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SECTORAL FISCAL MULTIPLIERS AND TECHNOLOGY IN OPEN ECONOMY*

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Abstract

Our evidence reveals that the rise in real GDP is uniformly distributed across sectors following a government spending shock while labor growth is concentrated in non-traded industries. A rationale behind these two findings lies in technology which responds endogenously to the government spending shock. While technology improvements are concentrated in traded industries, technological change is biased toward labor (capital) in non-traded (traded) industries. To account for our evidence, we consider a semi-small open economy model with tradables and non-tradables where both capital and technology can be used more intensively. While financial openness amplifies the biasedness of the demand shock toward non-traded goods, labor mobility costs, imperfect substitutability between home- and foreign-produced traded goods and endogenous capital utilization are necessary conditions for giving rise to traded technology improvement. The model can reproduce the size of fiscal multipliers once we let technology adjustment costs together with factor-biased technological change vary across sectors.

Keywords: Sector-biased government spending shocks; Endogenous technological change; Factor-augmenting efficiency; Open economy; Labor reallocation; CES production function; Labor income share.
JEL Classification: E25; E62; F11; F41; O33

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1 Introduction

Until recently, fiscal policy was considered to be effective only through higher private consumption or greater labor supply or both. In their article, Delong and Summers [2012] set out the possibility of a persistent increase in productivity following a rise in government spending. The evidence recently documented by D'Alessandro et al. [2019] and Jørgensen and Ravn [2022] on quarterly U.S. data reveals that an exogenous and temporary shock to government consumption significantly increases aggregate total factor productivity (TFP), lending credence to Delong and Summers's hypothesis. If TFP increases, the aggregate fiscal multiplier is higher than initially thought. Because the ability of firms to increase efficiency in the use of capital and labor may vary across industries, we address the following questions: Does the positive effect of a rise in government consumption on technology vary across traded and non-traded sectors? How does the technology channel modify the distribution of government spending multipliers across sectors in open economy? We find that shocks to government consumption significantly increase traded TFP relative to non-traded TFP, thus pushing up the value added multiplier of tradables relative to non-tradables. While technology improvements are concentrated in the traded sector, non-traded industries bias technological change toward labor, which increases the labor multiplier of non-tradables relative to tradables.

As exemplified by D'Alessandro et al. [2019], Jørgensen and Ravn [2022], Antolin-Diaz and Surico [2022], Klein and Linnemann [2022], the literature highlighting the technology channel of fiscal policy remains distinct from a second strand emphasizing the role of the sector's intensity in the government spending shock and barriers to mobility in determining sectoral and aggregate fiscal multipliers, see e.g., Boehm [2020], Bouakez et al. [2022b], Cardi et al. [2020], Cox et al. [2020], Lambertini and Proebsting [2022], Proebsting [2022]. Our paper brings these two distinct threads in the existing literature together in an open economy setting. We show that the technology channel is a key driver of the distribution of government spending multipliers across sectors, and that both the multi-sector aspect and the (financial and trade) openness dimension play a critical role.

Investigating the link between technology and fiscal policy at a sectoral level is important as it has a major economy policy implication. Evidence documented by Hlatshwayo and Spence [2014], Mian and Sufi [2014], Aghion et al. [2021], Beraja and Wolf [2022] show that labor in non-exporting industries experiences the largest drop (relative to traded labor) during downturns while value added of sectors producing traded goods declines more than that of sectors producing non-traded goods.[1] The fact that sectors are not symmetrically

[1] In Online Appendix A, we document evidence for the eighteen OECD countries of our sample which shows that non-traded hours worked fall more than traded hours worked during downturns while the reverse is true for traded value added as its decline is more pronounced than non-traded value added. By keeping in mind that the variations in the value added of tradables are driven by Manufacturing which is made up of industries producing durable goods, our empirical facts square well with the evidence documented by

affected by recessions raises the question of the capacity of fiscal policy to mitigate such a differential response of non-tradable versus tradable industries. Our work suggests that the technology channel of fiscal policy can accomplish this task as it encourages traded firms to improve technology and has an expansionary effect on non-traded labor through factor-biased technological change (FBTC henceforth).

To guide our quantitative analysis, we estimate the sectoral value added and sectoral hours effects of a shock to government consumption for eighteen OECD countries over the period running from 1970 to 2015, using Jordà's [2005] local projection method. We find empirically that the real GDP multiplier averages 1.4 during the first six years after the shock and importantly, 39% of the increase in real GDP is driven by TFP gains. Our estimates reveal that the aggregate multiplier is uniformly distributed across sectors, i.e., in accordance with the sectoral value added share. The muted response of the value added share of non-tradables to the government spending shock at any horizon is puzzling since according to the data taken from the World Input-Output Database, government purchases are concentrated in non-traded industries. A rationale behind this finding lies in the technology channel of fiscal transmission. We find that technology improvements are concentrated in the traded sector which offset the impact of the biasedness of the government spending shock, thus leaving the value added share of non-tradables unchanged.

The allocation of labor across sectors is quite distinct from the sectoral distribution of value added. While the labor multiplier averages 1.15 during the first six years after the shock, 88% of the rise in total hours worked is concentrated in non-traded industries. Our evidence suggests that the disproportionate increase in non-traded hours is driven by the biasedness of technological change toward labor in non-traded industries which amplifies the impact on non-traded hours of the biasedness of the demand shock toward non-tradables.

To account for the role of technology in determining the magnitude of government spending multipliers, we put forward a two-sector open economy model with tradables and non-tradables which is similar to the model version by Chodorow-Reich et al. [2021] except that we abstract from nominal and financial frictions and instead consider the specific elements detailed below.[2] Building on Bianchi et al. [2019], we endogenize technological change at a sectoral level by allowing for endogenous utilization of technology. This modelling is based on our evidence which reveals that changes in capital-utilization-adjusted-TFP of tradables are not associated with an increase in the stock of R&D. By taking advantage of the panel data dimension of our sample, we conduct a split-sample analysis which reveals that effi-

Beraja and Wolf [2022] who show that recessions are deeper and the recovery greater in U.S. States where the share of sectors producing durable goods is larger.

[2]Like Kehoe and Ruhl [2009] and Bertinelli et al. [2022], we assume that the economy is small in world capital markets so that the world interest rate is given, but large enough in the world goods market to influence the relative price of its export good so that terms of trade are endogenous. Our model can also be viewed as a two-sector extension with endogenous technology and capital use of the open economy RBC model developed by Ferrara et al. [2021].

ciency gains we estimate following a government spending shock are based on the internal organization of firms which is a mediating factor through which demand conditions affect technology adjustment. Building on our model's predictions, we show that the variations in technology are influenced by three sets of elements which includes the biasedness of the demand shock, barriers to factors' mobility and technology factors.

First, because the decision to improve technology is pro-cyclical and sectoral value added is affected by the reallocation of productive resources, the biasedness of the government spending shock towards non-traded goods impacts negatively the incentive to increase efficiency in the traded sector. Financial openness amplifies the impact of the biasedness of the demand shock towards non-traded goods. Because traded goods can be imported while non-traded goods must be produced by domestic firms, the open economy finds it optimal to borrow from abroad which further shifts productive resources away from traded industries. The reallocation of capital and labor is hampered however by barriers to factors' mobility which include workers' switching costs and imperfect substitutability between home-and foreign-produced traded goods. As home- and foreign-produced tradables are more differentiated, households are more reluctant to substitute imported for domestic goods which dramatically reduces the current account deficit and thus the reallocation of resources toward non-traded industries.

We find quantitatively that frictions into factors' mobility are not sufficient on their own to generate an increase in traded production efficiency. To give rise to an increase in overall efficiency of tradables, we have to allow for either an endogenous capital utilization rate or FBTC. To account for the magnitude of technology improvement in tradables relative to non-tradables we estimate empirically, both elements are necessary. Intuitively, a higher capital utilization rate in both sectors reduces the need to shift capital toward the non-traded sector and increases capital services rented by traded firms which has a positive impact on traded value added. Capital shifts toward tradable industries only once we assume that sectoral goods are produced by means of CES production functions and we let the mix of labor- and capital-augmenting efficiency vary along the technology frontier, along the lines of Caselli [2016]. Intuitively, because technological change is biased toward labor in the non-traded sector and biased toward capital in the traded sector, tradable industries experience a large capital inflow giving rise to incentives for improving technology.

To assess quantitatively the contribution of technology in determining the magnitude of aggregate and sectoral fiscal multipliers, we start with a simplified version of our model which collapses to the semi-small open economy model developed by Kehoe and Ruhl [2009] with capital adjustment costs and imperfect mobility of labor across sectors. In this restricted version, we shut down endogenous capital and technology utilization in both sectors and assume that sectoral goods are produced from Cobb-Douglas production functions. Under these assumptions, the restricted model considerably understates both real GDP and labor multipliers that we estimate empirically. By assuming fixed sectoral TFPs, the model also predicts a fall in traded value added and a disproportionate increase in non-traded value added, in contradiction with our evidence while the shift of labor toward the non-traded sector falls short of our estimates.

Once we let capital-utilization-adjusted-technology respond endogenously to the rise in government spending and allow firms to change the mix of labor- and capital-augmenting efficiency over time, the model can account for both the aggregate and sectoral effects that we estimate empirically. By increasing real GDP directly and also indirectly through higher wages which provide more incentives to increase labor supply, the rise in aggregate TFP allows the model to generate government spending multipliers larger than one in line with our evidence. Although the government spending shock is biased toward non-tradables, technology improvements are concentrated in tradables because the cost of adjusting technology is lower in the traded than in the non-traded sector, conditionally on barriers to factors' mobility together with FBTC and increased capital utilization. The TFP differential leads the real GDP multiplier to be symmetrically distributed across sectors. Conversely, the bulk of the rise in total hours worked is concentrated in the non-traded sector which biases technological change toward labor.

Literature. Our paper fits into several different literature strands, as we bring several distinct threads in the existing literature together.

Recently, Jordà et al. [2020] and Baqaee et al. [2021] have documented evidence pointing at the presence of a technology channel brought about by monetary policy and proposed an interpretation of TFP gains based on the shifts in the allocation of resources across firms, respectively. In contrast, in our paper, variations in TFP are the result of changes in endogenous utilization of existing technologies, along the lines of Bianchi et al. [2019]. This modelling strategy has been already introduced in a New Keynesian model by Jørgensen and Ravn [2022]. Differently, to generate a rise in aggregate TFP following a government spending shock, D'Alessandro et al. [2019] endogenize technological progress by assuming skill accumulation through past work experience, echoing the learning-by-doing mechanism. In contrast to these two papers, we show that the technology channel determines the distribution of real GDP and labor multipliers across sectors and that the decision to improve technology depends on the biasedness of the demand shock, barriers to factors' mobility, and sector-specific FBTC.

In this regard, we also contribute to a growing literature investigating fiscal transmission at a sectoral level, both empirically and theoretically. Ramey and Shapiro [1998] find that a rise in military spending (which is intensive in traded goods) reallocates labor toward traded industries. Benetrix and Lane [2010] document evidence which reveals that a government spending shock disproportionately increases non-traded value added. As shown by Cardi et al. [2020] and Lambertini and Proebsting [2022], to account for the fiscal transmission mechanism, the open economy model with tradables and non-tradables must allow for both a non-traded bias in government spending and imperfect mobility of labor across sectors. In contrast to both these aforementioned works, we highlight empirically the technology channel of government spending shocks and quantify its role in determining the size of sectoral government spending multipliers.

Third, our paper also relates to a growing literature which highlights the role of subcategories of aggregate government spending. Like us, research works by Boehm [2020], Bouakez et al. [2022a] and Proebsting [2022] stress the role of labor mobility costs in a multi-sector model and place the emphasis on the composition of government spending. Differently, we show that abstracting from the technology channel leads to understate the actual magnitude of both output and labor multipliers. Our results also point out that shutting down technology at a sectoral level would lead to overstate the value added multiplier of non-tradables and understate its labor multiplier. Bouakez et al. [2022b] and Cox et al. [2020] use granular U.S. data and find that government spending shocks are concentrated towards a few industries. Like them, we emphasize that government purchases do not mimic private demand in OECD countries. These two papers have a different strategy as they seek to determine the aggregate effects of sector-specific government spending shocks. Instead, our main objective is to analyze the distribution of real GDP and labor multipliers across sectors and thus we have to let all components of government spending vary at the same time. When we adopt the same strategy as Bouakez et al. [2022b] and Cox et al. [2020], we find that the multiplier is maximized as long as government consumption is allocated to the sector with the highest labor compensation share when technology is shut down, and differently that government consumption must be allocated to the sector with the lowest technology adjustment cost once we allow for the technology channel. In contrast to Bouakez et al. [2022b] and Cox et al. [2020] who estimate the multipliers only numerically, we quantify the multipliers at an aggregate and sectoral level, both empirically and numerically, like Bouakez et al. [2022a] who estimate the sectoral effects of a shock to government consumption.

Outline. In section 2, we document evidence about the technology channel of fiscal policy at both aggregate and sectoral levels. In section 3, we develop a semi-small open economy model with tradables and non-tradables, endogenous technology choices and FBTC. In section 4, we uncover the factors giving rise to technology improvements in the traded sector and quantify the role of the technology channel in driving aggregate and sectoral spending multipliers. The Online Appendix contains more empirical results, conducts robustness checks, details the solution method, and shows extensions of the baseline model.

2 Sectoral Fiscal Multipliers and Technology: Evidence

In this section, we document evidence for eighteen OECD countries about the role of the technology channel in determining the allocation of government spending multipliers across sectors. Below, we denote the percentage deviation from initial steady-state (or the rate of change) with a hat.

2.1 Preliminaries

To discipline our empirical investigation, we first develop intuition about how technology affects government spending multipliers at a sectoral level. We consider a two-sector open economy and make a distinction between a traded (indexed by the superscript H) vs. a non-traded sector (indexed by the superscript N).

Sectoral decomposition of the change in real GDP. Real GDP denoted by Y_R is the sum of value added at constant prices, i.e., $Y_{R,t} = P^H Y_t^H + P^N Y_t^N$ where Y_t^j is the real value added of sector $j = H, N$ at time t evaluated at the base year price P^j.[3] Log-linearizing in the neighborhood of the initial steady-state shows that the deviation of real GDP relative to its initial steady-state in percentage, $\hat{Y}_{R,t}$, is the sum of percentage deviations of value added (at constant prices) relative to their initial steady-state:

$$\hat{Y}_{R,t} = \nu^{Y,H} \hat{Y}_t^H + \nu^{Y,N} \hat{Y}_t^N, \tag{1}$$

where $\hat{Y}_t^j = \frac{Y_t^j - Y^j}{Y^j}$ and we denote the value added share of sector j by $\nu^{Y,j}$. Note that $\nu^{Y,H} + \nu^{Y,N} = 1$.

Definition of government spending multipliers. We denote government final consumption expenditure by G_t. We calculate the government spending multiplier over a t-year horizon as the ratio of the present value of the cumulative change in value added to the present value of the cumulative change in G_t over t years. Denoting the world interest rate by $r^\star$, pre-multiplying both sides of (1) by the discount factor, integrating over $(0, t)$ and denoting the multiplier by a superscript G leads to:

$$\hat{Y}_{R,t}^G = \nu^{Y,H} \hat{Y}_t^{H,G} + \nu^{Y,N} \hat{Y}_t^{N,G}, \tag{2}$$

where $\hat{X}_t^G = \frac{\int_0^t \hat{X}_\tau e^{-r^\star \tau} d\tau}{\omega_G \int_0^t \hat{G}_\tau e^{-r^\star \tau} d\tau}$ for $X = Y_R, L$, and $\hat{X}_t^{j,G} = \frac{\int_0^t \hat{X}_\tau^j e^{-r^\star \tau} d\tau}{\omega_G \int_0^t \hat{G}_\tau e^{-r^\star \tau} d\tau}$ for $X^j = Y^j, L^j$ with $\omega_G = G/Y$. Because the shock to G_t is normalized to 1% of GDP at time $t = 0$, impact effects collapse to impact multipliers, i.e., $\hat{X}_0 = \hat{X}_0^G$. We do not refer exclusively to the concept of spending multiplier below and also employ interchangeably the terms of growth or cumulative change over a t-year horizon (implicitly conditional on a shock to G_t) because the concept of multiplier is less meaningful for prices, wages, labor income shares, hours worked shares or value added shares.

[3] We consider an initial steady-state where prices are those at the base year so that real GDP, Y_R, collapses to nominal GDP, Y, initially.

Value added share at constant prices and its determinants. A sufficient statistic determining the degree of asymmetry in the distribution of real GDP growth across sectors is the change in the value added share at constant prices $\nu_t^{Y,j}$ in sector $j = H, N$; the change in $\nu_t^{Y,j}$ is defined as the excess (measured in ppt of GDP) of real value added growth in sector j over real GDP growth, i.e., $d\nu_t^{Y,j} = \nu^{Y,j}\left(\hat{Y}_t^j - \hat{Y}_{R,t}\right)$. Rearranging the latter equality as follows $\nu^{Y,j}\hat{Y}_t^j = \nu^{Y,j}\hat{Y}_{R,t} + d\nu_t^{Y,j}$ reveals that when $d\nu_t^{Y,j} = 0$, we have $\nu^{Y,j}\hat{Y}_t^j = \nu^{Y,j}\hat{Y}_{R,t}$, so that the cumulative change in real GDP caused by a shock to G_t is uniformly distributed across sectors, i.e., in accordance with their value added share. Conversely, value added of sector j increases disproportionately when $d\nu_t^{Y,j} > 0$.

The change in the value added share of sector j is determined by the reallocation of productive resources across sectors and the TFP growth differential caused by a government spending shock, see Online Appendix B.1 for a formal derivation. Using data from the World Input-Output Database (WIOD) [2013], [2016], we constructed time series for sectoral government consumption and find empirically that the non-traded sector receives on average 80% of government consumption (see column 4 of Table 1).[4] As demonstrated in Online Appendix R.1, see also Cardi et al. [2020], Proebsting [2022], when the intensity of the non-traded sector in the government spending shock, denoted by ω_{G^N}, is higher than the share of non-tradables in GDP (which averages 64%, see column 1 of Table 1), the demand shock moves productive resources toward the non-traded sector, as long as technology is kept fixed. The impact of the biasedness of the demand shock toward non-tradables can be neutralized however if the rise in G_t leads to endogenous TFP gains which vary across sectors.

