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The aggregation of individual distributive preferences through the distributive liberal social contract: Normative analysis.

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Abstract

We consider abstract social systems of private property, made of *n* individuals endowed with non-paternalistic interdependent preferences, who interact through exchanges on competitive markets and Pareto-efficient lump-sum transfers. The transfers follow from a distributive liberal social contract defined as a redistribution of initial endowments such that the resulting market equilibrium allocation is both Pareto-efficient relative to individual interdependent preferences, and unanimously weakly preferred to the initial market equilibrium. We notably elicit two global properties of Pareto-efficient redistribution in smooth differentiable social systems of the type above. The first one is the separability of allocation and distribution: Pareto-efficient redistribution leaves unaltered the role of market prices in the coordination of market exchanges, as expressed, notably, by the existence and efficiency properties of competitive equilibrium. The second one is the global structure of the set of Pareto-efficient allocations: its relative interior is a simply connected smooth manifold of dimension *n*-1, homeomorphic to the relative interior of the unit-simplex of $\mathbb{R}^n$. Both properties obtain under three suitable conditions on the partial preordering of Pareto associated with individual interdependent preferences, which essentially state that: the social utility functions built from weighted sums of individual interdependent utilities, by means of arbitrary positive weights, exhibit a property of differentiable nonsatiation and some suitably defined property of inequality aversion; and individuals have diverging views on redistribution, in some suitable sense, at (inclusive) distributive optima. The set of market equilibrium allocations associated with the transfers of the inclusive distributive liberal social contracts then consists of the maxima, in the set of attainable allocations unanimously weakly preferred to the initial market equilibrium, of the weighted sums of individual interdependent utilities derived from arbitrary vectors of positive weights of $\mathbb{R}_{++}^n$. Its relative interior is a simply connected smooth manifold of dimension *n*-1 whenever the initial market equilibrium is not Pareto-efficient relative to individual interdependent preferences. It is shown, finally, that the liberal social contract's inclusive solutions for redistribution, so characterized, maximize a family of social welfare functionals that verify Arrow's non-dictatorship axiom and Sen's liberty axiom for the social systems to which it applies.

JEL codes

A10, A12, A13, D02, D31, D41, D51, D60-64, D70, D71, D78, H10, H11, H21, H23, H41, H42, H44, H53, I30, I38, P10, P11, P14, P16, P35, Z13.

Keywords

Walrasian equilibrium; Pareto-efficiency; liberal social contract; social preferences; social choice; allocation; distribution.

1- Distribution in the liberal social contract

This article examines the possibility of rationally founding the distribution institution on the liberal social contract.

The liberal social contract (Kolm, 1985 and, notably, 1996: 5, and 2004: Chap.3; see also the related construct of Nozick, 1974) is a normative reference, corresponding to the unanimous agreement of individuals derived from the sole consideration of their preferences and rights by abstracting away all conceivable impediments to the achievement of this agreement or implementation of its contents, that is, notably, informational and other obstacles to the elaboration of the clauses of the social contract, and difficulties with their enforcement.

It differs from alternative normative theories such as Harsanyi's derivation of utilitarianism (1955) or Rawl's *Theory of Justice* (1971) by deducing the normative reference from *actual* individual preferences and rights.

Harsanyi and Rawls use the fiction of the veil of ignorance of the original position for abstracting away all possible sources of alteration of the impartiality of individual judgment that may follow from individual's actual position in society, his "interests" in an all-inclusive sense, comprehending not only material wealth (the rich and the poor), but also human wealth (the sick and the healthy, the smart and the dull), distinctions, set of interpersonal relations etc. Individuals, so abstractly placed in a position of objectivity, form their impartial judgment over social states by means of acts of imaginative sympathy, which consist of imagining themselves successively occupying all actual positions in society. The norms of justice are unanimous agreements of such individual impartial judgments, obtained from rational deliberation and bargaining in the construct of Rawls, and from the axioms of rational decision under uncertainty in the construct of Harsanyi.

The liberal social contract, by contrast, is a unanimous agreement of individuals in their actual position in society. It is a *positive* theory in that respect. It becomes normative, hence a theory of justice, only insofar as the process of contracting, and subsequent implementation of clauses, are concerned. The operation of abstraction that is performed at this level extends to the whole space of social contracting, as an "as if" or ceteris paribus proviso, the abstract characteristics of perfectly competitive market exchange, notably costless and immediate information, bargaining and enforcement. The norms of justice are the unanimous agreements that obtain in these ideal conditions of perfect social contracting. They define (ideal) objectives for collective action.

Actual collective action inspired by the liberal social contract fills in the gap between the norm of the social contract and the reality of society by means of actual contractual arrangements, or institutional substitutes for them, which permit the achievement, partial or complete, of some of its ideal objectives subject to the constraints associated with the actual costs of corresponding action. These modalities of collective action include state intervention, but do not reduce to the latter, in principle at least. In other words, the liberal social contract is mute, by construction, on the modalities of its implementation, as the costs of the latter proceed from the circumstances (information, transaction and enforcement costs) that are assumed away for its derivation. This characteristic makes the liberal social contract a possible explanation, in the normative, teleological sense, for a large variety of actual institutions and arrangements of institutions, including the distribution institution, to which we now turn.

The distribution institution consists of the sets of legitimate acts of redistribution of individually consumable wealth, and rights relative to individually consumable wealth.

So defined without making explicit the principle that legitimates some redistributive transfers and forbids others, the distribution institution seems to be a universal trait of human societies, large or small, across time and places. As such, it must probably be included in the small set of universal characteristics which, like language, should feature in any definition of a general notion of human society.

We are specifically interested, in this article, in the distribution institution that is founded on the liberal social contract as the principle of legitimacy of redistributive transfers. That is: the set of acts of redistribution of individually consumable wealth which meet the unanimous agreement of individual members of society, appreciated from their actual preferences and rights (this makes legitimate redistribution); the set of acts of redistribution of individually consumable wealth vetoed by some individual members of society from their actual preferences and rights (this makes illegitimate redistribution; the disagreement of a single individual suffices, in principle, to make a transfer illegitimate); and the subset of the latter which consists of *forbidden* acts of redistribution, and so defines individual rights to redistribute.

If legitimate redistributive transfers are imaginary, deduced from the abstract assumption of perfect social contracting, we have the *norm* of the distributive liberal social contract. If these transfers are the actual, partial achievements of this norm that obtain in the practical conditions of collective action, we have the *practical* distributive liberal social contract.

This article derives some general properties of the norm of the distributive liberal social contract.

The features of individual characteristics that are specifically relevant for our purposes are their preferences relative to the interpersonal distribution of individually consumable wealth on the one hand, and their rights relative to distribution (in short, distributive rights) on the other hand.

Individual preferences over distribution (in short, distributive preferences) make distribution a public good, as an object of common concern of the individual members of society (Kolm, 1966, Hochman and Rodgers, 1969). Any individual wealth is also a public good (or bad) in the same sense for the set of individuals who feel concerned about it, whenever this set does not reduce to the wealth owner himself.

An individual concern about another's wealth can be of the benevolent type, also called altruistic concern, or of the malevolent type, in situations of envy or of ill-intended gift.[1]

Individual distributive concerns are the basic source and condition for the existence of the distributive liberal social contract

Altruistic concerns, if they are strong and widespread enough, induce willingness to give. If the gifts are properly oriented and intended, they can be accepted by beneficiaries. Subject to the same condition, they may also arouse no frustration and cause no objection from those who do not take part in the gift-giving relationship as donor or beneficiary. They may, therefore, meet unanimous agreement (the latter understood in the wide, or *weak*, sense that includes indifference as a case of agreement). This makes the positive, active side of the

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[1] Envy is defined by economic theory as a situation where an individual prefers another's position (here, another's wealth) to his own. Envy in this sense does not imply malevolence; nor does malevolence imply envy in this sense. They can be associated, though, in the psychological attitudes of some relative to the wealthy, when the consideration of wealthy positions creates both dissatisfaction with one's own and subsequent resentment for the source of painful comparison. Malevolent distributive concern does not reduce to the case of envy, although the latter certainly has a great practical importance. Another important case is ill-intended gift (see Kolm, 2006: 4.2, for a comprehensive classification of gift motives, including the types of malevolent gift-giving).

distribution institution of the liberal social contract, that is, legitimate *re*distribution according to this ethical principle.

Malevolent redistribution is normally vetoed by solicited "donors", or by solicited receivers in the case of ill-intended gifts. The achievement of unanimous agreement from a given distribution of individual wealth supposes, therefore, a type of individual right, namely, *equal individual distributive liberty*, which consists of the liberty, for anyone, to decide what to transfer (that is, to choose the magnitude and contents of the transfer of individually consumable wealth) and to whom, and also to decide what to receive, and from whom; in other words, the ability of anyone to accept or refuse gifts, and to make or not to make gifts, subject to beneficiary's acceptance when a gift is made. This makes the negative, constitutional side of the distribution institution of the liberal social contract, that is, illegitimate redistribution forbidden by individual distributive rights. The latter rest on a double individual-based ethical foundation: unanimous agreement as principle of legitimacy of transfers (*objective* individual-based ethical principle); and negative moral appreciation of redistributive acts driven by evil intentions (malevolence), considered from the subjective perspective of involved individuals (*subjective* individual-based ethical principle).

To sum up, the distributive liberal social contract proceeds from a self-consistent system of individual preferences and rights relative to the distribution of individually consumable wealth. Individual *distributive preferences* convey distributive concerns of the benevolent and malevolent kinds. The type of illegitimate redistribution that proceeds from malevolent distributive concerns defines individual *distributive right* as the right of any individual to accept or refuse gifts, and to make or not to make gifts, subject to beneficiary's acceptance when a gift is made. Legitimate redistribution proceeds from altruistic distributive concerns. It consists of the set of gifts that are unanimously weakly preferred to distributive status quo.

Vetoed altruistic redistribution is illegitimate in the distributive liberal social contract, but is not forbidden by individual rights when disagreement is not the fact of the donors or beneficiaries involved in the gift-giving relationship. There remains, therefore, by construction, the possibility of a logical inconsistency of the distributive liberal social contract in situations where some individuals are willing to make gifts, which beneficiaries accept, and which are vetoed, notably for reasons of jealousy, by individuals who are neither donors nor beneficiaries in those gifts. In such cases, and in them only, the distributive liberal social contract is empty, because individual rights produce illegitimate actions. The distribution institution cannot, then, be founded on it (see the Example 14 of Mercier Ythier, 2006: 6.1.2.1).[2]

Both sides of the distribution institution of the liberal social contract involve sizeable costs for their practical elaboration and implementation.

On the active side of it, the public good characters of common altruistic distributive concerns raise the usual difficulties for optimal provision, that is, notably, indivisible information and transaction costs that steeply increase with the size of the pool of interdependent donors and beneficiaries, and enforcement costs which might follow up accordingly if free-riding behaviour increases with pool size as conjectured by Olson, 1965. A large variety of solutions is conceivable and effectively practised for these problems, from state intervention (e.g. public assistance) to the multifarious institutions and organizations of

[2] This example refers, metaphorically, to the American secession war of 1860-1865. The reference to the abolition of slavery is only partly relevant as an example in our context, because it involves much more, naturally, than a redistribution of property rights over individually consumable wealth. For a definition of jealousy, and a discussion of its relations with envy in the context of distributive theory, see Mercier Ythier, 2006, footnote [58].

philanthropic economy (notably charities), and also including important aspects of the economics of the family.

On the constitutional side, distributive rights imply the right of anyone to retain one's own wealth for one's own use, which is an essential constituent of individual property rights. Elaboration here refers to the whole apparatus of law (including common law, if any, and case law) and legislation relative to private property. Implementation reduces, in the main, to enforcement by specialized state institutions, essentially the police and the law.

The practical distributive liberal social contract consists of the set of solutions developed in order to achieve the norm of the distributive liberal social contract, and constraining this achievement, in a particular society. As already mentioned above, the particular emphasis of this article is on the norm of the distributive liberal social contract. We will therefore refer to the practical aspects of it only incidentally in the sequel.

The distributive right derived above is only one aspect, albeit essential, of the right of private property, namely, the aspect that refers to gift-giving. Its other aspects, relating to individual consumption, exchange, production and disposal activities, can be derived in the same way, by logical deduction from the individual-based ethical principles of the liberal social contract, of the objective type (unanimous preference) and subjective type (moral appreciation derived from the subjective perspective of individual participants in interactions). It is not the place here to proceed to a systematic exercise of this type. We will, instead, summarize the results that are useful for the understanding of the norm of the distributive liberal social contract, and comment two aspects of them which are critical for an appropriate definition and interpretation of the whole construct.

All relevant aspects of the norm of private property of the liberal social contract (that is, private property with perfect contracting) distinct from gift-giving are summarized in the norm of market exchange, as perfect competitive market exchange with free disposal. The self-consistency of the construct supposes that individual distributive concerns are non-paternalistic. We briefly comment the critical features of free disposal and non-paternalism.

Free disposal refers to the possibility, for any individual or agent, of disposing of commodities, that is, of either destroying them without any counterpart in terms of welfare or production, or transferring them to nature, and of so doing at no cost. This feature can be viewed as a closure of the definition of perfect contracting, supposing that *all* consumption or transfer activities are costless per se: consuming, selling or purchasing commodities, giving them, or disposing of them only "costs" the market value of consumed, transferred or disposed commodities (with the usual sign convention for quantities, namely: quantities entering in (resp. getting out of) individual property are positive (resp. negative)). An important implication of free disposal is that gift-giving necessarily increases the wealth of the beneficiary, or at least does not diminish it, from the perspective of price-taking individuals (Mercier Ythier, 2006: footnote [11]).

Non-paternalism analyzes in two complementary assumptions, stating respectively: that any individual has well-defined preferences over his own individual consumption, defining an ordinal index of private welfare independent of the individual consumption of others (his private utility or, to use the terms of Pareto, 1913 and 1916, his ophelimity); and that individual distributive concerns, if any, are specified over the private welfare of others, so defined. The second aspect of non-paternalism deduces from the first and perfect contracting (Mercier Ythier, 2006: 4.2.4). Its correct interpretation supposes a clear notion of the relations between individually consumable wealth, on the one hand, and human wealth on the other hand.

The human wealth of an individual consists of the various possible occupations of his time, including the various types of leisure (Becker, 1964 and 1965). The whole construct developed in this article views the human wealth of society, that is, the number of its individual members and the human wealth of each, as fixed relative to redistributive transfers. This means, in other words, that the distribution institution of the liberal social contract, such as analyzed here, is determined by human wealth, and does not determine it in return. Redistribution, in particular, consists of transfers of individually consumable wealth between pre-existing individuals, or, possibly also, between individual members of non-coexisting generations provided that, in the latter case, the transfers do not condition or influence birth or migration. Redistributive transfers do not alter the human wealth of donors or beneficiaries either, by assumption, although a significant part of them may follow from the necessity, endorsed by unanimous weak agreement, to provide for the basic needs in private wealth and welfare of individuals who have temporarily lost their autonomy on these grounds. In sum, the distribution of individually consumable wealth is analytically distinguished from the gross and net production of human wealth[3].

We may now summarize the main features and complete the definition of the analytical notion of distributive liberal social contract which this article purports to study.

The distributive liberal social contract is defined relative to a fixed human wealth of society, corresponding to a fixed population of individuals and a fixed human wealth of each of them.

The notion corresponds to the norm of the distributive liberal social contract, that is, we suppose perfect social contracting relative to distribution, perfect competition in market exchange, and free disposal.

The distributive liberal social contract so understood consists of a distribution of individually consumable wealth which meets the unanimous agreement of the individual members of society, appreciated from their non-paternalistic preferences and their private property rights.

There remains to specify an original position, and to derive an exact formulation of the corresponding norm of the distributive liberal social contract.

The original position of the social system consists of any fixed initial distribution of individual endowments of individually consumable wealth, and any fixed associate competitive market equilibrium (or Walrasian equilibrium). By original position, we simply mean the situation of the social system prior social contract redistribution. This is *logical*, not *chronological* anteriority, time being abstracted by definition in this rational construction of the distribution institution within the liberal social contract. The redistributive transfers of the social contract, as, more generally, any individual or collective acts derived in the norm of the liberal distributive social contract "before" or "after" social contract redistribution, are imaginary by construction, hence reversible.

The precise formulation of the contract deduces then from the general definition above, applied to the original position, that is:

The (norm of) distributive liberal social contract, relative to an original position, consists of the set of transfers achieved from the endowment distribution of the original position, and associate Walrasian equilibrium, such that the latter is a strong Pareto optimum relative to individual non-paternalistic preferences, and is unanimously weakly preferred to the Walrasian equilibrium of the original position.

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[3] This notably excludes from the field of liberal social contract redistribution education and health investments, support to persons in situation of long-run dependence (young children, disabled aged, dependent handicapped etc.) or family allowances. Foundations for this type of productive transfers within the liberal social contract resort to other components of the latter, namely, the parts which, such as fundamental insurance, deal with provision for basic needs other than income (Kolm, 1985, 1996 and 2004).

In the remainder of this article, we: provide a formal definition of the notions and fundamental assumptions above (section 2); set and interpret the working assumptions of differentiability and convexity (section 3); derive and interpret, as first fundamental property, the separability of allocation and distribution (section 4); characterize the set of inclusive distributive liberal social contract solutions and associate notions of equilibrium (sections 5 and 7); define and interpret, as second fundamental property, the regularity of the distributive liberal social contract solution (section 6); situate the distributive liberal social contract relative to social choice theory (section 8); and briefly return, finally, on the epistemological status of the whole construct (section 9). An appendix (section 10) recalls some useful fundamental properties of differentiable Walrasian economies.

2- Formal definitions and fundamental assumptions

We consider the following simple society of individual owners, consuming, exchanging and redistributing commodities.[4]

There are n individuals denoted by an index i running in $N=\{1,...,n\}$, and l goods and services, denoted by an index h running in $L=\{1,...,l\}$. We let $n\geq 2$ and $l\geq 1$ in the sequel, that is, we consider social systems with at least two agents and at least one commodity (the special case l=1 is studied in Mercier Ythier, 1997, whose main results are subsumed in the results of the present study, and notably in Theorems 2 and 5).

The final destination of goods and services is individual consumption. A consumption of individual i is a vector $(x_{i1},...,x_{il})$ of quantities of his consumption of commodities, denoted by x_i. The entries of x_i are nonnegative by convention, corresponding to demands in the abstract exchange economy outlined below. An allocation is a vector $(x_1,...,x_n)$, denoted by x.

Individuals exchange commodities on a complete system of perfectly competitive markets. There is, consequently, for each commodity h, a unique market price, denoted by p_h, which agents take as given (that is, as independent from their consumption, exchange or transfer decisions, including their collective transfer decisions if any). We let $p=(p_1,\ldots,p_l)$.

Transfer decisions are made by coalitions, formally defined as any nonempty subset I of N, which may possibly be reduced to a single individual. A transfer of commodity h from individual i to individual j is a nonnegative quantity t_{ijh}. We let: $t_{ij}=(t_{ij1},\ldots, t_{ijl})$ denote i's commodity transfers to j; $t_i=(t_{ij})_{j:j\neq i}$ denote the collection of i's transfers to others (viewed as a row-vector of $\mathbb{R}_+^{l(n-1)}$). A collection of transfers of the grand coalition N is denoted by t, that is: $t=(t_1,\ldots,t_n)$.

We make the following assumptions on commodity quantities: (i) they are perfectly *divisible*;

[4] We abstract from production for simplicity. The introduction of privately owned, price-taking, profit-maximizing firms with well-behaved (notably convex) production sets does not imply any significant change for the analysis below. Ophelimity-maximizing owners of firms unanimously wish, in particular, that the firms they own maximize their profits. This holds true also for utility-maximizing owners endowed with non-paternalistic interdependent utilities (because utility maximization supposes ophelimity maximization for such individuals). This conformity of views of any individual in his different economic and social positions and roles of firm owner, consumer and (potential) donor supposes perfect competitive exchange, that is, price-taking behavior of individuals and firms, and complete markets (with or without uncertainty). It does not hold true anymore, in general, in cases of imperfect competition or incomplete markets. But, in the latter case, we are outside the enchanted world of Arrow-Debreu economy which, we argued in section 1, is an essential part of the more general notion of perfect social contracting that underlies the norm of the distributive liberal social contract. Note, finally, that the types of activities that are really essential for the functioning of the distributive liberal social contract are the transfer activities of social contract gift-giving and market exchange. Production, consumption and disposal activities are only subsidiary in this respect.

(ii) the total quantity of each commodity is given once and for all (*exchange economy* with *fixed total resources*) and equal to 1 (the latter is a simple choice of units of measurement of commodities); (iii) an allocation x is attainable if it verifies the aggregate resource constraint of the economy, specified as follows: $\sum_{i\in N} x_{ih} \leq 1$ for all h (this definition of attainability implies *free disposal*).

The vector of total initial resources of the economy, that is, the diagonal vector (1,…,1) of $\mathbb{R}^l$, is denoted by ρ. The set of attainable allocations $\{x\in\mathbb{R}_+^{ln}: \sum_{i\in N} x_i \leq \rho\}$ is denoted by A.

The society is a *society of private property*. In particular, the total resources of the economy are owned by its individual members. The initial ownership or endowment of individual i in commodity h is a nonnegative quantity ω_{ih}. The vector $(\omega_{i1},...,\omega_{il})$ of i's initial endowments is denoted by ω_i. We have $\sum_{i\in N} \omega_i=\rho$ by assumption. The initial distribution $(\omega_1,\dots,\omega_n)$ is denoted by ω.

Individuals have preference preorderings over allocation, which are well defined (that is, reflexive and transitive) and complete. The allocation preferences of every individual i are assumed *separable* in his own consumption, that is, i's preference preordering induces a unique preordering on i's consumption set for all i. We suppose that preferences can be represented by utility functions. In particular, the preferences of individual i over his own consumption, as induced by his allocation preferences, are represented by the ("private", or "market") utility function $u_i:\mathbb{R}_+^l\to\mathbb{R}$, which we will sometimes also name ophelimity function by reference to Pareto (op. cit.). The product function $(u_1\circ \mathrm{pr}_1,\dots,u_n\circ \mathrm{pr}_n)$: $(x_1,...,x_n)\to(u_1(x_1),...,u_n(x_n))$, where pr_i denotes the i-th canonical projection $(x_1,...,x_n)\rightarrow x_i$, is denoted by u. Finally, we suppose that individual allocation preferences verify the following hypothesis of *non-paternalistic utility interdependence*: for all i, there exists a ("social", or "distributive") utility function w_i:$u(\mathbb{R}_+^n)\to\mathbb{R}$, increasing in its i-th argument, such that the product function $w_i\circ u$: $(x_1,...,x_n)\to w_i(u_1(x_1),...,u_n(x_n)))$ represents i's allocation preferences. Whenever i's distributive utility is increasing in j's ophelimity, this means that individual i endorses j's consumption preferences within his own allocation preferences ("non-paternalism"). Note, nevertheless, that non-paternalistic utility interdependence does not imply *distributive benevolence*, in the sense of individual distributive utilities increasing in some others' ophelimities. It is compatible, in particular, with the *distributive indifference* of an individual i relative to any other individual j, that is, the constancy of i's distributive utility in j's ophelimity in some open subset of domain $u(\mathbb{R}_+^n)$ ("local" distributive indifference of i relative to j) or in the whole of it ("global" indifference). It is compatible, also: with local or global *distributive malevolence,* in the sense of individual distributive utilities decreasing in some others' ophelimities; and, naturally, with any possible combination of local benevolence, indifference or malevolence of any individual relative to any other. For the sake of clarity, we reserve the terms "individual distributive utility function" for functions of the type w_i and "individual social utility function" for functions of the type $w_i\circ u$. The terms "individual distributive preferences" and "individual social preferences", on the contrary, are used as synonymous, and designate individual preference relations over allocation, in short, individual allocation preferences.

Individual private utilities are normalized so that $u_i(0)=0$ for all i. Naturally, this can be done without loss of generality, due to the ordinal character of allocation preferences.

We let w denote the product function $(w_1,\dots,w_n)$: $\hat{u}\to(w_1(\hat{u}),...,w_n(\hat{u}))$, defined on $u(\mathbb{R}_+^n)$.

We use as synonymous the following pairs of properties of the preference preordering and its utility representations: *smooth* (C^r, with $r\geq1$) preordering, and smooth (C^r) utility representations; *monotone* (resp. strictly monotone, resp. differentiably strictly monotone) preordering, and *increasing* (resp. strictly increasing, resp. differentiably strictly increasing) utility representations; *convex* (resp. strictly convex, resp. differentiably strictly convex) preordering, and quasi-concave (resp. strictly quasi-concave, resp. differentiably strictly quasi-concave) utility representations. Their definitions are recalled, for the sole utility representations, in a footnote of section 3.

