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Pollution effects on consumption demand and fertility: Reconciling environmental and demographic policies

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Abstract

We reconsider the pollution-fertility interplay in light of the effect of pollution on the marginal utility of consumption in a simple overlapping generations economy. Each household lives two periods. During the youth, the agent divides her time between work and raising children. Income from work is saved and consumed in retirement. The old are also exposed to a pollution externality generated by production activities. The state finances pollution abatement expenditures through a proportional tax levied on production.

Notable results emerge, both in the short and long run. In the long run, a higher green-tax rate decreases (increases) the population size when household environmental concerns are low (high). Surprisingly, in the case of higher environmental concern, as the data show, environmental policy can help counter the demographic decline being experienced, notably, by the European Union.

In the short run, endogenous fluctuations can arise around the unique steady state when pollution increases the marginal utility of consumption (the so-called compensation effect). More precisely, a persistent two-period cycle can emerge through a flip bifurcation, or a limit cycle through a Neimark-Sacker bifurcation. Interestingly, the equilibrium becomes locally indeterminate in the vicinity of the limit cycle, which promotes self-fulfilling expectations and greater macroeconomic instability.

Keywords: *Overlapping generations model; flip and Neimark-Sacker bifurcations; local indeterminacy; pollution; endogenous fertility.*

JEL codes: *C62, H23, O44.*

1 Introduction

As Raffin (2026) pointed out in a recent study, human population dynamics (fertility) and the environment interact through two main channels: reproductive health and household environmental concerns.

Regarding reproductive health, a large body of medical evidence highlights the adverse effects of air pollution on male (Martenies and Perry, 2013) and female (Mendola et al., 2008) reproductive health. These adverse effects could partly explain the low fertility rates observed in Western countries. This involuntary infertility has recently been the subject of a theoretical study by Etner et al. (2020). More precisely, they developed a simple Overlapping Generations (hereafter OG) model in which households can be either fertile or infertile. Unlike Becker and Barro (1988), fertility decisions do not result from an economic trade-off but depend entirely on the reproductive status of households. Infertility leads to a loss of utility, while the probability of being fertile decreases with capital intensity. This negative relationship is introduced to account for the detrimental impact of pollution on reproductive health. Pollution is modeled as a pure flow externality. By incorporating assisted reproductive treatments aimed at overcoming infertility, the authors examine the most effective way to correct the externality.

The second channel, namely environmental concern, is also supported by evidence. For instance, Schneider-Mayerson and Leong (2020) report that 96.5% of the 607 Americans surveyed, aged 27 to 45, expressed concern about the negative effects of climate change on the welfare of future generations. From a theoretical perspective, incorporating environmental concern into fertility choices requires moving away from the endogenous fertility models developed by Becker and Barro (1988) or Galor and Weil (1996), and instead assuming that pollution or environmental quality directly affects household welfare. Adopting this perspective, Melindi-Ghidi and Seegmuller (2025), consider an OG model in which the household lives two periods: youth and adulthood. Youth is a period of inactivity, while during adulthood, households choose their level of consumption, the number of children to raise, and the amount of capital passed on per child. Consumption is divided between green and polluting goods. Furthermore, an environmental index, defined as the ratio of environmentally friendly to polluting goods, directly affects the marginal utility of having children. As in Etner et al. (2020), pollution is thus modeled as a pure flow externality. In the long term, Melindi-Ghidi and Seegmuller (2025) show that, when environmental concerns and fertility are substitutes, an inverted U-shaped relationship emerges between fertility and the environmental index, a result consistent with data.

As Raffin (2026) and Melindi-Ghidi and Seegmuller (2025) point out, the theoretical literature linking environmental concern and fertility decisions remains relatively sparse. Most existing works on this topic focus instead on the effects of the environment on longevity (Varvarigos and Zakaria, 2017), labor productivity (Bosi and Desmarchelier, 2013), or paternalistic altruism (Constant et al., 2014). Unlike these models, our article reconsiders the relationship between environmental concern and fertility decisions. However, unlike Melindi-

Ghidi and Seegmuller (2025), we do not model environmental concern by the direct effect of pollution on the marginal utility of having children. Rather, we assume that pollution affects the marginal utility of consumption. This alternative channel matters as long as, at optimum, households balance the marginal utility of children with the marginal utility of consumption. Consequently, any pollution-induced change in consumption demand is likely to influence fertility choices. Understanding this mechanism is all the more relevant as there is a substantial body of literature showing that a positive effect of pollution on consumer demand (the so-called compensation effect) can generate cycles through flip or Neimark-Sacker bifurcations, both in the OG and Ramsey frameworks.[1]

To study how environmental concern, understood through the effect of pollution on the marginal utility of consumption, affects fertility decisions and economic dynamics, our article develops a simple OG model with endogenous fertility à la Galor and Weil (1996). Preferences are non-separable in consumption and pollution, but separable in the number of children. Following Etner et al. (2020) and Melindi-Ghidi and Seegmuller (2025), pollution is modeled as a pure flow generated by production. Furthermore, the government levies a proportional tax on production to finance pollution abatement according to a balanced-budget rule.

Focusing on the long run, we show that, under isoelastic preferences, a unique nontrivial steady state exists. Interestingly, the effect of a higher green-tax rate on long-run population size depends on the degree of environmental concern. More precisely, this effect is negative (positive) if and only if environmental concern is low (high). The explanation is straightforward. Indeed, a higher green tax rate provides an incentive to reduce production. As a result, income declines, leading to lower consumption possibilities. At the same time, the reduction in production decreases the level of pollution. While lower consumption generates a utility loss, the improvement in environmental quality yields a utility gain. Therefore, the household's response ultimately depends on the degree of environmental concern. More precisely, when environmental concern is low, households primarily react to the loss of consumption induced by higher taxation. In order to preserve their consumption, they increase their labor supply and reduce the desired number of children. Conversely, when environmental concern is high, the welfare gains associated with a cleaner environment predominate. Households then choose to increase their fertility and reduce their labor supply (and therefore their consumption), leading to a larger population in the long run. According to empirical evidence, the degree of environmental concern is relatively high: a recent study by Zhang et al. (2025) concludes that "rising temperatures significantly reduce individual fertility intentions". In particular, they say, "for every 1 °C increase in temperature, respondents' fertility intentions decrease by 0.3082 units". Since temperature and pollution are positively correlated, this clearly means that a higher green-tax rate would increase the long-run population size.

[1] Interested readers are referred to Zhang (1999) and Bosi et al. (2019) for the occurrence of a NS bifurcation in the OG and Ramsey frameworks, respectively.

