Tie-breaking Rules and Informational Cascades: A Note*

Frédéric Koessler
Anthony Ziegelmeyer †

BETA – Theme
Université Louis Pasteur, 61 Avenue de la Forêt-Noire
F-67085 Strasbourg Cedex (France)

June 13, 2000

Abstract

In Bikhchandani, Hirshleifer, and Welch's (1992) specific model, it is showed that conformist behaviors can emerge due to information externalities. In this note we establish that this result, based on 'informational cascades', heavily depends on the choice of a particular tie-breaking convention. Relaxing this assumption allows for other equilibria to exist, in which informational cascades are not necessarily observed. Our findings also have implications for the analysis of experimental data on informational cascades. In this respect, we argue that further experiments should be based on other experimental designs.

Keywords: Tie-breaking rules, informational cascades, experimental economics.
JEL classification: C72, C92, D82.

Résumé

Bikhchandani, Hirshleifer, et Welch (1992) ont montré qu'en présence d'externalités informationnelles des comportements grégaires peuvent émerger. Nous montrons qu'un tel phénomène, appelé 'cascade informationnelle', repose sur un choix particulier d'une convention de 'tie-break'. Le relâchement d'une telle hypothèse génère une multiplicité d'équilibres, équilibres dans lesquels des cascades informationnelles n'apparaissent pas nécessairement. Ce résultat permet une nouvelle interprétation des données expérimentales. Par ailleurs, nous suggérons l'utilisation d'autres protocoles expérimentaux.

Mots clés: Règles de 'tie-break', cascades informationnelles, économie expérimentale.
Classification JEL : C72, C92, D82.

*We especially thank Nicolas Jonard and Jully Jeunet for helpful suggestions. We also received useful comments from Gael Giraud, Guillaume Haeringer, Patrick Roger, Gisèle Umbhauer, and Marc Willinger. Moreover, we are grateful to Lisa Anderson, Charles Holt, Angela Hung and Charles Plott for sharing their experimental data with us.

†Corresponding author: sas@cournot.u-strasbg.fr

1 Introduction

In their seminal paper Anderson and Holt (1997) designed experiments, inspired by a specific model taken from Bikhchandani, Hirshleifer, and Welch (1992) (henceforth BHW), to study empirically the emergence of information cascades.[1] Though Anderson and Holt claimed their data supported the theory pretty well, they also noted that the incidence of observed cascades was lower than predicted due to inconsistent Bayesian updating decisions.

This conclusion heavily depends on the tie-breaking rule assumed by BHW. Indeed, in their specific model, BHW supposed that *any* agent, when indifferent between two actions, randomizes with equal probability. This calls for two comments. First this tie-breaking rule is only one among many other possible modelling assumptions. Second they asserted that each agent is endowed with the *same* tie-breaking rule, which amounts to saying that there is actually a tie-breaking convention.

In this note, we show that by relaxing BHW's hypothesis of a tie-breaking convention, we allow for other equilibria to exist, in which informational cascades are not necessarily observed.[2] More precisely, we consider a new tie-breaking rule: the *non-confident* tie-breaking rule. Under this rule, an indifferent agent simply imitates the action of his predecessor. Doing so, he takes an action that contradicts his private information, hence the label 'non-confident'. Considering a new tie-breaking rule also has implications for the analysis of Anderson and Holt's (1997) experimental data. In this respect, we argue that further experiments on information cascades should be based on other experimental designs.

Section 2 gives a general statement of the model. Section 3 discusses different tie-breaking rules, and experimental results taken from Anderson and Holt (1997) are reinterpreted in section 4. We conclude in Section 5.

2 The model

In BHW's specific model, each agent is given an independent private signal —taking a binary form— about the state of Nature. Agents are then arrayed in a randomly determined order and sequentially make a publicly announced decision about the state of Nature. Thus, at the time of his decision each agent has a private signal and also knows the decisions of all preceding agents. Anderson and Holt (1997) implemented this setup in the following way: balls tagged a or b were put in urns labeled A and B and one of these urns was selected at random; subjects were then chosen in a random order to observe a single draw from the selected urn; subjects were finally asked to make a public prediction about the identity of the selected urn. In the following sections, we proceed along the same line with an infinite ordered sequence of agents.