Sectoral decomposition of the change in hours. While changes in sectoral TFPs influence the distribution of real GDP growth across sectors, technology adjustment also shapes the responses of sectoral hours worked as a result of factor-biased technological change (FBTC henceforth). To shed some light on the impact of FBTC on the responses of sectoral hours worked, we start with the sectoral decomposition of the percentage deviation of total hours worked relative to its initial steady-state:

$$\hat{L}_t = \alpha_L^H \hat{L}_t^H + \alpha_L^N \hat{L}_t^N, \tag{3}$$

where L and L^j are total and sectoral hours worked, respectively, α_L^j is the labor compensation share in sector j and $\alpha_L^H + \alpha_L^N = 1$. Note that α_L^j collapses to L^j/L when we impose perfect mobility of labor across sectors.

Determinants of the change in the labor share of non-tradables. Like the value added share, the change in the labor share of sector j computed as $d\nu_t^{L,N} = \alpha_L^N\left(\hat{L}_t^N - \hat{L}_t\right)$, see Online Appendix B.2, indicates whether the rise in L_t is uniformly distributed across sectors. Because a shock to G_t is biased toward non-tradables, as evidence suggests, labor

[4]Bussière et al. [2013] also find that government spending mostly includes non-tradables.

shifts toward the non-traded sector, i.e., $d\nu_t^{L,N} > 0$, which has a positive impact on the labor multiplier of non-tradables measured by $\alpha_L^N \hat{L}_t^N = \alpha_L^N \hat{L}_t + d\nu_t^{L,N}$. Both barriers to mobility and factor-augmenting technology influence the magnitude of $d\nu_t^{L,N} > 0$.

Frictions in the movement of labor between sectors, caused by imperfect substitutability between traded and non-traded hours worked and between home- and foreign-produced traded goods, mitigate the rise in $\nu_t^{L,N}$ while FBTC may amplify it. We derive below a formal expression for L_t^N/L_t when labor market clears. Denoting the elasticity of labor supply across sectors by ϵ, the share of hours worked supplied to sector j is increasing in the wage differential, i.e., $\frac{L_t^j}{L_t} = \vartheta^j \left(\frac{W_t^j}{W_t}\right)^{\epsilon}$ where ϑ^j stands for the weight attached to labor supply in sector $j = H, N$, W_t^j and W_t are sectoral and aggregate wage rates, respectively. We assume perfectly competitive markets and constant returns to scale in production. Under these assumptions, labor is paid its marginal product. Denoting the labor income share by s_L^j, the marginal revenue product of labor, $s_{L,t}^j \frac{P_t^j Y_t^j}{L_t^j}$, must equate the wage rate W_t^j. The same logic applies at an aggregate level, i.e., $s_{L,t} \frac{Y_t}{L_t} = W_t$ where $s_{L,t}$ is the aggregate labor income share (LIS henceforth) and Y_t is GDP at current prices. Dividing W_t^N by W_t and making use of labor supply to sector j to eliminate W_t^N/W_t leads to a formal expression for the equilibrium non-traded-goods-share of total hours worked (see Online Appendix B.2):

$$L_t^N/L_t = (1-\vartheta)^{\frac{1}{1+\epsilon}} \left(s_{L,t}^N/s_{L,t}\right)^{\frac{\epsilon}{1+\epsilon}} \left(\omega_t^{Y,N}\right)^{\frac{\epsilon}{1+\epsilon}}, \tag{4}$$

where $\omega_t^{Y,N}$ is the value added share of non-tradables at current prices. In a model where production functions are Cobb-Douglas, LISs remain fixed. Eq. (4) states that by pushing up $\omega_t^{Y,N}$, a demand shock biased toward non-tradables increases L_t^N/L_t. The rise in $\omega^{Y,N}$ is curbed however by imperfect substitutability between home- and foreign-produced traded goods as households are reluctant to substitute imported for domestic goods; this gives rise to a terms of trade appreciation that raises the return on capital and labor in the traded sector. For a given change in $\omega_t^{Y,N}$, a lower labor mobility (i.e., ϵ takes lower values) mitigates the rise in L_t^N/L_t.

When sectoral goods are produced by means of CES production functions, the technology of production can become more labor (capital) intensive if technological change is biased toward labor (capital). If non-traded firms decide to bias technological change toward labor and traded firms to bias technological change toward capital, the non-traded LIS, $s_{L,t}^N$, increases relative to the aggregate LIS, $s_{L,t}$, which tilts the demand for labor toward the non-traded sector as shown on the RHS of (4). As shall be useful later, we draw on Caselli and Coleman [2006] and Caselli [2016] to construct time series for FBTC which must be adjusted with the capital utilization rate, as explained in section 2.3. Denoting the elasticity of substitution between capital and labor by σ^j, capital- and labor-augmenting efficiency by B_t^j and A_t^j, respectively, our measure of capital-utilization-adjusted-FBTC, denoted by $\text{FTBC}^j_{t,adjK}$, reads (see Online Appendix G for a formal derivation):

$$\text{FBTC}^j_{t,adjK} = \left(\frac{B^j_t/\bar{B}^j}{A^j_t/\bar{A}^j}\right)^{\frac{1-\sigma^j}{\sigma^j}} = \frac{S^j_t}{\bar{S}^j}\left(\frac{k^j_t}{\bar{k}^j}\right)^{-\frac{1-\sigma^j}{\sigma^j}}\left(\frac{u^{K,j}_t}{\bar{u}^{K,j}}\right)^{-\frac{1-\sigma^j}{\sigma^j}}, \tag{5}$$

where a bar refers to averaged values of the corresponding variable over 1970-2015. To construct time series for $\text{FTBC}^j_{t,adjK}$, we plug time series for the ratio of the labor to the capital income share, $S^j_t = s^j_{L,t}/\left(1-s^j_{L,t}\right)$, the capital-labor ratio, k^j_t, the capital utilization rate defined later, $u^{K,j}_t$. We also plug values for σ^j we have estimated for each country of our sample (see columns 14 and 15 of Table 1). We find values for σ^j smaller than one for the whole sample (and most of countries/sectors), thus corroborating the gross complementarity between capital and labor documented by Klump et al. [2007], Herrendorf et al. [2015], Oberfield and Raval [2021], Chirinko and Mallick [2017]. When $\text{FBTC}^j_{t,adjK}$ increases, technological change is biased toward labor while a fall indicates that technological change is biased toward capital.

2.2 VAR Model and Identification

To conduct our empirical study, we follow Corsetti et al. [2012] and compute the responses of selected variables by using a two-step estimation procedure. Like Bernardini et al. [2020] and Liu [2022], we first identify shocks to government consumption by considering a baseline VAR model where government spending is ordered before the other variables which amounts to adopting the standard Cholesky decomposition pioneered by Blanchard and Perotti [2002]. In the second step, we trace out the dynamic effects of the identified shock to government consumption by using Jordà's [2005] local projections.

First step. In the first step, we identify the government spending shocks by estimating the reduced-form VAR model in panel format on annual data:

$$Z_{i,t} = \alpha_i + \alpha_t + \beta_i t + \sum_{k=1}^{2} A^{-1}B_k Z_{i,t-k} + A^{-1}\epsilon_{i,t}, \tag{6}$$

where subscripts i and t denote the country and the year, k is the number of lags and $Z_{i,t}$ is the vector of endogenous variables; the specification includes country fixed effects, α_i, which control for time-invariant countries' characteristics, time dummies, α_t, which account for macroeconomic shocks common to OECD countries, and country-specific linear time trends; A is a matrix that describes the contemporaneous relation among the variables collected in vector $Z_{i,t}$, B_k is a matrix of lag-specific own- and cross-effects of variables on current observations, and the vector $\epsilon_{i,t}$ contains the structural disturbances which are uncorrelated with each other. In line with current practice, we include two lags in the regression model and use a panel OLS regression to estimate the coefficients $A^{-1}B_k$ and the reduced-form innovations $A^{-1}\epsilon_{i,t}$. The VAR model we estimate in the first step includes government final consumption expenditure, real GDP, total hours worked, the real consumption wage, and aggregate total factor productivity, where all variables are logged, while all quantities are expressed in real terms and scaled by the working-age population. Like Blanchard and Perotti [2002], we base the identification scheme on the assumption that there are some delays inherent to the legislative system which prevents government spending from responding endogenously to contemporaneous output developments.

Second step. Once we have identified government spending shocks, $\epsilon_{i,t}^{G}$, from (6), in the second step, we estimate the effects on selected variables (detailed later) by using Jordà's [2005] single-equation method. The local projection method amounts to running a series of regressions of each variable of interest on a structural identified shock for each horizon $h = 0, 1, 2, ...$:

$$x_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h} t + \psi_h (L) z_{i,t-1} + \gamma_h \epsilon_{i,t}^{G} + \eta_{i,t+h}, \tag{7}$$

where $\alpha_{i,h}$ are country fixed effects, $\alpha_{t,h}$ are time dummies, and we include country-specific linear time trends; x is the logarithm of the variable of interest, z is a vector of control variables (i.e., past values of government spending and of the variable of interest), $\psi_h (L)$ is a polynomial (of order two) in the lag operator. The coefficient γ_h gives the response of x at time $t + h$ to the government spending shock at time t. We compute heteroskedasticity and autocorrelation robust standard errors based on Newey-West.

Robustness checks. We have conducted a series of robustness checks related to several aspects of our VAR identification of government spending shocks and measures of technology which are detailed in Online Appendices P and Q. First, because using annual data makes the Blanchard-Perotti identification less natural, we have alternatively identified fiscal shocks by using quarterly data or by using military expenditure to instrument government consumption and find that our results are robust to our identification assumption and data frequency (see Online Appendices P.1, P.2, P.3). Second, instead of adopting a two-step approach where generated residuals are used as regressors, we alternatively considered the one-step method of Ramey and Zubairy [2018] and find that our results are unchanged (see Online Appendix P.6). Third, Online Appendix P.4 shows that the concern related to the potential presence of anticipation effects is substantially mitigated. Fourth, in Online Appendix P.5, we find that controlling for monetary policy does not affect our estimates. Fifth, the evidence documented in Online Appendices Q.1 and Q.3 reveals that the technology channel of fiscal transmission we highlight is robust to alternative measures of technology such as Fernald's [2014] or Basu's [1996] time series for utilization-adjusted-TFP.

2.3 Data Construction

Before presenting evidence on fiscal transmission across sectors, we briefly discuss the dataset we use. We take data from EU KLEMS [2011],[2017] and OECD STAN [2011],

[2017]. Our sample contains annual observations and consists of a panel of 18 OECD countries. The period runs from 1970 to 2015. In Online Appendix D, we detail the source and the construction of time series for value added at constant prices, Y_{it}^{j}, hours worked, L_{it}^{j}, the hours worked share, $\nu_{it}^{L,j}$, the value added share at constant prices, $\nu_{it}^{Y,j}$, the labor income share, $s_{L,it}^{j}$, of sector $j = H, N$.

Classification of industries as tradables or non-tradables. Since our primary objective is to quantify the role of the technology channel in determining the sectoral effects of a government spending shock, we describe below how we construct time series at a sectoral level. Our sample covers eleven 1-digit ISIC-rev.3 industries. Following De Gregorio et al. [1994], we define the tradability of an industry by constructing its openness to international trade given by the ratio of total trade (imports plus exports) to gross output, see Online Appendix O.1 for more details. Data for trade and output are taken from WIOD [2013], [2016]. "Agriculture, Hunting, Forestry and Fishing", "Mining and Quarrying", "Total Manufacturing" and "Transport, Storage and Communication" exhibit high openness ratios and are thus classified as tradables. At the other end of the scale, "Electricity, Gas and Water Supply", "Construction", "Wholesale and Retail Trade" and "Community Social and Personal Services" are considered as non-tradables since the openness ratio in this group of industries is low (i.e., less than 10% for most of the countries in our sample). For the three remaining industries "Hotels and Restaurants", "Financial Intermediation", "Real Estate, Renting and Business Services" the results are less clearcut since the openness ratio averages (across countries) 14% for the former and 20% for the last two sectors. In the benchmark classification, we adopt the standard classification of De Gregorio et al. [1994] by treating "Real Estate, Renting and Business Services" and "Hotels and Restaurants" as non-traded industries. Given the dramatic increase in financial openness that OECD countries have experienced since the end of the eighties, we allocate "Financial Intermediation" to the traded sector. This choice is also consistent with the classification of Jensen and Kletzer [2006] who categorize "Financial Intermediation" as tradable.[5]

Utilization-adjusted sectoral TFPs. Sectoral TFPs are Solow residuals calculated from constant-price (domestic currency) series of value added, Y_{it}^{j}, capital stock, K_{it}^{j}, and hours worked, L_{it}^{j}, i.e., $\hat{\text{TFP}}_{it}^{j} = \hat{Y}_{it}^{j} - s_{L,i}^{j}\hat{L}_{it}^{j} - \left(1 - s_{L,i}^{j}\right)\hat{K}_{it}^{j}$ where $s_{L,i}^{j}$ is the LIS in sector j averaged over the period 1970-2015. To obtain series for the capital stock in sector j, we first compute the overall capital stock by adopting the perpetual inventory approach, using constant-price investment series taken from the OECD's Annual National Accounts. Following Garofalo and Yamarik [2002], we split the gross capital stock into traded and non-traded industries by using sectoral valued added shares.[6] Once we have constructed

[5] Evidence documented in Online Appendix O.1 shows that treating "Financial Intermediation" as non-tradables or classifying "Hotels and Restaurants" or "Real Estate, Renting and Business Services" as tradables does not affect our main results.

[6] In Online Appendix O.5, we use EU KLEMS [2011], [2017] which provide disaggregated capital stock

the Solow residual for the traded and the non-traded sectors, we construct a measure for technological change by adjusting the Solow residual with the capital utilization rate, denoted by $u_{it}^{K,j}$:

$$\hat{Z}_{it}^{j} = \hat{\mathrm{TFP}}_{it}^{j} - \left(1 - s_{L,i}^{j}\right) \hat{u}_{it}^{K,j}, \tag{8}$$

where we follow Imbs [1999] in constructing time series for $u_{it}^{K,j}$ because time series for utilization-adjusted TFP are not available at a sectoral level for most of the OECD countries of our sample; see Online Appendix E where we detail the adaptation of Imbs's [1999] method to our case where sectoral goods are produced from CES production functions.

2.4 Sectoral Effects of Government Spending Shocks: VAR Evidence

We generated impulse response functions by means of local projections. The dynamic adjustment of variables to an exogenous increase in G_{it} by 1 percentage point of GDP is displayed by the solid blue line in Fig. 1. The shaded areas indicate 90% confidence bounds. The horizontal axis of each panel measures the time after the shock in years and the vertical axis measures deviations from trend. Responses of sectoral value added and sectoral hours worked are re-scaled by the sample average of sectoral value added to GDP and sectoral labor compensation share, respectively. As such, on impact, the responses of sectoral value added at constant prices and sectoral hours worked can be interpreted as value added and labor multipliers as they are expressed in percentage points of GDP and total hours worked, respectively. We also compute the government spending multipliers over a six-year horizon by setting the interest rate to 3% in accordance with the value shown in column 20 of Table 1 in section 4.1.

Aggregate effects. The first row of Fig. 1 displays the aggregate effects of a shock to government consumption. As shown in Fig. 1(a), government consumption, G_{it}, follows a hump-shaped response and displays a high level of persistence. Fig. 1(b) and Fig. 1(c) reveal that a rise in G_{it} has a strong expansionary effect on L_{it} and real GDP. Total hours worked increase by 0.9% on impact, while real GDP increases by 1.2%. The real GDP and labor multipliers over a six-year horizon average 1.4 and 1.15, respectively, the responses of L_{it} and $Y_{R,it}$ being statistically significant over this period. One key driver of a real GDP multiplier larger than one is technology, since 39% of real GDP growth is driven by TFP gains (see Fig. 1(d)) over a six-year horizon. To further check the importance of technology improvement in driving real GDP growth, we have adapted the methodology proposed by Sims and Zha [2006] to estimate empirically the government spending multiplier if the technology channel were shut down. As detailed in Online Appendix J, we find that the fiscal multiplier is reduced by 42% when the response of TFP to a shock to G_{it} is shut

data (at constant prices) at the 1-digit ISIC-rev.3 level for thirteen countries of our sample over the period 1970-2015. Our estimates show that our empirical findings are robust and unsensitive to the way the sectoral capital stocks are constructed in the data.

down.[7]