It is interesting to note that a higher environmental tax reduces both consumption and pollution, while its impact on fertility depends on households' environmental concern. Specifically, when this concern is sufficiently grave, the welfare gain associated with reduced pollution outweighs the negative effect of lower consumption. Households then choose to have more children, implying that a higher environmental tax ultimately increases population size in the long run. This finding is of major importance for economic policy. Indeed, the European Union (EU-28) is facing demographic decline, and the literature identifies immigration as one possible remedy (Marois et al., 2020). Our findings suggest that environmental policy could offer an additional avenue. Moreover, our theoretical result appears to be supported by empirical evidence. In a recent study, Zhang et al. (2025) show that rising temperatures reduce fertility intentions in China. If rising temperatures are interpreted as a deterioration in environmental quality, our result, combined with that of Zhang et al. (2025), suggests that stricter environmental policy could boost fertility and, consequently, increase long-run population size by mitigating pollution.

Focusing on the short run, we find the classic result that a compensation effect promotes endogenous fluctuations. More precisely, we show the existence of deterministic cycles via flip and Neimark-Sacker bifurcations. Importantly, we also observe local indeterminacy around the NS limit cycle.

The remainder of the article is organized as follows. Sections 2 and 3 present the model and equilibrium. Sections 4 and 5 study the steady state and local dynamics. Section 6 concludes. All the technical proofs are gathered in the Appendix.

2 Model

In the spirit of Galor and Weil (1996), we consider an OG market economy where households live two periods. When young, they work and have children. When old, they consume. As in Melindi-Ghidi and Seegmuller (2025), each household consists of a single individual who reproduces on her own. Firms use capital and labor to produce a good intended for both consumption and saving. Furthermore, production generates a pollution externality, assumed to be a pure flow affecting the household only during her old age. Finally, the government levies a tax on production to finance depollution, according to a simple balanced-budget rule.

2.1 Firms

m small enough (price-taker) firms share the same CRS technology F. At time t, the competitive firm j hires capital K_{jt} and labor L_{jt} to produce a quantity $Y_{jt} = F(K_{jt}, L_{jt})$ of good. The aggregate inputs and output are given by $K_t = \sum_{j=1}^{m} K_{jt}$, $L_t = \sum_{j=1}^{m} L_{jt}$ and $Y_t = \sum_{j=1}^{m} Y_{jt}$.

Because of the CRS technology, the profit maximization

$$\max\left[(1-\tau) F(K_{jt}, L_{jt}) - r_t K_{jt} - w_t L_{jt}\right]$$

leads to the same capital intensity $k_t \equiv K_{jt}/L_{jt} = K/L$ across the firms:

$$r_t = (1-\tau) f'(k_t) \equiv (1-\tau)\rho(k_t) \quad (1)$$

$$w_t = (1-\tau)\left[f(k_t) - f'(k_t) k_t\right] \equiv (1-\tau)\omega(k_t) \quad (2)$$

where r_t and w_t denote the real interest rate and wage rate, $\tau \in [0,1)$ is the tax rate and $f(k_t) \equiv F(k_t, 1)$, the average productivity.

Assumption 1 *The intensive production function f is twice continuously differentiable, strictly increasing and strictly concave: $f \in C^2$ and $f''(k_t) < 0 < f'(k_t)$. The Inada conditions are also satisfied: $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$.*

The elasticities of interest and wage rate are given by

$$\frac{k_t \rho'(k_t)}{\rho(k_t)} = -\frac{1-\alpha(k_t)}{\sigma(k_t)} < 0 \text{ and } \frac{k_t \omega'(k_t)}{\omega(k_t)} = \frac{\alpha(k_t)}{\sigma(k_t)} \quad (3)$$

where

$$\alpha(k_t) \equiv \frac{k_t f'(k_t)}{f(k_t)} \in (0,1) \text{ and } \sigma(k_t) = -\frac{f'(k_t)\left[f(k_t) - k_t f'(k_t)\right]}{k_t f(k_t) f''(k_t)} > 0 \quad (4)$$

are the capital share in total income and the Allen-Hicks elasticity of capital-labor substitution.

2.2 Households

A household lives two periods. When young, at time t, she supplies l_t units of labor, has n_t children and saves s_t. When old, at time $t+1$, she dissaves and consumes c_{t+1}.

Time at her disposal to work and rear children is normalized to one:

$$l_t + z n_t = 1$$

where $z \in (0,1)$ is the constant time to rear one child.

In this simple economy, the first and second-period budget constraints are given by

$$s_t = w_t l_t = w_t (1 - z n_t) \quad (5)$$

$$c_{t+1} = (1 - \delta + r_{t+1}) s_t \quad (6)$$

where δ represents the capital depreciation rate.

Replacing (5) in (6), we get the intertemporal budget constraint:

$$c_{t+1} = (1 - \delta + r_{t+1}) w_t (1 - z n_t) \quad (7)$$

Households share the same preferences:

$$v(n_t) + \beta u(c_{t+1}, P_{t+1}) \quad (8)$$

where $\beta \in (0, 1)$ is a measure of patience (discounting).

Assumption 2 *The utility functions* u and v are *twice continuously differentiable, strictly increasing and strictly concave:* $u, v \in C^2$, $v''(n_t) < 0 < v'(n_t)$, $u_{cc}(c_{t+1}, P_{t+1}) < 0 < u_c(c_{t+1}, P_{t+1})$. *Moreover, pollution is a negative externality:* $u_P(c_{t+1}, P_{t+1}) < 0$, *with a negative or positive impact on consumption demand:* $u_{cP} < 0$ *or* $u_{cP} > 0$.

The effect of pollution on utility reflects the environmental concern. Unlike Melindi-Ghidi and Seegmuller (2025), we assume that pollution does not affect the marginal utility of having children, but rather the marginal utility of consumption. In accordance with Michel and Rotillon (1995), $u_{cP} < 0$ captures a distaste effect: households consume less in a more polluted environment. Conversely, a $u_{cP} > 0$ indicates a compensation effect: households compensate for the loss of utility induced by pollution by increasing their consumption.

Furthermore, unlike Melindi-Ghidi and Seegmuller (2025), we consider households to be totally selfish. In particular, no bequest motive is taken into account and environmental concern reflects only the effects of pollution on their health. Medical data suggest that older people are more vulnerable to pollution than younger and justify that pollution affects only the old through their consumption demand. For instance, Jansomboon et al. (2025) show that pollution increases the prevalence of respiratory and cardiovascular symptoms in adults aged 55 and over, suggesting that pollution primarily leads to a welfare loss in old age.

Here, pollution is a pure externality. Its direct impact on consumption demand ($u_{cP} \neq 0$) generates an indirect impact on fertility through the consumption-fertility arbitrage:

$$v'(n_t) = z\beta (1 - \delta + r_{t+1}) w_t u_c(c_{t+1}, P_{t+1}) \tag{9}$$

resulting from utility maximization (8) under the intertemporal budget constraint (7).

