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Documents de travail

« Mobile money et performance des entreprises de la zone CEDEAO : le rôle de l'écosystème de l'innovation »

Auteurs

Mawuli Kodjovi COUCHORO, Agbessi Augustin DOTO, Tchapo GBANDI,Blaise GNIMASSOUN

Document de Travail n° 2025 – 11

Mars 2025

Bureau d'Économie Théorique et Appliquée BETA

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Contact :
jaoulgrammare@beta-cnrs.unistra.fr

Monopolistic Competition with large firms[*]

Claude d'Aspremont[†] Rodolphe Dos Santos Ferreira[‡]

April 27, 2025

Abstract

We consider the concept of *Cournotian monopolistic competition equilibrium* as a tractable way of taking the strategic behaviour of large firms into account in a general equilibrium framework. Existence is obtained under simple assumptions, ensuring in particular uniqueness of Cournot equilibrium for each group of firms. An extension of the concept, allowing intrasectoral competitive behaviour to vary in intensity is also examined.

Keywords: Oligopolistic and monopolistic competition. Uniqueness of Cournot equilibrium.

JEL classification: D43, D51.

1 Introduction

It is a general observation nowadays that economies are dominated by large firms and that many industries compete under oligopolistic conditions. This is well documented in the recent empirical literature showing that, in more and more markets, large firms compete strategically, a typical industry configuration associating a dominant group of large firms and a competitive fringe of small firms (Hottman *et al.*, 2016). Often, a small number of "superstar firms" dominate the market (Autor *et al.*, 2020). Concentration has substantially increased in several sectors and several countries. This appears clearly in the US as shown in several empirical studies (see, *e.g.*, Grullon *et al.*, 2019 and Gutiérrez and Philippon, 2017). There has been an important fattening of the upper tail of the distribution of markups since the 80s and a shift of market share from low to high markup firms (De Loecker *et al.*, 2020). This has ultimately resulted in "an increase in market power and a decline in competition," the share of pure profits having substantially increased at the expense of both the labour share and the capital share (Barkai, 2020). Finally, as well emphasised and empirically documented by Gabaix (2011), "many economic fluctuations are attributable to

[*]We would like to recognize our great debt to Louis-André Gérard-Varet, with whom we first developed the concept analysed in this paper and with whom it would have been such a pleasure to keep working.

[†]CORE Université Catholique de Louvain, Belgique.

[‡]BETA Université de Strasbourg, and Católica Lisbon School of Business and Economics.

the incompressible 'grains' of economic activity, the large firms". He calls this view the “granular” hypothesis. For instance, the idiosyncratic movements of the largest 100 firms in the United States appear to explain about one-third of variations in output growth.

These facts clearly justify the efforts that should be made to incorporate oligopolistic behaviour of firms in general equilibrium for macroeconomic analysis and international trade theory. For analytical tractability, these fields have been and still are largely dominated by models of either perfect or monopolistic competition, assuming symmetric preferences and a large number of negligible firms with symmetric costs. In particular, these assumptions are present in the simplest version of the Dixit-Stiglitz (1977) pathbreaking monopolistic competition model that has been successfully used in many fields. It is a two-sector general equilibrium model, with an imperfectly competitive sector producing differentiated goods (each firm acting as a monopoly in its own niche) and a perfectly competitive sector producing a homogenous good. The main assumption is the weak separability assumption introducing a subutility function on the bundles of differentiated goods, implying the analysis "to depend on the intra- and intersector elasticities of substitution" (Dixit and Stiglitz 1977, p.297). In the popular version of the model, the intrasectoral elasticity of substitution is assumed constant (the CES case). Recently, this monopolistic competition approach has been extended in two directions, either by relaxing the CES assumption[1] or by introducing heterogenous firms[2] but, still, without taking explicitly into account large firms strategic interactions.

In this chapter we want to go further and show that the Dixit-Stiglitz model can be extended to allow for such interactions. After recalling the *monopolistic competition* framework, which puts into play firms competing in prices, we adopt a Cournotian point of view and consider first the *Cournot-Walras* concept introduced by Gabszewicz and Vial (1972) for an economy with production. This concept has the advantage of taking into account all firm strategic interdependencies. It is interesting to note that this concept has an analogous definition for exchange economies with a continuum of traders, almost all behaving as price-takers but with some traders (called oligopolists) behaving strategically (Codognato and Gabszewicz, 1993). In such a model, the Cournot-Walras equilibria coincide with the Walrasian equilibria unless the oligopolists are represented by "atoms" in the measure space of traders. This is a way to define "the incompressible 'grains' of economic activity", hence to satisfy the Granularity Hypothesis.

However, to obtain a more tractable model of oligopoly in general equilibrium, we propose to use a closely related but less demanding concept of equilibrium, that is, requiring less computational ability from the firms: firms will not be required to inverse the complete demand system in the economy. For this purpose, we present the concept of *Cournotian Monopolistic Competition equilbrium*, first introduced in a multisectoral partial equilibrium model

[1] See, *e.g.*, Feenstra (2003), Zhelobodko *et al.* (2012), Parenti *et al.* (2017) and Matsuyama and Ushchev (2022).

[2] See, *e.g.*, Melitz (2003), Melitz and Ottaviano (2008) and Autor *et al.* (2020).

