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Pollution Effects on Public Capital and Social Capital Accumulation

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Abstract

The interactions between the accumulation of public capital, the formation and environmental degradation are largely unexplored in the literature on growth. Each of these dimensions has received considerable attention in isolation, but no existing model treats them as state variables evolving jointly within a unified intertemporal framework. This paper fills this gap by developing a theoretical model, in which public capital, social capital and a stock of pollution accumulate simultaneously beside private capital. Social capital formation is endogenised via a meeting opportunity parameter linked to the availability of public infrastructure, whilst pollution appears as a by-product of productive activity, without entering into household utility. Firstly, the model admits a unique internal steady state. Secondly, a Hopf bifurcation occurs near this state, allowing the rise of a stable limit cycle and ensuring that the economy converges towards a regime of persistent endogenous oscillations. These results are established analytically and confirmed by numerical simulations.

Keywords: Social Capital, Public Capital, Hopf Bifurcation, Endogenous Depreciation

JEL Code: O41, Q58, C62, H41

1 Introduction

Since the late 1980s, the French Government has been gradually reducing its investment in rural and peri-urban areas (Ecalte, 2016). Examples include the closure of railway lines, the decline in local public services, and the deterioration (or privatisation) of the road network (Yilmazer, 2026). This withdrawal from infrastructure investment is causing the social ties to thin out, leading to relatively fewer interactions between people (Putnam, 2000; Gray et al., 2006). Consequently, networks of trust, which are key to cooperation, have become less robust. Thus, when in autumn 2018 the French Government increased taxes on petroleum products as part of the ecological and environmental transition, the message for low-income rural households was clear: they have no alternative to the car (no developed public transport service), and no neighbours with whom

to carpool (breakdown of social ties) (Labrousse and Perdereau, 2025). The mass protests that followed led to the measure being scrapped. However, it should be noted that the demonstrations against the tax did not arise from a lack of awareness among households of the ecological challenges of the 21st century (Levain et al., 2023). They emerged due to the absence of the public and social capital necessary for the measure to be properly implemented, as well as for it to be understood by the population (Ewald et al., 2022).

This episode in the history of French popular discontent is, however, not a political anomaly. Indeed, it can even be seen as revealing an unexplored theoretical space: standard models of environmental taxation (in this case, increasing the tax on polluting petroleum products to make them more expensive and reduce their use, and thus the pollution they generate) are constructed within economies where public capital, social capital and pollution are independent of one another. A first strand of the literature has extended endogenous growth models to incorporate public capital and pollution jointly, showing that the two interact through the production function and through optimal fiscal policy design (Fisher and Turnovsky, 1998; Greiner, 2005). Yet, these models, however sophisticated, retain the assumption that social capital plays no role in shaping households' responsiveness to the tax: pollution is treated as a technical externality to be corrected, and the erosion of public capital is not allowed to feed back into the erosion of social capital, nor into the price-elasticity of households' demand for the taxed good. In reality, we see that this is precisely where the paradox lies: it is the joint, prior withdrawal of public capital and the resulting thinning of social capital that leaves rural households capital to car use. The very mechanism the standard model assumes away (the interdependence between public capital, social capital and the elasticity of polluting behaviour) is what determines whether the tax is politically and economically viable in the first place. This article therefore seeks to formalise this intuition.

Governments devote a significant portion of their budgets to public investment in infrastructure (with subnational governments alone accounting for nearly 60% of total public investment across OECD countries on average (OECD, 2022)), the effects of which on growth and productivity have been documented since Barro (1990). He shows that, in an endogenous growth model, productive public expenditure can sustain long-term growth by offsetting the decreasing returns on private capital. Barro (1990) assumes, however, that public capital is a flow, i.e. that only the level of current public expenditure enters the production function, and not the stock of infrastructure. This would even be equivalent saying, from one period to the next, all public infrastructure is destroyed and must be rebuilt. In contrast, Futagami et al. (1993) drop the flow assumption, treating capital as a stock, and thus a state variable. The implications of this modelling choice are numerous. Firstly, it substantially alters the transition to the steady state, introducing an additional dynamic equation and, unlike Barro (1990), genuine transitional dynamics: the system now converges gradually to the balanced growth path rather than reaching it instantly. Secondly, Futagami et al. (1993) show that the long-run growth rate is qualitatively lower than that of Barro (1990). The refinement of Barro's (1990) model through the inclusion

of public capital as a stock allows for a more realistic transition path, with a gradual convergence of the system rather than a sudden and instantaneous adjustment. Numerous papers following Futagami et al. (1993), such as Miyamoto et al. (2020), have examined public capital as a stock: optimal financing, redistributive effects, interactions with human capital, etc. Thus, this body of literature recommends modelling public capital as a state variable in order to accurately capture its accumulation dynamics and long-term effects.