Second-order characteristics (decreasing marginal utilities and nonzero effects of pollution on consumption) are better understood in terms of elasticities:

$$\epsilon \equiv \frac{nv''}{v'} < 0,\ \varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c} < 0,\ \varepsilon_{cP} \equiv \frac{Pu_{cP}}{u_c} \neq 0 \tag{10}$$

Therefore, preferences are characterized by a distaste (compensation) effect when $\varepsilon_{cP} < 0$ ($\varepsilon_{cP} > 0$).

As seen above, we assume a self-replicating individual. Agents share the same preferences and those born in the same period adopt the same behavior: thus, each household has the same number of children n_t and the size of the population N_t follows endogenous demographic growth:

$$N_{t+1} = n_t N_t \tag{11}$$

2.3 Pollution

As in Etner et al. (2020) and Melindi-Ghidi and Seegmuller (2025), global pollution P_t is a flow.[2] It originates from aggregate production Y_t and is mitigated by public spending G_t:

$$P_t = bY_t - \gamma G_t \tag{12}$$

where $b > 0$ is the dirtiness of production (pollution per output unit) and $\gamma > 0$ is the efficiency of public spending.

To finance pollution abatement, the government uses all tax revenues according to a budget balance rule:

$$G_t = \tau Y_t \tag{13}$$

Putting together equations (12) and (13), we get

$$P_t = (b - \gamma\tau)\, Y_t \tag{14}$$

Pollution cannot be negative. To ensure this, we introduce a new restriction.

Assumption 3 *The tax rate is bounded above: $\tau < b/\gamma$.*

This restriction is not too demanding since $\tau < 1$ and, presumably, $b > \gamma$.

3 General equilibrium

In general equilibrium, all markets clear. The economy is composed of three markets: capital, labor and consumption good: let us focus on market clearing.

3.1 Capital market

Aggregate capital demand by firms equals aggregate savings from households:

$$K_{t+1} = N_t s_t \tag{15}$$

3.2 Labor market

At time t, the aggregate labor supply is given by

$$N_t l_t = N_t\,(1 - zn_t) \tag{16}$$

In equilibrium, firms' labor demand L_t equals this aggregate supply:

$$L_t = N_t\,(1 - zn_t) \tag{17}$$

Dividing (15) by (17), we obtain

$$\frac{k_{t+1}L_{t+1}}{L_{t+1}} = \frac{N_t s_t}{N_{t+1}\,(1 - zn_{t+1})}$$

[2] According to Seegmuller and Verchère (2007), modeling pollution as a pure flow can be justified in an OG economy because the duration of a period is long enough for the pollution to be almost entirely absorbed in the interval.

that is, since $N_{t+1} = n_t N_t$,

$$s_t = n_t \left(1 - z n_{t+1}\right) k_{t+1} \tag{18}$$

According to equations (5) and (2), we get

$$s_t = w_t l_t = w_t \left(1 - z n_t\right) = \left(1 - \tau\right) \omega \left(k_t\right) \left(1 - z n_t\right) \tag{19}$$

Replacing (18) in (19), we find

$$n_t \left(1 - z n_{t+1}\right) k_{t+1} = \left(1 - \tau\right) \omega \left(k_t\right) \left(1 - z n_t\right) \tag{20}$$

which summarizes the market clearing in both the capital and the labor market.

Replacing (17) in (14), we obtain the equilibrium pollution:

$$P_t = \left(b - \gamma\tau\right) Y_t = \left(b - \gamma\tau\right) L_t f\left(k_t\right) = \left(b - \gamma\tau\right) N_t \left(1 - z n_t\right) f\left(k_t\right) \equiv P\left(n_t, k_t, N_t\right) \tag{21}$$

which is positive under Assumption 3.

3.3 Goods market

According to a corollary of Walras' law, equilibrium in the capital and labor markets leads to equilibrium in the last market, that of goods. To convince the reader, let us verify this corollary.

Corollary 1 *The equilibrium in the markets of capital and labor implies the equilibrium in the goods market.*

3.4 Dynamic general equilibrium

Since the length of a period is the half-life, in the sequel, for simplicity, we will assume full capital depreciation.

Assumption 4 *Capital fully depreciates between two periods:* $\delta = 1$.

Under Assumption 4, we have the following proposition.

Proposition 2 *An intertemporal equilibrium is a sequence* $\left(n_t, k_t, N_t\right)_{t=0}^{\infty}$ *solution to the three-dimensional system*

$$N_{t+1} = n_t N_t \tag{22}$$

$$n_t \left(1 - z n_{t+1}\right) k_{t+1} = \left(1 - \tau\right) \omega \left(k_t\right) \left(1 - z n_t\right) \tag{23}$$

$$v'\left(n_t\right) = z\beta \left(1 - \tau\right)^2 \rho\left(k_{t+1}\right) \omega\left(k_t\right) u_c\left(c\left(n_t, k_t, k_{t+1}\right), P\left(n_{t+1}, k_{t+1}, N_{t+1}\right)\right) \tag{24}$$

given the initial conditions N_0 *and* k_0*, where*

$$c_{t+1} = c\left(n_t, k_t, k_{t+1}\right) \equiv \left(1 - \tau\right)^2 \rho\left(k_{t+1}\right) \omega\left(k_t\right) \left(1 - z n_t\right) \tag{25}$$

and $P\left(n_{t+1}, k_{t+1}, N_{t+1}\right)$ *is given by (21).*

4 Steady state

Definition 3 (steady state) *A steady state is an equilibrium sequence* $(n_t, k_t, N_t)_{t=0}^{\infty}$ *such that* $(n_t, k_t, N_t) = (n_{t+1}, k_{t+1}, N_{t+1}) \equiv (n, k, N)$ *for any* t*, where* $n = 1$ *since, according to (22),* $N = nN$*.*

Assumption 5 *The production function is Cobb-Douglas:* $f(k_t) = Ak_t^{\alpha}$*.*
In this case, $\alpha(k_t)$ is a constant, say α, and $\sigma(k_t) = 1$. Moreover,

$$\rho(k_t) = f'(k_t) = \alpha A k_t^{\alpha-1} \tag{26}$$
$$\omega(k_t) = f(k_t) - f'(k_t)\, k_t = (1-\alpha)\, A k_t^{\alpha} \tag{27}$$