(d'Aspremont *et al.*, 1991) and then extended to the context of general equilibrium (d'Aspremont *et al.*, 1997). We further formulate conditions for equilibrium existence in a Dixit-Stiglitz economy. Finally, we show that this equilibrium is one among a continuum of equilibria that may be characterised by varying the intensity of competition, from a minimum (Cournot) to a maximum (Bertrand).

In the following section, we start by specifying the Dixit-Stiglitz framework for a finite set of goods and, after recalling the standard Monopolisic Competition and the Cournot-Walras equilibrium concepts, define the Cournotian Monopolistic Competition equilibrium. We prove existence of such an equilibrium. Then, in section 3, we enlarge the equilibrium concept to allow for varying degrees of competitive toughness. The conclusion follows.

2 Cournotian Monopolistic Competition

2.1 The model and the concept of equilibrium

Suppose there is a finite set $\mathcal{N}$ of n firms in the economy divided in K *groups*, the n_k firms ($n_k \geq 1$) in the group $\mathcal{N}_k$ producing the homogeneous *normal* good k ($n = \sum_{k=1}^{K} n_k$). Firm i in group k has a labour cost function $C_{ki} : \mathbb{R}_+ \to \mathbb{R}_+$.

Adopting the Dixit and Stiglitz model with several groups of firms (according to their terminology), with each group producing the same homogeneous good (the intragroup elasticity of substitution is infinite), the utility function of the representative consumer can be simply defined as $U(X_1, ..., X_K, z)$ where each X_k is the consumption of good k (a subutility in the general model with different varieties) with $X_k = \sum_{i \in \mathcal{N}_k} x_{ki}$ and x_{ki} the quantity supplied by firm i in group k and where z is the numeraire good. We take it to be leisure and assume it is a *luxury* good, an assumption ensuring labour supply $L - z$ (with L the time endowment) to be a decreasing function of income Y, for Y large enough. The budget constraint of the consumer is

$$\sum_{k=1}^{K} P_k X_k + z \leq Y,$$

where P_k is the price of good k and Y total income in terms of the numeraire. In each group k the law of one price applies if there is more than one firm: "the price is necessarily the same for each proprietor" supplying competitively the same market (Cournot, 1838, p.88). Solving the representative consumer's program

$$\max_{(\mathbf{X},z) \in \mathbb{R}_+^K \times [0,L]} U(X_1, ..., X_K, z)$$
$$\text{s.t.} \sum_{k=1}^{K} P_k X_k + z \leq Y,$$

we obtain the demand function as the vector-valued function $\mathbf{D}$ of the vector of prices $\mathbf{P} = (P_k)_{k=1}^K$ of all the homogeneous goods

$$\mathbf{D}(\mathbf{P}, Y) = (D_1(\mathbf{P}, Y), ..., D_k(\mathbf{P}, Y), ..., D_K(\mathbf{P}, Y)),$$

with $P_k \geq 0$ and $D_k(\mathbf{P}, Y) \geq 0$ for $k = 1, ..., K$, $Y \geq 0$ and $z = Y - \mathbf{PD}(\mathbf{P}, Y)$.

The limit case of a single firm in each group ($n_k = 1$ and $C_{ki} \equiv C_k$ for any k) leads directly to the standard understanding of *monopolistic competition*: a finite set of monopolies competing among themselves. For this limit case we have:

Definition 1 *A* monopolistic competition equilibrium *is a vector* $\widetilde{\mathbf{P}} \in \mathbb{R}_+^K$ *such that, for each firm k,*

$$\widetilde{P}_k \in \arg\max_{P_k \geq 0} P_k D_k\left(P_k, \widetilde{\mathbf{P}}_{-k}, \widetilde{Y}\right) - C_k\left(D_k\left(P_k, \widetilde{\mathbf{P}}_{-k}, \widetilde{Y}\right)\right),$$

with $\widetilde{Y} = L + \sum_{k=1}^K \left(\widetilde{P}_k D_k\left(\widetilde{\mathbf{P}}, \widetilde{Y}\right) - C_k\left(D_k\left(\widetilde{\mathbf{P}}, \widetilde{Y}\right)\right)\right)$.

The last condition is equivalent to

$$\sum_{k=1}^K C_k\left(D_k\left(\widetilde{\mathbf{P}}, \widetilde{Y}\right)\right) = L - \left(\widetilde{Y} - \widetilde{\mathbf{P}}\mathbf{D}\left(\widetilde{\mathbf{P}}, \widetilde{Y}\right)\right) = L - \widetilde{z},$$

the equality of labour demand and supply.

This definition characterises price competition among firms producing different goods. It is here applied to the producers of a finite set of goods, but the conventional approach to monopolistic competition (as opposed to oligopolistic competition in prices) refers rather to a continuum of differentiated goods.