Beyond public capital, literature in endogenous growth has progressively diversified the set of factors as sources of growth in the long-run. Following Romer (1986) and Lucas (1988), human capital and knowledge were the first to be modelled as state variables besides physical capital, followed by institutional quality (Acemoglu et al., 2001), and more recently, social capital. This expansion reflects a broader recognition that growth depends not only on the accumulation of tangible factors of production, but also on intangible assets that shape how agents coordinate and interact with one another. Putnam (1993, 2000) shows that cooperation, enabled by the density of social networks and mutual trust, is strongly correlated with the economic performance of the regions studied (Italy and then the United States). According to Putnam (1993), social capital is the set of norms, networks and relationships of trust that facilitate cooperation between individuals. Thus, social capital appears at first glance to constitute a collective factor of production, which could accumulate and therefore erode. This factor of production could therefore have lasting effects on an economy's capacity for coordination. Prior to Putnam, Coleman (1988) had highlighted the importance of norms of reciprocity and informal sanctions in the production of public goods and the reduction of transaction costs. Thus, social capital as defined by Putnam (1990) is necessary for the production of public infrastructure and for maintaining it in good working order, through norms and self-restraint in the face of, for example, the temptation to cause deliberate damage. Empirically, Knack and Keefer (2005) show that there is a strong positive correlation between social capital and economic growth across several countries, whereas Algan and Cahuc (2010) show the causality. These studies have shown that social capital can be treated as an accumulative stock, with its own dynamics, which depend on the number of interactions between agents and on public policies. In particular, public infrastructure determines the frequency and quality of interactions between individuals. This is the approach we have adopted in this paper: the formation of social capital depends on opportunities for interactions between agents, which are themselves an increasing function of the level of public capital available in the economy.

Finally, the integration of pollution into various growth models constitutes another strand of the literature (Gradus and Smulders, 1993, for instance). Contributions in this field focus on the interactions between growth, environmental taxation and long-term well-being. These articles demonstrate an incompatibility between preserving environmental quality and a long-term growth objective (Smulders and Gradus, 1996 ; Elbasha and Roe, 1996). Fiscal instruments appear to be central to the internalisation of environmental externalities. A recurring conclusion is that modelling household preferences

is essential to the effectiveness of fiscal instruments. When pollution does not directly affect utility, public authorities have no incentive to regulate pollution (John and Pecchenino, 2017). We find the same paradoxical results in this article.

Although the literature is extensive in the disciplinary fields mentioned above, two gaps remain. On the one hand, there is no model that simultaneously combines social capital, public capital and pollution as state variables, all within a Ramsey-style framework. These three concepts are systematically treated in pairs, but not as a whole. Secondly, public capital, which is clearly distinct from private physical capital and is financed solely through taxation, is rarely addressed in the literature, even though it seems obvious that it should be included.

Our paper seeks to address these two gaps by answering the following question: how does the inclusion of public capital alter the structural dynamics of a Ramsey-style economy incorporating social capital and a pollution stock, within a framework where households derive no disutility from pollution?

To this end, we have constructed a theoretical model containing three state variables: social capital, pollution and public capital; and one control variable, labour time (which allows for trade-offs between consumption and social networks). The system is therefore four-dimensional. Drawing on the methodology of Bosi and Desmarchelier (2019), we show that the unique steady state undergoes a Hopf bifurcation, giving rise to a stable limit cycle. This result ensures that the economy converges towards a regime of persistent endogenous oscillations, with pollution alternately accumulating and declining over the cycle.

The paper is organised as follows. Section 2 focuses on describing the behaviour of the agents. Section 3 sets out the existence and uniqueness conditions for the steady state, as well as public policy recommendations. Section 4 explores the stability of the steady state. Section 5 concludes and suggests avenues for future research. All proofs are gathered in the Appendix.

2 The Model

2.1 The Representative Household

Each household shares one unit of time between time spent at work z and time spent socializing $(1 - z)$. The effort they make in order to socialize allows households to accumulate social capital, s , which follows a linear process, in the spirit of Glaeser et al. (2002):

$$\dot{s} = B(1 - z) - \delta s \quad (1)$$

B represents households' ability to socialize, which can also be interpreted as opportunities they have to meet. Moreover, δ represents social capital's depreciation. Indeed, if households don't make an effort to take care of their social capital, it will depreciate.

Social capital is here assumed to generate market returns. To some extent, one can consider that workers are more efficient when they work with people they know (Mas and Moretti, 2009; Bandiera et al., 2009). A network effect can also be considered here. Indeed, an individual may be hired precisely for their network, or their address book, so that the firm itself can benefit from these connections. Social capital thus becomes an argument in favour of an individual's employability, which in turn gives individuals an incentive to develop their network, at least *a priori*.

Efficient labour is then denoted $l = zs$. We assume, in line with this reasoning, that social capital doesn't generate direct utility for the households¹.

