} D$$

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<sup>22</sup>Negative profit means that shareholders will have to pay out of their pocket when the amount of damages is higher than the firm's net value in the case of an accident.

<sup>23</sup>This step is treated in [Jacob, 2011].

So we have:  $p_B^{TU} = \frac{q^2}{4Z_B} - \frac{q^2}{4Z_A} + p_A$ , with  $p_B^L < p_B^{TU} < p_B^H$ .

Note that the value of this threshold probability is independent from the firm's decision to invest or not to invest in information seeking.

A consequence of this definition is that state  $l$  is: "the true  $p_B$  lies in  $[p_B^L, p_B^{TU}]$ "; state  $h$  is such that  $p_B \in [p_B^{TU}, p_B^H]$ .

**Definition 2:** Let  $P(h, \alpha)$  be the firm's *prior* belief of being in state  $h$ ,  $(1 - P(h, \alpha))$  be the firm's *prior* belief of being in state  $l$ , with  $0 \leq P(h, \alpha) \leq 1$ ,  $\frac{\partial P(h, \alpha)}{\partial \alpha} > 0$ ,  $P(h, 0) = 0$ ,  $P(h, 1) = 1$ .

Hence, the firm's *prior* belief about the true  $p_B$  can be written as:

$$\hat{p}_B(\alpha) = P(h, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TU}] + (1 - P(h, \alpha))[\alpha p_B^{TU} + (1 - \alpha)p_B^L]$$

This corresponds to the firm's *prior* belief of being in state  $h$  times the firm's belief about  $p_B$  *knowing that* state  $h$  is the true one, plus the firm's *prior* belief of being in state  $l$  times the firm's belief about  $p_B$  *knowing that* state  $l$  is the true one.

If the firm has invested in information seeking ( $I^* = \bar{I}$ ), then she receives a signal  $\theta^j$  ( $j = l, h$ ) about the true state, with a reliability  $f \in [\frac{1}{2}, 1]$ . This signal permits the firm to update her *prior* beliefs of being in  $l$  or in  $h$  thanks to Bayes' rule<sup>24</sup>. As a result, we obtain the following *updated* beliefs:

- If  $\theta^h$  is received:

- $P(h|\theta^h, \alpha) = \frac{P(h, \alpha)f}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}$ ,
- $(1 - P(h|\theta^h, \alpha)) = \frac{(1 - P(h, \alpha))(1 - f)}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}$

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<sup>24</sup>Until now, there is no unique proper rule of conditioning under ambiguity (see [Chateauneuf *et al.*, 2010], [Jaffray, 1992]). But here, we can remark that the new information ( $\theta^j$ ) relates to the *probability* of being in state  $j = l, h$  (i.e.  $P(h, \alpha)$  and  $(1 - P(h, \alpha))$ ). Hence conditioning holds on events that are not affected by ambiguity, so Bayes' rule may apply.

- If  $\theta^l$  is received:

- $P(h|\theta^l, \alpha) = \frac{P(h, \alpha)(1-f)}{P(h, \alpha)(1-f) + (1-P(h, \alpha))f}$ ,
- $(1 - P(h|\theta^l, \alpha)) = \frac{(1-P(h, \alpha))f}{P(h, \alpha)(1-f) + (1-P(h, \alpha))f}$

And then, for a given signal  $\theta^j$  ( $j = l, h$ ), the updated belief about the true  $p_B$  becomes:

$$\hat{p}_B(\theta^j, \alpha) = P(h|\theta^j, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TU}] + (1 - P(h|\theta^j, \alpha))[\alpha p_B^{TU} + (1 - \alpha)p_B^L]$$

**Lemma 1** *If the firm has invested in information seeking then, for a given signal  $\theta^j$  ( $j = l, h$ ), we have:  $\hat{p}_B(\theta^l, \alpha) \leq \hat{p}_B(\alpha) \leq \hat{p}_B(\theta^h, \alpha)$*

**Proof.:** see the Appendix ♦

So, whatever  $\alpha \in ]0, 1[$ , a  $\theta^l$  signal reduces the firm's belief about  $p_B$  when  $f > \frac{1}{2}$ .  $\theta^h$  increases the firm's belief when  $f > \frac{1}{2}$ . When  $f = \frac{1}{2}$ , the signal is not informative and the firm keeps her prior belief.

Now we turn to the study of the firm's choices under unlimited liability.

### 2.2.2 Firm's choices under unlimited liability

In this part, we determine the firm's technical choice strategy for a given available information ( $\theta^l, \theta^h$ , no additional information). From Definition 1, we can deduce that the firm chooses the technology  $B$  ( $A$ ) if her belief about  $p_B$  is smaller (higher) than the threshold probability  $p_B^{TU}$ .

**Proposition 1** *Let us assume that (strict and) unlimited liability holds*

*When the firm has not invested in information seeking ( $I^* = 0$ ), there exists a value of  $\alpha$ , namely  $\check{\alpha} \in ]0, 1[$ , such that:*

- *The technology  $B$  is chosen by firms with  $\alpha < \check{\alpha}$*
- *The technology  $A$  is chosen by firms with  $\alpha > \check{\alpha}$*
- *Firms with  $\alpha = \check{\alpha}$  are indifferent between both technologies.*

$$\text{with } \check{\alpha} = \frac{(1-P(h,\check{\alpha}))(p_B^{TU}-p_B^L)}{P(h,\check{\alpha})(p_B^H-p_B^{TU})+(1-P(h,\check{\alpha}))(p_B^{TU}-p_B^L)}$$

**Proof.:** see the Appendix  $\blacklozenge$

In the following, we denote as “optimistic” (“pessimistic”) a firm characterized by an  $\alpha$  lower than  $\check{\alpha}$  (higher than  $\check{\alpha}$ ). In the following Proposition we consider the firms that have invested in information seeking.

**Proposition 2** *Let us assume that an unlimited liability regime holds and that the firm has invested in information seeking ( $I^* = \bar{I}$ )*

(i) *If the firm is optimistic ( $\alpha < \check{\alpha}$ ):*

- *Then she adopts the technology B when  $\theta^l$ , whatever  $f$*
- *Then she adopts the technology A when  $\theta^h$  and  $f > f_{omin}$ , she adopts B otherwise.*

$$\text{with } f_{omin} = \frac{(1-\alpha)(1-P(h,\alpha))(p_B^{TU}-p_B^L)}{\alpha P(h,\alpha)(p_B^H-p_B^{TU})+(1-\alpha)(1-P(h,\alpha))(p_B^{TU}-p_B^L)}, f_{omin} \in [\frac{1}{2}, 1], f_{omin} \text{ decreasing in } \alpha.$$

(ii) *If the firm is pessimistic ( $\alpha > \check{\alpha}$ ):*

- *Then she adopts the technology A when  $\theta^h$ , whatever  $f$*
- *Then she adopts the technology B when  $\theta^l$  and  $f > f_{pmin}$ , she adopts A otherwise.*

$$\text{with } f_{pmin} = \frac{\alpha P(h,\alpha)(p_B^H-p_B^{TU})}{\alpha P(h,\alpha)(p_B^H-p_B^{TU})+(1-\alpha)(1-P(h,\alpha))(p_B^{TU}-p_B^L)}, f_{pmin} \in [\frac{1}{2}, 1], f_{pmin} \text{ increasing in } \alpha.$$

**Proof.:** see the Appendix  $\blacklozenge$

From Proposition 2 point (ii), the adoption of the technology B by a pessimistic firm when  $\theta = \theta^l$  is subject to a minimum reliability of the information signal. On the contrary, from point (i), an optimistic firm can adopt the technology B even after having received a  $\theta^h$  signal if its reliability  $f$  is weak.

Besides it is easy to check that an increase in  $p_B^H$  leads to an increase in  $f_{pmin}$  and to a decrease in  $f_{omin}$ . This means the more dangerous the technology B (the higher  $p_B^H$ ), the higher the precision needed of  $\theta^l$  to convince the pessimistic firms to adopt the technology B (and a lower precision needed of  $\theta^h$  to convince the optimistic firms to adopt the technology A). Similarly, a decrease in  $p_B^L$  leads to an increase in  $f_{omin}$  and to a

decrease in  $f_{pmin}$ : when the technology  $B$  represents potentially a higher improvement in safety (the lower  $p_B^L$ ), pessimistic firms do not need a high precision of  $\theta^l$  to adopt the technology  $B$ , while a higher reliability of  $\theta^h$  is needed to convince the optimistic firms to adopt the technology  $A$ .

Now we turn to determine the firm's choices under limited liability.

## 2.3 Behavior under limited liability

As in the unlimited liability case, we first present the implications of limited liability on the model. Then we study the firm's behavior under this liability regime.

