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Voluntary simplicity, the Laffer Curve and the Green Paradox

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Abstract

This paper develops a simple general equilibrium model with social capital accumulation. The representative household chooses how much time to allocate between work and social capital accumulation. Social capital generates satisfaction, but this satisfaction is affected by pollution. This paper considers two cases: (1) pollution reduces the marginal utility of social capital (social withdrawal effect) and (2) pollution increases the marginal utility of social capital (social engagement effect). Pollution, treated as a pure externality, is assumed to originate from production. In line with Pigouvian principles, the government introduces a proportional tax on production to finance depollution expenditures under a balanced budget rule. When preferences exhibit a social engagement effect or a weak social withdrawal effect, the economy has a unique steady state, which may experience a Laffer Curve. When preferences exhibit a strong social withdrawal effect, two steady states can coexist: one characterized by a high level of social capital and low consumption (voluntary simplicity steady state), and the other by a low level of social capital and high consumption (consumerist steady state). As in the case of a unique steady state, a Laffer Curve may emerge at the consumerist steady state but never at the voluntary simplicity steady state. However, the voluntary simplicity steady state always exhibits a Green Paradox, a phenomenon that never occurs at the consumerist steady state. Regarding the dynamics, the unique steady state that arises when preferences exhibit a social engagement effect or a weak social withdrawal effect is saddle-path stable. When preferences are described by a strong withdrawal effect, we prove that the consumerist steady state is also always saddle-path stable while the voluntary simplicity steady state is always locally indeterminate.

Keywords: Green Paradox, Laffer Curve, local indeterminacy and social capital.

JEL code: C62, H23, O44.

1 Introduction

Avoiding overconsumption is often highlighted as a necessity for making economic growth sustainable (Demirel, 2022). The idea is simple: reducing consumption leads to lower production and, consequently, lower pollution emissions. Limiting consumption to satisfy only essential needs in order to devote more time to non-materialistic sources of satisfaction is known in sociology as voluntary simplicity (Osikominu and Bocken, 2020). More specifically, as noted by Rich et al. (2017), relationships and community, what Bourdieu (1980) refers to as "social capital", are at the core of the voluntary simplicity lifestyle. This way of living thus proposes substituting consumption with social capital.

The first attempt to propose a theoretical framework explaining social capital accumulation as a voluntary process was made by Glaeser et al. (2002). They develop a life-cycle model in which the economic agent invests in social capital because it provides satisfaction (a non-market return). Within their framework, they replicate an important stylized fact: social capital first increases with age and then declines. The rationale for this outcome is as follows: toward the end of life, the benefits of social capital accumulation decrease, and the agent loses incentives to engage in socialization.

Since Glaeser et al. (2002), several contributions have incorporated social capital into general equilibrium models and highlighted its positive effects on the long-run growth rate of the economy. This is particularly the case when social capital interacts with human capital (Chou, 2006; Dinda, 2008, 2014) or when it facilitates innovation (Thompson, 2018). Interestingly, these theoretical contributions have not considered the environment. However, the existence of the voluntary simplicity movement underscores the importance of examining the interplay between social capital and pollution. The objective of this paper is to fill this gap by explicitly considering this interaction and analyzing its implications for green fiscal policy.

From an empirical perspective, there is no direct evidence of a possible effect of pollution on the incentive to accumulate social capital. However, linking medical and sociological findings provides an interesting insight. Indeed, pollution is known to exacerbate depressive symptoms (Fan et al., 2020), while individuals experiencing depression tend to spend less time in social interactions (Elmer and Stadtfeld, 2020). Connecting these findings suggests that pollution reduces the incentive to accumulate social capital, thereby lowering its marginal utility. Hereafter, we refer to this negative relationship as the *social withdrawal effect*. Despite the lack of empirical evidence, from a purely theoretical standpoint, the opposite effect could also be considered. Indeed, households might choose to increase their social capital accumulation to compensate for the utility loss induced by rising pollution. In this case, pollution increases the marginal utility of social capital. Hereafter, we refer to this positive relationship as the *social engagement effect*. Even though the social withdrawal effect is the only empirically supported effect, we propose to consider both effects in this study.

As previously discussed, the preference structure considered in this paper is general enough to encompass both the social withdrawal and social engagement effects. However, to keep the analysis as simple as possible, we exclude physical capital, meaning that the production sector generates a pure consumption good using labor supply as the sole productive input. The environmental dimension is captured by a pollution externality arising from this production activity. To further simplify the analysis, pollution is introduced as a pure flow. Finally, in line with Pigouvian principles, the government is assumed to levy a proportional tax on production to finance depollution expenditures under a balanced budget rule.

This simple framework yields interesting results for both the long run and the short run. In the long run, both the social engagement effect and the weak social withdrawal effect ensure the existence of a unique steady state. While a higher green tax rate always reduces pollution at this steady state, we highlight the possible existence of a Laffer Curve[1]. The rationale behind this result is as follows. At the steady state, if the consumption level is relatively high compared to social capital, the marginal utility of consumption is low, while the marginal utility of social capital is high. A higher tax rate reduces household income, prompting the household to reallocate time previously devoted to labor supply toward social capital accumulation. If the reduction in labor supply is sufficiently large, the resulting decline in production can outweigh the direct effect of a higher tax rate on tax revenue, leading to a decrease in total tax income and thus generating a Laffer Curve.

When preferences are characterized by a strong social withdrawal effect, two steady states coexist. The first is marked by a high level of social capital and a low level of consumption, while the second features a low level of social capital and high consumption. The first steady state clearly describes a voluntary simplicity regime and is therefore referred to as the voluntary simplicity steady state. The second is called the consumerist steady state. Given the characteristics of the consumerist steady state (high consumption and low social capital), it is not surprising that we recover the Laffer Curve observed when preferences are shaped by a social engagement effect (or a weak social withdrawal effect). Interestingly, this Laffer Curve never arises around the voluntary simplicity steady state, because in this case, a higher tax rate always leads to an increase in labor supply (as the marginal utility of consumption remains high compared to the low marginal utility of social capital). However, another adverse effect emerges: higher labor supply leads to higher production and, consequently, higher pollution. In other words, at the voluntary simplicity steady state, an increase in the green tax rate always results in higher pollution, a Green Paradox[2] is systematically at play.