2.1 The game

We call player 1 the player who decides first, player 2 the player who decides second, and so on. Each player $i \in N = \{1, 2, \dots\}$ chooses an action $s_i \in S = \{A, B\}$ where A stands for 'predicting urn A' and B for 'predicting urn B'. Let $\mathcal{H} = \{\emptyset\} \cup \{(s_i)_{i \leq l} :$

[1] The same experimental design was also used by Willinger and Ziegelmeyer (1998) and Hung and Plott (2000) in order to find some evidence for herd behavior.

[2] A tie-breaking convention can be thought of as an equilibrium selection device.

$l \in N,\ s_i \in S\}$ be the *set of histories*. Denote by $h^l \in \mathcal{H}^l = \{h \in \mathcal{H} : |h| = l\}$ a history of length l, and write $h^0 = \emptyset$. When agent i takes an action, he observes a history h^{i-1}, i.e., he observes the actions taken by agents $1, \dots, i-1$.

Agents have a common prior belief p on a payoff relevant state space $\Omega = \{\alpha, \beta\}$, where α stands for 'urn A has been selected', β stands for 'urn B has been selected', and $p(\alpha) = p(\beta) = 1/2$. Each agent i has a set of possible signals $T_i = \{a_i, b_i\}$. Conditionally to the realization of a state of Nature, the agents' signals are i.i.d and the conditional probabilities are given by $p(a_i \mid \alpha) = p(b_i \mid \beta) = q$ and $p(a_i \mid \beta) = p(b_i \mid \alpha) = 1 - q$, where $q > 1/2$. That is, signal a_i is favorable to α and signal b_i is favorable to β.

Because players' actions do not directly influence the utility of the others, we can define the vNM utility function of each player on $S \times \Omega$. More precisely, for all $i \in N$, we assume $u_i(A, \alpha) = u_i(B, \beta) = g$ and $u_i(A, \beta) = u_i(B, \alpha) = g'$, where $g > g'$.[3]

2.2 The equilibrium

A *behavioral strategy* of player i is given by a function $\sigma_i : T_i \times \mathcal{H}^{i-1} \to \Delta(S)$, where $\Delta(S)$ is the set of probability distributions over S. A profile of behavioral strategies is denoted by $\sigma = (\sigma_1, \dots, \sigma_N)$. For a given signal t_i and a given history h^{i-1}, $\sigma_i(s_i \mid t_i, h^{i-1})$ is the probability that player i chooses action $s_i \in S$. Let $\phi_i(s_i \mid \alpha, h^{i-1})$ (respectively $\phi_i(s_i \mid \beta, h^{i-1})$) be the probability that player i chooses action s_i given history h^{i-1} and the realization of α (respectively β). Given a behavioral strategy of player i, these probabilities are given by

$$\begin{aligned}
\phi_i(s_i \mid \alpha, h^{i-1}) &= q\sigma_i(s_i \mid a_i, h^{i-1}) + (1-q)\sigma_i(s_i \mid b_i, h^{i-1}), \\
\phi_i(s_i \mid \beta, h^{i-1}) &= q\sigma_i(s_i \mid b_i, h^{i-1}) + (1-q)\sigma_i(s_i \mid a_i, h^{i-1}).
\end{aligned}$$

Let $\mu_i : T_i \times \mathcal{H}^{i-1} \to [0,1]$ be player i's *belief* (conditional probability given past observed actions and his private signal) that the state of Nature is α. A system of beliefs is denoted by $\mu = (\mu_1, \dots, \mu_N)$.[4]

Given a history h^{i-1}, a signal t_i and a belief $\mu_i(t_i, h^{i-1})$, player i's expected utility is given by

$$\begin{aligned}
EU_i(A, t_i, h^{i-1}) &= g\mu_i(t_i, h^{i-1}) + g'(1 - \mu_i(t_i, h^{i-1})), \\
EU_i(B, t_i, h^{i-1}) &= g(1 - \mu_i(t_i, h^{i-1})) + g'\mu_i(t_i, h^{i-1}).
\end{aligned}$$

Hence, predicting urn A is relevant for player i if he believes α with probability greater than 1/2.

In a *perfect Bayesian equilibrium*, players rationally update their beliefs, by observing their signal and previously taken actions, and they act rationally given these beliefs.