### 2.3.1 Introducing the limited liability regime

We still consider a *strict* based liability in the sense that the firm is always liable when damage occurs, whatever her behavior. But we consider now that liability is *limited*. Under such a regime, the injurer (firm) can benefit from an *ex post financial legal protection* in the case of an accident. More precisely, the firm is *ex ante* liable for the entire damage but, *ex post*, she will have to pay only up to her net present value. As a consequence, if the amount of damages exceeds the firm's net value, only this value can be confiscated for compensation: shareholders' personal assets are protected<sup>25</sup> and part of the damage is borne by the victims or Society. Therefore, the *ex post* profit of the firm can never be negative. In the paper at hand, we consider an amount of damage  $D$  such that the firm is pushed into bankruptcy in the case of an accident<sup>26</sup>: the firm always benefits from the legal protection, her profit is always equal to zero in the case of an accident.

As a result, for a given technology  $i$ , the firm's expected profit is:

$$E[\tilde{\Pi}_i^L(\alpha)] = (1 - p_i)(qy_i - Z_i y_i^2 - I^*) \quad (2)$$

<sup>25</sup>Shareholders' liability is limited to their investment in the firm.

<sup>26</sup>So we consider  $D \geq qy_i - Z_i y_i^2 - I^*$ ,  $i = A, B$ . In the opposite case, the firm's profit is strictly positive in the case of an accident: the firm internalizes the risk in full, which is equivalent to unlimited liability (see the previous case).

with  $I^*$  the optimal information seeking policy ( $I^* = 0, \bar{I}$ ) defined in Step 1.<sup>27</sup> This profit function means that profit is positive ( $qy_i - Z_i y_i^2 - I^* > 0$ ) when no accident occurs (probability  $(1 - p_i)$ ), and the profit is zero in a case of an accident ( $p_i$ ).

The private level of activity  $y_i^{L^*}$  that is solution to (2) is:  $y_i^{L^*} = \frac{q}{2Z_i}$ . The optimal expected profit can be rewritten as:  $(1 - p_i)(\frac{q^2}{4Z_i} - I^*)$ .

**Definition 3:** Let  $p_B^{TL}(I^*)$ , with  $I^* = (0, \bar{I})$ , be the *threshold probability* of accident for the technology  $B$ , under limited liability, that induces the same expected profit with technology  $A$  and technology  $B$ , depending on the firm's information seeking policy. So we have:

- $p_B^{TL}(I^* = 0)$  such that:  $(1 - p_B^{TL}(I^* = 0))(\frac{q^2}{4Z_B}) = (1 - p_A)(\frac{q^2}{4Z_A})$ , leading to 
$$p_B^{TL}(I^* = 0) = 1 - \left[ \frac{(1-p_A)\frac{q^2}{4Z_A}}{\frac{q^2}{4Z_B}} \right]$$
- $p_B^{TL}(I^* = \bar{I})$  such that:  $(1 - p_B^{TL}(I^* = \bar{I}))(\frac{q^2}{4Z_B} - \bar{I}) = (1 - p_A)(\frac{q^2}{4Z_A} - \bar{I})$ , leading to 
$$p_B^{TL}(I^* = \bar{I}) = 1 - \left[ \frac{(1-p_A)(\frac{q^2}{4Z_A} - \bar{I})}{\frac{q^2}{4Z_B} - \bar{I}} \right]$$

Contrary to what prevails under unlimited liability, the threshold probability determining the technical choice is here different as the firm invests or not in information seeking. This phenomenon appears because of the firm's insolvency in the case of an accident. Indeed, investing in information seeking represents a monetary investment that reduces the firm's wealth. As a consequence, this reduces the amount that can be confiscated for compensation in the case of an accident: investing \$1 in information seeking reduces by \$1 the amount of damages that can be claimed to the firm. Hence, the marginal cost of increasing the budget allocated to information seeking is  $(1 - p_i)$ ; it varies depending on the technology used<sup>28</sup>. As a consequence, this investment has an impact on the threshold probability that must induce the same expected profit with both technologies.

Hence under limited liability the firm's "vision" about both technologies is not the same,

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<sup>27</sup>See [Jacob, 2011].

<sup>28</sup>This phenomenon is similar to that described in [Beard, 1990] with monetary prevention (that reduces the firm's wealth) under limited liability. Note that under unlimited liability, the amount invested in information seeking also reduces the firm's wealth but it does *not* reduce the amount of damages that can be claimed because of the possibility to confiscate shareholders' personal assets.



as she invests or not in information seeking. Consider a firm which has invested in information seeking ( $I^* = \bar{I}$ ). State  $l$  is “the true  $p_B$  lies in  $[p_B^L, p_B^{TL}(I^* = \bar{I})]$ ”, state  $h$  is such that  $p_B \in [p_B^{TL}(I^* = \bar{I}), p_B^H]$ . When a signal  $\theta^j$  ( $j = l, h$ ) arrives, she updates her *prior* beliefs of being in state  $l$  or  $h$  thanks to Bayes’ rule. Consider now a firm which has not invested in information seeking ( $I^* = 0$ ). For her, state  $l$  is “the true  $p_B$  lies in  $[p_B^L, p_B^{TL}(I^* = 0)]$ ”, state  $h$  is such that  $p_B \in [p_B^{TL}(I^* = 0), p_B^H]$ . She builds *prior* beliefs about these two states (that will never be updated). Note that, depending on whether the firm invests or not in information seeking, her prior beliefs of being in state  $l$  or  $h$  are not the same because these states are different depending on whether she has invested or not.<sup>29</sup>

**Definition 4:** Consider a firm which has invested in information acquisition under a limited liability regime. Before receiving any signal,  $P^L(h, \alpha)$  is her *prior* belief of being in state  $h$ ,  $(1 - P^L(h, \alpha))$  is her *prior* belief of being in state  $l$ , with  $0 \leq P^L(h, \alpha) \leq 1$ ,  $\frac{\partial P^L(h, \alpha)}{\partial \alpha} > 0$ ,  $P^L(h, 0) = 0$ ,  $P^L(h, 1) = 1$ .

Hence, under limited liability, the *prior* belief about the true  $p_B$  of a firm which decides to invest in information seeking can be written as:

$$\hat{p}_B(\alpha) = P^L(h, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TL}(I^* = \bar{I})] + (1 - P^L(h, \alpha))[\alpha p_B^T(I^* = \bar{I}) + (1 - \alpha)p_B^L] \quad (3)$$

When the firm receives a signal, she updates her *prior* beliefs. We obtain:

- If  $\theta^h$  is received:

- $P^L(h|\theta^h, \alpha) = \frac{P^L(h, \alpha)f}{P^L(h, \alpha)f + (1 - P^L(h, \alpha))(1 - f)}$ ,
- $(1 - P^L(h|\theta^h, \alpha)) = \frac{(1 - P^L(h, \alpha))(1 - f)}{P^L(h, \alpha)f + (1 - P^L(h, \alpha))(1 - f)}$

- If  $\theta^l$  is received:

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<sup>29</sup>However, note that the firm’s prior belief about the true value of  $p_B$ , namely  $\hat{p}_B(\alpha)$ , is not affected by the decision to invest or not in information seeking, nor by the liability regime ( $\hat{p}_B(\alpha)$  is the same under unlimited and limited liability). The same remark applies for updated beliefs about  $p_B$  (i.e.  $\hat{p}_B(\theta^j, \alpha)$ ).

- $P^L(h|\theta^l, \alpha) = \frac{P^L(h, \alpha)(1-f)}{P^L(h, \alpha)(1-f) + (1-P^L(h, \alpha))f}$ ,
- $(1 - P^L(h|\theta^l, \alpha)) = \frac{(1-P^L(h, \alpha))f}{P^L(h, \alpha)(1-f) + (1-P^L(h, \alpha))f}$

Her updated belief about the true  $p_B$  is:

$$\hat{p}_B(\theta^j, \alpha) = P^L(h|\theta^j, \alpha)[\alpha p_B^H + (1-\alpha)p_B^{TL}(I^* = \bar{I})] + (1-P^L(h|\theta^j, \alpha))[\alpha p_B^{TL}(I^* = \bar{I}) + (1-\alpha)p_B^L]$$

As for Lemma 1, we can easily check that receiving  $\theta^l$  decreases (receiving  $\theta^h$  increases) the firm's belief about the true  $p_B$  when  $f > \frac{1}{2}$ .

Regarding a firm which has not invested in information seeking, the expression of her prior belief (that will never be updated) about the true  $p_B$  is close to (3), with  $p_B^{TL}(I^* = 0)$  as threshold probability and different prior beliefs about states  $l$  and  $h$ .

Now we turn to the study of the firm's choices when a limited liability is enforced.

### 2.3.2 Firm's choices under limited liability

As for the unlimited liability case, we determine the firm's technical choice strategy given the available information, i.e. when she has invested in information acquisition and receives  $\theta^l$ , when she receives  $\theta^h$  and when she has not invested. From Definition 3, we can deduce the firm's decision criterion concerning the technical choice. When the firm has invested ( $I^* = \bar{I}$ ), she chooses the technology  $B$  ( $A$ ) if her updated belief  $\hat{p}_B(\theta^j, \alpha)$ ,  $j = l, h$ , is smaller (higher) than  $p_B^{TL}(I^* = \bar{I})$ . When  $I^* = 0$ , she chooses the technology  $B$  ( $A$ ) if her belief  $\hat{p}_B(\alpha)$  is smaller (higher) than  $p_B^{TL}(I^* = 0)$ .

