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A note on pollution inertia and endogenous cycles in Ramsey economies

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Abstract

The literature has highlighted the potential occurrence of a limit cycle through a Hopf bifurcation near the steady state of a competitive Ramsey economy when pollution significantly increases the marginal utility of consumption (compensation effect). This latter condition is necessary but not sufficient. More specifically, pollution inertia must be strong when pollution originates from production but not when it stems from consumption. This paper investigates the reasons for this difference and emphasizes the role of decreasing marginal productivity of capital in explaining it.

JEL Classification: E32, O44.

Keywords: Ramsey model, Pollution inertia, Hopf bifurcation.

1 Introduction

The literature has highlighted the possible existence of a limit cycle through a Hopf bifurcation near the steady state of the competitive Ramsey model when a pollution externality, viewed as a stock variable, sufficiently increases the marginal utility of consumption (compensation effect[1]). To the best of our knowledge, the first paper to point out this possibility is Heal (1982). He developed a Ramsey-type model where pollution comes from consumption. Even though he assumed that pollution is a stock variable, he only referred to the characteristics of the utility function to discuss the existence of the Hopf bifurcation. A similar conclusion was reached by Bosi and Desmarchelier (2017). Their model is close to the one proposed by Heal (1982) but with endogenous labour supply and they also observe that a strong compensation effect leads to the existence of a Hopf bifurcation. Interestingly, when pollution originates from production rather than from consumption, Bosi and Desmarchelier (2018)

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[1] Michel and Rotillon (1995).

pointed out that a strong compensation effect is not sufficient for a Hopf bifurcation to occur, it is also necessary that the pollution stock possesses strong inertia, a condition that does not appear in Heal (1982) nor in Bosi and Desmarchelier (2017). The objective of the present paper is to propose a model where pollution arises from both consumption and production in order to precisely compare the conditions under which a Hopf bifurcation is possible when pollution solely originates from production or consumption with a special focus on pollution inertia. Additionally, this paper aims to propose an explanation for this difference.

In this unified framework, we recover the results of both Heal (1982) and Bosi and Desmarchelier (2018): a strong compensation effect is a necessary condition for a Hopf bifurcation to arise whether pollution solely comes from consumption or from production. Moreover, as expected, we observe that the pollution stock must also possess strong inertia when pollution originates solely from production while no such restriction is required when pollution originates solely from consumption.

How can we explain this result? It appears that the decreasing marginal productivity of capital plays a central role. Indeed, the pollution process is influenced by two opposing forces: an attractive force (natural pollution absorption) and a repulsive force (pollution emissions resulting from consumption or production). The existence of endogenous cycles (Hopf bifurcation) arises from the interaction between these two opposing forces. Interestingly, the repulsive forces are affected by households' choices regarding consumption and saving. In the literature, as well as in the present paper, the pollution accumulation process is assumed to be linear with respect to consumption (Heal, 1982) or production (Bosi and Desmarchelier, 2018). In other words, higher consumption demand has a direct positive effect on pollution accumulation, thereby increasing the future pollution stock. However, when pollution originates from production, a higher pollution level results from an increase in saving: higher saving implies greater capital accumulation, which in turn raises production and pollution. Nevertheless, in the long run, the capital stock becomes large, and the marginal productivity of capital declines. As a result, higher saving has only a moderate effect on the pollution level. To ensure that pollution effectively increases, the attractive force must also be weak which implies that pollution inertia has to be strong. Without this latter condition, the repulsive force is not strong enough to destabilize the economy and, consequently, insufficient to allow for the emergence of a limit cycle (Hopf bifurcation).

The paper is organized as follows. Section 2 presents the model. Section 3 describes the equilibrium, while Section 4 examines the local dynamics. Section 5 concludes the paper.

2 The model

We propose to consider a simple competitive Ramsey-Cass-Koopmans economy in which a pollution externality comes from both production and consumption.