Proposition 4 (steady state) *Let Assumptions 1 to 5 hold. There exists a unique*[3] *non-trivial steady state* $(n_t, k_t, N_t) = (1, k, N)$ *with*

$$k = [A(1-\alpha)(1-\tau)]^{\frac{1}{1-\alpha}} \tag{29}$$

and N *the unique solution to*

$$u_c\left(c, \tilde{P}(N)\right) = \kappa \tag{30}$$

where

$$c = k(1-z)\frac{\alpha}{1-\alpha} \tag{31}$$
$$\tilde{P}(N) \equiv kN\frac{1-z}{1-\alpha}\frac{b-\gamma\tau}{1-\tau} \tag{32}$$

are the stationary consumption demand and pollution flow, and

$$\kappa \equiv \frac{1}{k}\frac{v'(1)}{z\beta}\frac{1-\alpha}{\alpha} \tag{33}$$

provided that

$$u_c(c, \infty) < \kappa < u_c(c, 0) \text{ if } \varepsilon_{cP} < 0 \text{ (distaste effect)} \tag{34}$$
$$u_c(c, 0) < \kappa < u_c(c, \infty) \text{ if } \varepsilon_{cP} > 0 \text{ (compensation effect)} \tag{35}$$

Proposition 5 (comparative statics) *The effects of the tax rate* τ *on capital and consumption are negative:*

$$\frac{\tau}{k}\frac{\partial k}{\partial \tau} = \frac{\tau}{c}\frac{\partial c}{\partial \tau} = -\frac{1}{1-\alpha}\frac{\tau}{1-\tau} < 0 \tag{36}$$

[3] The capital of steady state is solution to

$$\frac{\omega(k)}{k} = \frac{1}{1-\tau} \tag{28}$$

and, in the general isoelastic case, $[\omega(k)/k]' < 0 \Leftrightarrow \alpha(k) < \sigma$. Thus, if $\sigma \geq 1$, the LHS of (28) is strictly decreasing and the steady state k is unique (for instance, in our Cobb-Douglas case, $\sigma = 1$ and k is given by (29)). In the case $\sigma < 1$, the LHS of (28) can be U-shaped or inverted U-shaped in some interval and equation (28) can have multiple solutions k. Indeed, it is possible to have $\alpha(k) < \sigma$ for some values of k and $\alpha(k) > \sigma$ for other values.

while the effects on pollution and children depend on the second-order elasticities ε_{cc} *and* ε_{cP} *(distaste vs compensation effect):*

$$\frac{\tau}{P}\frac{\partial P}{\partial \tau} = \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\frac{1+\varepsilon_{cc}}{\varepsilon_{cP}} < 0 \Leftrightarrow \frac{1+\varepsilon_{cc}}{\varepsilon_{cP}} < 0 \tag{37}$$

$$\begin{aligned}\frac{\tau}{N}\frac{\partial N}{\partial \tau} &= \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\left(\alpha + \frac{1+\varepsilon_{cc}}{\varepsilon_{cP}}\right) + \frac{\gamma\tau}{b-\gamma\tau} < 0 \\ &\Leftrightarrow \frac{1+\varepsilon_{cc}}{\varepsilon_{cP}} < -\frac{\gamma-\gamma\tau}{b-\gamma\tau} - \alpha\frac{b-\gamma}{b-\gamma\tau}\,(<0)\end{aligned} \tag{38}$$

Assumption 6 *Preferences are isoelastic:*

$$v(n_t) \equiv \frac{n_t^{1-1/\varphi}}{1-1/\varphi} \text{ and } u(c_{t+1}, P_{t+1}) \equiv \frac{\left(c_{t+1}P_{t+1}^{-\eta}\right)^{1-1/\varepsilon}}{1-1/\varepsilon} \tag{39}$$

with $\varepsilon, \varphi, \eta > 0$.

ε and φ are the elasticities of intertemporal substitution of the composite good $c_{t+1}P_{t+1}^{-\eta}$ and the number of children n_t, and $\eta > 0$ is a measure of the impact of pollution on consumption demand.

We observe that

$$\epsilon = -\frac{1}{\varphi},\ \varepsilon_{cc} = -\frac{1}{\varepsilon} \text{ and } \varepsilon_{cP} = \eta\frac{1-\varepsilon}{\varepsilon} \tag{40}$$

When $\varepsilon < 1$ (> 1), $\varepsilon_{cP} > 0$ (< 0), preferences exhibit a compensation (distaste) effect.

Under Assumption 3, let

$$\eta^* \equiv \frac{1}{\alpha + (1-\alpha)\frac{1-\tau}{\tau}\frac{\gamma\tau}{b-\gamma\tau}} > 0$$

to be the critical value of the impact of pollution on consumption demand.

Proposition 6 *Let Assumptions 3 to 6 hold.*

The fiscal effects on capital and consumption are not affected by the structure of preferences and are always given by (36).

Taxation has a negative impact on pollution:

$$\frac{\tau}{P}\frac{\partial P}{\partial \tau} = -\frac{1}{\eta}\frac{1}{1-\alpha}\frac{\tau}{1-\tau} < 0 \tag{41}$$

while the taxation impact on fertility depends on the magnitude of the pollution impact η*:*

$$\frac{\tau}{N}\frac{\partial N}{\partial \tau} = \frac{\gamma\tau}{b-\gamma\tau} + \frac{1}{1-\alpha}\frac{\tau}{1-\tau}\left(\alpha - \frac{1}{\eta}\right) < 0 \Leftrightarrow \eta < \eta^* \tag{42}$$

Proposition 6 deserves an economic interpretation. First, since the tax is levied on production activities, a higher rate entails a reduction in production. The resulting contraction in income leads to a decrease in the long-run capital stock and a decline in the long-run level of consumption. Furthermore, the decrease in the level of production results in a pollution reduction.

Let us now explain why a strong environmental concern leads to a larger population in the long run when the eco-tax rate increases. Before proceeding, note that in light of (25), (24) can be written as:

$$(1 - zn)\, v'(n) = z\beta c u_c(c, P) \tag{43}$$

Let us first assume that preferences are characterized by compensation effect (i.e., $u_{cP} > 0$). As previously noted, a higher environmental tax rate reduces both consumption and pollution. Lower consumption leads to higher marginal utility of consumption (see Assumption 2), whereas reduced pollution implies lower marginal utility of consumption due to the compensation effect. If the household's environmental concern is sufficiently strong (i.e., $\eta > \eta^*$), the household is highly sensitive to pollution; hence, the decline in the marginal utility of consumption more than offsets the increase resulting from reduced consumption. In other words, a higher environmental tax rate leads to a decrease in both consumption and the marginal utility of consumption: the RHS of equation (43) decreases. Optimally, the household adjusts its fertility choice to reduce the LHS of equation (43) accordingly, thereby increasing the number of children. This higher fertility rate translates into a larger population size in the long run.