#### Lemma 2

If  $Z_B > Z_A$ , then  $p_B^{TL}(I^* = \bar{I}) < p_B^{TL}(I^* = 0)$

**Proof.** :  $\frac{\partial p_B^{TL}(I^* = \bar{I})}{\partial I} = -\frac{(1-p_A)[\frac{q^2}{4Z_A} - \bar{I} - (\frac{q^2}{4Z_B} - \bar{I})]}{(\frac{q^2}{4Z_B} - \bar{I})^2} < 0$  ♦

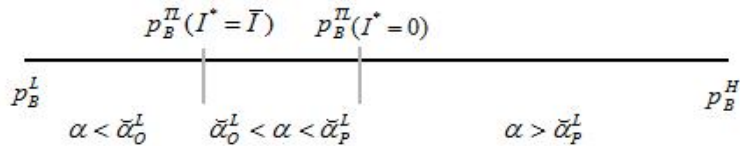
This Lemma states that, under limited liability, if the ambiguous technology has the highest marginal operating cost (i.e.  $Z_B > Z_A$ ), then the threshold probability that makes the firm indifferent between the two available technologies decreases when the firm invests in information seeking. In other words, when the firm invests in information

seeking, there is a smaller set of parameters (fewer beliefs about the true  $p_B$ ) allowing for the adoption of the technology  $B$ .

This phenomenon can be explained in the following manner. Recall that we assume the firm goes bankrupt in the case of an accident. So the expected profit under limited liability can be written as:  $(1 - p_i)(qy_i^* - Z_i y_i^{*2} - I^*)$ . Given  $Z_B > Z_A$ , the net revenue from activity,  $qy_i^* - Z_i y_i^{*2}$ , is lower with the technology  $B$  than with the technology  $A$ . Thus,  $p_B^{TL}(I^*)$  must be smaller than  $p_A$  to ensure equality between the expected profits of both technologies. Hence, for all beliefs about  $p_B$  that ensure the adoption of the technology  $B$ , the likelihood of bankruptcy is smaller with the technology  $B$  than with the technology  $A$ . As a consequence, the marginal cost of increasing the budget allocated to information seeking,  $(1 - p_i)$ , is higher with the technology  $B$  than with the technology  $A$ .<sup>30</sup> Investing in information seeking being more costly with the technology  $B$ , investors thus demand as a “compensation” a smaller cost of the risk.

As a consequence, contrary to what prevails under unlimited liability, we can distinguish three types of firms (see the Diagram and the Definition below).

**Diagram 2: types of firms under limited liability**



**Definition 5:** Let us define three types of firms under limited liability:

- We call “optimistic” a firm characterized by  $\alpha < \check{\alpha}_O^L$ , with  $\check{\alpha}_O^L$  such that  $\hat{p}_B(\check{\alpha}_O^L) = p_B^{TL}(I^* = \bar{I})$
- We call “moderate” a firm characterized by  $\check{\alpha}_O^L < \alpha < \check{\alpha}_P^L$ , with  $\check{\alpha}_P^L$  such that  $\hat{p}_B(\check{\alpha}_P^L) = p_B^{TL}(I^* = 0)$
- We call “pessimistic” a firm characterized by  $\alpha > \check{\alpha}_P^L$

These three types of firms have different attitudes regarding the technical choice strategy.

<sup>30</sup>Indeed, the decrease in the marginal cost of investing (because of lower damages to be paid in the case of an accident) is higher with the technology  $A$  than with the technology  $B$ .

**Proposition 3** *Let us assume that a limited liability regime holds.*

(i) *If the firm is optimistic ( $\alpha < \check{\alpha}_O^L$ ):*

- *Then she adopts the technology B when  $I^* = 0$*
- *Then she adopts the technology B when  $\theta^l$ , whatever  $f$  ( $I^* = \bar{I}$ )*
- *Then she adopts the technology A when  $\theta^h$  and  $f > f_{omin}^L$ , she adopts the technology B otherwise ( $I^* = \bar{I}$ )*

with  $f_{omin}^L = \frac{(1-\alpha)(1-P^L(h,\alpha))(p_B^{TL}(I^*=\bar{I})-p_B^L)}{\alpha P^L(h,\alpha)(p_B^H-p_B^{TL}(I^*=\bar{I}))+(1-\alpha)(1-P^L(h,\alpha))(p_B^{TL}(I^*=\bar{I})-p_B^L)}$ ,  $f_{omin}^L \in [\frac{1}{2}, 1]$ ,  $f_{omin}^L$  decreasing in  $\alpha$

(ii) *If the firm is moderate ( $\check{\alpha}_O^L < \alpha < \check{\alpha}_P^L$ ):*

- *Then she adopts the technology B when  $I^* = 0$*
- *Then she adopts the technology A when  $\theta^h$ , whatever  $f$  ( $I^* = \bar{I}$ )*
- *Then she adopts the technology B when  $\theta^l$  and  $f > f_{pmin}^L$ , she adopts the technology A otherwise ( $I^* = \bar{I}$ )*

with  $f_{pmin}^L = \frac{\alpha P^L(h,\alpha)(p_B^H-p_B^{TL}(I^*=\bar{I}))}{\alpha P^L(h,\alpha)(p_B^H-p_B^{TL}(I^*=\bar{I}))+(1-\alpha)(1-P^L(h,\alpha))(p_B^{TL}(I^*=\bar{I})-p_B^L)}$ ,  $f_{pmin}^L \in [\frac{1}{2}, 1]$ ,  $f_{pmin}^L$  increasing in  $\alpha$

(iii) *If the firm is pessimistic ( $\alpha > \check{\alpha}_P^L$ ):*

- *Then she adopts the technology A when  $I^* = 0$*
- *Then she adopts the technology A when  $\theta^h$ , whatever  $f$  ( $I^* = \bar{I}$ )*
- *Then she adopts the technology B when  $\theta^l$  and  $f > f_{pmin}^L$ , she adopts the technology A otherwise ( $I^* = \bar{I}$ )*

**Proof.** : see the Appendix ♦

We can highlight the “particular” behavior of “moderate” firms. Indeed, these firms *a priori* (before investing) prefer the technology B but, *ex post* (after investment and

reception of a signal), they are more likely to adopt the technology  $A$  in the sense that they impose a minimum reliability on a  $\theta^l$  signal to adopt the technology  $B$ , but not on a  $\theta^h$  signal (to adopt the technology  $A$ ). This particular case is only the consequence of  $p_B^{TL}(I^* = \bar{I}) < p_B^{TL}(I^* = 0)$ , i.e. the higher requirement to technology  $B$  in terms of cost of the risk when the firm invests in information acquisition.

Now we turn to directly compare the impact of both liability regimes on the firms' behavior.

### 3 Comparison of both liability regimes

In this section, we compare the firms' behavior under each liability regime. Given that the technical choice is driven by the value of the firm's belief about  $p_B$  relatively to a *threshold* probability, we first compare the different threshold probabilities that can be encountered, depending on the liability regime that is in force.

#### Lemma 3

If  $Z_B > Z_A$ , then  $p_B^{TL}(I^* = 0) < p_B^{TU}$

**Proof.:** see the Appendix ♦

In words, Lemmas 2 and 3 state that, if  $Z_B > Z_A$ , the threshold probabilities prevailing under limited liability are smaller than the probability prevailing under unlimited liability. To explain this difference, we have to consider different situations regarding the magnitude of the potential damage  $D$ , from  $D$  close to zero to higher values.

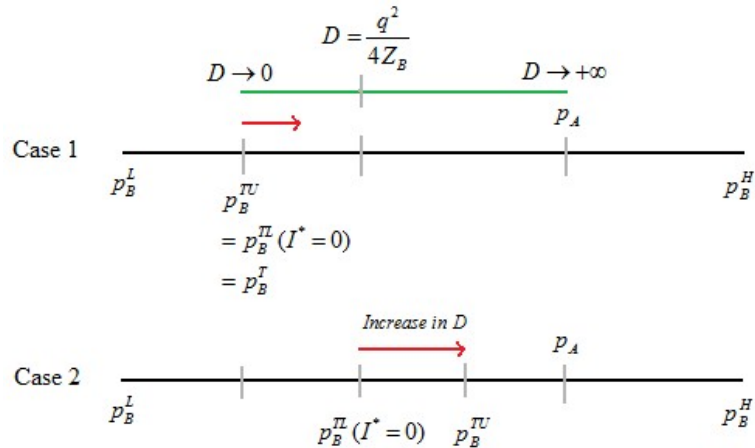
Diagram 3 below represents the possible values of the threshold probabilities (for both liability regimes) depending on the values of  $D$ . These values lie within the interval represented by the green line. In case 1 of Diagram 3 (below), we consider values of  $D$  lying in  $]0, \frac{q^2}{4Z_B}]$ . For these values of  $D$ , the firm is able to internalize the risk in full, even under a limited liability regime. As a consequence, expected profits are the same under both liability regimes; a unique threshold probability,  $p_B^T$  say, prevails<sup>31</sup>. Because