[1]The Laffer Curve is a bell-shaped relationship between the tax rate and tax revenue. It indicates that two different tax rates, a low one and a high one, can generate the same tax revenue. This concept was popularized by Arthur Laffer in the late 1970s (Wanniski, 1978) and is based on an idea expressed several centuries earlier by Ibn Khaldun:

"It should be known that at the beginning of a dynasty, taxation yields a large revenue from small assessments. At the end of the dynasty, taxation yields a small revenue from large assessments" (Khaldun, 1377, p. 230).

[2]The Green Paradox was initially introduced by Sinn (2008) and refers to a situation where a higher green tax rate worsens environmental quality.

From a public policy perspective, if the economy is at a voluntary simplicity steady state, the existence of a Green Paradox indicates that the green tax is clearly a bad instrument to control pollution. In all other cases, the green tax has to be tuned to avoid the economy being located along the decreasing branch of the Laffer Curve.

The long-run results reveal an incompatibility between the Laffer Curve and the Green Paradox. This incompatibility is similar to the one identified by Bosi and Desmarchelier (2017) in a very different context. Specifically, they developed a Ramsey economy with physical capital but without social capital, where pollution results from consumption and is treated as a stock variable. They find that this economy always has a unique steady state and highlight that a weak positive effect of pollution on the marginal utility of consumption leads to the existence of a Laffer Curve at the steady state, while a strong effect implies the presence of a Green Paradox. They also emphasize that the existence of a Laffer Curve excludes the possibility of a Green Paradox, and vice versa. Similarly, in the present paper, we observe that the Green Paradox emerges only at the voluntary simplicity steady state, whereas the Laffer Curve appears exclusively at the consumerist steady state (or also at the unique steady state when preferences are characterized by a social engagement effect or a weak social withdrawal effect). This incompatibility, observed in distinct contexts, underscores its robustness.

Concerning the short run, we prove that the unique steady state that arises when preferences are characterized by either a social engagement effect or a weak social withdrawal effect is always saddle-path stable. When preferences are instead characterized by a strong social withdrawal effect, we show that the consumerist steady state is also always saddle-path stable. However, we also find that the voluntary simplicity steady state is always locally indeterminate, allowing for fluctuations driven by self-fulfilling expectations.

The paper is organized as follows. Section 2 presents the model, section 3 studies both steady state existence and uniqueness, and section 4 studies the dynamics of this economy. Finally, section 5 concludes the paper. All proofs are gathered in the Appendix.

2 The model

2.1 The representative household

At each moment in time, the representative household shares one unit of time between working (l) and socialization ($1-l$). Her income only comes from labor supply and is fully consumed, namely:

$$c = wl \tag{1}$$

where c and w represent respectively consumption and the wage rate. The socialization effort allows to accumulate social capital s according to the following simple linear process:

$$\dot{s} = B(1-l) - \delta s \quad (2)$$

where B represents the ability of the household to socialize and δ is the depreciation rate of social capital. (2) is analogous to the process introduced by Glaeser et al. (2002). Following their approach, social capital s can be defined as the size of a household's contact list (the "Rolodex" in Glaeser et al. (2002)) and is assumed to generate non-market returns, that is, a higher s increases the household's utility.

Moreover, empirical evidence suggests that air pollution (P) increases depressive behavior (Fan et al., 2020), while Elmer and Stadtfeld (2020) have shown that depressive symptoms are associated with less time spent in social interactions. Considering the findings of Fan et al. (2020) and Elmer and Stadtfeld (2020) together, it follows that higher pollution levels reduce the incentives to accumulate social capital, indicating that pollution lowers the marginal utility of social capital (social withdrawal effect). On the other hand, since pollution reduces utility, households might also choose to compensate for this decline by increasing their social interactions. In this case, pollution would increase the marginal utility of social capital (social engagement effect). While the social withdrawal effect appears to be empirically supported, to the best of our knowledge, there is no empirical evidence for the social engagement effect. Nevertheless, we consider both effects in the following analysis to explore their implications. More precisely, the utility function U takes the following form:

$$U(c, s, P) \equiv u(c) + v(s, P)$$

The next assumption sums up properties of both u and v.

Assumption 1 *$u''(c) < 0 < u'(c)$ and*[3] *$v_{ss} < 0 < v_s$ such that $\lim_{c\to 0} u'(c) = \lim_{s\to 0} v_s = +\infty$ and $\lim_{c\to +\infty} u'(c) = \lim_{s\to +\infty} v_s = 0$.*

Assumption 1 does not preclude any sign for the cross derivative v_{sP}. Indeed, as discussed before, $v_{sP} < 0$ corresponds to a social withdrawal effect while $v_{sP} > 0$ corresponds to a social engagement effect. While pollution is able to affect marginal utility of social capital, it remains a pure externality for the representative household. More precisely, the representative household chooses l in order to maximize her intertemporal utility:

$$\max_{l} \int_0^{+\infty} e^{-\theta t} \left[u(c) + v(s, P)\right] dt$$

subject to (1) and (2) and taking as given both w and P. $\theta > 0$ represents the household's rate of time preference. The Hamiltonian writes $H = u(c) + v(s, P) + \lambda \left[B(1-l) - \delta s\right]$ where $\lambda > 0$ is the Lagrangian multiplier. First order conditions are given by:

[3]For notational parsimony, in the rest of this paper, $v_s \equiv \partial v/\partial s$, $v_{ss} \equiv \partial^2 v/\partial s^2$ and $v_{sP} \equiv \partial^2 v/(\partial s \partial P)$.

$$\frac{\partial H}{\partial l} = u'(c)\,w - \lambda B = 0 \tag{3}$$

$$\frac{\partial H}{\partial s} = v_s(s,P) - \delta\lambda = \theta\lambda - \dot{\lambda} \tag{4}$$

jointly with $\partial H/\partial \lambda = \dot{s}$ as well as the transversality condition $\lim_{t\to+\infty} e^{-\theta t}\lambda s = 0$.