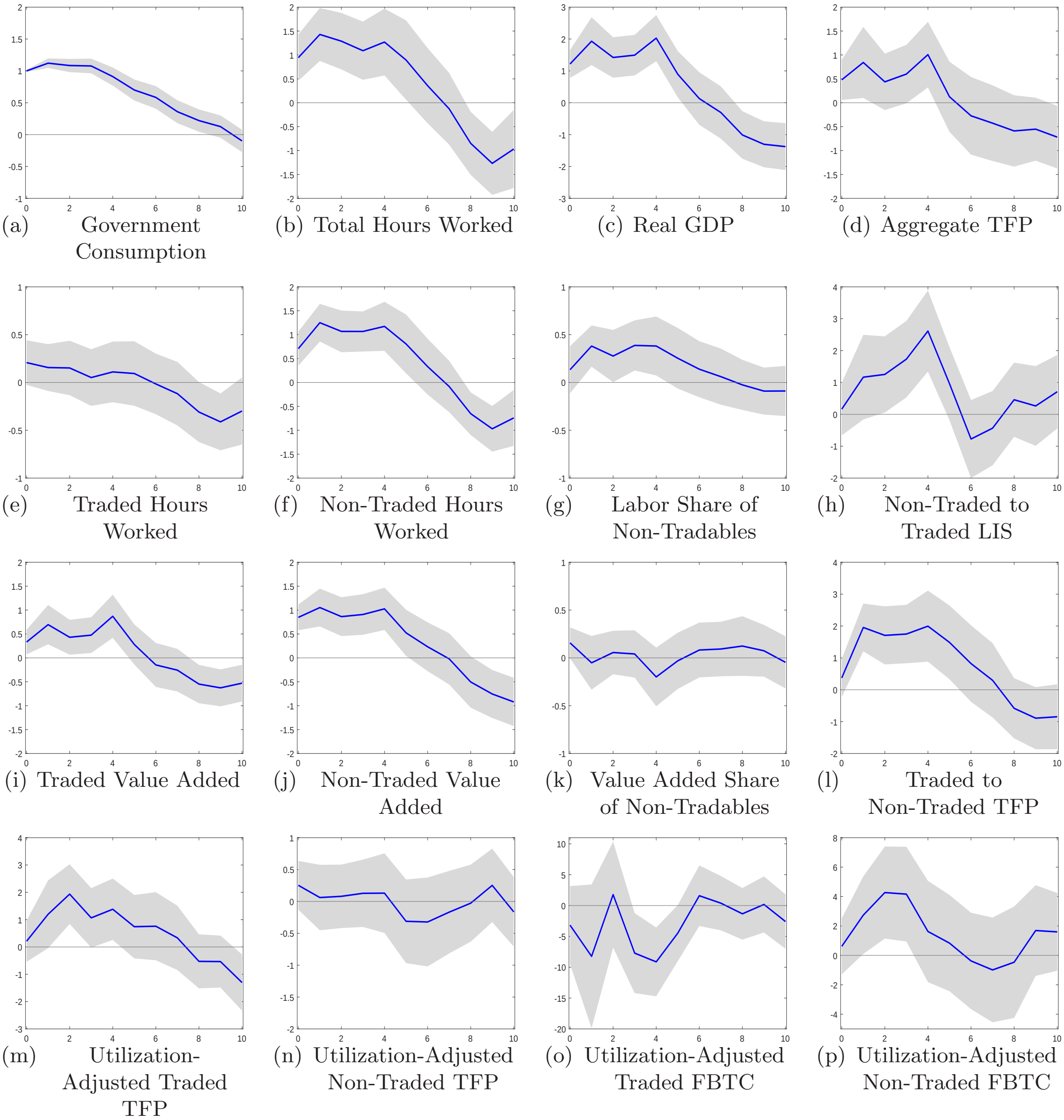
Sectoral labor multipliers and LISs. The second row of Fig. 1 displays the dynamic adjustment of sectoral hours worked. Fig. 1(e) and 1(f) reveal that a shock to G_{it} by 1 ppt of GDP increases both traded and non-traded hours but only the latter is statistically significant. More specifically, the six-year-horizon labor multipliers of non-tradables and tradables average 1.02 ppt and 0.13 ppt of total hours worked, respectively. Therefore, the rise in L^N_{it} contributes 88% to the increase in L_{it}. As displayed by Fig. 1(g), the non-traded goods-sector-share of total hours worked increases by 0.3 ppt of total hours worked on average over the first six years, which implies that the reallocation of labor toward the non-traded sector contributes 29% to the rise in L^N. The expansionary effect of the biasedness of the demand shock toward non-tradables on $\nu^{L,N}_{it}$ is amplified by the increased labor intensity of non-traded production, as reflected in a non-traded LIS which builds up relative to the traded LIS, see Fig. 1(h). When we apply the Sims and Zha [2006] methodology and assume that the ratio of the non-traded to the traded LIS is unresponsive to the government spending shock, the rise in $\nu^{L,N}_{it}$ is found to be almost twice smaller. Our evidence shown below reveals that the rise in s^N_L/s^H_L is brought about by technological change biased toward labor.[8]

Sectoral value added multipliers and TFPs. The third row of Fig. 1 shows that a rise in G_{it} increases both traded and non-traded value added at constant prices. Both responses are statistically significant. Over the first six years, the value added multipliers of tradables and non-tradables average 0.52 ppt and 0.89 ppt of GDP, respectively. In contrast to labor, the non-traded sector contributes 64% only to the cumulative change in real GDP, a value which collapses to its share in GDP. In accordance with this observation, Fig. 1(k) reveals that $\nu^{Y,N}_{it}$ remains unresponsive to the shock which is puzzling because the government spending shock is strongly biased toward non-tradables. As shown in Fig. 1(l), the solution to this puzzle lies in the technology channel. On average, over the first six years, the TFP differential between tradables and non-tradables amounts to 1.5% per year. The technology gap between sectors is large enough to neutralize the impact of the reallocation of productive resources toward the non-traded sector, thus leaving $\nu^{Y,N}_{it}$ unaffected. The Sims and Zha [2006] approach corroborates the role of technology as we find empirically that $\nu^{Y,N}_{it}$ increases by 0.26 ppt per year over a six-year horizon when the ratio of traded to non-traded TFP is kept fixed but rises by only 0.09 ppt once we allow for the technology channel.

Utilization-adjusted TFP. The last row of Fig. 1 displays the dynamic adjustment

[7] We estimate a VAR model with three variables, government consumption, aggregate TFP and real GDP. We find a six-year-horizon-government spending multiplier of 1.25. When we shut down technological change, the real GDP multiplier averages 0.73 only.

[8] We compute the LIS like Gollin [2002], i.e., labor compensation is defined as the sum of compensation of employees plus compensation of the self-employed. We find that our results are robust to alternative constructions of the LIS, see Online Appendix O.2.



(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP

(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS

(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP

(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 1: Sectoral Effects of a Shock to Government Consumption. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP. Shaded areas indicate the 90 percent confidence bounds based on Newey-West standard errors. To estimate the dynamic responses to a shock to government consumption, we adopt a two-step method. In the first step, the government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, real GDP, total hours worked, the real consumption wage, and aggregate TFP. In the second step, we estimate the effects by using Jordà's [2005] single-equation method. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTCs). Sample: 18 OECD countries, 1970-2015, annual data.

of TFP and FBTC for tradables and non-tradables, which are both adjusted with capital utilization to reflect the true variations of technological change. After correcting for capital utilization, Fig. 1(m) and Fig. 1(n) reveal that technology improves in the traded sector and is essentially unchanged in the non-traded sector, respectively. Because the capital utilization rate increases in the traded relative to the non-traded sector, these findings indicate that the rise in the relative TFP of tradables shown in Fig. 1(l) is driven by both a higher utilization of capital and a technology improvement in the traded sector, thus explaining the muted response of $\nu_{it}^{Y,N}$.

Utilization-adjusted FBTC. The last two panels of the last row of Fig. 1 show the responses of utilization-adjusted FBTC. Because higher values for FBTC^j_{adjK} imply that production turns out to be more labor intensive, evidence displayed by Fig. 1(o) and Fig. 1(p) reveal that technological change is biased toward labor in the non-traded sector and biased toward capital in the traded sector. These findings are consistent with the rise in the non-traded LIS relative to the traded LIS shown in Fig. 1(h).

Since $\text{FBTC}^j_{t,adjK} = \left(\frac{B^j_t/\bar{B}^j}{A^j_t/\bar{A}^j}\right)^{\frac{1-\sigma^j}{\sigma^j}}$ (see eq. (5)) where both $\sigma^H < 1$ and $\sigma^N < 1$ for the whole sample, the traded sector lowers B^H/A^H and non-traded firms increase capital-relative to labor-augmenting productivity B^N/A^N. In Online Appendix I, we document evidence which rationalizes the decision to bias technological change toward one factor of production. Because the non-traded sector must pay higher wages to encourage workers to shift, non-traded firms increase labor-augmenting productivity to mitigate the rise in the labor cost. Since our estimates show that labor- and capital-augmenting productivity are strong complements along the technology frontier, capital-augmenting efficiency disproportionately increases, thus generating a rise in B^N/A^N. The other way around is true in the traded sector.

Technology channel at a disaggregated level. Our dataset covers eleven industries and in Online Appendix O.3, we conduct the same empirical analysis as in the main text, except that we consider a more disaggregated industry level. We find that the five industries classified as tradables increase their TFP, which confirms that the rise in traded TFP is driven by a technology improvement within each industry. Conversely, the responses of TFP in non-traded industries are more heterogenous and clustered around the horizontal axis.

Technology improvements are driven by a cost-minimization strategy. The evidence documented above raises two important questions: does the technology channel vary across countries and which factors cause such international differences? Our evidence relegated to Online Appendix K shows that technology tends to increase more in the traded than in the non-traded sector but the responses of utilization-adjusted TFP to a government spending shock display a wide cross-country dispersion. We also find empirically that technology improvements are driven by a cost-minimization strategy, as sector $j = H, N$ increases utilization-adjusted-TFP in countries where the unit cost for producing rises following a government spending shock. In accordance with this observation, we model the decision to increase the utilization of available technology as a trade-off between the rise in output generated by enhanced productivity and the cost associated with a higher utilization rate of technology within each sector $j = H, N$.

Potential determinants of technology adjustment costs. Before discussing factors that could potentially influence the variations in technology following a government spending shock, it is important to clarify the concept of technology utilization rate. Variations in utilization-adjusted TFP in sector j, Z_{it}^j, can be driven by a change in the stock of knowledge, $\bar{Z}_{it}^j$, or by a change in the rate of utilization of the stock of ideas, $u_{it}^{Z,j}$, or both. Using data from Stehrer et al. [2019] (EU KLEMS database) we construct time series for both gross fixed capital formation and capital stock in R&D in the traded and non-traded sectors. Data are available for twelve countries over 1995-2015. As detailed in Online Appendix Q.4, our estimates reveal that neither investment in R&D nor the stock of knowledge respond to the government spending shock. These evidence thus suggest that the variations in the utilization-adjusted TFP reflect a better firm's organization leading to efficiency gains necessary to curb upward pressure on production costs instead of pure innovation that would be reflected into a rise in the stock of knowledge. In our model, higher production efficiency is thus captured by a rise in the use of existing technology.

Since we interpret the technology utilization rate as the capacity of firms to increase overall production efficiency to meet higher demand, firms' characteristics such as the intensity of production in capital and/or skilled labor and/or R&D are likely to influence their ability to adjust efficiency. In this regard, the literature on institutions and international trade provides an insightful link between production/goods' characteristics and efficiency. The complexity of a good is captured by its intensity in relationship-specific efforts or investment and according to the evidence documented by Nunn [2007], goods' complexity is strongly and positively correlated with skill intensity but negatively correlated with capital intensity. In the same vein, Costinot [2009] shows that more complex goods involve a more fragmented chain of production with a higher number of tasks, all of them being essential. Because transaction and coordination costs become larger as the chain of production becomes more fragmented, we expect capital intensive industries to be more prone to improve efficiency to meet higher demand while industries which are more intensive in skilled labor or in R&D should experience a larger cost of adjusting technology. These predictions also echo evidence documented by Adão et al. [2022] who find that skills' specificity tend to slow productivity gains.

To test our hypothesis, we perform a split-sample based on the median of the intensity of value added in tangible assets, skilled labor, and intangible assets, see Online Appendix Q.4 which details the source and construction of data. For each group of countries, we plot the technology multiplier against time. The technology multiplier is calculated as the cumulative response of utilization-adjusted aggregate TFP divided by the cumulative response of government consumption. In line with our assumptions, we find that the long-run technology multiplier is significantly positive in the group of countries where traded and non-traded industries are relatively more intensive in physical capital and relatively less intensive in skilled labor or in R&D. More specifically, the technology multiplier is significantly larger than one after six years for industries relatively more intensive in capital, or relatively less intensive in skilled labor or in the the stock of knowledge. Conversely, in the second group of countries where the intensity in capital is lower and the intensity in skilled labor or R&D is larger, the technology multiplier is not statistically different from zero in the long-run.

3 A Semi-Small Open Economy Model with Tradables and Non-Tradables

We consider a semi-small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. Like Kehoe and Ruhl [2009], Chodorow-Reich et al. [2021], the country is assumed to be semi-small in the sense that it is a price-taker in international capital markets, and thus faces a given world interest rate, $r^{\star}$, but is large enough on world good markets to influence the price of its export goods. The open economy produces a traded good which can be exported, consumed or invested and imports consumption and investment goods. While the home-produced traded good, denoted by the superscript H, faces both a domestic and a foreign demand, a non-traded sector produces a good, denoted by the superscript N, for domestic absorption only. Households supply labor and capital services to firms and must decide about the intensity in the use of tangible and intangible assets. We add a tilde below when the variable is augmented with the rate of utilization of (tangible or intangible) assets. Firms rent capital and labor services and choose a mix of capital- and labor- along the technology frontier. The foreign good is chosen as the numeraire. Time is continuous and indexed by t.

3.1 Households

At each instant the representative household consumes traded and non-traded goods denoted by $C^T(t)$ and $C^N(t)$, respectively, which are aggregated by means of a CES function:

$$C(t) = \left[\varphi^{\frac{1}{\phi}} \left(C^T(t)\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(C^N(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{9}$$

where $0 < \varphi < 1$ is the weight of the traded good in the overall consumption bundle and ϕ corresponds to the elasticity of substitution between traded goods and non-traded goods.

The traded consumption index $C^T(t)$ is defined as a CES aggregator of home-produced traded goods, $C^H(t)$, and foreign-produced traded goods, $C^F(t)$:

$$C^T(t) = \left[\left(\varphi^H\right)^{\frac{1}{\rho}} \left(C^H(t)\right)^{\frac{\rho-1}{\rho}} + \left(1-\varphi^H\right)^{\frac{1}{\rho}} \left(C^F(t)\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{10}$$

where $0 < \varphi^H < 1$ is the weight of the home-produced traded good and ρ corresponds to the elasticity of substitution between home- and foreign-produced traded goods.

The representative household supplies labor to the traded and non-traded sectors, denoted by $L^H(t)$ and $L^N(t)$, respectively. To put frictions into the movement of labor across sectors, we build on Horvath [2000] and assume that sectoral hours worked are imperfect substitutes:

$$L(t) = \left[\vartheta^{-1/\epsilon} \left(L^H(t)\right)^{\frac{\epsilon+1}{\epsilon}} + (1-\vartheta)^{-1/\epsilon} \left(L^N(t)\right)^{\frac{\epsilon+1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon+1}}, \tag{11}$$

where $0 < \vartheta < 1$ parametrizes the weight attached to the supply of hours worked in the traded sector and ϵ is the elasticity of substitution between sectoral hours worked.

The representative agent is endowed with one unit of time, supplies a fraction $L(t)$ as labor, and consumes the remainder $1 - L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^\infty \left\{ \frac{1}{1-\frac{1}{\sigma_C}} C(t)^{1-\frac{1}{\sigma_C}} - \frac{1}{1+\frac{1}{\sigma_L}} L(t)^{1+\frac{1}{\sigma_L}} \right\} e^{-\beta t} \mathrm{d}t, \tag{12}$$

where $\beta > 0$ is the discount rate, $\sigma_C > 0$ the intertemporal elasticity of substitution for consumption, and $\sigma_L > 0$ the Frisch elasticity of (aggregate) labor supply.

Households receive a wage rate $\tilde{W}(t)$ and a capital rental rate $\tilde{R}(t)$, in exchange for labor $L(t)$ and capital services $K(t)$. We assume that households choose the level of capital utilization $u^{K,j}(t)$ in sector j. They also own the stock of intangible capital $\bar{Z}^j$ and decide about the level of utilization $u^{Z,j}(t)$ of existing technology in sector j. In the sequel, we normalize the stock of knowledge, $\bar{Z}^j$, to one as we abstract from endogenous choices on the stock of knowledge.[9] Because households may decide to use more intensively the stock of knowledge in sector j which increases the efficiency in the use of inputs, the counterpart is a rise in factor prices, since factors are paid their marginal product. In accordance with the Euler Theorem, we have $P^j(t)u^{Z,j}(t)Y^j(t) = \tilde{W}^j(t)L^j(t) + \tilde{R}^j(t)u^{K,j}(t)K^j(t)$ where $\tilde{R}^j(t) = u^{Z,j}(t)R(t)$ is the capital rental rate, $\tilde{W}^j(t) = u^{Z,j}(t)W^j(t)$ is the wage rate, $P^j(t)$ is the value added deflator and Y^j stands for technology-utilization-adjusted real value added in sector j. Both the capital $u^{K,j}(t)$ and the technology utilization rate $u^{Z,j}(t)$

[9] Bianchi et al. [2019] assume that firms can choose both the technology utilization rate and the stock of knowledge. In accordance with the evidence documented in Online Appendix Q.4 which reveals that capital stocks in R&D in both sectors are unresponsive to a government spending shock, we assume that the stocks of knowledge are constant over time. In addition, our estimates show that utilization-adjusted-sectoral TFP is restored back toward its initial steady-state level in both sectors which is consistent with a time-varying technology utilization rate at a sectoral level.

collapse to one at the steady-state. We let the functions $C^{K,j}(t)$ and $C^{Z,j}(t)$ denote the adjustment costs associated with the choice of capital and technology utilization rates, which are increasing and convex functions of utilization rates:

$$C^{K,j}(t) = \xi_1^j \left(u^{K,j}(t) - 1\right) + \frac{\xi_2^j}{2}\left(u^{K,j}(t) - 1\right)^2, \tag{13a}$$

$$C^{Z,j}(t) = \chi_1^j \left(u^{Z,j}(t) - 1\right) + \frac{\chi_2^j}{2}\left(u^{Z,j}(t) - 1\right)^2, \tag{13b}$$

where $\xi_2^j > 0$, $\chi_2^j > 0$ are free parameters; as $\xi_2^j \to \infty$, $\chi_2^j \to \infty$, utilization is fixed at unity. It is worth mentioning that while the technology utilization rate is assumed to be Hicks-neutral and factor-biased technological change is recovered by using a wedge analysis as detailed later, we could alternatively assume that households choose the utilization rate of factor-augmenting technology. We have considered this possibility both theoretically and numerically. The model fails to reproduce our evidence however as it can account for neither the technology improvement in the traded sector nor the magnitude of technological change biased toward labor which is necessary to generate a rise in the non-traded LIS.