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<sup>31</sup>So we have:  $p_B^{TL}(I^*) = p_B^{TU} = p_B^T = \frac{q^2}{4Z_B} - \frac{q^2}{4Z_A} + p_A$

of  $Z_B > Z_A$  leading to a smaller net operating revenue with the technology  $B$ ,  $p_B^T$  must be smaller than  $p_A$  to ensure equality of expected profits between both technologies. Starting from  $D$  close to zero, a higher value of  $D$  increases the importance of the cost of the risk. This reduces the relative advantage of the technology  $A$  in terms of operating cost, so that the value of  $p_B^T$  increases (the red arrow on the Diagram)<sup>32</sup>. In case 2 of Diagram 3, we consider values of  $D$  higher than  $\frac{q^2}{4Z_B}$ . For these values of  $D$ , a distinction between limited liability and unlimited liability appears, the first regime being now characterized by a partial internalization of the risk. In this interval, higher the value of  $D$ , the higher the value of the threshold probability under unlimited liability,  $p_B^{TU}$ , because the “operating-cost advantage” of the technology  $A$  decreases further. But the value of the threshold probability under limited liability,  $p_B^{TL}(I^* = 0)$ ,<sup>33</sup> remains constant for  $D \geq \frac{q^2}{4Z_A}$ :<sup>34</sup> firms under limited liability do not care about  $D$  when  $D > \frac{q^2}{4Z_A}$  because their profit falls to zero in the case of an accident, whatever the value of  $D$ . Hence, the difference between  $p_B^{TU}$  and  $p_B^{TL}(I^* = 0)$  is explained by the fact that under unlimited liability, when  $D \geq \frac{q^2}{4Z_A}$ , an increase in the value of  $D$  reduces the “operating cost advantage” of the technology  $A$ , thus leading to an increase in  $p_B^{TU}$ ; while this phenomenon does not hold under limited liability because the firm’s profit in the case of an accident remains constant, whatever  $D$ .

**Diagram 3: Distinction between  $p_B^{TU}$  and  $p_B^{TL}(I^*)$**



<sup>32</sup>This relative decrease in the “operating-cost advantage” of the technology  $A$  permits the firms to relax the stronger requirement in terms of cost of the risk for the technology  $B$ .

<sup>33</sup>For this explanation we consider a firm which has not invested in information seeking.

<sup>34</sup> $p_B^{TL}(I^* = 0)$  is thus equal to the value of  $p_B^T$  in the case when  $p_B^T$  is such that:  $(1 - p_B^T) \frac{q^2}{4Z_B} = \frac{q^2}{4Z_A} - p_A D$ , with  $D = \frac{q^2}{4Z_A}$ .

A direct consequence of this Lemma is that, for a given  $\alpha$ , we have  $P^L(h, \alpha) > P(h, \alpha)$ : the *prior* belief of being in state  $h$  (higher expected profit with technology  $A$ ) is higher under limited liability than under unlimited liability<sup>35</sup>. Also, the following situation holds:

**Diagram 4: Types of firms depending on the liability regime**

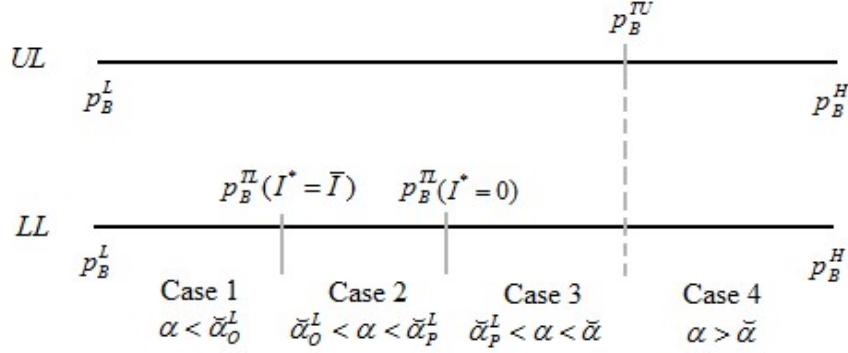


Diagram 4 permits to compare different “types of firms” depending on the value of their pessimism index  $\alpha$  and depending on the liability regime. Before explaining the four cases we find, note that the Diagram 4 permits us to make a conjecture. Given that the threshold probabilities under limited liability (LL) are smaller than the threshold probability prevailing under unlimited liability (UL), there exists a smaller set of parameters allowing for the adoption of the *ambiguous* technology ( $B$ ) under LL compared to under UL. This reasoning is more salient when considering that the types of firms ( $\alpha$ ) are uniformly distributed within the concerned industry: in that case, a smaller number of firms adopts the technology  $B$  under LL than under UL. But we are going to show that this assertion is valid whatever the firm’s information policy: it is less likely that a firm adopts the technology  $B$  under LL than under UL, whether this firm invests or not in information seeking.

**Lemma 4**

If  $p_B^{TL}(I^* = \bar{I}) < p_B^{TU}$ , then  $f_{pmin}^L > f_{pmin}$  and  $f_{omin}^L < f_{omin}$

**Proof.:** it directly follows from Lemma 2, Lemma 3 and  $P^L(h, \alpha) > P(h, \alpha)$ .

<sup>35</sup>Indeed, in a given interval  $[p_B^L, p_B^H]$ , if the threshold probability increases, then the belief to be in  $h$  decreases: there is a smaller set of beliefs for which the adoption of the technology  $A$  is possible; state  $h$  is thus less likely to occur. Hence  $p_B^{TL}(I^* = \bar{I}) < p_B^{TU}$  implies  $P^L(h, \alpha) > P(h, \alpha)$ .

**Proposition 4** *Let us consider that  $Z_B > Z_A$  and that  $D$  is sufficiently high to push the firm into bankruptcy in the case of an accident:*

(i) *when  $I^* = 0$ , it is less likely that a firm adopts the technology  $B$  under limited liability than under unlimited liability*

(ii) *when  $I^* = \bar{I}$ , whatever the signal  $(\theta^l, \theta^h)$ , the likelihood that a given firm (for a given value of  $\alpha$ ) adopts the ambiguous technology is smaller (or equal) under limited liability than under unlimited liability.*

The explanation of this Proposition is as follows. We consider all the possible situations regarding the information available to the firm. First, point (i) considers the case where the firm has not invested in information acquisition (and thus keeps her *prior* beliefs). For a given firm (with  $\alpha$  given), to change the liability regime does not change her technical choice except if  $\alpha \in [\check{\alpha}_P^L, \check{\alpha}]$  (i.e. if  $\hat{p}_B(\alpha) \in [p_B^{TL}(I^* = 0), p_B^{TU}]$ , case 3 in Diagram 4): in this case, a firm prefers the technology  $A$  under LL but adopts the technology  $B$  under UL. If we consider a population of sufficiently heterogeneous (and numerous) firms, having  $p_B^{TL}(I^* = 0) < p_B^{TU}$  ensures that it is less likely that a (randomly drawn) firm adopts the technology  $B$  under LL than under UL when  $I^* = 0$ .

Now we consider the cases where the firm has invested in information seeking (point (ii)). Contrary to the previous case, to switch from LL to UL when  $I^* = \bar{I}$  increases the likelihood of all types of firms to adopt the ambiguous technology,  $B$ . When  $\theta^l$  is received, we know from Lemma 4 that  $f_{pmin}^L > f_{pmin}$ . This implies that a higher reliability of a  $\theta^l$  signal is needed for the *most pessimistic* firms ( $\alpha > \check{\alpha}$ , i.e.  $\hat{p}_B(\alpha) > p_B^{TU}$ ], case 4 in Diagram 3) to adopt the technology  $B$  under LL relatively to UL. Concerning firms in cases 2 and 3 ( $\alpha \in [\check{\alpha}_O^L, \check{\alpha}]$ , or  $\hat{p}_B(\alpha) \in [p_B^{TL}(I^* = \bar{I}), p_B^{TU}]$ ), there is no condition on the reliability of  $\theta^l$  to adopt the technology  $B$  under UL while there exists such a condition under LL. So, for all firms of these types<sup>36</sup>, there is a smaller likelihood to adopt the technology  $B$  when  $\theta^l$  under LL than under UL.

When  $\theta^h$  is received, we know from Lemma 4 that  $f_{omin}^L < f_{omin}$ . This implies that a higher reliability of a  $\theta^h$  signal is needed for the *most optimistic* firms ( $\alpha < \check{\alpha}_O^L$ ,

<sup>36</sup>Obviously, the firms characterized by  $\alpha < \check{\alpha}_O^L$  (case 1 in Diagram 4) adopt the technology  $B$  when  $\theta^l$ , whatever  $f$ . Hence, in that case, the liability regime has no impact on their technical choice.