Instead of considering price competition, we can refer to quantity competition, with n_k not necessarily restricted to 1. This leads us to define a *Cournot-Walras* equilbrium. For that purpose we need to assume, whenever $n_k > 1$ for some k, that the demand system can be inverted, that is, that for each k there is a well-defined function D_k^{-1} such that

$$\begin{aligned}
P_k &= D_k^{-1}\left(\sum_{i\in\mathcal{N}_1} x_{1i}, ..., \sum_{i\in\mathcal{N}_k} x_{ki}, ..., \sum_{i\in\mathcal{N}_K} x_{Ki}, Y\right) \text{ if and only if} \\
\sum_{i\in N_k} x_{ki} &= D^k(P_1, ..., P_k, ..., P_K, Y).
\end{aligned}$$

This is a strong assumption, equivalent to requiring the existence of a unique *Walrasian* equilibrium for each quantity choices of the producers. Assuming in addition that each firm knows this inverted demand system we have the following:

Definition 2 *A* Cournot-Walras equilibrium *is a vector* $\widetilde{\mathbf{x}} \in \mathbb{R}_+^n$ *(with* $n = \sum_k n_k$*) such that, for each firm i in each group k,*

$$\widetilde{x}_{ki} \in \arg\max_{x_{ki} \geq 0} \left\{ \begin{array}{c} x_{ki} D_k^{-1} \left(\sum_{j \in \mathcal{N}_1} \widetilde{x}_{1j}, ..., x_{ki} + \sum_{j \in \mathcal{N}_k \smallsetminus \{i\}} \widetilde{x}_{kj}, ..., \sum_{j \in \mathcal{N}_K} \widetilde{x}_{Kj}, \widetilde{Y} \right) \\ -C_{ki}(x_{ki}) \end{array} \right\},$$

with $\widetilde{Y} = L + \sum_{k=1}^K \sum_{i \in \mathcal{N}_k} \left(\widetilde{P}_k \widetilde{x}_{ki} - C_{ki}(\widetilde{x}_{ki}) \right)$.

Again, the last condition is equivalent to $\sum_{k=1}^K \sum_{i \in \mathcal{N}_k} C_{ki}(x_{ki}) = L - \left(\widetilde{Y} - \sum_{k=1}^K \widetilde{P}_k \sum_{i \in \mathcal{N}_k} \widetilde{x}_{ki} \right)$, the equality of labour demand and supply.

To simplify firms conjectures, we may however suppose that, in each group k, each firm i acts as a monopolist choosing its optimal price P_k and quantity x_{ki} against the residual group demand $D_k \left(P_k, \widetilde{\mathbf{P}}_{-k}, \widetilde{Y} \right) - \sum_{j \in \mathcal{N}_k \smallsetminus \{i\}} \widetilde{x}_{kj}$ when taking as given its *direct* rivals' equilibrium quantities and the equilibrium market price of each other good. In other words, firms play Cournot in the markets for their own products while taking other goods prices as given. This leads to the following definition.

Definition 3 *A* Cournotian monopolistic competition equilibrium *is a K-tuple of* $(1 + n_k)$*-tuples* $\left(P_k^C, \mathbf{x}_k^C \right)_k$ *in* $\mathbb{R}_+^{K+n}$ *such that, for any i in group k,*

$$\begin{aligned} \left(P_k^C, x_{ki}^C \right) &\in \arg\max_{(P_k, x_{ki}) \in \mathbb{R}_+^2} P_k x_{ki} - C_{ki}(x_{ki}) \\ s.t.\ x_{ki} + \sum_{j \neq i} x_{kj}^C &\leq D_k \left(P_k, \mathbf{P}_{-k}^C, Y^C \right), \end{aligned}$$

with $Y^C = L + \sum_{k=1}^K \sum_{i \in \mathcal{N}_k} \left(P_k^C x_{ki}^C - C_{ki}\left(x_{ki}^C \right) \right)$.

The last condition is equivalent to the equality of labour demand and supply $\sum_{k=1}^K \sum_{i \in \mathcal{N}_k} C_{ki}\left(x_{ki}^{\mathrm{C}} \right) = L - \left(Y^{\mathrm{C}} - \sum_{k=1}^K P_k^{\mathrm{C}} \sum_{i \in \mathcal{N}_k} x_{ki}^{\mathrm{C}} \right)$.

This concept has been already used for macro-modelling[3] but assuming a continuum of goods, which rationalises each producer's conjecture that the prices of other products are to be taken as independent of his own action, as each firm is negligible in the economy. The concept is further closely related to Neary's *General Oligopolistic Equilibrium* (GOLE), also referring to a continuum of goods, and used to explore positive and normative aspects of international trade (Neary, 2016).

2.2 Existence of equilibrium

We present three assumptions sufficient to prove that a Cournotian monopolistic competition equilbrium exists. The proof follows the following two steps. As a first step, using the first two assumptions, which ensure strict quasi-concavity of

[3]See *e.g.* Costa (2004), Costa and Dixon (2011).

firms payoff functions, we can show that a unique Cournot equilbrium exists for each group whatever the income and whatever the market prices in the other groups. Then, as a second step, we use the third assumption and resort to Tarski (1955) fixed point theorem to get existence of Cournotian monopolistic competition equilbrium.

The first assumption introduces a finite price upper bound common to all groups and formulates for each group the first and second Marshall laws of demand. The second postulates cost convexity. The third assumption formulates conditions affecting intergroup competition. It implies that a price decrease in some group, when effective on other groups, diminishes both their market size and their market power.