A social system is a list (w,u,ρ) of social and private utility functions of individuals, and aggregate initial resources in consumption commodities. A social system of private property is a list (w,u,ω), that is, a social system where the total resources of society are owned by individuals and initially distributed between them according to distribution ω.

It will not be necessary, for the definite purposes of this article, to develop a fully explicit concept of social interactions, synthesized in a formal notion of social equilibrium, such as those of Debreu, 1952, Becker, 1974 or Mercier Ythier, 1993 or 1998a for example (see Mercier Ythier, 2006: 3.1.1, 4.2.1 and 6.1.1 for a review of such notions). The following informal description, and set of partial definitions, will suffice.

Market exchange is operated by individuals, who interact “asympathetically” (Edgeworth, 1881) or “nontuistically” (Wicksteed, 1913) on anonymous markets, through ophelimity-maximizing demands determined on the sole basis of market prices.

Sympathetic or altruistic interactions take place in redistribution. They may proceed, in principle, from a whole range of moral sentiments of individuals, from individual sentiments of affection between relatives, to individual moral sentiments of a more universal kind such as philanthropy or individual sense of distributive justice. They may, likewise, find their expression in a large variety of actions, from individual gift-giving to family transfers, charity donations, or public transfers. We concentrate, in this article, on *lump-sum redistribution which meets the (weak) unanimous agreement of the grand coalition*, that is, redistribution of initial endowments that is approved by some individual members of society (one of them at least) and is disapproved by none. Note that, due to distributive indifference, any bilateral transfer so (weakly) preferred by the unanimity of individuals may be an object of effective concern for only a very limited number of persons, possibly reduced to the donor and the beneficiary of transfer. In other words, the abstract notion of altruistic transfer that we use here covers a wide spectrum of possibilities of voluntary redistribution, such as individual gifts, or collective transfers within groups of any possible size from families to society as a whole.

These elements of social functioning are summarized in the formal definitions below, of a *competitive market equilibrium*, and a *distributive liberal social contract*. They are complemented by the two notions of Pareto efficiency naturally associated with them, that is, respectively, the Pareto-efficiency relative to individual private utilities (in short, *market efficiency*, or *market optimum*), and the Pareto-efficiency relative to individual social utilities (in short, *distributive efficiency*, or *distributive optimum*).

Definition 1: A pair (p,x) such that $p\geq0$ is a *competitive market equilibrium* (also called *Walrasian equilibrium*) with free disposal of the social system of private property (w,u,ω) if: (i) x is attainable; (ii) $p_h(1-\sum_{i\in N} x_{ih})=0$ for all h; (iii) and x_i maximizes u_i in $\{z_i\in\mathbb{R}_+^l$: $\sum_{h\in L} p_h z_{ih} \leq \sum_{h\in L} p_h\, \omega_{ih}\}$ for all i.

Definition 2: An allocation x is a *strong* (resp. *weak*) *market optimum* of the social system (w,u,ρ) if it is attainable and if there exists no attainable allocation x' such that $u_i(x_i')\geq u_i(x_i)$ for all i, with a strict inequality for at least one i (resp. $u_i(x_i')>u_i(x_i)$ for all i). The set of weak (resp. strong) market optima of (w,u,ρ) is denoted by P_u (resp. $P_u^* \subset P_u$).

Definition 3: An allocation x is a *strong* (resp. *weak*) *distributive optimum* of the social system (w,u,ρ) if it is attainable and if there exists no attainable allocation x' such that $w_i(u(x'))\geq w_i(u(x))$ for all i, with a strict inequality for at least one i (resp. $w_i(u(x'))> w_i(u(x))$ for all i). The set of weak (resp. strong) distributive optima of (w,u,ρ) is denoted by P_w (resp. $P_w^* \subset P_w$).

Definition 4: Let (p,x) be a competitive market equilibrium with free disposal of the social system of private property (w,u,ω). Pair $(\omega',(p',x'))$ is a *distributive liberal social contract* of (w,u,ω) relative to market equilibrium (p,x) if (p',x') is a competitive market equilibrium with free disposal of (w,u,ω') such that: (i) x' is a strong distributive optimum of (w,u,ρ); (ii) and $w_i(u(x'))\geq w_i(u(x))$ for all i.

For the sake of brevity, the competitive market equilibrium with free disposal of Definition 1 will often be referred to as Walrasian equilibrium or even simply as "market equilibrium" in the sequel. Likewise, we will often refer to the distributive liberal social contract simply as the "social contract".

Whenever a pair $(\omega',(p',x'))$ is a distributive liberal social contract of (w,u,ω) relative to market equilibrium (p,x), we also refer to ω' as a distributive liberal social contract of (w,u,ω) relative to (p,x), and to x' as a *distributive liberal social contract solution* of (w,u,ω) relative to (p,x).

3- Differentiable, convex social systems

In this section, we first present our differentiability and convexity hypotheses, summarized in Assumption 1 below. The definitions of corresponding standard properties of utility functions, such as differentiability, quasi-concavity, strict quasi-concavity and other, are recalled in the associate footnote, with brief comments on their relations and on some of their elementary consequences.

We next discuss the general significance and justifications of the hypothesis, with a particular emphasis on its application to individual social preferences in parts (ii) and (iii) of the assumption.

We use the following standard notations. Let $z=(z_1,\ldots,z_m)$ and $z'=(z'_1,\ldots,z'_m)\in\mathbb{R}^m$, $m\geq 1$: $z\geq z'$ if $z_i\geq z_i'$ for any i ; $z>z'$ if $z\geq z'$ and $z\neq z'$; $z>>z'$ if $z_i>z_i'$ for any i ; $z.z'$ is the inner product $\sum_{i=1}^m z_i z'_i$; z^T is the transpose (column-) vector of z; $\mathbb{R}_+^m=\{z\in\mathbb{R}^m: z\geq 0\}$; $\mathbb{R}_{++}^m=\{z\in\mathbb{R}_+^m: z>>0\}$. Let $f=(f_1,\ldots,f_q):V\to\mathbb{R}^q$, defined on open set $V\subset\mathbb{R}^m$, be the Cartesian product of the C^2 real-valued functions $f_i:V\to\mathbb{R}$: ∂f and $\partial^2 f$ denote its first and second derivative respectively; $\partial f(x)$, viewed in matrix form, is the $q\times m$ (Jacobian) matrix whose generic entry $(\partial f_i/\partial x_j)(x)$, also denoted by $\partial_j f_i(x)$ (or, sometimes, by $\partial_{x_j} f_i(x)$), is the first partial derivative of f_i with respect to its j-th argument at x; the transpose $[\partial f_i(x)]^T$ of the i-th row of $\partial f(x)$ is the gradient vector of f_i at x; finally, $\partial^2 f_i(x)$, viewed in matrix form, is the $m\times m$ (Hessian) matrix whose generic entries $(\partial^2 f_i/\partial x_j\partial x_k)(x)$, also denoted by $\partial^2_{jk} f_i(x)$, are the second partial derivatives of f_i at x.

Assumption 1[5]: Differentiable convex social system: (i) For all i, u_i is: (a) continuous, increasing, and unbounded above ; (b) C^2 in $\mathbb{R}_{++}^l$; (c) differentiably strictly quasi-concave in $\mathbb{R}_{++}^l$, and, in particular, differentiably strictly concave in an open, convex neighborhood of $\{x_i \in \mathbb{R}_{++}^l : x_i \leq \rho\}$ in $\mathbb{R}_{++}^l$; (d) and such that $x_i >> 0$ whenever $u_i(x_i) > 0 (= u_i(0))$.(ii) For all i, w_i is: (a) increasing in its i-th argument and continuous; (b) C^2 in $\mathbb{R}_{++}^n$; (c) quasi-concave; (d) and such that $w_i(\hat{u}) > w_i(0)$ if and only if $\hat{u} >> 0$. (iii) For all i, $w_i \circ u$ is quasi-concave.

Assumption 1 will be maintained throughout the sequel.

The use of private utility functions endowed with properties of smoothness (that is, C^r utility functions, with $r \geq 2$), rather than the weaker property of continuity, is generally justified by reference to approximation theory (see notably Balasko, 1988: 2.3, for an informal discussion, and Mas-Colell, 1985: 2.8 and 8.4, for formal properties). The latter states, essentially, that any continuous function can be approximated arbitrarily close by a smooth function. This basic density property can, moreover, in some cases of theoretical interest, be complemented with a property of openness of relevant dense subsets of smooth utility functions, yielding a statement of genericity of the corresponding smoothness properties. The latter holds true, notably, for differentiably strictly convex monotone private preferences, which make an open and dense subset of the set of convex monotone private preferences (Mas-Colell, op. cit.: 8.4.1).

This general principle of method applies to individual social preferences and utilities as well. Note nevertheless that, for reasons made explicit below, we do assume that individual distributive utilities w_i are smooth (Assumption 1-(ii)-(b)), but do *not* suppose them differentiably strictly quasi-concave (Assumption 1-(ii)-(c)).

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[5] Recall that u_i is defined on $\mathbb{R}_+^l$, the nonnegative orthant of $\mathbb{R}^l$. We say that such a function is *increasing* (resp. strictly increasing) if $x_i \gg x_i'$ (resp. $x_i > x_i'$) implies $u_i(x_i) > u_i(x_i')$. It is: *quasi-concave* if $u_i(x_i) \geq u_i(x_i')$ implies $u_i(\alpha x_i + (1-\alpha) x_i') \geq u_i(x_i')$ for any $1 \geq \alpha \geq 0$; *strictly quasi-concave* if $u_i(x_i) \geq u_i(x_i')$, $x_i \neq x_i'$ implies $u_i(\alpha x_i + (1-\alpha) x_i') > u_i(x_i')$ for any $1 > \alpha > 0$; *differentiably strictly quasi-concave* in an open, convex set $V \subset \mathbb{R}_{++}^l$ if its restriction to V is C^2 (that is, twice differentiable with continuous second derivatives), strictly quasi-concave, and has a nonzero Gaussian curvature everywhere in V (or equivalently a nonzero determinant of the bordered Hessian $\begin{matrix} \partial^2 u_i(x_i) & [\partial u_i(x_i)]^T \\ \partial u_i(x_i) & 0 \end{matrix}$ for every x_i in V); *differentiably strictly concave* in an open, convex set $V \subset \mathbb{R}_{++}^l$ if its restriction to V is C^2 and such that the Hessian matrix $\partial^2 u_i(x_i)$ is negative definite for every x_i in V. Note that the differentiable strict quasi-concavity of u_i in $\mathbb{R}_{++}^l$ implies the existence of a differentiably strictly concave utility representation of the underlying preference preordering on any compact, convex subset of $\mathbb{R}_{++}^l$ (Mas-Colell, op. cit.: 2.6.4), so that the second part of assumption 1-(i)-(c) does not imply any additional restriction, relative to the first part of the same assumption. Note also that an increasing u_i which also is differentiably strictly quasi-concave in $\mathbb{R}_{++}^l$ must be *differentiably strictly increasing* in $\mathbb{R}_{++}^l$, that is, such that $\partial u_i(x_i) >> 0$ everywhere in $\mathbb{R}_{++}^l$ (hence strictly increasing in $\mathbb{R}_{++}^l$). And note, finally, that in the special case of a single market commodity (that is, $l=1$), we can let $u_i(x_i) = \text{Log}(1+x_i)$ without loss of generality (as "C^2 differentiable strictly quasi-concave" degenerates, in this simple case, to "C^2 strictly increasing").

Suppose, next, that utility representation u_i is bounded above and verifies all other Assumptions 1-(i). Let $\sup u_i(\mathbb{R}_+^l) = b > a > u_i(\rho)$. Note that $a \in u_i(\mathbb{R}_+^l) = [0,b)$,since u_i is continuous and increasing. Define ξ: $[0,b) \to \mathbb{R}_+$ by: $\xi(t) = t$ if $t \in [0,a)$; and $\xi(t) = t + (t-a)^3 \exp(1/(b-t))$ if $t \in [a,b)$. One verifies by simple calculations that ξ is strictly increasing, and that $\xi \circ u_i$ is C^2, unbounded above, and therefore represents the same preordering as u_i and verifies assumption 1-(i). That is, there is no loss of generality in supposing u_i unbounded above.

Assumption 1-(i) notably implies that $u: \mathbb{R}_+^{ln} \to \mathbb{R}_+^n$ is onto (since u_i is a continuous, increasing, unbounded above function $\mathbb{R}_+^l \to [0, \infty)$ for all i), so that the domain $u(\mathbb{R}_+^l)$ of individual distributive utility functions coincides with the nonnegative orthant of $\mathbb{R}^n$. The definitions above extend readily to functions w_i and $w_i \circ u$.

The convexity (resp. strict convexity) of preferences is sufficient, and in general necessary, for the continuity (resp. continuity and determinacy) of individual preference-maximizing behaviour relative to the parameters of the individual's environment, such as market prices, the distribution of wealth, or the actions of others.

What appears essential, strictly speaking, for general equilibrium analysis, is not so much the continuity of individual demand behaviour or convexity of individual private preferences per se, as the continuity or convexity of suitable aggregate counterparts, such as, notably: for the existence of general equilibrium, the continuity of aggregate excess demand relative to prices; and, for the price-supportability of market-efficient allocations, the convexity of the set of aggregate demands associated with allocations unanimously preferred to the market optimum (for private preferences). Moreover, approximate continuity or convexity follow naturally from aggregation when the economic and social systems consist of a large number of negligible individual agents. In other words, nonconvexities in individual private preferences, if any, can be safely neglected, as far as the global properties of the social system are concerned, when the latter is large and made of individual agents who are small relative to it (such as households, for example).

The argument above applies, in similar terms, both to the market (i.e. private) preferences of individuals, and to their distributive (i.e. social) preferences. There is, moreover, an additional argument in favour of the convexity assumption, which specifically applies to the latter type of preferences, namely, the natural connections between the convexity of distributive preferences, and inequality aversion. Quasi-concavity implies a "preference for averaging" (in the sense that, if z and z' are indifferent for the preference relation, then $\alpha z+(1-\alpha)z'$ is weakly preferred to both z and z' for any α in $[0,1]$) which, applied to distribution issues, admits a natural interpretation in terms of a weak preference for equality.

We have recorded, at this point, the main reasons for the use of the smoothness and convexity hypotheses of Assumption 1 in the context of this study. There remains to give the reasons for: (i) the boundary conditions 1-(i)-(d) and 1-(ii)-(d), which state that positive utility implies that all the arguments of the utility function are positive; (ii) and the weakness of the monotonicity and convexity assumptions on social preferences, relative to similar assumptions on private preferences.

The boundary condition on private utilities is a standard technical convenience, designed to eliminate inessential singularities associated with corner solutions in individual consumption behaviour (that is, with zero equilibrium consumption of some commodities by some individual).

The boundary condition on distributive utilities is a substantial assumption on the contrary. Associated with the former, it implies that all individuals strictly prefer allocations where every individual is enjoying a positive wealth and welfare, to allocations where any individual is starving to death.

The hypothesis of increasing private utilities (1-(i)-(a)), which, combined with differentiable strict quasi-concavity, actually implies that private utilities are differentiably strictly increasing (see the footnote of Assumption 1), is used as a peculiarly simple and direct way of eliminating such borderline cases of little practical interest as zero equilibrium prices, and associate pathologies such as, notably, individual budget sets with empty interiors (for individuals whose endowment is not strictly positive) and associate discontinuities in demand. It can be relaxed to a mere assumption of differentiable nonsatiation (that is, $\partial u_i(x_i) \neq 0$ for all $x_i \in \mathbb{R}_+^l$), provided that one moreover supposes, for example, that all individuals have strictly positive endowments (that is, $\omega_i >> 0$ for all i), and that, at any system of nonnegative prices, all commodities are desired by some individuals (linked allocation: Mas-Colell, op. cit.: Chap. 4).

Monotonicity assumptions on individual social preferences obey quite different considerations, narrowly conditioned by their object, and notably by the large-scale character of the latter (the allocation of resources in society as a whole).

We only assume that an individual's distributive utility is increasing in his own private utility (see section 2). The latter follows from the basic hypothesis of separability of individual allocation preferences in own consumption, and interprets as a basic consistency requirement, stipulating that an individual's "social" view on his own consumption, as induced by his allocation preferences, must coincide with his "private" view on the same object, as represented by his private utility function.

We mentioned in section 2 that our formulation of the hypothesis of non-paternalistic utility interdependence was compatible with the distributive malevolence or indifference of any individual relative to any other, in a local or in a global sense. The casual observation of social life suggests that none of such psychological attitudes can be excluded on a priori grounds. It is also a commonplace of the stylized psychological theory of economists, elaborately expressed in Adam Smith's *Theory of Moral Sentiments*, 1759, that individuals should, in most circumstances of ordinary life, be more sensitive to their own welfare (in the sense of their ophelimity) than to the welfare of others (at least "distant" others), notably because the psychological perception of others' welfare proceeds, to a large extent, from acts of imaginative sympathy (imagining oneself in the other's skin), which tend to be associated with less powerful affects in terms of frequency and average intensity, hence to produce less vivid and enduring perceptions, than the perception of one's own welfare through one's own senses[6]. Considered from this elaborate theoretical perspective, or from flat factual evidence, individual social preferences should notably exhibit wide ranges of indifference, distributive or else, due to the large-scale character of their object. It seems natural to expect, for example, that an individual will ordinarily feel indifferent relative to reallocations between individuals of close observable characteristics, such as similar ways of life for instance, if these characteristics are very different from his own and if he has no personal relationship with these fellows. Such indifference is inconsistent, in general, with strictly monotone, or strictly convex preferences.

We chose, therefore, to keep to a minimum the monotonicity and convexity assumptions on social preferences at the individual level. Meaningful hypotheses of this type, if any, must be stated directly at the aggregate level, for the Paretian partial preordering induced by individual social preferences. This we do in section 4, for monotonicity, and in section 6 for differentiable strict convexity.

4- The separability of allocation and distribution

The first general property of the abstract social systems outlined in section 2 is the separability of allocation and distribution. The property states, essentially, that the redistribution of the social contract does not alter the fundamental features of the allocation of resources through the market, which follow from the role of market prices in the coordination of individual supplies and demands, namely, the existence of market equilibrium, the Pareto-

[6] See Lévy-Gargoua et alii, 2006, for a comprehensive review of the literature, and also for original views on the formation of the social preferences of individuals, developed notably (but not only) from the economists' perspective.

efficiency of equilibrium allocations relative to private utilities ("market-efficiency") and the price-supportability of market optima.

The existence of market equilibrium, and the so-called first and second fundamental theorems of welfare economics (that is, respectively, in our terms, the market-efficiency of equilibrium allocation and the price-supportability of market optima), are well-known consequences of Assumption 1-(i). Social contract redistribution was characterized, in section 2, as a redistribution of individual endowments yielding a market equilibrium that is both Pareto-efficient relative to individual social utilities and unanimously (weakly) preferred to the initial market equilibrium. The separability property readily follows, therefore, from the notion of distributive liberal social contract itself, provided that the latter is consistently defined, that is, provided that there always exists a market equilibrium which is a distributive optimum unanimously preferred to the initial market equilibrium for individual social preferences.

The section is organized as follows. We first establish the inner consistency of the definition of the distributive liberal social contract in Theorem 1. We next provide a useful characterization of distributive optima as the maxima of averages of individual social utility functions (Theorem 2). We then proceed to the elicitation of an important consequence of separability, namely, the equivalence of cash and in-kind transfers for Pareto-efficient redistribution (Theorem 3). And we conclude with an analysis of the significance and scope of separability.

The inner consistency of the definition of the distributive liberal social contract is a simple consequence of the well-known fact that distributive optima are necessarily also market optima, provided that: (i) utility interdependence is non-paternalistic; (ii) and the partial preordering of Pareto associated with distributive utilities verifies some suitable property of nonsatiation (see notably Rader, 1980 and Lemche, 1986; a detailed account of this literature is provided in Mercier Ythier, 2006: 4.1.2). The theorem below first fits this basic property into the differentiable setup of the present article, and next draws its consequences for the existence of the distributive liberal social contract.

The strong (resp. weak) partial preordering of Pareto relative to distributive utilities (in short, strong (resp. weak) distributive preordering of Pareto), denoted by $\succ_w$ (resp. $\succ^*_w$), is defined on the set $u(\mathbb{R}_+^n)$ of ophelimity distributions by: $\hat{u} \succ_w \hat{u}'$ (resp. $\hat{u} \succ^*_w \hat{u}'$) if $w(\hat{u}) >> w(\hat{u}')$ (resp. $w(\hat{u}) > w(\hat{u}')$). The weak (resp. strong) ophelimity distributions associated with the distributive optima of (w,u,ρ) are, by definition, the maximal elements of $\succ_w$ (resp. $\succ^*_w$) in the set $u(A)$ of attainable ophelimity distributions, that is, the elements $\hat{u}$ of $u(A)$ such that there exists no $\hat{u}'$ in $u(A)$ such that $\hat{u}' \succ_w \hat{u}$ (resp. $\hat{u}' \succ^*_w \hat{u}$).

Note that weak and strong distributive efficiency are not equivalent, in general, under Assumption 1. We will therefore maintain the distinction between the weak and strong notions of distributive optimum throughout this article. On the contrary, as is well-known, weak and strong market efficiency are equivalent under Assumption 1-(i) (see Proposition 6 in the Appendix). Therefore we shall not distinguish between them anymore in the sequel.

For any integer $m \geq 2$, we denote by S_m the unit-simplex of $\mathbb{R}^m$, that is, set $\{z=(z_1,\ldots,z_m) \in \mathbb{R}_+^m : \sum_{i=1}^m z_i = 1\}$.

Theorem 1: Let (w,u,ρ) verify Assumption 1, and suppose moreover that, for all $\mu \in S_n$ and all $\hat{u} \in u(A) \cap \mathbb{R}_{++}^n$, $\sum_{i \in N} \mu_i \partial w_i(\hat{u}) \neq 0$ (differentiable nonsatiation of the weak distributive preordering of Pareto). Then: (i) any distributive optimum is a market optimum; (ii) there exists a distributive liberal social contract for any initial distribution ω, relative to any market equilibrium of (w,u,ω).

Proof: (i) A distributive optimum x is by definition a local weak maximum of the product function $(w_1\circ u,\ldots,w_n\circ u)$ in the set of attainable allocations A. Assumptions 1-(i)-(d) and 1-(ii)-(d) readily imply x>>0 and $u(x)$>>0. The first-order necessary conditions (f.o.c.) for this smooth optimization problem (e.g. Mas-Colell, op. cit.: D.1) then state that there exists $(\mu,p)\in\mathbb{R}_+^n\times\mathbb{R}_+^l$ such that: (i) $(\mu,p)\neq 0$; (ii) $p.(\rho-\sum_{i\in N}x_i)=0$; (iii) $\sum_{i\in N}\mu_i\partial_j w_i(u(x))\partial u_j(x_j)-p=0$ for all $j\in N$. We must have $\mu>0$, for otherwise $p=0$ by f.o.c. (iii), which contradicts f.o.c. (i). Since $\mu>0$, (μ,p) can be replaced by $(\mu/\sum_{i\in N}\mu_i\,,\,p/\sum_{i\in N}\mu_i)$ in the f.o.c., that is, we can suppose from there on that $\mu\in S_n$. F.o.c. (iii) is equivalent to: $(\sum_{i\in N}\mu_i\partial_j w_i(u(x)))\partial u_j(x_j)=p$ for all j. Differentiable nonsatiation of the Paretian preordering and strictly increasing private utilities then imply that p>>0 and $\sum_{i\in N}\mu_i\partial_j w_i(u(x))>0$ for all j. The necessary first-order conditions reduce therefore to the following, equivalent proposition: x is >>0, such that $\sum_{i\in N}x_i=\rho$, and there exists $(\mu,p)\in S_n\times\mathbb{R}_{++}^l$ such that, for all $j\in N$, $\sum_{i\in N}\mu_i\partial_j w_i(u(x))>0$ and $\partial u_j(x_j)=(1/\sum_{i\in N}\mu_i\partial_j w_i(u(x))p$. The latter system of conditions characterizes a market optimum of (w,u,ρ) under Assumption 1-(i), by application of standard results on the characterization of Pareto optima of differentiable economies (see Proposition 6 of the Appendix). This establishes the first part of Theorem 1.
(ii) Let (p,x) be a competitive market equilibrium with free disposal of (w,u,ω). The set $A(x)=\{z\in A: w_i(u(z))\geq w_i(u(x))$ for all $i\in N\}$ of attainable allocations unanimously weakly preferred to x is nonempty (it contains x), and compact (as a subset of compact set A, which is closed by continuity of $w_i\circ u$ for all i). Continuous function $\sum_{i\in N}\mu_i(w_i\circ u)$ therefore has at least one maximum in $A(x)$, for any given $\mu\in S_n$. Let ω' be such a maximum, that is: $\sum_{i\in N}\mu_i(w_i(u(\omega'))\geq\sum_{i\in N}\mu_i(w_i(u(z))$ for all $z\in A(x)$, for a given $\mu\in S_n$. We suppose moreover that μ>>0. We want to prove that there exists a price system p' such that $(\omega',(p',\omega'))$ is a distributive liberal social contract of (w,u,ω) relative to (p,x).