$\hat{p}_B(\alpha) \in [p_B^L, p_B^{TL}(I^* = \bar{I})]$ , case 1 in Diagram 4) to adopt the technology  $A$  under UL relatively to LL. Concerning firms in cases 2 and 3, there exists no condition on the reliability of a  $\theta^h$  signal (to adopt  $A$ ) under LL while such a condition exists when UL is applied. So, for all firms of these types<sup>37</sup>, there is a smaller likelihood to adopt the technology  $A$  when  $\theta^h$  under UL than under LL. By complementarity, there is a smaller likelihood to adopt the technology  $B$  when  $\theta^h$  under LL than under UL.

Hence, we have to underline the fact that all these results crucially depend on the *relative values* of the threshold probabilities under the different liability regimes. As stated in different Lemmas and Propositions, the relative values of the different threshold probabilities depend on the marginal operating costs of the technologies. Thus, if we assume instead that the technology  $B$  is more “cost-efficient” than the technology  $A$  ( $Z_B < Z_A$ ), then opposite results to those highlighted in this paper hold.

Consider the Lemma 3 when  $Z_B < Z_A$ . Again, for small values of  $D$  there exists a unique threshold probability  $p_B^T$  because the risk is fully internalized whatever the liability regime. The “cost-operating advantage” of the technology  $B$  explains a weaker requirement in terms of cost of the risk, so we have  $p_B^T > p_A$ . For higher values of  $D$ , this cost advantage is smaller: the value of  $p_B^T$  is smaller, closer to  $p_A$  (stronger requirement in cost of the risk for the technology  $B$ ). For  $D \geq \frac{q^2}{4Z_B}$ , there is a distinction between both liability regimes: high values of  $D$  lead to a decline in the cost advantage of  $B$  under unlimited liability (and to a small threshold probability  $p_B^{TU}$ ) while this phenomenon does not occur under limited liability because of insolvency in the case of an accident, whatever  $D \geq \frac{q^2}{4Z_B}$ . Finally consider Lemma 2 when  $Z_B < Z_A$ . We know that  $p_B^{TL}(I^* = 0) > p_A$ . Consider a prior belief  $\hat{p}_B(\alpha)$  such that  $\hat{p}_B(\alpha) = p_B^{TL}(I^* = 0)$ . Thus, the marginal cost of increasing the budget allocated to information seeking,  $(1 - p_i)$ , is smaller with the technology  $B$  than with the technology  $A$ . This lower cost with the technology  $B$  permits the firm to bear a higher cost of the risk with this technology (relatively to the technology  $A$ ), thus leading to  $p_B^{TL}(I^* = \bar{I}) > p_B^{TL}(I^* = 0)$

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<sup>37</sup>It is obvious that firms characterized by  $\alpha > \check{\alpha}$  (case 4 in Diagram 4) adopt the technology  $A$  when  $\theta^h$ , whatever  $f$  and whatever the liability regime in force.

### Corollary 1

*If the ambiguous technology has the highest (smallest) marginal operating cost, then the limited liability (the unlimited liability) promotes more “conservative” behavior.*

Indeed, generally speaking, we have shown that a firm is less likely to adopt the technology  $B$  under limited liability (than under UL) if this technology is costly to use: that’s we call *more conservative behavior*. In a similar manner, it is possible to show the opposite statement if the ambiguous technology is the less costly to use: the limited liability then promotes *less conservative* behavior than unlimited liability.

Finally, we can remark that a change in the liability regime has a higher impact on behavior of firms characterized by an “intermediate” value of  $\alpha$ , i.e.  $\alpha \in [\check{\alpha}_O^L, \check{\alpha}]$  (cases 2 and 3 in Diagram 4).

Firms characterized by  $\alpha \in [\check{\alpha}_O^L, \check{\alpha}_P^L]$  (case 2) are called *moderate* under LL and are called *optimistic* under UL. The liability regime has no impact on these firms when they have not invested in information seeking, but it has an impact when they decide to invest. In this latter case, under LL, a firm with  $\alpha \in [\check{\alpha}_O^L, \check{\alpha}_P^L]$  adopts an attitude similar to the firms that are called *pessimistic* under LL (i.e. to adopt the technology  $A$  except if a sufficiently reliable  $\theta^l$  is received) but, under UL, this kind of firm behave as the firms that are called *optimistic* (i.e. to adopt the technology  $B$  except if a sufficiently reliable  $\theta^h$  is received). Concerning the firms characterized by  $\alpha \in [\check{\alpha}_P^L, \check{\alpha}]$  (case 3), they adopt diametrically opposed behavior depending on whether a UL or a LL regime holds, even when they decide to not invest in information seeking. They are indeed called *optimistic* under UL but *pessimistic* under LL.

However the firms that have a more “extreme” attitude towards ambiguity are almost not affected by the liability regime, they keep a similar attitude whatever the liability regime. In case 1 ( $\alpha < \check{\alpha}_O^L$ ), the firms always prefer the technology  $B$  except if a sufficiently reliable  $\theta^h$  is received, whatever the liability regime. In case 4 ( $\alpha > \check{\alpha}$ ), the firms always prefer the technology  $A$  except if a sufficiently reliable  $\theta^l$  is received, whatever the liability regime. Nevertheless the liability regime has an impact on the level of the

required reliability of information signals to switch from one technology to the other.

## 4 Conclusion and discussion

This paper deals with a firm the activity of which drives a technological, large-scale risk of accident for Society that can push her into bankruptcy in the case of an accident. The firm has to choose between two technologies, one of them being characterized by an *imprecise* probability of accident in the sense of [Jaffray, 1989a]. This means that the true probability of accident lies in an interval of *objectives* probabilities in a manner that this *ambiguous* technology can be more or less profitable than the other one, depending on the value of the probability. Nevertheless the firm was given the possibility to invest in information acquisition to obtain a signal about the riskiness (and the profitability) of the ambiguous technology. The decision model we use is inspired by Jaffray's work on imprecise probabilities ([Jaffray, 1989a], [Jaffray, 1989b]) and by Orset [[Chemarin & Orset, 2010],[Orset, 2010]]. An advantage of Jaffray's modeling is that it is specifically designed to deal with *imprecise risks*; it permits to capture both attitude towards risk, with a VNM utility function, and attitude towards ambiguity, thanks to an Hurwicz's optimism-pessimism index.

Our main objective is to study the influence of the liability regime on the firm's technical choice strategy in the presence of ambiguity, *given* the information available to her. Two *strict based* liability regimes are compared: unlimited and limited. A strict liability rule holds the firm liable whenever damage occurs, regardless of her behavior. The limited liability regime permits the firm to limit the damages up to her net present value, while unlimited liability can lead to a negative profit when the value of the damages is higher than the firm's net present value; shareholders' personal assets are thus confiscated for compensation.

The important result of this paper is the following: depending on the relative values of the marginal operating costs of the two available technologies, limited liability can

promote a more or a less *conservative* attitude than unlimited liability, whatever the information available to the firm. In the presence of a (potential) damage that can push the firm into bankruptcy in the case of an accident, if the ambiguous technology has the highest marginal operating cost, then a firm is less likely to adopt the ambiguous technology under limited liability than under unlimited liability. We show that this assertion is valid whatever the information available to the firm. The opposite result holds if the ambiguous technology has the smallest marginal operating cost: the firm is more likely to adopt the ambiguous technology under limited liability than under unlimited liability.

An important feature underlying this result is the following: under unlimited liability, an increase in the amount of damages above the value that can push the firm into bankruptcy decreases the relative “operating-cost advantage” of one technology over the other one; while this feature does not hold under limited liability because of the insolvency of the firm (*judgment-proof*). As a consequence, for sufficiently important amounts of damage, there is a higher set of parameters allowing for the adoption of the less “cost-efficient” technology under unlimited liability relatively to limited liability (or a smaller set of parameters allowing for the adoption of the most “cost efficient” technology under unlimited liability relatively to limited liability).

Another important feature is the fact that, under limited liability, for a high level of damage, the criterion driving the technical choice is different depending on whether the firm invests or not in information acquisition, while this is not the case under unlimited liability. If the ambiguous technology is more costly to use, we find that investing in information seeking under limited liability reduces the set of parameters allowing for the adoption of this technology. The opposite result holds if the ambiguous technology is the less costly to use. This feature is explained by a change in the firm’s requirement on the cost of the risk of the ambiguous technology consequently to the investment in information seeking; the marginal cost of this investment under limited liability is indeed different depending on the technology used.

These results can provide new elements to the debate on the relevance of the limited liability regime for firms operating in high-risk sectors. Indeed, we can see that limited liability (relatively to unlimited liability) can promote a conservative attitude and discourage to adopt an ambiguous technology which could be more risky than the default one. This result weakens the traditional arguments about the detrimental effect of limited liability in terms of suboptimal prevention of the risk, or in terms of “too strong” incentives to participate in risky activities (see [Brown, 1973] or [Shavell, 1986] for instance). Here, in the presence of imprecise risk, limited liability does not always lead to adopt the technology that could be the more risky for Society, despite the possibility for the firm to benefit from gains (if the new technology is less risky than the default one) and to externalize part of the cost of the risk on Society (if the new technology is more dangerous than the default one). This is an important point concerning a subject of growing concern, namely the regulation of innovations the potential dangerousness of which is misperceived.

