BETA
Bureau
d'économie
théorique
et appliquée

Documents de travail

« Political cycles around roundabouts»

Auteurs

Stefano BOSI, David DESMARCHELIER

Document de Travail n° 2024 – 55

Novembre 2024

Bureau d'Économie Théorique et Appliquée
BETA

www.beta-economics.fr

@beta_economics

Contact :
jaoulgrammare@beta-cnrs.unistra.fr

Université de Strasbourg
cnrs
UNIVERSITÉ DE LORRAINE
INRAE
AgroParisTech

Political cycles around roundabouts

Stefano BOSI[*] David DESMARCHELIER[†]

EPEE, Université Paris-Saclay BETA, Université de Lorraine

November 24, 2024

Abstract

We develop a unified framework at the crossroads of economics, political and environmental science, and, to some extent, epidemiology. Populism is equated with climate skepticism and seen as an opinion that spreads through the population. Drawing on compartmental models in epidemiology, the population is divided into two groups that interact with each other: climate skeptics, almost always populists, and environmentalists. The political building block is integrated into a Ramsey model with a pollution externality originated from production. We introduce a Pigouvian tax to finance depollution according to a balanced-budget rule. To take account of populist pressure against environmental policies, we assume also that the tax rate decreases in the share of skeptics in population. Our unified approach reveals an interesting result: populism generates stable limit cycles through a Hopf bifurcation around the steady state, whatever the pollution effect on the consumption demand. Importantly, without populism, it was not the case under a negative distaste effect. Thus, populism exacerbates pollution-induced volatility: populist parties focusing on economic issues should manage excess volatility without rejecting environmental policies out of hand.

Keywords: *ecotax, populism, political cycles.*
JEL codes: *C62, H23, O44.*

1 Introduction

According to Buzogány and Mohamad-Klotzbach (2021), both populism and climate change represent two major threats for contemporary democracies. While the definition of climate change is now well-established, populism remains more difficult to identify, as it encompasses both left and right.[1] For Lockwood (2018)

[*]Université Paris-Saclay, EPEE, 91025, Evry, France. E-mail: stefano.bosi@univ-evry.fr.

[†]Université de Lorraine, Université de Strasbourg, CNRS, BETA, 54000, Nancy, France. E-mail: david.desmarchelier@univ-lorraine.fr.

[1]According to United Nations/Climate Action, "climate change refers to long-term shifts in temperatures and weather patterns" (https://www.un.org/en/climatechange/what-is-climate-change).

and Guriev and Papaioannou (2022), right- or left-wing populism are ideologies that divide the population into two groups: “the pure people” and a “corrupted elite”, who oppose each other. However, as Huber (2020) points out, far-right parties reject environmental policies, while far-left parties broadly supports them.

Right-wing populism places great importance on economic issues rather than environmental ones (Lockwood, 2018). Two recent examples are Donald Trump in the USA, who withdrew the US from the Paris Agreement, and Jair Bolsonaro in Brazil, who authorized massive deforestation of the Amazon rainforest (Pereira et al., 2019). Buzogány and Mohamad-Klotzbach (2021) point out that right-wing populism is associated with antiscience, which partially explains why populists reject environmental policies. Two other explanations are proposed by Lockwood (2018): (1) his "structuralist" argument rests on fact that right-wing populism stems from the rejection of past structural changes such as globalization, and that climate change is seen as a new structural change that worsens the economic situation of those left behind, the so-called pure people populists seek to represent. The "ideological" explanation lies instead in the fact that right-wing populism is nationalist, and environmental concern is seen as a counter-national interest.

Over the past decade, right-wing populism has taken hold of the political landscape in European countries and the United States. The Brexit referendum in the UK in 2016, the elections of Donald Trump in 2017 and 2024 in the US, the recent electoral performances of the *Rassemblement National* in France or *Alternative für Deutschland* in Germany are all examples of this political breakthrough. It warns on the social acceptance of environmental policies.

The yellow-vests crisis in France is a good example of the possible social distrust of environmental policies: the massive protests that began in November 2018 convinced the French government to abandon the increase in the carbon tax on fuel. While the *gilets-jaunes* movement is clearly populist, it is far from clear whether it belongs to the right or the left (Bourdin and Torre, 2023). Nevertheless, from a theoretical point of view, it seems interesting to consider together populism and climate skepticism when discussing environmental policy. This article is a first attempt to introduce them in a dynamic general equilibrium context, while focusing only on right-wing populism: from now on, climate skepticism will be synonymous with right-wing populism.

To represent the evolution of public opinion, we propose to apply a simple model of disease spread.[2] Specifically, the population is divided into two groups: “environmentalists” and “skeptics”. On each date, the agents meet and influence each other. During a bilateral encounter, a skeptic may become environmentalist and an environmentalist skeptic, depending on the relative power of persuasion. This power depends on the level of pollution: more (less) pollution gives credit to environmentalists (skeptics). We also assume that an agent can change her mind spontaneously. Our political model is close to the SIS epi-

[2] We adapt Desmarchelier and Lanzi’s (2023) opinion dynamics model to an environmental context.

demiological model but not identical.[3] Indeed, if we equate skeptics with sick agents and environmentalists with healthy agents, here, the latter can “contaminate” the former who, then, become environmentalists, whereas, in a SIS model, a healthy agent is unable to "contaminate" a sick agent. In addition, the government is assumed to introduce a green tax levied on the production level to finance clean-up according to a balanced-budget rule. We suppose that the tax rate decreases with the share of skeptics in population: the more skeptics there are, the greater the pressure for a tax cut. This is what we call the Yellow-Vest Effect (hereafter YVE). This contagion model adapted to climate populism is integrated into a Ramsey model to understand the interaction between economic variables (capital and consumption), opinion dynamics (climate skepticism/populism) and pollution dynamics.

Our model, at the crossroads of economics, political and environmental sciences and, to some extent, epidemiology, reveals that populism is not only bad for the environment, but also for economic stability. Populist parties that prioritize economic issues should manage excess volatility without rejecting environmentally-friendly solutions out of hand.

For the sake of precision, we know that limit cycles can appear via a Hopf bifurcation in a Ramsey model where pollution increases consumption demand (compensation effect), while they are ruled out when pollution lowers demand (distaste effect).[4] The rationale for the existence of cycles is quite simple: higher pollution today implies higher consumption demand (compensation effect) which reduces savings and, then, the capital stock of the next period. A lower capital stock decreases production possibilities and the stock of pollution in turn. And so on.

Let us now take into account the additional dynamics resulting from the spread of climate skepticism. The initially higher level of pollution strengthens the persuasive power of environmentalists who lobby to increase the ecotax rate and, ultimately, reduces production and pollution. So, now, in both cases, compensation or distaste effect with populism, cycles take place: higher pollution today is followed by lower pollution tomorrow, and so on.

We conclude by observing that not only does populism generate permanent cycles in the less favorable case of a distaste effect, where cycles are normally impossible, but it also exacerbates macroeconomic volatility in the more favorable case of a compensation effect, where cycles arise naturally. Populism implies that fluctuations generated by pollution can occur regardless of the effect of pollution on the marginal utility of consumption.

The remainder of the article is organized as follows. Section 2 presents the model, Section 3 studies the equilibrium, Section 4 proposes two simulations, while Section 5 concludes.

---

[3] In the SIS model, infection does not confer long-term immunity and individuals become susceptible again when they recover from infection. The population is divided into labeled compartments: S and I stand for susceptible and infectives. Readers interested in a simple presentation of this model are referred to Hethcote (2009).

[4] Distaste and compensation effects were introduced by Michel and Rotillon (1995). See also Bosi and Desmarchelier (2018).

2 Fundamentals

We consider a Ramsey-type market economy where a productive pollution externality affects household utility. Agents have different ecological attitudes. Environmentalists are confronted with skeptics. The government levies a green tax to finance environmental maintenance and clean-up.

2.1 Producers

The production sector consists of a continuum of price-taker firms. Their output is produced according to a Constant-Returns-to-Scale (CRS) technology using capital and labor. Because of the CRS, it is equivalent to consider a single firm who behaves competitively:

$$Y \equiv F(K, L) = Lf(k) \tag{1}$$

where Y, K and L denote the aggregate supply of output and the aggregate demands for capital and labor, $k \equiv K/L$ and $f(k) \equiv F(k, 1)$ represent the capital intensity and the average output.

Assumption 1 $f''(k) < 0 < f'(k)$. *The Inada conditions hold:* $\lim_{k\to 0} f'(k) = \infty$ *and* $\lim_{k\to\infty} f'(k) = 0$.

The government levies a proportional tax on output at the rate $\tau \in (0, 1)$. This rate can vary over time: $\tau = \tau(t)$. Taking the announced tax rate as given at date t, the representative firm chooses the capital and labour demand to maximize its static profit: $\max_{(K,L)} [(1 - \tau) Lf(k) - rK - wL]$. Capital and labour demand are set to equalize marginal productivity and prices:

$$r = (1 - \tau) f'(k) = (1 - \tau) \rho(k) \tag{2}$$

$$w = (1 - \tau) [f(k) - kf'(k)] = (1 - \tau) \omega(k) \tag{3}$$

where $\rho(k) \equiv f'(k)$ and $\omega(k) \equiv f(k) - kf'(k)$.

The capital share in total income is given by

$$\alpha(k) \equiv \frac{kf'(k)}{f(k)} \in (0, 1) \tag{4}$$

while the Allen-Hicks' elasticity of capital-labour substitution by

$$\varepsilon(K, L) = \frac{\frac{\partial F}{\partial K}\frac{\partial F}{\partial L}}{F\frac{\partial^2 F}{\partial K \partial L}}$$

In the CRS case, this elasticity becomes:

$$\varepsilon(K, L) = -\frac{f'(k)[f(k) - kf'(k)]}{kf(k) f''(k)} \equiv \sigma(k) > 0$$

and the elasticities of factor prices:

$$\frac{k\rho'(k)}{\rho(k)} = -\frac{1 - \alpha(k)}{\sigma(k)} < 0 \text{ and } \frac{k\omega'(k)}{\omega(k)} = \frac{\alpha(k)}{\sigma(k)} > 0 \tag{5}$$

Under a constant elasticity of capital-labor substitution:

$$F(K,L) \equiv \left(aK^{\frac{\sigma-1}{\sigma}} + bL^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

we get $\varepsilon(K,L) = \sigma(k) = \sigma$ and

$$\alpha(k) = \frac{a}{a + bk^{\frac{1-\sigma}{\sigma}}} \in (0,1)$$

In the Cobb-Douglas case, $\sigma = 1$ and α are constant.