From (3), we get $\lambda = (w/B)\,u'(c)$ and then, (4) rewrites:

$$\dot{l} = \frac{1}{\varepsilon_{cc}}\left[(\delta+\theta) - \frac{B}{w}\frac{v_s(s,P)}{u'(c)}\right] l \tag{5}$$

With $\varepsilon_{cc} \equiv cu''(c)/u'(c) < 0$. $-1/\varepsilon_{cc}$ is the usual intertemporal elasticity of substitution of consumption. (5) is the Euler equation of the household's program, it captures how the household's smooths her labor supply through time.

2.2 The production sector

To keep things as simple as possible, let us assume that the consumption good is produced in quantity Y by using a simple linear technology which uses labor as single productive input:

$$Y = AL$$

where $A > 0$ stands for labor productivity and L represents the labor demand. Moreover, we assume that production generates pollution and then, in order to regulate it, the Government introduces a tax rate $\tau \in (0,1)$ on the production level. The profit π is then given by:

$$\pi \equiv (1-\tau)\,AL - wL$$

The firm behaves competitively and then, chooses the amount of labor L which maximizes π. The linearity of π implies a zero-profit condition:

$$w = (1-\tau)\,A \tag{6}$$

2.3 Pollution and the Government

To keep the spirit of a very simple framework, let us consider that pollution is a pure flow[4], namely:

$$P = bY - \gamma G \tag{7}$$

[4]This strong assumption is sometime considered in Ramsey-Cass-Koopmans framework (see for instance Fernandez et al., 2012 or Itaya, 2008). The reader interested in a model with pollution accumulation is refereed to Bosi and Desmarchelier (2018).

where $b > 0$ represents the environmental impact of production, G stands for depollution expenditures while $\gamma > 0$ is the depollution efficiency. To keep things as simple as possible, depollution expenditures are financed by the Government using the tax revenue according to a balanced budget rule:

$$G = \tau Y \tag{8}$$

Considering (8), (7) writes:

$$P = (b - \gamma\tau) Y$$

Assumption 2 $\tau < b/\gamma$

Assumption 2 ensures that $P > 0$.

2.4 Equilibrium

This economy is composed by two markets: the labor market and the goods market. At the equilibrium, total labor demand (L) is equal to total labor supply (Nl where $N > 0$ stands for the population size), that is $L = Nl$. To simplify the exposition, we assume that the population remains constant over time ($\dot{N} = 0$) and we normalize the population size to the unity ($N = 1$). On the goods market, at the equilibrium, the total supply (Y) is just equal to the total demand ($Nc + G$), that is:

$$c = (1 - \tau) Al \tag{9}$$

In addition, one can remark that considering (6), then (1) gives (9).

The dynamical system representing the evolution of the economic equilibrium is then simply given by:

$$\dot{l} = f_1(l, s) = \frac{1}{\varepsilon_{cc}} \left[(\delta + \theta) - \frac{B}{(1 - \tau) A} \frac{v_s(s, P(l))}{u'(c(l))} \right] l \tag{10}$$

$$\dot{s} = f_2(l, s) = B(1 - l) - \delta s \tag{11}$$

with:

$$c(l) \equiv (1 - \tau) Al$$
$$P(l) \equiv (b - \gamma\tau) Al$$

3 Steady state and comparative static

In order to explore both existence and unicity/multiplicity of a steady state, let us consider now the following functional forms:

$$u(c) \equiv \frac{c^{1-\varepsilon}}{1-\varepsilon} \text{ and } v(s, P) \equiv \alpha \frac{\left(sP^{-\eta}\right)^{1-\varphi}}{1-\varphi} \tag{12}$$

with $\varepsilon > 0$, $\eta > 0$, $\varphi > 0$ and $\alpha > 0$. Considering (12), $\varepsilon_{cc} = -\varepsilon < 0$ and:

$$\frac{Pv_{sP}}{v_s} = \eta\left(\varphi - 1\right)$$

it follows that $v_{sP} < 0$ if and only if $\varphi < 1$ (social withdrawal effect) and $v_{sP} > 0$ if and only if $\varphi > 1$ (social engagement effect). Interestingly, η captures the magnitude of both social withdrawal and social engagement effects.

To discuss conditions under which a steady state exists, let:

$$\begin{aligned}
\sigma\left(l\right) &\equiv \alpha^{\frac{1}{\varphi}}\kappa l^{\frac{\varepsilon-\eta(1-\varphi)}{\varphi}} + l - 1 \\
\kappa &\equiv \frac{\delta B^{\frac{1-\varphi}{\varphi}} A^{\frac{\varepsilon-1-\eta(1-\varphi)}{\varphi}}}{\left(\delta+\theta\right)^{\frac{1}{\varphi}}} \frac{\left(1-\tau\right)^{\frac{\varepsilon-1}{\varphi}}}{\left(b-\gamma\tau\right)^{\eta\left(\frac{1-\varphi}{\varphi}\right)}} > 0 \\
\bar{l} &\equiv \alpha^{\frac{1}{\eta(1-\varphi)-\varepsilon+\varphi}} \left[\frac{\varphi}{\left(\eta\left(1-\varphi\right)-\varepsilon\right)\kappa}\right]^{\frac{\varphi}{\varepsilon-\eta(1-\varphi)-\varphi}} > 0
\end{aligned}$$

Proposition 1 *Consider functional forms (12).*

(1) If $\varepsilon - \eta\left(1-\varphi\right) > 0$, there exists a unique steady state for this economy $\left(l^*, s^*\right)$ with $s^* = \left(B/\delta\right)\left(1-l^*\right)$ provided that $l = l^*$is the unique solution of $\sigma\left(l\right) = 0$.