Definition 1 A *perfect Bayesian equilibrium* of the game $G \equiv \langle N, \mathcal{H}, \Omega, p, (T_i), (u_i) \rangle$ is a profile of behavioral strategies σ and a system of beliefs μ such that the following properties are satisfied for all $i \in N$, $t_i \in T_i$ and $(s_i)_{i \in N} \in \prod_{i \in N} S$:

[3] Payoffs are defined slightly differently in BHW's specific model. Indeed, they posited that for all $i \in N$, $u_i(A, \alpha) = 1/2$, $u_i(A, \beta) = -1/2$ and $u_i(B, \alpha) = u_i(B, \beta) = 0$. Our results are maintained with this payoff function.

[4] To simplify the notations we write $\sigma_1(t_1, \emptyset) = \sigma_1(t_1)$ and $\mu_1(t_1, \emptyset) = \mu_1(t_1)$ for all $t_1 \in T_1$.

(i) Bayes' Rule. $\mu_1(t_1) = p(\alpha \mid t_1)$ and for all $i \geq 2$,

$$\mu_i(t_i, h^{i-1}) = \frac{p(t_i \mid \alpha) \prod_{j<i} \phi_j(s_j \mid \alpha, h^{j-1})}{p(t_i \mid \alpha) \prod_{j<i} \phi_j(s_j \mid \alpha, h^{j-1}) + p(t_i \mid \beta) \prod_{j<i} \phi_j(s_j \mid \beta, h^{j-1})},$$

where $h^j = (s_k)_{k \leq j}$ and $s_j \in supp(\phi_j(\cdot \mid \alpha, h^{j-1}))$.[5]

(ii) Sequential rationality.

$$\sigma_i(A \mid t_i, h^{i-1}) = \begin{cases} 1, & \text{if } \mu_i(t_i, h^{i-1}) > 1/2; \\ 0, & \text{if } \mu_i(t_i, h^{i-1}) < 1/2. \end{cases}$$

Sequential rationality is inconclusive when a player's belief equals 1/2. Thus each player has to be endowed with a tie-breaking rule that specifies his behavior in case of indifference. Obviously, different tie-breaking rules might yield different equilibrium outcomes, which clearly shows that common knowledge of rationality is not sufficient to determine a unique equilibrium outcome.

3 Tie-breaking rules

The specification of particular tie-breaking rules has an impact on information aggregation and the occurrence of informational cascades. According to BHW, an informational cascade occurs when it is optimal for a player, having observed his predecessors' actions, to follow the behavior of the preceding player without regard to his own information. Unfortunately, such a definition is not appropriate if one considers heterogeneity in terms of tie-breaking rules. In this respect, we alternatively define an informational cascade in terms of beliefs. A cascade is said to occur when public information (the sequence of past actions) overwhelms private information in agents' beliefs. This means that whatever his private signal, the *only* optimal action of a player is to follow the established pattern. Let (σ, μ) be a perfect Bayesian equilibrium, $h^{i-1} = (s_j)_{j<i}$ and $s_j \in supp(\phi_j(\cdot \mid \alpha, h^{j-1}))$, for all $j \in N$.

Definition 2 An *informational cascade* occurs in period k if and only if for each player $i \geq k$ (but not for players $i < k$), whatever his signal t_i, either $\mu_i(t_i, h^{i-1}) > \frac{1}{2}$ or $\mu_i(t_i, h^{i-1}) < \frac{1}{2}$.

In this section we first consider two tie-breaking rules which are in favor of the emergence of informational cascades. Second we introduce a third tie-breaking rule which produces arbitrarily late informational cascades. We then show that if no homogeneous tie-breaking rule is assumed, any sequence of actions might be observed.

3.1 Tie-breaking conventions favoring informational cascades

As a tie-breaking convention, BHW assumed that each player who is indifferent between the two actions predicts each urn with equal probability. Formally, if for a given pair (t_i, h^{i-1}) we have $\mu_i(t_i, h^{i-1}) = 1/2$, they posited that $\sigma_i(A \mid t_i, h^{i-1}) = 1/2$. Another tie-breaking convention has been proposed by Anderson and Holt (1997)

[5] For all $j \geq 1$, $supp(\phi_j(\cdot \mid \alpha, h^{j-1}))$ is the support of $\phi_j(\cdot \mid \alpha, h^{j-1})$.

(hereafter denoted AH's tie-breaking convention). They assumed that each player who is indifferent between the two actions follows his private signal.[6] Formally, if for a given history h^{i-1} we have $\mu_i(a_i, h^{i-1}) = 1/2$ then $\sigma_i(A \mid a_i, h^{i-1}) = 1$. Likewise if for a given history h^{i-1} we have $\mu_i(b_i, h^{i-1}) = 1/2$ then $\sigma_i(A \mid b_i, h^{i-1}) = 0$.