This framework will be used in order to compare conditions under which a limit cycle, through a Hopf bifurcation, can arise around the steady state of the economy with a special attention to pollution inertia.

2.1 The representative household

Let h be the individual wealth. The representative household uses her income to finance both consumption (c) and saving ($\dot{h}$). For simplicity, we assume that labour (l) is supplied inelastically and normalized to the unity ($l = 1$). Denoting by r, w and δ respectively the real interest rate, the wage rate and the capital depreciation rate, the household's budget constraint is simply given by:

$$\dot{h} \leq (r - \delta)\, h + w - c \tag{1}$$

Preferences are rationalized by a non-separable utility function $u(c, P)$ where P represents the pollution stock. The next assumption sums up its properties.

Assumption 1 $u_{cc} < 0 < u_c$, $u_P < 0$. *Moreover,* $\lim_{c \to 0} u_c = +\infty$ *and* $\lim_{c \to +\infty} u_{cc} = 0$.

Throughout this paper, pollution is viewed as a pure externality. At this step of the reasoning, nothing is said concerning the cross derivative of the utility function, that is $u_{cP} \lesseqgtr 0$. Following Michel and Rotillon (1995), $u_{cP} < 0$ represents the so-called *distaste effect*. The rational is the following: if the representative household enjoys to consume in a clean environment, a higher pollution level reduces the incentive to consume. Conversely, $u_{cP} > 0$ represents the so-called *compensation effect* (Michel and Rotillon, 1995). The rational is also simple to capture: since a higher pollution level reduces the household's utility (namely, $u_P < 0$), she can decide to compensate this utility loss by increasing her consumption demand.

The compensation effect is a well-known necessary condition for the occurrence of a limit cycle (Hopf bifurcation) near the steady state of a competitive Ramsey economy, both when pollution comes from consumption (Heal, 1982) or from production (Bosi and Desmarchelier, 2018). Interestingly when pollution comes from production, pollution inertia has to be strong for a Hopf bifurcation to occur. However, this is not the case when pollution comes from consumption. The following paper aims to explain this difference.

For further references, let us introduce two second order elasticities:

$$\varepsilon_{cc} \equiv \frac{c u_{cc}}{u_c} < 0 \text{ and } \varepsilon_{cP} \equiv \frac{P u_{cP}}{u_c}$$

$-1/\varepsilon_{cc}$ represents the so-called elasticity of intertemporal substitution in consumption and ε_{cP} captures the pollution effect on marginal utility of consumption. That is, if preferences depict a distaste effect (compensation effect), then $\varepsilon_{cP} < 0\,(> 0)$.

As usual in an economy à la Ramsey, the representative household chooses the consumption path which maximizes her intertemporal utility $\int_0^\infty e^{-\rho t} u(c, P)\, dt$ under the budget constraint (1). $\theta > 0$ represents the rate of time preference.

To solve this program, which is well-defined under Assumption 1, we maximize the Hamiltonian $H = u(c, P) + \mu[(r - \delta)h + w - c]$ where μ is the Lagrangian multiplier. Using the first-order conditions $\partial H/\partial c = 0$, $\partial H/\partial h = \theta\mu - \dot{\mu}$ and $\partial H/\partial \mu = \dot{h}$. Those conditions lead to a static relation:

$$u_c(c, P) = \mu \tag{2}$$

to a dynamic Euler equation:

$$\dot{\mu} = (\theta + \delta - r)\mu \tag{3}$$

and the budget constraint (1) is now binding:

$$\dot{h} = (r - \delta)h + w - c \tag{4}$$

The optimal path satisfies also the transversality condition: $\lim_{t\to+\infty} e^{-\theta t}\mu h = 0$.