To keep things as simple as possible, we assume that there is no physical capital in our economy. Thus, there are no savings. The whole wage w is then allocated to consumption c :

$$c = wl \quad (2)$$

Households' preferences are represented by a simple utility function $u(c)$. The next assumption sums up its properties.

Assumption 1. $u_{cc} < 0 < u_c$ and Inada's conditions hold.

Let us introduce a second-order elasticity, which represent the usual elasticity of intertemporal substitution in consumption:

$$\varepsilon_{cc} = -\frac{u_c}{cu_{cc}} > 0$$

Households choose the path of time allocated to labour in order to maximize their intertemporal utility $\int_0^{+\infty} e^{-\theta t} u(c) dt$, under the constraint of social capital accumulation (1) and (2), where $\theta > 0$ represents the rate of time preference. The Hamiltonian writes $H = u(c) + \lambda(B(1-z) - \delta s)$, with λ the Lagrangian multiplier. First order conditions

$$\begin{aligned} \frac{\partial H}{\partial z} &= wsu_c - \lambda B = 0 \\ \frac{\partial H}{\partial s} &= wz u_c - \delta \lambda = \theta \lambda - \dot{\lambda} \\ \frac{\partial H}{\partial \lambda} &= \dot{s} \end{aligned}$$

give the Euler equation for time spent at work :

$$\dot{z} = -\varepsilon_{cc} \left((\delta + \theta) - B \frac{z}{s} - \left(\frac{\dot{w}}{w} + \frac{\dot{s}}{s} \right) \frac{\varepsilon_{cc} - 1}{\varepsilon_{cc}} \right) z \quad (3)$$

¹The reader interested in a model where social capital affects utility is referred to Campeniet et al. (2026).

2.2 The Representative Firm

As said previously, we assume there is no physical capital. Consumption good is then produced only thanks to labour, in quantity Y :

$$Y = AL$$

with $A > 0$ the labour productivity, and L the labour demand.

Let us assume that production generates pollution. In accordance with Pigouvian principles, the Government introduces a tax $\tau \in (0, 1)$, proportionally to the production level. The profit is given, as usual:

$$\Pi = (1 - \tau)Y - wL$$

As the firm behaves competitively by choosing the amount of labour L maximizing the profit, we can use the zero-profit condition, thanks to the linearity of production function:

$$w = (1 - \tau)A$$

2.3 Pollution and Government

Pollution is assumed to be a stock. Its accumulation is often depicted as linear (Michel and Rotillon, 1995; Stern, 2002). The only source of pollution is production². Pollution stock evolves according to:

$$\dot{P} = -aP + bY$$

where $a > 0$ is the natural absorption rate, and $b > 0$ is the environmental impact of production.

Moreover, public capital M also accumulates, following a linear process (Futagami et al., 1993):

$$\dot{M} = \beta G - \rho M$$

with G represents public spendings. $\beta > 0$ then represents the productive efficiency of public expenditure, and then the rate at which Government's spendings actually converts to public capital (one unit of public spending generates β units of public capital). At the same time, $\rho > 0$ is the depreciation rate of public capital, through physical deterioration, obsolescence or insufficient maintenance.

The specification of public capital's role deserves a particular attention, as it distinguishes from Barro (1990) and Futagami et al. (1993). In their contributions, public capital is introduced as a productive input, which enters the production function, alongside physical (private) capital and labour (Arrow and

²The interested reader may refer to Campenot and Desmarchelier (2025) for a related model featuring two distinct sources of pollution, arising from both production and consumption, and a detailed discussion of the inertia conditions required for the stability of the steady state.

Kurz, 1970). This modelling choice has become the benchmark in literature linking endogenous growth and public capital, such as Glomm and Ravikumar (1994): public capital is generally treated as a complementary factor of production raising marginal productivities of other factors, ensuring long-run growth.

In our model, public capital does not enter the production function. This departure from the standard formulation is central to our contribution. Public capital is instead assumed to affect both the formation and depreciation of social capital.

Indeed, one might assume that households are better able to interact when there is more public capital (Putnam, 2000). Indeed, examples of public capital include parks, streets and sport facilities, and according to Durlauf and Fafchamps (2005) and Combes et al. (2008), local public goods appear as coordination mechanisms. These are all forms of public infrastructure that enable interactions between individuals (or facilitate it, in the case of roads). Thus, public infrastructure increases the likelihood of households meeting, which ultimately promotes the formation of social capital. We can then assume a certain degree of complementarity between public infrastructure and the formation of social capital, the intensity of which is measured by $\sigma_B > 0$.

Assumption 2. $B \equiv B(M)$ such that $B'(M) > 0$.