2.2 Households

The representative household are represented by a continuous-time utility functional

$$\int_0^\infty e^{-\theta t} u(c(t), P(t))\, dt$$

where θ denotes her time preference, u is a strictly increasing and concave, and non-separable felicity function depending on her consumption level $c(t)$ and an aggregate pollution externality $P(t)$.

Assumption 2 *$u_{cc} < 0 < u_c$ and $u_P < 0$ for any $(c,P) \in \mathbb{R}^2_+$. The Inada conditions hold: $\lim_{c\to 0} u_c = \infty$, $\lim_{c\to\infty} u_c = 0$.*

Remark 1 *The cross effect u_{cP} can be negative or positive under non-separability. Following Michel and Rotillon (1995), $u_{cP} < 0$ captures the distaste effect (a higher pollution level reduces consumption demand, namely when the household likes to consume in a pleasant environment). Conversely, $u_{cP} > 0$ represents a compensation effect (the household compensates the utility drop entailed by a higher pollution level ($u_P < 0$), by an increase of her consumption demand).*

Let $h(t)$ and $l(t)$ denote the household's individual wealth in terms of capital and labor supply, and $r(t)$ and $w(t)$ be the corresponding prices, that is the interest rate and wage. Let also δ and $n(t) \equiv \dot{N}(t)/N(t)$ denote the constant capital depreciation rate and the growth rate of population.

She spends her income $r(t)h(t) + w(t)l(t)$, to consume and to save $\dot{h}(t) + [n(t)+\delta]h(t)$, where $\dot{h}$ denote the time derivative. Indeed, denoting $H(t) = h(t)N(t)$ the aggregate wealth and $\dot{H}(t)$ the variation of aggregate wealth, we have that the individual gross investment is given by

$$\frac{\dot{H}(t)}{N(t)} + \frac{\delta H(t)}{N(t)} = \frac{\dot{H}(t)}{N(t)} + \delta h(t) = \dot{h}(t) + [n(t)+\delta]h(t)$$

since $\dot{H}(t)/N(t) = \dot{h}(t) + n(t)h(t)$.

For simplicity, capital and consumption are the same good, and labor supply is inelastic and equal to one: $l(t) = 1$. Therefore, her budget constraint becomes

$$\dot{h}(t) \leq r(t)h(t) + w(t) - n(t)h(t) - \delta h(t) - c(t) \quad (6)$$

In the rest of the paper, we will assume the population to be constant over time: $N(t) = N$, that is $n(t) = 0$ for any t. From now on, for notational parsimony, we will omit also the time argument.

Therefore, the household' program becomes

$$\max_c \int_0^\infty e^{-\theta t} u(c, P)\, dt \tag{7}$$
$$\dot{h} \leq (r - \delta) h + w - c$$

She chooses the entire path $c = c(t)$ in order to maximize this utility functional.

Lemma 2 *The first-order conditions of program (7) are given by*

$$\frac{\dot{\mu}}{\mu} = \delta + \theta - r \tag{8}$$
$$\dot{h} = (r - \delta) h + w - c \tag{9}$$

where

$$\mu = u_c(c, P) \tag{10}$$

is the marginal utility of consumption. The transversality condition is given by: $\lim_{t\to\infty} e^{-\theta t} \mu h = 0$.

We introduce two second-order elasticities of utility:

$$\varepsilon_{cc} \equiv \frac{c u_{cc}}{u_c} < 0 \text{ and } \varepsilon_{cP} \equiv \frac{P u_{cP}}{u_c}$$

$-1/\varepsilon_{cc}$ represents the intertemporal elasticity of substitution in consumption while ε_{cP} captures the pollution effect on marginal utility of consumption. More precisely, in the spirit of Michel and Rotillon (1995) a compensation (distaste) effect takes place when $\varepsilon_{cP} > 0$ (< 0). Applying the implicit function theorem to (10), we obtain the consumption function

$$c \equiv c(\mu, P) \tag{11}$$

with elasticities:

$$\frac{\mu}{c}\frac{dc}{d\mu} = \frac{1}{\varepsilon_{cc}} < 0 \tag{12}$$
$$\frac{P}{c}\frac{dc}{dP} = -\frac{\varepsilon_{cP}}{\varepsilon_{cc}} \tag{13}$$

Focusing on elasticity (13), we observe that the impact of pollution on consumption is negative if $\varepsilon_{cP} < 0$ (distaste effect) and positive if $\varepsilon_{cP} > 0$ (compensation effect).

The elasticities of the explicit utility function:

$$u(c, P) \equiv \frac{\left(cP^{-\eta}\right)^{1-\varepsilon}}{1 - \varepsilon} \tag{14}$$

with $\varepsilon > 0$ and $\eta > 0$, are constant:

$$\varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c} = -\varepsilon < 0 \quad (15)$$

$$\varepsilon_{cP} \equiv \frac{Pu_{cP}}{u_c} = \eta(\varepsilon - 1) < 0 \Leftrightarrow \varepsilon < 1 \quad (16)$$

In this case, the impact of pollution on consumption is negative if $\varepsilon < 1$ (distaste effect) or positive if $\varepsilon > 1$ (compensation effect). We observe that η captures the degree of pollution externality. Since $\varepsilon_{cP} = \eta(\varepsilon - 1)$, it amplifies the compensation effect or the distaste effect.

Moreover, in the case of function (14), using (10), we obtain (11):

$$c(\mu, P) = \mu^{-\frac{1}{\varepsilon}} P^{\eta \frac{\varepsilon - 1}{\varepsilon}}$$

2.3 Government and pollution

Pollution P is a stock coming from production. To keep things as simple as possible, we assume a linear accumulation process:

$$\dot{P} = -aP + bY - dG \quad (17)$$

where a, b and d represent, respectively, the rate of natural pollution absorption, the environmental impact per unit of production, and the depollution efficiency. In a world with no human, from a date 0 on, pollution follows an exponential decay: $P(t) = P(0)e^{-at}$.

The government uses the whole tax revenue to finance depollution:

$$G = \tau Y \quad (18)$$

2.4 Political contagion

In the spirit of Desmarchelier and Lanzi (2023), a model of opinion dynamics, we represent the spread of skepticism through the economy. The population is divided in two groups, the "environmentalists" and the "environmental skeptics": $N = E + S$, where E and S denote the size of these two groups.

People can change their opinions. While, for simplicity, the size N of population is constant, the share $s \equiv S/N$ varies over time.

At any time, each agent interacts and exchanges opinions with another agent on climate change. The probability for a skeptic to meet an environmentalist is given by E/N. The number of skeptics meeting an opponent at each moment is given by $S(E/N)$. Symmetrically, the number of environmentalists meeting a skeptic is given by $E(S/N)$. Of course, $E(S/N) = S(E/N)$.

During a bilateral meeting, the degree of persuasiveness of an environmentalist to convince a skeptic is given by $\beta_1 \geq 0$, while $\beta_2 \geq 0$ represents the degree of persuasiveness of a skeptic to convince an environmentalist.

Furthermore, we suppose that both the types can also change their opinion spontaneously. We denote by $\gamma_1 \geq 0$ the share of skeptics who, spontaneously, become environmentalists and, by $\gamma_2 \geq 0$, the share of environmentalists who, spontaneously, become skeptics.

Proposition 3 *Opinion dynamics are given by*

$$\dot{s} = s(1-s)\left(\beta_2 - \beta_1 + \frac{\gamma_2}{s} - \frac{\gamma_1}{1-s}\right) \tag{19}$$

If $\beta_2 > \beta_1$, skeptics' persuasion force is stronger than the environmentalists' one. Conversely, if $\beta_1 > \beta_2$, the environmentalists' persuasion force is dominant.

Environmental quality influences the spread of climate skepticism. Specifically, a higher pollution level proves that economic activities are harmful to the environment. On the one hand, this reduces the persuasiveness of skeptics and increases that of environmentalists (respectively, $\beta_2'(P) < 0$ and $\beta_1'(P) > 0$. On the other hand, the share of skeptics who, spontaneously, become environmentalists, increases ($\gamma_1'(P) > 0$) and the share of environmentalists who, spontaneously, become skeptics, decreases ($\gamma_2'(P) < 0$).

Assumption 3 *For any* $P \geq 0$, $\beta_1'(P) > 0$ and $\beta_2'(P) < 0$ *and with*

$$\begin{aligned}
\lim_{P\to 0} \beta_1(P) &= 0 \text{ and } \lim_{P\to\infty} \beta_1(P) = \infty \\
\lim_{P\to 0} \beta_2(P) &= \infty \text{ and } \lim_{P\to\infty} \beta_2(P) = 0
\end{aligned}$$

and $\gamma_1'(P) > 0$ *and* $\gamma_2'(P) < 0$ *with*

$$\begin{aligned}
\lim_{P\to 0} \gamma_1(P) &= 0 \text{ and } \lim_{P\to\infty} \gamma_1(P) = \infty \\
\lim_{P\to 0} \gamma_2(P) &= \infty \text{ and } \lim_{P\to\infty} \gamma_2(P) = 0
\end{aligned}$$

Let us introduce the political elasticities:

$$\begin{aligned}
\varepsilon_1(P) &\equiv \frac{P\beta_1'(P)}{\beta_1(P)} > 0 \text{ and } \varepsilon_2(P) \equiv \frac{P\beta_2'(P)}{\beta_2(P)} < 0 \\
\eta_1(P) &\equiv \frac{P\gamma_1'(P)}{\gamma_1(P)} > 0 \text{ and } \eta_2(P) \equiv \frac{P\gamma_2'(P)}{\gamma_2(P)} < 0
\end{aligned}$$

For instance, the following political functions

$$\beta_1(P) = B_1 P^{\varepsilon_1} \text{ with } \varepsilon_1 > 0 \tag{20}$$
$$\beta_2(P) = B_2 P^{\varepsilon_2} \text{ with } \varepsilon_2 < 0 \tag{21}$$
$$\gamma_1(P) = C_1 P^{\eta_1} \text{ with } \eta_1 > 0 \tag{22}$$
$$\gamma_2(P) = C_2 P^{\eta_2} \text{ with } \eta_2 < 0 \tag{23}$$

have constant elasticities: $\varepsilon_1(P) \equiv \varepsilon_1 > 0$, $\varepsilon_2(P) \equiv \varepsilon_2 < 0$, $\eta_1(P) \equiv \eta_1 > 0$, $\eta_2(P) \equiv \eta_2 < 0$.

