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## « If the Alliance Fits ... : Innovation and Network Dynamics »

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# If the Alliance Fits . . . : Innovation and Network Dynamics \*

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## Abstract

Network formation is often said to be driven by social capital considerations. A typical pattern observed in the empirical data on strategic alliances is that of small world networks: dense subgroups of firms interconnected by (few) clique-spanning ties. The typical argument is that there is social capital value both to being embedded in a dense cluster, and to bridging disconnected clusters. In this paper we develop and analyze a simple model of joint innovation where we are able to reproduce these features, based solely on the assumption that successful partnering demands some intermediate amount of similarity between the partners.

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# 1 INTRODUCTION

In this paper we are interested in strategic alliance networks. Two firms form an R&D alliance with the goal of innovating, and the expected profitability of this alliance depends on the very nature of joint innovation. We develop a simple model of joint innovation to which firms respond, forming partnerships, and thereby creating an industry alliance network. We show that when firms make partnership choices based solely on the nature of innovation, the emerging network displays all the properties characteristic of observed strategic alliance networks. The networks that arise in the model are small worlds with skewed link distributions. This is the case in spite of the fact that in the model firms pay no attention to issues of social capital while making alliance decisions.

## 1.1 Alliance Networks

Networks of strategic alliances have been studied in many industries.<sup>1</sup> Several properties seem to be very common: the networks are sparse; they tend to be small worlds; and the link distributions are skewed to the right. Sparseness is easily explained by the fact that links are costly to create and maintain. Skewness is explainable through the fact that in most industries firm size distribution is skewed. If links have costs, it is likely that a larger firm has more resources with which to create or maintain links than does a smaller firm. So the distribution of links should reflect the distribution of firm size.

The small world properties of alliance networks have been more challenging to explain. Typically the explanation invokes social capital.

Clustering arises from the tendency of firms to partner with past partners and with partners of partners. The former creates inertia; the latter closes triangles in the network. The reasons given for these tendencies come from a form of social control. Partnering is risky, and information is a good way to reduce that risk. If a firm forms a link with a past partner, that link is said to be “relationally embedded”; a link with partners of partners is “structurally embedded”. In both cases embeddedness is a source of information about potential partners; in the first case from our history together; in the second from our com-

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<sup>1</sup>See for instance the studies of Powell et al. (2005) on the biotech industry, Baum et al. (2003) on bank syndicates, Riccaboni and Pammolli (2002) on the life sciences and ICT industry, Ahuja (2000) on the international chemicals industry.

mon partners. The information value of structurally embedded links serves as an incentive for firms to create closed triangles in their local networks. Structural embeddedness is also valuable as a source of social control. If a firm behaves badly towards one of its partners, that behaviour will be reported to, and presumably punished by the local community. If the local community is dense (which is the case when links are structurally embedded) this is an effective way of creating incentives to behave well. The social capital of embeddedness (see Coleman, 1988) works to create cliques of densely connected agents at the local level.<sup>2</sup>

But small worlds are not only cliquish, they also have enough clique-spanning ties that the average length of the shortest path between two firms is low. To explain clique-spanning ties, a different sort of social capital is invoked (Burt, 1992). A firm with links outside its local neighbourhood is in a position to access information originating in distant parts of the network. It might also be in a position to act as an information broker between different parts of the network. Both provide incentives for firms to look for partners that are not embedded in their local networks, and not to be themselves embedded in dense local networks. But of course, as more and more firms engage in these strategies, the value of the strategy falls. Thus we might expect to see some, but not too many firms forming clique-spanning ties.<sup>3</sup>

## 1.2 Innovation

Joint innovation involves two agents combining forces to create new knowledge. If the agents are identical, there is little value in combining forces — they can only duplicate each others thoughts and actions. If the agents are extremely different, they will have difficulties exploiting each other's competences. Thus we might expect that when a firm evaluates potential partners it would find desirable firms that are somewhat similar but not too similar.

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<sup>2</sup>The value of that form of social capital is empirically observed by Dyer and Nobeoka (2000) in the automobile industry; Gulati and Gargiulo (1999) in several industries; Powell et al. (1996) in the biotechnology sector; Rowley et al. (2000) in the steel and semiconductor industries. See also the findings and detailed discussions on the value of embeddedness in Walker et al. (1997).

<sup>3</sup>The value of structural holes is examined in Ahuja (2000) in the context of the international chemical industry (structural holes have a negative impact on industry performance, whereas indirect and direct ties have a positive impact on firm innovative performance); Gargiulo and Bennis (2000) in a study of Italian IT firm, find that a trade-off emerges, which is associated with the safety conferred by cohesive ties — social capital — and the flexibility conferred by ties that connect different parts of a network; Baum et al. (2003) in the Canadian merchant banking industry.

There are many ways in which firms can be similar or dissimilar: in terms of product portfolios; production technologies; organizational structures; organizational culture; competences generally and so on. For our purpose we can focus on whether or not firms are similar in their knowledge portfolios. Several studies have examined this question empirically.<sup>4</sup>

If we consider that firms are located in some underlying knowledge space, the general conclusion of these studies is that when firms ally (or merge) the success of their venture is an inverted-U in the distance in that space.<sup>5</sup>

This is the result on which we base our model of innovation. We show that when this is what drives success in joint innovation, the networks the results from strategic alliance formation can be small worlds with a skewed link distribution.

## 2 THE MODEL

Consider a finite population of firms located in a single, highly innovative industry. We assume that innovation is necessary for survival and growth, so the immediate goal of every firm is to accumulate knowledge. We model knowledge as a set  $\mathcal{W}$  of discrete facts or ideas, each of which a firm either knows or does not know. The knowledge endowment of any firm  $i$  can be represented by a binary vector  $v_i$  of length  $\#\mathcal{W}$ , in which  $v_i^z = 1$  indicates that the firm knows fact  $z$ . The knowledge of a firm changes over time as the firm innovates.

Innovation can take place in isolation, or in a partnership between two firms. Because our main concern is with joint innovation, we have a simple, reduced form model of autarchic innovation: autarchic innovations arrive at a rate  $\lambda$ , independently of a firm's alliance activities. When two firms form an alliance to innovate, the probability of success depends on how well their knowledge stocks complement each other. Following the empirical literature, we model this as an inverted-U relationship in the distance between the two firms in knowledge space. We measure distance by “overlap” — that is, the number of facts

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<sup>4</sup>See for example Ahuja and Katila (2001), Gulati and Gargiulo (1999), Mowery et al. (1998, 1996), or Schoenmakers and Duysters (2006). In this volume, Reagans and McEvily use “expertise overlap” as a control variable, and find that it is strongly significant for formation both of knowledge transfer and knowledge seeking links. They include only a linear effect, though, so we cannot tell whether it is possible to have “too much” overlap from their results.

<sup>5</sup>Distance is measured differently in different studies, though often based on patent data. The inverted-U relationship seems to hold nonetheless.

which both firms know.<sup>6</sup> We assume a fixed cost of alliance formation, so two firms will form an alliance if (and only if) the overlap of their knowledge stocks is not “too far” from the optimal overlap. Innovation success implies the discovery of a fact new to the innovator, or in the case of joint innovation, new to both of the innovation partners. The new fact is incorporated into the knowledge stocks of (each of) the innovator(s). Every period firms evaluate all possible alliances and form exactly those that have positive expected value, and within those alliances attempt to innovate. At the end of each period, all alliances dissolve; and in the subsequent period the process repeats, firms having possibly added knowledge to their portfolios. The network thus evolves; co-evolving with the changing knowledge portfolios of the firms, but the network structure itself at any point in time is determined entirely by the knowledge held by firms and by the nature of joint innovation.

We have made the very simple assumption that firms form alliances purely on the basis of whether or not the knowledge portfolios of the prospective partners are complementary. Based only on this assumption we are able to derive several structural properties of the strategic alliance network. The network responds in predictable ways to changes in the amount of knowledge firms hold on average, to the size of the optimal overlap, and to how strictly the optimal overlap must be met.

## 2.1 Innovation and equilibrium

In this section we derive a description of the equilibrium network, as determined by our assumptions on joint innovation.