Applying the implicit function theorem on the static relation (2) gives that $c \equiv c(\mu, P)$ with:

$$\frac{\mu}{c}\frac{\partial c}{\partial \mu} = \frac{1}{\varepsilon_{cc}} < 0$$
$$\frac{P}{c}\frac{\partial c}{\partial P} = -\frac{\varepsilon_{cP}}{\varepsilon_{cc}} \lesseqgtr 0$$

It follows that consumption demand is an increasing (a decreasing) function with respect to pollution if and only if preferences are described by a compensation effect (distaste effect).

2.2 The production sector

The production sector produces a quantity Y of a composite good which can be consumed or saved. Production is made by using a constant returns to scale technology F which combines labour L and capital K:

$$Y = F(K, L) = Lf(k)$$

with $k \equiv K/L$ and $f(k) \equiv F(k, 1)$. The next assumption sums up properties of f.

Assumption 2 $f''(k) < 0 < f'(k)$ and Inada conditions hold: $\lim_{k\to 0} f'(k) = +\infty$ and $\lim_{k\to+\infty} f'(k) = 0$.

Before going further, let us introduce two useful elasticities:

$$\alpha \equiv \frac{kf'(k)}{f(k)} \in (0, 1) \text{ and } \sigma \equiv -\frac{f'(k)[f(k) - kf'(k)]}{kf(k)f''(k)} > 0$$

As usual, α represents the share of capital income into the total income of the economy while σ is the so-called elasticity of capital-labour substitution.

The production sector chooses K and L in order to maximize its profit, taking prices as given:

$$\max_{(K,L)} Lf(k) - wL - rK$$

First order conditions give as usual:

$$r = f'(k) \equiv r(k) \tag{5}$$
$$w = f(k) - kf'(k) \equiv w(k) \tag{6}$$

Elasticities of factor prices are simply functions of both α and σ:

$$\frac{kr'(k)}{r(k)} = \frac{\alpha - 1}{\sigma} < 0 \text{ and } \frac{kw'(k)}{w(k)} = \frac{\alpha}{\sigma} > 0$$

2.3 Pollution

Pollution P is a stock coming from both production and consumption. It is assumed to evolve as follow:

$$\dot{P} = -aP + b_1 C + b_2 Y \tag{7}$$

where $a > 0$, $b_1 \geq 0$ and $b_2 \geq 0$ represent, respectively, the natural pollution absorption, the environmental impact of consumption, and the environmental impact of production. It appears that a accounts for pollution inertia. In particular, when $a \to 0$, pollution tends to behave as a pure stock and pollution inertia is maximal. As discussed previously a is an important parameter for the occurrence of a Hopf bifurcation when pollution originates from production (Bosi and Desmarchelier, 2018). The pollution process (7) allows to encompass the two configurations: (1) $b_1 > 0$ jointly with $b_2 = 0$ implies that pollution only comes from consumption and (2), $b_1 = 0$ jointly with $b_2 > 0$ implies that pollution only comes from production.

3 Equilibrium

3.1 Dynamical system

At the equilibrium, all markets clear together. Let N be the size of the (constant) population. Focusing on the labour market, demand for labour L is equal to supply, namely $L = Nl$. To simplify the exposition, we normalize the population size to the unity ($N = 1$). Since in addition $l = 1$, it follows that $L = 1$. On the capital market, the demand for capital K is equal to the supply Nh, it then appears that $k = h$. Finally, with $N = 1$, the household's budget constraint (4) represent the good market clearing condition.

Considering jointly (3), (4), (5), (6) and (7) give the dynamical system describing the evolution of all variables of this economy:

$$\dot{\mu} = f_1(\mu, k, P) \equiv [\theta + \delta - r(k)]\,\mu \tag{8}$$

$$\dot{k} = f_2(\mu, k, P) \equiv [r(k) - \delta]\,k + w(k) - c(\mu, P) \tag{9}$$

$$\dot{P} = f_3(\mu, k, P) \equiv -aP + b_1 c(\mu, P) + b_2 f(k) \tag{10}$$

Equations (8) and (9) are the usual equations of the Ramsey model while equation (10) captures the environmental block of the model. Clearly, μ is a jump variable while both k and P are predetermined variables. As usual, the system (8)-(10) is not analytically solvable. Therefore, to capture the dynamics, we proceed in two steps: first, we prove the existence of a steady state; second, we linearize the system around this steady state.

