Bureau d'économie théorique et appliquée (BETA)
UMR 7522

Documents de travail

« Adverse Selection, Emission Permits and Optimal Price Differentiation ».

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Document de Travail n° 2010 - **07**

Mars 2010

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Adverse Selection, Emission Permits and Optimal Price Differentiation[1]

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8th March 2010

[1]We are particularly grateful to Yannick Gabuthy, Bertrand Koebel, Anta Ndoye and to participants at the 2009 AFSE Congress for their comments on this work.

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Abstract

In this paper, we focus on the adverse selection issue that prevails in an economy when the regulator is not able to observe the type of the abatement costs of the firms. The regulator decides the total level of emission that minimizes the total social cost and he sells them to the firms at some differentiated prices. When firms can hide their type relative to their true abatement costs, prices must not only minimize the social cost of the environmental policy. They must also induce the firms to reveal their true type. A striking point of our model is that there is no participation constraint for firms are compelled to be actors of the environmental policy. Another original result concerns the rent, which still benefits to low-cost types, but which appears to be a fee paid by high-cost types.

Key words: Regulation; adverse selection; emission permits; abatement costs; price differentiation.

JEL Classification: D82; H23; Q52.

1 Introduction

The number of articles focusing on the efficiency of instruments for controlling diffuse pollution has significantly increased in the last three decades. From some of them ((Weitzman 1974), (Roberts and Spence 1976), (Baumol and Oates 1988) and (Cropper and Oates 1992)) we know that regulating pollution by using taxes or emission permits reach the same optimum if information is complete and perfect in the economy. Indeed, if the regulator knows the abatement costs of the firms, he is able to fix the optimal total level of emission for Society and the price at which each unit of permits should be exchanged (this corresponds to the tax rate or to the price of an initial allocation).

From a practical view, this economic tool has been used in different countries to regulate nitrate use by the farmers. It is also considered by Sweden as a way to regulate CO_2 emissions. And it is currently being discussed in France. Indeed the french governement has been implementing a tax on each ton of emitted carbon, whatever its origin. Namely, consumers and firms of some sectors will have to pay (trough additional tax on gas, on heating pollutant energy, on goods which production emits CO_2, etc). A large debate is taking place about the 'right' level of the tax rate and also about the concerned sectors. While experts announced that a minimum price should be 30 euros per ton, the government has decided to propose 14 euros per ton, essentially for social acceptability and solvency features (those arguments prevailing in the citizens' mind during economic recession or stagnation). The french government has also proposed to redistribute the income tax to the consumers that are not able to adopt less pollutant technologies (in rural areas without collective transport networks or for households with low revenue, who are not able to change their heating technology in the short term,

...).

For the sake of simplicity and because it is an environmental measure rather than a fiscal one, the tax rate is unique and, thus, not differentiated with regard to the abatement costs of the firms or the consumers (even if, for this last category of agents, part of the tax will be redistributed). Nevertheless, we know from the theory that fixing a unique price can create some difficulties for firms (or consumers) with high abatement costs to achieve the appropriate level of emissions. In the meantime, other firms, with low abatement costs, could abate more pollution from a financial point of view, but they have no incentives to do so if the cost of pollution is not too high. One solution consists to differentiate unit prices in order to control for pollution internalization at a minimum cost. Prices should be related to the abatement costs of each type of firm.

In the french policy, policymakers try to implement such a kind of differentiation within the consumers population. The redistribution process aims at differentiating between the different types of consumers. Unfortunately, this will also call for large information seeking on their effective costs, their effective unability to change their behaviors, etc.

And this may lead to more costs than the expected benefits. Finally, because heterogeneity exists between the abatement technologies of the economic agents, they will also choose different levels of emissions or pollutant goods consumptions. Thus, in order to collect the tax, the agency must also have some reliable information on these different consumptions and emissions.

From a theoretical point of view, the situation we just described above deals with an adverse selection problem. And we know, since ((Akerlof 1970))and the numerous papers focusing on this issue that a solution consists in defining some separate contracts so that each type reveals himself by choosing the contract that has been bult for him. Hence the regulator does no longer need to know who is who and he can save some information costs.

In the frame of imperfect competition and monopoly tarification, some important results have been derived in a setting where the Agent (a monopolistic firm) has an informational advantage over the Principal (the regulator) ((Loeb and Magat 1979), (Baron and Myerson 1982), (Laffont and Tirole 1990)). In particular, ((Baron and Myerson 1982)) proposes to give a premium to firms which accept to reveal *ex ante* their production plans and when these plans are effectively realized *ex post*.