With either BHW's or AH's tie-breaking convention, equilibrium beliefs and behavioral strategies are unique and our definition of an informational cascade is equivalent to the one given by BHW. Therefore, we have the following proposition (proofs of the propositions are in appendix).

Proposition 1 *With either BHW's or AH's tie-breaking convention, the probability that an informational cascade eventually occurs approaches one as the number of periods increases.*

It should be noted that in BHW's specific model, either with BHW's or AH's tie-breaking convention assumption, it takes an imbalance of two decisions in one direction to ensure uniformity of the ongoing sequence. For example, if the first two agents make identical predictions then all subsequent agents have to follow the established pattern.

3.2 Non-confident tie-breaking rule

In this section we introduce a new tie-breaking rule which induces a player who is indifferent between the two actions to predict the same urn as his predecessor. We call such a tie-breaking rule the non-confident tie-breaking rule. Formally, if for a given pair (t_i, h^{i-1}) we have $\mu_i(t_i, h^{i-1}) = 1/2$ then we posit that $\sigma_i(A \mid t_i, h^{i-1}) = 1$ if $s_{i-1} = A$ and $\sigma_i(A \mid t_i, h^{i-1}) = 0$ if $s_{i-1} = B$. It should be noted that, in such a case, a player's prediction do not reveal any information. Therefore, we have the following proposition.[7]

Proposition 2 *If agents $i \leq n$, where $n \in N$ is an arbitrary integer, are endowed with the non-confident tie-breaking rule, no informational cascade occurs before period n.*

Proposition 2 implies that if agent $n + 1$, endowed with AH's tie-breaking rule, observes a long sequence of identical actions he can *rationally* follow his private signal and take a different action.

3.3 The general case

We now show that if no tie-breaking convention is assumed any sequence of actions can be observed. For this to be true, we only need to guarantee that, for a particular subset of agents $N^* \subseteq N$, the labels of signals and actions coincide.[8] For example, the sequence of actions AAB is an outcome of a perfect Bayesian equilibrium if agent 1's signal is a_1 and agent 3's signal is b_3. In addition, agent 2 has to be endowed with

[6] Anderson and Holt argued that "This assumption is reasonable when there is a positive probability that the [previous] person makes an error [...] This assumption is also supported by an econometric analysis of the error rates" (Anderson and Holt, 1997 p. 849).

[7] If each player $i \leq n$, where $n \in N$ is an arbitrary integer, is endowed with the non-confident tie-breaking rule then equilibrium beliefs and behavioral strategies of agents $i \leq n$ are unique.

[8] Of course, agents who are indifferent have to be endowed with a tie-breaking rule.

the non-confident tie-breaking rule, whereas agent 3 has to be endowed with the AH's tie-breaking rule. It should be noted that this last requirement only ensures that a perfect Bayesian equilibrium is completely defined, and should not be seen as a weakness of our approach. As in proposition 3 we show that any observed sequence of actions is the outcome of a particular perfect Bayesian equilibrium, endowing agents who are indifferent with the appropriate tie-breaking rule is not demanding.

Proposition 3 *Let* $(s_i)_{i\in N} \in \prod_{i\in N} S$ *be an arbitrary sequence of actions. If each agent* $i \in N^*$ *receives a signal which matches the label of his action, then* $(s_i)_{i\in N}$ *is an outcome of a perfect Bayesian equilibrium.*

Propositions 2 and 3 also apply when the set of agents is finite. Therefore, our findings have implications for the analysis of experimental data on informational cascades as shown more in details in the next section.

4 Reinterpretation of experimental data

In their first two experiments, Anderson and Holt (1997) considered a symmetric design related to the framework described in section 2, in which $q = 2/3$, a correct prediction yields \$2, and a wrong prediction yields \$0. Moreover, the game was repeated fifteen times for each group of six subjects with a new die throw to select the urn at the beginning of each repetition.[9] Anderson and Holt (1997) concluded that information cascades are not due to irrational behavior, or caused by a taste for conformity, but are rather due to rational inference of previous choices. Nevertheless, some inconsistencies with Bayesian updating were observed: "Individuals generally used information efficiently and followed the decisions of others when it was rational. There were, however, some errors, which tended to make subjects rely more on their own private information" (Anderson and Holt, 1997 p. 859). Table 1 shows some of Anderson and Holt's first experiment data (session 2).