Finally, we assume that the carbon tax rate depends on the degree of environmental skepticism: $\tau'(s) < 0$. Let us call it Yellow-Vest Effect (YVE). The green policy of a non-populist government is limited by the rise of a populist party or, when a populist party comes to power, it reduces or abolishes green taxes.

Assumption 4 $\tau'(s) < 0$ *with* $\tau(0) = \bar{\tau} \leq 1$ *and* $\tau(1) = 0$.

We introduce the fiscal elasticity:

$$\varepsilon_\tau(s) \equiv \frac{s\tau'(s)}{\tau(s)} < 0$$

For instance, Assumption 4 is satisfied by the explicit function τ:

$$\tau(s) = \bar{\tau}(1 - s^\pi) \tag{24}$$

with $\pi > 0$. We observe that $\tau(0) = \bar{\tau} \leq 1$, $\tau(1) = 0$ and

$$\varepsilon_\tau(s) = -\pi \frac{s^\pi}{1 - s^\pi}$$

In particular, when $\pi = 1$, we obtain

$$\begin{aligned} \tau(s) &= \bar{\tau}(1 - s) \\ \varepsilon_\tau(s) &= -\frac{s}{1 - s} \end{aligned} \tag{25}$$

3 Equilibrium

The economy is made up of three markets: the labor market, the capital market and the goods market. In general equilibrium, these markets clear together.

Proposition 4 *The dynamic general equilibrium is a represented by the following system:*

$$\dot{\mu} = (\delta + \theta - [1 - \tau(s)]\rho(k))\mu \tag{26}$$

$$\dot{k} = [1 - \tau(s)] f(k) - \delta k - c(\mu, P) \tag{27}$$

$$\dot{P} = -aP + [b - d\tau(s)] f(k) \tag{28}$$

$$\dot{s} = s(1 - s)\left[\beta_2(P) - \beta_1(P) + \frac{\gamma_2(P)}{s} - \frac{\gamma_1(P)}{1 - s}\right] \tag{29}$$

The system results form the addition of three blocks: economic, ecological and political. More precisely, equations (26) and (27) represent the Ramsey model, augmented by the pollution process (28) and the opinion dynamics (29). μ is a jump variable while k, P and s are three predetermined variables.

3.1 Steady state

Let us suppose that production has a larger impact than depollution on pollution.

Assumption 5 $b \geq d$.

Assumption 5 ensures the existence of a non-negative pollution stock at the steady state.

Proposition 5 *Under Assumptions 1 to 5, there exists at least a non-trivial steady state (μ^*, k^*, P^*, s^*). This steady state is unique.*

Capital k^* and pollution P^* depend on populism s^*

$$k^* = \rho^{-1}\left(\frac{\delta+\theta}{1-\tau(s^*)}\right) \equiv k(s^*) > 0 \tag{30}$$

$$P^* = \frac{b-d\tau(s^*)}{a} f(k(s^*)) \equiv P(s^*) > 0 \tag{31}$$

where s^* is the unique solution to

$$\beta_2(P(s)) - \beta_1(P(s)) = \frac{\gamma_1(P(s))}{1-s} - \frac{\gamma_2(P(s))}{s} \tag{32}$$

The consumption demand and its marginal utility are given by

$$c^* = [1-\tau(s^*)] f(k^*) - \delta k^* > 0 \tag{33}$$

$$\mu^* = u_c(c^*, P^*) > 0 \tag{34}$$

In the isoelastic case (14), only μ^* depends on (ε, η), while (k^*, P^*, s^*, c^*) don't.

3.2 Comparative statics

Consider the impact of YVE, that is $\bar{\tau}$, on the steady state. Let us define the steady state as a function of YVE: $s^* = \hat{s}(\bar{\tau})$, $k^* = \hat{k}(\bar{\tau})$, $P^* = \hat{P}(\bar{\tau})$, and introduce the following positive blocks under Assumption 5:

$$S_0 \equiv \frac{1-s^\pi}{s^\pi} > 0$$

$$S_1 \equiv \frac{1}{\bar{\tau}s^\pi} - \frac{1-s^\pi}{s^\pi} = \frac{1-\tau(s)}{\bar{\tau}s^\pi} > 0$$

$$S_2 \equiv \frac{b}{d}\frac{1}{\bar{\tau}s^\pi} - \frac{1-s^\pi}{s^\pi} > 0$$

$$Q_1 \equiv \frac{s}{1-s}\frac{\gamma_1(P)}{1-s} + \frac{\gamma_2(P)}{s} > 0$$

$$Q_2 \equiv \beta_1(P)\varepsilon_1(P) - \beta_2(P)\varepsilon_2(P) + \eta_1(P)\frac{\gamma_1(P)}{1-s} - \eta_2(P)\frac{\gamma_2(P)}{s} > 0$$

Proposition 6 *The impacts of YVE $\bar{\tau}$ on s^*, k^* and P^* are given by*

$$\frac{\bar{\tau}}{s^*}\frac{d\hat{s}}{d\bar{\tau}} = \frac{S_0 Q_2\left[\alpha(k) S_2 + \frac{1-\alpha(k)}{\sigma(k)} S_1\right]}{\alpha(k)\pi Q_2 S_2 + \frac{1-\alpha(k)}{\sigma(k)} S_1(\pi Q_2 + Q_1 S_2)} > 0 \tag{35}$$

$$\frac{\bar{\tau}}{k^*}\frac{d\hat{k}}{d\bar{\tau}} = -\frac{Q_1 S_0 S_2}{\alpha(k)\pi Q_2 S_2 + \frac{1-\alpha(k)}{\sigma(k)} S_1(\pi Q_2 + Q_1 S_2)} < 0 \tag{36}$$

$$\frac{\bar{\tau}}{P^*}\frac{d\hat{P}}{d\bar{\tau}} = -\frac{Q_1 S_0\left[\alpha(k) S_2 + \frac{1-\alpha(k)}{\sigma(k)} S_1\right]}{\alpha(k)\pi Q_2 S_2 + \frac{1-\alpha(k)}{\sigma(k)} S_1(\pi Q_2 + Q_1 S_2)} < 0 \tag{37}$$

Proposition 6 deserves an economic interpretation. Since the tax is levied on the level of production, a higher rate provides an incentive to reduce production. This implies a lower demand for inputs, which translates into a lower level of capital in the long run. In addition, the lower level of production leads to a reduction in pollutant emissions and therefore in the level of pollution in the long term. In line with Assumption 3, the decrease in pollution induced by a higher tax rate increases the persuasiveness of skeptics and decreases that of environmentalists: the share of skeptics increases in the long term.

3.3 Opinion cycles

To study the local dynamics, that is the occurrence of local bifurcations, we apply a methodology based on the study of the minors of the Jacobian matrix, and developed by Bosi and Desmarchelier (2019) among others. In this respect, we linearize the dynamical system (26)-(29) around the non-trivial steady state given by (30), (31), (32) and (34), and obtain a four-dimensional Jacobian matrix. In the following, let λ_1, λ_2, λ_3 and λ_4 be the corresponding eigenvalues.

Since the seminal Heal's (1982) contribution, it is known that a limit cycle can arise near the steady state of a Ramsey economy when a pollution externality increases the marginal utility of consumption (compensation effect). Our goal now is to understand how robust this result is under populism.

Let us introduce the new parameter blocks:

$$\begin{aligned}
A_1 &\equiv \frac{\delta+\theta}{\alpha\left(k^*\right)} > 0 \\
A_2 &\equiv \frac{\tau\left(s^*\right)\varepsilon_\tau\left(s^*\right)}{1-\tau\left(s^*\right)} < 0 \\
A_3 &\equiv \frac{b-d\tau\left(s^*\right)}{1-\tau\left(s^*\right)} > 0 \\
A_4 &\equiv -\frac{1}{\varepsilon_{cc}}\left[\frac{\delta+\theta}{\alpha\left(k^*\right)}-\delta\right] = -\frac{1}{\varepsilon_{cc}}\frac{c^*}{k^*} > 0 \\
A_5 &\equiv a\frac{\varepsilon_{cP}}{\varepsilon_{cc}}\frac{1-\tau\left(s^*\right)}{b-d\tau\left(s^*\right)}\left[1-\delta\frac{\alpha\left(k^*\right)}{\delta+\theta}\right] > 0 \Leftrightarrow \varepsilon_{cP} < 0 \qquad (38) \\
A_6 &\equiv \varepsilon_2\left(P^*\right)\beta_2\left(P^*\right)-\varepsilon_1\left(P^*\right)\beta_1\left(P^*\right)+\eta_2\left(P^*\right)\frac{\gamma_2\left(P^*\right)}{s^*}-\eta_1\left(P^*\right)\frac{\gamma_1\left(P^*\right)}{1-s^*} < 0 \\
A_7 &\equiv -s^*\frac{\gamma_1\left(P^*\right)}{1-s^*}-\left(1-s^*\right)\frac{\gamma_2\left(P^*\right)}{s^*} \leq 0
\end{aligned}$$

Lemma 7 *The sums of the principal minors of the Jacobian matrix of system*

(26)-(29) are given by:

$$
\begin{aligned}
S_1 &= \lambda_1+\lambda_2+\lambda_3+\lambda_4=\theta-a+A_7 && (39)\\
S_2 &= \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4=\Sigma_2-\alpha(k^*)A_1A_3A_5 && (40)\\
S_3 &= \lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4=\Sigma_3-\alpha(k^*)A_1A_3A_5A_7 && (41)\\
S_4 &= \lambda_1\lambda_2\lambda_3\lambda_4\\
&= a\alpha(k^*)A_1A_4\left(A_7\frac{1-\alpha(k^*)}{\sigma(k^*)}-B(1-s^*)\left[\alpha(k^*)A_3+d\frac{1-\alpha(k^*)}{\sigma(k^*)}\right]\right) && (42)
\end{aligned}
$$

where $S_1=T$ is the trace and $S_4=D<0$ is the determinant, and

$$
\begin{aligned}
\Sigma_2 &\equiv (\theta-a)A_7+adB(1-s^*)-a\theta-\alpha(k^*)A_1A_4\frac{1-\alpha(k^*)}{\sigma(k^*)}\\
\Sigma_3 &\equiv a(1-s^*)[d\theta-\alpha(k^*)A_1A_3]B-a\theta A_7+\alpha(k^*)A_1A_4(a-A_7)\frac{1-\alpha(k^*)}{\sigma(k^*)}
\end{aligned}
$$

with

$$B\equiv A_6\frac{A_2}{A_3}=A_6\frac{\tau(s^*)\varepsilon_\tau(s^*)}{b-d\tau(s^*)}>0$$

Proposition 8 *The equilibrium is locally unique.*

Proposition 8 rules out local indeterminacy and fluctuations due to self-fulfilling expectations.