### 2.1.1 Innovation

Initially each firm knows each idea with probability  $p_0$ . This probability is independent both of knowing other facts, and of the state of knowledge of other firms. Innovation is defined as the discovery of a previously unknown (to the innovators) fact, and has, without loss of generality, a value of 1.<sup>7</sup>

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<sup>6</sup>In fact “overlap” is a negative measure of distance, but given the symmetry in the inverted-U, the intuition and arguments are unaffected. Overlap also implies similarity, and we argued above that both similarity and complementarity were necessary for successful alliances. But complementarity is guaranteed to exist, except in the exceptional case where the two partners have exactly the same knowledge stocks (which happens with very small probability, approximately  $p^w$ , where  $p$  is the probability that a firm knows any particular fact).

<sup>7</sup>Implicitly we are assuming that a fact new to a firm and a fact new to the world have the same value. It could be argued that in a second stage in which firms use the knowledge they have discovered to create

Define  $v_{ij} = \{z : v_i^z = v_j^z = 1\}$  as the intersection, or overlap, of the knowledge portfolios of  $i$  and  $j$ , and define  $y_{ij} = \#v_{ij}$  to be the size of the overlap, or the number of facts known to both  $i$  and  $j$ . If the partnership  $ij$  forms, both  $i$  and  $j$  pay a fixed cost  $c$ , and the partnership innovates with probability  $f(y_{ij})$ , independently of the other alliances formed by  $i$  and  $j$ .<sup>8</sup> The function  $f(y)$  is the inverted-U discussed above, so  $f(y)$  is positive, symmetric about the unique optimal value,  $y = \delta$ , with  $c < f(\delta) \ll 1$ , increasing (decreasing) monotonically on the left (right) of  $\delta$ . If the partnership  $ij$  innovates, it discovers a new fact  $z$  which both partners receive, implying that their post-innovation knowledge portfolios become  $v_i + \{z\}$  and  $v_j + \{z\}$ .

As stated above, an innovation can only take place in a location in the knowledge vector where the innovator(s) is(are) currently ignorant. Thus some innovations will be “new to the innovator” but not new to the world: the innovation takes place within the knowledge frontier. We also allow the frontier to expand, that is, innovations can take place beyond the frontier. The innovation takes place at location  $z$ ,  $1 \leq z \leq w + \Delta$ , where  $\Delta$  measures the extent to which an innovation can extend the frontier. If the innovation is within the frontier,  $i$ ’s innovation has no effect on the knowledge stocks of other firms. However, if  $i$ ’s innovation expands the frontier, we assume that this makes redundant older knowledge. We make the assumption that if an innovation takes place at location  $z$ ,  $w+1 \leq z \leq w+\Delta$ , then the first  $z - w$  knowledge elements become obsolete. The frontier is thus pushed forward by the discovery of ideas beyond  $w$ . As innovation consists in drawing the location of an “empty” slot uniformly at random in  $\{1, \dots, w + \Delta\}$ , the frontier expansion is more likely

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competitive advantage, facts “new to the world” are more valuable than facts “new to me”, but here we only focus on the immediate production of knowledge. A second implicit assumption is that firms are indifferent about which facts they discover. To include the more realistic idea that firms get more value from one fact than another, would demand both a detailed specification of how firms turn facts into profits and how facts interact in that process, and a detailed specification of how firms turn existing knowledge into new knowledge. These are of course possible in principle, and material for an extension of the model, but the complication they would add at this point would distract significantly from the main message.

<sup>8</sup>There is a second order effect that we ignore. If  $i$  has a partnership with  $j$ , the partnership  $ij$  can innovate in any location in  $L = \mathcal{W} - v_i - v_j$ . If  $i$  now considers  $k$  as a potential partner, any innovations that  $ik$  might make in  $L$  would be duplicates, and therefore of less value than innovations that take place outside  $L$ . Thus the evaluation of  $k$  as a partner should involve this second order effect, and in general, the evaluation of a portfolio of partners should include these interactions. We ignore this here and in what follows, since in the numerical experiment below success probabilities are small enough that these second order effects will have very little effect on decisions. In the experiment below, a firm innovates on average once per period, thus the risk of duplicate innovations is very remote. Including this effect would on average lower the expected value of an alliance, and so decrease the degree of the network.

to be caused by more knowledgeable firms, and the innovative potential of the industry is never exhausted.

We can characterize this in the following way. In principle there are a countably infinite number of facts in the world. At any time  $t$  there is a relevant set, which we denote  $\mathcal{W}_t = \{\underline{w}_t, \dots, \bar{w}_t\}$ , with  $\bar{w}_t - \underline{w}_t = w$  for any  $t$ . This relevant set evolves: if at  $t$  an innovation takes place within the frontier (or no innovation takes place),  $\mathcal{W}_{t+1} = \mathcal{W}_t$ . If an innovation takes place at  $z > \bar{w}_t$  then  $\bar{w}_{t+1} = z$  and  $\underline{w}_{t+1} = z - w$ .

### 2.1.2 Equilibrium

In any period, the independent alliance formation results in a network  $g$ . Define the neighbourhood of firm  $i$  as  $N_i^g = \{j \neq i : ij \in g\}$ , that is, the set of the agents to whom  $i$  is directly connected. The degree of a firm is the size of its neighbourhood, which we denote  $n_i^g = \#N_i^g$ .

The value of forming a link lies purely in its potential to produce an innovation. In this model, inter-firm links are only useful for creating knowledge. Unlike, for example, the communication model of Jackson and Wolinsky (1996), links here are not conduits for knowledge spillovers from other (distant) firms, nor do they perform any other task (control, information brokerage, etc.). Thus the value of a link is simply the expected value of the potential innovation less the costs of the link. So given a network  $g$ , we can write the value to firm  $i$  of its links as

$$\pi_i^g = \sum_{j \in N_i^g} f(y_{ij}) - cn_i^g. \quad (1)$$

Every firm faces the same problem when evaluating potential links, and a link only forms if both partners agree. Thus a network  $g$  is stable if and only if every existing alliance is beneficial to both partners, and the creation of a (currently) non-existent link would reduce the net profits of at least one of the partners. In the present model, the simple form of firms' profits implies that both existing and non existing links have their potential value determined in exactly the same way, the value of  $ij$  being  $f(y_{ij}) - c$  to both  $i$  and  $j$ . As a consequence, the stable network  $g$  is simply  $\{ij : f(y_{ij}) \geq c\}$ .



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Include Figure 1 here.

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The equilibrium network structure is determined by the interaction of  $c$  with  $f(y)$ . The effect of  $c$  is clear: if  $c > f(\delta)$ , no partnership forms and the network is empty; if  $c < \min(f(0), f(w))$  then the network is complete; but for intermediate values of  $c$  ( $\min\{f(0); f(w)\} < c < f(\delta)$ ), all partnerships between firms having overlap  $y_{ij}$  such that  $f(y_{ij}) \geq c$  form. In each case, we observe a unique equilibrium network.

This can be seen relatively easily graphically, by recalling that  $f(y)$  is an inverted-U. Define  $\rho \geq 0$  by  $f(\delta \pm \rho) = c$ . Two firms  $i$  and  $j$ , with an overlap less than  $\rho$  find it profitable to form an alliance since  $f(y_{ij}) < c$ . Figure 1 above illustrates this result and Proposition 1 below states it.

**Proposition 1** *For any  $c \geq 0$ , there exists a unique equilibrium network  $g$ . When  $c > f(\delta)$  the empty network is (uniquely) stable; when  $c < \min\{f(0); f(w)\}$  the complete network is stable; when  $\min\{f(0); f(w)\} < c < f(\delta)$  the stable network is  $g = \{ij : |y_{ij} - \delta| \leq \rho\}$ .*

## 2.2 Digression on a special case

Consider for the moment a special case of this model, in which the inverted-U is actually a spike: joint innovation is possible if and only if partners' knowledge vectors overlap in exactly  $\delta$  positions. In addition, assume that firms knowledge vectors are independent both internally and with respect to those of other firms. In other words, firm  $i$  knows fact  $k$  with some fixed probability  $p$ , where  $p$  is independent of what other facts the firm knows and what other firms know. In this case,  $p$  is both the probability that a firm knows some particular fact, and a measure of the prevalence of knowledge in the industry — if  $p$  is small, the typical firm is ignorant of many potentially relevant facts; if  $p$  is large, most firms know most things.