Households can accumulate internationally traded bonds (expressed in foreign good units), $N(t)$, that yield net interest rate earnings of $r^\star N(t)$. Denoting lump-sum taxes by $T(t)$, and the aggregate consumption and investment price index by $P_C(t)$ and $P_J(t)$, respectively, the household's flow budget constraint states that real disposable income can be saved by accumulating traded bonds, $\dot{N}(t)$, consumed, $P_C(t)C(t)$, invested, $P_J(t)J(t)$, or can cover (capital and technology) utilization adjustment costs:

$$\begin{aligned}\dot{N}(t) &+ P_C(t)C(t) + P_J(t)J(t) + P^H(t)C^{K,H}(t)\alpha_K(t)K(t) + P^N(t)C^{K,N}(t)\left(1-\alpha_K(t)\right)K(t) \\ + &\quad P^H(t)C^{Z,H}(t) + P^N(t)C^{Z,N}(t) = \left[\alpha_L(t)u^{Z,H}(t) + \left(1-\alpha_L(t)\right)u^{Z,N}(t)\right]W(t)L(t) \\ + &\quad \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + \left(1-\alpha_K(t)\right)u^{K,N}(t)u^{Z,N}(t)\right]R(t)K(t) + r^\star N(t) - T(t), \end{aligned}\tag{14}$$

where P^H is the price of home-produced traded goods or the terms of trade and P^N is the price of non-traded goods; we denote the share of traded capital in the aggregate capital stock by $\alpha_K(t) = K^H(t)/K(t)$ and the labor compensation share of tradables by $\alpha_L(t) = \frac{W^H(t)L^H(t)}{W(t)L(t)}$.

The investment good is (costlessly) produced using inputs of the traded good and the non-traded good by means of a CES technology:

$$J(t) = \left[\varphi_J^{\frac{1}{\phi_J}}\left(J^T(t)\right)^{\frac{\phi_J-1}{\phi_J}} + \left(1-\varphi_J\right)^{\frac{1}{\phi_J}}\left(J^N(t)\right)^{\frac{\phi_J-1}{\phi_J}}\right]^{\frac{\phi_J}{\phi_J-1}}, \tag{15}$$

where $0 < \varphi_J < 1$ is the weight of the investment traded input and ϕ_J corresponds to the elasticity of substitution between investment traded goods and investment non-traded goods. The index $J^T(t)$ is defined as a CES aggregator of home-produced traded inputs, $J^H(t)$, and foreign-produced traded inputs, $J^F(t)$:

$$J^T(t) = \left[\left(\iota^H\right)^{\frac{1}{\rho_J}}\left(J^H(t)\right)^{\frac{\rho_J-1}{\rho_J}} + \left(1-\iota^H\right)^{\frac{1}{\rho_J}}\left(J^F(t)\right)^{\frac{\rho_J-1}{\rho_J}}\right]^{\frac{\rho_J}{\rho_J-1}}, \tag{16}$$

where $0 < \iota^H < 1$ is the weight of the home-produced traded input and ρ_J corresponds to the elasticity of substitution between home- and foreign-produced traded inputs.

Installation of new investment goods involves convex costs, assumed to be quadratic. Thus, total investment $J(t)$ differs from effectively installed new capital:

$$J(t) = I(t) + \frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)^2 K(t), \tag{17}$$

where the parameter $\kappa > 0$ governs the magnitude of adjustment costs to capital accumulation. Denoting the fixed capital depreciation rate by $0 \leq \delta_K < 1$, aggregate investment, $I(t)$, gives rise to capital accumulation according to the dynamic equation:

$$\dot{K}(t) = I(t) - \delta_K K(t). \tag{18}$$

Households choose consumption, hours, capital and technology utilization rates, investment in capital and traded bonds by maximizing lifetime utility (12) subject to (14) and (18) together with (17). Denoting by λ and Q' the co-state variables associated with (14) and (18), the first-order conditions characterizing the representative household's optimal plans are:

$$(C(t))^{-\frac{1}{\sigma_C}} = P_C(t)\lambda(t), \tag{19a}$$

$$\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}} = \lambda(t)\tilde{W}(t), \tag{19b}$$

$$Q(t) = P_J(t)\left[1 + \kappa\left(\frac{I(t)}{K(t)} - \delta_K\right)\right], \tag{19c}$$

$$\dot{\lambda}(t) = \lambda\left(\beta - r^\star\right), \tag{19d}$$

$$\begin{aligned}\dot{Q}(t) = \left(r^\star + \delta_K\right)Q(t) - \Big\{ & \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + \left(1 - \alpha_K(t)\right)u^{K,N}(t)u^{Z,N}(t)\right]R(t) \\ & -P^H(t)C^{K,H}(t)\alpha_K(t) - P^N(t)C^{K,N}(t)\left(1 - \alpha_K(t)\right) - P_J(t)\frac{\partial J(t)}{\partial K(t)}\Big\},\end{aligned} \tag{19e}$$

$$R(t)u^{Z,j}(t) = P^j(t)\left[\xi_1^j + \xi_2^j\left(u^{K,j}(t) - 1\right)\right], \qquad j = H, N, \tag{19f}$$

$$Y^j(t) = \chi_1^j + \chi_2^j\left(u^{Z,j}(t) - 1\right), \qquad j = H, N, \tag{19g}$$

and the transversality conditions $\lim_{t\to\infty} \bar{\lambda}N(t)e^{-\beta t} = 0$ and $\lim_{t\to\infty} Q(t)K(t)e^{-\beta t} = 0$; to derive (19c) and (19e), we used the fact that $Q(t) = Q'(t)/\lambda(t)$. To determine (19g), we made use of the fact that value added is exhausted by factor payments $P^j(t)Y^j(t) = R(t)\tilde{K}^j(t) + W^j(t)L^j(t)$ where $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$.

The technology channel of fiscal policy is captured by eq. (19g) which states that the decision to improve technology in sector j is pro-cyclical. As detailed later in sections 4.2 and 4.3, the adjustment in $Y^j(t)$ depends on the intensity (measured by ω_{G^j}) of the sector in the government spending shock, the degree of labor mobility across sectors (measured by ϵ), the substitutability between home- and foreign-produced traded goods (measured by ρ and ρ_J), the intensity in the use of tangible assets, $u^{K,j}(t)$, and FBTC (presented later).

In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta = r^\star$ in order to generate an interior solution. Setting $\beta = r^\star$ into (19d) implies that the shadow value of wealth is constant over time, i.e., $\lambda(t) = \bar{\lambda}$. When new information about the fiscal shock arrives, $\bar{\lambda}$ jumps to fulfill the intertemporal solvency condition and remains constant afterwards. While the rise in taxes to balance the government budget puts upward pressure on $\bar{\lambda}$, the technology channel lowers it.

Once aggregate consumption and investment have been chosen, the allocation of expenditure across goods, i.e., C^g and I^g (with $g = F, H, N$), is determined by applying Shephard's lemma. The same logic applies to labor, i.e., $L^H(t) = \vartheta \left(\tilde{W}^H(t)/\tilde{W}(t)\right)^\epsilon L(t)$ and $L^N(t) = (1-\vartheta)\left(\tilde{W}^N(t)/\tilde{W}(t)\right)^\epsilon L(t)$. As the elasticity of labor supply across sectors, ϵ, takes higher values, workers experience lower mobility costs and thus more labor shifts from one sector to another.

3.2 Firms

We denote by $\tilde{Y}^j(t)$ the value added of sector j inclusive of technology utilization, i.e., $\tilde{Y}^j(t) = u^Z(t)Y^j(t)$. Both the traded and non-traded sectors use physical capital (inclusive of capital utilization), denoted by $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$, and labor, L^j, according to a constant returns-to-scale technology described by a CES production function:

$$\tilde{Y}^j(t) = \left[\gamma^j \left(\tilde{A}^j(t)L^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(\tilde{B}^j(t)\tilde{K}^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}, \tag{20}$$

where $0 < \gamma^j < 1$ and $0 < 1-\gamma^j < 1$ are the weight of labor and capital in the production technology, respectively, σ^j is the elasticity of substitution between capital and labor in sector $j = H, N$. We allow for labor- and capital-augmenting efficiency denoted by $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. We assume that factor-augmenting productivity has a symmetric time-varying component which collapses to $u^{Z,j}(t)$, such that $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$ and $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$. For given Hicks-neutral technology improvement, the mix of labor and capital-augmenting efficiency can change at each point of time along the technology frontier described later.

Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to $R(t)$, and a labor cost equal to the wage rate $W^j(t)$. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given. While capital can move freely between the two sectors, costly labor mobility implies a wage differential across sectors:[10]

$$P^j(t)\gamma^j\left(A^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}}\left(y^j(t)\right)^{\frac{1}{\sigma^j}} = W^j(t), \tag{21a}$$

$$P^j(t)\left(1-\gamma^j\right)\left(B^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}}\left(u^{K,j}(t)k^j(t)\right)^{-\frac{1}{\sigma^j}}\left(y^j(t)\right)^{\frac{1}{\sigma^j}} = R(t), \tag{21b}$$

[10] Since the profit function is a linear function of the technology utilization rate, i.e., $\tilde{\Pi}^j(t) = u^{Z,j}(t)\Pi^j(t)$, $u^{Z,j}(t)$ does not show up in the first-order conditions shown in (21).

where we denote by $k^j(t) \equiv K^j(t)/L^j(t)$ the capital-labor ratio for sector $j = H, N$, and $y^j(t) \equiv Y^j(t)/L^j(t)$ refers to value added per hours worked.

Demand for inputs can be rewritten in terms of their respective cost in value added; for labor, we have $s_L^j(t) = \gamma^j \left(A^j(t)/y^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}}$. Applying the same logic for capital and denoting the ratio of labor to capital income share by $S^j(t) \equiv s_L^j(t)/\left(1 - s_L^j(t)\right)$, we have:

$$S^j(t) \equiv \frac{s_L^j(t)}{1 - s_L^j(t)} = \frac{\gamma^j}{1-\gamma^j} \left(\frac{B^j(t) u^{K,j}(t) K^j(t)}{A^j(t) L^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}. \tag{22}$$

When technological change is assumed to be Hicks-neutral, productivity increases uniformly across inputs, i.e., $\hat{A}^j(t) = \hat{B}^j(t)$. Sectoral LISs are thus affected only through changes in $u^{K,j}(t)k^j(t)$. By contrast, when technological change is factor-biased, higher values for capital relative to labor efficiency (i.e., a rise in $B^j(t)/A^j(t)$) increases capital-utilization-adjusted-FBTC in sector j, i.e., $\left(B^j(t)/A^j(t)\right)^{\frac{1-\sigma^j}{\sigma^j}}$, because $\sigma^j < 1$ as evidence suggests. Higher values of capital-utilization-adjusted-FBTC has an expansionary effect on the demand for labor and thus generates a rise in the sectoral LIS, $s_L^j(t)$. By encouraging firms to increase the intensity of production either in labor or in capital, FBTC influences the reallocation of factors across sectors. Because $u^{Z,j}(t)$ is pro-cyclical and $Y^j(t)$ is affected by the shift of inputs, FBTC impinges on $u^{Z,j}(t)$.

Finally, aggregating over the two sectors gives us the resource constraint for capital:

$$K^H(t) + K^N(t) = K(t). \tag{23}$$

3.3 Technology Frontier

While households choose capital and technology utilization rates, firms within each sector $j = H, N$ decide about the split of capital-utilization-adjusted-TFP, denoted by $Z^j(t) = u^{Z,j}(t)\bar{Z}^j$ where $\bar{Z}^j$ is normalized to one, between labor- and capital-augmenting efficiency $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. Following Caselli and Coleman [2006] and Caselli [2016], we assume that firms choose a mix of $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$ along a CES technology frontier:

$$\left[\gamma_Z^j \left(\tilde{A}^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} + \left(1-\gamma_Z^j\right)\left(\tilde{B}^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}\right]^{\frac{\sigma_Z^j}{\sigma_Z^j-1}} \leq Z^j(t), \tag{24}$$

where $Z^j(t) > 0$ is the height of the technology frontier, $0 < \gamma_Z^j < 1$ is the weight of labor efficiency in utilization-adjusted-TFP and $\sigma_Z^j > 0$ corresponds to the elasticity of substitution between labor- and capital-augmenting productivity. Firms choose $\tilde{A}^j$ and $\tilde{B}^j$ along the technology frontier described by eq. (24) that minimizes the unit cost function. The unit cost minimization requires that (see Online Appendix I):

$$\frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{\tilde{A}^j(t)}{\tilde{B}^j(t)}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} = \frac{s_L^j(t)}{1-s_L^j(t)} \equiv S^j(t). \tag{25}$$

Solving (25) for the LIS in sector j leads to $s^j_L = \gamma^j_Z \left(\tilde{A}^j/Z^j\right)^{\frac{\sigma^j_Z-1}{\sigma^j_Z}}$. Inserting this equality into the log-linearized version of the technology frontier (24) shows that technological change in sector j is a factor-income-share-weighted sum of changes in factor-augmenting efficiency:

$$\hat{Z}^j(t) = s^j_L \hat{\tilde{A}}^j(t) + \left(1 - s^j_L\right) \hat{\tilde{B}}^j(t). \tag{26}$$

As shown in eq. (26), while we assume that technology improvement is Hicks-neutral within each sector j, i.e., $Z^j(t) = u^{Z,j}(t)$, the stock of knowledge is made up of a mix of labor- and capital-augmenting productivity which can be modified at each point in time, thus leading technological change to be factor-biased.

3.4 Government

The final agent in the economy is the government. Government spending includes expenditure on non-traded goods, G^N, home- and foreign-produced traded goods, G^H and G^F, respectively. The government finances public spending, G, by raising lump-sum taxes, T. As a result, Ricardian equivalence obtains and the time path of taxes is irrelevant for the real allocation. We may thus assume without loss of generality that government budget is balanced at each instant:

$$G(t) \equiv P^N(t)G^N(t) + P^H(t)G^H(t) + G^F(t) = T(t). \tag{27}$$

In Online Appendix V, we allow for distortionary labor and consumption taxation and consider a rise in $G(t)$ which is debt-financed. Quantitative results displayed in Online Appendix V.7 show that results are similar to those obtained when assuming a balanced-budget government spending shock.

3.5 Model Closure and Equilibrium

Denoting exports of home-produced goods by X^H, the goods market clearing conditions for non-traded and home-produced traded goods read:

$$Y^N(t) = C^N(t) + J^N(t) + G^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t), \tag{28a}$$

$$Y^H(t) = C^H(t) + J^H(t) + G^H(t) + X^H(t) + C^{K,H}(t)K^H(t) + C^{Z,H}(t), \tag{28b}$$

where exports are assumed to be decreasing in the terms of trade, P^H:

$$X^H(t) = \varphi_X \left(P^H(t)\right)^{-\phi_X}, \tag{29}$$

where $\varphi_X > 0$ is a scaling parameter, and ϕ_X is the elasticity of exports w.r.t. P^H. Using the properties of constant returns to scale in production, identities $P_C(t)C(t) = \sum_g P^g(t)C^g(t)$ and $P_J(t)J(t) = \sum_g P^g(t)J^g(t)$ (with $g = F, H, N$) along with market clearing conditions (28), the current account equation (14) can be rewritten as a function of the trade balance:

$$\dot{N}(t) = r^\star N(t) + P^H(t)X^H(t) - M^F(t), \tag{30}$$

where $M^F(t) = C^F(t) + G^F(t) + J^F(t)$ stands for imports of foreign-produced consumption and investment goods.

We drop the time index below to denote steady-state values. In order to account for the dynamic adjustment of $G(t)$ (see Fig. 1(a)), we assume that the deviation of government spending relative to its initial value, i.e., $dG(t) = G(t) - G$, as a percentage of initial GDP is governed by the law of motion:

$$dG(t)/Y = e^{-\xi t} - (1-g)\, e^{-\chi t}, \tag{31}$$

where $g > 0$ parametrizes the magnitude of the exogenous fiscal shock, $\xi > 0$ and $\chi > 0$ are (positive) parameters which are set in order to capture the hump-shaped endogenous response of $G(t)$. We assume that the rise in $G(t)$ is split into non-traded, ω_{G^N}, and home-produced traded goods, $\omega_{G^H} = P^H G^H/G$, and foreign-produced traded goods, ω_{G^F}. Formally, we have $dG(t)/Y = \sum_{g=F,H,N} \omega_{G^g} dG(t)/Y$. In line with the evidence we document in Appendix F, ω_{G^N} refers to the non-tradable content of government consumption, as well as the intensity of the government spending shock in non-traded goods.

To recover the dynamics of factor-augmenting productivity, we adopt a wedge analysis. As detailed in subsection 4.2, we estimate the shifts of $A^j(t)$ and $B^j(t)$ along the technology frontier (24), which are consistent with the demand for labor relative to the demand for capital described by (22). Denoting $X^j = A^j, B^j$, to achieve a perfect match with the data, we specify the law of motion for labor- and capital-augmenting efficiency expressed as a percentage deviation relative to the initial steady-state:

$$\hat{X}^j(t) = e^{-\xi^j_X t} - \left(1 - x^j\right) e^{-\chi^j_X t}, \tag{32}$$

and choose x^j to reproduce the impact response of factor-augmenting technological change while $\xi^j_X > 0$ and $\chi^j_X > 0$ are chosen to reproduce the shape of factor-augmenting productivity together with their cumulative change following a shock to government consumption that we infer from (22) and (26).

The adjustment of the open economy toward the steady-state is described by a dynamic system which comprises two equations. The first dynamic equation corresponds to the non-traded goods market clearing condition (28a) and the second dynamic equation corresponds to (19e) which is a no-arbitrage condition. To solve the model, we adopt the solution method by Buiter [1984] for continuous time models. See Online Appendix U which details the solution method.

4 Quantitative Analysis

In this section, we take the model to the data. For this purpose we solve the model numerically. Therefore, first we discuss parameter values before turning to the effects of an exogenous temporary increase in government consumption.

4.1 Calibration

Calibration strategy. At the steady-state, utilization rates for technology, $u^{Z,j}$, and capital, $u^{K,j}$, collapse to one so that $\tilde{Y}^j = Y^j$ and $\tilde{K}^j = K^j$. We consider an initial steady-state with Hicks-neutral technological change and normalize $A^j = B^j = Z^j$ to 1. To ensure that the initial steady-state with CES production functions is invariant when σ^j is changed, we normalize CES production functions by choosing the initial steady-state in a model with Cobb-Douglas production functions as the normalization point. Once we have calibrated the initial steady-state with Cobb-Douglas production functions, we assign values to σ^j in accordance with our estimates and the CES economy is endogenously calibrated to reproduce the ratios of the Cobb-Douglas economy, including the sectoral LISs.

To calibrate the reference model that we use to normalize the CES economy, we have estimated a set of ratios and parameters for the eighteen OECD economies in our dataset, see Table 7 relegated to Online Appendix L.1. Our reference period for the calibration corresponds to the period 1970-2015. Because we calibrate the reference model to a representative OECD economy, we take unweighted average values of ratios and parameters which are summarized in Table 1. Among the 26 parameters that the model contains, 12 have empirical counterparts while the remaining 14 parameters must be endogenously calibrated to match ratios.