Let us now note that (39) implies:

$$c u_c(c, P) = \left(cP^{-\eta}\right)^{\frac{\varepsilon-1}{\varepsilon}} \tag{44}$$

Next, let us consider the case where preferences are characterized by a distaste effect (i.e., $u_{cP} < 0$ or, equivalently, $\varepsilon > 1$). As before, a higher eco-tax rate reduces both consumption and pollution. According to (44), lower consumption leads to a decrease in the RHS of (43), whereas lower pollution leads to an increase in that same term. To determine which effect prevails, it is useful to note that the reduction in pollution induced by a higher green-tax rate is inversely proportional to the household's environmental concern (see (41)). Thus, high environmental concern (i.e., $\eta > \eta^*$) implies a moderate reduction in pollution. Consequently, the drop in the right-hand side of (44) associated with the decrease in consumption more than offsets the increase linked to the reduction in pollution. Thus, a higher eco-tax rate reduces the RHS of (43). As before, the household adjusts the number of children in order to reduce the LHS of (43) accordingly. This higher fertility rate translates into a larger population in the long run.

The positive effect of environmental taxation on fertility is noteworthy, as it suggests that environmental policy could offer a solution to the demographic decline facing developed countries. In a recent article, Marois et al. (2020)

report that the share of the population aged 65 and over in the European Union (EU-28) stood at 18.2%, with projections indicating a rise to approximately 30% by 2050. Marois et al. (2020) consider immigration as a means to counter this demographic decline. Our results clearly suggest that environmental policy could serve as another potential solution. This alternative is particularly relevant as it relies on a strong environmental concern (i.e., $\eta > \eta^*$) within the population. In this regard, Zhang et al. (2025) report that rising temperatures reduce fertility intentions in China. Given the positive correlation between temperature and pollution, combining our findings with the empirical observations of Zhang et al. (2025) suggests that a higher green-tax rate, by mitigating pollution, would lead to an increase in long-run population size.

5 Local dynamics

To study the local bifurcations, we linearize the dynamic system (24)-(22) around the unique non-trivial steady state $(n, k, N) = (1, k, N)$.

Lemma 7 *Let Assumption 3 to 6 hold. Linearizing system (22)-(24) around the steady state, we obtain*

$$\begin{bmatrix} \frac{dn_{t+1}}{n} \\ \frac{dk_{t+1}}{k} \\ \frac{dN_{t+1}}{N} \end{bmatrix} = J \begin{bmatrix} \frac{dn_t}{n} \\ \frac{dk_t}{k} \\ \frac{dN_t}{N} \end{bmatrix}$$

where J is the Jacobian matrix whose trace, sum of principal minors of order two and determinant are given by

$$T = \frac{1}{z} + E_1 \tag{45}$$

$$S = \alpha E_1 + E_2 \tag{46}$$

$$D = \alpha E_2 \tag{47}$$

where

$$E_1 = \frac{z + (1-z)\frac{1}{\varphi}}{z(1-\alpha)(1-\eta)\left(1-\frac{1}{\varepsilon}\right)}$$

$$E_2 = \frac{1 + (1-z)\left(\frac{1}{\varphi} - \frac{1}{\varepsilon}\right)}{z(1-\alpha)(1-\eta)\left(1-\frac{1}{\varepsilon}\right)}$$

A flip bifurcation generates a two-period cycle and generically arises when one eigenvalue goes through -1. We can choose a meaningful fundamental parameter and compute its critical flip bifurcation value ensuring that one eigenvalue is equal to -1. In our model, the impact of pollution on consumption demand η is key to understand the interplay between environment concern and fertility. The following proposition provides the critical value for η.

Proposition 8 *A two-period cycle generically arises around the steady state through a flip bifurcation at*

$$\eta = \eta_F \equiv 2\frac{1+z-(1-\alpha z)\frac{1}{\varepsilon}+(1+\alpha)(1-z)\frac{1}{\varphi}}{(1-\alpha)(1+z)\left(1-\frac{1}{\varepsilon}\right)} \tag{48}$$

solution to $1+T+S+D=0$. η_F *is positive (and the flip bifurcation is possible) if and only if*

$$\varepsilon < \frac{1-\alpha z}{1+z+(1+\alpha)(1-z)\frac{1}{\varphi}} \,(<1) \quad \textit{or } \varepsilon > 1 \tag{49}$$

In our model, not only flip bifurcations (and period-doubling bifurcations and, eventually, chaos) are generically possible, but also Neimark-Sacker bifurcations generating limit cycles.

Let

$$\varepsilon_N \equiv \frac{\varphi(1-z)(1-\alpha z)}{\varphi(1+z^2)+(1-z)(1-\alpha z+\alpha+z)} > 0$$

Proposition 9 *Let Assumptions 3 to 6 hold, and fix* $\varepsilon = \varepsilon_N$. *A limit cycle generically arises around the steady state through a Neimark-Sacker bifurcation at*

$$\eta = \eta_N \equiv 2\frac{(1-\alpha)z-(1-z)\alpha^2}{1-\alpha z+(1-\alpha)z-(1-z)\alpha^2} \tag{50}$$

provided that η_N *is positive or, equivalently,*

$$z > \frac{\alpha^2}{1-\alpha+\alpha^2} \in (0,1) \tag{51}$$

Moreover, the equilibrium is locally indeterminate around the Neimark-Sacker bifurcation.

Remark 10 *We observe that* ε_N *does not depend on* η *and it is positive: then, it is well-defined. Proposition 9 means that, given a pair of parameters* (α, z) *satisfying inequalities (51), there exists a positive continuous function* $\tilde{\eta}$ *in a neighborhood* N_ε *of* ε_N *such that, for any* $\varepsilon \in N_\varepsilon$, *the system undergoes a Neimark-Sacker bifurcation at* $\eta = \tilde{\eta}(\varepsilon)$ *and a limit cycle generically arises around the steady state. The existence of such a function depends on the continuity properties of technology and preferences. In other words, there exists a nonzero measure subset of the parameter space* $\{(\alpha, \varepsilon, \eta, \varphi, z)\}$ *with a limit cycle around the steady state for any point in this subset.*

Remark 11 *We do not know whether the limit cycle is stable (supercritical) or unstable (subcritical). Assume that you are in the right or in the left neighborhood of* η_N *where the cycles exist.*

If $|\lambda_1| < 1$ *and* $|\lambda_2| = |\lambda_3| < 1$, *then the cycle is unstable and the steady state inside is stable: the multiple equilibrium trajectories converge from points inside the invariant closed curve to the steady state.*

If $|\lambda_1| < 1 < |\lambda_2| = |\lambda_3|$*, then the cycle is stable and the steady state inside is unstable: the multiple equilibrium trajectories converge from points inside the invariant closed curve to the limit cycle.*

$\varphi(1-z)(1-\alpha z) < \varphi(1+z^2)$ implies $\varepsilon_N < 1$: the Neimark-Sacker bifurcation arises only if preferences exhibit a compensation effect. Thus, according to Propositions 8 and 9, $\varepsilon < 1$ is a necessary condition for the occurrence of both a flip and a Neimark-Sacker bifurcation.