Cowan and Jonard (2008) examine this model analytically, asking about the degree distribution and clustering of the emergent network. Here we simply show their results graphically. Figure 2 shows the degree distribution (left panel) and expected degree (right panel). (In each case there are 100 firms in the industry and 100 potentially relevant facts.)

The left panel shows the link distribution for different values of  $p$ , with  $\delta$  fixed at 15. What we see in general is that the distributions are skewed to the right (note that this is a log-linear plot), having long tails, but that typically these tails are not as heavy as a power law. For values of  $p$  near 0.36 the distribution is not monotonic, having a strictly positive mode. The right panel shows expected degree as a function of  $p$ , for 4 values of  $\delta$ . We observe that for every value of  $\delta$ , the expected degree rises and then falls with  $p$ . When  $p$  is small, firms have trouble finding firms with the right overlap. For example if  $p$  is very small, almost every firm will have fewer than  $\delta$  pieces of knowledge, and thus will not be able to participate in any alliance. When  $p$  is smaller, firms will have close to  $\delta$  pieces of knowledge so (in the case easiest to see), if a firm has exactly  $\delta$  pieces, it can only partner with a firm that knows the same pieces, and possibly more. At the other extreme, when  $p$  is large, firms know too much, and typically any pair of firms matches in too many places to have a partnership.

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Include Figure 2 here.

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Even in the restricted model it is not possible to calculate clustering coefficients directly. An indication of whether or not a network will be clustered though, can be obtained by examining the ratio of two probabilities: the probability that a link  $ij$  forms conditional on its being a triangle closing link (that is,  $\Pr\{ij|ik \text{ and } jk \text{ exist}\}$ ) to the unconditional probability that  $ij$  forms. Figure 3 shows this ratio as a function of  $p$  for 4 values of  $\delta$  (again there are 100 firms and 100 facts). What we observe is that except when  $p \approx \sqrt{\delta/w}$  this ratio is very large, and no matter the value of  $p$ , always greater than one. This implies that the networks will always be clustered, and for large regions of the parameter space even extremely clustered.

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Include Figure 3 here.

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These results show, or suggest that in this special case, the alliance networks that emerge will have skewed link distributions, and are likely to display small world properties. By generating numerically many networks, and covering the parameter space, Cowan and

Jonard show that this is indeed the case. When  $\delta$  is extremely large or small, there is almost no alliance activity (it is too hard to find partners); when  $p \approx \sqrt{\delta/w}$  the network is almost complete (it is too easy to find partners), but for other values of  $p$ , the network is in fact a small world with a skewed link distribution, corresponding nicely to what is observed empirically.

Finally, an interesting question has to do with “who is partnering”; in particular, whether it is firms with a lot or a little knowledge that form partnerships. Again this is possible to answer analytically in the special case, and we show the result in Figure 4. What we observe is that when the industry average knowledge levels are low relative to the optimal overlap, firms that are knowledge-rich form many partnerships; when industry knowledge levels are high, firms that are knowledge-poor are the ones that partner.

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Include Figure 4 here.

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This discussion concerned a special case of the model. We return now to the full model as described. It is complex enough that it cannot be treated analytically, but numerical analysis of the model yields results that are both consistent with the analysis of the special case, and interesting empirically.

## 2.3 Computational experiment

A challenge with numerical simulation generally is to disentangle general effects from artefacts due to, for example, idiosyncracies in the initial conditions that have been assigned. In the model described above, however, this is not an issue, as the model represents a stationary Markov process. Standard results from Markov process theory imply that initial conditions are unimportant — in the long run the system has a unique steady state, regardless of where it starts. We must be careful with the expression “steady state”, though, as this does not imply that the system stops moving, but rather that the proportion of time the system spends at each point of the state space settles down to an unchanging value.

To expand slightly: Each firm has a binary knowledge vector  $v_i$  of fixed length  $w$ . The state space of each *firm* is the set of all possible values of that vector,  $\mathcal{V}_i$ . Since for

every firm  $v_i \in \mathcal{V}_i$ , the state space of our *system* of firms is the product of firms' state spaces:  $\mathcal{V} = \times_j \mathcal{V}_j$ . Any time a firm innovates, knowledge vectors change, and the system moves from one state  $(v_1, \dots, v_n)$  to another state  $(v'_1, \dots, v'_n)$  in the system state space  $\mathcal{V}$ . (We move to a point where the innovating firm has one more piece of knowledge, and possibly where other firms have less knowledge, if the innovation has changed the frontier.) The transition from one point to another in this space is random, but with well-defined transition probabilities. Further, these probabilities do not depend on which period we are in; nor do they depend on where the system was at any point in the past other than where it was yesterday. Thus the system is a (finite) first-order stationary Markov chain.

If we ignore the unreachable states (examples of which: no firm knows any fact; no firm knows the frontier fact) it is easy to show that if we choose any two states of the system  $(v_1, \dots, v_n)$  and  $(v'_1, \dots, v'_n)$ , there is some (possibly several) finite sequence(s) of joint and autarchic innovations that will take us from one to the other.<sup>9</sup> This is the definition of an irreducible Markov chain. As a consequence the system has a unique stationary distribution. A convenient implication is that the expected value of any statistic of interest is arbitrarily well approximated by its time average. For instance, the expected number of ideas held by agents in the stationary distribution can be estimated by the time average of the number of ideas held. For this reason, numerical (Monte Carlo) simulation can be efficiently applied to study the complex dynamic system representing the industry.

## 2.4 Parameters effects

The characteristics of the industry network and the knowledge production process are affected by 5 parameters, which can essentially be discussed in terms either of their effect on the profitability of allying ( $\delta$  and  $\rho$ ), or in terms of their effect on the dynamics of innovation ( $\Delta$  and  $\lambda$ ). The fifth parameter,  $p_0$ , will be seen to have only a transient influence.

First, joint innovation is profitable in expected terms (and so the alliance is worth creating for both partners) if and only if the partners' overlap lies in  $[\delta - \rho, \delta + \rho]$ . This interval depends on  $\delta$  and  $\rho$ , which control the position and width of the inverted U.

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<sup>9</sup>This can be seen by construction. Find a firm,  $i$ , in  $(v'_1, \dots, v'_n)$  that knows the frontier fact. From  $(v_1, \dots, v_n)$  have firm  $i$  alone innovate at  $w + \Delta$  until all other firms vectors are empty. Then have each firm innovate autarchically in the locations where it has knowledge under state  $(v'_1, \dots, v'_n)$ . Each of these events has positive probability, so the transition from  $(v_1, \dots, v_n)$  to  $(v'_1, \dots, v'_n)$  has positive probability.

$\delta$  controls the optimal overlap when combining knowledge endowments. Suppose  $i$  and  $j$  hold  $a_i$  and  $a_j$  ideas, and suppose further these ideas are uniformly distributed over the possible  $w$  locations for the two firms. Then the number of hits between  $i$  and  $j$  is binomially distributed with parameters  $w$  and  $a_i a_j / w^2$ . The expected size of  $i$  and  $j$ 's intersection is  $a_i a_j / w$ , and thus the likelihood of them allying peaks when  $\delta = a_i a_j / w$ . At the outset, all firms hold each idea independently with probability  $p_0$ . Thus partnering and structural similarity will peak when  $\delta = w p_0^2$ , or equivalently  $p_0 = \sqrt{\delta / w}$  (see also the discussion in Section 2.2). As time passes, firms knowledge endowments change, and the ratio of what they know to the total relevant knowledge changes, (which is roughly equivalent to changing  $p$ ), which will affect their networking decisions and innovative success.

$\rho$  controls the width of the inverted U, i.e. the extent to which deviating from the optimal distance is acceptable for joint innovation. Having  $\rho$  close to 0 implies very little networking, as the condition for establishing an alliance is very tight. Larger values of  $\rho$  sustain partnering, possibly to an artificially important extent.

Second, innovation itself, whether joint or autarchic, depends on  $\Delta$ , which scales the magnitude of innovative jumps, and  $\lambda$ , which scales innovation rates, i.e. the pace of technical progress.