Fourteen parameters must be set to target ratios. Out of fourteen parameters, φ, φ^H, ι, ι^H, ϑ, δ_K, G, G^N, G^H and initial conditions (N_0 and K_0), must be set to target a non-tradable content of consumption and investment expenditure of $1 - \alpha_C = 56\%$ and $1 - \alpha_J = 69\%$, respectively, a home content of consumption and investment expenditure in tradables of $\alpha^H = 66\%$ and $\alpha^H_J = 43\%$, respectively, a weight of labor supply to the non-traded sector of $L^N/L = 62\%$, an investment-to-GDP ratio of $\omega_J = 24\%$, a ratio of government spending to GDP of $\omega_G = 19\%$ $(= G/Y)$, a non-tradable and home-tradable share of government spending of $\omega_{G^N} = 80\%$ $(= P^N G^N/G)$, and $\omega_{G^H} = 18\%$ $(= P^H G^H/G)$, and we choose initial conditions so as trade is balanced, i.e., $\upsilon_{NX} = \frac{NX}{P^H Y^H} = 0$ with $NX = P^H X^H - C^F - I^F - G^F$. Because $u^{K,j} = u^{Z,j} = 1$ at the steady-state, four parameters related to capital ξ^H_1, ξ^N_1, and technology, χ^H_1, χ^N_1, adjustment cost functions are set to be equal to R/P^H, R/P^N, Y^H, Y^N, respectively.

Five parameters are assigned values which are taken directly or estimated from our own data. We choose the model period to be one year. In accordance with the column 20 of Table 1, the world interest rate, $r^\star$, which is equal to the subjective time discount rate, β, is set to 3%. In line with mean values shown in columns 10 and 11 of Table 1, the shares of labor income in traded and non-traded value added, θ^H and θ^N, are set to 0.63 and 0.69, respectively, which leads to an aggregate LIS of 66% (see column 17 of Table 1).

Table 1: Data to Calibrate the Two Open Economy Sector Model

Non-tradable share					Home share				Labor Share	
GDP	Cons.	Inv.	Gov.	Labor	X^H	C^H	I^H	G^H	LIS^H	LIS^N
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0.64	0.56	0.69	0.80	0.62	0.13	0.66	0.43	0.18	0.63	0.69
Elasticities					Aggregate ratios					
ϕ	ϵ	σ^H	σ^N	ϕ_X	LIS	I/Y	G/Y	r		
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)		
0.77	0.83	0.64	0.80	1.70	0.66	0.24	0.19	0.030		

Notes: Columns 1-5 show the GDP share of non-tradables, the non-tradable content of consumption, investment and government expenditure, the share of non-tradables in labor. Column 6 gives the ratio of exports of final goods and services to GDP; columns 7 and 8 show the home share of consumption and investment expenditure in tradables and column 9 shows the content of government spending in home-produced traded goods; ϕ is the elasticity of substitution between traded and non-traded goods in consumption; ϵ is the elasticity of labor supply across sectors; σ^j is the elasticity of substitution between capital and labor in sector $j = H, N$; estimates of the elasticity of exports w.r.t. terms of trade, ϕ_X, are taken from Imbs and Mejean [2015]. LIS^j stands for the labor income share in sector $j = H, N$ while LIS refers to the aggregate LIS; I/Y is the investment-to-GDP ratio and G/Y is government spending as a share of GDP. The real interest rate is the real long-term interest rate calculated as the nominal interest rate on 10 years government bonds minus the rate of inflation which is the rate of change of the Consumption Price Index.

Because barriers to labor mobility play a key role in our model and estimates for OECD countries are not available, we have estimated empirically the elasticity of labor supply across sectors, ϵ, for each OECD economy. As shown in Online Appendix L.2, we pin down ϵ from a testable equation obtained by combining labor supply and labor demand and run the regression in panel format on annual data of the percentage change in the labor share of sector j on the percentage change share in the relative share of value added paid to workers in sector j over 1970-2015. Building on our estimates, the degree of labor mobility across sectors is set to 0.83, in line with the average of our estimates (see column 13 of Table 1). Note that this value is close to the value of 1 estimated by Horvath [2000] on US data over 1948-1985 and commonly chosen in the literature allowing for imperfect mobility of labor.

While there is a consensus in the open-economy macroeconomics literature that C^T and C^N are gross complements and thus ϕ should take a value lower than one, precise estimates for OECD countries are still lacking. In the same spirit as Mendoza [1992], we use the first-order condition for C^N and run the regression of the logged share of non-tradables $1 - \alpha_C(t)$ on logged $P^N(t)/P_C(t)$. Time series for $1 - \alpha_C(t)$ are constructed by using the market clearing condition for non-tradables. Building on our panel data estimates, the elasticity of substitution ϕ between traded and non-traded goods is set to 0.77, since this value corresponds to the average of estimates (see column 12 of Table 1). It is worth mentioning that our value is close to the estimated elasticity by Mendoza [1992] who reports an estimate of 0.74 for thirteen OECD countries by using cross-section data for the year 1975.

Seven parameters are taken from external research works. As pointed out recently by Best et al. [2020], there exists no consensus on a reasonable value for the intertemporal elasticity of substitution for consumption as estimates in the literature range between 0 and 2. We choose a value of one for σ_C. Estimates reported by empirical studies for the Frisch elasticity of labor supply range from 0 to 3, see Peterman [2016] for a survey. We set σ_L to 1. These values for σ_C and σ_L are a typical choice in the business cycle literature and have the advantage of making our quantitative results directly comparable with other macroeconomic studies. We choose the value of parameter κ which captures the magnitude of capital adjustment costs so that the elasticity of I/K with respect to Tobin's q, i.e., Q/P_J, is equal to the value implied by estimates in Eberly et al. [2008]. The resulting value of κ is equal to 17.

In line with the empirical findings documented by Bems [2008] who finds that the non-tradable content of investment expenditure is stable in OECD countries, we set the elasticity of substitution, ϕ_J, between J^T and J^N to 1. Following Backus et al. [1994], we set the elasticity of substitution in consumption (investment), ρ (ρ_J), between home- and foreign-produced traded goods (inputs) to 1.5 which fits estimates by Bertinelli et al. [2022] who find a vale of 1.48 for ρ from a panel of seventeen OECD countries. Building on structural estimates of the price elasticities of aggregate exports documented by Imbs and Mejean [2015], we set the export price elasticity, ϕ_X, to 1.7 in the baseline calibration (see column 16 of Table 1).

Calibrating the CES economy. To calibrate the CES economy, we proceed as follows. First, we choose the same values for the twelve parameters which have empirical counterparts as above, except for the labor income shares which are now endogenously calibrated. Thus in addition to σ_C, σ_L, κ, ϕ_J, ρ, ρ_J, ϕ_X, $r^\star$, ϵ, ϕ, we have to choose values for the elasticity of substitution between capital and labor for tradables and non-tradables, σ^H and σ^N. We estimate σ^H and σ^N over 1970-2015 on panel data so as to have consistent estimates in accordance with our classification of industries as tradables and non-tradables and sample composition. Drawing on Antràs [2004], we run the regression of value added per hours worked on the real wage in sector j by adopting cointegration methods, see Online Appendix L.3 which details the empirical strategy. In line with our panel data estimates, we choose $\sigma^H = 0.64$ and $\sigma^N = 0.80$ (see columns 14 and 15 of Table 1).

Given the set of elasticities above, the remaining parameters are set so as to maintain the steady-state of the CES economy equal to the normalization point. Therefore, we calibrate the model with CES production functions so that fifteen parameters φ, ι, φ^H, ι^H, ϑ, δ_K, G, G^N, G^H, N_0, K_0, Z^H, Z^N, γ^H, γ^N are endogenously set to target $1-\bar{\alpha}_C$, $1-\bar{\alpha}_J$, $\bar{\alpha}^H$, $\bar{\alpha}^H_J$, $\bar{L}^N/\bar{L}$, $\bar{\omega}_J$, $\bar{\omega}_G$, $\bar{\omega}_{G^N}$, $\bar{\omega}_{G^H}$, $\bar{\upsilon}_{NX}$, $\bar{K}$, $\bar{y}^H$, $\bar{y}^N$, $\bar{s}^H_L = \theta^H$, $\bar{s}^N_L = \theta^N$, respectively, where a bar indicates that the ratio is obtained from the Cobb-Douglas economy. In addition, four parameters, including ξ^H_1, ξ^N_1, χ^H_1, χ^N_1, are endogenously set to target R/P^H, R/P^N, Y^H, Y^N.

4.2 Government Spending Shock and Technology: Calibration

In this subsection, we detail how we calibrate the endogenous responses of $G(t)$, $u^{K,j}(t)$, $u^{Z,j}(t)$ to the exogenous fiscal shock, and factor-biased technological adjustment.

Endogenous response of $G(t)$ to exogenous fiscal shock. In order to capture the endogenous response of government consumption to the exogenous fiscal shock we have identified, we assume that the dynamic adjustment of $G(t)$ is governed by eq. (31). In the quantitative analysis, we set $g = 0.01$ so that $G(t)$ increases by 1 ppt of initial GDP. To calibrate ξ and χ which parametrize the shape of the dynamic adjustment of $G(t)$ along with its persistence, we proceed as follows. Because $G(t)$ peaks after one year, we have $\dot{G}(1)/Y = -\left[\xi e^{-\xi} - \chi\,(1-g)\,e^{-\chi}\right] = 0$. In addition, the cumulative response of $G(t)$ over a ten-year horizon is $\int_0^9 \left[dG(\tau)/Y\right] e^{-r^\star \tau} d\tau = g'$ with $g' = 6.56$ ppt of GDP. We choose $\xi = 0.430$ and $\chi = 0.439$. Left-multiplying eq. (31) by ω_{G^g} (with $g = F, H, N$) gives the dynamic adjustment of sectoral government consumption to the exogenous fiscal shock:

$$\omega_{G^g}\left(dG(t)/Y\right) = \omega_{G^g}\left[e^{-\xi t} - (1-g)\,e^{-\chi t}\right], \tag{33}$$

where ω_{G^g} is the share of government final consumption expenditure in good g. To determine (33), we assume that the parameters that govern the persistence and shape of the response of sectoral government consumption are identical across sectors, while the sectoral intensity of the government spending shock is constant over time and thus collapses to ω_{G^j}.[11]

Capital utilization adjustment costs. Log-linearizing (19f) shows that it is profitable to increase $u^{K,j}(t)$ when the real capital rental rate goes up

$$\hat{u}^{K,j}(t) = \frac{\xi_1^j}{\xi_2^j}\left(\hat{\tilde{R}}^j(t) - \hat{P}^j(t)\right). \tag{34}$$

The parameter ξ_2^j determines the magnitude of the adjustment in $u^{K,j}(t)$. We set $\xi_2^H = 0.27$ and $\xi_2^N = 0.03$ so as to account for empirical responses of $u^{K,j}(t)$ conditional on the government spending shock.[12]

Technology utilization adjustment costs. Log-linearizing (19g) shows that the intensity in the use of existing technologies is pro-cyclical:

$$\hat{u}^{Z,j}(t) = \frac{\chi_1^j}{\chi_2^j}\hat{Y}^j(t). \tag{35}$$

Intuitively, since $Y^j(t) = \frac{W^j(t)L^j(t)+R(t)\tilde{K}^j(t)}{P^j(t)}$, it is profitable to increase the technology rate when the real cost of producing goes up. The parameter χ_2^j determines the magnitude of the response of the technology utilization rate $u^{Z,j}(t)$. We choose 0.8 and 2.85 for χ_2^H and χ_2^N, respectively, in order to reproduce the empirical responses of the capital-utilization-adjusted TFP, $Z^j(t)$, see Fig. 2(a) and Fig. 2(d). The model produces a cumulative change in $Z^H(t)$ and $Z^N(t)$ over a 10-year horizon of 5.84% and 2.12%, respectively, close to what we estimate in the data, i.e., 6.12% and 2.15%.

[11] Assuming that the intensity of the non-traded sector in the government spending shock collapses to the non-tradable content of $G(t)$ is in line with the evidence documented in Online Appendix F. In particular, Fig. 7 shows that a shock to government consumption by 1% of GDP is associated with a rise in G^N by 0.8% of GDP on impact.

[12] For reasons of space, the responses of $u^{K,j}$ are relegated to the Online Appendix E, see Fig. 6.

Building intuition on the determinants of the technology channel. Using the fact that $Y^j(t) - \chi_1^j = dY^j(t)$ (since $\chi_1^j = Y^j$) and making use of the expression for the capital income share which implies that $Y^j(t)$ co-moves with $\frac{R(t)}{P^j(t)} \frac{\tilde{K}^j(t)}{1-s_L^j(t)}$, we derive a simple expression which links the change in the technology utilization rate and the variation in the real cost of capital services (instead of the real cost of producing):

$$du^{Z,j}(t) = \frac{1}{\chi_2} d\left(\frac{R(t)}{P^j(t)} u^{K,j}(t) \alpha_K^j(t) \frac{K(t)}{1 - s_L^j(t)} \right), \tag{36}$$

where $\alpha_K^j(t) \equiv \frac{K^j(t)}{K(t)}$. When terms of trade are exogenous (i.e., $P^H(t) = P^H = 1$), production functions are Cobb-Douglas (i.e., $s_L^j(t) = \theta^j$ is fixed), and the capital utilization rate is shut down (i.e., $u^{K,j}(t) = 1$), the change in the technology utilization rate in the traded sector on impact, i.e., $du^{Z,H}(0)$, depends only on the response of the capital rental rate $dR(0)$ and the reallocation of capital across sectors captured by $d\alpha_K^H(0)$ because the aggregate capital stock is predetermined (i.e., $K(0) = K_0$). A one-sector model ignores capital reallocation so that α_K^j collapses to one. Under this assumption, since a government spending shock leads households to supply more labor which lowers the capital-labor ratio, the capital cost increases and thus firms improve technology. In a two-sector model, technology decisions are influenced by the reallocation of factors. A rise in $G(t)$ which is biased toward non-traded goods produces a dramatic shift of capital away from traded industries (i.e., $\alpha_K^H(t)$ falls). Because workers do not experience labor mobility costs (i.e., $\epsilon \to \infty$), the traded sector also experiences a dramatic labor outflow that increases its capital-labor ratio k^H and thus lowers the capital rental rate $R(t)$. Since the capital cost $R(t)K^H(t)$ declines, traded firms reduce $u^{Z,H}(t)$. By mitigating the shift of labor toward non-traded industries, labor mobility costs (i.e., $0 < \epsilon < \infty$) lead $k^H(t)$ to fall. Because $k^H(t)$ declines, $R(t)$ increases. However, the traded sector experiences a dramatic capital outflow that lowers $R(t)K^H(t)$ because capital can move freely across sectors.

When we assume that home- and foreign-produced traded goods are imperfect substitutes (i.e., $0 < \rho < \infty$ and $0 < \rho_J < \infty$), the terms of trade $P^H(t)$ appreciate as households are less inclined to substitute imported for home-produced traded goods. While the appreciation in $P^H(t)$ mitigates the shift of capital toward non-traded industries, numerical results in the next subsection show that the real cost of capital services increases only once we let tangible assets to be used more intensively in both sectors. FBTC also matters. When we allow for CES production functions, technological change biased toward capital in the traded sector amplifies the rise in the capital cost which leads traded firms to improve technology by a magnitude that squares well with our evidence.

Factor-augmenting efficiency. To set the adjustment of factor-augmenting efficiency, we first recover their dynamics in the data. Using the fact that factor-augmenting productivity has a symmetric time-varying component denoted by $u^{Z,j}(t)$ such that $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$ and $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$, log-linearizing the demand for labor relative to the demand for capital (22) and using the log-linearized version of the technology frontier (26), we can solve for deviations of $A^j(t)$ and $B^j(t)$ relative to their initial values:

$$\hat{A}^j(t) = -\left(1 - s_L^j\right)\left[\left(\frac{\sigma^j}{1-\sigma^j}\right)\hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t)\right], \tag{37a}$$

$$\hat{B}^j(t) = s_L^j\left[\left(\frac{\sigma^j}{1-\sigma^j}\right)\hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t)\right]. \tag{37b}$$

Plugging estimated values for σ^j and empirically estimated responses for $s_L^j(t)$, $k^j(t)$, $u^{K,j}(t)$ into above equations enables us to recover the dynamics for $A^j(t)$ and $B^j(t)$ consistent with the demand for factors of production (22) and the technology frontier (26). Then we choose values for exogenous parameters x^j (for $x = a, b$), ξ_X^j and χ_X^j (for $X = A, B$) of the continuous time paths (32) within sector $j = H, N$, which are consistent with the estimated paths (37a)-(37b) for $A^j(t)$ and $B^j(t)$.

4.3 Drivers of the Technology Channel

As suggested by our empirical evidence and corroborated by our numerical results in the next subsection, the technology channel of fiscal transmission plays a key role in determining the size of aggregate and sectoral fiscal multipliers. In this subsection, we highlight the factors driving the technology decisions following a government spending shock. We show that both the two-sector dimension and the open economy aspect of our model matter in determining the adjustment in the technology utilization rate.

Our baseline model includes three sets of elements. The first set is related to the biasedness of the demand shock toward non-traded goods. The second set of elements is related to barriers to factors' mobility which include labor mobility costs and imperfect substitutability between home- and foreign-produced traded goods. The third set of elements is related to sector-specific capital utilization rates together with factor-biased technological change. To understand (and quantify) the role of each element, we first consider the simplest model and add one ingredient at a time. Table 2 reports the cumulative change of selected variables, including value added, $\tilde{Y}^j(t)$, the value added share of non-tradables, $\nu^{Y,N}(t)$, the technology utilization rate $u^{Z,j}(t)$, and the current account $CA(t)$, over a six-year horizon.