If $z\in A(x)$ is not a strong distributive optimum, that is, if there exists $z'\in A$ such that $w(u(z'))>w(u(z))$, then $z'\in A(x)$, and $\sum_{i\in N}\mu_i(w_i(u(z'))>\sum_{i\in N}\mu_i(w_i(u(z))$ (since μ>>0), so that z does not maximize $\sum_{i\in N}\mu_i(w_i\circ u)$ in $A(x)$. Therefore, ω' is a strong distributive optimum of (w,u,ρ), unanimously weakly preferred to x by construction. It suffices to establish, to finish with, that there exists a price system p' such that (p',ω') is a competitive market equilibrium with free disposal of (w,u,ω'). But this readily follows from the first-order conditions of the end of part (i) of this proof (recall that $P_w^*\subset P_w$), by application of standard results on the characterization of competitive equilibria of differentiable economies.■

An important by-product of the proof of Theorem 1 is the characterization of distributive optima as maxima of weighted averages of individual social utilities (see the first part of Theorem 2 below). The latter extends to distributive optima and utilities, with similar arguments, the familiar characterization of market optima as maxima of weighted averages of individual private utilities.

The Pareto-efficient redistribution of the distributive liberal social contract, in particular, implicitly supposes a process of identification of socially desirable allocations by: (i) aggregation, first, of "individual-social" utilities into a "social-social" utility function

$\sum_{i\in N} \mu_i(w_i \circ u)$ by means of arbitrary vectors of weights $\mu\in S_n$; (ii) and maximization, second, of these "social-social" utility functions in the set of attainable allocations unanimously weakly preferred to some original equilibrium position (see our constructive proof of the existence of a distributive liberal social contract, in part (ii) of the proof of Theorem 1). Note, nevertheless, that the distributive liberal social contract, such as defined in section 2, does not itself implement the distributive optimum. It only redistributes endowments, and leaves to the market the task of achieving the equilibrium allocation[7].

Theorem 2: Let (w,u,ρ) verify Assumption 1, and suppose moreover that, for all $\mu\in S_n$ and all $\hat{u}\in u(A)\cap\mathbb{R}_{++}^n$, $\sum_{i\in N} \mu_i\partial w_i(\hat{u})\neq 0$. The following three propositions are then equivalent: (i) x is a weak distributive optimum (w,u,ρ); (ii) x is >>0, such that $\sum_{i\in N} x_i=\rho$, and there exists $(\mu,p)\in S_n\times\mathbb{R}_{++}^l$ such that, for all $j\in N$, $\sum_{i\in N} \mu_i\partial_j w_i(u(x))>0$ and $(\sum_{i\in N} \mu_i\partial_j w_i(u(x))\partial u_j(x_j)= p$; (iii) there exists $\mu\in S_n$ such that x maximizes $\sum_{i\in N} \mu_i(w_i \circ u)$ in A.

Proof: The proof of Theorem 2 is a simple extension of an argument developed in the first part of the proof of Theorem 1, where we already established that (i)$\Rightarrow$(ii). We now prove that (ii)$\Rightarrow$(iii)$\Rightarrow$(i).

If x is not a weak distributive optimum, that is, if $x\notin A$, or if $x\in A$ and there exists $x'\in A$ such that $w(u(x'))>>w(u(x))$, then, clearly, x is not a maximum of $\sum_{i\in N} \mu_i(w_i \circ u)$ in A, whatever $\mu\in S_n$. Therefore, (iii)$\Rightarrow$(i).

We establish, to finish with, that (ii)$\Rightarrow$(iii). Suppose that x is >>0, such that $\sum_{i\in N} x_i=\rho$, and that there exists $(\mu,p)\in S_n\times\mathbb{R}_{++}^l$ such that, for all $j\in N$, $\sum_{i\in N} \mu_i\partial_j w_i(u(x))>0$ and $\sum_{i\in N} \mu_i\partial_j w_i(u(x)\partial u_j(x_j)= p$. Fixing μ, this set of conditions coincides with the first-order necessary conditions for a local maximum of $\sum_{i\in N} \mu_i(w_i \circ u)$ in A (apply to the latter program the argument developed in the proof of Theorem 1 for the derivation of the f.o.c. for a weak distributive optimum). The proof will be completed, therefore, if we establish that these necessary conditions for a local maximum of $\sum_{i\in N} \mu_i(w_i \circ u)$ in A are also sufficient conditions for a global maximum of the same program. But this readily follows from our assumptions and the Theorem 1 of Arrow and Enthoven, 1961 (notably their conditions (b) or (c), which are both verified under our assumptions).■

The third property outlined in this section is the equivalence of cash and in-kind Pareto-efficient redistribution.

We first introduce a notion of price-wealth distributive optimum, on a pattern similar to the price-wealth equilibrium of market equilibrium theory, and next prove that this notion is equivalent to the distributive optimum defined in section 2.

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[7] Except, of course, in the special case, where, as in part (ii) of the proof of Theorem 1, endowment redistribution achieves market equilibrium. This special case is theoretically interesting, because it is always accessible in theory (by the second fundamental theorem of welfare economics), and therefore provides an easy and simple way for establishing the existence of a distributive liberal social contract. The corresponding market equilibrium is the autarkic equilibrium, that is, a market equilibrium where each individual demands and consumes his own endowment. This equilibrium is necessarily unique under Assumption 1-(i), due notably to the regularity assumption of differentiable strict convexity of individual private preferences (Balasko, op. cit.: 3.4.4). This means, in particular, that social contract redistribution fully crowds out market exchange in this case, which therefore appears empty on practical grounds, as actual economies hardly reach or even approach any state of reasonable economic efficiency without large market exchanges.

We use standard definitions and properties of demand and indirect utility functions, which hold true under Assumption 1-(i). Notably, there exists, for each individual i, a C^1 demand function $f_i:\mathbb{R}_{++}^l\times\mathbb{R}_{++}\to\mathbb{R}_{++}^l$, that is, a C^1 function such that, for any price-wealth vector $(p,r_i)>>0$, $f_i(p,r_i)$ is the (unique) consumption bundle that maximizes the private utility of individual i subject to this individual's budget constraint $p.x_i\leq r_i$. The (private) indirect utility function of individual i, defined as $v_i=u_i\circ f_i$, also is a C^1 function $\mathbb{R}_{++}^l\times\mathbb{R}_{++}\to\mathbb{R}_{++}$. Moreover, f_i and v_i are both well defined and continuous on $\mathbb{R}_{++}^l\times\mathbb{R}_{+}$, with $f_i(p,0)=0$ and $v_i(p,0)=0$ for all $p>>0$. Demand functions are: positively homogeneous of degree 0 (that is, $f_i(\alpha p,\alpha r_i)= f_i(p,r_i)$ for all $(p,r_i)\in\mathbb{R}_{++}^l\times\mathbb{R}_{+}$ and all $\alpha\in\mathbb{R}_{++}$); and such that $p.f_i(p,r_i)=r_i$ for all $(p,r_i)\in\mathbb{R}_{++}^l\times\mathbb{R}_{+}$ (the so-called additivity property of Walrasian demand). Indirect utility functions are positively homogeneous of degree 0, and strictly increasing with respect to wealth. Since the money wealth of an individual reduces, in our setup, to the market value of his endowment $r_i=p.\omega_i$, we get $\sum_{i\in N} p.f_i(p,p.\omega_i)=\sum_{i\in N} p.\omega_i=p.\rho$ as the expression of Walras Law for aggregate demand, verified for any system of positive market prices $p>>0$ and any distribution of initial endowments $\omega\in\{z\in\mathbb{R}_{+}^{ln}:\sum_{i\in N} z_i=\rho\}$. From Walras Law and the homogeneity properties of individual demands, a system of equilibrium market prices is defined only up to a positive multiplicative constant. In the sequel, market prices are normalized so that $p\in S_l$ (that is, we replace p by the equivalent $p/\sum_{i\in L} p_i$; this always is possible since $\sum_{i\in L} p_i$ necessarily is >0 at equilibrium with our definitions and assumptions). With this normalization, we get $p.\rho=1$ for any p, which means that the market value of the aggregate resources of the economy is constant relative to normalized market prices, equal to 1. We let: the distribution of money wealth $(r_1,\ldots,r_n)$ be denoted by r; the product function $(p,r)\to(f_1(p,r_1),\ldots, f_n(p,r_n))$ be denoted by f; the product function $(p,r)\to(v_1(p,r_1),\ldots, v_n(p,r_n))$ be denoted by v.

There is a well-known one-to-one correspondence, in differentiable economies, between market optima $x\in P_u$ and the systems of prices and wealth distribution (p,r) such that $\sum_{i\in N} f_i(p,r_i)=\rho$ (price-wealth market equilibria). Precisely, under Assumption 1-(i): for any $x\in P_u$, there exists a unique $p\in S_l$ such that the pair $(p,r)=(p,(p.x_1,\ldots,p.x_n))$ is a price-wealth market equilibrium (and the equilibrium p is then >>0); conversely, if (p,r) is a price-wealth market equilibrium, then $x=f(p,r)$ is a market optimum, p is >>0 and $r=(p.x_1,\ldots,p.x_n)$ (see the Appendix: Proposition 6). The notion of price-wealth market equilibrium yields a natural alternative definition of distributive optimum as a price-wealth market equilibrium which is not Pareto-dominated, relative to individual social utilities, by any other price-wealth market equilibrium. Formally:

Definition 5: A *price-wealth market equilibrium* of social system (w,u,ρ) is a pair $(p,r)\in S_l\times S_n$ such that $\sum_{i\in N} f_i(p,r_i)=\rho$.

Definition 6: A pair $(p,r)\in S_l\times S_n$ is a (weak) *price-wealth distributive optimum* of social system (w,u,ρ) if: (i) (p,r) is a price-wealth equilibrium of (w,u,ρ); (ii) and there exists no price-wealth equilibrium (p',r') of (w,u,ρ) such that $w(v(p',r'))>>w(v(p,r))$.

The following theorem establishes the equivalence of the two notions of distributive efficiency for the social systems that verify Assumption 1 and differentiable nonsatiation of the weak distributive preordering of Pareto. Necessity straightforwardly follows from definitions. The proof of sufficiency, on the contrary, is far from immediate. Its complexity stems from the definition of price-wealth distributive optimum as a maximum of $w\circ v$ in the set of price-wealth equilibria of (w,u,ρ). The latter set has a complex structure. Under Assumption 1-(i), its intersection with $\mathbb{R}_{++}{}^{l}\times\mathbb{R}_{++}{}^{n}$ is a C^1 manifold of dimension n-1 C^1-diffeomorphic to $P_u\cap\mathbb{R}_{++}{}^{ln}$ (as implied by the proofs of Balasko, op. cit.: 4.7.1, 5.2.1 and 5.2.4 with suitable adjustments in definitions and assumptions; see the Appendix of the present article for a discussion of the relations between our setup and Balasko's). This complexity finds its expression in the set of first-order necessary conditions for the maxima of $w\circ v$, derived in the first step of the second part of the proof below. Their computation involves some of the fundamental properties of Walrasian demand, such as additivity, Walras Law and the rank of the substitution matrices of Slutsky.

Theorem 3: Let (w,u,ρ) verify Assumption 1, and suppose moreover that, for all $\mu\in S_n$ and all $\hat{u}\in u(A)\cap\mathbb{R}_{++}{}^{n}$, $\sum_{i\in N}\mu_i\partial w_i(\hat{u})\neq 0$. Let x be a market optimum of (w,u,ρ). The following two propositions are then equivalent: (i) x is a weak distributive optimum; (ii) the unique $(p,r)\in S_l\times S_n$ such that $x=f(p,r)$ is a weak price-wealth distributive optimum.

Proof: We begin with a short summary of standard results concerning the f.o.c. for market optima of exchange economies verifying Assumption 1-(i) (see the Proposition 6 of the Appendix). A first familiar result states that x is a market optimum of (w,u,ρ) if and only if $\sum_{i\in N}x_i$ =ρ, and there exists a price system $(\lambda,p)\in\mathbb{R}_{+}{}^{n}\times\mathbb{R}_{++}{}^{l}$ such that, for all i: either x_i=0; or x_i>>0 and $\lambda_i\,\partial u_i(x_i)=p$. In particular, if x is a >>0 market optimum, we must have λ>>0 and $\lambda_i\partial u_i(x_i)=p$ for all i. In the latter case, the vector of multipliers (λ,p) of the f.o.c. is unique up a positive multiplicative constant, and there is a unique (λ,p) that verifies the f.o.c. with $p\in S_l$. In the general case (x>0 market optimum), p is unique up to a positive multiplicative constant, proportional to $(1/\sum_{j\in L}\partial_{j\in L}u_i(x_i))\partial u_i(x_i)\in S_l$ for any i such that x_i>>0 (there necessarily exists at least one such i). Vectors (p,x) that verify the f.o.c. above are competitive market equilibria with free disposal of the social system of private property (w,u,x), and one and only one of them is such that $p\in S_l$. Consequently, for any market optimum x of a social system (w,u,ρ) that verifies Assumption 1-(i), there exists a unique price-wealth equilibrium $(p,r)\in S_l\times S_n$ such that $x=f(p,r)$; and p is then necessarily >>0. If, moreover, x is >>0, this unique equilibrium (p,r) is >>0, because $r_i=p.f_i(p,r_i)>0$ for all i as a joint consequence of the additivity property of Walrasian demand, the assumption that x is >>0, and the fact that p is >>0. Finally, the envelope theorem implies that, for any (p,r)>>0, the inverse of multiplier λ_i is equal to $\partial_{r_i}v_i(p,r_i)$, the partial derivative of i's indirect utility function with respect to its wealth argument r_i (i's "marginal ophelimity of wealth").

Suppose, now, that proposition (ii) of Theorem 3 is not verified, that is, in view of the former paragraph, suppose that the unique $(p,r)\in S_l\times S_n$ such that $x=f(p,r)$ is not a price-wealth distributive optimum. Definitions 3 and 6 then readily imply that x is not a weak distributive optimum. Therefore (i)⇒(ii).

We now prove the converse. Let $(p^*,r^*)\in S_l\times S_n$ denote the unique (p,r) such that $x=f(p,r)$, and suppose that it is a weak price-wealth distributive optimum. The proof proceeds in three steps. We first derive the first-order necessary conditions for a local weak maximum of $w\circ v$ in $\mathbb{R}_{++}{}^{l}\times\mathbb{R}_{++}{}^{n}$ subject to constraint $\rho-\sum_{i\in N}f_i(p,r_i)\geq 0$. We next establish that (p^*,r^*) is a local weak maximum of this program. And we finally prove that the f.o.c. of the program characterize a weak distributive optimum.

The f.o.c. read as follows: there exists $(\mu,\alpha)\in\mathbb{R}_{+}{}^{n}\times\mathbb{R}_{+}{}^{l}$ such that:

(i) $(\mu,\alpha)\neq 0$;

(ii) $\alpha.(\rho-\sum_{i\in N} f_i(p,r_i))=0$;

(iii) $\sum_{i\in N} \mu_i \sum_{j\in N} \partial_j w_i(v(p,r))\partial_p v_j(p,r_j)=\sum_{i\in L} \alpha_i \sum_{j\in N}\partial_p f_{ji}(p,r_j)$;

(iv) $\sum_{i\in N} \mu_i \partial_j w_i(v(p,r))\partial_{r_j} v_j(p,r_j)=\sum_{i\in L} \alpha_i \partial_{r_j} f_{ji}(p,r_j)$ for all $j\in N$.

Suppose that $\alpha=\gamma p$, with $\gamma\in\mathbb{R}_{++}$. Since $p>>0$ by assumption, f.o.c. (ii) implies $\sum_{i\in N} f_i(p,r_i)=\rho$. The differentiation of Walras Law with respect to p (resp. r_j) moreover yields identity $p.\sum_{i\in N}\partial_p f_i(p,r_i)=-\sum_{i\in N} f_i(p,r_i)$ (resp. $\sum_{i\in L} p_i\partial_{r_j} f_{ji}(p,r_j)=1$). The right-hand side of f.o.c. (iii) then reduces to $-\gamma\sum_{i\in N} f_i(p,r_i)$, which is $=-\gamma\rho$ by f.o.c. (ii), while the right-hand side of f.o.c. (iv) reduces to γ. F.o.c. (iv) therefore implies $\mu>0$ (since $\gamma>0$). Using Roy's identity (which states that $f_i(p,r_i) = -(\partial_p v_i(p,r_i)/\partial_{r_i} v_i(p,r_i))$ for all i), we can write the left-hand side of f.o.c. (iv) equivalently as:
$-\sum_{j\in N} \sum_{i\in N} \mu_i\partial_j w_i(v(p,r))\partial_{r_j} v_j(p,r_j) f_j(p,r_j)$. Substituting f.o.c. (iv) into f.o.c. (iii) and using f.o.c. (ii) then yields the redundant equality $\rho=\rho$, that is, f.o.c. (ii) and (iv) together imply f.o.c. (iii). Summarizing, if $\alpha=\gamma p$ with $\gamma\in\mathbb{R}_{++}$, the system of first-order necessary conditions reduces to the following equivalent expression: $(p,r)>>0$ is such that $\sum_{i\in N} f_i(p,r_i)=\rho$, and there exists $\mu>0$ in $\mathbb{R}^n$ such that $\sum_{i\in N} \mu_i\partial_j w_i(v(p,r))\partial_{r_j} v_j(p,r_j)=1$ for all j. It will suffice, therefore, for completing this first step, to establish that α is nonzero and proportional to p. This we do in the next paragraph.

In view of f.o.c. (iv), $\alpha=0$ is inconsistent with the differentiable nonsatiation of the weak distributive preordering of Pareto. Let us prove that α is proportional to p. Using Roy's identity, rewrite, as above, the left-hand side of f.o.c. (iii) equivalently as: $-\sum_{j\in N} \sum_{i\in N} \mu_i\partial_j w_i(v(p,r))\partial_{r_j} v_j(p,r_j) f_j(p,r_j)$. Consider f.o.c. (iv) for all j and add up over j. Substituting the result into the left-hand side of f.o.c. (iii) rewritten as above, we get: $-\sum_{j\in N} (\sum_{i\in L} \alpha_i\partial_{r_j} f_{ji}(p,r_j)) f_j(p,r_j)=\sum_{i\in L} \alpha_i\sum_{j\in N}\partial_p f_{ji}(p,r_j)$. Rewriting in matrix form and rearranging yields the following equivalent expression for the latter equation: $\alpha.\sum_{j\in N}\partial_p f_j(p,r_j)+\partial_{r_j} f_j(p,r_j)=0$. Matrix $\sum_{j\in N}\partial_p f_j(p,r_j)+\partial_{r_j} f_j(p,r_j)$ is the sum of the matrices of coefficients of substitution of Slutsky $\partial_p f_j(p,r_j)+\partial_{r_j} f_j(p,r_j)$. Its rank is l-1 under Assumption 1-(i) (see the end of the proof of Lemma 5.2.1 in Balasko, 1988). Equation $\alpha.\sum_{j\in N}\partial_p f_j(p,r_j)+\partial_{r_j} f_j(p,r_j)=0$, viewed as a linear equation in α, therefore defines a linear space of dimension 1, which contains p since $p.\sum_{j\in N}\partial_p f_j(p,r_j)+\partial_{r_j} f_j(p,r_j)$ is identically =0, from the identities derived from Walras Law recalled in the beginning of the former paragraph. This completes the first step.

The f.o.c. obtained in the first step finally reduce to the following: $(p,r)>>0$ is such that $\sum_{i\in N} f_i(p,r_i)=\rho$, and there exists $\mu>0$ in $\mathbb{R}^n$ such that $\sum_{i\in N} \mu_i\partial_j w_i(v(p,r))\partial_{r_j} v_j(p,r_j)=1$ for all j. In view of the homogeneity and additivity properties of Walrasian demand, they define (p,r) only up to a positive multiplicative constant. In particular, if (p,r) verifies the f.o.c., then $(p/\sum_{i\in L} p_i, r/\sum_{i\in L} p_i)\in S_l\times S_n$ is a price-wealth equilibrium that also verifies the f.o.c.. This means, in other words, that any local weak maximum of $w \circ v$ in $\{(p,r) \in \mathbb{R}_{++}^{l} \times \mathbb{R}_{++}^{n}: \rho - \sum_{i \in N} f_i(p,r_i) \geq 0\}$ is, up to a positive multiplicative constant, a price-wealth equilibrium of (w,u,ρ). As a partial converse, any price-wealth equilibrium (p,r) of (w,u,ρ) such that $r>>0$ belongs to $\{(p,r) \in \mathbb{R}_{++}^{l} \times \mathbb{R}_{++}^{n}: \rho - \sum_{i \in N} f_i(p,r_i) \geq 0\}$ (because p necessarily is >>0, as recalled above). Therefore, the local weak maxima of $w \circ v$ in $\{(p,r) \in \mathbb{R}_{++}^{l} \times \mathbb{R}_{++}^{n}: \rho - \sum_{i \in N} f_i(p,r_i) \geq 0\}$ normalized so that $(p,r) \in S_l \times S_n$ coincide with the local weak maxima of $w \circ v$ in $\{(p,r) \in \mathbb{R}_{++}^{l} \times \mathbb{R}_{++}^{n}: (p,r)$ is a price-wealth equilibrium of $(w,u,\rho)\}$. In view of Definition 6, it suffices, consequently, for completing the second step of the proof, to establish that all price-wealth distributive optima (p,r) are >>0.

If (p,r) is a price-wealth distributive optimum, then it is a price-wealth equilibrium by Definition 6, and p must therefore be >>0 as recalled above. We conclude the proof of the second step by establishing the following: if $(p,r) \in S_l \times S_n$ is such that $p>>0$ and $r_i=0$ for some i, then there exists a >>0 price-wealth equilibrium $(p',r') \in S_l \times S_n$ of (w,u,ρ) such that $w(v(p',r'))>>w(v(p,r))$. Note first that if $(p,r) \in S_l \times S_n$ is such that $p>>0$ and $r_i=0$ for some i, and if $(p',r') \in S_l \times S_n$ is >>0, then: $v_i(p,r_i)=0<v_i(p',r')$ (see the properties of indirect utility functions recalled above); hence $w(v(p,r)) \leq w(0) << w(v(p',r'))$ by Assumption 1-(ii)-(d). We prove, to finish with, that strictly positive price-wealth equilibria exist under Assumption 1-(i): pick any initial distribution ω such that $\omega_i>>0$ for all i; (w,u,ω) has at least one competitive market equilibrium with free disposal (p',x') by application of the general existence theorem of Arrow and Debreu, 1954; as already noticed, we necessarily have $p'>>0$ at equilibrium under Assumption 1-(i); therefore $(p'.\omega_1,\ldots,p'.\omega_n)$ is >>0, and $(p', (p'.\omega_1,\ldots,p'.\omega_n))$ is a >>0 price-wealth equilibrium of (w,u,ρ). This completes the second step.

Recall that we denoted by (p^*,r^*) the unique price-wealth equilibrium such that $x=f(p^*,r^*)$, where x is some fixed market optimum, and that we supposed that (p^*,r^*) is a weak price-wealth distributive optimum. We established in the second step above that (p^*,r^*) necessarily is a local weak maximum of $w \circ v$ in $\{(p,r) \in \mathbb{R}_{++}^{l} \times \mathbb{R}_{++}^{n}: \rho - \sum_{i \in N} f_i(p,r_i) \geq 0\}$. The f.o.c. obtained in the first step then imply that: $x>>0$ (since $x=f(p^*,r^*)$ and $(p^*,r^*)>>0$); $\sum_{i \in N} x_i = \rho$; and there exists $\mu>0$ in $\mathbb{R}^n$ such that $\sum_{i \in N} \mu_i \partial_j w_i(v(p^*,r^*)) \partial_{r_j} v_j(p^*,r_j^*) = 1$ for all j. Since x is a >>0 equilibrium allocation, we must have $\partial u_j(x_j) = \partial_{r_j} v(p^*,r_j^*) p^*$ for all j. The f.o.c. therefore imply, equivalently, the following: x is >>0, such that $\sum_{i \in N} x_i = \rho$, and there exists $\mu>0$ in $\mathbb{R}^n$ such that $\sum_{i \in N} \mu_i \partial_j w_i(u(x))>0$ and $\sum_{i \in N} \mu_i \partial_j w_i(u(x)) \partial u_j(x_j)=p^*$ for all j. The conclusion then follows from Theorem 2, with suitable adjustments in the normalizations of μ and p (we can freely normalize either μ or p in the f.o.c., but cannot normalize them simultaneously).■

We finally return, to conclude this section, to the meaning and scope of the separability property (see also Mercier Ythier, 2006: 2.2, on the same object).