Assumption 1 **a)** There is $\overline{P} \in \mathbb{R}_{++}$ such that, for each k and for any $Y \in \mathbb{R}_{++}$, the demand function $D_k(\cdot, Y) : [0, \overline{P}]^K \to [0, \infty)$ is continuous, twice continuously differentiable whenever positive, and satisfies, for any $\mathbf{P}_{-k} \in [0, \overline{P}]^{K-1}$, $D_k(\overline{P}, \mathbf{P}_{-k}, Y) = 0$. Also, for any $\mathbf{P} \in [0, \overline{P}]^K$, the demand function $\mathbf{D}(\mathbf{P}, \cdot)$ is continuous and increasing and the budget share $\mathbf{PD}(\mathbf{P}, Y)/Y$ is decreasing in Y, for Y large enough.

b) For each k and any $(\mathbf{P}_{-k}, Y) \in [0, \overline{P}]^{K-1} \times \mathbb{R}_{++}$, the Marshallian demand elasticity $\sigma_k(\mathbf{P}, Y) \equiv -(\partial D_k(\mathbf{P}, Y)/\partial P_k)(P_k/D_k(\mathbf{P}, Y))$ is positive and increasing in P_k whenever $D_k(\mathbf{P}, Y) > 0$ (*first and second Marshall laws of demand*). Also, $\lim_{P_k \to 0} \sigma_k(P_k, \mathbf{P}_{-k}, Y) = 0$.

Assumption 2 For each k and each $i \in \mathcal{N}_k$, the cost function $C_{ki} : [0, \infty) \to [0, \infty)$ is twice continuously differentiable in $(0, \infty)$, non-decreasing, convex and such that, for any i, $C_{ki}(0) = 0$ and, for some i,

$$C'_{ki}(0) < \inf \left\{ P_k \in [0, \overline{P}(Y)] \,\middle|\, D_k(P_k, \mathbf{P}_{-k}, Y) = 0 \right\}.$$

Assumption 3 For $Y \in \mathbb{R}_{++}$, for each k and any $h \neq k$, $\partial D_k(\mathbf{P}, Y)/\partial P_h \geq 0$ and $\partial \sigma_k(\mathbf{P}, Y)/\partial P_h \leq 0$ whenever $D_k(\mathbf{P}, Y) > 0$.[4]

We next prove existence and uniqueness of a non-trivial Cournot equilibrium for each group.

Lemma 4 *Under Assumptions 1 and 2, there exists a unique non-trivial* Cournot equilibrium $\mathbf{x}_k^C$ *for each group k, which is a jointly continuous function of* $(\mathbf{P}_{-k}, Y) \in [0, \overline{P}]^{K-1} \times \mathbb{R}_{++}$.

Proof. (i) *Existence*: For each k and any $(\mathbf{P}_{-k}, Y) \in [0, \overline{P}]^{K-1} \times \mathbb{R}_{++}$, each firm $i \in \mathcal{N}_k$ can be assigned a compact convex strategy set $[0, \overline{x}_k(\mathbf{P}_{-k}, Y)]$.

[4] Assumption 3 first states that any good is a gross substitute to (or independent from) any other good. It results further from any good being at least as substitutable to any other good as it becomes more expensive, since

$$\frac{\partial \sigma_k(\mathbf{P}, Y)}{\partial P_h} = -\frac{1}{D_k(\mathbf{P}, Y)} \left(P_k \frac{\partial^2 D_k(\mathbf{P}, Y)}{\partial P_h \partial P_k} + \sigma_k(\mathbf{P}, Y) \frac{\partial D_k(\mathbf{P}, Y)}{\partial P_h} \right).$$

As already stated, this assumption implies that a price decrease in some group diminishes both the market size and the market power in other dependent groups.

Given the sum $X_{k(-i)} = \sum_{j \neq i} x_{kj}$ of the strategies $\mathbf{x}_{k(-i)}$ of its competitors, its payoff function is

$$\Psi_k \left(x_{ki} + X_{k(-i)}, \mathbf{P}_{-k}, Y \right) x_{ki} - C_{ki} \left(x_{ki} \right),$$

where $\Psi_k \left(x_{ki} + X_{k(-i)}, \mathbf{P}_{-k}, Y \right) = P_k > 0$ iff $D_k \left(P_k, \mathbf{P}_{-k}, Y \right) = x_{ki} + X_{k(-i)} > 0$, with $\Psi_k \left(x_{ki} + X_{k(-i)}, \mathbf{P}_{-k}, Y \right) = 0$ if $D_k \left(0, \mathbf{P}_{-k}, Y \right) \leq x_{ik} + X_{-ik}$, and $\Psi_k \left(0, \mathbf{P}_{-k}, Y \right) = \inf \left\{ P_k \in \left[0, \overline{P} \right] \middle| D_k \left(P_k, \mathbf{P}_{-k}, Y \right) = 0 \right\}$. This payoff function is clearly continuous in $\mathbf{x}_{k(-i)}$ and the corresponding marginal revenue, omitting reference to $\mathbf{P}_{-k}$ and Y for brevity, is

$$\frac{\partial \Psi_k \left(x_{ki} + X_{k(-i)} \right)}{\partial x_{ki}} x_{ki} + \Psi_k \left(x_{ki} + X_{k(-i)} \right) = P_k \left(1 - \frac{1}{\sigma_k \left(P_k \right)} \frac{x_{ki}}{x_{ki} + X_{k(-i)}} \right),$$

which is negative for any $x_k \geq \overline{x}_k$, since $\left(1/\sigma_k \left(P_k \right) \right) \left(x_k / \left(x_k + X_{k(-i)} \right) \right) \geq \left(1/\sigma_k \left(\Psi_k \left(n_k \overline{x}_k \right) \right) \right) \left(1/n_k \right) > 1$ if $\overline{x}_k$ is chosen so as to verify $\sigma_k \left(\Psi_k \left(n_k \overline{x}_k \right) \right) < 1/n_k$ (recall that, by Assumption 1, $\lim_{P_k \to 0} \sigma_k \left(P_k \right) = 0$). Thus, no firm would indeed want to choose $x_k \geq \overline{x}_k$. The marginal revenue is decreasing when non-negative if $x_{ki} > 0$, since, by Assumption 1,