$\Delta$  is the largest possible innovative step taken from the current frontier at any point in time. It exerts a central influence on the processes of knowledge accumulation and networking. Accumulated knowledge dictates how easily a firm can find partners, in turn partnering affects the speed of knowledge accumulation, which again affects the firm's standing in terms of future partnering. The effects of increasing  $\Delta$  are easily understood. All else equal, a larger  $\Delta$  increases the probability of the next innovation being beyond the current frontier, which increases the probability that every firm's endowment falls. So at any point in time we would expect on average a smaller number of ideas held by the firms when  $\Delta$  is larger.

Consider a single firm  $i$ . Suppose it innovates. This can happen either within the frontier or beyond the frontier. If it holds a total of  $a_t = a$  elements at time  $t$ , there are  $w - a + \Delta$  possible locations at which it can innovate, of which  $w - a$  are within the frontier. Thus the probability it innovates inside the frontier is  $(w - a) / (w - a + \Delta)$ . If this happens

then the variation in the firm's knowledge stock is one:  $a_{t+1} = a + 1$ . With probability  $\Delta/(w - a + \Delta)$  the firm innovates outside the frontier. In this case, the location of the innovation is  $l$ ,  $w + 1 \leq l \leq w + \Delta$ , and the first  $l - w$  locations are dropped and replaced by zeroes. Assuming that the  $a$  facts it knows are uniformly distributed over the  $w$  locations,<sup>10</sup> in expected value the first  $l - w$  locations hold  $(l - w) \cdot a/w$  pieces of knowledge. As the expected value of  $l$  is  $(2w + \Delta + 1)/2$ , the expected change in the knowledge stock is simply  $1 - (\Delta + 1)/2 \cdot a/w$ . Thus if a firm innovates, the expected variation of its knowledge stock is

$$\frac{w - a}{w - a + \Delta} + \frac{\Delta}{w - a + \Delta} \times \left(1 - \frac{a(\Delta + 1)}{2w}\right).$$

In a world of homogeneous firms, if there is an innovation the probability that firm  $i$  makes it is simply  $1/n$ . If a different firm,  $j$ , makes it,  $i$ 's knowledge stock is unaffected unless  $j$  innovates beyond the frontier, which it does with probability  $\Delta/(w - a + \Delta)$ .<sup>11</sup> In this case, in expected value,  $i$ 's knowledge stock varies by  $-a(\Delta + 1)/2w$ .

Thus following any innovation, the expected change in  $i$ 's knowledge stock from period  $t$  to period  $t + 1$  is

$$\frac{1}{n} \frac{1}{w - a + \Delta} \left[ w - a + \Delta \left(1 - \frac{a(\Delta + 1)}{2w}\right) \right] - \frac{n - 1}{n} \frac{\Delta}{w - a + \Delta} \frac{a(\Delta + 1)}{2w}.$$

Defining a stationary number of ideas as a value of  $a$  such that the expected change is zero yields

$$a^* = \frac{2w(w + \Delta)}{2w + \Delta^2 n + n\Delta}, \quad (2)$$

It is immediately seen that  $a^*$  is falling with  $\Delta$ , and decreasing with  $n$ . Both effects are intuitive: a larger upper bound to innovative jumps means a more rapid obsolescence of ideas, and thus on average an emptier idea set; similarly, more innovations made by more other firms in the industry means fewer ideas entering my vector relative to the number

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<sup>10</sup>At first glance this assumption seems incorrect, since after an innovation that takes place at  $w + z$ , expanding the frontier, all of  $i$ 's knowledge is held in the first  $w - z$  places. However, the position of  $i$ 's knowledge in the long run is determined by when he innovates relative to when other firms have frontier expanding innovations. All firms are equally likely to innovate in each period, and so there is no pattern.  $i$ 's knowledge, on average, will be spread evenly over the  $w$  positions.

<sup>11</sup>The assumption of homogeneous firms seems strong. In section 3.3.1 below we show that in terms of the number of facts held by a firm, there is in fact little variation, so this assumption is not as strong as it seems at first glance.

of ideas exiting my vector, and so on average an emptier idea set. We will see in Section ?? that the stationary value  $a^*$  is a fairly good approximation of the real behaviour of the system. From the above calculation we can also immediately derive a few straightforward implications. The stationary number of ideas will interact with  $\delta$  the optimal overlap required for innovation. Suppose the variance of the distribution of the number of ideas across firms is small, so that every firm holds roughly the same number of ideas  $a^*$ . If  $a^*$  is much less or much more than  $\sqrt{w\delta}$  it will be very unlikely that networking is active. By contrast, if the stationary number of ideas is close to  $\sqrt{w\delta}$ , networking will be more intense (see also the discussion in Section 2.2).<sup>12</sup>

A second aspect of innovation is encapsulated in  $\lambda$ , the arrival rate of innovations. This parameter controls the probability (independent across firms and partnerships) of each innovative attempt yielding the discovery of a new idea. The larger  $\lambda$ , the more rapid the pace of innovation and the obsolescence of previous ideas. However, the stationarity of industry behaviour (in the sense of it being a stationary Markov process) is unaffected by the specific value of  $\lambda$ , and so the only real impact of  $\lambda$  on the system is that changes in knowledge stocks and thus alliance networks from one period to the next are of greater magnitude. As a result, industry statistics would behave much less smoothly over time with larger  $\lambda$  than with smaller.

Finally, the initial amount of knowledge in the industry is controlled by  $p_0$ , the probability of any firm holding any idea at the industry birth independently from other ideas and firms. A larger  $p_0$  implies on average a higher number of ideas held by the firms early in history. This parameter however only has a transient effect. Indeed, as argued in Section 2.3, the industry is a stationary Markov process and such processes are initial-conditions independent. However, how fast a particular firm and as a consequence the industry as a whole will approach (and then fluctuate around) the stationary knowledge stock identified above will be affected by  $p_0$ , through the effect of the latter on the intensity of networking and thus on the number of innovations per period.

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<sup>12</sup>The above calculation is an approximation of what happens in the more complex situation in which firms partner and jointly innovate, but the general intuitions are correct, as will be seen below. We have also made the implicit assumption that firms are homogeneous in terms of where innovation takes place in their idea sets, so  $a^*$  should be seen as the expected value of the stationary number of ideas rather than the stationary value itself.

## 2.5 Settings

We consider a population of  $n = 100$  firms and a relevant knowledge base of  $w = 100$  ideas. At the outset individual knowledge endowments are randomly drawn over  $\{0, 1\}$  with initial probability  $p_0 = 1/2$ , independently for every element in each firm's idea set. The autarchic innovation rate  $\lambda$  is set to 5% (20 periods is the expected waiting time between two consecutive innovations for a given firm or partnership). For the inverted-U we use

$$f(y) = \frac{1}{\lambda\rho^2} (-y^2 + 2\delta y - \delta^2 + 2\rho^2), \quad \rho > 0. \quad (3)$$

At the optimal overlap  $f(\delta) = 2/\lambda$  (twice the innovation rate of internal R&D) and  $c = 1/\lambda$ . Let  $f(\delta - \rho) = f(\delta + \rho) = c$ , using  $\rho$  to control the width of the inverted U. As alliances form if  $f(y) \geq c$ , the base width of the inverted-U ( $2\rho$ ) and  $c$  play equivalent roles: a larger  $c$  is equivalent to a smaller  $\rho$ , both reducing the number of partnerships that can form. The equilibrium network in each period is thus simply  $g = \{ij : f(y_{ij}) \geq c\} = \{ij : |y_{ij} - \delta| \leq \rho\}$ . Finally set  $\rho$  to 10, so that 10% of the knowledge base is the largest acceptable deviation from the optimal amount of overlap.

Each period firms form pairs (or stay alone), attempt to innovate, possibly discover new knowledge which makes old ideas obsolete, and part company. In the first stage of the analysis, our interest will be static: given the state of firms' knowledge, what are the characteristics of the network that forms? Then we will turn to a longer term perspective. Preliminary experiments have suggested that the system reaches its stationary regime after roughly speaking 500 periods. So we will focus on the periods 500 to 800, collecting averages of the statistics each period. We will also look at time series, to understand the type of cyclical behaviour displayed by network structure.

There remain two independent parameters,  $\delta$  and  $\Delta$ , which we vary from 0 to 60 and 1 to 10 respectively. For each point in the parameter space, we generate one history of length 800 periods and retain first period results and fluctuations around the stationary values from the periods posterior to 500.



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Include Figure 5 here.