Additionally, the depreciation of social capital may also depend on the level of public capital. Indeed, social capital naturally erodes through distance, a lack of contact or mistrust. Public capital can therefore serve as a mechanism for maintaining social ties. Thus public capital acts as a catalyst for reactivating social ties: well-maintained parks, efficient public transport and accessible urban facilities are places where households can meet and, consequently, rekindle their bonds (Putnam et al. 1994 ; Putnam, 1995 ; Bowles and Gintis, 2002). The concept differs slightly from the creation of new social capital, as explained above, since here we are concerned with the maintenance of existing social capital. Furthermore, in the absence of public capital, social ties rapidly decrepit, a phenomenon observable in contexts of massive urban decay (Storper and Venables, 2004 ; Acemoglu and Dell, 2010). Conversely, highly developed public capital can lead to the near-elimination of the depreciation of social capital. The sensitivity of social capital depreciation to public capital is measured by the elasticity $\sigma_\delta < 0$.

Assumption 3. $\delta \equiv \delta(M)$ such that $\delta'(M) < 0$.

Finally, pollution damages public facilities, through the corrosion of structures or the deterioration of green spaces. Thus, pollution reduces the effective use of public capital. A polluted park tends to attract fewer visitors, just as a neglected public space fosters a sense of insecurity and encourages social withdrawal (Kahn, 2000 ; Heckbert et al., 2010 ; Banzhaf and Walsh, 2008). In this way, we can endogenise ρ , assuming that the higher the pollution, the greater the rate of erosion of public capital. When pollution is very high, public infrastructure can be completely destroyed. This rate of depreciation is represented by the parameter $\sigma_\rho > 0$.

Assumption 4. $\rho \equiv \rho(P)$ such that $\rho'(P) > 0$.

Assumptions 2, 3 and 4 imply respectively $\sigma_B \equiv \frac{MB'(M)}{B(M)} > 0$, $\sigma_\delta \equiv \frac{M\delta'(M)}{\delta(M)} < 0$, $\sigma_\rho \equiv \frac{P\rho'(P)}{\rho(P)} > 0$.

3 Equilibrium

3.1 Dynamic System

At the equilibrium, all markets clear.

By denoting $N > 0$ the population's size, total labour demand equals total labour supply: $Nl = L$. Let us assume that population size is constant over time ($\dot{N} = 0$), and equal to unity ($N = 1$), in order to simplify the understanding. We then conclude that $L = l$.

Finally, on the goods market, at the equilibrium, total supply Y equals total demand (private and public) $Nc + G$. Hence, we can write, thanks to (2):

$$c = (1 - \tau) Azs$$

Finally, we can write the dynamic system, which represents the evolution of the economic equilibrium:

$$f_1(z, s, P, M) \equiv \dot{z} = -\varepsilon_{cc} \left((\delta(M) + \theta) - B(M) \frac{z}{s} - \left(\frac{\dot{w}}{w} + \frac{\dot{s}}{s} \right) \frac{\varepsilon_{cc} - 1}{\varepsilon_{cc}} \right) \quad (4)$$

$$f_2(z, s, P, M) \equiv \dot{s} = B(M) (1 - z) - \delta(M)s \quad (5)$$

$$f_3(z, s, P, M) \equiv \dot{P} = -aP + bAzs \quad (6)$$

$$f_4(z, s, P, M) \equiv \dot{M} = \beta\tau Azs - \rho(P)M \quad (7)$$

This system includes one control variable (z), and three predetermined state variables (s, P, M).

3.2 Steady State and Comparative Statics

In order to facilitate the exploration of the possible existence and unicity of a steady state, we will, for further sections, consider the following functional form:

$$\rho(P) = \kappa P \quad (8)$$

with $\kappa > 0$.

A steady state for this economy is a quadruplet (z^*, s^*, P^*, M^*) such that $\dot{z} = \dot{s} = \dot{P} = \dot{M} = 0$, and $z^* \in (0, 1)$, $s^* > 0$, $P^* > 0$, $M^* > 0$.

(6), (7), and (8) give:

$$P = \frac{b}{a} Azs \quad (9)$$

$$M^* = \tau \frac{a\beta}{\kappa b} \quad (10)$$

According to (10), M^* is fully parametric, and hence unique.
At the same time, (4), (5) and (8) give:

$$s^* = \frac{B(M^*)}{2\delta(M^*) + \theta} \quad (11)$$

$$z^* = \frac{\delta(M^*) + \theta}{2\delta(M^*) + \theta} < 1 \quad (12)$$

It implies that the steady state is also unique, and then:

$$P^* = \frac{b}{a} A z^* s^* \quad (13)$$

Previous discussion leads to the following proposition.

Proposition 1 *Consider functional form (8). There always exists a unique positive steady state for the economy, given by (z^*, s^*, P^*, M^*) .*

We can furthermore explore the way the tax rate affects main variables at the steady state.