In insurance economics, adverse selection is also the topic of numerous papers. In particular, it is shown that high risks must obtain a full insurance contract with a price determined with respect to the high loss probability. Low risks have access to partial insurance, but tarified at a low rate ((Rothschild and Stiglitz 1976), (Wilson 1977), (Stewart 1994)). In such a setting, contracts are differentiated with respect to prices and to quantities.

Unfortunately, and even if the topic of heterogenous cost abatement is recognized as being an important issue, there are not so many papers that deal with it. And results are sometimes rather different from one paper to another. [1]

[1] For instance, and contrary to what is often obtained in the litterature ((Mougeot and Schwartz 2008)) show that the firm emits less pollution in incomplete information setting than with complete information. Moreover a rent is given to firms with high costs in order to induce them to not imitate low costs firms. In the meantime, ((Bulckaen 1997)) estimates that adverse selection should not be consider as a really worrying issue in environmental economics. He derives some conditions that have to be met by marginal costs and benefits of emissions in order to be almost sure that firms will naturally tell the truth.

In this paper, we consider a regulator who wants the firms to internalize the negative externalities generated by their pollutant emissions. The environmental policy that is implemented is close to an emission permits market, but there is only one seller of permits, namely the regulator, and only one time at which permits are sold. At the beginning of the period, the regulator offers some contracts to the firms that stipulate quantities of permits and selling prices. Depending on their abatement cost structure, they choose one contract in the menu. Such an environmental policy leaves some room for different depollution strategies. Indeed, the firm makes a trade off between not emitting a unit of pollution and bearing the internal cost of depollution, or emitting it but then buying the equivalent in permits. The regulator is in a monopolistic situation regarding the permits selling. Even so, he faces some kind of competition since firms can decide to abate some units of pollution rather than buying permits. Hence the optimal prices will depend on the elasticity of the demand for each type of firm.

More precisely, two types of firms are distributed within the population, with high and low abatement costs. First, we characterize the optimal menu of contracts that prevails in a complete information setting and we show that, contrary to a pooling contract, price discrimination takes already place because the regulator is looking for all the surplus of each firm. Thus not only quantities of permits but also prices are different from one type of firm to the other one. In a second step, we focus on adverse selection and we show first that optimal quantities of emission remain the same as in the first-best optimum and contrary to what is obtained by ((Mougeot and Schwartz 2008)) and ((Shah 2005)). Second, that all the incentives to reveal the true type are carried out by the prices. In our setting, the firms with high abatement costs pay a higher price than in the complete information setting, while the situation for the firm with low abatement costs remains unchanged: this result can be interpreted to the existence of a negative rent borne by the high-cost type. Moreover, in both frameworks (with and without complete information about the types), prices can be negative: for low elasticities of deman, firms must be sibsidized for their depollution acitivity. Different possible situations are discussed (positive prices for both types, negative prices for both types, positive price for the good type and negative price for the high type).

From a methodological point of view, one main difference appears when comparing with the Rotschild and Stiglitz formalization: There is no participation constraints in our model for the firms are compelled to be actors of the environmental policy implemented by the regulator. Nevertheless, this policy is applied at its minimum possible cost. This feature may explain why the rent of information we obtain be different from a standard one.

The oganization of the paper is as follows. Before presenting the formal model, we describe the main insights of our story and our results in Section 2. Then Section 3 focuses on the first-best optimum, which prevails in a complete information setting. Section 4 deals with adverse selection. Section 5 concludes the paper.

2 Main insights

As mentioned in the introduction, the advese selection problem we focus on in this paper is not a standard one, even if the beginning of the story seems to be classiċ. We consider an economy in which two types of firms produce some goods and emit some pollutants. Without any environmental policy, none of them internalize the negative externality that their activities induce and they emit the maximum possible level of emissions (with respect to their production plans). Firms are on competitive markets so that maximizing profits is equivalent to minimizing costs, as shown by (Roberts and Spence 1976). Our study focuses on cost minimizations.

A regulator, in charge of the social welfare maximization, decides to compell the firms to buy a quantity of permits in couterpart of their individual emissions. This sale takes place between each firm and the regulator, so that there is only one seller on this market. He is responsible for the liquidity of the market and he decides the prices.