Proposition 9 *There is no room for zero local bifurcations (saddle-node, transcritical, pitchfork). There is no room for codimension-two local bifurcations (Bogdanov-Takens, Gavrilov-Guckenheimer and double-Hopf).*

A suitable parameter to study the occurrence of limit cycles is η. Indeed, η captures the degree of pollution externality and, since $\varepsilon_{cP}=\eta(\varepsilon-1)$, it amplifies the compensation effect or the distaste effect, that is the forces at work responsible for the fluctuations. Let

$$A_5^H\equiv\frac{T\Sigma_3+TA_7\Sigma_2-2A_7\Sigma_3-T\sqrt{(A_7\Sigma_2-\Sigma_3)^2+4DA_7(\theta-a)}}{2\alpha(k^*)A_1A_3A_7(\theta-a)} \quad (43)$$

Proposition 10 *If $a<\theta$, a limit cycle arises through a Hopf bifurcation around the non-trivial steady state when*

$$\varepsilon_{cP}=\varepsilon_{cP}^H\equiv A_5^H\frac{\varepsilon_{cc}}{a}\frac{b-d\tau(s^*)}{1-\tau(s^*)}\frac{\theta+\delta}{\theta+\delta[1-\alpha(k^*)]} \quad (44)$$

or, more explicitly, in the isoelastic case (14), when

$$\eta=\eta_H\equiv\frac{A_5^H}{a}\frac{\varepsilon}{1-\varepsilon}\frac{b-d\tau(s^*)}{1-\tau(s^*)}\frac{\theta+\delta}{\theta+\delta[1-\alpha(k^*)]} \quad (45)$$

Remark 11 *In the general case,* ε_{cc} *and* ε_{cP} *are related by some fundamental parameter of preferences and equation (44) remains implicit. Conversely, in the isoelastic case,* η_H *that is the RHS of (45) does not depend on* η *and, therefore, the bifurcation point in terms of* η *is unambiguously and explicitly defined.*

Lemma 12 *If* $a > \theta$*, there exists a critical value of the cross effect* ε_{cP} *such that a limit cycle generically arises through a Hopf bifurcation around the non-trivial steady state if and only if*

$$\frac{DA_7}{a-\theta} \leq \left[\frac{A_7\Sigma_2 - \Sigma_3}{2(a-\theta)}\right]^2 \text{ and } A_7\Sigma_2 - \Sigma_3 > 0 \tag{46}$$

In the particular case $\gamma_1(P) = \gamma_2(P) = 0$ *for any* P*, there exists a critical value of* ε_{cP} *such that a limit cycle generically arises through a Hopf bifurcation around the non-trivial steady state if and only if* $\Sigma_3 < 0$*.*

Let

$$\tilde{A}_5^H \equiv \frac{1}{\alpha(k^*) A_1 A_3}\left(\Sigma_2 + \frac{\Sigma_3}{a-\theta} + \frac{a-\theta}{\Sigma_3}D\right) \tag{47}$$

Proposition 13 *Let* $a > \theta$ *and* $\gamma_1(P) = \gamma_2(P) = 0$ *for any* P*. A limit cycle generically arises through a Hopf bifurcation around the non-trivial steady state when*

$$\varepsilon_{cP} = \tilde{\varepsilon}_{cP}^H \equiv \varepsilon_{cc}\frac{\tilde{A}_5^H}{a}\frac{b-d\tau(s^*)}{1-\tau(s^*)}\frac{\theta+\delta}{\theta+\delta[1-\alpha(k^*)]} \tag{48}$$

provided that

$$-\varepsilon_{cc} > \frac{1}{A_6}\frac{1-\alpha(k^*)}{\alpha(k^*)\sigma(k^*)}\frac{1-\tau(s^*)}{(1-s^*)\tau(s^*)\varepsilon_\tau(s^*)}\frac{\theta+\delta[1-\alpha(k^*)]}{1-\frac{\theta}{\delta+\theta}\frac{d-d\tau(s^*)}{b-d\tau(s^*)}}\,(>0) \tag{49}$$

More explicitly, in the isoelastic case (14), a limit cycle generically arises when

$$\eta = \tilde{\eta}_H \equiv \frac{\tilde{A}_5^H}{a}\frac{\varepsilon}{1-\varepsilon}\frac{b-d\tau(s^*)}{1-\tau(s^*)}\frac{\theta+\delta}{\theta+\delta[1-\alpha(k^*)]} \tag{50}$$

provided that inequality (49) holds with $\varepsilon = -\varepsilon_{cc}$ *in the LHS. If* $\tilde{A}_5^H < 0$*, a limit cycle generically arises at* $\tilde{\eta}_H$ *under a compensation effect* ($\varepsilon > 1$)*, while, if* $\tilde{A}_5^H > 0$*, under a distaste effect* ($\varepsilon < 1$)*.*

Remark 14 *In the isoelastic case (14), the RHS of (49) does not depend on* ε *nor on* η*, while the RHS of (50) does not depend on* η*. In other terms, the critical value is explicit and well-defined.*

To be able to interpret propositions 10 and 12, we need to determine the sign of ε_{cP}^H (when $a < \theta$) and $\tilde{\varepsilon}_{cP}^H$ (when $a > \theta$). These expressions are cumbersome and computations are far from being easy. However, according to the existing literature on the cross effects in preferences without opinion dynamics, the existence of limit cycles through a Hopf bifurcation requires a compensation effect

($\varepsilon_{cP} > 0$) under sufficient high degree of pollution inertia ($a < \theta$). Conversely there is no room for Hopf bifurcations under a distaste effect ($\varepsilon_{cP} < 0$), whatever the degree of pollution inertia.[5] Our model generalizes this basic framework with opinion dynamics and, unsurprisingly, we recover the occurrence of limit cycles under a compensation effect. What is new is the possibility of limit cycles under the distaste effect because of the opinion dynamics.

Focus first on the case of compensation effect to understand how populism makes cycles more likely. Assume an exogenous increase in the level of pollution today. As a result of the compensation effect, the household increases its current consumption and reduces its savings, thereby reducing tomorrow's capital stock. Less capital also means less production and, therefore, a lower level of pollution, and so on. As seen above, this explanation is standard in Ramsey economies with pollution.[6]

However, populism makes these cycles more likely. Indeed, the more pollution there is, the more convincing the environmentalists are, which reduces the proportion of skeptics in the population. The pressure for an environmental policy becomes higher and the green tax increases, reducing the level of pollution. As before, an increase in pollution is followed by a decrease in pollution: the two mechanisms, consumption and populism, move in the same direction.

Focus now on the case of distaste effect. The previous mechanism suggests that a green taxation highly sensitive to populism can promote the occurrence of cycles even in the unfavorable case of distaste. As seen above, because of the complicated expressions of ε_{cP}^{H} and $\tilde{\varepsilon}_{cP}^{H}$, it is not possible to prove analytically this conjecture. However, it is numerically. More precisely, we will show later that there is room for stable cycles under distaste effect ($\varepsilon < 1$) if the rate of pollution absorption is sufficiently high ($a > \theta$). In this respect, we can affirm that populism always exacerbates economic volatility in a polluted world.

3.4 A simple model with constant elasticities

So far, we have developed a general model of opinion dynamics with two channels for change in opinion: (1) contagion through β_1 and β_2 and (2) spontaneous change through γ_1 and γ_2. In our interpretation of fluctuations following Proposition 13, the contagion channel plays the main role.

Now, we want to deepen our analysis of local bifurcations, namely Propositions 9 and 13, by focusing on the particular case where contagion is the only possible mechanism for opinion change.

The fundamentals of this particular economy are the following.

(1) Isoelastic political functions (20) and (21).

(2) Isoelastic political functions (22) and (23) with $C_1 = C_2 = 0$ entailing $\gamma_1(P) = \gamma_2(P) = 0$ for any P (we neutralize the spontaneous opinion change).

(3) Cobb-Douglas production function with $\sigma(k^*) = 1$ and $\alpha(k^*) = \alpha$.

(4) Isoelastic utility function (14).

[5] The reader is referred to Propositions 11 and 12 in Bosi and Desmarchelier (2018).

[6] See Bosi and Desmarchelier (2018) among others.

(5) Linear tax rate $\tau(s) = 1 - s$ corresponding to (25) with $\bar{\tau} = 1$.

This simple model will allow us to perform numerical simulations and check the stability properties of cycles.

3.4.1 Long run

We compute the steady state of the simple model.

Proposition 15 *In the case of constant elasticities with $\gamma_1 = \gamma_2 = 0$ and $\sigma = 1$, and linear taxation $\tau(s) = 1 - s$, the steady state s^* is the unique solution to*

$$\left(\frac{B_2}{B_1}\right)^{\frac{1}{\varepsilon_1-\varepsilon_2}} = (b - d + ds)\frac{A}{a}\left(\frac{s\alpha A}{\delta+\theta}\right)^{\frac{\alpha}{1-\alpha}} \tag{51}$$

while the corresponding stocks of pollution and capital are given by

$$P^* = \left(\frac{B_2}{B_1}\right)^{\frac{1}{\varepsilon_1-\varepsilon_2}} \text{ and } k^* = \left(\frac{s^*\alpha A}{\delta+\theta}\right)^{\frac{1}{1-\alpha}} \tag{52}$$

Unsurprisingly, in this simplified framework, we find a unique, non-trivial equilibrium state, as indicated by Proposition 5.