The existence of endogenous cycles in these two cases warrants an economic interpretation. Suppose the economy is initially at the steady state at time t and consider an exogenous increase in the pollution level at that same time. Due to the compensation effect, the old at time t react by increasing their consumption demand. The level of production also increases, leading to a higher demand for labor. The young at time t respond by increasing labor supply, thus reducing the number of children they wish to have. This decreases future production opportunities and, consequently, future pollution. As a result, higher pollution today is followed by lower pollution tomorrow: endogenous cycles emerge.

The emergence of self-fulfilling prophecies relies on the existence of multiple equilibria. Local indeterminacy is a form of multiplicity that allows for the existence of sunspot equilibria.[4]

Let us illustrate how this mechanism works in our economy. Suppose that households born at time t anticipate an increase in future pollution. Due to the compensation effect ($\varepsilon < 1$), they will want to raise their future consumption demand. To do that, they will have to increase their income by working more and reducing the number of children today. Larger savings and higher consumption demand tomorrow will induce firms to produce and, therefore, pollute more. Thus, under the compensation effect, rational expectations of higher future pollution will lead the economic system to experience an increase in pollution tomorrow, and will become self-fulfilling.

6 Conclusion

We have considered the effects of households' environmental concerns on their fertility decisions, in the spirit of Melindi-Ghidi and Seegmuller (2025). Unlike their approach, we have focused on the effect of pollution on the marginal utility of consumption, rather than on the marginal utility of having children. Our approach has allowed us to bridge research on endogenous fertility with research on pollution-driven endogenous cycles. The latter has highlighted the destabilizing role of the compensation effect. Our article confirms this result in an endogenous fertility context. Furthermore, we have disentangled the ambiguous impact of the green-tax rate on the population size in the long run. More precisely, we have found that a higher rate decreases (increases) the population size when households' environmental concerns are low (high). The rationale is quite simple: an increase in the green-tax rate reduces both consumption and

[4] See Azariadis (1981), among others.

pollution. The decrease in consumption leads to a utility loss, while the drop in pollution, to a utility gain.

When environmental concern is low, households place little importance on environmental quality, and the utility loss associated with reduced consumption becomes predominant. To maintain their income and consumption standards and, thus, compensate for the utility loss, households increase their working hours. Consequently, they devote less time to raising children, which ultimately leads to a smaller population. Conversely, when environmental concern is high, the utility gain from a cleaner environment becomes dominant. In this case, households have less incentive to increase their working hours and can devote more time to raising children, resulting in a larger population in the long run.

As European countries face demographic decline (Marois et al., 2020), this result highlights a potential additional solution: the adoption of stricter environmental policies. Such policies could complement the solution already identified in the literature, namely, immigration (Marois et al., 2020).

7 Appendix

Proof of Corollary 1

At time t, the aggregate supply of goods is given by Y_t while the aggregate demand by $I_t + \delta K_t + C_t + G_t$, where $I_t = K_{t+1} - K_t$ is the gross investment and $C_t = N_{t-1}c_t$ the aggregate consumption. At equilibrium,

$$Y_t = K_{t+1} - (1-\delta) K_t + N_{t-1}c_t + \tau Y_t \tag{52}$$

Considering the capital market clearing (15) jointly with (6), (52) writes:

$$(1-\tau) Y_t = N_t s_t + r_t N_{t-1} s_{t-1} \tag{53}$$

According to (5) and (15), (53) becomes:

$$(1-\tau) Y_t = w_t N_t (1 - z n_t) + r_t K_t \tag{54}$$

Finally, the equilibrium in the labour market (17) and (54) implies

$$(1-\tau) Y_t = w_t L_t + r_t K_t \tag{55}$$

According to (1) and (2), (55) represents precisely the exhaustion of product due to the CRS: the corollary of the Walras' law is verified. ■

Proof of Proposition 2

Considering together equations (7), (1) and (2), we obtain (25).

According to (11), (20) and (9), we obtain the three-dimensional dynamic system (22)-(24) in the variables (n_t, k_t, N_t). ■

Proof of Proposition 4

At the steady state $(1, k, N)$, equations (23) and (24) write

$$k = (1-\tau) \omega(k) \tag{56}$$

$$v'(1) = z\beta (1-\tau)^2 \rho(k) \omega(k) u_c\left(c, \tilde{P}(N)\right) \tag{57}$$

Replacing (27) in (56) and solving for k, we obtain (29).

c is given by (25), that is now by

$$c = (1-z)(1-\tau)^2 \rho(k)\,\omega(k) \tag{58}$$

where

$$\rho(k) = \frac{1}{1-\tau}\frac{\alpha}{1-\alpha} \tag{59}$$

$$\omega(k) = \frac{k}{1-\tau} \tag{60}$$

Then, (58) becomes (31).

At the steady state, (21) gives the pollution flow $P = N(1-z)(b-\gamma\tau)Ak^\alpha$. Noticing that $Ak^\alpha = k\rho(k)/\alpha$ and using (59), we obtain the pollution function (32).

Replacing (59) and (60) in (57), the consumption-children arbitrage becomes (30).

Finally, we observe that u_c is strictly decreasing (increasing) with respect to P if $u_{cP} < 0$ (> 0), that is if $\varepsilon_{cP} < 0$ (> 0).

Moreover, u is C^2 and and that P is linear in N.

Thus, (34) and (35) ensure that the steady state exists and it is unique. ■

Proof of Proposition 5

Taking the logarithm of (29) and (31), we find

$$\begin{aligned}\ln k &= \frac{\ln[A(1-\alpha)] + \ln(1-\tau)}{1-\alpha} \\ \ln c &= \ln k + \ln\frac{\alpha(1-z)}{1-\alpha}\end{aligned}$$

Deriving these equations and noticing that $\partial \ln c/\partial\tau = \partial \ln k/\partial\tau$, we obtain (36).