Perfect mobility of labor across sectors. In columns 1-4 of Table 2, we consider a model with perfect mobility of labor (PML) across sectors (i.e., we let $\epsilon \to \infty$), assume that home- and foreign-produced traded goods are perfect substitutes (i.e., we let ρ, ρ_J and ϕ_X tend toward infinity) so that terms of trade (TOT) are exogenous and fixed, and allow for Cobb-Douglas (CD) production functions (so that FBTC is shut down). In column 1, we abstract from capital adjustment costs (i.e., we set $\kappa = 0$) and shut down the technology channel. As shown in panel A, a rise in $G(t)$ has a negative impact on traded value added. Intuitively, productive resources shift toward the non-traded sector because this sector is highly intensive in the government spending shock. Financial openness and the tradability of goods further bias the demand shock toward non-tradables. Intuitively, when home- and foreign-produced traded goods are perfect substitutes, it is optimal for the open economy to import traded goods and reallocate capital and labor to produce additional units of non-traded goods. The open economy thus runs a large current account deficit (see panel C). Because workers are not subject to switching costs, the labor inflow experienced by the non-traded sector is such that it keeps non-traded prices and thus factor prices unchanged.[13]

When we allow for capital adjustment costs (CAC), as considered in column 2, the (slight) appreciation in the relative price of non-tradables produces stronger incentives to shift productive resources toward the non-traded sector which in turn amplifies the expansionary effect of the government spending shock on the non-traded sector at the expense of traded industries. As shown in column 3, when we allow for an endogenous intensity in the use of existing technologies, the dramatic decline in traded value added produces a fall in $u^{Z,H}(t)$. As highlighted in the previous section, the fall in $\tilde{Y}^H$ mimics the decline in the real capital cost which leads traded firms to reduce their efforts to improve production efficiency. As shown in column 4, the decline in the technology utilization rate $u^{Z,H}$ is still large but mitigated once we allow for endogenous capital utilization. Intuitively, in face of a higher real capital rental rate (driven by the rise in $u^{Z,N}(t)$), non-traded firms increase $u^{K,N}(t)$, while the other way around is true for traded firms which lower $u^{K,H}(t)$. Because non-traded firms use more intensively the physical capital stock while traded firms reduce $u^{K,H}(t)$, less capital shifts away from traded industries which softens the fall in $\tilde{Y}^H(t)$ and thereby in $u^{Z,H}(t)$.

Imperfect mobility of labor across sectors and technology. In column 5, we consider the same model as in column 4 except that we now assume that traded and non-traded hours worked are imperfect substitutes. When workers experience costs of switching sectors, less labor shifts away from traded industries. Therefore both $\tilde{Y}^H(t)$ and the use of existing technologies $u^{Z,H}(t)$ decline by a lower magnitude.

Imperfect mobility of labor across sectors, endogenous terms of trade and technology. In columns 6-9, we assume that home- and foreign-produced traded goods are imperfect substitutes. This assumption leads to a significant reduction in the magnitude of the current account deficit (see panel C) because households are reluctant to substitute imported for home-produced traded goods. Higher demand for domestic goods appreciates $P^H(t)$ which increases the return on factors. The real capital rental rate increases in the traded sector which makes it more profitable to raise $u^{K,H}(t)$. The combined effect of labor mobility costs, imperfect substitutability between home- and foreign-produced traded goods

[13] Because factor prices remain fixed as the result of the immediate reallocation of inputs, as pointed out by Baqaee [2018] and Bouakez et al. [2022a], the absence of frictions in a multi-sector model implies that the allocation of government spending across sectors does not impact the aggregate outcome, i.e., the magnitude of aggregate fiscal multipliers. This is true in a closed economy but not in an open economy because a demand shock biased toward non-traded goods produces a current account deficit which impinges on the equilibrium value of the marginal utility of wealth and thus on aggregate labor supply.

Table 2: Cumulative Effects on Technology of a Government Spending Shock

	CD: PML				CD: IML	CD: IML & TOT		CES: FBTC & IML & TOT	
	No CAC	CAC	$u^{Z,j}$	$u^{K,j}$	exo TOT	endo TOT	No $u^{K,j}$	No $u^{K,j}$	Bench
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A.Sectoral Value Added									
T VA, $d\tilde{Y}^H(t)$	-3.52	-3.45	-8.26	-4.86	-1.40	1.10	-0.84	1.61	3.12
NT VA, $d\tilde{Y}^N(t)$	4.04	4.15	5.75	7.89	5.35	6.08	3.85	3.54	4.24
VA share of NT, $d\nu^{Y,N}(t)$	3.73	3.75	8.06	6.94	2.97	1.48	1.96	0.24	-0.41
B.Technology									
T technology, $du^{Z,H}(t)$	0.00	0.00	-12.14	-7.55	-2.57	1.50	-1.27	2.26	4.35
NT technology, $du^{Z,N}(t)$	0.00	0.00	2.71	3.60	2.17	2.79	1.81	1.62	1.97
C.Current Account									
Current Account, $dCA(t)$	-4.80	-4.80	-5.82	-4.18	-1.96	-0.19	-0.42	-0.09	-0.02

Notes: This table shows cumulative effects over a six-year horizon of a 1% temporary increase in government consumption in the baseline model (column 9) and in restricted versions of the model (columns 1-8). Impact and cumulative effects for additional variables are relegated to Online Appendix N for reasons of space. 'T' refers to traded industries while 'NT' refers to non-tradables. Panel A shows the cumulative effects for traded and non-traded value added, panel B displays the cumulative effects for traded and non-traded technology, panel C reports the cumulative change in the current account (in percentage point of GDP). Across all scenarios, we consider a 1% increase in government consumption on impact which gives rise to a (present discounted value of) cumulative change of $G(t)$ by 5.46 ppt of GDP over a six-year horizon. In columns 1-5, we consider a small open economy model where home-produced and foreign-produced traded goods are perfect substitutes so that terms of trade are exogenous. In columns 1-4 we consider four variants of a model with perfect mobility of labor while in column 5 we assume imperfect mobility of labor across sectors. In column 1, we abstract from capital installation costs and shut down the technology channel. In column 2, we allow for capital installation costs and switch off the technology channel. In column 3, we allow for capital installation costs and endogenous technology utilization rate. In column 4, we allow for capital installation costs, endogenous technology and capital utilization rates. In column 5, we assume imperfect mobility of labor across sectors and allow for the technology channel. In columns 6-7, we allow for capital installation costs, imperfect substitutability between home- and foreign- produced traded goods and assume Cobb-Douglas production functions. In column 6, we allow for endogenous technology and capital utilization rates while in column 7, we shut down capital utilization rates. In column 8, we consider the baseline model with CES production functions, factor-biased technological change, and endogenous technology utilization rates except that we shut down the capital utilization rate.

together with higher $u^{K,H}(t)$ pushes $\tilde{Y}^H(t)$ up and encourages traded firms to increase production efficiency (see panel B).

Endogenous capital utilization is key to giving rise to technology improvement in the traded sector. As shown in column 7, when we consider the same setup as in column 6 whilst shutting down $u^{K,j}(t)$ (i.e., $\chi_2^j \to \infty$), the model fails to account for the rise in $u^{Z,H}(t)$ because the capital outflow experienced by the traded sector is too large.

Imperfect mobility of labor across sectors, endogenous terms of trade, FBTC and technology. In column 8, we keep the same model as in column 7 (i.e., $\chi_2^j \to \infty$), except that we allow for CES production functions and FBTC. These two elements result in changes in the intensity of production technology in capital and labor in both sectors. Because technological change is biased toward capital in the traded sector and biased toward labor in the non-traded sector, traded industries experience a capital inflow instead of a capital outflow. Since capital is not subject to mobility costs across sectors, the capital inflow more than offsets the greater labor outflow experienced by traded industries, thus raising $\tilde{Y}^H(t)$ and encouraging traded firms to increase $u^{Z,H}(t)$.

Taking stock. In column 9 of Table 2, we consider our baseline model. As displayed by panel B, allowing for endogenous capital utilization (column 6) or FBTC (column 8) in addition to labor mobility costs and endogenous terms of trade are necessary elements to give rise to technology improvements in the traded sector. However, our empirical evidence over a six-year horizon shows that overall efficiency gains are 2.8 larger in traded than in non-traded industries. As shown in column 9, it is only once we allow for both endogenous

$u^{K,j}(t)$ and FBTC in addition to $\epsilon < \infty$, $\rho < \infty$, $\rho_J < \infty$, that the model can account for the concentration of technology improvements in the traded (relative to the non-traded) sector we estimate.

4.4 Government Spending Shock and Technology: Model Performance

Our objective is now to isolate quantitatively the role of the technology channel in determining the size of fiscal multipliers in an open economy. In our baseline calibration, we assume that capital and technology utilization rates (i.e., $u^{K,j}(t)$ and $u^{Z,j}(t)$) respond endogenously to the government spending shock, and allow for time-varying FBTC in sector j driven by the dynamic adjustment of labor- and capital-augmenting efficiency, while sectoral goods are produced from CES production functions. To gauge the quantitative implications of technology for fiscal transmission, we contrast our results with those obtained in a restricted model with Cobb-Douglas production functions (i.e., $\sigma^j = 1$) where we shut down the endogenous response of capital and technology utilization by letting ξ_2^j and χ_2^j tend towards infinity and impose $\xi_A^j = \xi_B^j = \chi_A^j = \chi_B^j = 0$ so that $u^{K,j}(t) = u^{Z,j}(t) = A^j(t) = B^j(t) = 1$.[14]

In Table 3, we report the simulated impact (i.e., at $t = 0$) and six-year cumulative (i.e., at $t = 0, ..., 5$) effects following an exogenous temporary increase in $G(t)$ by 1 ppt of GDP. Cumulative effects are expressed in present discounted value terms. The six-year horizon fiscal multiplier is obtained by dividing the cumulative change in value added (or labor) by the cumulative change in $G(t)$.

While columns 1 and 4 show impact and cumulative responses from local projection for comparison purposes, columns 2 and 5 show results for the baseline model. We contrast the benchmark results with those shown in columns 3 and 6 for impact and cumulative effects, respectively, which are obtained in the restricted model where technology is shut down. While in Table 3, we focus on value added and hours worked, numerical results for sectoral TFPs, utilization-adjusted-TFPs, and sectoral LISs are relegated to Online Appendix M for reasons of space.

Adjustment in $G(t)$. As can be seen in the first row of panel A of Table 3, the baseline (and the restricted) model generates a cumulative change in $G(t)$ of 5.46 ppt of GDP (see columns 5-6), close to our estimation of 5.51 ppt (see column 4). As shown in Fig. 3(a), the endogenous response of $G(t)/Y$ to an exogenous fiscal shock that we generate theoretically (black line with squares) by specifying the law of motion (31) reproduces well the dynamic adjustment we estimate empirically (blue line).

Restricted model. Results for the restricted model where technology is shut down are

[14] While in the main text we assume MaCurdy [1981] preferences, perfect mobility of capital across sectors and flexible prices, in Online Appendices W and X we allow for non-separability in utility between consumption and leisure in the lines of Shimer [2009], imperfect mobility of capital, and nominal price rigidities on non-traded goods. We contrast the predictions of the baseline model with those of its variants in Online Appendix Y.

reported in columns 3 and 6 of Table 3. Because the capital and technology utilization rates remain fixed, sectoral TFPs are unchanged, see the first two columns of Fig. 2. Because the elasticity of value added w.r.t. inputs is fixed (i.e., $s_L^j = \theta^j$) the labor income shares are constant, see the last column of Fig. 2.

We start with the aggregate effects. By producing a negative wealth effect, a balanced-budget government spending shock leads agents to supply more labor, which in turn increases real GDP. As shown in panel A of Table 3, a rise in $G(t)$ by 1% of GDP generates an increase in $L(t)$ by 0.63% and a rise in real GDP by 0.42% on impact, the latter value being almost three times smaller what we estimate empirically (i.e., 1.18%, see column 1).

Panel B of Table 3 shows that non-traded and traded hours increase by 0.54 ppt and 0.09 ppt of total hours worked, respectively. Formally, the rise in non-traded hours worked, i.e., $\alpha_L^N \hat{L}^N(t) = \alpha_L^N \hat{L}(t) + d\nu^{L,N}(t)$ (see section 2.1), is driven by higher labor supply $\hat{L}(t)$ and the reallocation of hours across sectors, as captured by $d\nu^{L,N}(t)$. In the model and the data, the labor multiplier of non-tradables is larger than that of tradables because the non-traded sector accounts for almost two-third (i.e., $\alpha_L^N = 63\%$) of total labor and also experiences a labor inflow (i.e., $d\nu^{L,N}(t) > 0$) as a result of the biasedness of the government spending shock towards non-tradables which is reinforced by the current account deficit.

Although the demand boom for non-traded goods is amplified by financial openness as discussed in the previous subsection, the restricted model substantially understates the cumulative change in $L^N(t)$ (2.87 ppt of total hours vs. 5.64 ppt in the data, see the first row of panel B). The reason is twofold. By shutting down technology, the restricted model produces a labor multiplier over a six-year horizon of 0.6 (= 3.34/5.46) only which is two times smaller what we estimate because the rise in $W(t)$ is not large enough to encourage households to supply more labor. The restricted model also substantially understates the reallocation of labor toward the non-traded sector (0.71 ppt against 1.68 ppt of total hours worked, see the third row of panel B).

Turning to value added, the model shutting down technology and capital utilization rates generates a real GDP multiplier of 0.4 over a six-year horizon (= 2.14/5.46), which is more than three times smaller its estimated value 1.4 (= 7.74/5.51). As a matter of consequence, the restricted model predicts a value added multiplier of non-tradables which is twice smaller what we estimate empirically at any horizon. By overestimating the shift of capital toward the non-traded sector, the restricted model also generates a decline in traded value added both on impact and over a six-year horizon (see the first row of panel C) which is at odds with our empirical findings.

Baseline model. The performance of the model increases when the capital and the technology utilization rates are allowed to respond endogenously to the government spending shock and firms bias technological change toward production factors. Quantitative

Table 3: Impact and Cumulative Effects of an Increase in $G(t)$ by 1% of GDP

	LP $t=0$	Impact Responses		LP $t=0...5$	Cumulative Responses	
	Data	CES-TECH	CD	Data	CES-TECH	CD
	(1)	(2)	(3)	(4)	(5)	(6)
A.Aggregate Multipliers						
Gov. spending, $dG(t)$	1.00	1.00	1.00	5.51	5.46	5.46
Total hours worked, $dL(t)$	0.91	0.97	0.63	6.37	5.61	3.34
Real GDP, $dY_R(t)$	1.18	1.07	0.42	7.74	7.36	2.14
B.Sectoral Labor						
Traded labor, $dL^H(t)$	0.21	0.18	0.09	0.73	0.69	0.47
Non-traded labor, $dL^N(t)$	0.71	0.78	0.54	5.64	4.92	2.87
Labor share of non-tradables, $d\nu^{L,N}(t)$	0.13	0.16	0.14	1.68	1.26	0.71
Decomposition						
Caused by $d\omega^{Y,N}(t)$		0.14	0.14		0.31	0.71
Caused by cap. deep. differential		0.01	0.00		-0.24	0.00
Caused by **FBTC** differential		0.01	0.00		1.20	0.00
C.Sectoral Value Added						
Traded VA, $dY^H(t)$	0.33	0.20	-0.04	2.86	3.12	-0.28
Non-traded VA, $dY^N(t)$	0.85	0.88	0.46	4.88	4.24	2.41
Non-traded VA share, $d\nu^{Y,N}(t)$	0.16	0.20	0.20	-0.01	-0.41	1.07
Decomposition						
Caused by TFP differential		0.03	0.00		-0.47	0.00
Caused by labor reallocation		0.16	0.14		1.30	0.72
Caused by capital reallocation		0.01	0.06		-1.23	0.35

Notes: Impact ($t=0$) and cumulative ($t=0...5$) effects of an exogenous temporary increase in government consumption by 1% of GDP. Panels A,B,C show the deviation in percentage relative to the steady-state for aggregate and sectoral variables. Sectoral value added and the value added share are both expressed as a percentage of initial GDP, while sectoral labor and the labor share are both expressed as a percentage of initial total hours worked. Columns 2 and 5, labelled 'CES-TECH', show predictions of the baseline model while columns 3 and 6, labelled 'CD', shows predictions of the restricted version of the model. In the restricted model, we impose $\sigma^j = 1$ so that production functions are Cobb-Douglas, let ξ_2^j, χ_2^j tend toward infinity so that the capital and technology utilization rate collapses to one, and set ξ_A^j, χ_A^j, ξ_B^j, χ_B^j to zero so that the labor- and capital-augmenting technological rate remain fixed. In columns 1 and 4, we report point estimates from local projections. Since there is a (slight) discrepancy between the response of aggregate real GDP (total hours worked) and the sum of the responses of traded and non-traded value added (hours worked), columns 1 and 4 report the sum of responses of Y^H and Y^N (L^H and L^N, resp.) to ensure consistency between aggregate and sectoral responses.

results are shown in column 2 for impact effects and in column 5 for the cumulative effects.