3.4.2 Opinion cycles

As in the general Proposition 10, we choose η as bifurcation value. Let

$$A_1 = \frac{\delta+\theta}{\alpha} > 0 \tag{53}$$

$$A_2 = -1 < 0 \tag{54}$$

$$A_3 = \frac{b-d+ds^*}{s^*} > 0 \tag{55}$$

$$A_4 = \frac{1}{\varepsilon}\frac{\theta+\delta(1-\alpha)}{\alpha} > 0 \tag{56}$$

$$A_5 = a\eta\frac{1-\varepsilon}{\varepsilon}\frac{\theta+\delta(1-\alpha)}{\delta+\theta}\frac{s^*}{b-d+ds^*} > 0 \Leftrightarrow \varepsilon < 1 \tag{57}$$

$$A_6 = \beta(\varepsilon_2-\varepsilon_1) < 0 \tag{58}$$

$$A_7 = 0 \tag{59}$$

and

$$\Sigma_2 = a\beta(\varepsilon_1-\varepsilon_2)(1-s^*)\frac{ds^*}{b-d+ds^*} - a\theta - \frac{1-\alpha}{\alpha}\frac{\delta+\theta}{\varepsilon}[\theta+\delta(1-\alpha)] \tag{60}$$

$$\Sigma_3 = a\frac{1-\alpha}{\alpha}\frac{\delta+\theta}{\varepsilon}[\theta+\delta(1-\alpha)] - a\beta(\varepsilon_1-\varepsilon_2)(1-s^*)\left(\delta+\theta\frac{b-d}{b-d+ds^*}\right) \tag{61}$$

with

$$\begin{aligned} \beta &\equiv B_2^{\frac{\varepsilon_1}{\varepsilon_1-\varepsilon_2}} B_1^{1-\frac{\varepsilon_1}{\varepsilon_1-\varepsilon_2}} \\ B &= \frac{(\varepsilon_1-\varepsilon_2)\,\beta s^*}{b-d+ds^*} > 0 \end{aligned} \tag{62}$$

As seen in the general case, η is a suitable parameter to study the occurrence of cycles because it captures the degree of pollution externality and it amplifies the compensation or the distaste effect according to equation $\varepsilon_{cP} = \eta\,(\varepsilon - 1)$.

Proposition 16 *The Hopf bifurcation value η_H is given by*

$$\begin{aligned} \eta_H \equiv\; & \frac{\varepsilon}{1-\varepsilon}\frac{\delta+\theta}{\theta+\delta\,(1-\alpha)}\frac{b-d+ds^*}{s^*}\frac{1}{a\alpha A_1 A_3} \\ & * \left(\Sigma_2 - \frac{\Sigma_3}{\theta-a} + (\theta-a)\,\frac{a\alpha B A_1 A_4\,(1-s^*)\,[\alpha A_3 + (1-\alpha)\,d]}{\Sigma_3}\right) \end{aligned} \tag{63}$$

provided that:
(1) if $a < \theta$ (low natural pollution absorption), $\Sigma_3 > 0$,
(2) if $a > \theta$ (high natural pollution absorption), $\Sigma_3 < 0$, that is

$$\varepsilon > \frac{1-\alpha}{\alpha}\frac{1}{1-s^*}\frac{1}{\beta\,(\varepsilon_1-\varepsilon_2)}\frac{\theta+\delta\,(1-\alpha)}{1-\frac{\theta}{\delta+\theta}\frac{ds^*}{b-d+ds^*}} \equiv \varepsilon^* \tag{64}$$

As was the case in the general model, we can compute explicitly the bifurcation value η_H. However, as above, we can't check analytically the positivity of η_H. We can no longer verify whether the restriction 89 holds under a compensation or a distaste effect.

To address these issues, we need to simulate the model under opportune parameter calibrations.

4 Simulation

In the previous sections, we have provided the necessary and sufficient generic conditions for the occurrence of limit cycles through a Hopf bifurcation around the non trivial-steady state both in the general case and in a simpler isoelastic model. However, our analysis has not permitted to know whether the occurrence of cycles requires a distaste ($\varepsilon < 1$) or a compensation effect ($\varepsilon > 1$). In the current section, we show numerically that both these effects can lead to a Hopf bifurcation.

This is an important and new result because, it is known, there is no room for Hopf bifurcation under a distaste effect with no opinion dynamics. Simulations give also us the opportunity to study the stability of the limit cycle arising through the Hopf bifurcation, that is its supercriticity. Importantly, using the Matcont package for Matlab, we are able to simulate directly the original non-linear system (26)-(29) instead of a linear approximation. These simulations are fully consistent with our analytical results, obtained through the (Jacobian) linearization.

Focus on the explicit functional forms (14), (20), (21), (22) and (23). To simplify the simulation, we normalize the political and fiscal parameters:

Parameter	ε_1	ε_2	η_1	η_2	π	$\bar{\tau}$	B_1	B_2	C_1	C_2
Value	1	-1	1	-1	1	1	1	1	1	1

(65)

Under calibration (65), we get $\beta_1(P) = \gamma_1(P) = P$, $\beta_2(P) = \gamma_2(P) = 1/P$ and $\tau(s) = 1 - s$.

The non-trivial steady state becomes:

$$\begin{aligned} k^* &= \left(\frac{\alpha A s^*}{\delta + \theta}\right)^{\frac{1}{1-\alpha}} \\ P^* &= P(s^*) = A\frac{b - d(1-s^*)}{a}\left(\frac{\alpha A s^*}{\delta+\theta}\right)^{\frac{\alpha}{1-\alpha}} \\ \mu^* &= \left(\frac{P^{*\eta\frac{\varepsilon-1}{\varepsilon}}}{s^* A k^{*\alpha} - \delta k^*}\right)^{\varepsilon} \end{aligned}$$

where $s^* \in (0,1)$ is solution to (51), that is to

$$P(s)^2 = \frac{1-s^2}{s(2-s)} \tag{66}$$

4.1 Case $a < \theta$ and $\varepsilon > 1$

Consider a Ramsey model with pollution and without populism. The traditional case of a strong pollution inertia ($a < \theta$) under a compensation effect ($\varepsilon > 1$) is known to generate a Hopf bifurcation.[7]

We complete the calibration (65) as follows:

Parameter	θ	a	A	δ	α	ε	b	d
Value	0.01	0.005	1	0.025	1/2	4	0.001	0.0005

(67)

$\alpha = 1/2$ simplifies the computation of the roots of equation (66). We observe also that $\varepsilon = 4$ implies $\varepsilon_{cP} = \eta(\varepsilon - 1) > 0$ (compensation effect).

Calibrations (65) and (67) lead to the following steady state value:

$$(k^*, P^*, s^*) \approx (47.594, 1.023, 0.482\,92)$$

Using (45), we compute the critical value:

$$\eta_H = \frac{A_5^H}{a}\frac{\varepsilon}{1-\varepsilon}\frac{b - d(1-s^*)}{s^*}\frac{\delta+\theta}{\theta+\delta(1-\alpha)} \approx 13.693$$

[7] See Bosi and Desmarchelier (2018) among others.

where $A_5^H \approx -21.5$ is given by (43). Finally, using $\eta = \eta_H \approx 13.693$, we can also compute the stationary multiplier:

$$\mu^* = \left(\frac{P^* \eta^{\frac{\varepsilon - 1}{\varepsilon}}}{s^* A k^{*\alpha} - \delta k^*} \right)^{\varepsilon} \approx 0.120\,95$$

We implement the dynamic system (26)-(29) in Matcont. The software finds independently a Hopf bifurcation at $\eta_H \approx 13.693091$. When $\eta = \eta^H$ the real eigenvalues are given by:

$$(\lambda_1, \lambda_2) \approx (-1.99968, 0.00260269)$$

while the nonreal (purely imaginary) eigenvalues by

$$(\lambda_3, \lambda_4) \approx (-0.0340141, 0.0340141)\, i$$

The corresponding first Lyapunov coefficient evaluated with Matcont is negative: $l_1 \approx -3.406572 * 10^{-6}$, meaning that the Hopf bifurcation is supercritical.

The stable limit cycle arising around the non-trivial steady state is represented in Figure 1.

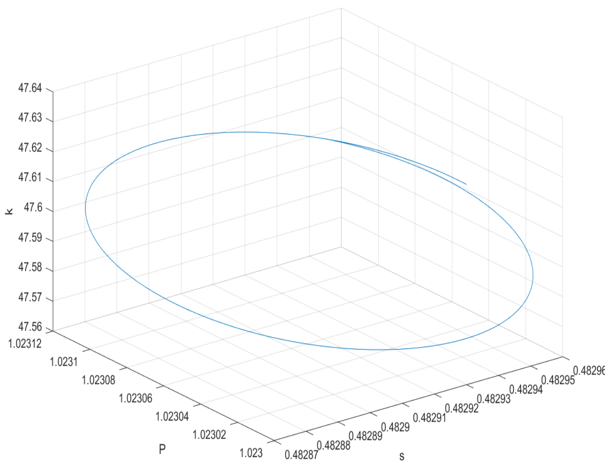

Fig.1 Stable limit cycle.

4.2 Case $a > \theta$ and $\varepsilon < 1$

We know that limit cycles arise through a Hopf bifurcation in the basic model without opinion cycles if $\varepsilon > 1$ (compensation effect) *and* $a < \theta$ (pollution inertia).[8] Conversely, if $\varepsilon < 1$ (distaste effect) *or* $a > \theta$ (fast pollution absorption), limit cycles are impossible.

[8] See Propositions 11 and 12 in Bosi and Desmarchelier (2018).

The introduction of political dynamics promotes the emergence of limit cycles even when they fail to exist in the basic model without political contagion. To illustrate how powerful opinion waves are, we show the possibility of cycles in the less favorable case: $\varepsilon < 1$ *and* $a > \theta$. A numerical exercise in the simple model is enough to highlight this possibility.