Separability states that any distributive optimum is a market optimum, and that it always is possible to redistribute endowments in such a way that the market equilibrium associated with the new distribution of endowments is a distributive optimum unanimously preferred to the market equilibrium associated with the initial distribution. In other words, the redistribution of endowments by the distributive liberal social contract, and the allocation of resources by competitive markets, can be viewed as autonomous processes, which articulate consistently in the sense that the allocation that they jointly produce (they do produce some, which is unanimously preferred to the initial market equilibrium) is Pareto-efficient relative to both the private and the social preferences of individuals.

Separability is closely related to a similar property of the Bergson-Samuelson social welfare function (Bergson, 1938; Samuelson, 1947: Chap. VIII). We established in Theorem 2 that Pareto-optimal redistribution was equivalent to the maximization, in the set of accessible allocations, of a weighted average $\sum_{i\in N} \mu_i(w_i \circ u)$ of individual social utility functions, with arbitrary weights $\mu \in S_n$. Such "social-social" utility functions $\sum_{i\in N} \mu_i(w_i \circ u)$ are Bergson-Samuelson social welfare functions whenever $\sum_{i\in N} \mu_i w_i$ are strictly increasing in ophelimities (that is, strictly increasing functions $u(\mathbb{R}_+^{ln}) \to \mathbb{R}$, or, in a restrictive but nevertheless possible interpretation of the construct, strictly increasing in some open neighbourhood of the set of distributive optima; note that the latter is implied by the f.o.c. of Theorem 2-(ii)). The distributive liberal social contract therefore yields solutions to the problem of the joint optimization of allocation and distribution, which belong to the general class of solutions defined by Bergson and Samuelson, provided that the social welfare functions that they implicitly maximize are increasing in all individual private utilities in some relevant subset of $u(\mathbb{R}_+^{ln})$. These solutions are distinguished, within the general framework of Bergson-Samuelson, by the following two specific features: the social welfare function is a weighted average of pre-existing individual social utility functions; and the equilibrium allocation of the social contract is required to be unanimously weakly preferred to the initial equilibrium allocation, that is, it must not be vetoed by any individual agent comparing it with this initial allocation. The distributive liberal social contract solutions, in other words, consists of a set of Bergson-Samuelson solutions constrained by existing individual views on distribution (through the aggregation of individual social preferences) and by initial equilibrium allocation (through individual rights of veto on redistribution).

The separability property relies upon a set of four main conditions: (i) Walrasian equilibrium; (ii) non-paternalistic utility interdependence; (iii) lump-sum endowment transfers; (iv) and nonsatiation of the distributive Pareto preordering. Each of them can be considered as essential for the property, independently of the three others; and they together delineate the scope and the limits of the property. We concentrate our comments on condition (iii) (lump-sum transfers), which requires a specific attention within the present construct, where redistribution proceeds from the distributive preferences of individuals.[8]

Strategic transfers of individuals or coalitions, if any, could operate, in principle, through two channels, namely: the strategic manipulation of others' transfers; or the strategic manipulation of market prices.

The distributive liberal social contract, such as defined in section 2, is a cooperative solution to the problem of optimal redistribution which, by construction, only considers the transfers preferred by the grand coalition, hence ignores the first channel. A full-fledged game-theoretical foundation for this cooperative solution is developed in Mercier Ythier, 1998ab, in the single-commodity setup (see also Mercier Ythier, 2006: 2.2 and 6.1). We provide natural assumptions on individual distributive preferences, implying that any system of redistributive

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[8] Conditions (i) and (iii) are traditional topics of distribution theory. Their relations with the separability of allocation and distribution are appropriately discussed in the formulation of this theory by Bergson and Samuelson in the references cited above, where the lump-sum transfers are construed as taxes maximizing a social welfare function which is given a priori. Condition (iii), in particular, reduces then to a mere technical characteristic of the tax system. Conditions (ii) and (iv) have been elaborated and discussed in the literature on general equilibrium with interdependent utilities referred to at the beginning of this section and reviewed in Mercier Ythier, 2006 (notably in sections 4.1.2, 4.2.3 and 4.2.4).

transfers blocked by an individual agent or by a coalition, in a cooperative game of simultaneous redistributive transfers, must be blocked by the grand coalition also. The assumptions are: that individuals do not want to redistribute wealth from themselves to the wealthier (*weak self-centredness*); and that they do not object to progressive (that is, inequality reducing) bilateral transfers whenever they are not the donor in the transfer (*non-jealousy*). Status quo (that is, zero transfers) then is the unique strong Nash equilibrium of the game of transfers at the distributive optimum of a liberal social contract[9]. This property in turn provides an ex post justification to the pattern of simultaneous interactions built in the game of transfers and associate equilibrium concept. In sum, once performed the lump-sum transfers of endowments of the distributive liberal social contract, and once achieved the associate Walrasian equilibrium, the resulting distributive optimum appears robustly self-sustained, as the unique status quo strong Nash equilibrium of a relevant cooperative game of simultaneous redistributive transfers.

The grand coalition could, in principle, "play with the market" when performing the endowment transfers of the social contract, but misses the reasons for it. Lump-sum redistribution suffices as an instrument for the achievement of a distributive Pareto optimum unanimously preferred to the initial market equilibrium, which is the object of the distributive liberal social contract. The considerable informational, computational and transactional difficulties of the task of solving the "millions of equations with millions of unknowns" of economic equilibrium and enforcing the associate transactions are adequately resolved by the markets in this construct. Put differently, the separability property, and notably the underlying condition (i) (Walrasian equilibrium) allows the grand coalition to save the considerable costs of discovering the equilibrium correspondence of the economy and of enforcing a particular equilibrium, by making these knowledge and enforcement costs useless for the achievement of the objectives of the social contract[10].

The second channel for strategic redistribution is not operative, therefore, at the level of the grand coalition under condition (i). But we cannot apply the same statement, in principle at least, to proper sub-coalitions of the latter. A well-known case for the strategic manipulation of equilibrium market prices by means of voluntary endowment transfers is made by the large body of literature on the transfer paradox (see the brief discussion of this literature, in relation to distribution theory, in Mercier Ythier, 2006: 4.3). The latter refers to the logical possibility that an agent or group of agents of a Walrasian economy withholding, destroying or transferring some fraction of their initial endowment ends up better off (and/or the recipients of transfers, if any, worse off) in ophelimity terms, due to the effects of their endowment manipulations on equilibrium market prices. This possibility has been amply

[9] This property implies that the transfers of the social contract crowd out all other voluntary transfers, whether individual or collective (see Mercier Ythier, 2006: 6.1.1, notably 6.1.1.2).

[10] There nevertheless remains some room for a price policy within this construct, when the endowment distribution of the social contract induces several market equilibria, such that some are distributive optima unanimously preferred to the initial equilibrium allocation, and others are not. Choosing a vector of market prices inside the (typically finite) set of equilibrium price systems then becomes a way of selecting an allocation endowed with the wished properties. An alternative way is lump-sum redistribution itself, since autarkic market equilibrium is generically unique (see footnote [7] above; note that it is not necessary to fully crowd out market exchange to make a market optimum the unique equilibrium of the economy: it suffices to select an endowment distribution in the fiber associated with the target allocation, and to take it "sufficiently close" to the latter, in the sense of belonging to a suitable connected component of the set of regular economies which contains the set of market optima (Balasko, op. cit.: 3.4.4 and 7.3.10)). The choice of the policy instrument (price policy versus lump-sum transfers) for removing the indeterminacy of market equilibrium (if any) should be determined by the comparison of their costs, notably informational, computational and enforcement costs. The latter are not modeled in the construct. This omission can be viewed as an assumption that these costs are not so large as to make the social contract unenforceable in such situations.

documented by the production of numerous theoretical examples, most often referring to the context of international trade (see the references of Mercier Ythier, 2006 above for a selected sample of such examples).

The embedding of such strategic manipulations of Walrasian equilibrium in the definition of social equilibrium confronts basic difficulties, which notably include the logical possibility of wars of gifts, and the discontinuities of the Walrasian equilibrium correspondence associated with variations in the number of equilibria. The discussion of these issues goes beyond the scope of this article. We will content ourselves here with the simple remark that the deliberate manipulation of Walrasian equilibrium by means of endowment transfers supposes very demanding conditions to become effective, notably: the existence of a coalition which, while a proper subset of the whole of society, nevertheless is large enough to be able to exert a sizeable influence on general equilibrium by means of transfers in the endowments of its members; the practical ability of this large coalition to act collectively through some adequate institutional representation, and to impose the effects of this collective action on the other members of society (effects which necessarily are detrimental to some of the latter); and a sufficient knowledge of the Walrasian equilibrium correspondence, that is, an ability to determine, with sufficient accuracy for conscious and deliberate action, the effects of endowment transfers on equilibrium prices. There seems to be little real basis, to say the least, for the three conditions to hold within a nation. And it is debatable whether the third one actually holds in the international context.

In sum, the manipulation of Walrasian equilibrium by means of endowment transfers appears pointless at society level, and seems generally unfeasible at sub-society level.

Note, to conclude this comment of condition (iii), that the exclusion of strategic transfers which it assumes, while it is not implied by the other conditions for separability, nevertheless heavily depends upon condition (i). In other words, lump-sum redistribution is an independent assumption, which is compatible with the other three conditions, and which generally becomes irrelevant in the presence of non-market exchange[11]. An analogous remark applies to non-paternalistic utility interdependence as well[12]. The set of four conditions underlying the separability property analyzes, therefore, to finish with, as follows: a basic hypothesis on the (ideal) organization and functioning of market exchange (condition (i)); the design of a redistribution institution exactly compatible with the former (conditions (ii) and (iii)); and the hypothesis of civil peace as a common foundation, and/or joint consequence, of market exchange and social contract redistribution (condition (iv)).

5- Supported distributive optima

This section draws the consequences of the public good characteristics of the distribution of ophelimity or wealth (Kolm, 1966, op. cit.), in terms of the latter's valuation by suitably defined supporting prices at distributive optimum.

We first recall the definition of a *market price equilibrium*, and then proceed to the construction, on an analogous pattern, of a notion of *social contract price equilibrium*.

Definition 7: Attainable allocation x is a *market price equilibrium* with free disposal of

[11] This reliance of condition (v) on conditions (i) and (ii) is nicely illustrated, notably, by the literatures on strategic bequests and on the Samaritan's dilemma (see Mercier Ythier, 2006: 2.1 and 7.1.3).

[12] Tutelary transfer motives are generally incompatible with Walrasian exchange (Mercier Ythier, 2006: 4.2.3 and 4.2.4) and with non-strategic redistribution (Mercier Ythier, 2006: 2.3 and 7.1.2).

(w,u,ρ) if there exists a vector of market prices $p \geq 0$ such that $p.(\rho - \sum_{i\in N} x_i)=0$ and x_i maximizes u_i in $\{z_i \in \mathbb{R}_+^l: p.z_i \leq p.x_i\}$ for all i.

Under Assumption 1-(i), market price equilibrium is equivalent to market optimum, as a consequence of the first and second theorems of welfare economics (Appendix: Proposition 6).

We saw in section 4 that, under Assumption 1 and the differentiable nonsatiation of the weak distributive preordering of Pareto, the weak distributive optima of (w,u,ρ) could be identified with the maxima of $\sum_{i\in N} \mu_i(w_i \circ u)$ in the set of attainable allocations $A=\{x \in \mathbb{R}_+^{ln}: \sum_{i\in N} x_i \leq \rho\}$, the vector of weights μ running over the unit-simplex S_n (Theorem 2). This fact yields the following definition of a *supported distributive optimum*:

Definition 8: A weak distributive optimum x of (w,u,ρ) is *supported* by vector $\mu \neq 0$ of $\mathbb{R}_+^n$ if x maximizes $\sum_{i\in N} \mu_i(w_i \circ u)$ in the set of attainable allocations of the social system.

The maxima of the "social-social" welfare functions $\sum_{i\in N} \mu_i(w_i \circ u)$ with strictly positive weights are of special interest from a normative perspective, as they take into account, to some extent at least, the distributive preferences of *all* individuals. For this reason, we label them *inclusive* distributive optima below, defined formally as follows:

Definition 9: A weak distributive optimum is *inclusive* if it is supported by a >>0 vector μ.

Supported distributive optima are identical to weak distributive optima by Theorem 2. The set of inclusive distributive optima is contained in the set of strong distributive optima as an immediate consequence of definitions. The latter inclusion is proper in general (see the remark following Theorem 5, in section 6 below). We denote by P_w^{**} the set of inclusive distributive optima. We therefore have $P_w^{**} \subset P_w^{*} \subset P_w$, with generally proper inclusions.

We know from Theorem 1-(i) and Theorem 2-(ii) that any weak distributive optimum is supported by a strictly positive vector of market prices. A pair $(\mu,p) \in \mathbb{R}_+^n \times \mathbb{R}_{++}^l$ (with $\mu \neq 0$) supporting any weak distributive optimum x is defined up to a positive multiplicative constant by the first-order conditions of Theorem 2-(ii), and therefore can be chosen so that either $\mu \in S_n$ or $p \in S_l$ (but not both, except by coincidence). Note that μ need not be unique, in general, for a given p, while p necessarily is unique for any given μ. If μ_i is >0, the term $\mu_i \partial_j w_i(u(x)) \partial_{r_j} v_j(p, p.x_j)$ of the first-order conditions interprets as the marginal valuation, by individual i, of individual j's wealth. The sum $\sum_{i\in N} \mu_i \partial_j w_i(u(x)) \partial_{r_j} v_j(p, p.x_j)$ is the "social-social" marginal valuation of j's wealth at the distributive optimum. It is constant over j (=1: see the proof of Theorem 3). The distinction of an individual and a "social-social" marginal valuation of individual wealth is a consequence of the public good character of wealth distribution in this setup. The f.o.c. $\sum_{i\in N} \mu_i \partial_j w_i(u(x)) \partial_{r_j} v_j(p, p.x_j) = 1$ derived in the proof of Theorem 3 correspond, in particular, to the Bowen-Lindahl-Samuelson condition for the optimal provision of "public good" r_j.[13]

"Social-social" marginal valuations of individual ophelimities are well-defined at any weak distributive optimum, while a complete system of individual marginal valuations of his and others' ophelimities is well-defined only for inclusive distributive optima (as the definition of a meaningful system of marginal valuations of any individual i supposes a positive supporting μ_i). These facts, and the normative reason for a special consideration of inclusive distributive optima, justify the introduction of the two additional notions below, which emphasize the inclusive outcomes of social contract redistribution.

Let π_{ij} denote i's marginal valuation of j's wealth, corresponding, in the former paragraph, to a term of the type $\mu_i\partial_j w_i(u(x))\partial_{r_j} v_j(p, p.x_j)$. This corresponds to i's *Lindahl price* of j's wealth, in a scheme of Lindahl pricing of wealth distribution as a public good. Note that π_{ii} necessarily is positive at inclusive distributive optimum under Assumption 1, but that π_{ij} could be negative (resp. =0) for a pair of distinct individuals i and j, if (and only if) i is malevolent (resp. indifferent) to j at this optimum that is, if $\partial_j w_i(u(x))<0$ (resp. =0). We let $\pi_i=(\pi_{i1},\ldots,\pi_{in})$ and $\pi=(\pi_1,\ldots,\pi_n)$ in the sequel. We then define an *inclusive distributive liberal social contract*, and a *social contract price equilibrium* as follows:

Definition 10: Pair $(\omega',(p',x'))$ is an *inclusive distributive liberal social contract* of (w,u,ω), relative to competitive market equilibrium with free disposal (p,x) of (w,u,ω), if (p',x') is a competitive market equilibrium with free disposal of (w,u,ω') such that: (i) $w(u(x'))\geq w(u(x))$; (ii) and x' is an inclusive distributive optimum of (w,u,ρ).

Definition 11: Market price equilibrium x' of (w,u,ρ) is a *social contract price equilibrium* of (w,u,ω), relative to competitive market equilibrium with free disposal (p,x) of (w,u,ω), if: (i) $w(u(x'))\geq w(u(x))$; (ii) there exists (p',π) such that: (a) p' supports x'; (b) $\sum_{i\in N}\pi_{ij}=1$ for all j; (c) and, for all i, $r'=(p'.x_1',\ldots,p'.x_n')$ maximizes $r\rightarrow w_i(v(p',r))$ in $\{r\in\mathbb{R}_+^n: \pi_i.r\leq\pi_i.r'\}$.

The next theorem establishes the necessary connections between these last two notions, and shows, as a by-product, that the set of (>>0) social contract price equilibria of a social system of private property, relative to a Walrasian equilibrium x of the latter, is the set of inclusive distributive optima unanimously weakly preferred to x.

Theorem 4: Let (w,u,ρ) verify Assumption 1, and suppose moreover that: for all $\mu\in S_n$ and all $\hat{u}\in u(A)\cap\mathbb{R}_{++}^n$, $\sum_{i\in N}\mu_i\partial w_i(\hat{u})\neq 0$; and for all $p>>0$ and all $i\in N$, function $r\rightarrow w_i(v(p,r))$ is quasi-concave in $\mathbb{R}_{++}^n$. The following propositions (i) and (ii) are then equivalent: (i) Allocation $x^*=\omega^*$ is a >>0 social contract price equilibrium of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω); (ii) Endowment distribution $\omega^*=x^*$ is, both: (a) an inclusive distributive optimum of (w,u,ρ); (b) and an inclusive distributive liberal social contract of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω). In particular, the set of >>0 social contract price equilibria of (w,u,ω) relative to (p^0,x^0) is equal to $\{x\in P_w^{**}:w(u(x))\geq w(u(x^0))\}$.

Proof: The last part of Theorem 4 is a simple consequence of the first part and Definition 10.

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[13] The f.o.c. ($\sum_{i\in N}\mu_i\partial_j w_i(u(x))\partial u_j(x_j)=p$ of Theorem 2-(ii) correspond formally, likewise, to Bowen-Lindahl-Samuelson conditions for "public good" x_j. For a detailed comment of the paradoxes associated with the formal identification of private wealth with a public good, see Mercier Ythier, 2006: 6, notably pp. 296-300.

Let us prove the first part, that is, (i)$\Leftrightarrow$(ii).
(i) We first prove that (i)$\Rightarrow$(ii). Let x^* be a >>0 social contract price equilibrium relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω). Then x^* is a market price equilibrium by Definition 11. It is supported by a >>0 system of market prices p^*, hence such that $\sum_{i\in N} x_i$ =ρ. Since x^* is >>0, we have $\partial u_i(x_i^*) = \partial_{r_i} v_i(p^*, p^*.x_i^*)p^*$ for all i. Moreover, for all i there exists $\nu_i\in \mathbb{R}_{++}$ such that $\partial_j w_i(v(p^*,r^*))\partial_{r_j} v_j(p^*,r_j^*)=\nu_i\pi_{ij}$ for all $j\in N$, by the first-order conditions for a >>0 maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in \mathbb{R}_+^n$: $\pi_i.r\leq\pi_i.r^*\}$ (where $r^*=(p^*.x_1^*,\ldots, p^*.x_n^*)$). Dividing both sides of the f.o.c. by ν_i, adding up over i for any given j, and using the fact that $\sum_{i\in N}$ π_{ij}=1 by Definition 11, one gets the set of Bowen-Lindahl-Samuelson conditions: $\sum_{i\in N}$ $(1/\nu_i)\partial_j w_i(v(p^*,r^*))\partial_{r_j} v_j(p^*,r_j^*)=1$ for all j. Letting $\mu=(1/\nu_1,\ldots,1/\nu_n)$, and combining the findings above, we end up with the following: x^* is >>0, such that $\sum_{i\in N} x_i^*$ =ρ, and there exists $(\mu,p^*)\in \mathbb{R}_{++}^n\times\mathbb{R}_{++}^l$ such that, for all $j\in N$, $\sum_{i\in N}$ $\mu_i\partial_j w_i(u(x^*))>0$ and $(\sum_{i\in N}$ $\mu_i\partial_j w_i(u(x^*))\partial u_j(x_j^*)=p^*$. The conclusion follows from Theorem 2 with a suitable normalization of μ.
(ii) We now prove the converse (ii)$\Rightarrow$(i). Let endowment distribution ω^* be an inclusive distributive optimum of (w,u,ρ) and an inclusive distributive liberal social contract of (w,u,ω) relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω). From Theorem 2 and the definition of an inclusive distributive optimum: ω^* is >>0, such that $\sum_{i\in N}$ ω_i^*=ρ, and there exists a $\mu\in \mathbb{R}_{++}^n$ and a unique $p^*\in S_l$ such that, for all $j\in N$, $\sum_{i\in N}$ $\mu_i\partial_j w_i(u(\omega^*))>0$ and $(\sum_{i\in N}$ $\mu_i\partial_j w_i(u(\omega^*))\partial u_j(\omega_j^*)=p^*$. We know that, consequently: ω^* is a market price equilibrium with free disposal of (w,u,ρ), supported by p^*, and that $\partial_{r_j} v_j(p^*,p^*.\omega_j^*)=1/\sum_{i\in N}$ $\mu_i\partial_j w_i(u(\omega^*))$ for all j. Let: $r^*=(p^*.\omega_1^*,\ldots, p^*.\omega_n^*)$; $\pi_{ij}=\mu_i\partial_j w_i(v(p^*,r^*))\partial_{r_j} v_j(p^*,r_j^*)$ for all (i,j). Then $\sum_{i\in N}$ π_{ij}=1 for all j. And for all (i,j): $\partial_j w_i(v(p^*,r^*))\partial_{r_j} v_j(p^*,r_j^*) = (1/\mu_i)\pi_{ij}$, with $1/\mu_i>0$.

At this stage, we have proved that: there exists a system of market prices p^*>>0 which supports ω^* as a market price equilibrium of (w,u,ρ), and a system of Lindahl prices π such that: $\pi_{ij}=\mu_i\partial_j w_i(v(p^*,r^*))\partial_{r_j} v_j(p^*,r_j^*)$ for all (i,j); $\sum_{i\in N}$ π_{ij}=1 for all j; and, for all i, r^* verifies the first-order necessary conditions for a local maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in \mathbb{R}_+^n$: $\pi_i.r\leq\pi_i.r^*\}$. There remains to establish that: $w_i(u(\omega^*))\geq w_i(u(x^0))$ for all i; and r^* is a global maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in \mathbb{R}_+^n$: $\pi_i.r\leq\pi_i.r^*\}$ for all i.

Endowment distribution ω^* being a market price equilibrium of (w,u,ρ) necessarily is the unique Walrasian equilibrium allocation of (w,u,ω^*) under Assumption 1-(i) (Balasko, op. cit.: 3.4.4). The definition of a liberal distributive social contract then readily implies that $w_i(u(\omega^*))\geq w_i(u(x^0))$ for all i.

Finally, the functions $r\rightarrow w_i(v(p^*,r))$ being quasi-concave in $\mathbb{R}_{++}^n$ by assumption, the first-order necessary conditions for a local maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in \mathbb{R}_+^n$: $\pi_i.r\leq\pi_i.r^*\}$ are also sufficient conditions for a global maximum of the same program, as a consequence of the Theorem 1 of Arrow and Enthoven, 1961.■

The assumption that functions $r\rightarrow w_i(v(p,r))$ are quasi-concave in $\mathbb{R}_{++}^n$ does not imply significant additional restrictions on individual preferences, relative to the quasi-concavity of distributive utility functions w_i, as established in the following proposition:

Proposition 1: Suppose that (w,u) verifies Assumption 1, and let $D_{ij}(\hat{u})$ (resp. $D_{ij}'(r)$) denote the j-th principal minor of the bordered Hessian of w_i (resp. $r \rightarrow w_i(v(p,r))$), evaluated at $\hat{u}>>0$ (resp. $r>>0$). Then: $D_{ij}'(r)=(\prod_{k\leq j}\partial_{r_k}v_k(p,r_k))^2 D_{ij}(v(p,r))$ for all $(p,r)>>0$, and all i and j. In particular, for all i: (i) principal minors $D_{ij}'(r)$ verify the necessary condition for the quasi-concavity of $r \rightarrow w_i(v(p,r))$ in $\mathbb{R}_{++}{}^n$; (ii) if principal minors $D_{ij}(\hat{u})$ verify the sufficient condition for the quasi-concavity of w_i in $\mathbb{R}_{++}{}^n$, then $r \rightarrow w_i(v(p,r))$ is quasi-concave in $\mathbb{R}_{++}{}^n$.