Proposition 2 *At the steady state, the tax rate affects the model's variables as follows:*

$$\begin{aligned} \frac{\tau}{M} \frac{\partial M}{\partial \tau} &= 1 \\ \frac{\tau}{s} \frac{\partial s}{\partial \tau} &= \sigma_B - \frac{2\delta(M^*)}{2\delta(M^*) + \theta} \sigma_\delta > 0 \\ \frac{\tau}{z} \frac{\partial z}{\partial \tau} &= -\frac{\theta\delta(M^*)}{(\delta(M^*) + \theta)(2\delta(M^*) + \theta)} \sigma_\delta > 0 \\ \frac{\tau}{P} \frac{\partial P}{\partial \tau} &= \frac{\tau}{z} \frac{\partial z}{\partial \tau} + \frac{\tau}{s} \frac{\partial s}{\partial \tau} > 0 \\ \frac{\tau}{c} \frac{\partial c}{\partial \tau} &= -\frac{\tau}{1-\tau} + \Omega \end{aligned}$$

with $\Omega \equiv \sigma_B - \sigma_\delta \frac{(3\theta+2\delta)\delta}{(\theta+2\delta)(\theta+\delta)} > 0$.

Moreover, $\frac{\tau}{c} \frac{\partial c}{\partial \tau} > 0 \Leftrightarrow \tau < \tau^* \equiv \frac{\Omega}{1+\Omega} < 1$. If $\sigma_B = \sigma_\delta = 0$, then $\Omega = 0$, $\tau^* = 0$ and $\frac{\tau}{c} \frac{\partial c}{\partial \tau} = -\frac{\tau}{1-\tau} < 0$ for all $\tau > 0$.

A change in the tax rate is found to generate a strictly proportional change in the stock of public capital. The entirety of the tax revenue is allocated to this single item of expenditure, implying the government does not trade off, and consequently use the whole additional revenue to its sole category of spending.

Moreover, social capital rises when the tax rate does. The mechanical increase in public capital causes social capital to accumulate faster ($\sigma_B > 0$), and

to depreciate slower ($\sigma_\delta < 0$). Both effects jointly raise the steady-state level of social capital, as the tax increases.

The intuition for z operates through a distinct channel from that of s . The rise in s draws on two channels: a higher accumulation rate (thanks to σ_B) and a slower depreciation (thanks to σ_δ). The response of time allocated to labour relies only on the second channel. The tax mechanically raises public capital. This increase in public capital still lower the depreciation rate of social capital $\delta(M)$. Households therefore need less efforts to offset this depreciation and to maintain a given level of social capital. They then reallocate this time towards labour, increasing z .

Finally, the long-run level of pollution depends positively on the level of working time and on social capital, in a linear way. Consequently, as the tax rate rises, the level of pollution increases. This follows from the fact that tax revenue is allocated entirely to investment in public capital, and not at all to pollution abatement.

For consumption, the effect is ambiguous. We can solve the ambiguity with the threshold τ , obtained below. A rise in τ mechanically reduces the households' income, but also stimulates production through Ω . As long as $\tau < \tau^*$, the productive effect dominates the income effect, and c increases with τ . Beyond τ^* , the rationale is the opposite. c , as a function of τ , is then bell-shaped, peaking at τ^* .

τ^* is the tax rate that maximizes consumption at the steady state. Since pollution does not enter the utility function, τ^* maximizes the household welfare in this economy. It then is a second-best optimum. The first-best optimum would be Pigouvian equating marginal cost of abatement to the marginal environmental damage, which does not exist in our model; but τ^* solves $\varepsilon_{c,\tau} = 0$, which only trades off the income effect, against the production effect.

4 Dynamics

4.1 Generalities

We have shown in previous sections that there always exists a unique steady state in the economy. We now focus on the dynamic around the steady state, following Bosi and Desmarchelier (2019).

In order to keep things as simple as possible, let us specify the utility function:

$$u(c) = \frac{c^{1-\varepsilon}}{1-\varepsilon}$$

It implies:

$$\varepsilon_{cc} = \frac{1}{\varepsilon} > 0$$

Let us compute the Jacobian matrix:

$$J^* = \begin{pmatrix} \Phi & -\varepsilon \frac{\Phi}{B(M^*)} (\theta - \delta(M^*) (\frac{1}{\varepsilon} - 2)) & 0 & \varepsilon (\theta \sigma_B - \frac{1-2\varepsilon}{\varepsilon} \Psi) \frac{z^*}{M^*} \\ -B(M^*) & -\delta(M^*) & 0 & \Psi \frac{s^*}{M^*} \\ a \frac{P^*}{z^*} & a \frac{P^*}{s^*} & -a & 0 \\ \kappa M^* \frac{P^*}{z^*} & \kappa M^* \frac{P^*}{s^*} & -\kappa M^* & -\kappa P^* \end{pmatrix}$$

with $\Psi \equiv \delta(M^*) (\sigma_B - \sigma_\delta) > 0$ and $\Phi \equiv (\theta + \delta(M^*)) > 0$.