(2) If $\varepsilon - \eta\left(1-\varphi\right) < 0$, then:

(a) $\alpha < \alpha^*$ implies that there exists two steady states $\left(l_1, s_1\right)$ and $\left(l_2, s_2\right)$ such that $0 < l_1 < \bar{l} < l_2 < 1$, and $s_1 = \left(B/\delta\right)\left(1-l_1\right) > s_2 = \left(B/\delta\right)\left(1-l_2\right)$.

(b) $\alpha = \alpha^*$ implies $l_1 = l_2 = \bar{l}$.

(c) $\alpha > \alpha^*$ implies that there is no steady state.

Provided that α^* is such that when $\alpha = \alpha^*$then $\sigma\left(\bar{l}\right) = 0$. Moreover, $l = l_1$ and $l = l_2$ are the two solutions of $\sigma\left(l\right) = 0$ when $\alpha < \alpha^*$.

Proof. See the Appendix. ∎

It is interesting to remark that the social engagement effect ($\varphi > 1$) ensures the existence of a unique steady state. By continuity, it follows from Proposition 1 that this property is preserved under a weak social withdrawal effect (i.e. $\varphi < 1$ such that $\varepsilon - \eta\left(1-\varphi\right) > 0$). However, under a strong effect (i.e. $\varphi < 1$ jointly with $\varepsilon - \eta\left(1-\varphi\right) < 0$), multiple steady states can arise, one with a high level of social capital (i.e. s_1) and a low consumption level (i.e. $c\left(l_1\right)$) and the other one with a low level of social capital (i.e. s_2) and a high consumption level (i.e. $c\left(l_2\right)$). In what follows, the first steady state will be called the voluntary simplicity[5] steady state and the second one will be called the consumerist steady state. The next proposition explores how the tax rate affects the main variables at each steady state.

[5]The concept of voluntary simplicity was popularized by Elgin (1981). The voluntary simplicity lifestyle consists to limit consumption to the essential needs in order to engage more time devoted to non-materialistic sources of satisfaction (Osikominu and Bocken, 2020).

Proposition 2 *Let $b = \gamma = 1$. The impact of the tax rate τ on the steady states is the following.*

Focus first on (l^*, s^*)

$$\frac{\tau}{l^*}\frac{\partial l^*}{\partial \tau} < (>)\, 0 \text{ iff } \varepsilon - \eta\,(1-\varphi) < (>)\, 1$$
$$\frac{\tau}{s^*}\frac{\partial s^*}{\partial \tau} > (<)\, 0 \text{ iff } \varepsilon - \eta\,(1-\varphi) < (>)\, 1$$
$$\frac{\tau}{P^*}\frac{\partial P^*}{\partial \tau} < 0 \text{ and } \frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} < 0$$

Focus on (l_1, s_1)

$$\frac{\tau}{l_1}\frac{\partial l_1}{\partial \tau} > 0 \text{ and } \frac{\tau}{s_1}\frac{\partial s_1}{\partial \tau} < 0$$
$$\frac{\tau}{P_1}\frac{\partial P_1}{\partial \tau} > 0 \text{ and } \frac{\tau}{c_1}\frac{\partial c_1}{\partial \tau} > 0$$

Focus on (l_2, s_2)

$$\frac{\tau}{l_2}\frac{\partial l_2}{\partial \tau} < 0 \text{ and } \frac{\tau}{s_2}\frac{\partial s_2}{\partial \tau} > 0$$
$$\frac{\tau}{P_2}\frac{\partial P_2}{\partial \tau} < 0 \text{ and } \frac{\tau}{c_2}\frac{\partial c_2}{\partial \tau} < 0$$

Proof. See the Appendix. ■

This proposition deserves economic interpretations. First, consider the case of a strong social withdrawal effect (i.e. $\varphi < 1$ jointly with $\varepsilon - \eta\,(1-\varphi) < 0$), which leads to a situation where two steady states coexist: the voluntary simplicity steady state ((l_1, s_1)) and the consumerist steady state ((l_2, s_2)). At the voluntary simplicity steady state, the level of social capital is high, while the consumption level is low. As a result, the marginal utility of social capital is low, whereas the marginal utility of consumption is high (see Assumption 1). An increase in the tax rate reduces household income. Given the comparison of marginal utilities, the household responds by increasing labor supply, which allows for a higher consumption level and a lower level of social capital. Interestingly, a higher labor supply leads to a higher production level and, consequently, a higher pollution level: a Green Paradox is at play. At the consumerist steady state, the relationships are completely reversed. Specifically, at this steady state, social capital is low while consumption is high, implying a high marginal utility of social capital and a low marginal utility of consumption (see Assumption 1). Since a higher tax rate reduces household income, the household reacts by decreasing labor supply to increase social capital accumulation while reducing consumption. Indeed, a lower consumption level has a limited impact on utility (due to the low marginal utility of consumption), whereas a higher level of social capital significantly boosts utility (due to its high marginal utility). Finally, a lower labor supply results in a lower production level and, consequently, a lower pollution level.

Focus on the social engagement effect (i.e. $\varphi > 1$). The steady state is then given by (l^*, s^*). As previously mentioned, a higher tax rate implies a lower income. If the magnitude of the social engagement effect is high[6] (low[7]), then the long-run level of social capital must be high (low), and its marginal utility must be low (high). In this case, the household must increase (decrease) her labor supply in response to the higher tax rate in order to reduce (increase) the level of social capital. Interestingly, while a higher tax rate tends to reduce the production level, the increase in labor supply tends to increase the production level. However, Proposition 2 points out that the direct negative effect of the tax rate on the production level always dominates the possible positive effect of the labor supply reaction, thus, a higher tax rate always reduces the production level. Ultimately, this also implies a lower consumption possibility as well as a lower pollution level.

From an economic policy perspective, it appears that if the economy is located in the long run at the voluntary simplicity steady state and if the government aims to control the pollution level, it is not a good idea to introduce a green tax, as it inevitably increases the pollution level (Green Paradox).