Moreover, the results found in this paper mainly depend on only two variables: the marginal operating costs of the two available technologies. Having knowledge of these variables could permit a Regulator of a high-risk industry to know the “general trend” concerning the firms’ technical choice, even if the firms have very different attitudes towards ambiguity. If the ambiguous technology is the more costly to use, the Regulator could promote a conservative (less conservative) attitude by enforcing limited liability (unlimited liability). The opposite treatment holds if the ambiguous technology is more cost-efficient. Note that we do not ask which is the optimal policy to induce. This depends on the Regulator’s (or Society’s) attitude towards ambiguity, which is out of the scope of our study. Indeed, the two possible policies (to promote a conservative attitude by encouraging to keep the technology *A*, or to facilitate the adoption of the technology *B*) could be justified because the ambiguous technology could be more dangerous, or it could represent a high improvement in safety compared to the “historical” technology.

In this paper we suppose a monetary cost of investment that reduces the firm’s

solvency. To consider instead a non-monetary investment effort has no impact under unlimited liability. Under this regime, all costs are perfectly internalized by firms. To consider or not to consider the cost of investment when comparing both technologies has no impact on the technical choice. However, under limited liability, we show that a monetary investment cost impacts the decision criterion concerning the technical choice (the “threshold probability” for technology  $B$ ). Under this regime, to consider a non-monetary effort in information seeking would lead to a unique threshold probability, as under unlimited liability. As a consequence, there would be no “moderate” firms. Nevertheless, because this unique threshold probability prevailing under limited liability is smaller than the one prevailing under unlimited liability, the main result of the paper would not be modified: whatever the firm’s information seeking policy, which type of liability regime promotes which technology depends on the relative value of the marginal operating costs of the two technologies.

Even if this result is interesting, this study could (and should) be extended. First, the paper at hand focus on the firms’ technical choice strategy given the available information to the firms. Even if this is an important point, above all a Regulator might want to promote information seeking, before driving the technical choice. Indeed, in such an ambiguous situation, a Regulator could be more or less hesitant concerning about which of the two available technologies to promote, but he might want that the firms adopt *precautionary* behavior by searching for more information before making any decision concerning their technical choice. This is equivalent to study the firms’ decision concerning the investment in information seeking, and thus to rolling back the decision tree of our model (resolving Step 1 in Diagram 1). This is the topic of another study (see [Jacob, 2011]). Secondly, this paper only deals with strict liability. Even if this kind of liability rule is increasingly used in the field of industrial and/or environmental risks, the negligence rule (i.e. when liability is subject to a fault of the injurer) is sometimes still applied, and its study would be desirable from a normative point of view. Thus it could be interesting to see to what extent a negligence rule could (or not) provide different

incentives, and to compare the results in the light of those from [Teitelbaum, 2007], who finds that a negligence rule is more robust to ambiguity than strict liability. Finally, some possible interactions are not taken into account. It could be interesting to consider an endogenous level of capital by allowing the firms to borrow funds from outside investors (like banks), and thus studying their behavior concerning risk management (technical choice and information seeking policy) especially when these investors can also be held (fully or partially) liable for damage.

## Appendix

**Proof of Lemma 1** Similarly to [Chemarin & Orset, 2010], Lemma 1:

After receiving  $\theta^h$ , the firm's belief about the true  $p_B$  is:

$$\hat{p}_B(\theta^h, \alpha) = P(h|\theta^h, \alpha)(\alpha p_B^H + (1 - \alpha)p_B^{TU}) + (1 - P(h|\theta^h, \alpha))(\alpha p_B^{TU} + (1 - \alpha)p_B^L)$$

Then we pose:

$$\begin{aligned} & P(h|\theta^h, \alpha)(\alpha p_B^H + (1 - \alpha)p_B^{TU}) + (1 - P(h|\theta^h, \alpha))(\alpha p_B^{TU} + (1 - \alpha)p_B^L) - \hat{p}_B(\alpha) \\ &= \frac{P(h, \alpha)f}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}(\alpha p_B^H + (1 - \alpha)p_B^{TU}) \\ &+ \frac{(1 - P(h, \alpha))(1 - f)}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}(\alpha p_B^{TU} + (1 - \alpha)p_B^L) \\ &- [P(h, \alpha)(\alpha p_B^H + (1 - \alpha)p_B^{TU}) + (1 - P(h, \alpha))(\alpha p_B^{TU} + (1 - \alpha)p_B^L)] \\ &\times \left[ \frac{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)} \right] \end{aligned}$$

After some algebraic manipulations, we obtain:

$$\frac{P(h, \alpha)(1 - P(h, \alpha))[(\alpha p_B^H + (1 - \alpha)p_B^{TU}) - (\alpha p_B^{TU} + (1 - \alpha)p_B^L)](2f - 1)}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)} \geq 0$$

because of  $f \geq \frac{1}{2}$ .

In the same manner, after receiving  $\theta^l$  the firm's belief about the true  $p_B$  is:

$$\hat{p}_B(\theta^l, \alpha) = P(h|\theta^l, \alpha)(\alpha p_B^H + (1 - \alpha)p_B^{TU}) + (1 - P(h|\theta^l, \alpha))(\alpha p_B^{TU} + (1 - \alpha)p_B^L)$$

After some algebraic manipulations, we find  $\hat{p}_B(\theta^l, \alpha) - \hat{p}_B(\alpha)$  equals to:

$$\frac{P(h, \alpha)(1 - P(h, \alpha))[(\alpha p_B^H + (1 - \alpha)p_B^{TU}) - (\alpha p_B^{TU} + (1 - \alpha)p_B^L)](1 - 2f)}{P(h, \alpha)(1 - f) + (1 - P(h, \alpha))f} \leq 0$$

because of  $f \geq \frac{1}{2}$ .

Finally we obtain:  $\hat{p}_B(\theta^l, \alpha) < \hat{p}_B < \hat{p}_B(\theta^h, \alpha)$ . ♦

### Proof of Proposition 1

$\alpha = \check{\alpha}$  is defined so that  $\hat{p}_B(\check{\alpha}) = p_B^{TU}$ , with  $\hat{p}_B(\check{\alpha}) = P(h, \check{\alpha})[(\check{\alpha} p_B^H + (1 - \check{\alpha})p_B^{TU}] + (1 - P(h, \check{\alpha}))(\check{\alpha} p_B^{TU} + (1 - \check{\alpha})p_B^L)$ . Some manipulations permit to obtain the value of  $\check{\alpha}$ . From Definition 1 and knowing  $\hat{p}_B(\check{\alpha})$  is increasing in  $\alpha$ , the Proposition is demonstrated. ♦

### Proof of Proposition 2

Point (i) considers optimistic firms. *Before* receiving any signal, the *prior* belief of an optimistic firm about the true  $p_B$  is such that:  $\hat{p}_B(\alpha) < p_B^{TU}$ . From Lemma 1 Point (i), we know that  $\hat{p}_B(\theta^l, \alpha) < \hat{p}_B(\alpha)$ . The first part of Point (i) is explained. Concerning the second part of this Point, an optimistic firm adopts the technology  $A$  iff  $\hat{p}_B(\theta^h, \alpha) > p_B^{TU}$ , i.e.:

$$\begin{aligned} & P(h|\theta^h, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TU}] + (1 - P(h|\theta^h, \alpha))[\alpha p_B^{TU} + (1 - \alpha)p_B^L] > p_B^{TU} \\ \Leftrightarrow & \frac{P(h, \alpha)f}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}(\alpha p_B^H + (1 - \alpha)p_B^{TU}) \\ & + \frac{(1 - P(h, \alpha))(1 - f)}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}(\alpha p_B^{TU} + (1 - \alpha)p_B^L) > p_B^{TU} \end{aligned}$$

After some algebraic manipulations we find:  $f_{omin} = \frac{(1 - \alpha)(1 - P(h, \alpha))(p_B^{TU} - p_B^L)}{\alpha P(h, \alpha)(p_B^H - p_B^{TU}) + (1 - \alpha)(1 - P(h, \alpha))(p_B^{TU} - p_B^L)}$

We can easily check that  $f_{omin} = \frac{1}{2}$  when  $\alpha = \check{\alpha}$ , and  $f_{omin} = 1$  when  $\alpha = 0$ . This complete Point (i).