$$\begin{aligned} & \underbrace{\frac{1}{\partial D_k \left(P_k \right) / \partial P_k}}_{<0} \underbrace{\left(1 - \frac{1}{\sigma_k \left(P_k \right)} \frac{x_{ki}}{x_{ki} + X_{k(-i)}} \right)}_{\geq 0} + \\ & \underbrace{\frac{P_k}{\sigma_k \left(P_k \right)}}_{>0} \underbrace{\left(\frac{\partial \sigma_k \left(P_k \right) / \partial P_k}{\sigma_k \left(P_k \right)} \frac{1}{\partial D_k \left(P_k \right) / \partial P_k} \frac{x_{ki}}{x_{ki} + X_{k(-i)}} - \frac{X_{k(-i)}}{\left(x_{ki} + X_{k(-i)} \right)^2} \right)}_{<0} < 0. \end{aligned}$$

Thus, if $X_{k(-i)} < D_k (0)$, $\Psi_k \left(x_{ki} + X_{k(-i)} \right) x_{ki}$ is first increasing and strictly concave in x_{ki}, and then decreasing. As a consequence, by Assumption 2, firm i's payoff function is first strictly concave and then decreasing, hence strictly quasi-concave. This ensures existence of an equilibrium in pure strategies.

(ii) *Uniqueness*: Suppose that, for some group k, there are two Cournot equilibria $\underline{\mathbf{x}}_k^{\mathrm{C}}$ and $\overline{\mathbf{x}}_k^{\mathrm{C}}$, with $\underline{X}_k^{\mathrm{C}} \leq \overline{X}_k^{\mathrm{C}}$. Then there is a non-empty set J such that, for any $i \in J$, $\underline{x}_{ki}^{\mathrm{C}} < \overline{x}_{ki}^{\mathrm{C}}$. We first show that the sum over J of the marginal revenues is larger in $\underline{\mathbf{x}}_k^{\mathrm{C}}$ than in $\overline{\mathbf{x}}_k^{\mathrm{C}}$, which, referring to the proof of (i), writes:

$$\begin{aligned} & \Psi_k \left(\underline{X}_k^{\mathrm{C}} \right) \left(|J| - \frac{1}{\sigma_k \left(\Psi_k \left(\underline{X}_k^{\mathrm{C}} \right) \right)} \frac{\sum_{i \in J} \underline{x}_{ki}^{\mathrm{C}}}{\sum_{i \in J} \underline{x}_{ki}^{\mathrm{C}} + \sum_{i \in \mathcal{N}_k \diagdown J} \underline{x}_{ki}^{\mathrm{C}}} \right) \\ > \quad & \Psi_k \left(\overline{X}_k^{\mathrm{C}} \right) \left(|J| - \frac{1}{\sigma_k \left(\Psi_k \left(\overline{X}_k^{\mathrm{C}} \right) \right)} \frac{\sum_{i \in J} \overline{x}_{ki}^{\mathrm{C}}}{\sum_{i \in J} \overline{x}_{ki}^{\mathrm{C}} + \sum_{i \in \mathcal{N}_k \diagdown J} \overline{x}_{ki}^{\mathrm{C}}} \right). \end{aligned}$$

As $\partial \Psi_k / \partial X_k < 0$ and $\partial \sigma_k / \partial P_k > 0$ by the two Marshall laws of demand, and since $\overline{x}_{ki}^{\mathrm{C}} \leq \underline{x}_{ki}^{\mathrm{C}}$ for any $j \in \mathcal{N}_k \diagdown J$, the preceding inequality is true. By convexity of the cost functions, the inequality on marginal revenues can be extended to marginal profits:

$$\begin{aligned} & \sum_{i\in J}\left(\Psi_k'\left(\underline{X}_k\right)\underline{x}_{ki}+\Psi_k\left(\underline{X}_k\right)\right)-\sum_{i\in J}C_{ki}'\left(\underline{x}_{ki}\right) \\ > \quad & \sum_{i\in J}\left(\Psi_k'\left(\overline{X}_k\right)\overline{x}_{ki}+\Psi_k\left(\overline{X}_k\right)\right)-\sum_{i\in J}C_{ki}'\left(\overline{x}_{ki}\right). \end{aligned}$$

This inequality violates the first order conditions for profit maximisation within the subgroup J, which impose nullity of the RHS and non-positivity of the LHS (since $\underline{x}_{ki}$ might be zero for all i in J). Hence, the Cournot equilibrium is unique. It is also non-trivial since, by Assumption 2, $\Psi_k\left(0,\mathbf{P}_{-k},Y\right)>C_{ki}'\left(x_{ki}\right)$ for some $i\in\mathcal{N}_k$. Finally, continuity of $\mathbf{x}_k^{\mathrm{C}}$ as a function of $\left(\mathbf{P}_{-k},Y\right)$ results from our assumptions by applying the maximum theorem. ■

The former uniqueness proof is an adaptation of the simple proof in Von Mouche and Quartieri (2015), requiring a strict concavity of the aggregate revenue function $P_kD\left(P_k\right)$ that can be reformulated as $\sigma_k'\left(P_k\right)P_k/\sigma_k\left(P_k\right)>\sigma_k\left(P_k\right)-1$ for any P_k. Our proof requires the alternative assumption that σ_k is an increasing function of P_k (the second Mashall law of demand).