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### 3 RESULTS

In this section we begin by examining the static, first period industry in which knowledge is identically and independently distributed across firms. Then we turn to the long run behaviour of the industry, both examining cyclical patterns in time series and analyzing the stationary distribution of several statistics of the system.

#### 3.1 First period outcome

The first period results are summarized in the two figures below. The intuitions gathered from the restricted, special case (Section 2.2) suggest that small worlds, as defined by the conjunction of high clustering and low average path lengths, are likely to be present in some parts of the parameter space. To confirm this conjecture numerically, we have created a large number of networks and computed several network statistics. Sticking to  $w = 100$  and  $n = 100$ , we generate 1,000 equilibrium networks for different values of  $p_0$  and with  $\delta = 20$ .

To abstract from issues of isolated firms, when discussing clustering and characteristic path length we focus on the largest component, i.e. the largest subset of the industry such that any two firms in it are linked by a path. Clustering coefficient and average distance have to be rescaled by a random “counterpart” in order to distinguish the presence of structure from mere randomness. The approximations used in the literature for path length and clustering of random graphs<sup>13</sup> are only reasonable for very large populations. Our smaller networks demand a specially tailored normalization. For each network we wish to rescale, we record density and then generate 1,000 random networks with exactly that density. Taking the average clustering and path length over that sample, we use these as the random counterpart with which to normalize our statistics. The small world ratio is then simply the ratio of rescaled clustering over rescaled distance. In addition we show the

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Include Figure 6 here.

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skewness of the link distribution and the correlation between knowledge levels and degree.

Figure 5 illustrates the special role of  $p_0 = \sqrt{\delta/w}$ , which is equal to  $\sqrt{20/100} = 0.447$  in the case at hand. At this level of initial endowment, the likelihood of partnering is maximized, and so is the value where the largest component size and average degree are maximized. Moving away from  $\sqrt{\delta/w}$  in either direction implies a decline in these two measures. Note that skewness (asymmetry in the degree distribution) and the SW ratio behave in the opposite way: an almost complete graph is uninteresting in terms of skewness (everyone has approximately  $n - 1$  connections) and structure. Rather, it is when initial endowments move away from  $\sqrt{\delta/w}$  that sparser networks can form, with a potential for richer architectures. Specifically, around 0.2 and 0.65 the equilibrium graphs displays the largest amounts of asymmetry and clustering. These are the regions where our model produces results that resemble the networks observed empirically.

Figure 6 shows four curves: rescaled clustering, rescaled distance, the proportion of joint discoveries is provided and the correlation between knowledge and degree. The first two simply decompose the ratio, shown in Figure 5. Not surprisingly the proportion of innovations that are made in an alliance as opposed to being made by firms acting individually tracks degree faithfully. The more alliances there are, the more innovations will be made jointly. Finally we can see that the correlation between knowledge and degree changes sign as  $p_0$  crosses the  $\sqrt{\delta/w}$  threshold. Left of that value firms hold on average too few ideas, so a more successful firm is one with more ideas than average. Right of that value firms hold on average too much knowledge, thus firms that are successful in finding partners are those with fewer than average ideas.

### 3.2 Fluctuations in the stationary regime

In this section we observe the fluctuations of the system in its stationary regime. Again (as stated in Section 2.3), stationary refers to the fact that the system spends a fixed proportion of time (encapsulated in the stationary distribution) in each possible state . It

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<sup>13</sup>If  $\bar{n}$  is average degree, the standard approximations are  $\bar{n}/(n-1)$  for the clustering coefficient and  $\ln(n)/\ln(\bar{n})$  for the characteristic path length.

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Include Figure 7 here.

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should not be understood as convergence to a single absorbing fixed state. Thus there will always be fluctuations for any statistic of interest. In addition, the time average of the fluctuations of a given statistic is an unbiased estimator of the expectation of that statistic computed with the stationary distribution. Thus we can proceed in two ways. In this section we freeze  $\delta$  and  $\Delta$ , (at  $\delta = 35$  and  $\Delta = 2$ ) and ask how the system changes over the longer time horizon, once it has reached its stationary distribution. In the next section we ask about how average behaviour (over time) responds to the two parameter  $\delta$  and  $\Delta$ . This we do by running the model for a large range of the two parameters and looking at averages of the interesting statistics (degree, clustering and so on) over the last 300 periods.

The left panel of Figure 7 displays the behaviour of the average number of ideas held and average degree over time. Both time series display ample fluctuations, with the average degree oscillating between 2 and 15, while the average number of ideas evolves between 10 and 12. The two statistics are negatively correlated, as illustrated by a scatter-plot of ideas versus degree provided in the right panel of Figure 7. So the industry displays a variety of patterns, marked with outbursts and collapses in network activity (for a discussion of a simple model generating comparable patterns see Marsili et al., 2004). A batch of innovations by several firms will impoverish the industry in general by making obsolete much pre-existing knowledge. This will trigger a decline in networking (as  $a^*$  has moved away from  $\sqrt{w\delta}$ ) and the slow accumulation of knowledge through internal R&D until the point where networking increases again, which creates the possibility for the next collapse through a lucky sequence of innovations.

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Include Figure 8 here.

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Additional aspects of network structure are shown in Figure 8. In the left panel, the clustering coefficient shows strong oscillations between 0.3 and 1, with larger values of clustering being more prevalent. In parallel, average distance fluctuates between 1 and 2. The general situation is strongly small world-ish, with the small world ratio of rescaled

clustering over rescaled path length being systematically large in the right panel in Figure 8.

We conclude this section with some elements on the occurrence of repeated ties. Figure 9 depicts the evolution of the proportion, at time point  $t$  in time, of ties existing at  $t$  which were in place at  $t - 1$ . Thus this proportion is a measure of the extent of turmoil at the micro-level, which direct observation of aggregate variables cannot capture: typically a constant number of edges in a graph does not imply that edges are held by the same firms over time.

The time series in Figure 9 (again) display significant fluctuations: periods of stability in the network (in which all of today's ties are repeats) can be followed by more disruptive periods where the network reorganizes. Again these changes are triggered by particular sequences of innovation, in which repeated ties tend to be associated with unlucky innovative attempts, while successful innovations in the knowledge space tend also to create innovations in network (re-)organization.

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Include Figure 9 here.

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### 3.3 Long run average behaviour

In this section time averages are computed to obtain a more compact representation of the relationship between our two independent parameters  $\delta$  and  $\Delta$ , and various industry statistics. We begin with a check of our intuitions and calculations regarding the existence of a stationary number of ideas with limited fluctuations around it.

#### 3.3.1 Knowledge

Figure 10 displays the characteristic features of knowledge accumulation: the average number of ideas and the stationary prediction derived in Section 2.4. To construct the box-plots below, we have pooled all the observations for the average number of ideas across time and all  $\delta$ -values. The middle of the box-plot is the sample median, the box top (bottom) is the 75th (25th) percentile and the whisker top (bottom) is the 90th (10th) percentile. The stationary value  $a^*$  turns out to be a very reliable upper bound to the average number of ideas

held by the firms for any  $\Delta$  value, and displays the same sensitivity to changes in  $\Delta$  as the computed average. The median number of ideas however is always significantly lower than  $a^*$ . The reason is that the calculation of  $a^*$  does not take into account the effects of alliance, namely joint innovation. We have derived  $a^*$  under the (strong) assumption of exclusively autarchic independent innovation. Adding joint innovation will result always, though to a varying extent, in more rapid technological change, that is, more rapid obsolescence and thus fewer relevant ideas held on average.

Regarding firms' heterogeneity, the coefficient of variation in number of ideas across firms (not shown here) is always small (below 1.5, for a maximum possible value of  $\sqrt{99} = 9.9$  with  $n = 100$  firms) so that the average number of ideas held is representative of the behaviour of the population of firms.

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 Include Figure 10 here.  
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### 3.3.2 Network

The 6-panel graph below displays for each network statistic the time average of the final 300 periods. As discussed in Section 2.3 above, this is a good representation of the general behaviour of the system.