Bosi and Desmarchelier (2017) have already highlighted the potential occurrence of a long-run Green Paradox in a Ramsey model with a pollution stock resulting from consumption in an endogenous labor supply context. In their model, the economy always possesses a unique steady state. However, they observe that when there is no room for a Green Paradox in the long run, the economy experiences a Laffer Curve. Could this incompatibility also be present in the current context? The existence of a Laffer Curve seems possible when $l = l^*$ (with $\varepsilon - \eta(1-\varphi) < 1$) or when $l = l_2$ (the consumerist steady state), because a higher tax rate implies a lower labor supply, which in turn leads to a lower production level, potentially resulting in lower tax revenue. However, at the voluntary simplicity steady state i.e. $l = l_1$), a higher tax rate always implies a higher labor supply and, therefore, a higher production level, leading to higher tax revenue, rendering impossible the existence of a Laffer Curve. In the following, we propose to explore this eventuality.

Let $G \equiv \tau Y = \tau A l$ and $G^* = \tau A l^*$, $G_1 = \tau A l_1$ and $G_2 = \tau A l_2$.

Proposition 3 *Let $b = \gamma = 1$. There is no room for a Laffer Curve at $l = l^*$ when $\varepsilon - \eta(1-\varphi) > 1$ as well as at $l = l_1$.*

Proof. See the Appendix. ■

As intuition suggested previously, cases where a higher tax rate increases the labor supply render impossible the existence of a Laffer Curve (i.e. when $l = l^*$ with $\varepsilon - \eta(1-\varphi) > 1$ and when $l = l_1$). Concerning $l^*$and l_2, it is not possible to discuss general conditions allowing to observe the existence of a Laffer Curve. Nevertheless, to convince the reader that a Laffer Curve can well arise in those two cases, we propose to develop two numerical examples.

Example 1: $l = l^*$ with $\varepsilon - \eta(1-\varphi) < 1$

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[6] i.e. $\varepsilon - \eta(1-\varphi) > 1$.

[7] i.e. $\varepsilon - \eta(1-\varphi) < 1$.

The aim is to illustrate the possible existence of a Laffer Curve at $l = l^*$ when $\varepsilon - \eta (1 - \varphi) < 1$. To do so, let us consider that:

$$\varepsilon = \varphi + \eta (1 - \varphi) \tag{13}$$

this implies,

$$l^* = \frac{1}{1 + \alpha^{\frac{1}{\varphi}} \kappa} < 1$$

Now consider the following calibration:

Parameters	δ	A	B	b	γ	θ	φ	α	η
Values	0.025	1	1	1	1	0.01	1/2	1	2

(14)

From (13), it follows that $\varepsilon = 3/2$. Calibration (14) implies $\varepsilon - \eta (1 - \varphi) = 1/2 < 1$. Fig.1 represents the bell-shaped relation linking τ and G at the steady state, that is the existence of a Laffer Curve.

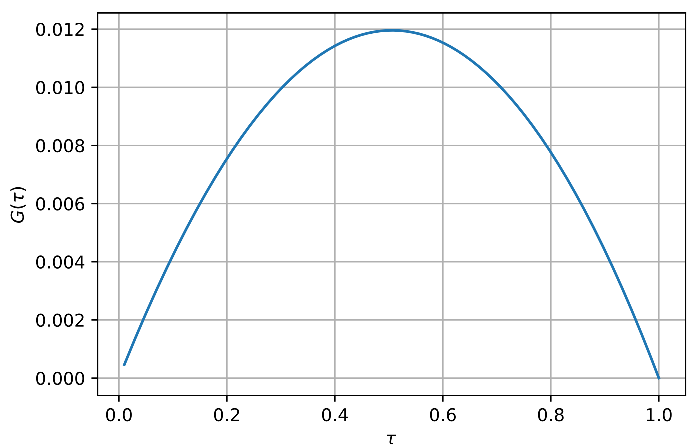

Fig. 1. Laffer Curve at $l = l^*$ and $\varepsilon - \eta (1 - \varphi) < 1$

Example 2: $l = l_2$ (consumerist steady state)

The aim is to illustrate the possible existence of a Laffer Curve at $l = l_2$. To do so, let us assume that:

$$\varepsilon = \eta (1 - \varphi) - \varphi \tag{15}$$

In this case,

$$l_1 = \frac{1}{2}\left(1 - \sqrt{1 - 4\alpha^{\frac{1}{\varphi}}\kappa}\right)$$

$$l_2 = \frac{1}{2}\left(1 + \sqrt{1 - 4\alpha^{\frac{1}{\varphi}}\kappa}\right)$$

Provided that $\alpha < \alpha^* = (1/4\kappa)^{\varphi}$.

Now consider the following calibration:

Parameters	δ	A	B	b	γ	θ	φ	α	η
Values	0.025	1	1	1	1	0.01	1/2	0.001	2

(16)

From (15), it appears that $\varepsilon = 1/2$. Interestingly, $\varepsilon - \eta(1-\varphi) = -1/2 < 0$. Fig.2 represents the bell-shaped relation linking τ and G at the steady state, that is the existence of a Laffer Curve.

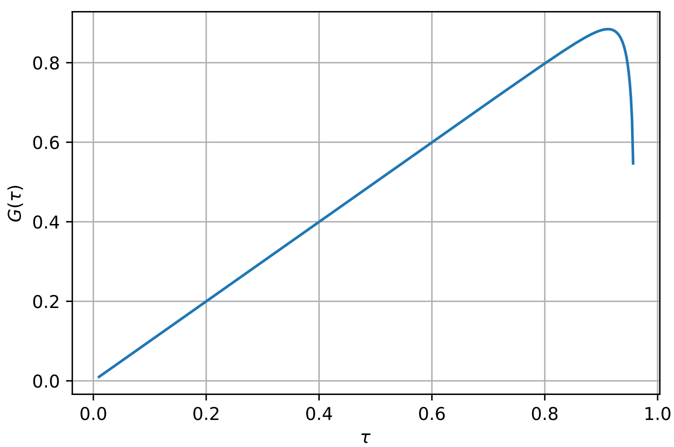

Fig. 2. Laffer Curve at $l = l_2$.