We then compute the minors of order one (trace, T), two (S_2), three (S_3) and four (determinant, D):

$$\begin{aligned} T &= \Upsilon_0 \\ S_2 &= \Upsilon_1 \varepsilon + \Upsilon_2 \\ S_3 &= \varepsilon \Upsilon_3 + \Upsilon_4 \\ D &= \Upsilon_5 \varepsilon < 0 \end{aligned}$$

with:

$$\begin{aligned} \Upsilon_0 &= -(a - \theta + \kappa P^*) \\ \Upsilon_1 &= -\Phi \Lambda < 0 \\ \Upsilon_2 &= -a\theta + P^* \kappa (a + 3\Psi + \theta (\sigma_B - 1)) \\ \Upsilon_3 &= (a\Phi \Lambda + P^* \kappa (-2\Psi\theta + \Phi \Lambda - \theta^2 \sigma_B)) < 0 \\ \Upsilon_4 &= -P^* \theta \kappa (a + \Psi) < 0 \\ \Upsilon_5 &= -P^* a \kappa \Phi \Lambda < 0 \\ \Lambda &\equiv (\theta + 2\delta(M^*)) > 0 \end{aligned}$$

Proposition 3 *There is no room for local indeterminacy.*

We can now study the occurrence of a Hopf bifurcation, whereby the stable eigenvalues become a pair of purely imaginary numbers, giving rise to deterministic cycles in the neighbourhood of the steady state. To investigate this possibility, we study under which conditions on ε a Hopf bifurcation may arise. Following Bosi and Desmarchelier (2019, Proposition 16), a Hopf bifurcation arises if and only if $S_2 = \frac{S_3}{T} + \frac{T}{S_3} D$ and $\frac{S_3}{T} > 0$. The Lemma follows.

Lemma 4 *There exists a second-degree polynomial in ε arising from the Hopf bifurcation condition stated above. Provided that $\Upsilon_3^2 - \Upsilon_0 \Upsilon_1 \Upsilon_3 \neq 0$ and $\Delta = \Upsilon_0^2 \Upsilon_5^2 + \Upsilon_1^2 \Upsilon_4^2 + \Upsilon_2^2 \Upsilon_3^2 + 4\Upsilon_3 \Upsilon_4 \Upsilon_5 - 2\Upsilon_0 \Upsilon_1 \Upsilon_4 \Upsilon_5 - 2\Upsilon_0 \Upsilon_2 \Upsilon_3 \Upsilon_5 - 2\Upsilon_1 \Upsilon_2 \Upsilon_3 \Upsilon_4$ is non-negative, (14) admits two real conjugate roots:*

$$\varepsilon^\pm = \frac{1}{2 \Upsilon_3^2 - \Upsilon_0 \Upsilon_1 \Upsilon_3} \left(\pm \Upsilon_0 \sqrt{\Delta} - 2\Upsilon_3 \Upsilon_4 - \Upsilon_0^2 \Upsilon_5 + \Upsilon_0 \Upsilon_1 \Upsilon_4 + \Upsilon_0 \Upsilon_2 \Upsilon_3 \right)$$

Each root $\varepsilon^\pm$ is thus a candidate value at which the condition (15) underlying a Hopf bifurcation is met. Since these values are not clearly real nor

positive, a calibration procedure is required to best illustrate. Indeed, any parameter configuration admitting a Hopf bifurcation belongs to an open set of such configurations. One may therefore conclude that a Hopf bifurcation occurs in the economy, generating a limit cycle in the neighbourhood of the steady state.

Consider an exogenous increase in pollution, P . Since $\kappa > 0$, the depreciation rate of public capital, κP , rises. Given that public capital can be written as $M = \frac{\beta\tau}{\kappa P} A z s$, the stock of public capital M falls, *ceteris paribus*. This has two effects : (i) fewer opportunities to socialize, since $B(M)$ decreases, (ii) faster erosion of social capital, since the depreciation rate of social capital, $\delta(M)$ increases. Taken together, these effects reduce the stock of social capital s . Labour time, z , also falls, as it depends on $\delta(M)$. Since P depends positively and linearly on both social capital and labour time, the fall in z and s reduces pollution. The initial rise in P then triggers a fall in P , which imply a negative feedback loop, implying, intuitively, a cycle.

This negative feedback loop described suggests a natural candidate for endogenous fluctuations. A plausible scenario is that the steady state loses its saddle-path stability through a Hopf bifurcation, at which a pair of complex conjugate eigenvalues crosses the imaginary axis. In the next section, we therefore compute the eigenvalues of the linearised system for candidate values of ε , thanks to numerical simulation.

4.2 Simulations

To illustrate and complement the preceding theoretical analysis, we propose a numerical simulation. Indeed, while the analytical study allows us to establish the theoretical conditions for the existence of a Hopf bifurcation, it does not show the either qualitative behaviour at the neighbourhood of the critical point or the stability of the limit cycle. Numerical simulation thus constitutes a useful tool, allowing us to visualize the emergence of the limit cycle as the bifurcation parameter crosses its critical value. At the same time, this simulation allows us to observe whether trajectories converge toward this cycle (orbital stability) or diverge from it (instability), confirming the subcritical or supercritical nature of the bifurcation identified analytically.