Average degree within the largest component indicates differentiated levels of network activity across the  $(\delta, \Delta)$  space. There is a region of intense partnering: a very dense graph (almost complete) obtains when  $\delta \leq 10$  and  $\Delta \geq 2$ . When  $\Delta > 2$  the average number of ideas held by firms is very small. Thus partnering is only possible if the demands of knowledge overlap are very weak. The width of the inverted-U is 20 ( $\delta \pm 10$ ) so when  $\delta \leq 10$  even zero overlap results in a profitable alliance. As  $\delta$  increases from 10, though, the probability of finding a partner falls very rapidly, especially for larger  $\Delta$  values. For lower values of  $\Delta$ , the average number of ideas becomes relatively large, and networking is sustainable at larger values of  $\delta$ .<sup>14</sup> Consequently, the case  $\Delta = 1$  is markedly different from

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<sup>14</sup>From Section 2.2 above, when the inverted-U is a spike of zero width, networking peaks when  $p \approx \sqrt{\delta/w}$ .  $\Delta$  determines the average steady state knowledge quantity  $a^*$ , which can be seen as  $a^* = p * w$ . Thus we can calculate directly the value for  $\delta$  for which networking should peak for different values of  $\Delta$ . A list of  $(\Delta, \delta)$  ordered pairs illustrates: (1, 25), (2, 7), (3, 2), (4, 1), (5, 0). The reason we see any networking

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Include Figure 11 here.  
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all other  $\Delta$  values, with an interior peak in networking (around  $\delta = 20$ ), and partnering existing for the whole range of  $\delta$  values. The logic of these patterns arise entirely from two properties of the model: As  $\Delta$  increases, the average number of ideas held falls, relatively quickly; and for a given average knowledge level (provided it is not too low), the probability of finding a partner (all else equal) rises and falls with  $\delta$ . This all follows from Section 2.2 above.

We can observe a frontier between a region of autarchy, in the upper right, and networking firms in the lower left. The system has interesting behaviour along the border between these two regions (the contour line 1, in the average degree panel). In this boundary region all indicators (rescaled path length and clustering, small world ratio of rescaled clustering over rescaled path length, skewness) point to the presence of skewed small worlds. Finally, in this zone of small worlds, the correlation between knowledge and degree is positive, showing the value to a firm of holding more ideas.

From the discussions in Section 2.2 (spike restriction) and Section 3.1 (period 0), the results reported here tell us that similarity-driven joint innovation still produces small world type of networks when embedded in a larger model of the creation (discovery) and destruction (obsolescence) of ideas. Over some parts of the parameter space an almost complete or an empty, and thus relatively uninteresting graph forms. However there are also regions where the emerging structure is sparser, leaving room for a richer organization of ties. There, clustered groups tend to form and persist, disbanding after a particular sequence of innovations, before such new groups form again.

## 4 CONCLUSION

Our model of joint innovation lies at the heart of the alliance formation process. It has the feature of being “tunable” in terms of the degree to which the knowledge frontier

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at all for values of  $\Delta$  above 4 has to do with the width of the inverted-U, and only for  $\Delta = 1$  will we observe the rise and fall of networking as  $\delta$  increases, that we would expect from the special case calculations.

can advance in response to a single innovation (our parameter  $\Delta$ ). One thing we observe here is that in an industry in which there is large innovative potential (large  $\Delta$ ) rapid progress in terms of the frontier implies that knowledge is made obsolete more quickly, and so at any point in time the average firm knows fewer of the relevant facts. (Recall that we have assumed that at the industry level there is a constant number of relevant facts,  $w$ .) This suggests greater differentiation among firms when the innovative potential is large. In addition, firms that are successful in pushing the frontier will have more relevant knowledge than other firms, and so will have two competitive advantages: they will possess the most recent discoveries, and they will be more knowledgeable generally. In industries with smaller innovative potential (as measured by size of potential jumps beyond the frontier) frontier firms will lose the second advantage, as the average firm knows a higher proportion of the relevant facts. Firms are more similar, and the differentiating feature generally is how recently a firm has pushed the frontier in a way that makes other knowledge obsolete.

A second observation regarding innovative potential is that over the life-cycle of an industry we might expect innovative potential to fall. Because our model is stationary, the results we have produced regarding how the model behaves in different parts of the states space can be used to make conjectures about how an industry network might change over time. If the industry transits through its life-cycle relatively slowly, then as parameters change due to its aging, it will fairly quickly respond and move to a new steady state. Thus in essence, as an industry ages it will move through the parameter space of Figure 11. Focussing on the declining innovative potential, as an industry ages  $\Delta$  will fall—we move vertically down the panels in Figure 11. As is well known, entry and exit form an important part of the patterns we observe over time, so the top left panel of the figure is likely to be misleading, due to its heavy dependence on the number of firms being constant. The others, however are less dependent on the number of firms. If it is generally the case that innovative potential falls over the life of an industry, then we can see, by tracing vertically down the panels of Figure 11 that we can expect clustering to rise and then fall as an industry ages (middle left panel). By contrast, average distances do not respond strongly to  $\Delta$ . These jointly suggest that a young industry will not look like a small world, a middle-aged one will, and an old one probably will, but in a less strong fashion (bottom

left panel of Figure 11). Finally, the skewness of the link distribution also rises and falls as time passes. Because many things apart from innovative potential change over the course of a life-cycle this is certainly not the whole story, but it could be suggestive of patterns that might emerge empirically.

In this paper we have argued that the nature of joint innovation alone is enough to produce industry networks that share many properties of empirically observed strategic alliance networks. When choosing alliance partners, firms clearly take many things into consideration. The empirical literature, however, has focussed very heavily on issues of social capital, and has paid much less attention to the fact that partnering firms must also have a “cognitive fit”. In this way our model is complementary to the work of Reagans and McEvily in this volume. They use knowledge overlap as a (linear) control in examining formation of knowledge transfer and knowledge seeking links within an organization. Their main interest is in network positioning effects, but this control is highly significant in their results. One interesting extension to their work would be to ask whether expertise overlap can “do more work” in explaining link formation with a richer (non-linear) specification.

Our model does present a strong contrast to two other papers in this volume, van Liere et al, and Amburgey et al. In both of those papers alliance formation is constrained by network distance — in effect firms cannot see very far along the network, and so partner choice is constrained to firms that are close by in network space (typically friends of friends). This is consistent with the idea that many alliances are formed through referrals by common third party ties. In our model, every firm has strong knowledge about every other firm in the industry, both that it exists, and precise details about its knowledge portfolio. This is clearly a strong assumption. But this assumption means that a firm can choose as a partner any other firm in the industry, not just those firms already close in network space. In a sense, this makes our results that much stronger: with no considerations of network distance constraining firms to form links within a small neighbourhood, we still see the emergence of clustered networks. While not denying the importance of social capital considerations, we show here that cognitive fit can be a very strong influence on the types of networks that form.



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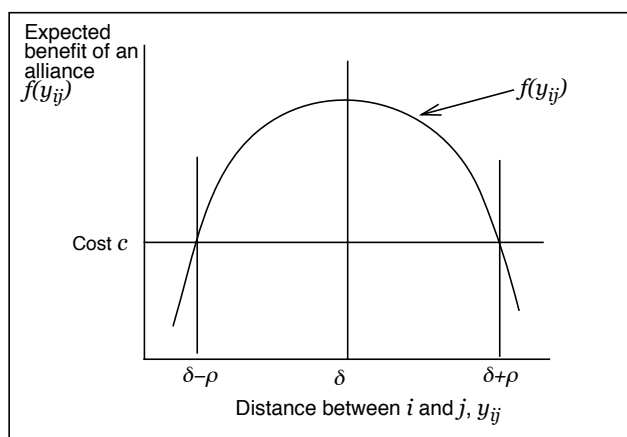


Figure 1: Costs and benefits of an alliance as a function of distance in knowledge space.