The timing of the decisions is as follows. First, the regulator must determine the level of total pollution that minimizes the social cost, which entails not only the social damage due to pollution but also the abatements costs of all the firms. Once it is determined, he must decide how to distribute this initial allocation among the firms knowing that they present some heterogeneous abatement costs and, as a direct consequence, different willingnesses to pay. Hence prices must be determined such as to minimize the total cost of abatement. But this may not be sufficient. Indeed, if the regulator is able to observe perfectly the type of each firm, he can associate each type of firm to an adequate contract, which stipulates quantities and prices. If asymmetric information prevails about the type of the firm, prices should also be determined in order to give incentives to the agents to reveal their true type. Thus, the objective can no longer be only cost minimizing. The proposed menu of contracts must also be incentive.

In this paper, our setting differs from a classical model. Because we are dealing with environmental insights, it is obvious that firms will have more costs to bear when a regulatory policy is implemented than in a context without any constraint. Obviously, the main objective of the policy is to induce the firms to bear the cost of the negative externality. Hence it is completely illusory to try to build contracts that are incentive and that simultanously guaranty the participation of the Agent, namely the firms. Thus the adverse selections that we address is a program with incentive constraints but without individual rationality constraints. Furthermore, and in the same spirit, the objective function of the regulator is not in conflict with the firms' ones. Indeed, it encompasses their objectives of abatement costs minimization, but also the cost of pollution for the victims. One important consequence of such a setting is that no positive rent is given to the good type in our model, contrary to what is usually observed in standard models "à la Rotschild et Stiglitz". Instead, a negative rent is given to the bad firm[2], namely the one with high abatement cost, while firms with low abatement costs obtain the same price as in a complete information model. They never have any interest to lie and to announce that they are of bad type since the price paid by the latter is always higher than the price she would pay as a good type.

Finally, all the distorsion due to imperfect information is driven by the prices. Quantities remain the same whether there is or there is no incomplete information.

3 The first-best optimum

Consider a population distributed over two types h and b and using two different technologies: there is a proportion π (respectively $1-\pi$) of firms of type h (respectively b) owning a technology with a high (respectively low)

[2]The unit price of permits she has to pay is higher in the adverse selection issue than with complete information.

abatement cost of pollution.

The regulator distributes a quantity l of permits to the firms in order to make them internalizing the negative effects of their production process. The cost of abatement decided by Firm i, $i = h, b$, is denoted as $C_i(l_i)$, with l_i being her individual level of emission and satisfying $l_i \in [0, \bar{l}]$. For each unit of emission, Firm i must buy one permit: l_i is also the quantity of permits she buys.

The cost structure i is common knowledge. Nevertheless, in the third section of the paper the regulator will have no information about which firm is of type h (respectively b).

Without any environmental regulation, it is still assumed that all firms adopt the same level of emissions, namely $l_i = \bar{l}$ and that this level induces an abatement cost equal to zero: $C_i(\bar{l}) = 0$. Moreover, we state the following fair assumptions.

Assumption 1.

$$C_i(l_i) \geq 0,\ C_i(\bar{l}) = 0,\ C_i^{'}(.) < 0,\ C_i^{''}(.) > 0 \quad i = h, b$$

Moreover, firms of type h have always a higher marginal cost of abatement than firms of type b.

Assumption 2. We assume that $C_h'(l) < C_b'(l)\ \forall l \in \left[0, \bar{l}\right[$.

A direct consequence of Assumptions 1 and 2 is that firms of type h have always a higher total cost of abatement for a given level of emission than firms of type b.

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Figure 1 about here

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Figure 1 displays both cost functions. It should be kept in mind that the marginal cost is negative in our setting: it is a marginal depollution cost rather than a cost of pollution. Thus the more the emission level for a given firm, the less the cost of depollution.

Finally, the total amount of pollutants emitted by all firms is

$$l = \pi l_h + (1 - \pi) l_b.$$

It induces an environmental damage to Society represented by an increasing and convex function D of l:

$$D \equiv D(l),\ D^{'}(.) > 0,\ D^{''}(.) > 0$$

This social damage function is common knowledge. The social total cost of pollution writes:

$$C^s(l, l_h, l_b) = \pi C_h(l_h) + (1 - \pi) C_b(l_b) + D(l) \tag{1}$$

The timing of decisions is as follows: first, the regulator decides the total level l of permits he allocates to the firms, knowing that the quantities l_h and l_b sold to the different types will depend on l. Then he determines the optimal allocation between the two types of firms and the optimal prices. The quantities $l_i,\ i = h, b$, minimize the private cost of each type of firm, while the prices minimize the total abatement costs. Finally, the regulator is able to propose a menu of contracts to the different types of firms.