In order to explicit every function in our system, let us use standard functional forms for $B(M)$ and $\delta(M)$:

$$\begin{aligned} B(M) &= B_M M^{\sigma_B} \\ \delta(M) &= \delta_M M^{\sigma_\delta} \end{aligned}$$

Consider the following calibration:

Parameter	θ	a	b	A	β	κ	B_M	δ_M	σ_B	σ_δ	τ
Value	0.18	0.6	0.44	6.97	0.87	0.18	0.48	0.3	1.75	-1.46	0.17

From (9),(10),(11) and (12), we get the following steady state:

Variable	z^*	s^*	P^*	M^*
Value	0.63078	0.85107	2.7439	1.1205

Implementing these values and our system in *MatLab*, and using *MatCont* package, we found, as expected, two values are possible for ε for a Hopf bifurcation to arise. These two values, satisfying (14), are:

$$\begin{aligned}\varepsilon_+ &= 0.96406 \\ \varepsilon_- &= 5.0683 \times 10^{-2}\end{aligned}$$

At ε_- , the eigenvalues of the Jacobian matrix are $(\lambda_1^-, \lambda_2^-, \lambda_3^-, \lambda_4^-)$ $= (0.22699, -0.10804, -0.22705, -0.80580)$, which are purely real eigenvalues, three of which are negative. As three eigenvalues are stable, matching the three predetermined variables s , M , P , the system remains saddle-path stable, with the unstable root corresponding to the single jump variable z , which adjusts instantaneously so as to place the economy on the unique convergent path. Since all four eigenvalues are real, convergence towards the steady state is strictly monotonic, with no oscillatory behaviour. The absence of a Hopf bifurcation is confirmed by the fact that ε_- does not satisfy condition (15), which is necessary for the emergence of limit cycles.

At ε_+ , the eigenvalues of the Jacobian matrix are $(\lambda_1^+, \lambda_2^+, \lambda_3^+, \lambda_4^+)$ $= (0.72455, -1.5385, -1.8934 \times 10^{-6} + 0.26812i, -1.8934 \times 10^{-6} - 0.26812i)$, which comprise one positive real root, one negative real root, and a pair of complex conjugate eigenvalues, whose real part is negative. The system therefore exhibits three stable roots, matching the three predetermined variables, so that saddle-path stability continues to hold. However, the presence of the complex pair implies that convergence towards the steady state is no longer monotonic, but takes the form of oscillations, with the predetermined variables spiralling towards the steady state before settling upon it. One can remark that the real part is numerically very close to zero, providing a numerical confirmation that ε_+ corresponds to a crossing of the imaginary axis. The occurrence of a Hopf bifurcation is confirmed by the fact that ε_+ satisfies condition (15), which is necessary for the emergence of limit cycles.

Moreover, calibration allowed the computation of the first Lyapunov coefficient by *MatCont*, which is $-6.782798 \times 10^{-3} < 0$. Its negativity implies that the Hopf bifurcation is supercritical, and the limit cycle is stable. It implies that the steady state loses its stability, and the system converges to a self-sustained periodic oscillation. This cycle is represented in the next figure.

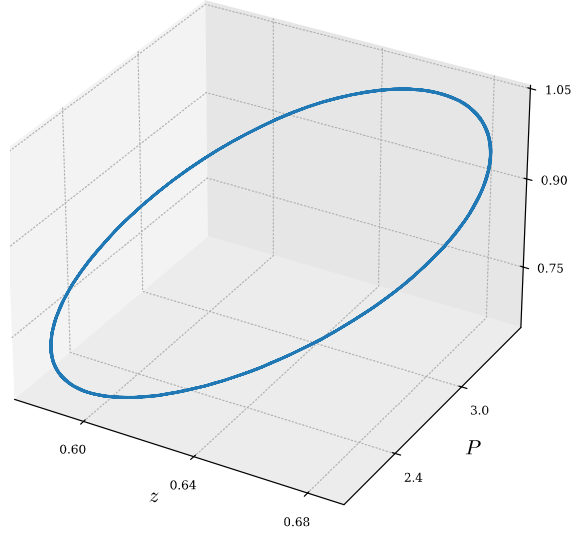


Fig. 1: Stable limit cycle around the steady state.

5 Conclusion

This paper examines how the inclusion of public capital alters the structural dynamics of an economy incorporating social capital and a pollution stock, within a framework where households derive no disutility from pollution nor utility from social capital.