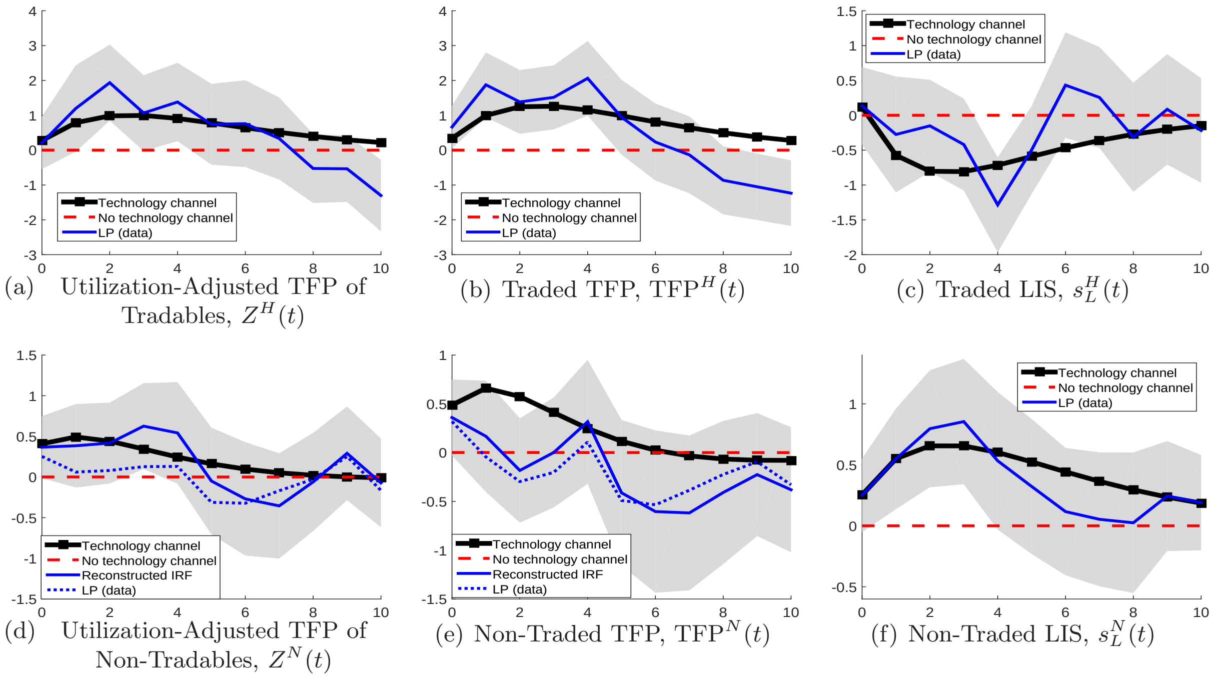
As can be seen in panel A of Table 3, the baseline model does a good job in reproducing the aggregate effects of a shock to $G(t)$. More specifically, along the transitional path, the baseline model produces a cumulative change in $L(t)$ and in real GDP of 5.61% and 7.36% (vs. 6.37% and 7.74% in the data), respectively, thus generating spending multipliers of 1.03 for labor and 1.35 for real GDP on average over the first six years close to the multipliers that we estimate empirically (i.e., 1.15 and 1.40). Three factors give rise to government spending multipliers above one like in the data. First, in the face of a higher real capital rental rate, both sectors, especially traded firms, increase the capital utilization rate, $u^{K,j}(t)$. By raising the demand for labor and the use of the capital input, higher capital utilization amplifies the rise in $L(t)$ and $\tilde{Y}_R(t)$. Second, because the rise in $G(t)$ puts upward pressure on the unit cost for producing, it is optimal to increase the technology utilization rate in both sectors. Efficiency gains increase real GDP directly and also through higher labor supply by putting upward pressure on $\tilde{W}(t)$. Third, as discussed below, the rise in $L(t)$ is amplified because the production technology becomes more labor intensive in the non-traded sector.

Panel B of Table 3 reveals that the baseline model reproduces well the cumulative rise in traded and non-traded hours worked which amounts to 0.69 ppt (vs. 0.73 ppt in the data) and 4.92 ppt (vs. 5.64 ppt in the data) of total hours worked, respectively. The reason is twofold. First, the model allowing for technological change can account for the increase in $L(t)$, each sector receiving a share (which collapses to their labor compensation share) of $\hat{L}(t)$. Second, because non-traded firms bias technological change toward labor and traded firms bias technological change toward capital, technology further tilts the demand of labor toward non-tradables which amplifies the shift of labor away from traded industries, as detailed below.

As shown in the third row of panel B, we have $d\nu^{L,N}(t) > 0$. The non-traded sector thus experiences a labor inflow which in turn increases disproportionately $L^N(t)$. The last three rows of panel B of Table 3 breaks down the change in $\nu^{L,N}(t)$ into three components, see Online Appendix C for a formal derivation. Focusing on cumulative changes, the decomposition shown in column 6 of panel B for the restricted model reveals that the rise in $\nu^{L,N}(t)$ by 0.71 ppt of total hours worked is only driven by the biasedness of the demand shock toward non-tradables. When we turn to the decomposition of $d\nu^{L,N}(t)$ for the baseline model shown in column 5, our findings show that the bulk of $d\nu^{L,N}(t)$ is driven by the FBTC differential between non-tradables and tradables. The combined effect of technological change biased toward labor in the non-traded sector and biased toward capital in the traded sector generates on its own a cumulative reallocation of labor of 1.2 ppt of total hours worked toward the non-traded sector. The biasedness of the demand shock toward non-tradables further increases $\nu^{L,N}(t)$ by 0.31 ppt of total hours worked. Conversely, capital deepening in the traded sector increases labor demand in this sector, which lowers $\nu^{L,N}(t)$ by -0.24 ppt of total hours worked. The sum of these three effects results in a cumulative increase in the labor share of non-tradables by 1.26 ppt of total hours worked (1.68 ppt in the data). Importantly, FBTC contributes 69% on its own to the change in $\nu^{L,N}(t)$ over a six-year horizon.

We turn to the adjustment in sectoral value added at constant prices, shown in panel C of Table 3. The baseline model generates a multiplier of 0.57 (= 3.12/5.46) for tradables and 0.78 (= 4.24/5.46) for non-tradables, respectively while we estimate empirically a government spending multiplier of 0.52 (= 2.86/5.51) for tradables and 0.89 (= 4.88/5.51) for non-tradables over a six-year horizon. The performance of the baseline model in reproducing the size of the government spending multiplier, i.e., $\hat{Y}_R^G(t)$, along with its distribution across sectors, as captured by $d\nu^{Y,N}(t)$, lies in the technology channel which varies across sectors as discussed below.

As shown in the third row of column 4, $\nu^{Y,N}(t)$ remains almost unchanged and thus the cumulative change in real GDP is distributed across sectors in accordance with their value added share. The last three rows of panel C of Table 3 provide a quantitative decomposition of the cumulative change in the value added share of non-tradables, see Online Appendix



(a) Utilization-Adjusted TFP of Tradables, $Z^H(t)$ (b) Traded TFP, $\text{TFP}^H(t)$ (c) Traded LIS, $s_L^H(t)$

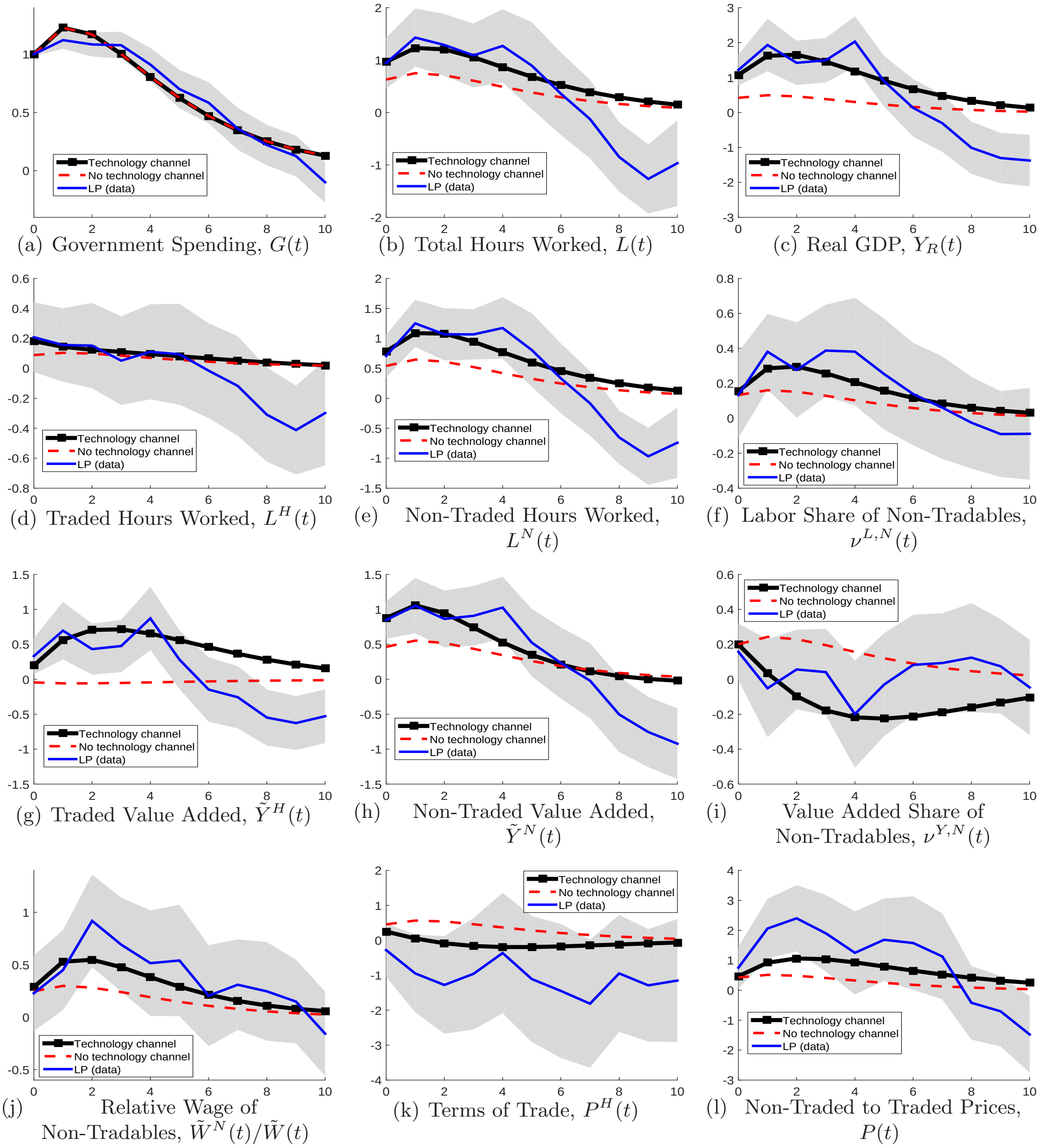
(d) Utilization-Adjusted TFP of Non-Tradables, $Z^N(t)$ (e) Non-Traded TFP, $\text{TFP}^N(t)$ (f) Non-Traded LIS, $s_L^N(t)$

Figure 2: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Technology Effects. Notes: 'LP (data)' refers to the solid blue line which displays point estimate from local projections with shaded areas indicating 90% confidence bounds except for Fig. 2(d) and 2(e) where we have reconstructed empirical responses of $\text{TFP}^N(t)$ and $Z^N(t)$ because we found a substantial discrepancy between the empirically estimated (dotted blue line) and reconstructed (solid blue line) responses. In the latter case, we use empirical responses of aggregate and traded TFP, which are both statistically significant, to reconstruct the dynamic responses of $\hat{\text{TFP}}^N(t)$ by using the fact that aggregate TFP growth is equal to the sum of traded and non-traded TFP growth weighted by the value added share of the corresponding sector, and once we have recovered $\hat{\text{TFP}}^N(t)$, we recover $\hat{Z}^N(t)$ by subtracting the response of $\hat{u}^{K,N}(t)$ weighted by $1 - s_L^N$, see eq. (8). The thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC, while the dashed red line shows predictions of a model with Cobb-Douglas production functions and abstracting from capital and technology utilization.

C for a formal derivation. As can be seen in column 6, when technological change is shut down, both labor and capital shift toward the non-traded sector, increasing $\nu^{Y,N}(t)$ by 1.07 ppt of GDP which in turn gives rise to a fall in traded value added, in contradiction with our evidence. In contrast, in the baseline scenario displayed by column 5, the labor inflow amplified by technological change biased toward labor in the non-traded sector is offset by the capital outflow caused by technological change biased toward capital in the traded sector. Because TFP gains are concentrated in the traded sector, $\nu^{Y,N}(t)$ slightly declines by 0.41 ppt, which in turn prevents traded value added from decreasing, in line with our evidence.

The ability of the model to account for the muted response of $\nu^{Y,N}(t)$ lies in the technology adjustment cost which is lower in the traded than in the non-traded sector. If instead we had set $\chi_2^H = \chi_2^N$ (and $\xi_2^H = \xi_2^N$), the cumulative change in $u^{Z,N}(t)$ would have been twice larger than that of $u^{Z,H}(t)$, thus producing a rise in $\nu^{Y,N}(t)$ by 2.7 ppt of GDP, in contradiction with our evidence.

Dynamics: Empirical vs. theoretical responses. In Fig. 2 and Fig. 3, we contrast theoretical predictions (displayed by solid black lines with squares) with empirical responses from local projections (displayed by solid blue lines). The shaded area indicates the 90%



(a) Government Spending, $G(t)$ (b) Total Hours Worked, $L(t)$ (c) Real GDP, $Y_R(t)$

(d) Traded Hours Worked, $L^H(t)$ (e) Non-Traded Hours Worked, $L^N(t)$ (f) Labor Share of Non-Tradables, $\nu^{L,N}(t)$

(g) Traded Value Added, $\tilde{Y}^H(t)$ (h) Non-Traded Value Added, $\tilde{Y}^N(t)$ (i) Value Added Share of Non-Tradables, $\nu^{Y,N}(t)$

(j) Relative Wage of Non-Tradables, $\tilde{W}^N(t)/\tilde{W}(t)$ (k) Terms of Trade, $P^H(t)$ (l) Non-Traded to Traded Prices, $P(t)$

Figure 3: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Hours and Value Added Effects. Notes: 'LP (data)' refers to the solid blue line which displays point estimate from local projections with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC, while the dashed red line shows predictions of a model with Cobb-Douglas production functions and abstracting from capital and technology utilization. The relative wage of non-tradables is constructed as the ratio of the non-traded wage to the aggregate wage, the relative price of non-tradables is computed as the ratio of the non-traded value added deflator to the traded value added deflator, and the terms of trade are constructed as the ratio of the traded value added deflator of the home country i to the geometric average of the traded value added deflator of the seventeen trade partners of the corresponding country i, the weight being equal to the share of imports from the trade partner k (averaged over 1970-2015).

confidence bounds. For comparison purposes and to assess the role of the technology channel in driving the effects, the dashed red lines show the predictions of the restricted model where technology is shutdown.

We start with the adjustment of technology displayed by Fig. 2. Because higher $G(t)$ increases the demand for traded and non-traded goods, both sectors find it profitable to raise their efficiency in the use of inputs to meet higher demand for sectoral goods. While the demand shock is biased toward non-traded goods, Fig. 2(a) and Fig. 2(d) show that technology improvements along the transitional path (i.e., $\hat{Z}^j(t) > 0$) are much more pronounced in the traded than in the non-traded sector because the former sector experiences a lower adjustment cost of technology. Since the demand for capital rises in both sectors, which puts upward pressure on the real capital rental rate, it is profitable to use the stock of capital more intensively (i.e., $u^{K,j}(t)$ rises) which results in higher sectoral TFPs as displayed by Fig. 2(b) and Fig. 2(e). Besides technology improvements, firms change the mix of labor- and capital-augmenting efficiency. Because traded firms bias technological change toward capital, the traded LIS falls below trend, as shown in Fig. 2(c). Conversely, non-traded firms bias technological change toward labor which increases the non-traded LIS, as displayed by Fig. 2(f).

As shown in Fig. 3, both the shift in the technology frontier and the change in the mix of labor- and capital-augmenting efficiency along the technology frontier increase the ability of the two-sector open economy model to account for the empirical evidence. As can be seen in Fig. 3(b) and Fig. 3(c), the model reproduces well the dynamics of $L(t)$ and real GDP once we allow for the technology channel. Intuitively, technology improvements and a higher labor intensity of production result in a higher wage rate which encourages agents to supply more labor. The combined effect of TFP gains and higher labor supply amplifies the increase in real GDP.

As displayed by Fig. 3(f), more labor shifts toward the non-traded sector in the baseline than in the restricted model because non-traded (traded) firms use labor (capital) more intensively. By further increasing labor supply and generating a labor inflow in the non-traded sector, the model can account for the dynamics of traded and non-traded hours worked (see Fig. 3(d) and 3(e)) we estimate empirically. As displayed by the dashed red line in Fig. 3(i), by producing a shift of capital toward non-traded industries the restricted model generates an increase in $\nu^{Y,N}(t)$, thus leading to a decline in $\tilde{Y}^H(t)$ in contradiction with our evidence, see Fig. 3(g). Conversely, by letting both capital and technology utilization rates increase endogenously, the baseline model can account for the hump-shaped dynamics of traded value added.

To gain a better understanding of fiscal transmission, in the fourth row of Fig. 3, we assess the ability of our model to account for the behavior of relative wages $\tilde{W}^j(t)/\tilde{W}(t)$

and relative prices. As shown in Fig. 3(j), non-traded firms increase wages above the aggregate wage to encourage workers (who experience mobility costs) to shift toward the non-traded sector. The fact that sectoral wages move away from the aggregate wage reveals the presence of switching costs. Because technological change is biased toward capital in the traded sector and biased toward labor in the non-traded sector, the adjustment in relative wages is more pronounced in the baseline model, in line with the evidence. As can be seen in Fig. 3(l), because the demand shock is biased toward non-tradables, the relative price of non-tradables appreciates. If home- and foreign-produced traded were perfect substitutes, the terms of trade would be unchanged. Instead, because households are reluctant to substitute imported for domestic goods and since $G^H(t)$ increases, the terms of trade slightly appreciate on impact and then depreciate as a result of technology improvements concentrated in the traded sector, see Fig. 3(k).

Distortionary labor and consumption taxation. While the dynamics of the baseline model shown in Fig. 2 and Fig. 3 can account for the empirical evidence until $t = 6$, it cannot account for the persistent decline in value added and in hours worked below trend after $t = 7$. As shown in Online Appendix V.7, the model can generate the adjustment in $L(t)$ and real GDP (and their sectoral counterparts) after $t = 7$ we estimate empirically once we relax the assumption of lump-sum taxes and allow for distortionary (labor and consumption) taxation.

Shocks to Sectoral government consumption. So far, we have investigated the dynamic effects of a shock to $G(t)$, assuming that 80% of $dG(t)/Y$ is spent on non-traded goods and 20% is spent on traded goods. Instead of highlighting the role of the technology channel in determining the allocation of spending multipliers across sectors, we could ask: which sector should be allocated government purchases so as to maximize the real GDP and total hours effects? Numerical results detailed in Online Appendix R.3 show that both labor (i.e., 0.62) and real GDP multipliers (i.e., 0.40) after a shock to G^N are larger than those after a shock to G^T. This conclusion is reversed (1.54 and 2.19 for labor and real GDP multipliers) once we allow for the technology channel because the cost of adjusting technology is smaller in the traded than in the non-traded sector.

5 Conclusion

This paper highlights the role of the technology channel in determining the size of the government spending multiplier and its sectoral decomposition. First, we find empirically that the government spending multiplier is higher than one and 39% of the cumulative change in real GDP over a six-year horizon is driven by TFP gains. Second, while the demand shock is biased toward non-traded goods, the concentration of technology improvements in traded industries implies that the government spending multiplier is distributed uniformly across sectors at any horizon. Conversely, 88% of the cumulative change in total hours worked is concentrated in the non-traded sector as non-traded (traded) production becomes more intensive in labor (capital).