3.1 Optimal agregate level of permits

The regulator must determine the level l^* of permits that minimizes the total social cost of pollution. Formally, by considering the best response functions $l_h^* = l_h^*(l)$ and $l_b^* = l_b^*(l)$, the program writes:

$$\underset{l}{Min}\ C^s(l, l_h^*(l), l_b^*(l)) = \pi C_h(l_h^*(l)) + (1-\pi)C_b(l_b^*(l)) + D(l) \quad (2)$$

$$\text{s.t. } l = \pi l_h^*(l) + (1-\pi) l_b^*(l) \quad (3)$$

The first order condition is such that:

$$\frac{\partial C^s}{\partial l} = 0 \ \Leftrightarrow 0 = \pi . l_h^{*'}(l).C_h^{'}(l_h^*) + (1-\pi).l_b^{*'}(l).C_b^{'}(l_b^*) + D^{'}(l)$$

From (3) we have $\pi l_h^{*'}(l) + (1-\pi) l_b^{*'}(l) = 1$, so that $l_h^{*'}(l) = \frac{1-(1-\pi).l_b^{*'}(l)}{\pi}$. Thus:

$$D^{'}(l^*) = (1-\pi) l_b^{*'}(l) \left(C_h^{'}(l_h^*) - C_b^{'}(l_b^*(l)) \right) - C_h^{'}(l_h^*(l)) \quad (4)$$

It is interesting to notice that Condition (4) holds even if the regulator is not able to associate a type to each firm, namely when he faces an adverse selection problem. Indeed this condition only requires that he knows perfectly the abatement cost structure of each type, but not necessarily that he knows who is who. One important implication of such a result is that the optimal total level l^* of permits that must be allocated to the economy will not differ from this one in Section 4. The agency costs and the incentives for type revelation will be carried out by the prices.

3.2 Optimal allocation of permits

Now the regulator must decide how to distribute the total optimal initial allocation l between both types. The optimal distribution minimizes the private costs of each type of firm, and the regulator must consider the following minimization program for type $i = b,\ h$:

$$\underset{l_i}{Min} C_i(l_i) + p_i.l_i \tag{5}$$
$$\text{s.t.} \quad l = \pi l_h + (1-\pi) l_b \tag{6}$$

The lagrangian $\mathcal{L}_i$ writes:

$$\mathcal{L}_i = C_i(l_i) + p_i.l_i + \lambda \left(\pi l_h + (1-\pi) l_b - l\right)$$

The first order condition for Firm h is:

$$\frac{\partial \mathcal{L}_h}{\partial l_h} = C_h^{'}(l_h) + p_h + \lambda \pi = 0$$

$$\Leftrightarrow \quad \lambda = -\frac{\left(C_h^{'}(l_h) + p_h\right)}{\pi} \tag{7}$$

The first order condition for Firm b is:

$$\frac{\partial \mathcal{L}_b}{\partial l_b} = C_b^{'}(l_b) + p_b + (1-\pi)\lambda = 0$$

$$\Leftrightarrow \quad \lambda = -\frac{\left(C_b^{'}(l_b) + p_b\right)}{(1-\pi)} \tag{8}$$

From (7) and (8) we obtain:

$$C_b^{'}(l_b) = \frac{(1-\pi)}{\pi}\left(C_h^{'}(l_h) + p_h\right) - p_b \tag{9}$$

Proposition 1 *For given prices p_h and p_b and for a given, optimal, initial and total allocation l^* of permits, the optimal distributions l_h^* and l_b^* among the firms are functions of best responses $l_h^* = l_h(p_h, p_b, l^*)$ and $l_b^* = l_b(p_h, p_b, l^*)$ that satisfy:*

$$\begin{cases} \ell^* = \pi \ell_h^* + (1-\pi)\ell_b^* \\ C_b^{'}(\ell_b^*) = \frac{(1-\pi)}{\pi}\left(C_h^{'}(\ell_h^*) + p_h\right) - p_b \end{cases}$$

3.3 Prices determination at the first-best optimum

Let us denote as p_b^{FB} and p_h^{FB} the prices of permits for firms of type b and h that prevail at first-best optimum, namely in this current subsection 3.3.