Moreover, taking the logarithm of (30) and (32), and considering (33), we have

$$\begin{aligned}\ln u_c(c,P) &= -\ln k + \ln\left[\frac{v'(1)}{z\beta}\frac{1-\alpha}{\alpha}\right] \\ \ln P &= \ln k + \ln N + \ln\frac{1-z}{1-\alpha} + \ln(b-\gamma\tau) - \ln(1-\tau)\end{aligned}$$

Deriving these equations, we find

$$\varepsilon_{cP}\frac{\tau}{P}\frac{\partial P}{\partial\tau} = -\frac{\tau}{k}\frac{\partial k}{\partial\tau} - \varepsilon_{cc}\frac{\tau}{c}\frac{\partial c}{\partial\tau} \tag{61}$$

$$\frac{\tau}{N}\frac{\partial N}{\partial\tau} = \frac{\tau}{P}\frac{\partial P}{\partial\tau} - \frac{\tau}{k}\frac{\partial k}{\partial\tau} + \frac{\tau\gamma}{b-\gamma\tau} - \frac{\tau}{1-\tau} \tag{62}$$

Replacing (36) in (61), we get (37). Finally, replacing (36) and (37) in (62), we obtain (38). ■

Proof of Proposition 6

We observe that

$$\frac{1+\varepsilon_{cc}}{\varepsilon_{cP}} = -\frac{1}{\eta} \tag{63}$$

Replacing (63) in (37) and (38), we obtain (41) and (42), respectively. ■

Proof of Lemma 7

Totally differentiating system (22)-(24) and noticing that, at the steady state,

$$\begin{aligned} n &= 1 \\ k &= (1-\tau)\,\omega \\ v'(1) &= z\beta(1-\tau)^2 \rho\omega u_c \end{aligned}$$

we obtain

$$\begin{aligned} \frac{dN_{t+1}}{N} &= \frac{dn_t}{n} + \frac{dN_t}{N} \\ z\frac{dn_{t+1}}{n} - (1-z)\frac{dk_{t+1}}{k} &= \frac{dn_t}{n} - (1-z)\frac{k\omega'(k)}{\omega(k)}\frac{dk_t}{k} \end{aligned}$$

$$\begin{aligned} &\frac{Pu_{cP}}{u_c}\frac{n}{P}\frac{\partial P}{\partial n_{t+1}}\frac{dn_{t+1}}{n} + \left[\frac{k\rho'(k)}{\rho(k)} + \frac{cu_{cc}}{u_c}\frac{k}{c}\frac{\partial c}{\partial k_{t+1}} + \frac{Pu_{cP}}{u_c}\frac{k}{P}\frac{\partial P}{\partial k_{t+1}}\right]\frac{dk_{t+1}}{k} \\ &+\frac{Pu_{cP}}{u_c}\frac{N}{P}\frac{\partial P}{\partial N_{t+1}}\frac{dN_{t+1}}{N} \\ = &\left[\frac{1v''(1)}{v'(1)} - \frac{cu_{cc}}{u_c}\frac{n}{c}\frac{\partial c}{\partial n_t}\right]\frac{dn_t}{n} - \left[\frac{k\omega'(k)}{\omega(k)} + \frac{cu_{cc}}{u_c}\frac{k}{c}\frac{\partial c}{\partial k_t}\right]\frac{dk_t}{k} \end{aligned}$$

According to (3) and (4), and definitions (10), we have

$$\frac{k\rho'(k)}{\rho(k)} = \alpha - 1,\ \frac{k\omega'(k)}{\omega(k)} = \alpha = \frac{kf'(k)}{f(k)}$$
$$\epsilon = \frac{1v''(1)}{v'(1)},\ \varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c},\ \varepsilon_{cP} \equiv \frac{Pu_{cP}}{u_c}$$

and we get

$$\begin{aligned} \frac{dN_{t+1}}{N} &= \frac{dn_t}{n} + \frac{dN_t}{N} \\ z\frac{dn_{t+1}}{n} - (1-z)\frac{dk_{t+1}}{k} &= \frac{dn_t}{n} - \alpha(1-z)\frac{dk_t}{k} \end{aligned}$$

that is

$$\begin{aligned} &\varepsilon_{cP}\frac{n}{P}\frac{\partial P}{\partial n_{t+1}}\frac{dn_{t+1}}{n} + \left(\alpha - 1 + \varepsilon_{cc}\frac{k}{c}\frac{\partial c}{\partial k_{t+1}} + \varepsilon_{cP}\frac{k}{P}\frac{\partial P}{\partial k_{t+1}}\right)\frac{dk_{t+1}}{k} \\ &+\varepsilon_{cP}\frac{N}{P}\frac{\partial P}{\partial N_{t+1}}\frac{dN_{t+1}}{N} \\ = &\left(\epsilon - \varepsilon_{cc}\frac{n}{c}\frac{\partial c}{\partial n_t}\right)\frac{dn_t}{n} - \left(\alpha + \varepsilon_{cc}\frac{k}{c}\frac{\partial c}{\partial k_t}\right)\frac{dk_t}{k} \end{aligned}$$

Reconsidering (21) and (25):

$$\begin{aligned} P\left(n_{t+1},k_{t+1},N_{t+1}\right) &= N_{t+1}f\left(k_{t+1}\right)\left(1-zn_{t+1}\right)\left(b-\gamma\tau\right) \\ c\left(n_{t},k_{t},k_{t+1}\right) &= \rho\left(k_{t+1}\right)\omega\left(k_{t}\right)\left(1-zn_{t}\right)\left(1-\tau\right)^{2} \end{aligned}$$

and observing that, at the steady state,

$$\begin{aligned} P &= Nf\left(k\right)\left(1-z\right)\left(b-\gamma\tau\right) \\ c &= \rho\omega\left(1-z\right)\left(1-\tau\right)^{2} \end{aligned}$$

we can compute the partial derivatives of $P\left(n_{t+1},k_{t+1},N_{t+1}\right)$ and $c\left(n_{t},k_{t},k_{t+1}\right)$:

$$\left[\begin{array}{ccc} \frac{n}{P}\frac{\partial P}{\partial n_{t+1}} & \frac{k}{P}\frac{\partial P}{\partial k_{t+1}} & \frac{N}{P}\frac{\partial P}{\partial N_{t+1}} \\ \frac{n}{c}\frac{\partial c}{\partial n_{t}} & \frac{k}{c}\frac{\partial c}{\partial k_{t}} & \frac{k}{c}\frac{\partial c}{\partial k_{t+1}} \end{array}\right]=\left[\begin{array}{ccc} -\frac{z}{1-z} & \alpha & 1 \\ -\frac{z}{1-z} & \alpha & \alpha-1 \end{array}\right]$$