We can now turn to the second step of the proof of existence of a Cournotian monopolistic competition equilibrium.

Proposition 5 *Under Assumptions 1, 2 and 3, there exists a Cournotian monopolistic competition equilibrium.*

Proof. We begin by taking income $Y\in\mathbb{R}_+$ as given, omitting reference to it in the following passage, in order to simplify notations. We prove this proposition by applying Tarski's fixpoint theorem: $\left[0,\overline{P}\right]^K$ is a complete lattice with respect to the natural order "$\geq$" and the Cournot equilibrium price mapping $\mathbf{P}^{\mathrm{C}}:\left[0,\overline{P}\right]^K\to\left[0,\overline{P}\right]^K$ is isotone, as we are going to show. Take $\mathbf{P}_{-k}^0$ and $\mathbf{P}_{-k}^1$ in $\left[0,\overline{P}\right]^{K-1}$ such that $\mathbf{P}_{-k}^0\leq\mathbf{P}_{-k}^1$. Then, as P_k^{C} can be taken as constant in P_k, it is enough to show that $P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right)\leq P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right)$. Suppose indeed that $P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right)>P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right)$. Then, by the first order condition for profit maximisation (equality of marginal revenue and marginal cost), we have by Assumptions 1 and 3 (on σ_k), for any $i\in\mathcal{N}_k$:

$$\begin{aligned} & \frac{1-\frac{C_{ki}'\left(x_{ki}^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right)\right)}{P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right)}}{\frac{x_{ki}^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right)}{D_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right),\mathbf{P}_{-k}^0\right)}}=\frac{1}{\sigma_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right),\mathbf{P}_{-k}^0\right)} \\ < \quad & \frac{1}{\sigma_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right),\mathbf{P}_{-k}^1\right)}=\frac{1-\frac{C_{ki}'\left(x_{ki}^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right)\right)}{P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right)}}{\frac{x_{ki}^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right)}{D_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right),\mathbf{P}_{-k}^1\right)}}. \end{aligned}$$

As $D_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}\right),\mathbf{P}_{-k}\right)=\sum_i x_{ki}^{\mathrm{C}}\left(\mathbf{P}_{-k}\right)$, we must have for some j

$$\frac{x_{kj}^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right)}{D_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^0\right),\mathbf{P}_{-k}^0\right)}\leq\frac{x_{kj}^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right)}{D_k\left(P_k^{\mathrm{C}}\left(\mathbf{P}_{-k}^1\right),\mathbf{P}_{-k}^1\right)},$$

hence, again by Assumptions 1 and 3 (now on D_k), $x^{\mathrm{C}}_{kj}\left(\mathbf{P}^{0}_{-k}\right) < x^{\mathrm{C}}_{kj}\left(\mathbf{P}^{1}_{-k}\right)$ and, by Assumption 2, $C'_{kj}\left(x^{\mathrm{C}}_{kj}\left(\mathbf{P}^{0}_{-k}\right)\right) \leq C'_{kj}\left(x^{\mathrm{C}}_{ki}\left(\mathbf{P}^{1}_{-k}\right)\right)$, contradicting the preceding inequality. We have thus proved that the Cournot equilibrium price mapping $\mathbf{P}^{\mathrm{C}}\left(\cdot, Y\right)$ has a fixed point for *any* given income Y: $\exists \mathbf{P}^*$ such that $\mathbf{P}^{\mathrm{C}}\left(\mathbf{P}^*, Y\right) = \mathbf{P}^*$, a function of Y. To conclude the existence proof, Y must be endogenised as a solution to the equation of labour demand with labour supply:[5]

$$\sum_{k=1}^{K}\sum_{i\in\mathcal{N}_k} C_{ki}\left(x^{\mathrm{C}}_{ki}\left(\mathbf{P}^*_{-k}\left(Y\right), Y\right)\right) = L - \left(Y - \sum_{k=1}^{K} P^*_k\left(Y\right)\sum_{i\in\mathcal{N}_k} x^{\mathrm{C}}_{ki}\left(\mathbf{P}^*_{-k}\left(Y\right), Y\right)\right).$$

By continuity of the functions x^{C}_{ki} and $\mathbf{P}^*$, both sides of this equation are continuous. For $Y = 0$, the LHS is zero whereas the RHS is $L > 0$. For Y large enough, the LHS remains non-negative whereas the RHS (labour supply) becomes negative since labour is a luxury good (by Assumption 1, the consumption budget share is a decreasing function of Y). Continuity ensures existence of a positive solution Y^*. ■

3 An extension: varying the intensity of competition within each group

The concept of Cournotian monopolistic competition supposes Cournot competition within each group. A natural extension of the concept results from allowing intrasectoral oligopolistic behaviour to take more competitive forms, like Bertrand conduct. Such extension has been explored by Neary and Tharakan (2012), where the mode of competition (Cournot vs. Bertrand) is endogenised by embedding firm behaviour in a two-stage model with investment in capacity at the first stage, along the lines of Kreps and Scheinkman (1983). Here, we shall develop this idea by enlarging the concept of oligopolistic equilibrium to allow for various intensities of competition, in fact a continuum, between Cournot and Bertrand. This results from adding to the consideration by Cournot's producers of the impact of their actions on *market size* (through the residual demand) that of Bertrand's producers fighting for their *market share*. (see d'Aspremont and Dos Santos Ferreira, 2021).