The analysis reveals that the model allows a unique steady state. At the steady state, a higher tax rate raises public capital in strict proportion, since the entire revenue is devoted to this single item of expenditure, and the government faces no trade-off. The resulting increase in public capital accelerates the accumulation of social capital and slows its depreciation, so the steady-state level of social capital rises. The fall in the depreciation rate also means that households need to devote less effort to maintaining thier social capital, and they reallocate this time to labour. As pollution depends positively and linearly on those both variables, and as no revenue is allocated to abatement, it rises with the tax rate. The effect on consumption is ambiguous: a hugher tax rate mechanically reduces household income, but stimulates production. The productive effect dominates below a threshold, beyond which the income effect prevails, so that consumption's reaction to tax rate is bell-shaped. Since pollution does not enter the utility function, the tax rate at which consumption peaks maximises household welafare and consitutes a second-best optimum, given that the first-best (Pigouvian) optimum is absent from the model.

The dynamic study of the unique steady state reveals a Hopf bifurcation giving rise to a stable limit cycle. An exogenous rise in pollution accelerates the

depreciatio of public capital, which reduces the opportunities to socialize and increases the depreciation rate of social capital, implying a fall in social capital and in labour time. Since pollution depends positively on social capital and labour time, it subsequently declines. An endogenous cycle therefore emerges.

This work opens up several natural avenues for further research. The most straightforward approach is to incorporate pollution into the households' utility function. As mentioned earlier, this modification would make it possible to test whether the planner develops a genuine preference for environmental quality, and thus whether the fight against pollution becomes a fully-fledged objective of public policy. A second extension would concern the structure of social capital. The current framework is based on a linear dynamic, which smooths out the threshold and saturation effects that are nevertheless documented in the literature. Introducing quadratic effects would allow us to capture non-linear behaviour, such as the possibility that social capital, beyond a certain level of accumulation, generates diminishing returns or even negative effects on collective productivity.

6 Appendix

Proof of Proposition 2. Simply differentiate (10),(11),(12) and (13) with respect to τ .

Let us, for c , begin from the result established earlier, stating that public capital increases exactly one-for-one with τ at the steady state, $\frac{\tau}{M} \frac{\partial M}{\partial \tau} = 1$. Since $P = Azs$, with A constant, then $\frac{\tau}{P} \frac{\partial P}{\partial \tau} = \frac{\tau}{z} \frac{\partial z}{\partial \tau} + \frac{\tau}{s} \frac{\partial s}{\partial \tau}$.

Moreover, we can write $c = (1 - \tau)P$, which is $\ln c = \ln(1 - \tau) + \ln P$. Differentiating with respect to τ , $\frac{\tau}{c} \frac{\partial c}{\partial \tau} = -\frac{\tau}{1-\tau} + \frac{\tau}{P} \frac{\partial P}{\partial \tau}$. We can write $\frac{\tau}{P} \frac{\partial P}{\partial \tau}$ explicit, as

$$\frac{\tau}{P} \frac{\partial P}{\partial \tau} = \sigma_B - \left(\frac{\theta \delta(M^*)}{(\delta(M^*) + \theta)(2\delta(M^*) + \theta)} + \frac{2\delta(M^*)}{2\delta(M^*) + \theta} \right) \sigma_\delta = \sigma_B - \sigma_\delta \frac{(3\theta + 2\delta(M^*))\delta(M^*)}{(\theta + 2\delta(M^*))(\theta + \delta(M^*))} = \Omega.$$

Then, $\frac{\tau}{c} \frac{\partial c}{\partial \tau} = -\frac{\tau}{1-\tau} + \Omega$.

The sign of $\frac{\tau}{c} \frac{\partial c}{\partial \tau} > 0$ follows : $\Omega > \frac{\tau}{1-\tau} \Leftrightarrow \tau < \frac{\Omega}{1+\Omega} \equiv \tau^* < 1$.

Finally, if $\sigma_B = \sigma_\delta = 0$, then $\Omega = 0$, then $\tau^* = 0$ and $\frac{\tau}{c} \frac{\partial c}{\partial \tau} = -\frac{\tau}{1-\tau} < 0$ for all $\tau > 0$. ■

Proof of Proposition 3. D is the the product of the four eigenvalues of the Jacobian matrix. A pair of complex conjugate eigenvalues always contributes positively to this product, no matter its stability. The sign of D is then solely governed by the real eigenvalues. $D < 0$ therefore rules out an even number of stable eigenvalues (with negative real part). In particular, all four eigenvalues cannot be stable at the same time. As the system features three predetermined variables (s, M, P) and one jump variable (z), local indeterminacy would require all four eigenvalues to be stable. Since $D < 0$ excludes this configuration, local indeterminacy cannot arise. ■

Proof of Lemma 4. Following Bosi and Desmarchelier (2019, Proposition 16), a Hopf bifurcation arises if and only if:

$$S_2 = \frac{S_3}{T} + \frac{DT}{S_3} \quad (14)$$

and:

$$\frac{S_3}{T} > 0 \quad (15)$$

Substituting the expressions of S_2 , S_3 , T and D into (14) and rearranging terms yields a second-degree polynomial in ε . The quadratic formula gives the two roots above. ■

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