Proof: The bordered Hessian of $\hat{u} \rightarrow w_i(\hat{u})$, evaluated at $\hat{u}>>0$, is matrix $H_i(\hat{u})=$ $\begin{matrix}\partial^2 w_i(\hat{u}) & [\partial w_i(\hat{u})]^T \\ \partial w_i(\hat{u}) & 0\end{matrix}$. The bordered Hessian of $r \rightarrow w_i(v(p,r))$, evaluated at $r>>0$, is matrix $H_i'(r)=$ $\begin{matrix}\partial^2(w_i\circ v)(p,r) & [\partial(w_i\circ v)(p,r)]^T \\ \partial(w_i\circ v)(p,r) & 0\end{matrix}$. The generic entry of $\partial^2(w_i\circ v)(p,r)$ which is located on the j-th row and k-th column of $H_i'(r)$ is $\partial^2_{jk}w_i(v(p,r))\partial_{r_j}v_j(p,r_j)\partial_{r_k}v_k(p,r_k)$. The generic entry of $\partial(w_i\circ v)(p,r)$ (resp. $[\partial(w_i\circ v)(p,r)]^T$) which is located on the k-th column (resp. j-th row) of $H_i'(r)$, with $k\leq n$ (resp. $j\leq n$), is $\partial_k w_i(v(p,r))\partial_{r_k}v_k(p,r_k)$ (resp. $\partial_j w_i(v(p,r))\partial_{r_j}v_j(p,r_j)$). The multilinearity of the determinant then implies: $D_{ij}'(r)=(\prod_{k\leq j}\partial_{r_k}v_k(p,r_k))^2 D_{ij}(v(p,r))$. The marginal ophelimities of wealth $\partial_{r_k}v_k(p,r_k)$ being >0 for all k, $D_{ij}'(r)$ is equal to 0 if and only if $D_{ij}(v(p,r))=0$, and otherwise has the same sign as $D_j(v(p,r))$. The second part of the proposition is a simple consequence of these facts and of the Theorem 5 of Arrow and Enthoven, 1961.■

Note, to conclude this section, that the concept of social contract price equilibrium introduced above endorses the separation of allocation and distribution as autonomous processes. There is not, and actually there cannot be, in this setup, any price system that would simultaneously coordinate the allocation and distribution choices of individuals. The reason for this is quite simple indeed, embodied in the basic structure of the construct: for any given endowment distribution, the systems of equilibrium market prices are entirely determined by individual private preferences, through the aggregate excess demand function that the latter induce. Symmetrically, the coordination of redistributive transfers by means of Lindahl prices, if any, must be made on the basis of given market prices. We develop an equilibrium concept of this type in section 7.

6- Global properties of regular distributive efficiency

This section characterizes the global structure of the sets of inclusive distributive optima and social contract price equilibria, which stems from the characterization of inclusive distributive optima as maxima of positively weighted sums of individual social utilities in the set of attainable allocations. We first elicit, in subsection 6.1, the regularity conditions on the system of individual social preferences ensuring that the sets of inclusive distributive optima and of social contract price equilibria are well-behaved in terms of dimension and connectedness. This general property is complemented, in subsection 6.2, with the presentation of examples of social systems where the social contract solution appears degenerate, for reasons rooted in their basic structure, that is, in the initial endowment distribution or in the system of individual social preferences. Subsection 6.3, finally, provides insights on the type of restrictions on social systems required to obtain a well-behaved social contract solution.

6.1- Regular distributive efficiency

In this subsection, we notably concentrate on correspondence $\varphi:S_n \to A$ defined by: $\varphi(\mu)=$ argmax$\{\sum_{i\in N}\mu_i w_i(u(x)):x\in A\}$. The correspondence is well-defined, and its values are contained in P_w, when the social system verifies Assumption 1 and the differentiable nonsatiation of the weak distributive preordering of Pareto (Theorem 2).We summarize some of its elementary properties in the next proposition:

Proposition 2: Let (w,u,ρ) verify Assumption 1, and suppose moreover that, for all $\mu\in S_n$ and all $\hat{u}\in u(A)\cap\mathbb{R}_{++}^n$, $\sum_{i\in N}\mu_i\partial w_i(\hat{u})\neq 0$. Then: P_w is a nonempty and compact subset of A; and φ is a well-defined, upper hemi-continuous, compact- and convex-valued correspondence $S_n \to P_w$.

Proof: The continuity of functions $\sum_{i\in N}\mu_i(w_i\circ u)$ for all $\mu\in S_n$ and compactness of A readily implies that φ is well-defined, that is, that argmax$\{\sum_{i\in N}\mu_i w_i(u(x)):x\in A\}$ is a nonempty subset of A for all $\mu\in S_n$. The convex-valuedness of φ is a straightforward consequence of the convexity of set A and quasi-concavity of functions $w_i\circ u$ for all i. $P_w=\cup_{\mu\in S_n}\varphi(\mu)$ by Theorem 2. It will suffice, therefore, to finish with, to establish that Graph φ is closed (see Mas-Colell, op. cit.: A.6). Let (μ^q,x^q) be a converging sequence of elements of Graph φ, and denote by (μ,x) its limit. We want to prove that $\mu=\varphi(x)$. From Theorem 2 and the continuity of functions ∂w_i, u_i and ∂u_i for all i: x is ≥ 0, such that $\sum_{i\in N}x_i=\rho$, and there exists $p\in\mathbb{R}_+^l$ such that, for all $(i,j)\in N\times N$, $(\sum_{i\in N}\mu_i\partial_j w_i(u(x))\partial u_j(x_j)=p$. μ belongs to S_n by closedness of the latter, so that $\mu>0$. Therefore, x verifies the first-order necessary conditions for a weak maximum of w in A. The f.o.c. are also sufficient, by Assumption 1 and the Theorem 1 of Arrow and Enthoven, 1961. Therefore $x\in P_w$, and the conclusion then comes as a simple consequence of Theorem 2.■

Correspondence φ will be viewed, consequently, as a correspondence $S_n\to P_w$ from there on. Let Int S_n denote the relative interior of S_n $(=S_n\cap\mathbb{R}_{++}^n)$. The restriction of φ to Int S_n appears as a natural candidate for a homeomorphism Int $S_n\to P_w^{**}$, provided notably that $\varphi(\mu)$ and $\varphi^{-1}(x)$ be single-valued for all $\mu\in$ Int S_n and all $x\in P_w^{**}$. This need not hold true in general. The following notion of regular distributive efficiency sets minimal sufficient conditions for φ to define such a homeomorphism.

Definition 12: The differentiable social system (w,u,ρ) is *regular with respect to distributive efficiency* if: (i) $\partial w(u(x))$ is nonsingular for all $x\in P_w^{**}$; (ii) and $\sum_{i\in N}\mu_i(w_i\circ u)$ is differentiably strictly concave at all $x\in\varphi(\mu)$ for all $\mu\in$ Int S_n.

We show in Theorem 5 below that the second regularity condition (differentiable strict concavity) is sufficient for $\varphi(\mu)$ to be single-valued for all $\mu\in$ Int S_n, and that the first regularity condition of Definition 12 (nonsingularity) is sufficient for $\varphi^{-1}(x)$ to be single-valued for all $x\in P_w^{**}$.

The manifold structure of the set of inclusive distributive optima of differentiable social systems, and of the set of social contract price equilibria of differentiable social systems of private property, then follows from the first regularity condition by means of the Regular Value Theorem.

Theorem 5: (i) Let (w,u,ρ) verify Assumption 1, and suppose that: for all $\mu\in S_n$ and all $\hat{u}\in u(A)\cap\mathbb{R}_{++}^{n}$, $\sum_{i\in N}\mu_i\partial w_i(\hat{u})\neq 0$; and (w,u,ρ) is regular with respect to distributive efficiency. Then P_w^{**} is a simply connected C^1 manifold of dimension n-1, homeomorphic to Int S_n. (ii) Suppose moreover that functions $r\rightarrow w_i(v(p,r))$ are quasi-concave in $\mathbb{R}_{++}^{n}$ for all p>>0 and all $i\in N$. Then, for any initial distribution $\omega\in A$ and any competitive market equilibrium with free disposal (p,x) of (w,u,ω) such that $x\notin P_w$, the relative interior of the set of social contract price equilibria of (w,u,ω) relative to (p,x) is a simply connected C^1 manifold of dimension n-1, whose inverse image by φ is a simply connected, open subset of Int S_n.

Proof: The proof proceeds in three steps.
(i) In Step 1, we prove that: *The restriction of φ to Int S_n is a homeomorphism Int $S_n\rightarrow P_w^{**}$ with a C^1 inverse; in particular, P_w^{**} is simply connected.*
We first prove that the second regularity condition implies that $\varphi(\mu)$ is single-valued for all $\mu\in$ Int S_n. Let $\mu\in$ Int S_n. We suppose that $\varphi(\mu)$ contains two distinct elements x and x', and derive a contradiction. The definition of φ and the quasi-concavity of functions $w_i\circ u$ together imply that $w(u(\alpha x+(1-\alpha)x'))\geq w(u(x))=w(u(x'))$ for all real number $\alpha\in[0,1]$. The second regularity condition readily implies that the C^2 functions $w_i\circ u$ are all strictly concave in some neighbourhood U of x in $\mathbb{R}^{ln}$. For α<1 sufficiently close to 1, we must therefore have $w(u(\alpha x+(1-\alpha)x'))>>w(u(x))$. But $\alpha x+(1-\alpha)x'\in A$, due to the convexity of the latter set. Therefore $x\notin\varphi(\mu)$, the wished contradiction.

We next prove that, for any $x\in P_w^{**}$, $\varphi^{-1}(x)$ is single-valued and C^1.

From Theorems 1 or 2: $x\in P_w^{**}$ is a >>0 market price equilibrium supported by a >>0 price system p which is unique up to a positive multiplicative constant. Let p^* denote the unique supporting price system of x that belongs to S_l. Theorem 2 implies that for any $\mu\in\varphi^{-1}(x)$ there exists a unique price system αp^*, proportional to p^* with $\alpha\in\mathbb{R}_{++}$, such that, for all $j\in N$, $\sum_{i\in N}\mu_i\partial_j w_i(u(x))=1/\partial_{r_j}v_j(\alpha p^*,\alpha p^*.x_j)$.

The homogeneity of degree 0 of indirect ophelimity functions imply that $\partial_{r_j}v_j(\beta\alpha p^*,\beta\alpha p^*.x_j)=(1/\beta)\partial_{r_j}v_j(\alpha p^*,\alpha p^*.x_j)$ for all β>0 (positive homogeneity of degree -1 of the derivative). Letting $\beta=\partial_{r_1}v_1(\alpha p^*,\alpha p^*.x_1)$ and applying f.o.c. $\partial u_1(x_1)=\partial_{r_1}v_1(\alpha p^*,\alpha p^*.x_1)\alpha p^*$, one gets: $\partial_{r_j}v_j(\alpha p^*,\alpha p^*.x_j)/\partial_{r_1}v_1(\alpha p^*,\alpha p^*.x_1)=\partial_{r_j}v_j(\partial u_1(x_1),\partial u_1(x_1).x_j)$ for all j>1.

Dividing f.o.c. $\sum_{i\in N}\mu_i\partial_j w_i(u(x))=1/\partial_{r_j}v_j(\alpha p^*,\alpha p^*.x_j)$ by f.o.c. $\sum_{i\in N}\mu_i\partial_1 w_i(u(x))=1/\partial_{r_1}v_1(\alpha p^*,\alpha p^*.x_1)$ for all j>1, and using the result of the former paragraph, one gets the following equivalent system of n-1 equations: $(1/\sum_{i\in N}\mu_i\partial_1 w_i(u(x)))\sum_{i\in N}\mu_i\partial_j w_i(u(x))=1/\partial_{r_j}v_j(\partial u_1(x_1),\partial u_1(x_1).x_j)$. Multiplying both sides by $\sum_{i\in N}\mu_i\partial_1 w_i(u(x))$ and rearranging, one finally gets: $\sum_{i\in N}\mu_i(\partial_j w_i(u(x))-(1/\partial_{r_j}v_j(\partial u_1(x_1),\partial u_1(x_1).x_j))\partial_1 w_i(u(x)))=0$, j>1.

Denote by $B(x)$ the $n\times n$ matrix obtained from Jacobian matrix $\partial w(u(x))$ by substracting column-vector $(1/\partial_{r_j}v_j(\partial u_1(x_1),\partial u_1(x_1).x_j))\partial_1 w(u(x))$ to the first and j-th columns of $\partial w(u(x))$ for all j>1, and by $C(x)$ the $n\times(n-1)$ matrix obtained from $B(x)$ by deleting its first column. The system of f.o.c. obtained at the end of the former paragraph writes, in matrix form: $\mu.C(x)=0$, or equivalently $[C(x)]^T.\mu^T=0$, which, for any given x, characterizes the kernel of the transpose of $C(x)$. The first regularity condition of Definition 12 and the multilinearity of the determinant imply $|\partial w(u(x))| = |B(x)| \neq 0$, hence rank $C(x)$=rank $[C(x)]^T=n-1$. Therefore dim Kernel $[C(x)]^T=n-(n-1)=1$, that is, the kernel of $[C(x)]^T$ is a homogeneous line of $\mathbb{R}^n$, which moreover admits a >0 directing vector since $\varphi^{-1}(x)\subset$Kernel $[C(x)]^T$. Its intersection with hyperplane $\{z\in \mathbb{R}^n: \sum_{i\in N} z_i = 0\}$ reduces, consequently, to {0}. This implies in turn that the $n\times n$ matrix $D(x)$ obtained from $B(x)$ by substituting the transpose of the unit diagonal row-vector (1,…,1) of $\mathbb{R}^n$ for its first column is nonsingular, for: rank $D(x)$= rank $[D(x)]^T$ $=n$-dim Kernel $[D(x)]^T$= n-dim $\{z\in \mathbb{R}^n: \sum_{i\in N} z_i = 0\}\cap$Kernel $[C(x)]^T= n$. Therefore equation $\mu.D(x)-(1,0,\ldots,0)=0$, viewed as a linear equation in μ for any fixed $x\in P_w^{**}$, admits a unique solution, $=(1,0,\ldots,0).[D(x)]^{-1}$. We can let $\varphi^{-1}(x)=(1,0,\ldots,0).[D(x)]^{-1}$. Moreover, φ^{-1} is C^1 by Assumptions 1-(i)-(b) and 1-(ii)-(b) (C^2 utility functions) and the implicit function theorem applied to function $\mathbb{R}^n\times\mathbb{R}_{++}{}^{ln}\to\mathbb{R}^n$: $(\mu,x)\to\mu.D(x)-(1,0,\ldots,0)$ at any point $(\mu,x)\in S_n\times P_w^{**}$ such that $\mu\in\varphi^{-1}(x)$.

From there on, the restriction of φ to Int S_n is denoted by φ'.

Theorem 2 and the definition of inclusive distributive optimum readily imply that $\varphi(\text{Int } S_n)= P_w^{**}$. Function φ' therefore is a one-to-one mapping Int $S_n\to P_w^{**}$ with a C^1 inverse. We now prove that φ' is continuous. Let sequence μ^q converge to μ in Int S_n. The compactness of A implies that sequence $\varphi'(\mu^q)$ admits a converging subsequence in A. Let x be the latter's limit. The continuity of $(\mu,x)\to\sum_{i\in N} \mu_i w_i(u(x))$ implies that inequalities $\sum_{i\in N} \mu_i^q w_i(u(\varphi'(\mu^q)))\geq \sum_{i\in N} \mu_i^q w_i(u(z))$, which hold true for all pairs $(\mu^q,\varphi'(\mu^q))$ and all $z\in A$ by definition of φ', extend to the limit pair (μ,x). That is, $x=\varphi'(\mu)$.

Finally, Int S_n is simply connected, as a convex set. Therefore, $P_w^{**}=\varphi'(\text{Int } S_n)$ is simply connected, as homeomorphic to the former. This completes the proof of the first step.

(ii) In Step 2, we prove that: *P_w^{**} is a C^1 manifold of dimension n-1.*

Let g denote the C^1 function $\mathbb{R}_{++}{}^n\times$Int $P_u\to\mathbb{R}^n$ defined by $g(\mu,x)=\mu.D(x)-(1,0,\ldots,0)$ (see Step 1 above). Under Assumption 1-(i), Int P_u is a C^1 manifold of dimension n-1 (Mas-Colell, op. cit.: 4.6.9). Function g therefore is a C^1 function on a C^1 manifold of dimension $2n$-1, mapping into a C^∞ manifold of dimension n. From Theorem 2, Graph $\varphi' = g^{-1}(0)$. $\partial_\mu g(\mu,x)=D(x)$, which is a nonsingular $n\times n$ matrix at any $x\in P_w^{**}$ by the first regularity condition (see Step 1 above). Therefore rank $\partial g(\mu,x)=n$ everywhere in Graph φ', that is, 0 is a regular value of g. The Regular Value Theorem (see Mas-Colell, op. cit.: H.2.2) then implies that Graph φ' is a C^1 manifold, whose dimension is equal to dim($\mathbb{R}_{++}{}^n\times$Int P_u)-dim $\mathbb{R}^n=n-1$. Finally, denote by : $h_{(\mu,x)}$ a local C^1 diffeomorphism $\mathbb{R}^{n-1}\to$ Graph φ' at some point (μ,x) of Graph φ'; pr_2 the projection Graph $\varphi'\to P_w^{**}$ defined by $pr_2(\mu,x)=x$; and Φ function $P_w^{**}\to$Graph φ' defined by $\Phi(x)=(\varphi'^{-1}(x),x)$. Note that pr_2 is C^∞, while Φ is C^1 by Step 1 of this proof. Therefore, $pr_2\circ h_{(\mu,x)}$ is a local C^1 diffeomorphism $\mathbb{R}^{n-1}\to P_w^{**}$ at (μ,x), whose C^1 inverse is $(h_{(\mu,x)})^{-1}\circ\Phi$. This completes the proof of Step 2.

(iii) In Step 3, finally, we prove the second part of Theorem 5.

Let L denote the set of social contract price equilibria of (w,u,ω) relative to the Walrasian equilibrium (p,x) of the latter, and suppose that $x\notin P_w$. From Theorem 4, $L\cap\mathbb{R}_{++}{}^{ln}$= $P_w^{**}\cap\{z\in\mathbb{R}^{ln}:w(u(z))\geq w(u(x))\}$. The continuity of w and u and the openness of P_w^{**} then imply that Int L is equal to $P_w^{**}\cap\{z\in\mathbb{R}^{ln}:w(u(z))>>w(u(x))\}$. Since $x\notin P_w$, open set $\{z\in\mathbb{R}^{ln}$:

$w(u(z))>>w(u(x))\}$ is nonempty. And $P_w^{**} \cap \{z\in \mathbb{R}^{ln}: w(u(z))\geq w(u(x))\}$ is nonempty by the proof of the second part of Theorem 1. Therefore, so is Int L (since P_w^{**} is open). Functions $w_i \circ u$ being quasi-concave, set $\{z\in \mathbb{R}^{ln}: w(u(z))>>w(u(x))\}$ is convex, and is therefore an open convex subset of $\mathbb{R}^{ln}$, hence is a simply connected C^∞ manifold of dimension ln. P_w^{**} being a simply connected C^1 manifold of dimension n-1<ln by Steps 1 and 2 above, so is its intersection with $\{z\in \mathbb{R}^{ln}: w(u(z))>>w(u(x))\}$. That is, Int L is a simply connected C^1 submanifold of P_w^{**}, of same dimension as the latter. Consequently, φ^{-1}(Int L) is a simply connected, open subset of Int S_n.■

Note, to conclude this first subsection, that, as a straightforward consequence of definitions, if $w_i \circ u$ is *strictly* quasi-concave for all i (an assumption that we are not willing to make in general, for the reasons discussed in the last paragraphs of section 3, but that proves useful below for illustrative purposes), then $P_w^* = P_w$. If, moreover, the social system is regular with respect to distributive efficiency, we have P_w^{**}=Int P_w by Theorem 5, so that, in particular, inclusion $P_w^{**} \subset P_w^*$ is proper in this case (see Proposition 2). Theorem 5 then yields a simple geometric representation of well-behaved social contract solutions for 3-agents social systems, illustrated in Figure 1.

The Figure exploits the following consequences of the assumptions of Theorem 5 and the strict quasi-concavity of functions $w_i \circ u$.

From Assumption 1-(i)[14]: $u(A)$ is a convex subset of dimension n of $u(\mathbb{R}_+^{ln})=\mathbb{R}_+^n$; function $x \rightarrow u(x)$ is a homeomorphism $P_u \rightarrow u(P_u)$ and a C^1 diffeomorphism Int $P_u \rightarrow$ Int $u(P_u)$; the set of market-efficient ophelimity distributions $u(P_u)$(=$u(P_u^*)$) coincides with the upper frontier of $u(A)$, that is, with set $\{\hat{u}\in \partial u(A): \hat{u}'>\hat{u} \Rightarrow \hat{u}'\notin \partial u(A)\}$; its relative interior is a smooth (C^1) hypersurface (that is, n-1 dimensional submanifold) of $\mathbb{R}^n$.

These facts and Theorem 5 then imply that $u(P_w^{**})$ is a smooth hypersurface of $\mathbb{R}^n$ contained in Int $u(P_u)$. The same property applies, essentially, to Int $u(L)=\{\hat{u}\in u(P_w^{**})$: $w(\hat{u})>>w(u(x^0))\}$, that is, to the interior of the set of ophelimity distributions of inclusive social contract solutions associated with initial market equilibrium allocation x^0, when the latter is not a distributive optimum: this set is a C^1 hypersurface of $\mathbb{R}^n$ contained in $u(P_w^{**})$.

Introducing the additional assumption of strict quasi-concavity of functions $w_i \circ u$ yields the following additional properties: the ophelimity distribution that maximizes w_i in P_u is unique; and Int $u(P_w)=u(P_w^{**})$ (for u is a homeomorphism $P_u \rightarrow u(P_u)$, and Int $P_w = P_w^{**}$ by the strict quasi-concavity assumption).

In Figure 1, we denote by $\hat{u}^i$ the maximum of w_i in P_u, and by $\hat{u}^0$ the ophelimity distribution associated with some market equilibrium allocation $x^0\notin P_w$. From the facts above, $u(P_w)$ is the subarea of surface Int $u(P_u)$ delimited by the continuous curves

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[14] The convexity of $u(A)$ is a simple consequence of Assumptions 1-(i)-(b) and –(c) and the normalization $u(0)=0$. Function $x \rightarrow u(x)$ is a homeomorphism $P_u \rightarrow u(P_u)$ as a consequence of Assumptions 1-(i)-(b) and -(c) (e. g. Mas-Colell, op. cit.: 4.6.2) and a C^1 diffeomorphism Int $P_u \longrightarrow$ Int $u(P_u)$ as consequence of Assumption 1-(i) (Mas-Colell, op. cit.: 4.6.9). Equality $u(P_u^*)=\{\hat{u}\in \partial u(A): \hat{u}'>\hat{u} \Rightarrow \hat{u}'\notin \partial u(A)\}$ follows from the definition of strong market optimum and the continuity of private preferences (as implied by Assumption 1-(i)-(a)), while equality $u(P_u)=u(P_u^*)$ follows from the strict monotonicity and continuity of private preferences (as implied by Assumptions 1-(i)-(a) and -(c)); its global structure of smooth n-1 dimensional manifold follows from Assumption 1-(i) by Mas-Colell, op. cit.: 4.6.9.

$\hat{u}^i\hat{u}^j$=argmax{($w_i(\hat{u})$,$w_j(\hat{u})$):$\hat{u}\in P_u$} for all pairs {i,j} of distinct individuals of N={1,2,3}. The set of ophelimity distributions associated with the inclusive distributive optima of the social system is the relative interior of the former surface, that is, surface $u(P_w)\backslash(\hat{u}^1\hat{u}^2\cup\hat{u}^2\hat{u}^3\cup\hat{u}^1\hat{u}^3)$. Finally, set $u(L\cap\mathbb{R}_{++}^{ln})$ is the subarea of the former delimited by the indifference curves of w_2 and w_3 through $\hat{u}^0$, and Int $u(L)$ is its relative interior.

[Figure 1 approximately here]

6.2- Examples [15]

The three examples that we develop in this subsection exhibit four cases of social systems where the distributive liberal social contracts, while well-defined in the formal sense of Definition 4, nevertheless appear degenerate in some important respects. We first briefly summarize their main characteristics, and next proceed to the detailed derivation of their salient properties.

The social systems of the first two examples have a representative agent, in the sense that they "behave" as single rational (i.e. preference-maximizing) agents.

In Example 1, all individuals have the same social utility function, while they may differ in their private preferences. These unanimous distributive preferences make a representative agent in the common sense of the notion. They also make a representative agent in the abstract sense above, as its individual optimum is the unique social contract solution, irrespective of the initial distribution. This case of degeneracy stems from a conspicuous violation of the first regularity condition of Definition 12.