4 Dynamics

The global dynamics of this economy can be simply studied through a phase diagram analysis. Indeed, considering functional forms (12), it appears that:

$$\dot{l} > (<)\, 0 \text{ iff } s < (>)\, \frac{B}{\delta}\alpha^{\frac{1}{\varphi}}\kappa l^{\frac{\varepsilon-\eta(1-\varphi)}{\varphi}} \equiv \phi_1(l)$$

And,

$$\dot{s} > (<)\, 0 \text{ iff } s < (>)\, \frac{B}{\delta}(1-l) \equiv \phi_2(l)$$

Fig. 3 represents the phase diagram when $\varepsilon - \eta(1-\varphi) > 0$, that is, when preferences are described by a social engagement effect or weak social withdrawal effect. Conversely, Fig. 4 represents the phase diagram when $\varepsilon - \eta(1-\varphi) < 0$ jointly with $\alpha < \alpha^*$(see Proposition 1), that is, when preferences are described by a high social withdrawal effect.

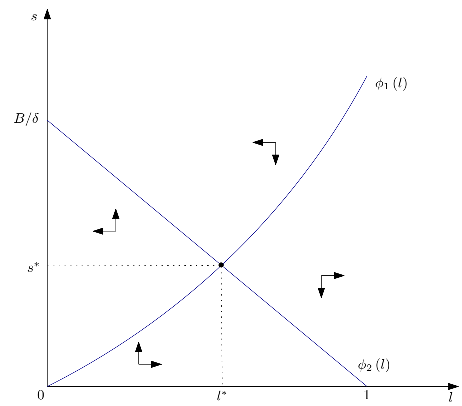

Fig.3. $\varepsilon - \eta(1-\varphi) > 0$.

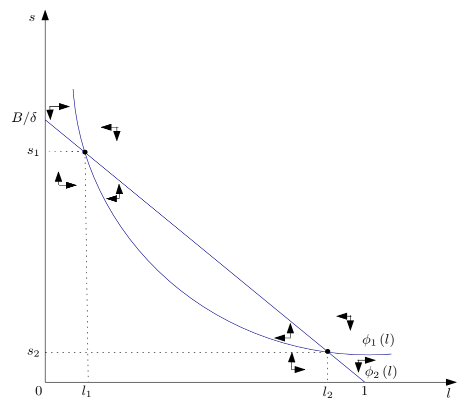

Fig.4. $\varepsilon - \eta(1-\varphi) < 0$.

From Fig. 3, it appears that preferences described by a social engagement effect or a weak social withdrawal effect lead to a saddle-path stable steady state. This means that, for a given initial level of social capital, there exists a unique initial level of labor supply that allows for reaching the steady state. Moreover, considering Fig. 4, preferences described by a strong social withdrawal effect imply that the consumerist steady state (i.e., (l_2, s_2)) is also saddle-path stable while the voluntary simplicity steady state is indeterminate. This means that for a given initial level of social capital, there exists a continuum of values for the initial level of labor supply that allows for convergence to the voluntary simplicity steady state (local indeterminacy). The existence of local indeterminacy is related to the existence of self-fulfilling prophecies (Azariadis, 1981). That is, let us show that expectations are self-fulfilling at the voluntary simplicity steady state. To do so, let us assume that the representative household expects an increase in pollution tomorrow. Because of the social withdrawal effect, the household knows that the marginal utility of social capital will be lower tomorrow, and thus, it has an incentive to reduce social capital accumulation by decreasing the time devoted to socializing and hence, increasing her labor supply. Such an incentive is magnified when the marginal utility of consumption is high, which precisely corresponds to the voluntary simplicity steady state because the consumption level is low. Additionally, more labor supply implies more production and, consequently, more pollution. It follows that expecting a higher pollution level results in pollution effectively increasing: expectations are self-fulfilling.

Moreover, from Fig. 4, it appears that for a given initial level of social capital, the household will choose the initial level of labor supply that allows for convergence toward the most relevant steady state, depending on her preferences. More precisely, if the household overvalues consumption (or social capital), she will choose her initial level of labor supply in order to reach the consumerist (or voluntary simplicity) steady state in the long run.

The saddle-path stability of l^* and l_2 (consumerist steady state) and the local indeterminacy of l_1 (i.e. voluntary simplicity steady state) can also be approached by studying the eigenvalues of the Jacobian matrix J of the system (10)-(11) evaluated at $l = l^*$, $l = l_1$or $l = l_2$. By linearizing the dynamical system (10)-(11) and using functional forms (12), it appears that:

$$J \equiv \begin{bmatrix} \frac{\partial f_1}{\partial l} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_2}{\partial l} & \frac{\partial f_2}{\partial s} \end{bmatrix} = \begin{bmatrix} (\delta + \theta)\left(1 - \frac{\eta(1-\varphi)}{\varepsilon}\right) & -(\delta + \theta)\frac{\varphi}{\varepsilon}\left(\frac{l}{1-l}\right)\frac{\delta}{B} \\ -B & -\delta \end{bmatrix}$$

Depending upon the steady state considered, $l = l^*$, $l = l_1$or $l = l_2$. The trace T and the determinant D of J are given by:

$$T = \frac{\theta\left(\varepsilon - \eta\left(1 - \varphi\right)\right) - \delta\eta\left(1 - \varphi\right)}{\varepsilon}$$
$$D = -\delta\left(\theta + \delta\right)\frac{\varphi l + \left(1 - l\right)\left(\varepsilon - \eta\left(1 - \varphi\right)\right)}{\varepsilon\left(1 - l\right)} \equiv D\left(l\right)$$

Proposition 4 *Both $l^*$and l_2 are saddle-path stable while l_1 is locally indeterminate.*

Proof. See the Appendix. ■

We obtain the same results as those pointed out by the phase diagram analysis. It follows that the voluntary simplicity steady state is characterized by fluctuations driven by the household's expectations (local indeterminacy), a situation that never occurs at the consumerist steady state.