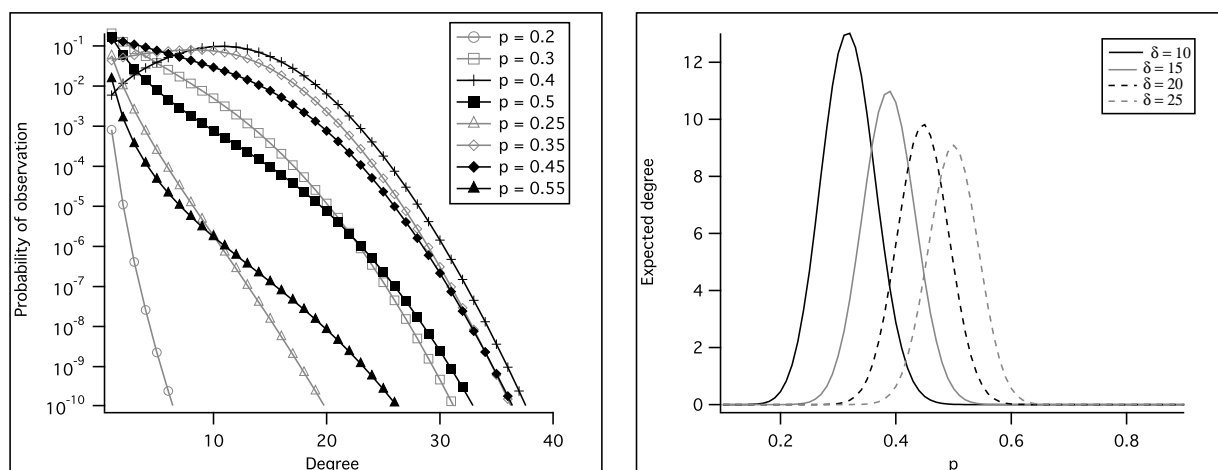


Figure 2: Left: Degree distribution for various values of  $p$ ; Right: expected degree and lower right: expected degree as function of  $p$  for four values of  $\delta$  (note that in both cases  $w = 100$ ,  $n = 100$ ).

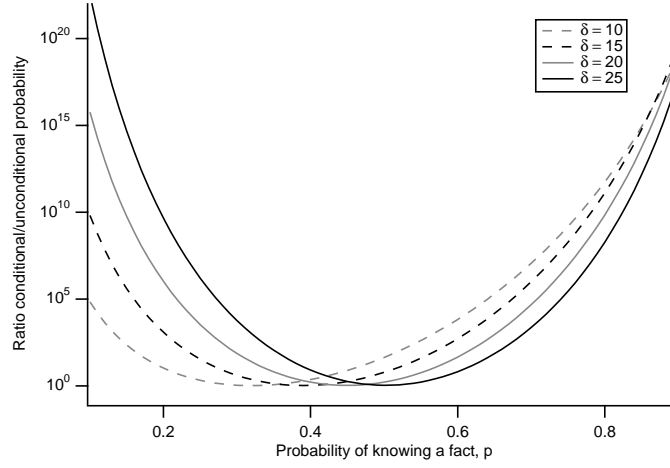


Figure 3: Ratio of conditional to unconditional probability as a function of  $p$  ( $w = 100$ ,  $n = 100$ ).

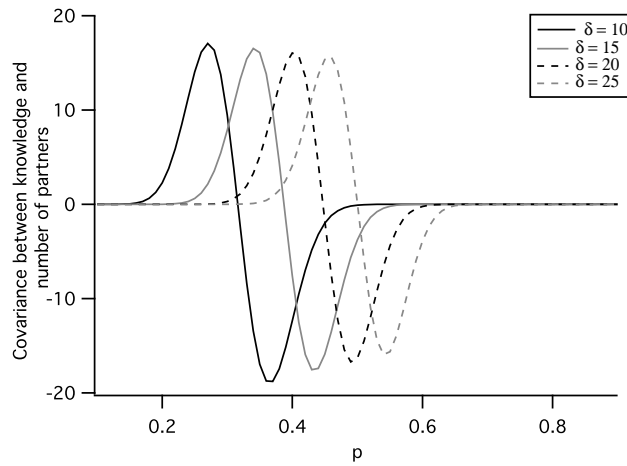


Figure 4: Correlation between knowledge levels and number of partners as a function of  $p$ .

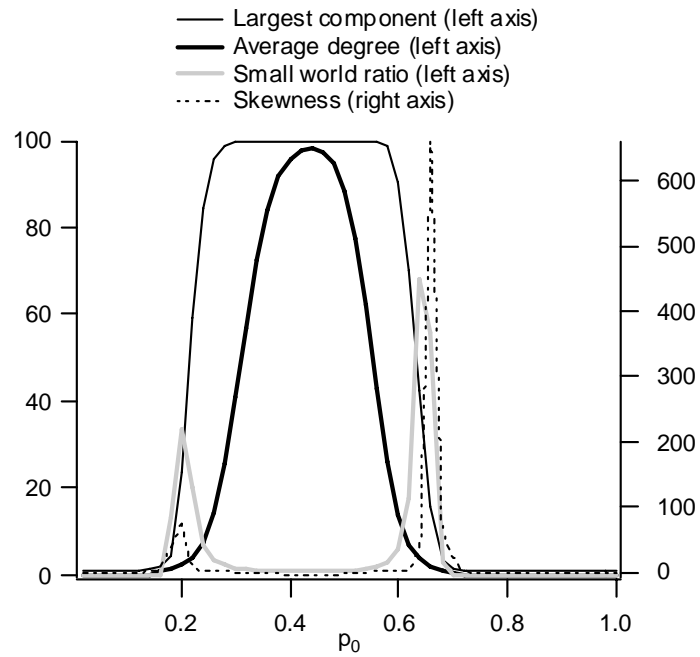


Figure 5: Network characteristics in the first period.

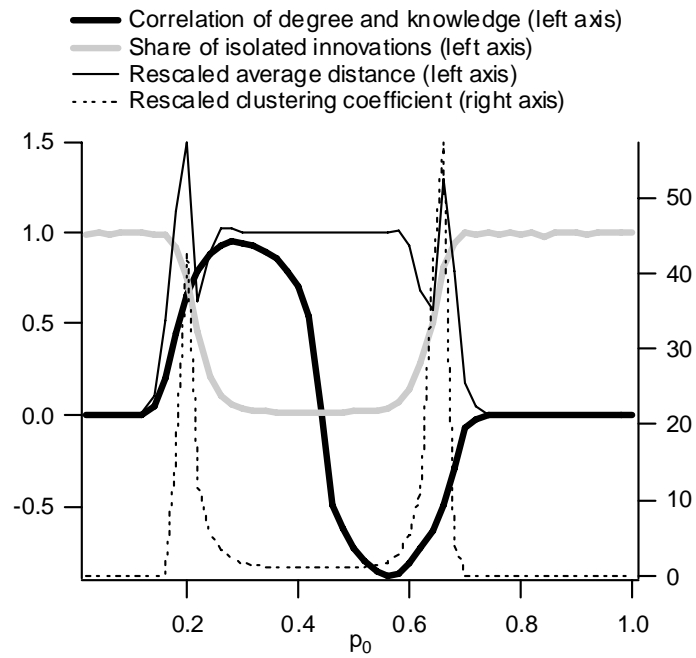


Figure 6: Network characteristics in the first period continued.

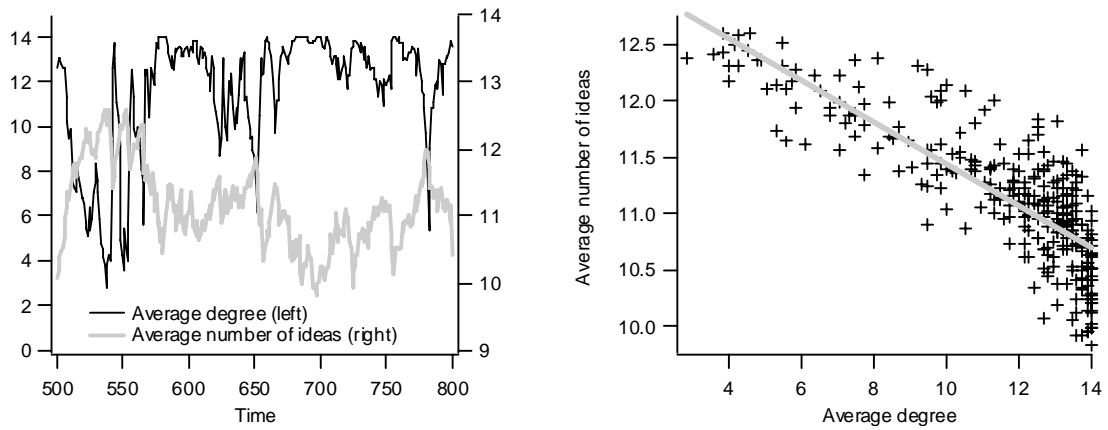


Figure 7: Average degree and number of ideas.