Proposition 17 *Let $a > \theta$. Under calibration (65) with $\alpha = 1/2$ and $b = d$, the steady state becomes*

$$s^* = \frac{1}{A}\sqrt{\frac{2a(\delta+\theta)}{d}} \tag{68}$$

$$k^* = \left[\frac{s^* A}{2(\delta+\theta)}\right]^2 \tag{69}$$

$$P^* = 1 \tag{70}$$

$$\mu^* = \left(s^* A k^{*1/2} - \delta k^*\right)^{-\varepsilon} \tag{71}$$

The Hopf bifurcation occurs at

$$\eta_H = \frac{1}{a}\frac{1-\alpha}{\alpha}\frac{\delta+\theta}{1-\varepsilon}\left(aZ + \frac{Z}{1-\alpha}\frac{\theta-a}{1-\delta Z} - 1 - a\frac{1-\delta Z}{\theta-a}\right) - \frac{\varepsilon}{1-\varepsilon}\frac{\theta}{\theta+\delta(1-\alpha)} \tag{72}$$

with

$$Z \equiv \frac{\alpha}{1-\alpha}\frac{\beta\varepsilon(\varepsilon_1-\varepsilon_2)}{(\delta+\theta)[\theta+\delta(1-\alpha)]}\left(1-\frac{1}{A}\sqrt{\frac{2a(\delta+\theta)}{d}}\right) \tag{73}$$

provided that

$$Z > \frac{1}{\delta} \tag{74}$$

Consider now calibration (65), that is $\beta = \varepsilon_1 = 1$ and $\varepsilon_2 = -1$, with

Parameter	θ	a	A	δ	α	ε	b	d
Value	0.005	0.01	1	0.025	1/2	6/100	0.001	0.001

(75)

According to (69)-(71) and to (68), the steady state becomes

$$(\mu^*, k^*, P^*, s^*) \approx (0.89959, 166.67, 1, 0.77460)$$

Using (72), we obtain the Hopf critical value $\eta_H = 2.3051$. We observe also that (74) is satisfied: $Z - 1/\delta = 11.521 > 0$, as well as condition (64): $\varepsilon = 0.06 > \varepsilon^* = 0.046584$. The real eigenvalues corresponding to this Hopf bifurcation value are given by

$$(\lambda_1, \lambda_2) \approx (-0.127\,63, 0.122\,63)$$

while the nonreal (purely imaginary) eigenvalues are given by

$$(\lambda_3, \lambda_4) \approx (-0.070995, 0.070995)\, i$$

The corresponding first Lyapunov coefficient evaluated with Matcont is negative: $l_1 \approx -1.668469 * 10^{-6} < 0$, meaning that the Hopf bifurcation is supercritical.

The stable limit cycle arising around the non-trivial steady state is represented in Figure 2.

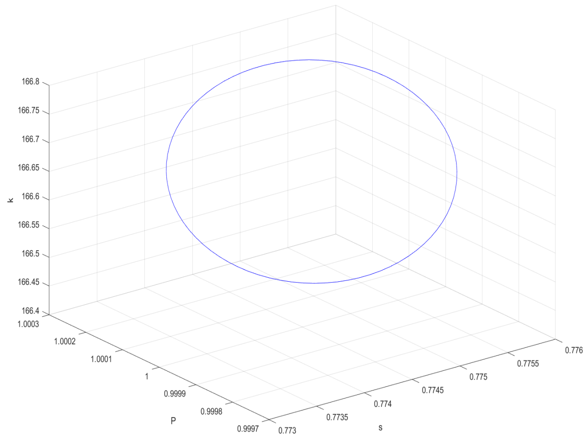

Fig. 2 Stable limit cycle (toy model).

5 Conclusion

This article is a first attempt to consider populism and pollution together in a dynamic general equilibrium model. We adapt a SIS model to represent the spread of climate skepticism and populism in society, dividing the population into two mutually influencing groups: climate skeptics and environmentalists.

Since right-wing populism is known for prioritizing economic over environmental issues (Lockwood, 2018), we equate populists with skeptics. This political bloc is part of a Ramsey model in which a stock of pollution is assumed to come from production. Environmental policy consists of a green tax levied at the production level, which finances depollution according to a balanced-budget rule. To take into account the political pressure of populists, we assume that the ecotax rate decreases in the share of skeptics in the population.

Our analysis, at the crossroads of economic, political and environmental sciences and epidemiology, reveals that populism promotes the emergence of stable limit cycles around the steady state through a Hopf bifurcation, regardless of the effects of pollution on consumption demand.

Interestingly, in the absence of populism, a Hopf bifurcation only appears in a Ramsey model when pollution increases the marginal utility of consumption (compensation effect). In other words, populism exacerbates pollution-induced volatility. In this regard, even if right-wing parties place a high priority on short-term economic performance, to manage macroeconomic volatility, they would be better off considering environmental policies rather than rejecting them *a priori*.

6 Appendix

Proof of Lemma 2

Maximizing the Hamiltonian $H \equiv e^{-\theta t}u(c,P)+\lambda\left[(r-\delta)h+w-c\right]$, we find the first-order conditions:

$$\frac{\partial H}{\partial c}=e^{-\theta t}u_c(c,P)-\lambda=0,\ \frac{\partial H}{\partial \lambda}=\dot{h}\text{ and }\frac{\partial H}{\partial h}=-\dot{\lambda}$$

These conditions result in the static relation (10) between the new multiplier $\mu \equiv \lambda e^{\theta t}$ and the consumption demand and, the dynamic Euler equation (8) and the budget constraint (9), now binding. The transversality condition is given by $\lim_{t\to\infty}\lambda h=\lim_{t\to\infty}e^{-\theta t}\mu h=0$. ■

Proof of Proposition 3

The opinion dynamics are given by:

$$\dot{E} = \beta_1 E\frac{S}{N}-\beta_2 S\frac{E}{N}+\gamma_1 S-\gamma_2 E \tag{76}$$

$$\dot{S} = \beta_2 S\frac{E}{N}-\beta_1 E\frac{S}{N}+\gamma_2 E-\gamma_1 S \tag{77}$$

Since $\dot{E}+\dot{S}=\dot{N}=0$, (76) and (77) are equivalent:

$$\dot{E}+\dot{S}=\left(\beta_1 E\frac{S}{N}-\beta_2 S\frac{E}{N}+\gamma_1 S-\gamma_2 E\right)+\left(\beta_2 S\frac{E}{N}-\beta_1 E\frac{S}{N}+\gamma_2 E-\gamma_1 S\right)=0$$

According to (77), $\dot{s}/s=\dot{S}/S-\dot{N}/N=\dot{S}/S$ implies

$$\frac{\dot{s}}{s}=(\beta_2-\beta_1)\frac{E}{N}+\gamma_2\frac{E}{S}-\gamma_1=(\beta_2-\beta_1)(1-s)+\gamma_2\frac{1-s}{s}-\gamma_1$$

and, finally, (19). ■

Proof of Proposition 4

Equilibrium in the labor market means: $L=Nl=N$ since $l=1$; while, in the capital market: $K=Nh=Lh$, that is $k=h$. In the good market, the aggregate demand is also equal to the aggregate supply: $C+\left(\dot{K}+\delta K\right)+G=Y$, that is

$$c+\left(\frac{\dot{H}}{N}+\delta h\right)+\tau y=c+\left(\dot{h}+nh+\delta h\right)+\tau f(k)=y=f(k)$$

Since $n=0$, we get $c+\left(\dot{h}+\delta h\right)=(1-\tau)f(k)$ or, equivalently,

$$\dot{k}=(1-\tau)f(k)-\delta k-c \tag{78}$$

Putting together (1), (2), (8), (11), (17), (18), (78), under Assumptions 3 and 4, we obtain system (26)-(29). Notice that, in equilibrium, the household's budget constraint (9) corresponds to the goods market clearing:

$$\dot{h} = (r - \delta) h + w - c = [(1 - \tau) f'(k) - \delta] k + (1 - \tau) [f(k) - kf'(k)] - c$$

that is to (78). ■

Proof of Proposition 5

At the steady state, according to equation (26), we have

$$\rho(k^*) = (\delta + \theta) / [1 - \tau(s^*)]$$

that is (30).

Under Assumption 1 and 4:

$$\lim_{s\to 0} \frac{\delta + \theta}{1 - \tau(s)} = \infty = \lim_{k\to 0} \rho(k) \text{ and } \lim_{s\to 1} \frac{\delta + \theta}{1 - \tau(s)} = \delta + \theta$$

We obtain $\lim_{s\to 0} k(s) = 0$ and $\lim_{s\to 1} k(s) = k_R$, where $k_R \equiv \rho^{-1}(\delta + \theta)$ is the Modified Golden Rule of the basic Ramsey model.

Focus now on equation (28). At the steady state, we get (31). We observe that $P'(s) > 0$. Moreover, $\lim_{s\to 0} P(s) = 0$. We find also

$$\lim_{s\to 1} P(s) = \frac{b}{a} f(k_R) = \frac{b}{a} f\left(\rho^{-1}(\delta + \theta)\right) \equiv \bar{P} > 0$$

Consider equation (29). At the steady state,

$$\varphi(s) \equiv (1 - s)(s[\beta_2(P(s)) - \beta_1(P(s))] + \gamma_2(P(s))) - s\gamma_1(P(s)) = 0 \quad (79)$$

Under Assumption 3,

$$\begin{aligned} \lim_{s\to 0} \varphi(s) &= \lim_{s\to 0} (s[\beta_2(P(s)) - \beta_1(P(s))] + \gamma_2(P(s))) = +\infty && (80) \\ \lim_{s\to 1} \varphi(s) &= -s\gamma_1\left(\bar{P}\right) < 0 && (81) \end{aligned}$$

Since $\varphi(s)$ is continuous, there exists at least one $s \in (0, 1)$ such that $\varphi(s) = 0$.

Let s^* be a steady state. According to (30) and (31), we obtain the corresponding values for capital intensity and pollution level: $k^* \equiv k(s^*)$ and $P^* \equiv P(s^*)$. Moreover, (27) entails (33) and, finally, (10) yields (34).

Focus now on uniqueness.