5 Conclusion

This paper explicitly considers the effect of pollution on the marginal utility of social capital. A negative effect is empirically supported and corresponds to a social withdrawal effect, whereas a positive effect, though not empirically grounded, makes theoretical sense and corresponds to a social engagement effect. In this economy, social capital is the only form of capital considered and the production sector relies solely on labor supply to produce a consumption good. Production generates a pollution externality, modeled as a pure flow. Finally, the government levies a proportional tax on production to finance depollution expenditures, following a balanced budget rule.

In the long run, when preferences are characterized by either a social engagement effect or a weak social withdrawal effect, the economy has a unique steady state. Interestingly, a Laffer Curve can emerge at this steady state. When preferences are characterized by a strong social withdrawal effect, two steady states can coexist. The first features a high level of social capital and low consumption (the voluntary simplicity steady state), while the second exhibits the opposite characteristics, with high consumption and low social capital (the consumerist steady state). Notably, the consumerist steady state can experience a Laffer Curve, whereas the voluntary simplicity steady state does not allow for one. However, a Green Paradox always arises at the voluntary simplicity steady state but never occurs at the consumerist steady state.

From a public policy perspective, the existence of a Green Paradox clearly indicates that introducing a green tax is a poor strategy for controlling pollution if the economy is at the voluntary simplicity steady state. In all other cases, the government must carefully adjust the green tax rate to prevent the economy from operating along the downward-sloping branch of the Laffer Curve.

Regarding the dynamics, we prove that the voluntary simplicity steady state is always indeterminate and thus subject to fluctuations driven by household expectations. Interestingly, both the consumerist steady state and the unique steady state that emerges when preferences are characterized by either a social engagement effect or a weak social withdrawal effect are always saddle-path stable.

6 Appendix

Proof of Proposition 1

At the steady state, (11) gives that:

$$s = \frac{B}{\delta}(1-l) \tag{17}$$

Considering (17) and using functional forms (12), equation (10) gives at the steady state that $\sigma(l) = 0$. It follows that a steady state for this economy exists if and only if there exists $l \in (0,1)$ such that $\sigma(l) = 0$.

First of all, assume that $\varepsilon - \eta(1-\varphi) > 0$. In this case,

$$\lim_{l \to 0} \sigma(l) = -1 \text{ and } \lim_{l \to 1} \sigma(l) = \alpha^{\frac{1}{\varphi}} \kappa > 0$$

In addition,

$$\sigma'(l) = 1 + \frac{\varepsilon - \eta(1-\varphi)}{\varphi} \alpha^{\frac{1}{\varphi}} \kappa l^{\frac{\varepsilon-\eta(1-\varphi)}{\varphi} - 1} > 0$$

It follows that there exists a unique $l \in (0,1)$ such that $\sigma(l) = 0$. Let us call this value l^*. The steady state in this case is then unique and given by $(l,s) = (l^*, s^*)$ such that $s^* = (B/\delta)(1-l^*)$.

Now, assume that $\varepsilon - \eta(1-\varphi) < 0$. In this case,

$$\lim_{l \to 0} \sigma(l) = +\infty$$

$$\lim_{l \to 1} \sigma(l) = \alpha^{\frac{1}{\varphi}} \kappa > 0$$

Moreover, $\sigma'(l) < (>) \, 0$ if and only if $l < (>) \, \bar{l}$.

At this step of the reasoning, it appears that two steady state values exist, namely $l_1$and l_2 such that $l_1 < \bar{l} < l_2$ if and only if the following two conditions are verified:

$$\bar{l} < 1 \text{ and } \sigma\left(\bar{l}\right) < 0$$

First of all, $\bar{l} < 1$ if and only if $\alpha < \bar{\alpha}$ with:

$$\bar{\alpha} \equiv \left[\frac{\varphi}{\left(\eta\left(1-\varphi\right)-\varepsilon\right)\kappa}\right]^{\varphi}$$

Furthermore,

$$\sigma\left(\bar{l}\right) = \alpha^{\frac{1}{\eta(1-\varphi)-\varepsilon+\varphi}} \left(\kappa\left[\frac{\varphi}{\left(\eta\left(1-\varphi\right)-\varepsilon\right)\kappa}\right]^{\frac{\varepsilon-\eta(1-\varphi)}{\varepsilon-\eta(1-\varphi)-\varphi}} + \left[\frac{\varphi}{\left(\eta\left(1-\varphi\right)-\varepsilon\right)\kappa}\right]^{\frac{\varphi}{\varepsilon-\eta(1-\varphi)-\varphi}}\right) - 1$$

and then,

$$\lim_{\alpha\to 0}\sigma\left(\bar{l}\right) = -1,\ \lim_{\alpha\to+\infty}\sigma\left(\bar{l}\right) = +\infty \text{ and } \frac{\partial\sigma\left(\bar{l}\right)}{\partial\alpha} > 0$$

That is, it exists a unique $\alpha > 0$, let us call it α^*, such that $\sigma\left(\bar{l}\right) = 0$ while $\alpha < (>) \, \alpha^*$ implies $\sigma\left(\bar{l}\right) < (>) \, 0$. Finally, remark that $\alpha = \bar{\alpha}$ implies $\bar{l} = 1$ and in this case, $\sigma\left(\bar{l}\right) = \bar{\alpha}^{\frac{1}{\varphi}}\kappa > 0$. It follows that $\alpha^* < \bar{\alpha}$. To sum up, it then appears that $\alpha < \alpha^*$ ensures that there exists two steady state l_1 and l_2 such that $0 < l_1 < \bar{l} < l_2 < 1$. When $\alpha = \alpha^*$, $l_1 = l_2 = \bar{l}$ and when $\alpha > \alpha^*$ there is no steady state. The corresponding values for s are obtained by considering $l = l_1$ or $l = l_2$ within equation (17). ■