The objective of the regulator is to implement the regulatory policy at a minimum cost for Society. In the complete information setting, prices are not used to give some incentives to firms since the regulator is able to observe the type of each firm and, then, to sell her the adequate contract (l_i, p_i^{FB}). Optimal prices are such that they simply minimize the sum of the private costs. Formally, the regulator has to solve the following program:

$$\underset{p_h, p_b}{Min} \pi \left(C_h(l_h^*) + p_h.l_h^*\right) + (1-\pi)\left(C_b(l_b^*) + p_b.l_b^*\right) \tag{10}$$

Let us denote as ε_b (respectively ε_h) the price elasticity of the demand of permits l_b^* with respect to the price p_b (respectively l_h^* with respect to the price p_h).

Proposition 2 *In a complete information setting, the first best prices satisfy:*

$$\begin{cases} p_h^{FB} = -C_h'(l_h^*)/(1-1/|\epsilon_h|) & (i) \\ p_b^{FB} = -C_b'(l_b^*)/(1-1/|\epsilon_b|) & (ii) \end{cases} \tag{11}$$

with ϵ_i the elasticity of demand l_i^* with respect to p_i, $i = h, b$.

Proof. The first order conditions of Program (10) are:

$$\begin{cases} \pi\left(C_h'(l_h^*) + p_h^{FB}\right).\frac{\partial l_h^*}{\partial p_h} + \pi l_h^* = 0 \\ (1-\pi)\left(C_b'(l_b^*) + p_b^{FB}\right).\frac{\partial l_b^*}{\partial p_b} + (1-\pi) l_b^* = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} p_h^{FB} = -C_h'(l_h^*) - \frac{l_h^*}{\partial l_h^*/\partial p_h} \\ p_b^{FB} = -C_b'(l_b^*) - \frac{l_b^*}{\partial l_b^*/\partial p_b} \end{cases}$$

By multiplying the right-hand-side term of the first (respectively the second) equation by $\frac{p_h^{FB}}{p_h^{FB}}$(respectively by $\frac{p_b^{FB}}{p_b^{FB}}$), we obtain Conditions (11.i) and (11.ii). Proposition 2 is demonstrated. ♦

The result of Proposition 2 is obtained without any constraint on the (negative) value of the elasticity of the demand. Yet these prices become negative if $|\varepsilon_i|$ is lower than one and they must be interpreted as subsidies in these cases. However it is fair to assume that the elasticity of firm b is higher, in absolute values, than elasticity of firm h: the higher the price, the higher the ability (in terms of costs) of Firm b to abate pollution instead of buying permits. Finally, three case can be considered. They are summarized in Proposition 3 hereafter.

Proposition 3 *In a complete information setting, the first best is one of the fowolling situations:*

(i) If $|\varepsilon_h| > 1$ *and* $|\varepsilon_b| > 1$*, both menues of contracts provide some positive prices: firms must pay for pollution.*

(ii) If $|\varepsilon_h| < 1$ *and* $|\varepsilon_b| < 1$*, prices are negative and all types are subsidized for their depollution activity.*

(iii) If $|\varepsilon_h| < 1$ *and* $|\varepsilon_b| > 1$*, firms of type* h *are subsidized for depollution while firms of type* b *have to pay for pollution.*

Proof. It is immediate from Proposition 2 and the assumption $|\varepsilon_h| < |\varepsilon_b|$. ♦

Even if there is no participation constraint, firms have the choice to stop their activity. Thus for small elasticities indicating that abatment costs are high, they must be subsidized for their depollution activity. In other cases, prices can be positive and they have to buy permits for each unit of pollution due to their production activity.

Now, we are able to compare prices and quantities between both types at the first-best optimum. In what follows we consider the case of elasticities lower than one (subsidy of the depollution activity). The analysis can easily been developed in the two other cases of Proposition 3.

Proposition 4 *Let us assume that elasticity demands are lower than one in absolute values.*

Let us consider the scalars δ *and* γ *such that* $\delta = \frac{1}{4}.\frac{(|\varepsilon_b|-|\varepsilon_h|)}{\left(1-\frac{(|\varepsilon_b|+|\varepsilon_h|)}{2}\right)}$ *and* $\gamma = \frac{1}{2}.\frac{(|\varepsilon_b|-|\varepsilon_h|)}{(|\varepsilon_b|+|\varepsilon_h|)}$.