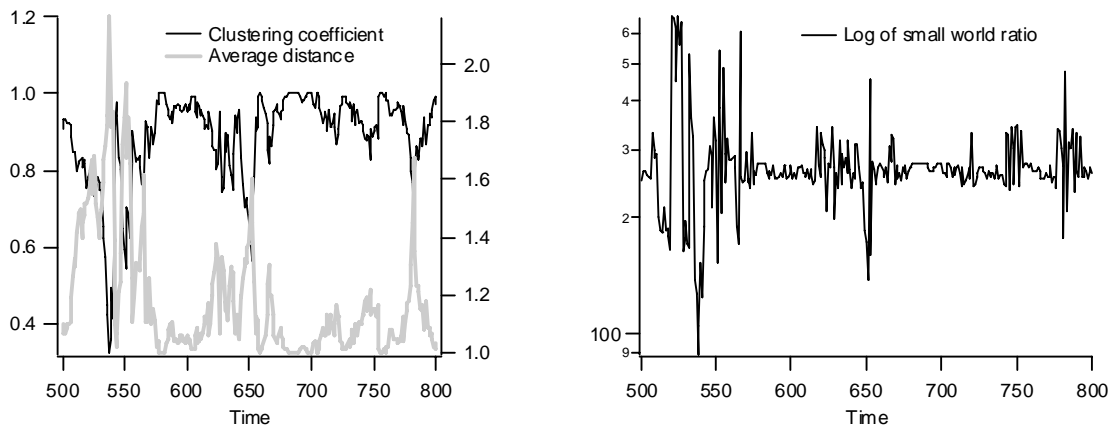


Figure 8: Average clustering and distance in the network, and the small world ratio.

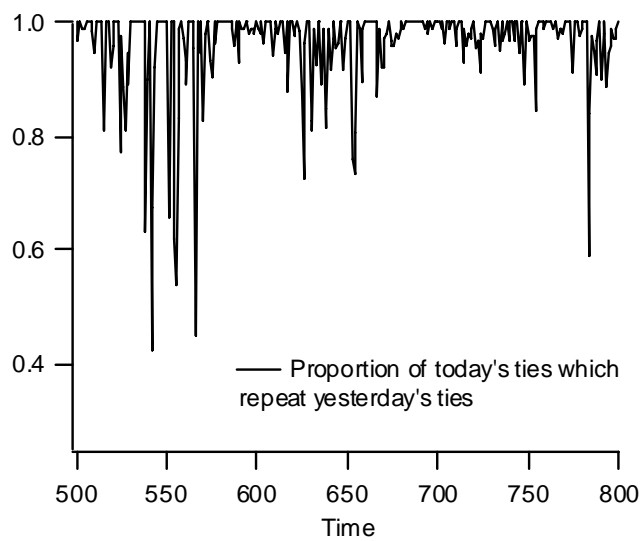


Figure 9: Stability at the micro-level measured by the proportion of repeats.

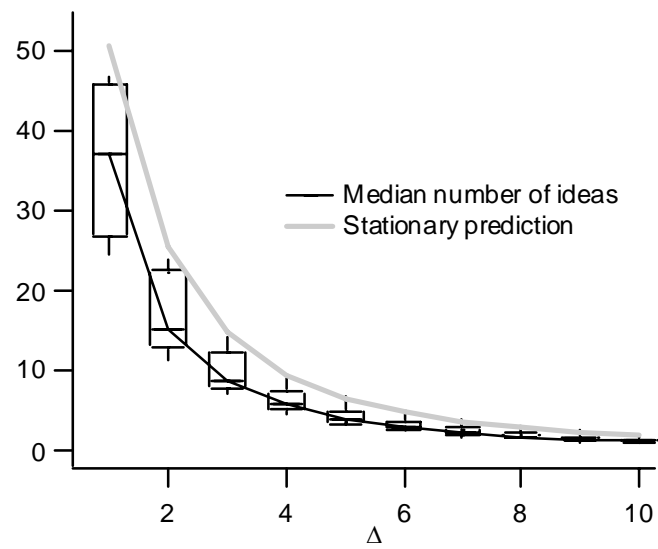


Figure 10: Box-plots for the population average number of ideas pooled across all values of  $\delta$  and all final 300 periods and the stationary value  $a^*$ .



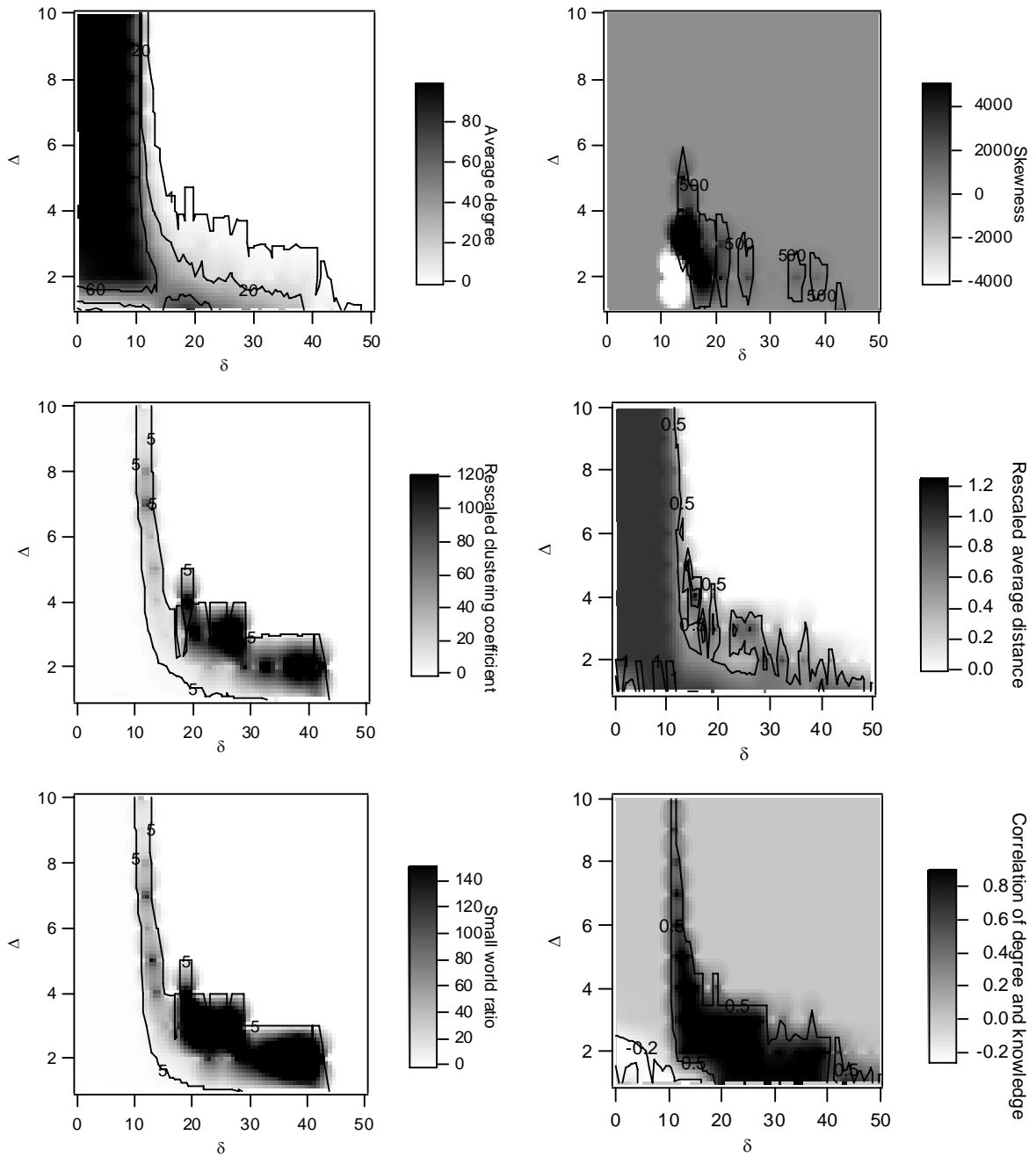


Figure 11: Network characteristics in the steady state.