Proof of Proposition 2

At the steady state,

$$\frac{\tau}{s}\frac{\partial s}{\partial\tau} = -\frac{l}{s}\frac{B}{\delta}\frac{\tau}{l}\frac{\partial l}{\partial\tau} \tag{18}$$

Moreover, differentiating $\sigma(l) = 0$ gives:

$$\frac{\tau}{l}\frac{\partial l}{\partial\tau} = -\frac{\alpha^{\frac{1}{\varphi}}\kappa l^{\frac{\varepsilon-\eta(1-\varphi)}{\varphi}}}{l\sigma'(l)}\frac{\tau}{\kappa}\frac{\partial\kappa}{\partial\tau}$$

When $b = \gamma = 1$, remark that:

$$\frac{\tau}{\kappa}\frac{\partial\kappa}{\partial\tau} = \frac{\tau}{\varphi}\left[\frac{1+\eta\left(1-\varphi\right)-\varepsilon}{1-\tau}\right]$$

Moreover, $\sigma(l) = 0$ implies that:

$$\alpha^{\frac{1}{\varphi}}\kappa l^{\frac{\varepsilon-\eta(1-\varphi)}{\varphi}} = 1 - l$$

and:

$$l\sigma'(l) = l + \frac{\varepsilon - \eta(1-\varphi)}{\varphi}(1-l)$$

When $\varepsilon - \eta(1-\varphi) > 0$, $l = l^*$ and $\sigma'(l^*) > 0$, that is,

$$\varphi l^* + (\varepsilon - \eta(1-\varphi))(1-l^*) > 0 \tag{19}$$

We know also from the proof of proposition (1) that when $\varepsilon - \eta(1-\varphi) < 0$, $l = l_1$ or $l = l_2$ such that $\sigma'(l_1) < 0 < \sigma'(l_2)$. That is,

$$\varphi l_1 + (\varepsilon - \eta(1-\varphi))(1-l_1) < 0 \tag{20}$$

$$\varphi l_2 + (\varepsilon - \eta(1-\varphi))(1-l_2) > 0 \tag{21}$$

And then,

$$\frac{\tau}{l}\frac{\partial l}{\partial \tau} = \frac{\tau(1-l)(\varepsilon - \eta(1-\varphi) - 1)}{(1-\tau)(\varphi l + (\varepsilon - \eta(1-\varphi))(1-l))}$$

That is,

$$\frac{\tau}{l^*}\frac{\partial l^*}{\partial \tau} < (>)\, 0 \text{ iff } \varepsilon - \eta(1-\varphi) < (>)\, 1$$

while,

$$\frac{\tau}{l_1}\frac{\partial l_1}{\partial \tau} > 0 \text{ and } \frac{\tau}{l_2}\frac{\partial l_2}{\partial \tau} < 0$$

From (18),

$$\frac{\tau}{s^*}\frac{\partial s^*}{\partial \tau} > (<)\, 0 \text{ iff } \varepsilon - \eta(1-\varphi) < (>)\, 1$$

while,

$$\frac{\tau}{s_1}\frac{\partial s_1}{\partial \tau} < 0 \text{ and } \frac{\tau}{s_2}\frac{\partial s_2}{\partial \tau} > 0$$

Focus on pollution. When $b = \gamma = 1$,

$$\frac{\tau}{P}\frac{\partial P}{\partial \tau} = -\frac{\tau}{1-\tau}\frac{((1-l)+\varphi l)}{\varphi l + (\varepsilon - \eta(1-\varphi))(1-l)}$$

It follows that,

$$\frac{\tau}{P^*}\frac{\partial P^*}{\partial \tau} < 0,\ \frac{\tau}{P_1}\frac{\partial P_1}{\partial \tau} > 0 \text{ and } \frac{\tau}{P_2}\frac{\partial P_2}{\partial \tau} < 0$$

with $P^* = P(l^*)$, $P_1 = P(l_1)$ and $P_2 = P(l_2)$.
Finally, since $b = \gamma = 1$, remark that $c(l) = P(l)$ and then,

$$\frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} < 0,\ \frac{\tau}{c_1}\frac{\partial c_1}{\partial \tau} > 0 \text{ and } \frac{\tau}{c_2}\frac{\partial c_2}{\partial \tau} < 0$$

with $c^* = c(l^*)$, $c_1 = c(l_1)$ and $c_2 = c(l_2)$. ■

Proof of Proposition 3

Assume that $b = \gamma = 1$:

$$\frac{\tau}{G}\frac{\partial G}{\partial \tau} = 1 + \frac{\tau}{l}\frac{\partial l}{\partial \tau}$$

From Proposition 2, $(\tau/l)(\partial l/\partial \tau) > 0$ when $l = l^*$if $\varepsilon - \eta(1-\varphi) > 1$ as well as when $l = l_1$. In those cases $(\tau/G)(\partial G/\partial \tau) > 0$ ruling out the existence of a Laffer Curve. ■

Proof of Proposition 4

Let us exploit the fact that $T = \lambda_1 + \lambda_2$ and $D = \lambda_1\lambda_2$ with λ_1 and λ_2 are the two eigenvalues of J. Following (19), (20) and (21), $D(l^*) < 0$, $D(l_1) > 0$ and $D(l_2) < 0$. Then, at $l = l^*$ and $l = l_2$, J possesses one stable eigenvalue and one unstable eigenvalue. Since the system (10)-(11) possesses one jump variable (i.e. l) and one predetermined variable (i.e. s), it follows that both l^* and l_2 are saddle-path stable. Moreover, when $l = l_1$, $\varepsilon - \eta(1-\varphi) < 0$ and then, $\varphi < 1$, that is, $T < 0$ meaning that $\lambda_1 < 0$ and $\lambda_2 < 0$. It follows that l_1 is locally indeterminate. ■

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