The first-best menu of contracts $\left\{\left(p_h^{FB}, l_h^*\right), \left(p_b^{FB}, l_b^*\right)\right\}$ *satisfies one of the three following cases:*

(i) If $\pi < 0.5 - \delta$, *then we have* $l_b^* < l_h^*$ *and* $p_b^{FB} > p_h^{FB}$.

(ii) If $0.5 - \delta \leq \pi \leq 0.5 + \gamma$, *then we have* $(l_b^* - l_h^*)$ *undetermined and* $p_b^{FB} > p_h^{FB}$.

(iii) If $0.5 + \gamma \leq \pi$, *then we have* $(l_b^* - l_h^*)$ *undetermined and* $p_b^{FB} < p_h^{FB}$.

Proof. See Appendix. ♦

Proposition 4 highlights the importance of the composition of the population. Indeed, if the population of low cost firms is large enough, they obtain a smaller individual part of permits over their total allocation $(1-\pi).l_b$. Then it becomes more costly for them to depollute (recall that we obtain this result with $-C_b'(l_b) > -C_h'(l_h)$). The regulator can fix a higher price of permits for them in order to capture all the surplus: this explains that $p_b > p_h$ in Point (i).

On the contrary, if π is high, it must become more expensive (compared to case (i)) for high cost firms to pollute. Thus $p_h > p_b$. This can lead to a lower level of permits for each firm, but it is not systematic. Indeed, in some cases high cost firms can prefer to pay a high price to obtain permits rather than to depollute more than low cost firms. Thus l_h can be either higher or lower than l_b. This is Point (iii).

Lastly, in the intermediate case (ii), $l_h - l_b$ is undetermined. But the population is sufficiently diversified to get the result $p_b > p_h$.

Finally, these optima are no longer systematic equilibria when information about cost types becomes imperfect.

4 Adverse selection and price distorsions

With adverse selection, the regulator can no longer fix prices equal to the preceding ones because one type of the firm, namely type b, will have some incentives to lie and to announce that she is of type h.

4.1 The second-best optimum

The incentive to lie can be removed by choosing an adequate price differentiation. Formally, the regulator must take into account the two incentive constraints associated to each type of firm. Besides, as explained in Section 2, the regulatory rules are costly for the firms and it is no longer consistent to write some participation constraints. If they would have the choice, firms would always decide not to participate in our static setting, since the environmental constraints induce some costs.[3] The objective function is the same as in the preceding section: the regulator wants to compell the firms

[3] Actually, the sole alternative that firms have is to decide to stop their activity. Here, we assume that it is always more profitable to produce with environmental constraints than to stop the production. In other words, profits are always non negative.

to internalize the pollution due to their activity at prices that minimize the total social cost of abatement.

Following this reasoning, the program of the regulator states as:

$$\underset{p_h,p_b}{Min} \quad \pi \left(C_h(l_h^*) + p_h.l_h^*\right) + (1-\pi)\left(C_b(l_b^*) + p_b.l_b^*\right) \tag{12}$$

$$\text{s.t.} \quad \left(C_h(l_h^*) + p_h.l_h^*\right) - \left(C_h(l_b^*) + p_b.l_b^*\right) \leq 0 \tag{13}$$

$$\left(C_b(l_b^*) \; + p_b.l_b^*\right) \; - \left(C_b(l_h^*) + p_h.l_h^*\right) \leq 0 \tag{14}$$

Constraints (13) and (14) state that prices must be determined in such a manner that no firm of a given type has an interest to choose the contract that has been built for the other type.

Denote as λ_1 and λ_2 the lagrangian scalars associated with (13) and (14). Three cases must be considered.

Case 1. Both constraints are binding in optimum. Thus we must have simultaneously:

$$\begin{aligned} C_h\left(l_h\right) + p_h.l_h - C_h\left(l_b\right) - p_b.l_b &= 0 \\ C_b\left(l_b\right) + p_b.l_b - C_b\left(l_h\right) - p_h.l_h &= 0 \end{aligned}$$

By summing them up, we obtain that

$$C_h\left(l_b\right) - C_b\left(l_b\right) = C_h\left(l_h\right) - C_b\left(l_h\right),$$

Having $l_h \neq l_b$,[4] this is not possible because of Assumption 2. Finally, both constraints cannot be simultaneously binding in optimum: the case ($\lambda_1 > 0$, $\lambda_2 > 0$) must be moved aside.