3.2 Steady state

A steady state for this economy is a triplet $(\mu, k, P) \in \mathbb{R}^3_+$ such that $\dot{\mu} = \dot{k} = \dot{P} = 0$. Considering $\dot{\mu} = 0$, (8) gives the capital level at the steady state:

$$k^* = r^{-1}(\theta + \delta) > 0 \tag{11}$$

Invertibility of $r(k)$ is ensured by assumption 2. This is the usual capital level given by the modified golden rule of the Ramsey model. It appears from (11) that k^* is not affected by pollution. This comes from the fact that we study a competitive economy, the representative household does not internalize pollution.

Now focus on (9) at the steady state ($\dot{k} = 0$) such that $k = k^*$, we obtain:

$$c^* = \theta k^* + w(k^*) \equiv c(k^*) > 0 \tag{12}$$

Considering jointly (11), (12) and (10) gives pollution at the steady state ($\dot{P} = 0$):

$$P^* = \frac{b_1}{a} c(k^*) + \frac{b_2}{a} f(k^*) \equiv P(k^*) > 0$$

To obtain the shadow price of capital at the steady state, focus on (2):

$$\mu^* = u_c(c(k^*), P(k^*)) \equiv \mu(k^*) > 0$$

This discussion leads to the following proposition.

Proposition 1 *Let assumptions 1 and 2 hold. There always exists a unique positive steady state for this economy given by* (μ^*, k^*, P^*).

4 Local dynamics

The previous section has established the existence of a unique steady state. We now aim to examine the dynamics around this steady state and to compare the conditions under which a Hopf bifurcation occurs when pollution originates solely from production versus when it comes solely from consumption. The literature has highlighted that strong pollution inertia is required in the former case (Bosi and Desmarchelier, 2018), while this condition has not been emphasized in the latter case (Heal, 1982). The purpose of this section is to clarify the reason for this difference.

As in Itaya (2008) or in Fernandez et al. (2012), let us consider the following functional form:

$$u(c,P) = \frac{\left(cP^{-\eta}\right)^{1-\varepsilon}}{1-\varepsilon} \tag{13}$$

with $\varepsilon > 0$ and $\eta > 0$. Functional form (13) is interesting because second order elasticities are fully expressed in terms of fundamental parameters:

$$\varepsilon_{cc} = -\varepsilon < 0 \text{ and } \varepsilon_{cP} = \eta(\varepsilon - 1) \lesseqgtr 0$$

Moreover, (13) account for both distaste and compensation effects, namely, $\varepsilon < 1\,(> 1)$, implies $\varepsilon_{cP} < 0\,(> 0)$, that is distaste effect (compensation effect) while η captures the magnitude of the distaste/compensation effect.

To study the dynamics around the steady state of this economy, we proceed as in Bosi and Desmarchelier (2019). That is, we compute the Jacobian matrix J, evaluated at the steady state, and we analyze the trace T, the determinant D and the sum of principle minors of order two S. Linearizing the dynamical system (8)-(9)-(10) around the steady state gives:

$$J \equiv \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} \end{bmatrix} = \begin{bmatrix} 0 & \frac{(1-\alpha)(\theta+\delta)}{\sigma}\frac{\mu}{k} & 0 \\ \frac{k}{\mu}\frac{\varphi}{\varepsilon} & \theta & a\gamma\frac{\eta(1-\varepsilon)}{\varepsilon} \\ -b_1\frac{k}{\mu}\frac{\varphi}{\varepsilon} & b_2(\theta+\delta) & -a\left(1 + b_1\gamma\frac{\eta(1-\varepsilon)}{\varepsilon}\right) \end{bmatrix}$$