In Example 2, we develop two variants of social systems from the same basic Walrasian exchange economy with transferable (quasi-linear) private utility.

The assumption of transferable utility implies the existence of a representative consumer, that is, the invariance of aggregate demand to redistribution.

In the first variant, the social system consists one self-centred utilitarian and n-1 egoistic individuals. Distribution is not a relevant object for the social contract, in the sense that, with these assumptions, any market optimum is a distributive optimum. The distributive liberal social contract then translates into the maximization of aggregate wealth on the one hand, and the status quo in distribution on the other hand. The social system is ruled, so to speak, according to the views of the representative consumer, which do not coincide with any of the individual views of actual consumers, but which, in a literal sense, coincide with their sum. This case of degeneracy involves the violation of the second regularity condition.

In the second variant, the social system is made of a benevolent Sovereign and his egoistic subjects. Individual preferences verify the first and second regularity conditions. The degeneracy of the social contract proceeds from the assumption that the Sovereign has complete control over the numeraire. He implements, consequently, his own optimum, with the effect of precluding the achievement of any inclusive social contract. The representative agent, in this last case, is the Sovereign.

The social system of Example 3 has no representative agent. It is made of unsympathetically isolated individuals, who only feel concerned with their own wealth and welfare. It identifies,

[15] This subsection owes much to my lecture notes from Mas-Colell's course on general equilibrium theory at Harvard, notably the part relative to representative consumer theory.

therefore, with the Walrasian exchange economy that it contains. It verifies all the assumptions of Theorem 5, and nevertheless exhibits, for obvious reasons, the same type of trivial status quo social contracts as the first variant of Example 2 above.

Example 1: *Unanimous distributive preferences*

Let (w,u,ρ), verifying Assumption 1, be such that all individuals have the same distributive utility function w^*. Distributive utility function w^* then is also the unique "social-social" utility function of the social system, that is, $\sum_{i\in N} \mu_i w_i = w^*$ for all $\mu\in S_n$. We suppose, moreover, that w^* is strictly increasing and strictly concave. The social system then verifies all assumptions of Theorem 5, except the first regularity condition which, clearly enough, is violated everywhere in P_w^{**}. Function w^* has a unique maximum in A, which we denote by x^*. One easily verifies that P_w, P_w^* and P_w^{**} then degenerate to the singleton $\{x^*\}$. The latter is also equal to $\varphi(\mu)$ for all $\mu\in S_n$, so that $\varphi^{-1}(x^*)=S_n$. This example therefore exhibits a simple (actually, a trivial) case of violation of the properties of Theorem 5 derived from the sole violation of the first regularity condition.

Example 2: *Transferable private utility*

In this example, it will be convenient to adopt the setup of Balasko, 1988, that is: individual private preferences are defined and C^∞ on the whole of $\mathbb{R}^l$, monotone, differentiably strictly convex and bounded from below, and the first commodity is selected as the numeraire (that is, its price is normalized to 1). Walrasian demand and indirect ophelimity functions are then well-defined C^∞ functions on $\{p\in\mathbb{R}_{++}^l : p_1=1\}\times\mathbb{R}$, and we moreover suppose that the restrictions of the latter to $\{p\in\mathbb{R}_{++}^l : p_1=1\}\times\mathbb{R}_+$ are of the type $v_i(p,r_i)=r_i+b_i(p)$, that is, we suppose that individuals' private preferences are quasi-linear in the numeraire for nonnegative consumption bundles. In other words, we consider a special case in the general class of exchange economies with transferable utility (Bergstrom and Varian, 1985).

Roy's identity and Walras Law readily imply that aggregate demand $\sum_{i\in N} f_i(p, p.\omega_i)$ is invariant to redistribution, that is: $\omega \to \sum_{i\in N} f_i(p, p.\omega_i)$ is constant in the set of nonnegative distributions ω such that $\sum_{i\in N} \omega_i=\rho$. There is, consequently, a unique equilibrium vector of market prices p^* such that $\sum_{i\in N} f_i(p, p.\omega_i)=\rho$ (from Balasko, op. cit.: 3.4.4), that is, this economy has a unique system of equilibrium prices, independent of distribution ω. Moreover, aggregate demand $\sum_{i\in N} f_i(p,r_i)$ writes $(r_1+\ldots+r_n+\sum_{i\in N}\sum_{k\in L,k\geq 2} p_k\partial_{p_k} b_i(p), -\sum_{i\in N}\partial_{p_2} b_i(p)), \ldots, -\sum_{i\in N}\partial_{p_n} b_i(p))$, hence is of the general type $G(p,r_1+\ldots+r_n)$, so that the economy has a representative consumer for nonnegative distributions (Balasko, op. cit.: 7.Ann.3). Finally, the set of market optima associated with nonnegative wealth distributions $(r_1,\ldots,r_n)\in S_n$ is: $\{(r_1+\sum_{k\in L,k\geq 2} p_k\partial_{p_k} b_1(p^*), -\partial_p b_1(p^*)),\ (r_2+\sum_{k\in L,k\geq 2} p_k\partial_{p_k} b_2(p^*), -\partial_p b_1(p^*))\ ,\ldots,\ (r_n+\sum_{k\in L,k\geq 2} p_k\partial_{p_k} b_n(p^*), -\partial_p b_1(p^*))) : (r_1,\ldots,r_n)\in S_n\}$, identical to S_n up to a simple one-to-one linear transformation. Abusing notations, we denote by P_u the intersection of the latter set with $\mathbb{R}_+^{ln}$, that is, the set of nonnegative market optima.

We now turn to the assumptions on distribution.

In a first variant of the Example, we suppose that agent 1 is a self-centred utilitarian, endowed with linear distributive utility function w_1:$\hat{u} \rightarrow \sum_{i\in N} \alpha_{1i}\hat{u}i$ such that $\alpha_{1i}=\beta<\alpha_{11}$ for all $i\geq2$. All other individuals are egoistic, that is: w_i is the i-th canonical projection pr_i: $\hat{u} \rightarrow \hat{u}_i$ for all $i\geq2$. The social system then verifies the first regularity condition, as $|\partial w(\hat{u})|=\alpha_{11}$ for all $\hat{u}$. But it violates the second one. In view of the characterization of the set P_u of nonnegative market optima above, "social-social" utility functions $\sum_{i\in N} \mu_i(w_i \circ u)$ appear essentially as linear functions of the distribution of wealth. In other words, the wealth of any pair of individuals are perfect substitutes relative to $\sum_{i\in N} \mu_i(w_i \circ u)$. One easily verifies, in particular, that the set of maxima of $\sum_{i\in N} \mu_i(w_i \circ u)$ in A ($\subset \mathbb{R}_+^{ln}$) is the whole set P_u if $\sum_{i\in N} \mu_i(w_i \circ u)$ puts the same weight on all ophelimities, that is, if $\mu_1\alpha_{11}= \mu_1\beta+\mu_2=\ldots= \mu_1\beta+\mu_n$. Simple calculations show that there exists one and only one such μ in S_n, which is >>0. Denoting by P_w^{**} the set of nonnegative market optima, we therefore have $P_w^{**}=P_u$, which contradicts the first property of Theorem 5. Distribution appears essentially irrelevant as an object of social contract in this social system. The sole basis for unanimous agreement is the concern for market efficiency, that is, to use Marshall's terminology (as this social system exhibits some of the main characteristics of Marshall's static equilibrium), the concern for the maximization of the sum of private surpluses, or, equivalently, for the maximization of aggregate wealth (the "wealth of nation", to use the words of Adam Smith). Moreover, the set of allocations unanimously weakly preferred to any given $x\in P_u$ reduces to $\{x\}$. The distributive liberal social contract therefore naturally leads to status quo in this setup, in spite of the existence of individual 1's distributive concerns.

The second variant of the Example is the macro-social transposition of Becker's theory of family interactions (1974). It is illustrated by Figure 2 for a 3-agents social system. Agent 1 (say, Pharao[16]) owns the numeraire (that is, $\omega_{11}=1$), and has a concave strictly increasing, differentiably strictly concave in $\mathbb{R}_{++}^n$ distributive utility function w_1. All other individuals are egoistic as above. The determinant of $\partial w(\hat{u})$ reduces to $|\partial w(\hat{u})| = \partial_1 w_1(\hat{u}) \neq 0$. The first regularity condition holds true, therefore, in this social system. The second regularity condition is also verified, by the Proposition 3 of subsection 6.3 below. We denote by x^* the unique maximum of Pharao's social utility in the set of feasible allocations, and suppose that it is >>0. If one moreover assumes, for simplicity, that the initial distribution ω is a Walrasian equilibrium, the achievement of Pharao's optimum then supposes some redistribution of wealth and numeraire from himself to all others. Therefore, $w(u(x^*))>>w(u(\omega))$, and $\omega\notin P_w$. Since Pharao has a complete control over the resources in numeraire, the natural distributive outcome for this social system is allocation x^*. The latter is a distributive optimum unanimously preferred to the initial Walrasian equilibrium. It corresponds, consequently, to a distributive liberal social contract in the formal sense of Definition 4. This social contract is not inclusive, and actually there cannot be any more exclusive social contract, in a formal sense, than this one, as the "social-social" utility function that it maximizes coincides with the sole social utility function of Pharao. Figure 2 displays the variant of Figure 1 that corresponds to this configuration of the social system: $u(P_u)$ is represented by an isosceles

[16] From Ramsey to Ramses II, so to speak : Barro's companion paper of Becker's in the 82nd issue of the JPE (1974) develops a macroeconomic analogue of the same model, where the representative agent is a dynastic sequence of altruistically related generations. This construct has often been compared, in subsequent literature on the same topic, with Ramsey's Mathematical Theory of Savings (1928). It seems to me that, besides their undeniable practical virtues in terms of legibility and tractability, these models draw much of their obvious power of seduction from their metaphorical resonance with an archetype, nicely characterized by Karl Polanyi under the label of "redistribution" (and contrasted by him with the market on the one hand, and with reciprocity on the other hand: *The Great Transformation*, 1944, Chap. 4; see also Max Weber, 1921).

triangle of base $\sqrt{2}$ obtained from S_3 by means of translation $(z_1,z_2,z_3)\rightarrow(z_1+b_1(p^*),z_2+b_2(p^*),z_3+b_3(p^*))$; $\hat{u}^*=u(x^*)$; the curve connecting points $u(\omega)$, $\hat{u}$' and $\hat{u}$'' is Pharao's indifference curve through $u(\omega)$; and the set of ophelimity distributions associated with the inclusive social contract solutions such that $w(u(x))>>w(u(\omega))$ is, consequently, the interior of surface $\hat{u}^*\hat{u}'\hat{u}''$.

[Figure 2 approximately here]

Example 3: *Unsympathetic isolation*

Let (w,u,ρ) verify Assumption 1, and suppose that $w_i=\text{pr}_i$ or all i, that is, all individuals are indifferent to the private wealth or welfare of others (universal distributive indifference). This social system verifies all the assumptions of Theorem 5, and notably, in particular, the first regularity condition, since $\partial w(\hat{u})=1_n$ for all $\hat{u}$, and the second regularity condition, for the differentiable strict concavity of all private utility functions implies the differentiable strict concavity of $x\rightarrow\sum_{i\in N}\mu_i(w_i(u(x))=\sum_{i\in N}\mu_i u_i(x_i)$ for all $\mu>>0$ (see Proposition 3 below). The social system (w,u,ρ) then identifies, essentially, with the Walrasian exchange economy (u,ρ). In particular: all market optima are distributive optima, that is, $P_w=P_u$; and, of course, the distributive liberal social contract implies status quo, that is, $\{z\in P_w:w(u(z))\geq w(u(x))\}=\{x\}$ for all $x\in P_u$. As is well-known, general Walrasian exchange economies, such as characterized by Assumption 1-(i), generally do not have representative agents (Balasko, op. cit.: 7.Ann.3).

6.3- Regular social systems

This last subsection makes a brief first exploration of the restrictions on admissible social systems required for a well-behaved liberal social contract solution to optimal redistribution. By social contract solution, we mean any distributive optimum unanimously weakly preferred to the initial market equilibrium (see the end of section 2), or the set they constitute. This makes the norm of the distributive liberal social contract defined in section 1.

The social contract solution is well-behaved if, notably: it is inclusive; it is not, or not always, a status quo; and it makes a simply connected subset of the set of market optima, of same dimension as the latter (that is, of dimension n-1). We consider each of these characteristics in turn, and some of their implications for the underlying social systems.

Inclusiveness is a basic normative requirement, designed to provide a universal foundation to the social contract, by ensuring the effective inclusion of all individual preferences in the design of aggregate social utility functions. It notably implies the use of the weak Pareto Principle (the weak distributive preordering of Pareto) for comparing allocations, and, consequently, of the strong Pareto optimum for the definition of distributive optimum, but actually demands still more than that (since the inclusion $P_w^{**}\subset P_w^*$ is proper, as noticed in 6.1 above).

The variant of Becker's social equilibrium analyzed in the Example 2 of subsection 6.2 suggests that the implementation of an inclusive social contract might require a sufficiently balanced initial distribution, or at least may be greatly eased by it. It should not be the case, in other words, that a single agent, or a group of agents (say, for example, "the Rich") are able and willing to take advantage of their dominant position at the initial allocation, to implement their own optimum, so performing a literal interpretation of redistribution as unilateral Charity from benevolent benefactors to passive and silent beneficiaries (see Mercier Ythier, 2006, notably 3.3.3 and 6.2, for a discussion of the theoretical literature on charitable donations). Note that such exclusive social contracts are always accessible from any initial market optimum $x \notin P_w$ (formally, $\partial P_w \cap \{z \in \mathbb{R}_+^{ln} : w(u(z)) \geq w(u(x))\}$ generally is nonempty, as clearly appears from Figure 1). The remark above, therefore, does not refer so much to the logical possibility or impossibility of exclusive solutions, as to the plausibility of the selection of an inclusive outcome, and to the general characteristics of the social system which condition the latter. A reasonably balanced initial distribution certainly is a favourable circumstance. A pervasive awareness of the robustness conferred to social contract by universal participation is another one, still more important than the former. It seems reasonable to think that the real counterpart of the abstract notion of v liberal social contract studied in this article, if any, supposes the both of them and their mutual reinforcement, in its state of maturity at least.

The second condition for a well-behaved social contract is that it explains effective redistribution, that is, that the social contract solution is not, or not always, the status quo. In a minimal interpretation of this requirement, this supposes that some market optima at least are not distributive optima, that is, formally, that inclusion $P_w \subset P_u$ is proper. The latter supposes in turn that preferences exhibit some taste for redistribution such as, for example, some degree of inequality aversion, at the individual level of course (see the social system of the Homo Economicus of Example 3), but also at the aggregate level (confer the Marshallian social system of Example 2). The second regularity condition of Definition 12 essentially supposes the latter, that is, a taste for averaging exhibited by the positively weighted sums of individual social utility functions at associate inclusive distributive optima. We establish below that this regularity condition does not impose any serious restrictions on *non-malevolent* individual distributive preferences, for two complementary reasons.

First of all, the set of smooth (C^2), monotone preference preorderings on $\mathbb{R}_+^{ln}\backslash\{0\}$ that are differentiably strictly convex in A is open and dense in the set of smooth monotone distributive preference preorderings on $\mathbb{R}_+^{ln}\backslash\{0\}$, as a consequence of Mas-Colell, op. cit.: 8.4.1, and its elements admit utility representations that are differentiably strictly concave in A, as a consequence of Mas-Colell, op. cit.: 2.6.4. In other words, the strict concavity of utility representations in the set of admissible allocations is a generic property of smooth convex monotone social preferences at the individual level, hence also at the aggregate level.

The genericity argument above is not completely satisfactory, nevertheless, as, first, it is mute on non-monotone (that is, malevolent) social preferences, and as, second, it derives the strict concavity of the "social-social" utility function from the strict concavity of individual social utility functions. We argued in section 3 that the latter was not realistic, due to the large-scale character of the object of preferences, and the distributive indifference that it seems normally to imply within widespread parts of their domain of definition. Fortunately enough, it can easily be established (see Proposition 3 below) that the *concavity* of individual *distributive* utility functions and *strict concavity* of *private* utility functions in A, which are much easier to defend, suffice for the strict concavity of *positively* weighted sums of individual social utilities in A, provided that individual distributive utility functions are monotone (non-malevolence) and increasing in own ophelimity.

The violation of the second regularity condition in the first variant of Example 2, therefore, is not robust, as it appears narrowly related with the specificities of quasi-linear ophelimity. Robust difficulties with this regularity condition, if any, will stem from distributive malevolence.

Proposition 3: Suppose that for all i: w_i is concave in A, increasing, and increasing in its i-th argument; u_i is strictly concave in A. Then $\sum_{i\in N} \mu_i(w_i \circ u)$ is strictly concave in A for all $\mu >> 0$.

Proof: For any pair of distinct attainable allocations (x,x') and any $0<\alpha<1$, we have $u(\alpha x+(1-\alpha)x') > \alpha u(x)+(1-\alpha)u(x')$ since the u_i are all strictly concave in A and x_i is different from x_i' for at least one i. Therefore, $w_i(u(\alpha x+(1-\alpha)x')) \geq w_i(\alpha u(x)+(1-\alpha)u(x'))$ for all i, with a strict inequality for any i such that $u_i(\alpha x+(1-\alpha)x') > \alpha u_i(x)+(1-\alpha)u_i(x')$, by the monotonicity assumptions. And $w_i(\alpha u(x)+(1-\alpha)u(x')) \geq \alpha w_i(u(x))+(1-\alpha)w_i(u(x'))$ by concavity for all i. Hence: $w(u(\alpha x+(1-\alpha)x')) > \alpha w(u(x))+(1-\alpha)w(u(x'))$. And therefore, for any $\mu >> 0$: $\mu.w(u(\alpha x+(1-\alpha)x')) > \alpha\mu.w(u(x))+(1-\alpha)\mu.w(u(x'))$.■

The third condition for a well-behaved social contract solution concerns the global structure of the solution set, as a simply connected set of dimension n-1 (Theorem 5-(ii)). The latter obtains as a simple consequence of the same properties of the set P_w^{**} of inclusive distributive optima (see Step 3 of the proof of Theorem 5).

The simple connectedness of P_w^{**} means, essentially, that this set has no "holes". The set of market optima P_u also is simply connected (Balasko, op. cit.: 3.2 and 3.3). This mathematical property is suggestive of the possibility of performing redistribution along a continuous path of minimal length of P_u or P_w^{**}, by means of continuous adjustments in the distribution of endowments (see Balasko, op. cit.: 3.2 for further developments of this interpretation). It follows from the first and second regularity conditions of Definition 12 (see Step 1 of the proof of Theorem 5).

The dimensional property $\dim P_w^{**} = n\text{-}1$ states that the set of inclusive distributive optima has the maximum dimension consistent with the separability property $P_w^{**} \subset P_u$ (since dim Int $P_u = n\text{-}1$). This corresponds to a property of non-degeneracy in the strict (mathematical) sense. The first regularity condition is the minimal sufficient condition for the latter, as appears clearly from Step 2 of the proof of Theorem 5. This regularity condition supposes, essentially, that individuals have diverging views on desirable redistribution at any inclusive distributive optimum. More formally, the rows of matrix $\partial w(u(x))$ at $x \in P_w^{**}$ are the Jacobian vectors $\partial w_i(u(x))$, pointing in the direction of the best (local) redistributions from $u(x)$ from the perspective of individual i. The first regularity condition therefore states, equivalently, that the families of Jacobian vectors $\{\partial w_i(u(x)) : i \in I\}$ have maximal rank for any nonempty $I \subset N$ at any inclusive distributive optimum. Hence the interpretation above.

The need for this regularity condition is a direct consequence of the public good character of private wealth and welfare distributions in this setup. The condition is automatically verified, for example, and can therefore remain implicit, in the social system of the Homo Economicus of Example 3 ($x \to u(x)$ is a homeomorphism $P_u \to u(P_u)$ for monotone strictly convex private preferences, as is well-known: see footnote 14 above). The very existence of a distributive liberal social contract, if any, supposes a balance between: (i) on the one hand, some degree of conformity in individuals' tastes for redistribution, which must be sufficient to imply unanimous agreement relative to some acts of redistribution at least; (ii) and, on the other hand, divergences in individual views relative to distribution, which must be sufficient to make a contractual solution meaningful, as opposed to the more centralized modes of collective action that would proceed from the exact conformity of individual distributive preferences in large subsets of N (with the social system of Example 1 as a limit case).This balance of the social contract deduces quite naturally from actual characteristics of individual preferences, which commonly balance propensities to redistribute associated with altruistic feelings, empathy, or sense of distributive justice, on the one hand, against care for own wealth and welfare on the other hand. To put it more completely, the liberal social contract most naturally interprets as the reflection, at the aggregate level, and translation into redistributive transfers, of these characteristics of actual individual preferences confronted with actual initial endowment distribution or actual pre-transfer market equilibrium allocation.

A major, if not unique, source of divergence of individual views on redistribution is self-centredness, which consists for an individual to put a larger weight on his own wealth than on the individual wealth of others, or of a suitable selection of the latter. The following Proposition derives, on this simple basic pattern, two assumptions on the system of individual social preferences which imply the first regularity condition, namely: the *distributive indifference to the wealthier*, which supposes that every individual puts, so to speak, a “null weight” on the wealth of any other individual at least as rich as himself at any inclusive distributive optimum; and the *positive diagonal dominance* of the Jacobian matrix of $r \rightarrow w(v(p,r))$ at any inclusive distributive optimum. These results should only be viewed as simple indications about a possible line of research for obtaining general characterizations of systems of preferences compatible with the first regularity condition. There seems to be scope for substantial improvements on this topic, quite clearly.

Proposition 4: Let (w,u,ρ) verify Assumption 1, and suppose that, for any weak price-wealth distributive optimum $(p,r)>>0$ such that $f(p,r)\in P_w^{**}$: (i) either $\partial_j w_i(v(p,r_i))=0$ for all pair of distinct individuals (i,j) such that $r_i \leq r_j$; (ii) or matrix $\partial w(v(p,r)).\partial_r v(p,r)$ has a positive dominant diagonal. Then (w,u,ρ) verifies the first regularity condition of Definition 12.

Proof: (i) Let (w,u,ρ) verify the first assumption, and suppose, without loss of generality, that $r_1 \geq r_2 \geq \ldots \geq r_n$. Then $\partial w(v(p,r))$ is a triangular matrix, whose sub-diagonal entries are all =0. Therefore: $\left|\partial w(v(p,r))\right| = \prod_{i\in N} \partial_i w_i(v(p,r))$, which is >0 by Assumption 1-(ii)-(a). The conclusion follows from the equivalence of weak price-wealth distributive and weak distributive optimum (Theorem 3).

(ii) Let (w,u,ρ) verify the second assumption. Note that the generic entry located on the i-th row and j-th column of matrix $\partial w(v(p,r)).\partial_r v(p,r)$ is $\partial_j w_i(v(p,r)).\partial_{r_j} v_j(p,r_j)$. The multilinearity of the determinant therefore implies that: $\left|\partial w(v(p,r)).\partial_r v(p,r)\right| =$ $(\prod_{i\in N} \partial_{r_i} v_i(p,r_i))\left|\partial w(v(p,r))\right|$, where $\prod_{i\in N} \partial_{r_i} v_i(p,r_i)$ is >0. The diagonal dominance assumption implies that $\left|\partial w(v(p,r)).\partial_r v(p,r)\right|$ is >0. Therefore $\left|\partial w(v(p,r))\right|$ is >0, and the conclusion follows from the equivalence of weak price-wealth distributive and weak distributive optimum as above.■

7- Social contract equilibrium

We very briefly return, to conclude the formal developments of this article, on the notion of social contract equilibrium.

The social contract solution developed in this article leaves, when it is well-behaved, a substantial amount of mathematical indeterminacy relative to distribution, as measured by the dimension (=n-1) of the manifold of price-wealth social contract equilibria or, equivalently, by the dimension of the set of supporting vectors of weights of the associate "social-social" utility functions (Theorem 5-(ii)).[17] A natural solution for removing this remaining indeterminacy in our setup is Lindahl equilibrium, construed as a process of social communication which uses Lindahl pricing to elicit and coordinate individual preferences relative to distribution treated as a public good. Mercier Ythier, 2004, defines the notion, and analyzes its existence and some of its determinacy properties in the one-commodity setup. We extend it to the present setup in Definition 13 below, and establish, as a corollary of Theorem 4, that it actually yields an inclusive social contract solution. The associate wealth distribution, moreover, is unanimously strictly preferred to the wealth distribution induced by the initial market equilibrium allocation evaluated at social equilibrium market prices, when the initial market equilibrium allocation is not itself an inclusive distributive optimum. These properties of social equilibrium hold true provided that indirect individual social utility functions $r \to w_i(v(p,r))$ exhibit suitable properties of preference for averages at social equilibrium market prices.