Equation (79) becomes (32). We know that $P'(s) > 0$. Then the LHS decreases, while the RHS increases. In addition, under Assumption 3,

$$\begin{aligned} \lim_{s\to 0} [\beta_2(P(s)) - \beta_1(P(s))] &= \infty \\ \lim_{s\to 1} [\beta_2(P(s)) - \beta_1(P(s))] &= \beta_2\left(\bar{P}\right) - \beta_1\left(\bar{P}\right) \\ \lim_{s\to 0} \left[\frac{\gamma_1(P(s))}{1 - s} - \frac{\gamma_2(P(s))}{s}\right] &= -\infty \\ \lim_{s\to 1} \left[\frac{\gamma_1(P(s))}{1 - s} - \frac{\gamma_2(P(s))}{s}\right] &= \infty \end{aligned}$$

with $\bar{P} \equiv f\left(\rho^{-1}(\delta+\theta)\right) b/a$. Therefore a unique non-trivial steady state exists. ■

Proof of Proposition 6

Consider $\tau(s) \equiv \bar{\tau}(1-s^{\pi})$. The steady state (s,k,P) is solution to system:

$$\begin{aligned}
\left[1-\bar{\tau}(1-s^{\pi})\right]\rho(k) &= \delta+\theta \\
\left[b-d\bar{\tau}(1-s^{\pi})\right] f(k) &= aP \\
\beta_2(P)-\beta_1(P) &= \frac{\gamma_1(P)}{1-s}-\frac{\gamma_2(P)}{s}
\end{aligned}$$

Totally differentiating with respect to $(s,k,P,\bar{\tau})$, we obtain

$$\begin{aligned}
\pi\frac{ds}{s}+S_1\frac{k\rho'(k)}{\rho(k)}\frac{dk}{k} &= S_0\frac{d\bar{\tau}}{\bar{\tau}} \\
\pi\frac{ds}{s}+S_2\frac{kf'(k)}{f(k)}\frac{dk}{k}-S_2\frac{dP}{P} &= S_0\frac{d\bar{\tau}}{\bar{\tau}} \\
Q_1\frac{ds}{s}+Q_2\frac{dP}{P} &= 0
\end{aligned}$$

and, replacing (4) and (5),

$$\begin{bmatrix} \pi & -\frac{1-\alpha(k)}{\sigma(k)}S_1 & 0 \\ \pi & \alpha(k)S_2 & -S_2 \\ Q_1 & 0 & Q_2 \end{bmatrix}\begin{bmatrix} \frac{\bar{\tau}}{s}\frac{ds}{d\bar{\tau}} \\ \frac{\bar{\tau}}{k}\frac{dk}{d\bar{\tau}} \\ \frac{\bar{\tau}}{P}\frac{dP}{d\bar{\tau}} \end{bmatrix}=\begin{bmatrix} S_0 \\ S_0 \\ 0 \end{bmatrix}$$

Solving the system, we find the elasticities (35) to (37). ■

Proof of Lemma 7

The dynamic system (26)-(29) writes $\left(\dot{\mu},\dot{k},\dot{P},\dot{s}\right)^T = f(\mu,k,P,s)$, where $f \equiv (f_1,f_2,f_3,f_4)^T$. The Jacobian matrix is given by

$$\begin{aligned}
J &\equiv \begin{bmatrix} \frac{\partial f_1}{\partial\mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_2}{\partial\mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_3}{\partial\mu} & \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial\mu} & \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \\
&= \begin{bmatrix} 0 & \alpha(k^*)A_1\frac{1-\alpha(k^*)}{\sigma(k^*)}\frac{\mu^*}{k^*} & 0 & \alpha(k^*)A_1A_2\frac{\mu^*}{s^*} \\ A_4\frac{k^*}{\mu^*} & \theta & A_5 & -A_1A_2\frac{k^*}{s^*} \\ 0 & \alpha(k^*)A_1A_3 & -a & -dA_1A_2\frac{k^*}{s^*} \\ 0 & 0 & a(1-s^*)\frac{A_6}{A_1A_3}\frac{s^*}{k^*} & A_7 \end{bmatrix}
\end{aligned}$$

where the partial derivatives are computed at the steady state. Computing

$S_1 = T$,

$$
\begin{aligned}
S_2 &= \left| \begin{bmatrix} \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} \end{bmatrix} \right| \\
&\quad + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial P} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} \end{bmatrix} \right| \\
S_3 &= \left| \begin{bmatrix} \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial P} & \frac{\partial f_3}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial P} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| \\
&\quad + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial s} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial s} \\ \frac{\partial f_4}{\partial \mu} & \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial s} \end{bmatrix} \right| + \left| \begin{bmatrix} \frac{\partial f_1}{\partial \mu} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial \mu} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial \mu} & \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} \end{bmatrix} \right|
\end{aligned}
$$

and $S_4 = D$, we obtain the sums of principal minors (39) to (42). ■

Proof of Proposition 8

The variables k, P and s are predetermined, while the multiplier μ isn't. We observe that $D = \lambda_1\lambda_2\lambda_3\lambda_4 < 0$. Let us show that at least one eigenvalue is real and positive.

There are three cases: (1) 4 real eigenvalues; (2) 2 real eigenvalues, say λ_1 and λ_2, and a pair of nonreal and conjugate eigenvalues; (3) two pairs of nonreal and conjugate eigenvalues, say (λ_1, λ_2) and (λ_3, λ_4).

(1) If $D = \lambda_1\lambda_2\lambda_3\lambda_4 < 0$, at least one eigenvalue is positive, otherwise $D \geq 0$.

(2) $\lambda_3\lambda_4 > 0$ and, therefore, $\lambda_1\lambda_2 < 0$, that is one real eigenvalue is positive.

(3) $\lambda_1\lambda_2 > 0$ and $\lambda_3\lambda_4 > 0$, that is $D > 0$, a contradiction.

The only possible cases are (1) and (2): at least one eigenvalue is real and positive, that is unstable.

Local indeterminacy of a four-dimensional system with three predetermined variables requires four stable eigenvalues. That is not the case in our model. Then, the equilibrium is locally determinate. ■

Proof of Proposition 9

We observe that, in our model, $D < 0$.

According to Propositions 15, 17 and 18 in Bosi and Desmarchelier (2019), zero, Bogdanov-Takens and Gavrilov-Guckenheimer bifurcations require $D = 0$.

According to Proposition 20 in Bosi and Desmarchelier (2019) a double-Hopf bifurcation requires $D > 0$. ■

Proof of Proposition 10

Notice that, at the steady state, (k^*, P^*, s^*) does not depend on the shape of the utility function u, while u and, therefore, μ depend on (c, P), that is on (k^*, P^*, s^*).

In the case of the isoelastic utility function (14), we have

$$\varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c} = -\varepsilon < 0$$
$$\varepsilon_{cP} \equiv \frac{Pu_{cP}}{u_c} = \eta(\varepsilon - 1) < 0 \Leftrightarrow \varepsilon < 1$$

In this case, ε_{cP} is a constant, independent on (k, P, s), and (k^*, P^*, s^*) is independent on ε_{cP}.

Notice that ε_{cP}, that is η, only appears in the block A_5.

According to Proposition 16 in Bosi and Desmarchelier (2019), generically, a Hopf bifurcation arises if and only if

$$x \equiv \frac{S_3}{T} > 0 \tag{82}$$

and

$$S_2 = x + \frac{D}{x} \tag{83}$$

Replacing (41) in (82), we get

$$A_5 = \frac{\Sigma_3 - Tx}{\alpha(k^*) A_1 A_3 A_7} \tag{84}$$

Replacing it in (40) and S_2 in (83), we find

$$(T - A_7) x^2 + (A_7 \Sigma_2 - \Sigma_3) x - DA_7 = 0 \tag{85}$$

Solving (85) for x and noticing that $D < 0$, $A_7 \leq 0$ and $T - A_7 = \theta - a > 0$, we obtain

$$x_- = \frac{-(A_7\Sigma_2 - \Sigma_3) - \sqrt{(A_7\Sigma_2 - \Sigma_3)^2 + 4DA_7(T - A_7)}}{2(T - A_7)} \leq 0$$
$$x_+ = \frac{-(A_7\Sigma_2 - \Sigma_3) + \sqrt{(A_7\Sigma_2 - \Sigma_3)^2 + 4DA_7(T - A_7)}}{2(T - A_7)} \geq 0$$

Thus, x_- does not satisfy inequality (82). Replacing x_+ in (84) and, finally, $T - A_7 = \theta - a$, we obtain (43).

According to expression (38), $A_5 = A_5^H$ is equivalent to (44) and, in the isoelastic case (14), according to (15) and (16), to (45). ■

Proof of Lemma 12

Solution to equation (83) are given by

$$x_- = \frac{A_7\Sigma_2 - \Sigma_3}{2(A_7 - T)} - \sqrt{\left[\frac{A_7\Sigma_2 - \Sigma_3}{2(A_7 - T)}\right]^2 - \frac{DA_7}{A_7 - T}} \tag{86}$$
$$x_+ = \frac{A_7\Sigma_2 - \Sigma_3}{2(A_7 - T)} + \sqrt{\left[\frac{A_7\Sigma_2 - \Sigma_3}{2(A_7 - T)}\right]^2 - \frac{DA_7}{A_7 - T}} \tag{87}$$

We observe that $A_7 - T = a - \theta > 0$ and, therefore, $DA_7/(A_7 - T) \geq 0$. Then,

$$x_-, x_+ \in \mathbb{R} \Leftrightarrow \frac{DA_7}{A_7 - T} \leq \left[\frac{A_7\Sigma_2 - \Sigma_3}{2(A_7 - T)}\right]^2$$

and, in this case, $A_7\Sigma_2 - \Sigma_3 > 0 \Rightarrow 0 < x_- < x_+$ and $A_7\Sigma_2 - \Sigma_3 < 0 \Rightarrow x_- < x_+ < 0$.

Since, according to (82), we require $x > 0$, a cycle through a Hopf bifurcation generically arises if and only if inequalities (46) hold.