		Urn decision (private draw)					
Period	Urn used	1st round	2nd round	3rd round	4th round	5th round	6th round
7	B	B (b)	A (a)	B (b)	B (b)	B (b)	B (a)
8	A	A (a)	A (a)	B (b)	A (a)	A (b)	A (a)

Table 1: Example of Anderson and Holt's experimental results.

Anderson and Holt interpreted the 6th decision in period 7 and the 5th decision in period 8 as Bayesian decisions inconsistent with private information. In this respect, they consider that cascade behavior was observed in these periods. On the contrary, they interpret the 3rd decision in period 8 as a decision based on private information, inconsistent with Bayesian updating. These interpretations are correct under either BHW's or AH's tie-breaking convention. For example, in period 7, since the first two predictions canceled each other out and the three following predictions create

[9] Public draws were introduced into the decision sequence in the second experiment.

an imbalance that can dominate the information contained in a single private draw, the last prediction is consistent with Bayes' rule but inconsistent with the subject's private draw.

New interpretations can be drawn from Anderson and Holt's data by considering the non-confident tie-breaking rule. The 6th prediction observed in period 7 may be due to the 6th subject's belief that the 4th and 5th subjects used the non-confident tie-breaking rule and the 6th subject's use of the non-confident tie-breaking rule. Similarly, the 3rd decision in period 8 should not be seen as inconsistent with Bayesian updating and the 5th prediction does not necessarily mean the birth of a cascade. More generally, any observed sequence of predictions including no decision which is inconsistent with both Bayes' rule and private information can be considered as an outcome of a perfect Bayesian equilibrium.[10] Nevertheless, decisions inconsistent with private information should not necessarily be considered as part of an informational cascade.

The reinterpretation of data taken from Anderson and Holt (1997) emphasizes the importance of subjects' beliefs on others' tie-breaking rules in laboratory experiments on informational cascades. Indeed, one can equally reinterpret Willinger and Ziegelmeyer's (1998) experimental data, and Hung and Plott's (2000) experimental data produced under the 'individualistic institution'. The overconfidence of subjects in their private information, due to, e.g., possible errors in previous decisions, has been mentioned as a plausible source of inconsistencies with Bayesian updating. Needless to say, such an explanation is unnecessary with the non-confident tie-breaking rule.

5 Conclusion

In this note we have emphasized the need for a particular tie-breaking convention in order to get informational cascades in the specific model taken from Bikhchandani, Hirshleifer, and Welch (1992). Indeed, if no homogeneous tie-breaking rule is specified any sequence of actions might be observed *at equilibrium*. Consequently, for many sequences of subjects' decisions observed in laboratory experiments on informational cascades (e.g., Anderson and Holt (1997), Willinger and Ziegelmeyer (1998), and Hung and Plott (2000)), no unequivocal interpretation can be given.

Anderson and Holt reported that, in their symmetric design, decisions were consistent with private information in most of the cases in which the posterior probability of each urn was equal to 1/2.[11] This remark, which supports AH's tie-breaking rule, does in no way imply that such a rule is common knowledge.

Fortunately, indifference cases and the need for a particular tie-breaking convention in order to get a unique prediction and to interpret unequivocally experimental data on informational cascades can be avoided. Indeed, breaking either the symmetry of common priors, of signals' precisions, or of payoffs functions rules out indifference

[10] Anderson and Holt reported that, in their symmetric design, only about 4 percent of such decisions were observed.

[11] We computed the proportion of decisions in accordance with AH's tie-breaking rule, by considering all indifference cases which did not rely on the specification of a particular tie-breaking rule. The following results were obtained: 83%, 75%, and 86% of decisions were in accordance with AH's tie-breaking rule respectively in Anderson and Holt's (1997) symmetric design, Willinger and Ziegelmeyer's (1998) experiments, and Hung and Plott's (2000) 'individualistic institution' experiment.

cases. For example, Anderson and Holt (1997) reported six additional sessions using an asymmetric design in which urns A and B contained different proportions of 'type a' and 'type b' balls. In this asymmetric setup, no tie-breaking convention has to be assumed because indifference situations cannot emerge. One can also rely on BHW's general model with more than two possible states and signals.[12]