We let Π denote set $\{\pi=(\pi_1,\ldots,\pi_n)\in \prod_{i\in N} \mathbb{R}^n: \sum_{i\in N} \pi_{ij}=1 \text{ for all } j\}$.

Definition 13: $(\pi,p^*,x^*)\in \Pi\times S_l \times A$ is a *social contract equilibrium* of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω), if: (i) $w(u(x^*))\geq w(u(x^0))$; (ii) x^* is a market price equilibrium supported by p^*; (iii) and for all i, $r^*=(p^*.x_1^*,\ldots,p^*.x_n^*)$ maximizes $r\to w_i(v(p^*,r))$ in $\{r\in \mathbb{R}_+^n: \pi_i.r\leq\pi_i.(p^*.x_1^0,\ldots,p^*.x_n^0)\}$.

The notion differs from the social contract price equilibrium of Definition 11 by maintaining the initial market equilibrium allocation x^0 in the specification of the right-hand side of individual "budget constraints". It shares with the former the fundamental feature of endorsing the separation of allocation and distribution as autonomous processes of coordination of: (i) on the one hand, individual decisions relative to market demand, coordinated by market prices for given distribution; (ii) and on the other hand, individual choices relative to distribution, coordinated by Lindahl shares for given market prices.

Corollary: Let (w,u,ρ) verify Assumption 1, and suppose moreover that: for all $\mu\in S_n$ and all $\hat{u}\in u(A)\cap\mathbb{R}_{++}^n$, $\sum_{i\in N} \mu_i\partial w_i(\hat{u})\neq 0$; and for all $p>>0$ and all $i\in N$, function $r\to w_i(v(p,r))$ is quasi-concave in $\mathbb{R}_{++}^n$. If (π,p^*,x^*) is a social contract equilibrium of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω), such that $x^*>>0$, then endowment distribution $\omega^*=x^*$ is, both: (a) an inclusive distributive optimum of (w,u,ρ); (b) and an inclusive distributive liberal social contract of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω). If, moreover, $x^0\notin P_w^{**}$ and $r\to w_i(v(p^*,r))$ is strictly quasi-concave for all i, then $w(u(x^*))>>w(v(p^*,(p^*.x_1^0,\ldots,p^*.x_n^0)))$.

[17] Note that indeterminacy in the sense above does not preclude a substantial explanation power of the notion, as measured by the ratio of the magnitude of hypersurface $u(L)$, computed from the relevant integral, relative to the magnitude of hypersurface $u(P_w)$ or $u(P_u)$ (see Figure 1 and the associate remarks, following the proof of Theorem 5). In other words, the set of social contract solutions could represent a very small fraction of the set of Pareto-efficient distributions in the distributive sense and, a fortiori, in the market sense. This will be the case, notably, if the initial market allocation is close to the set of distributive optima, or, equivalently, if the value of the transfers of the social contract represents a small fraction of the total value of the equilibrium allocation. This could very well be the case in practice, as redistributive transfers seem to represent only a small fraction of aggregate market wealth in real economies.

Proof: Let (π,p^*,x^*) be a social contract equilibrium of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω), such that $x^*>>0$, and denote $r^*=(p^*.x_1^*,...,p^*.x_n^*)$. Function $r\rightarrow w_i(v(p^*,r))$ being strictly increasing in r_i, the budget constraint must be satiated at any of its maxima in $\{r\in\mathbb{R}_+{}^n: \pi_i.r\leq\pi_i.(p^*.x_1^0,...,p^*.x_n^0)\}$. Therefore $\pi_i.r^*=\pi_i.(p^*.x_1^0,...,p^*.x_n^0)$, and r^* also is a maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in\mathbb{R}_+{}^n: \pi_i.r\leq\pi_i.r^*\}$. Hence x^* is a >>0 social contract price equilibrium of (w,u,ω), relative to competitive market equilibrium with free disposal (p^0,x^0) of (w,u,ω), and the first part of the Corollary follows from the application of Theorem 4.

Suppose that, moreover, $x^0\notin P_w^{**}$ and $r\rightarrow w_i(v(p^*,r))$ is strictly quasi-concave for all i. We have $w(u(x^*))\geq w(v(p^*,(p^*.x_1^0,...,p^*.x_n^0)))$, r^* being a maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in\mathbb{R}_+{}^n: \pi_i.r\leq\pi_i.(p^*.x_1^0,...,p^*.x_n^0)\}$ for all i by definition of a social contract equilibrium. Suppose that $w_i(u(x^*))=w_i(v(p^*,(p^*.x_1^0,...,p^*.x_n^0)))$ for some i, and let us derive a contradiction. The strict quasi-concavity of $r\rightarrow w_i(v(p^*,r))$ implies that any strict convex combination $\alpha r^*+(1-\alpha)(p^*.x_1^0,...,p^*.x_n^0)$, $0<\alpha<1$, is strictly preferred by i to both r^* and $(p^*.x_1^0,...,p^*.x_n^0)$ (since $w_i(v(p^*,r^*))=w_i(u(x^*))=w_i(v(p^*,(p^*.x_1^0,...,p^*.x_n^0)))$). Since moreover $\alpha r^*+(1-\alpha)(p^*.x_1^0,...,p^*.x_n^0)\in\{r\in\mathbb{R}_+{}^n:\pi_i.r\leq\pi_i.(p^*.x_1^0,...,p^*.x_n^0)\}$, r^* cannot be a maximum of $r\rightarrow w_i(v(p^*,r))$ in $\{r\in\mathbb{R}_+{}^n:\pi_i.r\leq\pi_i.(p^*.x_1^0,...,p^*.x_n^0)\}$, which yields the wished contradiction.■

8- Rational redistribution: the distributive liberal social contract and the axioms of social choice.

The distributive liberal social contract operates as an aggregator of individual distributive preferences. The inclusive variant of the notion, which, we argued above, is the most appropriate, maximizes positively weighted sums of individual social utility functions (that is, $\sum_{i\in N}\mu_i(w_i\circ u)$ with $\mu>>0$) in the set of attainable allocations unanimously weakly preferred to the initial market equilibrium (see Theorem 4). It provides a rational foundation for the distribution institution, based on the Pareto principle, that is, on the unanimous preference of the individual members of society. As such, it compares with the alternative solutions sharing the same basic feature, which are developed by social choice theory. The object of this section is to situate the distributive liberal social contract relative to some of the main pieces of this theory, namely, the social welfare functions of Bergson-Samuelson and social welfare functionals of Arrow-Sen, and the axiom of non-dictatorship of Arrow (1951) and liberty axiom of Sen (1970) that pertain to the latter. We use, for that purpose, the synthesis of Sen, 1986.

The distributive liberal social contract has already been related to the Bergson-Samuelson construct in section 4. We briefly summarize our conclusions as follows. The distributive liberal social contract solution shares with the Bergson-Samuelson solution the basic property of separability of allocation and distribution, that is, notably, the fact that the allocation solution necessarily is a market optimum. The elements of the family of “social-social” utility functions $\{\sum_{i\in N}\mu_i(w_i\circ u):\mu>>0\}$ associated with inclusive social contract solutions are Bergson-Samuelson social welfare functions. The liberal social contract construct differs from the Bergson-Samuelson construct in three respects: it generates a whole family of social welfare functions; its “social-social” welfare functions aggregate pre-existing individual social welfare functions, that is, the “social-social” preference relation is primarily deduced from individual *social* ones, and aggregates individual *private* preferences only as a consequence of this primary aggregation of individual *social* preferences; the set of admissible solutions is restricted to the set of attainable allocations unanimously weakly preferred to the initial market equilibrium. The latter two differences make the social contract solution a special case of the Bergson-Samuelson solution for any *given* vector of weights. The first feature (abstractness) can be conveniently reformulated within the framework of the social welfare functionals of Arrow-Sen, to which we now turn.

We show below that the inclusive distributive liberal social contract solutions can be viewed as proceeding from the maximization of the elements of the range a family of social welfare functionals of Arrow-Sen that verify the strong Pareto principle, the non-dictatorship axiom of Arrow and the liberty axiom of Sen for the social systems that verify the assumptions of Theorem 5. These social welfare functionals also verify ordinal non-comparability, but do not verify, in general, pairwise relational independence.

We first construct the relevant family of social welfare functionals, next establish their axiomatic properties (Proposition 5), and finally interpret the results in the light of the underlying definitions and assumptions.

The following assumptions and notations are maintained throughout the sequel: the economy (u,ω) is fixed; x^{ω} is a fixed market equilibrium allocation of (u,ω); (w,u,ω) verifies the assumptions of Theorem 5, that is, essentially, Assumption 1, the differentiable non-satiation of the distributive preordering of Pareto, and the distributive regularity of Definition 12. We denote by $L^{w,x^{\omega}}=\{x\in P_w^{**}:w(u(x))\geq w(u(x^{\omega}))\}$ the set of inclusive social contract solutions of (w,u,ω) relative to initial equilibrium allocation x^{ω}.

We now construct a family of social welfare functionals that generates, from the *n*-tuples of individual social utility functions, social preference relations which admit a Bergson-Samuelson social welfare function as utility representation, and whose maxima in the set of attainable allocations are the inclusive distributive optima of the liberal social contract.

Let Ψ be the set of *n*-tuples $\psi=(\psi_1,\ldots,\psi_n)$ of C^2 increasing functions $\psi_i:\mathbb{R}\rightarrow\mathbb{R}$ such that $\partial\psi_i>0$ for all *i*. We identify *n*-tuple $\psi=(\psi_1,\ldots,\psi_n)$ with function $\mathbb{R}^n\rightarrow\mathbb{R}^n:(s_1,\ldots,s_n)\rightarrow(\psi_1(s_1),\ldots,\psi_n(s_n))$. Proposition "There exists $\psi\in\Psi$ such that $w'=\psi\circ w$" defines an equivalence relation over the set of *n*-tuples of individual distributive utility functions *w*.[18] We denote by $\mathring{w}$ the equivalence class of *w* for this equivalence relation in $\{w:(w,u,\omega)$ verifies the assumptions of Theorem 5$\}$, that is, $\mathring{w}$ is the set of *n*-tuples of distributive utility functions that obtain from the application of *n*-tuples of increasing transformations of Ψ to an *n*-tuple *w* that verifies the assumptions of Theorem 5.[19] *W* denotes the quotient set $\{\mathring{w}:(w,u,\omega)$ verifies the assumptions of Theorem 5$\}$. The elements of *W* are in one-to-one correspondence with the *n*-tuples of individual social preference relations underlying the *n*-tuples of utility representations of $\{w\circ u:(w,u,\omega)$ verifies the assumptions of Theorem 5$\}$ (by Mas-Colell, op. cit.:2.3.11).[20]

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[18] One easily verifies that set Ψ, viewed as a set of functions $\mathbb{R}^n\rightarrow\mathbb{R}^n:(s_1,\ldots,s_n)\rightarrow(\psi_1(s_1),\ldots,\psi_n(s_n))$, and endowed with the composition of applications $\circ$, is a group: $\psi\circ\psi'\in\Psi$ for all $(\psi,\psi')\in\Psi\times\Psi$; the identity function $\mathbb{R}^n\rightarrow\mathbb{R}^n$ belongs to Ψ; $\psi\in\Psi$ implies $\psi^{-1}\in\Psi$; and $\circ$ is associative. The reflexivity, symmetry and transitivity of binary relation "*wRw*' if there exists $\psi\in\Psi$ such that $w'=\psi\circ w$" are simple consequences of the group structure of $(\Psi,\circ)$ (of the first three properties above, to be precise).

[19] Note that if *w* verifies the assumptions of Theorem 5, then so does $\psi\circ w$.

[20] Two C^2 utility functions w_i and w_i' with no critical point (that is, nonzero first derivative everywhere) represent the same underlying preference relation if and only if $w_i'=\psi_i\circ w_i$, where ψ_i is C^2 and has a positive first

Clearly, P_w^{**} and L^{w,x^ω} are invariant to any $\psi \in \Psi$. That is, inclusive distributive optimum and distributive liberal social contract solution verify the invariance property of ordinal non-comparability (see Sen, 1986: 6.1 for formal definitions of that and other invariance requirements). We can therefore let $P_w^{**} = P_{\mathring{w}}^{**}$ and $L^{w,x^\omega} = L^{\mathring{w},x^\omega}$ for any $w \in \mathring{w}$.

Pick a single w in each $\mathring{w} \in W$ and denote by Φ the resulting subfamily of $\{w{:}(w,u,\omega)$ verifies the assumptions of Theorem 5$\}$. Φ consists of an arbitrary selection of one single n-tuple of utility representations for each n-tuple of individual social preference relations. This family is maintained fixed in the sequel.

For any $w \in \Phi$, and any $x \in P_w^{**}$, let $F_{w,x}$ be a fixed C^2, concave, strictly increasing function $\mathbb{R}^n \to \mathbb{R}$ such that $F_{w,x} \circ w \circ u$ attains a unique maximum at x in A. For any $w' = \psi \circ w$, $\psi \in \Psi$ and $w \in \Phi$, define $F_{w',x}$ by: $F_{w',x} = F_{w,x} \circ \psi^{-1}$. Functions $F_{w,x} \circ w \circ u$ then define the same preference relation over allocations for all $w \in \mathring{w}$, which attains a unique maximum at x in A (formally: $F_{w',x} \circ w' \circ u = F_{w,x} \circ w \circ u$ for all $w' \in \mathring{w}$, for $w \in \mathring{w} \cap \Phi$). We denote this preference relation by $R_{\mathring{w},x}$, and the induced strict preference relation by $\succ_{\mathring{w},x}$ (that is $z \succ_{\mathring{w},x} z'$ if $z R_{\mathring{w},x} z'$ and non($z' R_{\mathring{w},x} z$)). The associate utility representation $F_{w,x} \circ w \circ u$, where $w \in \mathring{w} \cap \Phi$, is a Bergson-Samuelson social welfare function.

For any given $x \in \mathrm{Int}\, P_u$, function $w \circ u \to R_{\mathring{w},x}$, defined on $\{w \circ u : (\mathring{w}, x) \in W \times L^{\mathring{w},x^\omega}$ and (w,u,ω) verifies the assumptions of Theorem 5$\}$ is, by construction, a social welfare functional of Arrow-Sen that verifies ordinal non-comparability. Its range is made of Bergson-Samuelson social preference relations that attain their unique maximum in A at allocation x.

The maximization of the social preference relations $R_{\mathring{w},x}$ induced by the social welfare functionals for any fixed n-tuple of individual social preference relations, subject to the aggregate resource constraint of the social system, yields the set $L^{\mathring{w},x^\omega}$ of inclusive social contract solutions unanimously weakly preferred to fixed x^ω when x runs over $L^{\mathring{w},x^\omega}$, that is:

$$L^{\mathring{w},x^\omega} = \bigcup_{(\mathring{w},x):\, x \in L^{\mathring{w},x^\omega}} \{z \in A : z' \succ_{\mathring{w},x} z \Rightarrow z' \notin A\}.$$

We now proceed to the derivation of some axiomatic properties of this family of social welfare functionals. For the reader's commodity, the definitions of the axioms of social choice that we use below are recalled informally in footnote [21]. The social welfare functionals of

derivative (sufficiency is a straightforward consequence of definitions; necessity follows from Mas-Colell, op.cit.: 2.3.11).

[21] The *strong Pareto principle* states that a social state is preferred (resp. strictly preferred) to another for the social preference relation whenever the former is preferred to the latter for all individual social utility functions (resp. preferred to the latter for all individual social utility functions and strictly preferred to it for one individual social utility function at least). *Pairwise relational independence* states that the social preference relation over any pair of social states is a function of the sole restrictions of individual social utility functions over that pair. A social welfare functional of Arrow-Sen verifies *non-dictatorship* if there is no individual i such that, for all n-tuple of individual (social) utility functions in the domain of the social welfare functional and each pair of social states x and y, x is strictly preferred to y for the social preference relation whenever it is strictly preferred to y by individual i. Finally, the *liberty axiom* of Sen states that everyone is strongly decisive over one pair of social

family $\{w\circ u\rightarrow R_{\overset{\circ}{w},x} : (\overset{\circ}{w},x)\in W\times \overset{\circ}{L}^{w,x^{\omega}}\}$ clearly verify the strong Pareto principle. One can moreover make the following additional statements: these social welfare functionals verify Arrow's non-dictatorship axiom and the liberty axiom of Sen. Formally:

Proposition 5: Let market optimum $x\in$ Int P_u be fixed, and consider social welfare functional $w\circ u\rightarrow R_{\overset{\circ}{w},x}$ defined on $\{w\circ u : (\overset{\circ}{w},x)\in W\times \overset{\circ}{L}^{w,x^{\omega}}$ and (w,u,ω) verifies the assumptions of Theorem 5$\}$.

(i) *Strong Pareto principle*: For all $w\circ u$ in the domain above and all pair of allocations $(z,z')\in \mathbb{R}_+^{ln}\times\mathbb{R}_+^{ln}$, $w(u(z))\geq w(u(z'))$ implies $z\,R_{\overset{\circ}{w},x}\,z'$, the latter preference being strict whenever $w(u(z))>w(u(z'))$.

(ii) *Non-dictatorship*: There exists no i such that, for all $w\circ u$ in the domain above, $w_i(u(z))>w_i(u(z'))\Rightarrow z\succ_{\overset{\circ}{w},x} z'$.

(iii) *Liberty*: Let the domain of $w\circ u\rightarrow R_{\overset{\circ}{w},x}$ be restricted to $\{w\circ u : (\overset{\circ}{w},x)\in W\times \overset{\circ}{L}^{w,x^{\omega}}$, (w,u,ω) verifies the assumptions of Theorem 5, and $F_{w,x}\circ w:\hat{u}\rightarrow F_{w,x}(w(\hat{u}))$ is strictly increasing in $u(\mathbb{R}_+^{ln})\}$. Then: For all i, all nonnegative x_i' and x_i'', and all $w\circ u$ in the domain above, we have: $u_i(x_i')>u_i(x_i'')\Leftrightarrow w_i(u((x_{n/i},x_i'))>w_i(u((x_{n/i},x_i''))\Rightarrow(x_{n/i},x_i')\succ_{\overset{\circ}{w},x}(x_{n/i},x_i'')$, where $(x_{n/i},z_i)$ denotes the allocation obtained by substituting $z_i\in\{x_i', x_i''\}$ for x_i in x. This holds true, in particular, if the domain of the functional is further restricted to $\{w\circ u : (\overset{\circ}{w},x)\in W\times \overset{\circ}{L}^{w,x^{\omega}}$ and (w,u,ω) verifies the assumptions of Theorem 5 and distributive non-malevolence$\}$.

Proof: (i) is a simple consequence of inclusiveness, that is, strictly increasing $F_{w,x}$.

(iii) We show that if subfamily $w\circ u\rightarrow R_{\overset{\circ}{w},x}$ associated with fixed $x\in$ Int P_u has an Arrovian dictator then either the first regularity condition or inclusiveness is violated. Let i be a dictator, that is: for all $w\circ u$ in the domain of the social welfare functional, $w_i(u(z))>w_i(u(z'))\Rightarrow F_{w,x}(w(u(z))>F_{w,x}(w(u(z'))$. For any fixed $w\circ u$, Assumption 1 then readily implies that $w_i(u(z))\geq w_i(u(z'))\Rightarrow F_{w,x}(w(u(z))\geq F_{w,x}(w(u(z'))$, by the local nonsatiation of i's social utility function and continuity of individual social utility functions. That is, w_i and $F_{w,x}\circ w$ represent the same preference relation over $u(\mathbb{R}_+^{ln})$. In particular, their Jacobian vectors at any $\hat{u}\in u(\mathbb{R}_{++}^{ln})$ are positively proportional, that is, there exists $\alpha\in\mathbb{R}_{++}$ such that $\partial F_{w,x}(w(\hat{u})).\partial w(\hat{u})=\alpha\partial w_i(\hat{u})$. For $\hat{u}=u(x)$, the first regularity condition implies $\partial F_{w,x}(w(\hat{u}))=\alpha\partial w_i(\hat{u}).[\partial w(\hat{u})]^{-1}=\alpha e^i$, where e^i denotes the i-th vector of the canonical base of $\mathbb{R}^n$, whose i-th coordinate =1 and j-th coordinate =0 for all $j\neq i$. But this contradicts inclusiveness, which implies $\partial F_{w,x}(w(\hat{u}))>>0$.

(iv) Equivalence $u_i(x_i')>u_i(x_i'')\Leftrightarrow w_i(u((x_{n/i},x_i'))>w_i(u((x_{n/i},x_i''))$ is an immediate consequence of Assumption 1-(ii)-(a) (individual distributive utility increasing in own private utility). And $u_i(x_i')>u_i(x_i'')\Rightarrow(x_{n/i},x_i')\succ_{\overset{\circ}{w},x}(x_{n/i},x_i'')$ is an immediate consequence of the domain restriction

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states at least. A person i is *strongly decisive* over a pair of social states $\{x,y\}$ if the social preference relation induces the same strict ordering over the pair as the individual's social utility function whenever the individual's preference over the pair is strict. See Sen, 1986: 6.2 and 9.5 for formal definitions.

"$F_{w,x}\circ w$:$\hat{u}\rightarrow F_{w,x}(w(\hat{u}))$ is strictly increasing over $u(\mathbb{R}_+^{ln})$", which implies that $R_{\overset{\circ}{w},x}$ induces strictly monotone preferences over $u(\mathbb{R}_+^{ln})$ (since $u(x)>u(x')\Rightarrow x\succ_{\overset{\circ}{w},x} x'$). If, finally, w_i is non-malevolent (that is, nondecreasing) for all i, inclusiveness implies that $F_{w,x}\circ w$ is strictly increasing over $u(\mathbb{R}_+^{ln})$. ■

The social welfare functionals of Proposition 5 do not verify, in general, Arrow's axiom of pairwise relational independence. The reason for this is quite elementary: the functions $F_{w,x}$ used to construct the social preference relations $R_{\overset{\circ}{w},x}$, from the arbitray selection Φ of n-tuples of utility representations, are themselves arbitrary, except for their basic common characteristic of having a unique maximum at x in A. Consequently, the social preference relations $R_{\overset{\circ}{w},x}$ and $R_{\overset{\circ}{w}',x}$ associated with two distinct n-tuples of individual social preferences (two distinct equivalence classes $\overset{\circ}{w}$ and $\overset{\circ}{w}'$ of W) for a given x will generally be distinct. It will not be difficult, in particular, to find pairs of allocations which are ordered similarly by the individual social preferences of the n-tuples of preference relations associated with $\overset{\circ}{w}$ and $\overset{\circ}{w}'$, and are ordered differently by $R_{\overset{\circ}{w},x}$ and $R_{\overset{\circ}{w}',x}$. The possibility of selecting functions $F_{w,x}$ so that the associate functionals verify pairwise relational independence is an open question, that will not be examined here.

The strong Pareto principle of Proposition 5-(i) is an immediate, definitional consequence of inclusiveness, embodied in the definition of the social welfare functionals $w\circ u\rightarrow R_{\overset{\circ}{w},x}$ through strictly increasing aggregators $F_{w,x}$.

Since the social welfare functionals of family $\{w\circ u\rightarrow R_{\overset{\circ}{w},x}:(\overset{\circ}{w},x)\in W\times L^{\overset{\circ}{w},x^{\omega}}\}$ verify ordinal non-comparability, the strong Pareto principle and non-dictatorship (Proposition 5-(ii)), the relevant extension of Arrow's impossibility theorem to ordinal non-comparable social welfare functionals (Sen, 1986: 6.2) implies violations of independence and/or domain universality. The proof of non-dictatorship above explicitly relies on the latter. Namely, it establishes that the regularity of $\partial w(u(x))$ at inclusive distributive optimum x makes dictatorship incompatible with inclusiveness. The latter being a definitional feature of the social welfare functionals, the domain restriction that appears essential for non-dictatorship is the first condition of distributive regularity (the nonsingularity of the first derivative of $w\circ u$ in P_w^{**}). In other words, given the structural feature of inclusiveness of the functionals, some diversity of individual views on redistribution *within* some n-tuple of individual social utility functions of the domain of the functionals (intraprofile diversity, so to speak) suffices to exclude dictatorship in a formal sense. The first regularity condition of Definition 12 is one such sufficient condition. We argued in section 6 that this regularity property and inclusiveness were also the main conditions for a well-behaved liberal social contract solution to optimal redistribution (see subsection 6.3).