Point (ii) considers pessimistic firms. The *prior* belief of a pessimistic firm about the true  $p_B$  is such that:  $\hat{p}_B(\alpha) > p_B^{TU}$ . From Lemma 1, we know that  $\hat{p}_B(\theta^h, \alpha) > \hat{p}_B(\alpha)$ . The first part of Point (ii) is explained. Then, a pessimistic firm adopts the technology  $B$  iff  $\hat{p}_B(\theta^l, \alpha) < p_B^{TU}$ , i.e.:

$$\begin{aligned} & P(h|\theta^l, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TU}] + (1 - P(h|\theta^l, \alpha))[\alpha p_B^{TU} + (1 - \alpha)p_B^L] < p_B^{TU} \\ \Leftrightarrow & \frac{P(h, \alpha)(1 - f)}{P(h, \alpha)(1 - f) + (1 - P(h, \alpha))f}(\alpha p_B^H + (1 - \alpha)p_B^{TU}) \\ & + \frac{(1 - P(h, \alpha))f}{P(h, \alpha)f + (1 - P(h, \alpha))(1 - f)}(\alpha p_B^{TU} + (1 - \alpha)p_B^L) < p_B^{TU} \end{aligned}$$

After some algebraic manipulations we find:  $f_{pmin} = \frac{\alpha P(h, \alpha)(p_B^H - p_B^{TU})}{\alpha P(h, \alpha)(p_B^H - p_B^{TU}) + (1 - \alpha)(1 - P(h, \alpha))(p_B^{TU} - p_B^L)}$ . We can easily check that  $f_{pmin} = 1$  when  $\alpha = 1$ , and  $f_{pmin} = \frac{1}{2}$  when  $\alpha = \check{\alpha}$ . This complete Point (ii).

◆

### Proof of Proposition 3

Point (i) considers optimistic firms ( $\alpha < \check{\alpha}_O^L$ ). Their *prior* belief about the true  $p_B$  is such that:  $\hat{p}_B(\alpha) < p_B^{TL}(I^* = \bar{I})$ . From Lemma 2, we know that  $p_B^{TL}(I^* = \bar{I}) < p_B^{TL}(I^* = 0)$ . So, when  $I^* = 0$ , the optimistic firms adopt the technology  $B$ . This is the first part of this Point. Also we know that  $\hat{p}_B(\theta^l, \alpha) < \hat{p}_B(\alpha)$ , which explains the second part of Point (i). Concerning the third part of this Point, when  $I^* = \bar{I}$  an optimistic firm adopts the technology  $A$  iff  $\hat{p}_B(\theta^h, \alpha) > p_B^{TL}(I^* = \bar{I})$ , i.e.:

$$\begin{aligned} & P^L(h|\theta^h, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TL}(I^* = \bar{I})] \\ & + (1 - P^L(h|\theta^h, \alpha))[\alpha p_B^{TL}(I^* = \bar{I}) + (1 - \alpha)p_B^L] > p_B^{TL}(I^* = \bar{I}) \\ \Leftrightarrow & \frac{P^L(h, \alpha)f}{P^L(h, \alpha)f + (1 - P^L(h, \alpha))(1 - f)}(\alpha p_B^H + (1 - \alpha)p_B^{TL}(I^* = \bar{I})) \\ & + \frac{(1 - P^L(h, \alpha))(1 - f)}{P^L(h, \alpha)f + (1 - P^L(h, \alpha))(1 - f)}(\alpha p_B^{TL}(I^* = \bar{I}) + (1 - \alpha)p_B^L) > p_B^{TL}(I^* = \bar{I}) \end{aligned}$$

After some algebraic manipulations we find:

$$f_{omin}^L = \frac{(1 - \alpha)(1 - P^L(h, \alpha))(p_B^{TL}(I^* = \bar{I}) - p_B^L)}{\alpha P^L(h, \alpha)(p_B^H - p_B^{TL}(I^* = \bar{I})) + (1 - \alpha)(1 - P^L(h, \alpha))(p_B^{TL}(I^* = \bar{I}) - p_B^L)}$$

Following the same reasoning as in Proposition 1, we can easily find:

$$\check{\alpha}_O^L = \frac{(1 - P^L(h, \check{\alpha}_O^L))(p_B^{TL}(I^* = \bar{I}) - p_B^L)}{P^L(h, \check{\alpha}_O^L)(p_B^H - p_B^{TL}(I^* = \bar{I})) + (1 - P^L(h, \check{\alpha}_O^L))(p_B^{TL}(I^* = \bar{I}) - p_B^L)}$$

Then we can easily check that  $f_{omin}^L = \frac{1}{2}$  when  $\alpha = \check{\alpha}_O^L$ , and  $f_{omin}^L = 1$  when  $\alpha = 0$ . This complete Point (i).

Point (ii) considers moderate firms ( $\check{\alpha}_O^L < \alpha < \check{\alpha}_P^L$ ). Their *prior* belief about the true  $p_B$  is such that:  $p_B^{TL}(I^* = \bar{I}) < \hat{p}_B(\alpha) < p_B^{TL}(I^* = 0)$ . Thus, by definition, these firms prefer the technology  $B$  when  $I^* = 0$ . When  $I^* = \bar{I}$ , technical choice under limited liability is defined by the value of the *updated* belief,  $\hat{p}_B(\theta^j, \alpha)$ ,  $j = l, h$ , relatively to the threshold probability  $p_B^{TL}(I^* = \bar{I})$ . Knowing that  $\hat{p}_B(\theta^h, \alpha) > \hat{p}_B(\alpha)$ , a moderate firm adopts the technology  $A$  when  $\theta^h$  is received, whatever  $f$ . Also, a moderate firm adopts the technology  $B$  iff  $\hat{p}_B(\theta^l, \alpha) < p_B^{TL}(I^* = \bar{I})$ , i.e.:

$$\begin{aligned} & P^L(h|\theta^l, \alpha)[\alpha p_B^H + (1 - \alpha)p_B^{TL}(I^* = \bar{I})] \\ & + (1 - P^L(h|\theta^l, \alpha))[\alpha p_B^{TL}(I^* = \bar{I}) + (1 - \alpha)p_B^L] < p_B^{TL}(I^* = \bar{I}) \\ \Leftrightarrow & \frac{P^L(h, \alpha)(1 - f)}{P^L(h, \alpha)(1 - f) + (1 - P^L(h, \alpha))f}(\alpha p_B^H + (1 - \alpha)p_B^{TL}(I^* = \bar{I})) \\ & + \frac{(1 - P^L(h, \alpha))f}{P^L(h, \alpha)(1 - f) + (1 - P^L(h, \alpha))f}(\alpha p_B^{TL}(I^* = \bar{I}) + (1 - \alpha)p_B^L) < p_B^{TL}(I^* = \bar{I}) \end{aligned}$$

After some algebraic manipulations we find:

$$f_{pmin}^L = \frac{\alpha P^L(h, \alpha)(p_B^H - p_B^{TL}(I^* = \bar{I}))}{\alpha P^L(h, \alpha)(p_B^H - p_B^{TL}(I^* = \bar{I})) + (1 - \alpha)(1 - P^L(h, \alpha))(p_B^{TL}(I^* = \bar{I}) - p_B^L)}$$

We can easily check that  $f_{pmin}^L = 1$  when  $\alpha = 1$ , and  $f_{pmin}^L = \frac{1}{2}$  when  $\alpha = \check{\alpha}_O^L$ . This complete Point (ii).

Point (iii) considers pessimistic firms ( $\alpha > \check{\alpha}_P^L$ ). Their *prior* belief about the true  $p_B$

is such that:  $\hat{p}_B(\alpha) > p_B^{TL}(I^* = 0)$ . By definition, such a firm prefers the technology  $A$  when  $I^* = 0$ . When  $I^* = \bar{I}$ , to have  $p_B^{TL}(I^* = \bar{I}) < p_B^{TL}(I^* = 0)$  and  $\hat{p}_B(\theta^h, \alpha) > \hat{p}_B(\alpha)$  explain the second part of this Point. Finally, a pessimistic firm adopts the technology  $B$  iff  $\hat{p}_B(\theta^l, \alpha) < p_B^{TL}(I^* = \bar{I})$ . This condition is the same as for moderate firms: the minimum reliability of the signal  $f$  to adopt the technology  $B$  when  $\theta^l$  is equal to  $f_{min}^L$ . This complete Point (iii).

◆

### Proof of Lemma 3

$p_B^{TL}(I^* = 0) < p_B^{TU}$  requires:

$$\begin{aligned}
1 - \frac{(1 - p_A) \frac{q^2}{4Z_A}}{\frac{q^2}{4Z_B}} &< \frac{\frac{q^2}{4Z_B} - \frac{q^2}{4Z_A}}{D} + p_A \\
\Leftrightarrow (1 - p_A)D - \frac{(1 - p_A)D \frac{q^2}{4Z_A}}{\frac{q^2}{4Z_B}} &< \frac{q^2}{4Z_B} - \frac{q^2}{4Z_A} \\
\Leftrightarrow (1 - p_A)D \left( \frac{q^2}{4Z_A} - \frac{q^2}{4Z_B} \right) &> \left( \frac{q^2}{4Z_A} - \frac{q^2}{4Z_B} \right) \frac{q^2}{4Z_B} \\
\Leftrightarrow (1 - p_A)D > \frac{q^2}{4Z_B} &\Leftrightarrow D > \frac{q^2}{4Z_B} - p_A D
\end{aligned}$$

By assumption, we suppose  $qy_i^* - Z_i y_i^{*2} - D < 0 \Leftrightarrow D > \frac{q^2}{4Z_i}$ ,  $i = A, B$ , so that the firm goes to bankruptcy in a case of an accident. Thus this condition is satisfied. ◆

## References

- [Alexander, 1992] Alexander, J.C. 1992. Unlimited shareholder liability through a procedural lens. *Harvard Law Review*, **106 (2)**, 387–445.
- [Anonymous, 1986] Anonymous. 1986. Liability of Parent Corporations for Hazardous Waste Cleanup and Damages. *Harvard Law Review*, **99(5)**, 986–1003.
- [Arrow & Hurwicz, 1972] Arrow, K., & Hurwicz, L. 1972. An optimality criterion for decision-making under ignorance. *Uncertainty and Expectations in Economics*, in: Carter, C.F. & Ford, J.L. (eds), Oxford, England: Basil Blackwell & Mott Ltd.