[4] If quantities would be equal, all firms would choose the cheapest contract.

Case 2. **None of the constraints is binding.** This case would mean that all the firms are always better of by telling the truth about their type. None of them will lie and the first-best optimum applies. We have seen below that this is not possible under adverse selection. The case $\lambda_1 = 0$ and $\lambda_2 = 0$ must be moved aside.

Case 3. One, and only one, constraint is binding. Because Firm b is the type that has some interest in lying, the incentive constraint that is binding is (14). And Constraint (13) has no longer to be considered in the program. It will always be satisfied in (second-best) optimum.

Hence, we have to compute:

$$\begin{aligned} &\underset{p_h,p_b}{Min} \pi \left(C_h(l_h^*) + p_h.l_h^*\right) + (1-\pi)\left(C_b(l_b^*) + p_b.l_b^*\right) \\ &\text{s.t. } \left(C_b(l_b^*) + p_b.l_b^*\right) - \left(C_b(l_h^*) + p_h.l_h^*\right) = 0 \end{aligned} \tag{15}$$

When replacing the constraint into the objective function and deriving with respect to p_h, the first order condition gives

$$p_h = -\pi C_h'(l_h^*) - (1-\pi)C_b'(l_h^*) - l_h^* / \frac{\partial l_h^*}{\partial p_h}$$

$$\Leftrightarrow p_h = \frac{-C_h'(l_h^*)}{1 - 1/\left|\varepsilon_{l_h^*/p_h}\right|} + a, \tag{16}$$

with $a = \frac{(1-\pi).(C_h'(l_h^*) - C_b'(l_h^*))}{1-1/\left|\varepsilon_{l_h^*/p_h}\right|} > 0$.

Now, isolating $p_h.l_h$ in the contraint of Program (15), replacing it in the objective function and deriving it with respect to p_b lead to:

$$p_b = \frac{-C_b'(l_b^*)}{1 - 1/\left|\varepsilon_{l_b^*/p_b}\right|} \tag{17}$$

Proposition 5 *Assume that the regulator knows the structure of costs of type* h *and type* b *but that he is not able to know the type of a given firm. The second-best optimum is such that he offers two contracts, with differentiated unit prices of permits,* $(l_i^*, p_i); i = h, b$. *The levels of permits* l_i^* *and the associated unit prices* p_i *are determined by (6), (9), (16) and (17). They are such that*

(i) Firms of type h *emit* l_h^* *and buy the equivalent volume in permits at price* p_h,

(ii) Firms of type b *emit* l_b^* *and buy the equivalent volume in permits at price* p_b

(iii) The separate equilibrium is such that $p_h > p_h^{FB}$ *and* $p_b = p_b^{FB}$.

Proof. Since neither the private objectives (5-6) and (2-3) of the firms nor the social one have changed, the second-best quatities are those obtained in the first-best case: $l_i^* = l_i^*(l), i = h, b$. By comparing the prices obtained in Proposition 2 with (16) and (17) we have Point 3. Finally, the contracts have been built in such a manner that no type has an interest in lying. Thus each type chooses the contract that has been built for her (Point 1. and 2). ♦

Still here, prices can be negative if the elasticities of demand are lower than one. Nevertheless, from Point 3. of Proposition 5, it is possible that both firms have to pay a positive price for each unit of pollution, while in the complete information setting only the good type was in this situation (the high one being subsidized; Indeed, the strictly positive scalar a can lead to a positive price p_h even if p_h^{FB} is negative (the case for $|\varepsilon_h| < 1$).

As in a classical adversion selection model, a rent is given to the good type in order to avoid cheating. Nevertheless, this rent is not explicitly captured by the unit price of permits paid by firms of type b. Actually, it is borne by the high cost type and enters in her unit price as a fee that she has to pay in addition to the first-best price. Such a striking result is obtained because there is no participation constraint to be considered in our setting, as explained below. Nevertheless, the main characteristics of a separate equilibrium with positive rent for good types remains : quantities are not sufficient to separate types. Price differentiation is also needed. And a rent should be given to the good type in order to avoid cheating. Here all the distorsions are carried on by the prices and quantities are those that minimize the social cost of pollution.