Proposition 5-(iii) actually implies more than the strong decisiveness of every individual over his own consumption at distributive optimum, namely, the exact coincidence, for everyone, of his individual (private and social) preferences over his own consumption, with the "social-

social" preferences induced over it at distributive optimum x by the family of Bergson-Samuelson preference relations $R_{\overset{\circ}{w},x}$ associated with x.

The social welfare functional $w \circ u \to R_{\overset{\circ}{w},x}$ over $\{w \circ u : (\overset{\circ}{w}, x) \in W \times L^{\overset{\circ}{w},x^{\omega}}$, (w,u,ω) verifies the assumptions of Theorem 5, and $F_{w,x} \circ w$ is strictly increasing$\}$ verifies both the strong Pareto principle and the liberty axiom. Sen's impossibility theorem (1970 and 1986: 9.5) implies that this existence result is a consequence of the restrictions on the domain of admissible individual social preferences imposed by the assumptions of Theorem 5 and strictly increasing $F_{w,x} \circ w$. The latter means, essentially, that the distributive malevolence of individuals is not so intense or widespread as to imply that inclusive "social-social" preferences are nonincreasing in the *private* welfare of some. The aspect of the assumptions of Theorem 5 which appears critical for the existence property is non-paternalism, clearly, that is, the assumption that individuals do not meddle in the consumption choices of others, and derive, instead, their evaluation of social states from the sole consideration of ordinal private welfare. Within the reasonably protective social environment implied by "social-social" preferences that are both inclusive and strictly increasing, non-paternalism makes private consumption a protected sphere of individual choices. Combined with private property rights, which notably permits exchange and other alternatives to individual consumption, these assumptions are also essential conditions for the existence of competitive markets in consumption commodities. This existence property interprets, in other words, as an important psychological and social aspect of the separability property of the distributive liberal social contract (sections 1 and 4).

To sum up:
(i) the liberty axiom holds true due to domain restrictions which consist, essentially, of non-paternalistic individual social preferences and inclusive "social-social" preferences strictly increasing in private welfare; these features seem essential for the compatibility of the redistribution institution with market economy;
(ii) non-dictatorship proceeds from a domain restriction which, in addition to non-paternalism, supposes intraprofile diversity in individual views on redistribution at distributive optimum; and also from a structural feature of the social welfare functionals, namely, inclusiveness as strictly increasing aggregator functions $F_{w,x}$; intraprofile diversity and inclusiveness are also the two main conditions for well-behaved liberal social contract foundations for the redistribution institution;
(iii) the strong Pareto principle is a definitional consequence of the same structural feature (inclusiveness);

Although the whole set of liberal social contract solutions can be very easily generated from a family of social welfare functionals, as we just saw above, there remains, nevertheless, a fundamental difference between the liberal social contract and social welfare functional approaches to redistribution, namely, the basic reliance of the former on a notion of individual rights, which is absent from the latter.

This difference finds its formal expression in the fact that the liberal social contract solution typically consists of a large set of allocations for any given n-tuple of distributive preferences, while the social preference relation generated by the social welfare functional for the same system of individual preferences selects a single social optimum (its unique maximum in the set of attainable allocations).

The liberal social contract solution is constrained by individual rights in the sense that the selected distributive optima are required to be unanimously weakly preferred to the market equilibrium that prevails prior social contract redistribution. This condition or constraint can be construed as a definition of a liberal social contractual distribution of property rights (here understood as *endowments*[22]), in the sense that such rights exist, that is, are properly founded by the liberal social contract, if and only if they are endorsed by the unanimous weak agreement of the individual members of society. In other words, initial endowments, whatever they are, are redistributed by means of unanimously weakly preferred transfers until weak unanimous agreement is reached, which makes the endowment distribution a liberal social contractual distribution of property rights. Social contractual redistribution bears on endowments. Competitive market exchange achieves the final allocation from a distribution of rights in the sense above. There are, in general, many such distributions of rights that can be reached from an initial non-contractual endowment distribution, due to the character of partial preordering of the Paretian criterion of social choice that is maximized in social contract redistribution. This multiplicity of solutions is reflected in the structure of the solution set $L^{w,x^{\omega}}$ as the intersection of n-1 dimensional set of inclusive Pareto-optima P_w^{**} with nl-dimensional convex set $\{z\in \mathbb{R}_+^{ln}:w(u(z))\geq w(u(x^{\omega}))\}$ of allocations unanimously weakly preferred to initial market equilibrium x^{ω}. This solution set is typically (that is, under the assumptions of Theorem 5) a set of dimension n-1 if the initial market equilibrium is not a distributive optimum (the latter sufficient condition is also necessary: if x^{ω} is a distributive optimum, the social contract solution reduces to status quo, that is, to $\{x^{\omega}\}$). It is therefore, in that sense, a large set whenever social contract redistribution applies (see footnote [17] for a qualification, which relies on an alternative definition of the size of set $L^{w,x^{\omega}}$ as a hypersurface).

To sum up, the liberal social contract solution is free from the interpersonal comparisons of individual *distributive* welfare necessarily implied by the maximization of a Bergson-Samuelson social preference relation of the type above. Exactly as the Pareto optimum relative to *private* utilities is free from the interpersonal comparisons of *private* welfare implied by the maximization of a Bergson-Samuelson social welfare function of the conventional type.

The liberal social contract solution and social choice solution(s) can be articulated constructively in a simple way, by selecting a functional form for the utility representation of a Bergson-Samuelson preference relation (for example, a weighted average of individual social utilities, or a maximin criterion, or any C^2 approximation of the latter) that seems relevant, and maximizing it in $L^{w,x^{\omega}}$. That is, by selecting a social optimum relative to a Bergson-Samuelson social preference relation inside the set of multiple liberal social contract solutions. Note that this procedure applies, in principle, not only to the norm of the distributive liberal social contract (set $L^{w,x^{\omega}}$), but to practical distributive liberal social contract as well, provided that the latter does not reduce to a single allocation. In other words, if the actual constraints associated with imperfect social contracting (see section 1) do not suffice to determine a single outcome of the process of achievement of the norm of the distributive liberal social contract subject to the constraints of *actual* collective action, the remaining indeterminacy (i.e. multiplicity of solutions) interprets as a problem of social choice, which admits, in principle, a social choice solution, that is, the selection of a single outcome by maximization of a well-behaved social preference relation.

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[22] Property rights can be understood in two complementary senses: as general rules specifying the use an individual can make of his own resources (notably own consumption, gift-giving, selling, consumption in a production process and disposal); or as the set of resources that the individual can freely use in the alternative ways allowed by these general rules. In section 1, we used the term mainly in the first sense (legitimate rules). In this section, we understand it mainly in the second sense (legitimate endowments).

The liberal social contract solution is co-determined by individual preferences and by individual rights. We suggested in the discussion of the Beckerian social equilibrium of Example 2 (see subsection 6.3) that a sufficiently balanced initial endowment distribution could be an important favourable condition for a well-behaved liberal social contract solution, and notably for its inclusiveness. Another example of interaction of preferences and rights raising questions about the viability or sustainability of inclusive liberal social contract solutions is the so-called "poor white" problem (Mercier Ythier, 2006: 6.1.2.1, Example 14, referred to in section 1 above), corresponding to a configuration of the social system where transfers to the poorest are rejected by a sizeable fraction of the working and (possibly) middle classes, notably because they are not beneficiaries of the transfers, or because they fear a subsequent rise in the relative status of the beneficiaries. If a large set of donors nevertheless wants to perform such transfers from their own resources, then the liberal social contract solution, which here corresponds to status quo, cannot be sustained as a social equilibrium. That is, the coalition of donors will deviate from it, and the distribution of rights will be an object of social contest for some time. This example and the former illustrate the necessity of developing an approach in terms of social equilibrium, in the game-theoretic sense, for a complete analysis of the liberal social contract, fully taking into account the role of rights in the determination of the resulting social state, and notably the possible non-existence or non-sustainability of a liberal social contract solution in some configurations of rights and preferences.

9- An epistemological post-scriptum

The norm of the distributive liberal social contract derived in this article is a rational norm for collective action.

It is an element of social reality only insofar as it influences actual collective action inside state institutions or outside of them, that is, only insofar as elements of a practical distributive liberal social contract are implemented.

It certainly is possible to argue in that sense, and we began to do it elsewhere (Mercier Ythier, 2006: 2.3 and 7.2.2).

If practical distributive liberal social contracts are implemented, then the associate norm should by and large shape actual distribution institutions in the course of long-run economic and social development. It is interesting, in this respect, to note remarkable analogies between the norm and its separability and regularity properties on the one hand, and, on the other hand, the notion of organic solidarity used by Durkheim, 1893, to characterize and explain the long-run evolution of society which goes with economic progress (the division of labour) and population growth.

Durkheim's organic solidarity is a balance of individual differentiation and cooperation, which are co-determined in the sense that cooperation is both necessitated by and necessary for individual differentiation, and which find their main expression in the system of legal rules of society (see notably the table in the last page of the third chapter of Book I, op. cit.).

The norm of the distributive liberal social contract can be viewed as an ideal type of organic solidarity in the sense above, with two main characteristics: a consistent articulation of the unsympathetic isolation of individuals in market exchange, with their sympathetic relations in the cooperative redistribution of the social contract (this is the separability property); and, for cooperative redistribution itself, the inner balance of the regularity property, which combines maximal dimensional diversity of individual views on redistribution (first regularity condition) with the strict preference for averages of the "social-

social" welfare functions associated with the inclusive solutions of the distributive liberal social contract (second regularity condition).

10- Appendix: Differentiable Walrasian exchange economies

In this appendix, we first briefly discuss the relations between our assumptions on individual private preferences (Assumption 1-(i)) and Balasko's 1988 setup. For the reader's commodity, we next summarize in Proposition 6 and proof some useful standard results relative to the competitive equilibrium of differentiable exchange economies that verify Assumption 1-(i).

Balasko makes the following assumptions on individual consumption preferences (op. cit. Chap. 2): they are defined on the whole commodity space $\mathbb{R}^l$, and are supposed C^∞, monotone, differentiably strictly convex, and bounded from below.

The difference with Assumption 1-(i) relative to the "degree of smoothness" of preferences, so to speak, (C^∞ as opposed to C^2) can be safely neglected. Assumption 1-(i) implies monotonicity and boundedness from below in the nonnegative orthant of $\mathbb{R}^l$, and smoothness and differentiable strict convexity in its interior. The main difference between the two setups lies, therefore, in the domain of definition of the preferences and of their properties of smoothness and convexity. We first briefly recall the reasons for these differences, second provide a precise characterization of the difficulties that may follow from them in the transposition of Balasko's results into our setup, and third elicit the simple precautions that make this transposition valid.

The essential differences between the two setups interpret as different solutions to a same formal problem, namely, freeing analysis from inessential technicalities associated with non-negativity constraints in consumption choices. Balasko's solution simply consists of removing non-negativity constraints, by letting allocation and endowment distribution take on, a priori, any possible value in $\mathbb{R}^{ln}$. Another standard solution consists of designing preferences so that any consumer with a >0 endowment will always choose a >>0 consumption bundle. We adopted the latter for reasons that relate nicely with the object of this article (see section 3 above), namely: the null private welfare associated with zero consumption is a conventional definition of the state of starvation which appears both quite natural and very useful in the context of distributive social systems, as one of the basic normative justifications for a distributive social contract is, precisely, the removal of any situation of individual starvation. In other words: we need to refer to some definition of the state of starvation, because distributive social contracts are construed, notably, to remedy such situations; and one of the simplest ways of defining it is the conventional association of null private welfare with null consumption. It then appears most convenient, from this consideration, to suppose, in addition, that private welfare is >0 only if consumption is >>0.

Combining the boundary and monotonicity conditions of Assumption 1-(i) with the conventional association of zero private welfare with zero consumption yields an indifference class of 0 equal to $\{x_i \in \mathbb{R}^l : \prod_{k\in L} x_{ik} = 0\} = \partial\,\mathbb{R}_+^{\,l}$ for all i, which is neither smooth nor compatible with differentiable strict convexity. Consequently, one cannot view the systems of consumption preferences that verify Assumption 1-(i) as restrictions to $\mathbb{R}_+^{\,l}$ of the systems of consumption preferences of Balasko; while the properties of Balasko's systems of preferences are verified by the restrictions to $\mathbb{R}_{++}^{\,l}$ of the systems of preferences of Assumption 1-(i).

The general rule for a valid transposition of Balasko's results and arguments to the setup of Assumption 1-(i) is, therefore, to check that, as hopefully is generally the case, they remain valid when restricted to the interior of the nonnegative orthant, that is, notably, to >>0 Pareto optima, price-wealth equilibria or equilibrium allocations.

Proposition 6: Let (u,ρ) verify Assumption 1-(i). The following five propositions are then equivalent:
(i) x is a weak market optimum of (u,ρ);
(ii) x is a strong market optimum of (u,ρ);
(iii) $x\in A$ is such that $\sum_{i\in N} x_i$ =ρ, and there exists a price system p>>0 such that, for all i: either x_i=0; or x_i>>0 and $\partial u_i(x_i) = \partial_{r_i} v_i(p,r_i)p$;
(iv) there exists a price system p>>0 such that (p,($p.x_1$,…,$p.x_n$)) is a price-wealth market equilibrium of (u,ρ);
(v) x is a market price equilibrium of (u,ρ).

Proof: We prove (i)⇒(ii)⇒(iii)⇒(iv)⇒(v)⇒(i).
(i)⇒(ii). Note first that: if x is attainable and such that $x_i\in\partial\mathbb{R}_+^l$ for all i, then $u(x)=0<<u((1/n)(\rho,\ldots,\rho))$, and therefore x is not weak market optimum. That is, $x\in P_u$ implies x_i>>0 for some i. Note next that if x and x' are attainable allocations such that x_i>>0 for some i and $x<x'$, then the allocation $x+(2n)^{-1}(x_i'-x_i,\ldots, x_i'-x_i)$ obtained from x by distributing evenly to all individuals one half of the difference $x_i'-x_i$ is >>x, hence such that $u(x+(2n)^{-1}(x_i'-x_i,\ldots, x_i'-x_i))>>u(x)$, and is attainable, so that $x\notin P_u$. Consequently, $x\notin P_u$ whenever: $x\notin P_u^*$ is attainable and such that x_i>>0 for some i; $x\notin P_u^*$ is attainable and belongs to $\partial\mathbb{R}_+^l$; or $x\notin P_u^*$ is not attainable. That is, $x\notin P_u^* \Rightarrow x\notin P_u$.This concludes the proof of the first implication.■
(ii)⇒(iii). Let allocation x be attainable, such that: x_i>>0 for some i; x_j is a >0 element of $\partial\mathbb{R}_+^l$ for some j. Let x' be the allocation obtained from x by transferring x_j to individual i, that is: $x_i'=x_i+x_j$; $x_j'=0$; $x_k'=x_k$ for all $k\neq i, j$. Then: x' is attainable; $u_i(x_i')>u_i(x_i)$ because private utility is strictly increasing in $\mathbb{R}_{++}^l$; $u_j(x_j')=u_j(x_j)=0$; $u_k(x_k')=u_k(x_k)$ for all $k\neq i, j$. That is, x is not a strong market optimum, hence not a weak market optimum by the paragraph above. Therefore, for any $x\in P_u$: $N=\{i:x_i>>0\}\cup\{i:x_i=0\}$.

Let x be a strong market optimum. Definitions then imply that it is a weak market optimum. Let $I=\{i:x_i>>0\}$. From the paragraphs above, I is nonempty. And the definition of a weak optimum straightforwardly implies that x is a local weak maximum of u in $\{x\in\mathbb{R}_+^{ln}: \sum_{i\in I} x_i \leq\rho\}$. Suppose, without loss of generality, that $I=\{1,\ldots,m\}$, with $m\geq 1$. Then, x is a local weak maximum of u in $\{x\in\mathbb{R}_+^{ln}: \sum_{i\in I} x_i \leq\rho\}$ if and only if $(x_1,\ldots,x_m)$ is a local weak maximum of $(u_1,\ldots,u_m)$ in $\{(x_1,\ldots,x_m)\in\mathbb{R}_{++}^{lm}: \sum_{i\in I} x_i \leq\rho\}$. The first-order necessary conditions for the latter read as follows: there exists $(\mu,p)\in\mathbb{R}_+^m\times\mathbb{R}_+^l$ such that: (i) $(\mu,p)\neq 0$; (ii) $p.(\rho-\sum_{i\in I} x_i)=0$; (iii) $\mu_i\partial u_i(x_i)-p=0$ for all $i\in I$. We must have μ>0, for otherwise p=0 by f.o.c. (iii), which contradicts f.o.c. (i). Since μ_i>0 for some i, and $\partial u_i(x_i)$>>0 for all i, we must have (μ,p)>>0 by f.o.c. (iii). F.o.c. (ii) then implies in turn that $\sum_{i\in I} x_i$ =ρ. The f.o.c. reduce therefore to the following equivalent proposition: $\sum_{i\in I} x_i$ =ρ, and there exists $(\mu,p)\in\mathbb{R}_{++}^m\times\mathbb{R}_{++}^l$ such that $\mu_i\partial u_i(x_i)=p$ for all $i\in I$.

Proposition "There exists $(\mu_i,p)\in\mathbb{R}_{++}\times\mathbb{R}_{++}^l$ such that $\mu_i\partial_i(x_i)=p$" is the f.o.c. for a local maximum of u_i in $\{z_i\in\mathbb{R}_{++}^l:p.z_i\leq p.x_i\}$. The f.o.c. is also sufficient for a global maximum of the same program by the Theorem 1 of Arrow and Enthoven, 1961. The envelope theorem applied to this program then implies that $\partial_{r_i} v_i(p,r_i)=1/\mu_i$. This concludes the second proof.■
(iii)⇒(iv) is a simple consequence of definitions and the second proof.■
(iv)⇒(v) is a simple consequence of definitions.■

(v)$\Rightarrow$(i) is established by means of the standard argument: let x be a market price equilibrium, $p\geq 0$ be an associate price vector, suppose that there exists an attainable allocation x' such that $u(x')>>u(x)$. Then: $u_i(x_i')>u_i(x_i)=\max\{u_i(z_i):z_i\geq 0$ and $p.z_i\geq p.x_i\}$ implies $p.x_i'>p.x_i$. Summing up over i yields $p.\sum_{i\in N} x_i'>p.\sum_{i\in N} x_i=p.\rho$, while $\sum_{i\in N} x_i'\leq\rho$ and $p\geq 0$ imply $p.\sum_{i\in N} x_i'\leq p.\rho$, the wished contradiction.■

References

Arrow, K.J. (1951), Social Choice and Individual Values (Wiley, New York).

Arrow, K.J. and G. Debreu (1954), "Existence of an equilibrium for a competitive economy", Econometrica 22:265-290.

Arrow, K.J. and A.C. Enthoven (1961), "Quasi-concave programming", Econometrica 29: 779-800.

Balasko, Y. (1988), Foundations of the Theory of General Equilibrium (Academic Press, London).

Barro, R.J. (1974), "Are government bonds net wealth?", Journal of Political Economy 82: 1095-1117.

Becker, G.S. (1964), Human Capital (National Bureau of Economic Research, New York).

Becker, G.S. (1965) "A theory of the allocation of time", Economic Journal 75: 493-517.

Becker, G.S. (1974), "A theory of social interactions", Journal of Political Economy 82: 1063-1093.

Bergson, A. (1938), "A reformulation of certain aspects of welfare economics", Quaterly Journal of Economics 52: 310-334.

Bergstrom, T.C. and H.A. Varian (1985), "When do market games have transferable utility?", Journal of Economic Theory 35:222-223.

Debreu, G. (1952), "A social equilibrium existence theorem", Proceedings of the National Academy of Sciences 38: 886-893. Reprinted in G. Debreu (1983), Mathematical Economics (Cambridge University Press, Cambridge) 50-58.

Durkheim, E. (1893), De la Division du Travail Social (Presses Universitaires de France, Paris, 1998).

Edgeworth, F.Y. (1881), Mathematical Psychics (C. Keagan Paul and Co, London).

Hochman, H.M. and J.D. Rodgers (1969), "Pareto optimal redistribution", American Economic Review 59:542-557. Reprinted in H.M. Hochman (2002), Economic Behaviour and Distributional Choice, (Edward Elgar, Cheltenham, United Kingdom and Northampton, Massachusetts) 5-20.

Kolm, S.-Ch. (1966), "The optimal production of social justice", in H. Guitton, and J. Margolis eds: Proceedings of the International Economic Association on Public Economics, Biarritz; Economie Publique (CNRS, Paris, 1968) 109-177; Public Economics (Macmillan, London, 1969) 145-200.

Kolm, S.-Ch. (1985), Le Contrat Social Libéral (Presses Universitaires de France, Paris).

Kolm, S.-Ch. (1996), Modern Theories of Justice (MIT Press; Cambridge, Massachusetts and London, England).

Kolm, S.-Ch. (2004), Macrojustice: The Political Economy of Fairness (MIT Press; Cambridge, Massachusetts and London, England).

Kolm, S.-Ch. (2006), Reciprocity: Its scope, rationales, and consequences, Chap. 7 in S.-C. Kolm and J. Mercier Ythier eds., Handbook of the Economics of Giving, Altruism and Reciprocity (North-Holland, Amsterdam) 371-541.

Lemche, S.Q. (1986), "Remarks on non-paternalism and the second theorem of welfare economics", Canadian Journal of Economics 19:270-280.

Lévy-Garboua, L., Meidinger, C. and B. Rapoport (2006), The formation of social preferences: Some lessons from psychology and biology", Chap. 7 in S.-C. Kolm and J. Mercier Ythier eds., Handbook of the Economics of Giving, Altruism and Reciprocity (North-Holland, Amsterdam) 545-613.

Mas-Colell, A. (1985), The Theory of General Equilibrium. A Differentiable Approach (Cambridge University Press, London).

Mercier Ythier, J. (1993), "Equilibre général de dons individuels", Revue Economique 44: 925-950.

Mercier Ythier, J. (1997), "First-order conditions for distributive efficiency". Discussion paper 98A39. Marseille: GREQAM, EHESS and Université d'Aix-Marseille II et III.

Mercier Ythier, J. (1998a), "The distribution of wealth in the liberal social contract", European Economic Review 42:329-347.

Mercier Ythier, J. (1998b), "Existence of an equilibrium of property rights", Mathematical Social Sciences 35:261-272.

Mercier Ythier, J. (2004), "A limit theorem on the dual core of a distributive social system", Social Choice and Welfare 22:147-174.

Mercier Ythier, J. (2006), "The economic theory of gift-giving: Perfect substitutability of transfers and redistribution of wealth", Chap. 5 in S.-C. Kolm and J. Mercier Ythier eds., Handbook of the Economics of Giving, Altruism and Reciprocity (North-Holland, Amsterdam) 227-339.227-339.

Nozick, R. (1974). Anarchy, State and Utopia (New York, Basic Books).

Musgrave, R.A. (1959), The Theory of Public Finance (McGraw-Hill, New York).
Olson, M. (1965), The Logic of Collective Action (Harvard University Press, Cambridge).
Pareto, V. (1913), “Il massimo di utilità per una colletività », Giornale degli Economisti 3: 337-341.
Pareto, V. (1916), Traité de Sociologie Générale (Droz, Genève).
Polanyi, M. (1944), The Great Transformation (Beacon Press, Boston).
Rader, T. (1980), “The second theorem of welfare economics when utilities are interdependent”, Journal of Economic Theory 23: 420-424.
Ramsey, F.P. (1928), “A mathematical theory of savings”, Economic Journal 38: 543-559.
Rawls, J. (1971). A Theory of Justice (Cambridge, Harvard University Press).
Samuelson, P.A. (1947), Foundations of Economic Analysis (Harvard University Press, Cambridge).
Sen, A.K. (1970), “The impossibility of a Paretian liberal”, Journal of Political Economy 72: 152-157.
Sen, A.K. (1986), “Social choice theory”, Chap. 22 in K.J. Arrow and M.D. Intriligator eds., Handbook of Mathematical Economics (North-Holland, Amsterdam) 1073-1181.
Smith, A. (1759), The Theory of Moral Sentiments. Edited by D.D. Raphael and A.L. Macfie (Clarendon Press, Oxford,1976).
Weber, M. (1921), “Charity in Ancient Palestine”, Chap. 15 in R. Swedberg ed., Max Weber Essays in Economic Sociology (Princeton University Press, Princeton, New Jersey) 189-195.
Wicksteed, P.H. (1910), The Common Sense of Political Economy (Macmillan, London).

Documents de travail du BETA

2010–**01** *The Aggregation of Individual Distributive Preferences through the Distributive Liberal Social Contract : Normative Analysis*
Jean MERCIER-YTHIER, janvier 2010.

La présente liste ne comprend que les Documents de Travail publiés à partir du 1er janvier 2010. La liste complète peut être donnée sur demande.
This list contains the Working Paper writen after January 2010, 1rst. The complet list is available upon request.