The dynamics system becomes

$$\begin{aligned} \frac{dN_{t+1}}{N} &= \frac{dn_{t}}{n}+\frac{dN_{t}}{N} \\ z\frac{dn_{t+1}}{n}-\left(1-z\right)\frac{dk_{t+1}}{k} &= \frac{dn_{t}}{n}-\alpha\left(1-z\right)\frac{dk_{t}}{k} \end{aligned}$$

that is

$$\begin{aligned} & -\varepsilon_{cP}\frac{z}{1-z}\frac{dn_{t+1}}{n}+\left[\alpha\varepsilon_{cP}-\left(1-\alpha\right)\left(1+\varepsilon_{cc}\right)\right]\frac{dk_{t+1}}{k}+\varepsilon_{cP}\frac{dN_{t+1}}{N} \\ = & \left(\epsilon+\varepsilon_{cc}\frac{z}{1-z}\right)\frac{dn_{t}}{n}-\alpha\left(1+\varepsilon_{cc}\right)\frac{dk_{t}}{k} \end{aligned}$$

According to formulas (40), we obtain

$$\begin{aligned} \frac{dN_{t+1}}{N} &= \frac{dn_{t}}{n}+\frac{dN_{t}}{N} \\ z\frac{dn_{t+1}}{n}-\left(1-z\right)\frac{dk_{t+1}}{k} &= \frac{dn_{t}}{n}-\alpha\left(1-z\right)\frac{dk_{t}}{k} \end{aligned}$$

that is

$$\begin{aligned} & \eta z\frac{dn_{t+1}}{n}-\left(1-\alpha+\alpha\eta\right)\left(1-z\right)\frac{dk_{t+1}}{k}-\eta\left(1-z\right)\frac{dN_{t+1}}{N} \\ = & \frac{\varepsilon}{1-\varepsilon}\left(\frac{z}{\varepsilon}+\frac{1-z}{\varphi}\right)\frac{dn_{t}}{n}-\alpha\left(1-z\right)\frac{dk_{t}}{k} \end{aligned}$$

or, more compactly,

$$\left[\begin{array}{c} \frac{dn_{t+1}}{n} \\ \frac{dk_{t+1}}{k} \\ \frac{dN_{t+1}}{N} \end{array}\right]=J\left[\begin{array}{c} \frac{dn_{t}}{n} \\ \frac{dk_{t}}{k} \\ \frac{dN_{t}}{N} \end{array}\right]$$

where

$$J \equiv \begin{bmatrix} 0 & 0 & 1 \\ z & z-1 & 0 \\ \eta z & (1-\alpha+\alpha\eta)(z-1) & \eta(z-1) \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 1 & \alpha(z-1) & 0 \\ \frac{\varepsilon}{1-\varepsilon}\left(\frac{z}{\varepsilon}+\frac{1-z}{\varphi}\right) & \alpha(z-1) & 0 \end{bmatrix} \tag{64}$$

is the Jacobian matrix evaluated at the steady state.

The characteristic polynomial is given by:

$$C(\lambda) \equiv (\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3) = \lambda^3 - T\lambda^2 + S\lambda - D \tag{65}$$

where

$$\begin{aligned} T &\equiv \lambda_1+\lambda_2+\lambda_3 \\ S &\equiv \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3 \\ D &\equiv \lambda_1\lambda_2\lambda_3 \end{aligned}$$

are the trace, the sum of principal minors of order two, and the determinant.

Considering the Jacobian matrix (64) and computing the trace, the sum of principal minors of order two and the determinant, we obtain

$$T = \frac{1}{z} + \frac{z+(1-z)\frac{1}{\varphi}}{z(1-\alpha)(1-\eta)\left(1-\frac{1}{\varepsilon}\right)}$$

$$S = \frac{\alpha\left[z+(1-z)\frac{1}{\varphi}\right]+\left[1+(1-z)\left(\frac{1}{\varphi}-\frac{1}{\varepsilon}\right)\right]}{z(1-\alpha)(1-\eta)\left(1-\frac{1}{\varepsilon}\right)}$$

$$D = \alpha\frac{1+(1-z)\left(\frac{1}{\varphi}-\frac{1}{\varepsilon}\right)}{z(1-\alpha)(1-\eta)\left(1-\frac{1}{\varepsilon}\right)}$$

or, equivalently, (45), (46) and (47). ■

Proof of Proposition 8

A flip bifurcation generically arises when and eigenvalue, say λ_1, goes through -1. In this case, $C(-1) = (-1-\lambda_1)(-1-\lambda_2)(-1-\lambda_3) = 0$. As key parameter we choose η and compute its critical flip bifurcation value η_F as solution to $C(-1) = 0$, that is, according to (65), to $1+T+S+D = 0$.

Replacing (45), (46) and (47) in $1+T+S+D = 0$, and solving for η, we obtain (48).

We observe that $\eta_F > 0$ if and only if both the numerator and the denominator of (48) are negative, or both are positive, that is if and only if one of the two inequalities in (49) holds. ■

Proof of Proposition 8

According to Barinci and Drugeon (2017) or to Bosi et al. (2025), in the case of a three-dimensional system, a Neimark-Sacker bifurcation arises if and only if

$$S - 1 = D(T-D) \tag{66}$$

$$-2 < T - D < 2 \tag{67}$$

(ε_N, η_N) is the unique solution to the system

$$\begin{aligned} T &= D \\ S &= 1 \end{aligned}$$

In other terms, (ε_N, η_N) satisfies system (66)-(67), that is, given $\varepsilon = \varepsilon_N$, η_N is actually the corresponding Neimark-Sacker bifurcation value.

η_N is positive if and only if the nominator and the denominator in (50) are both negative or both positive, that is if and only if (51) holds.

Focus on the characteristic polynomial with $(\varepsilon, \eta) = (\varepsilon_N, \eta_N)$.

$$\begin{aligned} C(0) &= -D(\varepsilon_N, \eta_N) = -\frac{z\alpha + \alpha^2}{z(1+\alpha^2)} < 0 \\ C(1) &= 1 - T(\varepsilon_N, \eta_N) + S(\varepsilon_N, \eta_N) - D(\varepsilon_N, \eta_N) \\ &= 2\frac{z\left(1-\alpha+\alpha^2\right) - \alpha^2}{z(1+\alpha^2)} > 0 \Leftrightarrow z > \frac{\alpha^2}{1-\alpha+\alpha^2} \end{aligned}$$

Thus, under condition (51), we obtain $C(0) < 0 < C(1)$.

Then, at the bifurcation point, one eigenvalue, say λ_1, is real with $\lambda_1 \in (0, 1)$, while the other two, λ_2 and λ_3, are complex conjugates with $|\lambda_2| = |\lambda_3| = 1$.

Thus, in the right or in the left neighborhood of η_N there are three eigenvalues inside the unit circle: $|\lambda_1|, |\lambda_2|, |\lambda_3| < 1$. In this case, since there is one non-predetermined variable (n_t) and two predetermined variables (k_t and N_t), the equilibrium is locally indeterminate. ■

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