Definition 6 *An* oligopolistic equilibrium *is a $2n$-tuple* $(\mathbf{p}^*, \mathbf{x}^*)$ *such that, for*

[5] This last step is necessary since we cannot invoke here Walras' law, as firms do not adopt price-taking behaviour. They make decisions on the basis of a Marshallian (income-dependent) demand, not of a Walrasian demand depending exclusively on prices and verifying Walras' law.

each firm $i \in \mathcal{N}_k$ and each $k = 1, ..., K$, (p^*_{ki}, x^*_{ki}) is solution to the program

$$\begin{aligned}
(p^*_{ki}, x^*_{ki}) &\in \arg \max_{(p_{ki}, x_{ki}) \in \left[0, \overline{P}\right] \times \mathbb{R}_+} p_{ki} x_{ki} - C_{ki}(x_{ki}) \\
s.t.\ p_{ki} &\leq D_k^{-1}\left(x_{ki} + \sum_{j \neq i} x^*_{kj}, \mathbf{P}^*_{-k}, Y^*\right) \ and \\
p_{ki} &\leq \min_{j \in \mathcal{N}_k \setminus \{i\}} \left\{p^*_{kj}\right\}
\end{aligned}$$

with both constraints satisfied as equalities (implying $p^*_{ki} = P^*_k$ if $x^*_{ki} > 0$) and $Y^* = L + \sum_{k=1}^K \sum_{i \in \mathcal{N}_k} (p^*_{ki} x^*_{ki} - C_{ki}(x^*_{ki}))$.

The first order conditions of firm i in group k at an oligopolistic equilibrium (with multipliers $(\kappa_{ki}, \lambda_{ki}) \in \mathbb{R}^2_+ \setminus \{\mathbf{0}\}$ associated with the first and second constraints) require, by the positivity of p^*_{ki} and of x^*_{ki} (if firm i in k is active) that $x^*_{ki} - \kappa^*_{ki} - \lambda^*_{ki} = 0$, and $p^*_{ki} - C'_{ki}(x^*_{ki}) + \kappa^*_{ki}/D'(P^*_k\}) = 0$, with $P^*_k = \min_j\{p^*_{kj}\}$. If firm i is inactive, both constraints cease to bind, so that we let $\kappa^*_{ki} = \lambda^*_{ki} = 0$. Using the normalised parameter $\theta^*_{ki} \equiv \lambda^*_{ki}/(\kappa^*_{ki} + \lambda^*_{ki}) \in [0, 1]$, we can rewrite the first order conditions to characterise the (relative) markup of each active firm i in group k as a function of θ^*_{ki}:

$$\begin{aligned}
\frac{P^*_k - C'_{ki}(x^*_{ki})}{P^*_k} &= (1 - \theta^*_{ki}) \frac{x^*_{ki}/\sum_j x^*_{kj}}{-\left(\partial D_k\left(\mathbf{P}^*, Y^*\right)/\partial P_k\right)\left(P^*_k/D_k\left(\mathbf{P}^*, Y^*\right)\right)} \\
&\equiv (1 - \theta^*_{ki}) \frac{\alpha^*_{ki}}{\sigma_k(\mathbf{P}^*, Y^*)}.
\end{aligned}$$

The parameter θ^*_{ki} may be interpreted as measuring the *competitive toughness* of firm i in group k at the equilibrium $(\mathbf{p}^*, \mathbf{x}^*)$. For minimal competitive toughness of all active firms in group k ($\boldsymbol{\theta}^*_k = \mathbf{0}$), we obtain the standard markup formula for the Cournot equilibrium, so that we retrieve the Cournotian monopolistic equilibrium if this is verified for all groups. For overall maximal competitive toughness ($\boldsymbol{\theta}^* = \mathbf{1}$), each active firm equalising marginal cost to price, we obtain the perfectly competitive equilibrium. Of course, the concept of oligopolistic equilibrium is compatible with a diversified competitive toughness across firms and across groups. For tractability in macroeconomic applications, a relevant case results from taking competitive toughness to be uniform inside each group, characterising the corresponding market intensity of competition. However, variability of competitive toughness within each group may well respond to typical situations, for instance when a small number of superstar firms dominate a competitive fringe, as mentioned in the introduction.

4 Conclusion

We examined in this chapter the concept of Cournotian monopolistic competition equilibrium as a convenient and tractable way of taking the strategic behaviour of large firms into account, in a general equilibrium framework oriented to macroeconomic analysis or to international trade theory. Existence of such an equilibrium has been obtained under simple, easily interpretable, assumptions. The part of the proof concerning the uniqueness of Cournot equilibrium for each group of firms and using the two Marshall laws of demand is in particular a nice addition to the litterature on this topic.

A natural extension of the concept, reflecting the weight of large firms decisions in the economy, beyond their own market, is the integration in those firms conjectures of so-called *Ford effects*, namely the impact on demand ascribable to the income they generate (see *e.g.* d'Aspremont and Dos Santos Ferreira, 2017 and 2021, ch. 2). These income feedback effects, while keeping the structure of the markup formula unchanged, modify the relevant, intersectoral, elasticity of substitution, hence the equilibrium markup.