Appendix: Proofs

Proof of proposition 1. The sequences AAB and BBA cannot be observed under either the BHW or the AH tie-breaking convention. More generally, by denoting n_A (respectively n_B) the number of A (respectively B) actions in a history h^{i-1}, agent i starts a cascade if $|n_A - n_B| = 2$. Consider an agent $i \in N$, $i \geq 2$. Let $h^{i-1} \in \mathcal{H}^{i-1}$ be a history of length $i-1$ observed by agent i. From equilibrium conditions and the AH's tie-breaking convention, agent i's belief verifies

$$\begin{array}{ll}
\mu_i(a_i, h^{i-1}) > 1/2, & \text{if } n_A - n_B \geq 0, \\
\mu_i(a_i, h^{i-1}) = 1/2, & \text{if } n_A - n_B = -1, \\
\mu_i(a_i, h^{i-1}) < 1/2, & \text{if } n_A - n_B \leq -2, \\
\mu_i(b_i, h^{i-1}) < 1/2, & \text{if } n_A - n_B \leq 0, \\
\mu_i(b_i, h^{i-1}) = 1/2, & \text{if } n_A - n_B = 1, \\
\mu_i(b_i, h^{i-1}) > 1/2, & \text{if } n_A - n_B \geq 2.
\end{array}$$

Therefore, if $|n_A - n_B| \geq 2$, then $\mu_i(t_i, h^{i-1}) > 1/2$ for all $t_i \in T_i$, which implies that either an information cascade occurs in period i, or has already occurred. Thus, three cases remain: either $n_A = n_B$, or $|n_A - n_B| = 1$.

(i) Suppose that $t_i = a_i$.

(ia) If $n_A = n_B$, then $\mu_i(a_i, h^{i-1}) > 1/2$ and $\sigma_i(A \mid a_i, h^{i-1}) = 1$.

– If $t_{i+1} = a_{i+1}$, an informational cascade occurs in period $i+2$.

– If $t_{i+1} = b_{i+1}$, then we apply argument (iib) to player $i+1$.

(ib) If $n_A - n_B = -1$, then $\mu_i(a_i, h^{i-1}) = 1/2$ and $\sigma_i(A \mid a_i, h^{i-1}) = 1/2$. If $s_i = A$, then we apply argument (ia) if $t_{i+1} = a_{i+1}$ and (iia) if $t_{i+1} = b_{i+1}$ for agent $i+1$. If $s_i = B$, then an informational cascade occurs in period $i+1$.

(ic) If $n_A - n_B = 1$ then $\mu_i(a_i, h^{i-1}) > 1/2$, $\sigma_i(A \mid a_i, h^{i-1}) = 1$, and an informational cascade occurs in period $i+1$.

(ii) Suppose that $t_i = b_i$.

(iia) If $n_A = n_B$, then $\mu_i(b_i, h^{i-1}) < 1/2$ and $\sigma_i(B \mid a_i, h^{i-1}) = 1$.

– If $t_{i+1} = a_{i+1}$, then we apply argument (ib) to player $i+1$.

– If $t_{i+1} = b_{i+1}$, an informational cascade occurs in period $i+2$.

(iib) If $n_A - n_B = 1$, then $\mu_i(b_i, h^{i-1}) = 1/2$ and $\sigma_i(B \mid a_i, h^{i-1}) = 1/2$. If $s_i = B$, then we apply argument (ia) if $t_{i+1} = a_{i+1}$ and (iia) if $t_{i+1} = b_{i+1}$ for agent $i+1$. If $s_i = A$, then an informational cascade occurs in period $i+1$.

(iic) If $n_A - n_B = -1$ then $\mu_i(b_i, h^{i-1}) < 1/2$, $\sigma_i(B \mid a_i, h^{i-1}) = 1$, and an informational cascade occurs in period $i+1$.

[12] Though the general model of BHW generically has a unique pure-strategy equilibrium, some indifference situations can still occur with particular parameters. For example, assume that there are four states of the world (possible values of adoption) $\{v_1, v_2, v_3, v_4\} = \{7.5, -1.5, -3.5, -5.5\}$ with $p(v_i) = 1/4$ for all $i \in N$ and four signals $\{x_1, x_2, x_3, x_4\}$ such that $p(x_i \mid v_i) = 2/3$ and $p(x_i \mid v_j) = 1/9$ for all $i \neq j$. If the cost of adoption is equal to $C = 1/2$, then agent 2 is indifferent between adoption and rejection given that he observed a first adoption and received signal x_4.