APPENDIX

Proof of Proposition 4.

Replacing Equations (11.i) and (11.ii) into (9) yields:

$$\frac{C_b'(l_b^*)}{C_h'(l_h^*)} = \frac{(1-\pi)}{\pi}.\frac{1-\left|\varepsilon_{l_b^*/p_b}\right|}{1-\left|\varepsilon_{l_h^*/p_h}\right|} \tag{18}$$

With $1 > \left|\varepsilon_{l_b^*/p_b}\right| > \left|\varepsilon_{l_h^*/p_h}\right|$ by assumption, we have that $0 < \frac{1-\left|\varepsilon_{l_b^*/p_b}\right|}{1-\left|\varepsilon_{l_h^*/p_h}\right|} < 1$. Consider the positive scalar δ defined by:

$$\frac{1-(1/2-\delta)}{(1/2-\delta)}.\frac{1-\left|\varepsilon_{l_b^*/p_b}\right|}{1-\left|\varepsilon_{l_h^*/p_h}\right|} = 1$$

$$\Leftrightarrow \delta = \frac{1}{4}.\frac{\left(\left|\varepsilon_{l_b^*/p_b}\right| - \left|\varepsilon_{l_h^*/p_h}\right|\right)}{1-\frac{\left(\left|\varepsilon_{l_b^*/p_b}\right|+\left|\varepsilon_{l_h^*/p_h}\right|\right)}{2}}$$

If $\pi \leq 1/2-\delta$ we have that $\frac{C_b'(l_b^*)}{C_h'(l_h^*)} > 1$ and $l_b^* < l_h^*$. If $\pi > 1/2-\delta$, the sign of $(l_b^* - l_h^*)$ is undetermined.

Concerning the prices, we have from Proposition 2 that:

$$\frac{p_b^{FB}}{p_h^{FB}} = \frac{C_b'(l_b^*)}{C_h'(l_h^*)}.\frac{1 - 1/\left|\varepsilon_{l_h^*/p_h}\right|}{1 - 1/\left|\varepsilon_{l_b^*/p_b}\right|}$$

By replacing (18) into this expression, we obtain that:

$$\frac{p_b^{FB}}{p_h^{FB}} = \frac{(1-\pi)}{\pi}.\frac{\left|\varepsilon_{l_b^*/p_b}\right|}{\left|\varepsilon_{l_h^*/p_h}\right|}$$

We have $\frac{\left|\varepsilon_{l_b^*/p_b}\right|}{\left|\varepsilon_{l_h^*/p_h}\right|} > 1$. Consider the positive scalar γ defined by:

$$\frac{1-(1/2+\gamma)}{(1/2+\gamma)}.\frac{\left|\varepsilon_{l_b^*/p_b}\right|}{\left|\varepsilon_{l_h^*/p_h}\right|} = 1$$

If $\pi < 1/2 + \gamma$, we have that $p_b^{FB} > p_h^{FB}$ if and $p_b^{FB} \leq p_h^{FB}$ if $\pi \geq 1/2 + \gamma$. By computing these results with those obtained above for the quantities, we obtain the three cases of Proposition 4. ♦

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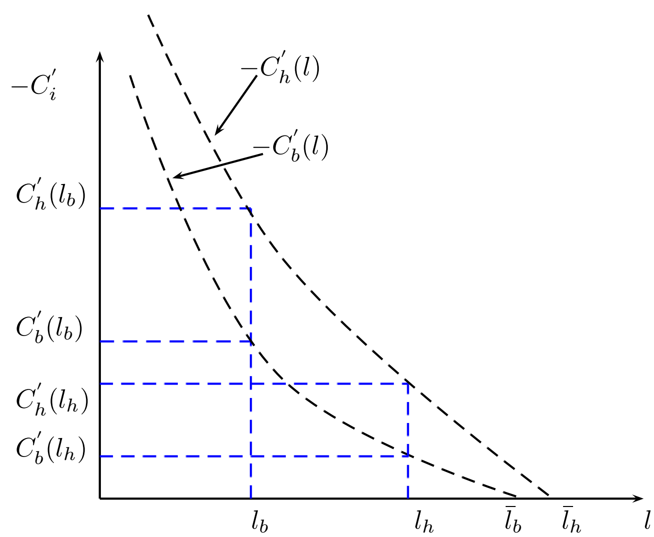

Abatement costs with $l_h > l_b$

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