Another natural extension, explored in our previous work, is the introduction of differentiated varieties within each group, generalising the approach to varying intensities of competition here restricted to the homogeneous oligopoly. Such extension completes the markup formula, making the markup explicitly appear as a weighted mean of two relevant elasticities of substitution. One is the intersectoral elasticity, across groups, reflecting the response of market size to price changes described by the Marshallian demand. The other is the intrasectoral elasticity, between the goods produced within the group, assumed infinite in the homogeneous good case.

References

[1] d'Aspremont, C., Dos Santos Ferreira, R. (2017). The Dixit-Stiglitz economy with a 'small group' of firms: A simple and robust equilibrium markup formula. *Research in Economics* 71: 729-739.

[2] d'Aspremont, C., Dos Santos Ferreira, R. (2021). *The Economics of Competition, Collusion and In-between.* Cham: Palgrave Macmillan.

[3] d'Aspremont, C., Dos Santos Ferreira, R., Gérard-Varet, L.-A. (1991). Pricing Schemes and Cournotian Equilibria. *American Economic Review* 81: 666-673.

[4] d'Aspremont, C., Dos Santos Ferreira, R., Gérard-Varet, L.-A. (1997). General Equilibrium Concepts under Imperfect Competition : A Cournotian Approach. *Journal of Economic Theory* 73: 199–230.

[5] Autor, D., Dorn, D., Katz, L. F., Patterson, C., Van Reenen, J. (2020). The Fall of the Labor Share and the Rise of Superstar Firms. *Quarterly Journal of Economics* 135: 645-709.

[6] Barkai, S. (2020). Declining Labor and Capital Shares. *Journal of Finance* 75: 2421-2463.

[7] Codognato, G., Gabszewicz, J. (1993). Cournot-Walras equilibria in markets with a continuum of traders. *Economic Theory* 3: 453-464.

[8] Costa, L. F. (2004). Endogenous Markups and Fiscal Policy. *Manchester School* 72: 55-71.

[9] Costa, L. F., Dixon, H. D. (2011). Fiscal policy under imperfect competition with flexible prices: An overview and survey. *Economics: The Open-Access* 5: 1-57.

[10] Cournot, Augustin (1838). *Recherches sur les principes mathématiques de la théorie des richesses*. Paris: Hachette. English translation by Nathaniel T. Bacon: *Researches into the Mathematical Principles of the Theory of Wealth*. New York: Macmillan, 1897.

[11] De Loecker, J., Eeckhout, J., Unger, G. (2020). The Rise of Market Power and the Macroeconomic Implications. *Quarterly Journal of Economics* 135: 561-644.

[12] Dixit, A. K., Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review* 67: 297-308.

[13] Feenstra, R.C. (2003). A homothetic utility function for monopolistic competition models, without constant price elasticity. *Economics Letters* 78: 79-86.

[14] Gabaix, X. (2011). The Granular Origins of Aggregate Fluctuations. *Econometrica* 79: 733-772.

[15] Gabszewicz, J., Vial, J.-P. (1972). Oligopoly "à la Cournot" in a general equilibrium analysis. *Journal of Economic Theory* 4: 381-400.

[16] Grullon, G., Larkin, Y., Michaely, R. (2019). Are US Industries Becoming More Concentrated? *Review of Finance* 23: 697-743.

[17] Gutiérrez, G., Philippon, T. (2017). Declining Competition and Investment in the U.S.. NBER Working Paper 23583.

[18] Hottman, C. J., Redding, S. J., Weinstein, D. E. (2016). Quantifying the Sources of Firm Heterogeneity. *Quarterly Journal of Economics* 131: 1291-1364.

[19] Kreps, D. M., Scheinkman, J. A. (1983). Quantity precommitment and Bertrand competition yield Cournot outcomes. *Bell Journal of Economics* 14: 326-337.

[20] Matsuyama, K., Ushchev, P. (2022). Destabilizing Effects of Market Size in the Dynamics of Innovation. *Journal of Economic Theory* 200: 105415.

[21] Melitz, M. J. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica* 71: 1695-1725.

[22] Melitz, M., Ottaviano, G. (2008). Market Size, Trade, and Productivity. *Review of Economic Studies* 75: 295-316.

[23] von Mouche, P., Quartieri, F. (2015). Cournot Equilibrium Uniqueness in Case of Concave Industry Revenue: a Simple Proof. *Economics Bulletin* 35 (2).

[24] Neary, J. P. (2016). International Trade in General Oligopolistic Equilibrium. *Review of International Economics* 24: 669-698.

[25] Neary, J. P., Tharakan, J. (2012). International Trade with Endogenous Mode of Competition in General Equilibrium. *Journal of International Economics* 86: 118-132.

[26] Parenti, M., Ushchev, P., Thisse, J.-F. (2017). Toward a theory of monopolistic competition. *Journal of Economic Theory* 167: 86-115.

[27] Tarski, A. (1955). A lattice-theoretical fixpoint theorem and its applications. *Pacific Journal of Mathematics* 5: 285-309.

[28] Zhelobodko, E., Kokovin, S., Parenti, M., Thisse, J.-F., 2012. Monopolistic competition: Beyond the constant elasticity of substitution. *Econometrica* 80: 2765-2784.