With:

$$\gamma \equiv \frac{\theta + \delta(1-\alpha)}{(\theta+\delta)(b_1+b_2) - \alpha\delta b_1} > 0 \text{ and } \varphi \equiv \frac{\theta + (1-\alpha)\delta}{\alpha} > 0$$

It follows that:

$$T = \theta - a\left(1 + \eta b_1 \gamma\left(\frac{1-\varepsilon}{\varepsilon}\right)\right)$$

$$D = a\varphi(1-\alpha)\frac{\theta+\delta}{\varepsilon\sigma} > 0 \tag{14}$$

$$S = a\gamma\eta\left(\theta b_1 + b_2(\theta+\delta)\right)\left(\frac{\varepsilon-1}{\varepsilon}\right) - \varphi\frac{1-\alpha}{\sigma}\frac{\theta+\delta}{\varepsilon} - a\theta \tag{15}$$

Following Bosi and Desmarchelier (2019, Proposition 4), a Hopf bifurcation occurs in a three dimensional system if and only if $D = ST$ such that $S > 0$. Considering (15), a necessary condition (but not sufficient) for $S > 0$ is that $\varepsilon > 1$ (i.e. compensation effect). That is, there is no room for a Hopf bifurcation when preferences are described by a distaste effect ($\varepsilon < 1$). To this respect, the rest of this paper is focused on the case where $\varepsilon > 1$.

Assumption 3 $\varepsilon > 1$.

Proposition 2 *Let assumption 3 holds and consider that pollution only comes from production (i.e. $b_1 = 0$ and $b_2 > 0$). There exists a unique positive value of η such that a Hopf bifurcation occurs around the steady state of this economy if and only if $a < \theta$.*

Proof. First of all, remark that D does not depend on η.

Let $b_1 = 0$ and $b_2 > 0$. It follows that:

$$T = \theta - a$$
$$S = a\gamma\eta b_2 (\theta + \delta) \left(\frac{\varepsilon - 1}{\varepsilon}\right) - \varphi \frac{1-\alpha}{\sigma} \frac{\theta + \delta}{\varepsilon} - a\theta$$

Following Bosi and Desmarchelier (2019, Proposition 4), a Hopf bifurcation occurs around the steady state if and only if $D = ST$ such that $S > 0$. Since $D > 0$ (see (14)), a necessary (but not sufficient) condition for which a Hopf bifurcation occurs is that $T > 0$ which is possible if and only if $a < \theta$. In this case, since $\varepsilon > 1$ (assumption 3):

$$\lim_{\eta \to 0} ST = (a - \theta) \left(\varphi \frac{1-\alpha}{\sigma} \frac{\theta + \delta}{\varepsilon} + a\theta\right) < 0 < D \quad (16)$$
$$\lim_{\eta \to +\infty} ST = +\infty > D > 0 \quad (17)$$

From the intermediate value theorem, (16) and (17) imply that there exists a least, one positive value of η for which a Hopf bifurcation occurs. Remark that ST is a linear function of η. This value is then unique. Interestingly, as discussed, $a < \theta$ ensures $T > 0$ and since $D > 0$, it follows that when η is such that $D = ST$, $S > 0$. The proposition follows. ■

Let us introduce a threshold value of η:

$$\eta_1 \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left(\frac{\varphi \frac{1-\alpha}{\sigma} \frac{\theta+\delta}{\varepsilon} + a\theta}{a\gamma\theta b_1}\right)$$

It is interesting to remark that assumption 3 ensures that $\eta_1 > 0$.

Proposition 3 *Let assumption 3 holds and consider that pollution only comes from consumption (i.e. $b_1 > 0$ and $b_2 = 0$). There exists, at least, one value of $\eta > \eta_1$ such that a Hopf bifurcation occurs around the steady state of this economy.*

Proof. First of all, remark that D does not depend on η.