For each player $i \geq 2$, there is a positive probability, independent of i, that an informational cascade occurs in period i, $i+1$ or $i+2$. Thus, the probability that an informational cascade eventually occurs approaches one as the number of periods increases. A similar proof applies with BHW's tie-breaking convention. ■

Proof of proposition 2. If $t_1 = a_1$, then $\mu_2(t_2, A) \geq 1/2$ for any $t_2 \in T_2$. Thus, $\sigma_2(A \mid t_2, A) = 1$ for any $t_2 \in T_2$, i.e., player 2 plays A whatever his signal. In the same manner, we get $\mu_3(t_3, A, A) \geq 1/2$ and $\sigma_3(A \mid t_3, A, A) = 1$ for any $t_3 \in T_3$, and so on. Therefore, for any $i \leq n$, we have $\mu_i(b_i, A, A, \dots) = 1/2$. Similarly, if $t_1 = b_1$, we get $\mu_i(a_i, B, B, \dots) = 1/2$ for all $i \leq n$. This completes the proof. ■

Proof of proposition 3. The proof, which is shown below, is constructive. For an arbitrary sequence of actions $(s_i)_{i \in N} \in \prod_{i \in N} S$, let n_1 be the number of consecutive actions A after history h^0, n_2 the number of consecutive actions B after history h^{n_1}, n_3 the number of consecutive actions A after history $h^{n_1+n_2}$, and so on. Without loss of generality, we assume that $n_1 \neq 0$. Let $N_{AH} = \{1+n_1, 1+n_1+n_2, \dots\}$ be the set of agents who do not follow the action taken by the agent just before them and $N_S = \{1, 2+n_1, 2+n_1+n_2, \dots\}$ be the first agent and all agents deciding just after agents of N_{AH}. We define $N^* = N_{AH} \cup N_S$. Let $N_1 = \{1, \dots, n_1\}$, $N_2 = \{1+n_1, \dots, n_1+n_2\} = \{1, \dots, n_1+n_2\} \backslash N_1$, $\dots$, $N_k = \{1, \dots, n_1 + \dots + n_k\} \backslash (\bigcup_{j=1}^{k-1} N_j)$ for all $k \in N$, $k \geq 2$. Let $N_A = \bigcup_{k \geq 0} N_{2k+1}$ be the set of agents choosing action A and $N_B = \bigcup_{k \geq 1} N_{2k}$ be the set of agents choosing action B. We impose that for all $i \in N^*$, $t_i = a_i$ if $s_i = A$ and $t_i = b_i$ if $s_i = B$.

By endowing each agent $i \in N_{AH}$ with AH's tie-breaking rule and each agent $i \in N \backslash N^*$ with the non-confident tie-breaking rule, agents' beliefs are the following ones:[13] if $i \in (N_A \backslash N^*) \cup (N_B \cap N_{AH} \backslash N_S)$ then $\mu_i(t_i, h^{i-1}) > 1/2$ if $t_i = a_i$ and $\mu_i(t_i, h^{i-1}) = 1/2$ if $t_i = b_i$; if $i \in (N_B \backslash N^*) \cup (N_A \cap N_{AH} \backslash N_S)$ then $\mu_i(t_i, h^{i-1}) = 1/2$ if $t_i = a_i$ and $\mu_i(t_i, h^{i-1}) < 1/2$ if $t_i = b_i$; if $i \in N_S$ then $\mu_i(t_i, h^{i-1}) > 1/2$ if $t_i = a_i$ and $\mu_i(t_i, h^{i-1}) < 1/2$ if $t_i = b_i$.

One can easily verify that all agents act rationally and according to their tie-breaking rules in case of indifference. This completes the proof. ■

References

Anderson, L., and C. Holt (1997): "Information Cascades in the Laboratory," *American Economic Review*, 87, 847–862.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1992): "A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades," *Journal of Political Economy*, 100, 992–1026.

Hung, A., and C. Plott (2000): "Information Cascades: Replication and an Extension to Majority Rule and Conformity Rewarding Institutions," *American Economic Review,* forthcoming.

Willinger, M., and A. Ziegelmeyer (1998): "Are More Informed Agents Able to Shatter Information Cascades in the Lab?," in P. Cohendet, P. Llerena, H. Stahn and G. Umbhauer (eds.), The Economics of Networks. Springer-Verlag.

[13] As each agent $i \in N_S$ is never indifferent, it is needless to endow him with a particular tie-breaking rule.