Consider now the particular case $\gamma_1(P) = \gamma_2(P) = 0$ for any P. In this case, $A_7 = 0$. Therefore, the first inequality in (46) is always satisfied and the second one reduces to $\Sigma_3 < 0$. ■

Proof of Proposition 13

$A_7 = 0$ implies

$$\begin{aligned}\Sigma_3 &\equiv a\left((1 - s^*)[d\theta - \alpha(k^*)A_1A_3]B + \alpha(k^*)A_1A_4\frac{1 - \alpha(k^*)}{\sigma(k^*)}\right) \\ &= a\left(A_4(\delta + \theta)\frac{1 - \alpha(k^*)}{\sigma(k^*)} - A_6(1 - s^*)\frac{\tau(s^*)\varepsilon_\tau(s^*)}{1 - \tau(s^*)}\left[\delta + \theta\frac{b - d}{b - d\tau(s^*)}\right]\right)\end{aligned}$$

with $A_6 = \varepsilon_2(P^*)\beta_2(P^*) - \varepsilon_1(P^*)\beta_1(P^*) < 0$. Therefore, condition for cycles $\Sigma_3 < 0$ is equivalent to (49).

Moreover, according to (86) and (87), noticing that $A_7 = 0$, $\Sigma_3 < 0$ and $A_7 - T = a - \theta > 0$, we have $x_- = 0 < x_+ = \Sigma_3/(\theta - a)$. Restriction (82) rules out the zero root x_-.

Replacing $S_2 = \Sigma_2 - \alpha(k^*)A_1A_3A_5$ and $x = x_+ > 0$ in equation (83), and solving for A_5, we obtain (47).

According to definition (38), we find the critical cross effect (48).

In the isoelastic case, $\varepsilon_{cc} = -\varepsilon$ and $\varepsilon_{cP} = \eta(\varepsilon - 1)$. Replacing ε_{cP} in (48) and solving for η, we obtain the explicit critical value (50).

To have a Hopf bifurcation, we need a positive critical value: $\tilde{\eta}_H > 0$, that is $\tilde{A}_5^H(1 - \varepsilon) > 0$. In other terms, there is room for limit cycles when $\tilde{A}_5^H < 0$ if $\varepsilon > 1$ (compensation effect) and, when $\tilde{A}_5^H > 0$ if $\varepsilon < 1$ (distaste effect). ■

Proof of Proposition 15

Focus on the steady state, that is equation (79). The non-trivial steady state is the unique solution to $\beta_1(P(s)) = \beta_2(P(s))$, that is P^* solution to $\beta_1(P) = \beta_2(P)$. In the case of the isoelastic political functions (20) and (21), we get P^* in (52) and, from $\rho(k) \equiv f'(k) = \alpha A k^{\alpha-1} = (\delta + \theta)/s$, k^* in (52). The non-trivial steady state s^* is the unique solution solution to $P(s) = P^*$ with

$$P(s) = (b - d + ds)\frac{A}{a}\left(\frac{s\alpha A}{\delta + \theta}\right)^{\frac{\alpha}{1-\alpha}}$$

that is to (51). ■

Proof of Proposition 16

The expressions A_i are now given by (53) to (59) with $\beta_1(P^*) = \beta_2(P^*) = \beta$ (notice that, in particular, when $B_1 = B_2$, then $P^* = 1$ and $\beta = B_1 = B_2$, that is β no longer depends on ε_1 and ε_2).

The moments of the original Jacobian matrix simplify:

$$\begin{aligned}
T &= \theta - a \\
S_2 &= \Sigma_2 - \alpha A_1 A_3 A_5 = \Sigma_2 - a\eta \left[\theta + \delta \left(1-\alpha\right)\right] \frac{1-\varepsilon}{\varepsilon} \\
S_3 &= \Sigma_3 \\
D &= -a\alpha B A_1 A_4 \left(1 - s^*\right) \left[\alpha A_3 + \left(1-\alpha\right) d\right] \\
&= -a\beta \left(\varepsilon_1 - \varepsilon_2\right) \left(1 - s^*\right) \left(1 + \frac{1-\alpha}{\alpha} \frac{ds^*}{b - d + ds^*}\right) \frac{\delta + \theta}{\varepsilon} \left[\theta + \delta \left(1-\alpha\right)\right]
\end{aligned}$$

with

$$\begin{aligned}
\Sigma_2 &= adB \left(1 - s^*\right) - a\theta - \alpha \left(1-\alpha\right) A_1 A_4 \\
\Sigma_3 &= a \left[B \left(1 - s^*\right) \left(d\theta - \alpha A_1 A_3\right) + \alpha \left(1-\alpha\right) A_1 A_4\right]
\end{aligned}$$

that is (60) and (**??**).

According to Proposition 16 in Bosi and Desmarchelier (2019), a limit cycle generically arises through a Hopf bifurcation if and only if

$$S_2 = \frac{S_3}{T} + \frac{TD}{S_3} \tag{88}$$

holds with

$$x \equiv \frac{S_3}{T} = \frac{\Sigma_3}{\theta - a} > 0 \tag{89}$$

(88) is equivalent to

$$\Sigma_2 - \alpha A_1 A_3 A_5 = \frac{\Sigma_3}{\theta - a} - \left(\theta - a\right) \frac{a\alpha B A_1 A_4 \left(1 - s^*\right) \left[\alpha A_3 + \left(1-\alpha\right) d\right]}{\Sigma_3} \tag{90}$$

Solving (90) for A_5, we find

$$A_5^H \equiv \frac{1}{\alpha A_1 A_3} \left(\Sigma_2 - \frac{\Sigma_3}{\theta - a} + \left(\theta - a\right) \frac{a\alpha B A_1 A_4 \left(1 - s^*\right) \left[\alpha A_3 + \left(1-\alpha\right) d\right]}{\Sigma_3}\right) \tag{91}$$

Interestingly, the RHS of (91) does not depend on η. That is, we can compute η_H by replacing (57) in (91), to obtain (63).

Thus, a Hopf bifurcation generically occurs around the non-trivial steady state if and only if $\eta = \eta_H$, provided that (89) holds.

We have to distinguish two cases in terms of natural pollution absorption a.

(1) In the case of low natural pollution absorption ($a < \theta$), restriction (89) becomes $\Sigma_3 > 0$.

(2) In the case of high natural pollution absorption ($a > \theta$), restriction (89) becomes $\Sigma_3 < 0$, that is (64). ■

Proof of Proposition 17

Reconsider this simplified model under calibration (65) with $\alpha = 1/2$. In this case, we obtain equations (69), (70) and (71). Calibration (65) implies $P^* = 1$. Thus,

$$P^* = \frac{b - d\left(1 - s^*\right)}{a} \frac{s^* A^2}{2\left(\delta + \theta\right)} = 1$$

implies that s^* is the non-negative solution to the quadratic equation

$$s^2 + \frac{b-d}{d} s - 2\frac{a}{d}\frac{\delta+\theta}{A^2} = 0$$

with roots:

$$\begin{aligned} s_1 &= -\frac{b-d}{2d} - \sqrt{\left(\frac{b-d}{2d}\right)^2 + 2\frac{a}{d}\frac{\delta+\theta}{A^2}} < 0 \\ s_2 &= -\frac{b-d}{2d} + \sqrt{\left(\frac{b-d}{2d}\right)^2 + 2\frac{a}{d}\frac{\delta+\theta}{A^2}} > 0 \end{aligned}$$

that is $s^* = s_2$. In the particular case $b = d$, we obtain (68).

A Hopf bifurcation occurs when $\eta = \eta^H$. In the case $b = d$, we have $A_3 = d$,

$$A_1 = \frac{\delta+\theta}{\alpha} > 0,\ A_4 = \frac{1}{\varepsilon}\frac{\theta+\delta(1-\alpha)}{\alpha} > 0 \text{ and } B = (\varepsilon_1 - \varepsilon_2)\frac{\beta}{d} > 0$$

and the critical value (63) becomes (72) with

$$Z = \frac{\alpha}{1-\alpha}\frac{\beta\varepsilon(\varepsilon_1-\varepsilon_2)}{(\delta+\theta)[\theta+\delta(1-\alpha)]}(1-s^*)$$

that is (73). We require also

$$\Sigma_3 = a(1-\delta Z)\frac{1-\alpha}{\alpha}\frac{(\delta+\theta)[\theta+\delta(1-\alpha)]}{\varepsilon} < 0$$

that is (74). ■

References

[1] Bosi S. and Desmarchelier D. (2018). Limit cycles under a negative effect of pollution on consumption demand: the role of an Environmental Kuznets Curve. *Environmental & Resource Economics* 69, 343-363.

[2] Bosi S. and Desmarchelier D. (2019). Local bifurcations of three and four-dimensional systems: A tractable characterization with economic applications. *Mathematical Social Sciences* 97, 38-50.

[3] Bourdin S. and Torre A. (2023). Geography of contestation: A study on the Yellow Vest movement and the rise of populism in France. *Journal of Regional Science* 63, 214-235.

[4] Buzogány A. and Mohamad-Klotzbach C. (2021). Populism and nature - the nature of populism: new perspectives on the relationship between populism, climate change, and nature protection. *Zeitschrift für Vergleichende Politikwissenschaft* 15, 155-164.

[5] Desmarchelier D. and Lanzi T. (2023). Opinion dynamics and political persuasion. *Revue d'Economie Politique* 133, 907-924.

[6] Guriev S. and Papaioannou E. (2022). The political economy of populism. *Journal of Economic Literature* 60, 753-832.

[7] Heal G. (1982). The use of common property resources. In Kerry Smith V. and Krutilla J. V., *Explorations in Natural Resource Economics*, The Johns Hopkins University Press for Resources for the Future, Baltimore.

[8] Hethcote H. W. (2009). The basic epidemiology models: models, expressions for R_0, parameter estimation, and applications. In *Mathematical Understanding of Infectious Disease Dynamics*, Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore 16, 1-61.

[9] Huber R. A. (2020). The role of populist attitudes in explaining climate change skepticism and support for environmental protection. *Environmental Politics* 29, 959-982.

[10] Lockwood M. (2018). Right-wing populism and the climate change agenda: exploring the linkages. *Environmental Politics* 27, 712-732.

[11] Michel P. and Rotillon G. (1995). Disutility of pollution and endogenous growth. *Environmental and Resource Economics* 6, 279-300.

[12] Pereira E. J. A. L., Ferreira P. J. S., Ribeiro L. C. S., Carvalho T. S. and Pereira H. B. B. (2019). Policy in Brazil (2016–2019) threaten conservation of the Amazon rainforest. *Environmental Science and Policy* 100, 8-12.