Let $b_1 > 0$ and $b_2 = 0$. It follows that:

$$T = \theta - a\left(1 + \eta b_1 \gamma \left(\frac{1-\varepsilon}{\varepsilon}\right)\right)$$
$$S = a\gamma\eta\theta b_1 \left(\frac{\varepsilon - 1}{\varepsilon}\right) - \varphi\frac{1-\alpha}{\sigma}\frac{\theta+\delta}{\varepsilon} - a\theta$$

Following Bosi and Desmarchelier (2019, Proposition 4), a Hopf bifurcation occurs around the steady state if and only if $D = ST$ such that $S > 0$. Remark that $S > (<)\, 0$ if and only if $\eta > (<)\, \eta_1$. That is, a necessary (but not sufficient) condition for a Hopf bifurcation to occur is that $\eta > \eta_1$. Moreover, since $\varepsilon > 1$ (assumption 3):

$$\lim_{\eta \to \eta_1} ST = 0 < D \tag{18}$$
$$\lim_{\eta \to +\infty} ST = +\infty > D > 0 \tag{19}$$

Remark that ST is a continuous function of η. From the intermediate value theorem, (18) and (19) imply that there exists a least, one positive value of $\eta > \eta_1$ for which a Hopf bifurcation occurs. Since both $D > 0$ and $S > 0$ when η is such that $D = ST$, it follows that, in this case, $T > 0$. ■

One can compare the conditions regarding pollution inertia under which a Hopf bifurcation occurs. Clearly, the only way to observe a limit cycle near the steady state when pollution originates solely from production is that $a < \theta$ (see Proposition 2). However, when pollution originates solely from consumption, there is no need to impose an upper bound on a (Proposition 3). In other words, a Hopf bifurcation can arise for a much more volatile pollution process when pollution comes from consumption than when it comes from production. This interesting result deserves an explanation. Endogenous cycles occur due to the interaction of two opposing forces: an attractive force and a repulsive force. According to (7), the attractive force is driven by natural pollution absorption, while the repulsive forces are generated by both consumption and production. The household is responsible for both repulsive forces. Indeed, by choosing, for instance, to increase its consumption level, the household can raise the future pollution level when pollution originates from consumption. However, if pollution comes from production, the household's saving decision influences the capital level in the next period. For instance, an increase in saving leads to a higher capital level, resulting in a higher production level. Thus, while household choices have a direct effect on the next period's pollution stock when pollution comes from consumption, this effect is indirect when pollution comes from production. Moreover, in the long run, the capital stock becomes relatively high, and due to decreasing marginal productivity, a higher capital level has only a moderate effect on production. As a result, the only way to observe a higher pollution level in the next period is if the attractive force is sufficiently weak ($a < \theta$). It follows that the decreasing marginal productivity of capital is responsible for the difference in the conditions regarding pollution inertia under which a Hopf bifurcation occurs when pollution originates from consumption versus production.

5 Conclusion

The present paper has investigated why strong pollution inertia appears to be a necessary condition for a Hopf bifurcation to occur in competitive Ramsey economies when pollution originates from production but not when it comes from consumption. To clarify this point and to connect these two strands of results, we have developed a very simple competitive Ramsey economy where pollution arises from both consumption and production. By successively comparing the conditions under which a limit cycle emerges near the steady state of the economy, we highlight that the decreasing marginal productivity of capital plays a central role. Indeed, at the steady state, when pollution originates from production, it increases if the household chooses to raise her saving which, in turn, increases the capital level. This contributes to higher the production level and, consequently, greater pollution. However, at the steady state, the capital level is already high and thus, the marginal productivity of capital is low. As a result, higher savings have only a limited effect on production. Consequently, pollution can increase significantly only if pollution inertia is strong. Without this last condition, the repulsive force is not strong enough to destabilize the economy and, therefore, insufficient to allow for the emergence of a limit cycle (Hopf bifurcation). This explanation helps to understand that the choice of modeling the source of pollution (consumption or production) has important consequences for the stability of the equilibrium.

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