- [Beard, 1990] Beard, T.R. 1990. Bankruptcy and Care Choice. *Rand Journal of Economics*, **21**, 626–634.
- [Boyer & Laffont, 1997] Boyer, M., & Laffont, J.J. 1997. Environmental Risks and Bank Liability. *European Economic Review*, **41**, 1427–1459.
- [Brown, 1973] Brown, J.P. 1973. Towards an economic theory of liability. *The Journal of Legal Studies*, **2**, 323–349.
- [Chateauneuf *et al.*, 2010] Chateauneuf, A., Gajdos, T., & Jaffray, J-Y. 2010. Regular Updating. *Theory and Decision*, forthcoming.
- [Chemarin & Orset, 2010] Chemarin, S., & Orset, C. 2010. Innovation and Information Acquisition under Time Inconsistency and Uncertainty. *The Geneva Risk and Insurance Review*, doi:10.1057/grir.2010.9.
- [Dempster, 1967] Dempster, A.P. 1967. Upper and Lower Probabilities induced by a Multivalued Mapping. *Annals of Mathematical Statistics*, **38**, 325–339.
- [Dionne & Spaeter, 2003] Dionne, G., & Spaeter, S. 2003. Environmental risk and extended liability: The case of green technologies. *Journal of Public Economics*, **87**, 1025–1060.
- [Eeckhoudt & Jeleva, 2004] Eeckhoudt, L., & Jeleva, M. 2004. Décision médicale et probabilités imprécises. *Revue Economique*, **55(5)**, 869–882.
- [Ellsberg, 1961] Ellsberg, D. 1961. Risk, Ambiguity, and the Savage Axioms. *Quarterly Journal of Economics*, **75(4)**, 643–669.
- [Endres & Bertram, 2006] Endres, A., & Bertram, R. 2006. The development of care technology under liability law. *International Review of Law and Economics*, **26**, 503–518.
- [Fallet *et al.*, 2010] Fallet, G., Duval, C., Weber, P., & Simon, C. 2010. Characterization and propagation of uncertainties in complex socio-technical system risk analyses. *Paper presented in Workshop on the Theory of Belief Functions, Brest, France.*

- [Fischer *et al.*, 2003] Fischer, C., Parry, I.W.H., & Pizer, W.A. 2003. Instrument choice for environmental protection when technological innovation is endogenous. *Journal of Environmental Economics and Management*, **87**, 1025–1060.
- [Giraud & Tallon, 2009] Giraud, R., & Tallon, J.M. 2009. Are Beliefs a Matter of Taste? A case for Objective Imprecise Information. *CES Working Papers/Documents de Travail du Centre d’Economie de la Sorbonne*, **2009.86**.
- [Grundfest, 1992] Grundfest, J.A. 1992. The Limited Future of Unlimited Liability : A Capital Market Perspective. *The Yale Law Review*, **102 (2)**, 387–425.
- [Halpern *et al.*, 1980] Halpern, P., Trebilcock, M., & Turnbull, S. 1980. An Economic Analysis of Limited Liability in Corporation Law. *University of Toronto Law Journal*, **30 (2)**, 117–150.
- [Hansmann & Kraakman, 1991] Hansmann, H., & Kraakman, R. 1991. Toward Unlimited Shareholder Liability for Corporate Torts. *The Yale Law Review*, **100 (7)**, 1878–1934.
- [Hurwicz, 1951] Hurwicz, L. 1951. Optimality criteria for decision making under ignorance. *Cowles Commission discussion paper, Statistics*, **370**.
- [Jacob, 2011] Jacob, J. 2011. “High risk” firms’ behavior under ambiguity: Liability regime and investment in information seeking. *mimeo BETA*.
- [Jaffray, 1989a] Jaffray, J. Y. 1989a. Généralisation du critère de l’utilité espérée aux choix dans l’incertain régulier. *Recherche opérationnelle*, **23**, 237–267.
- [Jaffray, 1989b] Jaffray, J. Y. 1989b. Linear Utility Theory and Belief Functions: A Discussion. *P & M Curie (Paris 6) University Discussion Paper*.
- [Jaffray, 1989c] Jaffray, J. Y. 1989c. Linear Utility Theory for Belief Functions. *Operations Research Letters*, **8**, 107–112.
- [Jaffray, 1992] Jaffray, J-Y. 1992. Bayesian Updating and belief functions. *IEEE Transactions on Systems, Man and Cybernetics*, **22(5)**, 1144–1152.

- [Jeleva, 1999] Jeleva, M. 1999. Demand for Insurance, Imprecise Probabilities and Ambiguity Aversion. *In: 1st International Symposium on Imprecise Probabilities and Their Applications, Ghent, Belgium.*
- [Klimek, 1990] Klimek, J. 1990. Liability for Environmental Damages in Insolvencies: Bulora, Panamerica and Lamford. *Journal of Environmental Law and Practice*, **2**, 257–269.
- [Leebron, 1991] Leebron, D.W. 1991. Limited liability, Tort Victims, and Creditors. *Columbia Law Review*, **91**, 1565–1650.
- [Lefèvre, 2001] Lefèvre, E. 2001. *Fusion adaptée d'informations conflictuelles dans le cadre de la théorie de l'évidence. Application au diagnostic médical.* Ph.D. thesis, Institut National des Sciences Appliquées de Rouen.
- [Magat, 1978] Magat, W.A. 1978. Pollution Control and Technological Advance : A Dynamic Model of the Firm. *Journal of Environmental Economics and Management*, **5(1)**, 1–25.
- [Magat, 1979] Magat, W.A. 1979. The effects of environmental regulation on innovation. *Law and Contemporary Problems*, **43(1)**, 3–25.
- [Magne & Vasseur, 2006] Magne, L., & Vasseur, D. 2006. Risques industriels. Complexité, incertitude et décision: une approche interdisciplinaire. *Collection EDF R&D, Edition Lavoisier, Paris, France.*
- [Martimort & Hiriart, 2006] Martimort, D., & Hiriart, Y. 2006. The Benefits of Extended Liability. *Rand Journal of Economics*, **37(3)**, 562–582.
- [McCardle, 1985] McCardle, K. F. 1985. Information Acquisition and the Adoption of New Technology. *Management Science*, **31(11)**, 1372–1389.
- [Milliman & Prince, 1989] Milliman, S.R., & Prince, R. 1989. Firm incentives to promote technological change in pollution control. *Journal of Environmental Economics and Management*, **17(3)**, 247–265.

- [Orset, 2010] Orset, C. 2010. Irreversible investment and information acquisition under uncertainty. *INRA-AgroParisTech, Joint Research Unit in Public Economics, Working Papers*, **2010/01**.
- [Parry, 1995] Parry, I.W.H. 1995. Optimal pollution taxes and endogenous technological progress. *Resource and Energy Economics*, **17**, 69–85.
- [Pitchford, 1995] Pitchford, R. 1995. How liable should a lender be? The case of judgment-proof firms and environmental risks. *American Economic Review*, **85**, 1171–1186.
- [Savage, 1954] Savage, L.J. 1954. The Foundations of Statistics. *Wiley, New-York, USA*.
- [Schwartz, 1985] Schwartz, A. 1985. Products Liability, Corporate Structure and Bankruptcy : Toxic Substances and the Remote Risk Relationship. *The Journal of Legal Studies*, **14(3)**, 689–736.
- [Shavell, 1980] Shavell, S. 1980. Strict Liability Versus Negligence. *The Journal of Legal Studies*, **9(1)**, 1–25.
- [Shavell, 1986] Shavell, S. 1986. The Judgment Proof Problem. *International Review of Law and Economics*, **6(1)**, 45–58.
- [Shavell, 1992] Shavell, S. 1992. Liability and the Incentive to Obtain Information about Risk. *The Journal of Legal Studies*, **21(2)**, 259–270.
- [Simon & Weber, 2008] Simon, C., & Weber, P. 2008. Analyse de la fiabilité imprécise des systèmes par les réseaux de fonctions de croyance. *Document HAL-INRIA, hal-00281395, publié lors des 4ème Journées Francophones sur les Réseaux Bayésiens, Lyon, France*.
- [Teitelbaum, 2007] Teitelbaum, J.C. 2007. A Unilateral Accident Model under Ambiguity. *The Journal of Legal Studies*, **36(2)**, 431–477.

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