To rationalize our evidence, we consider a semi-small open economy with tradables and non-tradables in the lines of Kehoe and Ruhl [2009]. Drawing on Bianchi et al. [2019], we assume that capital and technology can be used more intensively while the mix of labor- and capital-augmenting efficiency also vary, like Caselli and Coleman [2006]. We show that in a multi-sector open economy, the concentration of technology improvements in the traded sector following a rise in government consumption is conditional on barriers to factors' mobility and technological factors. Because the technology decision is pro-cyclical, the high intensity of non-traded industries in the government spending shock which is amplified by increased foreign borrowing produces a strong negative impact on traded overall efficiency. While both imperfect substitutability between traded and non-traded hours, and between home- and foreign-produced traded goods mitigate the shift of resources toward the non-traded sector, these two elements are not sufficient to generate a technology improvement in the traded sector. It is only once we allow capital to be used more intensively in both sectors and technological change to be biased toward labor in non-traded industries and to be biased toward capital in traded industries, that the model can account for the magnitude of the rise in overall efficiency in the traded relative to the non-traded sector.

To quantify the role of technology in determining the size of government spending multipliers and their distribution across sectors, we contrast the predictions of the baseline model with those of a restricted model where technological change is shut down and sectoral goods are produced from Cobb-Douglas production functions. Our quantitative analysis shows that a model abstracting from technological change cannot generate the rise in real GDP and in total hours worked that we estimate empirically, generates a disproportionate increase in non-traded relative to traded value added in contradiction with our evidence, understates the rise in non-traded hours worked, and cannot account for the dynamics of sectoral LISs. Conversely, the two-sector open economy model can account for the value added and labor effects of a government spending shock once we let the decision on technology improvement and factor intensity of production vary across sectors and time.

What are the determinants of technology adjustment costs at a sectoral level? While we document evidence which reveals that productivity gains (conditional on a government spending shock) are larger in industries which are relatively more intensive in tangible assets or less intensive in skilled labor or in intangible assets, we see these estimates as a first attempt to unravel the relationship between technology improvements and firm production structure. More work should be done to understand what is the optimal form of firm organization to adjust production along the business cycle in open economy.

References

Adão, Rodrigo, Martin Beraja, Nitya Pandalai-Nayar. (2022) Fast and Slow Technological Transitions. Mimeo.

Aghion, Philippe, Nicholas Bloom, Brian Lucking, Raffaella Sadun, John Van Reenen (2021) Turbulence, Firm Decentralization, and Growth in Bad Times. *American Economic Journal: Applied Economics*, 13(1), pp. 133-169.

Antolin-Diaz, Juan and Paolo Surico (2022) The Long-Run Effects of Government Spending (2022) Revised version of CEPR DP No. 17433.

Antràs, Pol (2004) Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *The B.E. Journal of Macroeconomics*, 4(1), pp. 1-36.

Backus, David K, Patrick J, Kehoe, Finn E. Kydland (1994) Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? *American Economic Review*, 84(1), pp. 84-103.

Baqaee, David (2018) Targeted Fiscal Policy. mimeo.

Baqaee, David, Emmanuel Farhi and Kunal Sangani (2021) The Supply-Side Effects of Monetary Policy. NBER WP 28345.

Basu, Susanto (1996) Procyclical Productivity: Increasing Returns or Cyclical Utilization? *The Quarterly Journal of Economics*, 111(3), pp. 719-751.

Bems, Rudolfs (2008) Aggregate Investment Expenditures on Tradable and Nontradable Goods. *Review of Economic Dynamics*, 4, pp. 852-883.

Benetrix, Agustin and Philip R. Lane (2010) Fiscal Shocks and the Sectoral Composition of Output. *Open Economies Review*, 21(3), pp. 335-350.

Beraja, Martin, and Christian K. Wolf. Demand Composition and the Strength of Recoveries. Mimeo.

Bernardini, Marco, Selien De Schryder and Gert Peersman, (2020) Heterogeneous Government Spending Multipliers in the Era Surrounding the Great Recession. *The Review of Economics and Statistics*, 102(2), pp. 304-322.

Bertinelli, Luisito, Olivier Cardi, and Romain Restout (2022) Labor Market Effects of Technology Shocks Biased toward the Traded Sector. *Journal of International Economics*, vol. 138(C).

Best, Michael Carlos, James S. Cloyne, E. Ilzetzki, H. J. Kleven (2020) Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches. *Review of Economic Studies*, 87(2), pp. 656-690.

Bianchi, Francesco, Howard Kung, and Gonzalo Morales (2019) Growth, Slowdowns, and Recoveries. *Journal of Monetary Economics*, 101, pp. 47-63.

Blanchard, Olivier J., and Roberto Perotti (2002) An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. *Quarterly Journal of Economcis* 177, pp. 1329-1368.

Boehm, Christoph E. (2020) Government Consumption and Investment: Does the Composition of Purchases Affect the Multiplier? *Journal of Monetary Economics*, 115(C), pp. 80-93.

Bouakez, Hafedh, Omar Rachedi, and Emiliano Santoro (2022a) The Government Spending Multiplier in a Multi-Sector Economy. *American Economic Journal: Macroeconomics, forthcoming.*

Bouakez, Hafedh, Omar Rachedi, and Emiliano Santoro (2022b) The Sectoral Origins of the Spending Multiplier. Mimeo.

Buiter, Willem H. (1984) Saddlepoint Problems in Continuous Time Rational Expectations Models: A General Method and some Macroeconomic Examples. *Econometrica*, 52, pp. 665-80.

Bussière, Matthieu, Giovanni Callegari, Fabio Ghironi, Giulia Sestieri, and Norihiko Yamano (2013) Estimating Trade Elasticities: Demand Composition and the Trade Collapse of 2008-2009. *American Economic Journal: Macroeconomics*, 5(3), pp. 118-151.

Cardi, Olivier, Romain Restout, and Peter Claeys (2020) Imperfect Mobility of Labor across Sectors and Fiscal Transmission in Open Economy. *Journal of Economic Dynamics and Control*, 111(C).

Caselli, Francesco (2016) Technology Differences over Time and Space. Princeton University Press.

Caselli, Francesco, and Wilbur John Coleman II (2006) The World Technology Frontier. *American Economic Review*, 96(3), pp. 499-522.

Chirinko, Robert S., and Debdulal Mallick (2017) The Substitution Elasticity, Factor Shares, and the Low-Frequency Panel Model. *American Economic Journal: Macroeconomics*, 9(4), pp 225-253.

Chodorow-Reich, Gabriel, Loukas Karabarbounis and Rohan Kekre (2021) The Macroeconomics of the Greek Depression. Mimeo.

Corsetti, Giancarlo, André Meier, and Gernot Müller (2012) What Determines Government Spending Multipliers? *Economic Policy*, 72, pp. 523-564.

Costinot, Arnaud (2009) On the Origins of Comparative Advantage. *Journal of International Economics*, 77(2), pp. 255-264.

Cox, Lydia, Gernot Müller, Ernesto Pastén, Raphael Schoenle, and Michael Weber (2020) Big G. *NBER Working Papers 27034.*

D'Alessandro, Antonello, Giulio Fella and Leonardo Melosi (2019) Fiscal Stimulus With Learning-By-Doing. *International Economic Review*, 60(3), pp. 1413-1432.

De Gregorio, Jose, Alberto Giovannini and Holger C. Wolf (1994) International Evidence on Tradables and Nontradables Inflation. *European Economic Review* 38, pp. 1225-1244.

Delong, Brad, and Lawrence H. Summers (2012) Fiscal Policy in a Depressed Economy *Brookings Papers on Economic Activity*, Spring, pp. 233-274.

Eberly, Janice Sergio Rebelo, and Nicolas Vincent (2008) Investment and Value: A Neoclassical Benchmark. *NBER Working Papers* 13866.

European Union KLEMS, 2011. Growth and productivity accounts.

European Union KLEMS, 2017. Growth and productivity accounts.

Fernald, John (2014) A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Federal Reserve Bank of San Francisco Working Paper Series 2012-19.

Ferrara, Laurent, Luca Metelli, Filippo Natoli and Daniele Siena (2021) Questioning the Puzzle: Fiscal policy, Real Exchange Rate and Inflation. *Journal of International Economics*, 133(C).

Garofalo, Gasper A., and Steven Yamarik (2002) Regional Convergence: Evidence From A New State-By-State Capital Stock Series. *Review of Economics and Statistics*, 84(2), pp. 316-323.

Gollin, D. (2002) Getting Income Shares Right. *Journal of Political Economy*, 110, pp. 458-474.

Hlatshwayo, Sandile, and Michael Spence (2014) Demand and Defective Growth Patterns: The Role of the Tradable and Non-tradable Sectors in an Open Economy. *American Economic Review*, 104(5), pp. 272-277.

Herrendorf, Berthold, Christopher Herrington, and Ákos Valentinyi (2015) Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics*, 7(4), pp. 104-133.

Horvath, Michael (2000) Sectoral Shocks and Aggregate Fluctuations. *Journal of Monetary Economics*, 45, pp. 69-106.

Imbs, Jean (1999) Technology, Growth and the Business Cycle. *Journal of Monetary Economics*, 44(1), pp. 65-80.

Imbs, Jean, and Isabelle Mejean (2015) Elasticity Optimism. *American Economic Journal: Macroeconomics*, 7(3), pages 43-83.

Jensen, Bradford J., and Lori G. Kletzer, (2006) Offshoring White-Collar Work, in Susan M. Collins and Lael Brainard, eds., *Brookings Trade Forum* 2005, Offshoring White-Collar Work. Brookings Institution: Washington, DC, pp. 75-134.

Jordà, Òscar (2005) Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review*, 95(1), pp. 161-182.

Jordà, Òscar, Sanjay R. Singh, and Alan M. Taylor (2020) The Long-Run Effects of Monetary Policy. NBER Working Papers 26666.

Jørgensen, Peter Lihn, and Søren Hove Ravn (2022) The Inflation Response to Government Spending Shocks: A Fiscal Price Puzzle? *European Economic Review*, forthcoming.

Kehoe, Timothy J. and Kim J., Ruhl (2009) Sudden Stops, Sectoral Reallocations, and the Real Exchange Rate. *Journal of Development Economics*, 89(2), pp. 235-249.

Klein, Mathias and Ludger Linnemann (2022) Fiscal Policy, International Spillovers, and Endogenous Productivity. Mimeo.

Klump, Rainer, Peter McAdam and Alpo Willman (2007) Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach. *Review of Economics and Statistics*, 89(1), pp. 183-92.

Lambertini, Luisa and Christian Proebsting (2022) Fiscal Policy, Relative Prices and Net Exports in a Currency Union. *American Economic Journal: Macroeconomics, forthcoming.*

Liu, Siming (2022) Government Spending during Sudden Stop Crises. *Journal of International Economics*, 135(C).

MaCurdy, Thomas (1981) An Empirical Model of Labor Supply in a Life-Cycle Setting. *Journal of Political Economy*, 89 (6), pp. 1059-1085.

Mendoza, Enrique, G. (1992) The Effects of Macroeconomic Shocks in a Basic Equilibrium Framework. *Staff Papers IMF*, 39(4), pp. 855-889.

Mian, Atif, and Amir Sufi (2014) Whar Explains the 2007-2009 Drop in Employment. *Econometrica*, 82(6), pp. 2197-2223.

Nunn, Nathan (2007) Relationship-Specificity, Incomplete Contracts, and the Pattern of Trade. *The Quarterly Journal of Economics*, 122(2), pp. 569-600.

Oberfield, Ezra, and Devesh Raval (2021) Micro Data and Macro Technology. *Econometrica*, 89(2), pp. 703-732.

Organization for Economic Cooperation and Development (2011), *Structural Analysis Database*, OECD, Paris.

Organization for Economic Cooperation and Development (2017), *Structural Analysis Database*, OECD, Paris.

Peterman, William B. (2016) Reconciling Micro And Macro Estimates Of The Frisch Labor Supply Elasticity. *Economic Inquiry*, 54(1), pp. 100-120.

Proebsting, Christian (2022) Market Segmentation and Spending Multipliers. *Journal of Monetary Economics*, forthcoming.

Ramey, Valerie A. (2011) Identifying Government Spending Shocks: It's All in the Timing *Quarterly Journal of Economics*, 126(1), pp. 1-50.

Ramey, Valerie A., and Matthew D. Shapiro (1998) Costly Capital Reallocation and the Effects of Government Spending. *Carnegie-Rochester Conference Series on Public Policy*, 48, pp. 145-194.

Ramey, Valerie A., and Sarah Zubairy (2018) Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data. *Journal of Political Economy*, 126(2), pp. 850-901.

Shimer, Robert (2009) Convergence in Macroeconomics: The Labor Wedge *American Economic Journal: Macroeconomics*, 1(1), pp. 280-297.

Sims, Chritopher, and Tao Zha (2006) Does Monetary Policy Generate Recessions? *Macroeconomic Dynamics*, 10(2), 231-272.

Stehrer, Robert, Alexandra Bykova, Kirsten Jäger, Oliver Reiter and Monika Schwarzhappel (2019) Industry Level Growth and Productivity Data with Special Focus on Intangible Assets. Statistical Report No. 8.

WIOD: Construction and Applications (2013). World Input-Output Database.

WIOD: Construction and Applications (2016). World Input-Output Database.

A Adjustment of Sectoral Hours and Value Added during Recessions and Expansions

In the introduction, we refer to several papers which document some evidence showing that traded and non-traded sectors are not affected symmetrically along the business cycle. Mian and Sufi [2014] for the U.S. (2007-2009) and De Ferra [2018] for Italy (2011-2013) find that non-traded firms experience the largest drop in labor during downturns. Evidence by Hlatshwayo and Spence [2014] on U.S. data reveals that tradable industries are the drivers of value added growth while employment growth originates from non-traded industries during expansions. Beraja and Wolf [2022] show that recessions are deeper and the recovery greater in U.S. States where the share of expenditure in durables is larger. Similar evidence are documented by Aghion et al. [2021] for a set of OECD countries. While we consider tradables and not durables, the variations in the value added of tradables are driven by Manufacturing which is made up of industries producing durable goods,

By using Hodrick-Prescott detrended U.S. and OECD data (for eighteen OECD countries) we also detect a strong negative correlation between the cyclical components of the ratio of traded to non-traded hours worked and real GDP after the great moderation. These findings for the U.S. (in the post-1984 period) and for our sample of eighteen OECD countries (in the post-1992 period) suggest that traded hours worked decline less than non-traded hours worked when the economy slows down. The conclusion is reversed when we focus on the cyclical component of traded to non-traded value added (at constant prices). We find empirically that traded value added declines more than non-traded value added during recessions in OECD countries. We provide more details below.

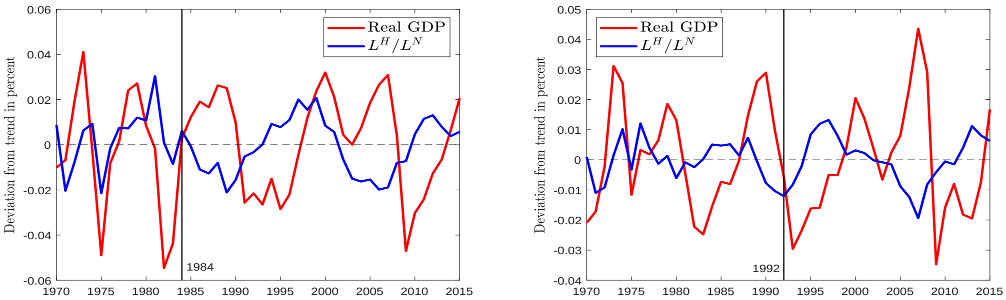
Fig. 4(a) plots the cyclical components of (logged) real GDP (displayed by the red line) and the (logged) ratio of traded to non-traded hours worked (displayed by the blue line) for (the market sector of) the United States. Over the period 1970-2015, the two series are uncorrelated, suggesting that the traded and the non-traded sectors are symmetrically affected during expansions and recessions. According to the evidence documented by Garìn et al. [2018] on U.S. data, the responses of sectors display more asymmetry along the business cycle in the post-1984 period, i.e., during the great moderation. When we split the whole period into two sub-samples, we find that the correlation between the cyclical components of real GDP and traded relative to non-traded hours worked moves from positive (at 0.43) in 1970-1984 to negative (at -0.30) in the post-1984 period. The negative correlation suggests that during recessions, non-traded industries have experienced a larger decline in hours worked than traded industries over the last thirty years.

This finding is not limited to the United States. Fig. 4(b) plots the cyclical components of real GDP and the ratio of traded to non-traded hours worked for the eighteen OECD countries in our sample. Choosing 1992 as the cutoff year for the whole sample, we find a correlation of 0.11 over 1970-1992 and a correlation of -0.44 in the post-1992 period. Two-thirds of our sample is made up of European countries for which the great moderation occurs in the post-1992 period, see e.g., Benati [2008] for the U.K., González Cabanillas and Ruscher [2008] for the euro area. Data on OECD countries thus further corroborates the finding that non-traded labor is more vulnerable to downturns than traded labor (during the great moderation). The conclusion is reversed when we focus on the cyclical component of traded to non-traded value added (at constant prices).[15] We find empirically that traded value added declines more than non-traded value added during recessions in OECD countries.[16] The fact that sectors are not symmetrically affected by recessions raises the question of the capacity of fiscal policy to mitigate such a differential response of non-tradable versus tradable industries. Our VAR evidence shows that the technology channel of fiscal policy can mitigate sector asymmetry along the business cycle by encouraging traded firms to improve their technology (which increases traded value added) and by leading non-traded firms to bias technological change toward labor (which increases non-traded hours worked).

Data on OECD countries reveals that sectors have not been symmetrically affected by recessions over the last thirty years as non-traded labor falls more than traded labor. Fig. 5 plots the cyclical component of real GDP in the solid red line against the cyclical component of traded to non-traded value added at constant prices in the solid blue line. For the eighteen OECD countries, as it stands out from Fig. 5(b), real GDP and traded relative to non-traded value added co-vary as they are strongly and positively correlated as the correlation stands at 0.74. This result suggests that recessions are associated with a larger decline in traded value added than in the non-traded value added. Although the non-traded sector experiences a larger decline in labor than the traded sector, the latter experiences a greater decline in productivity, thus explaining why real GDP and

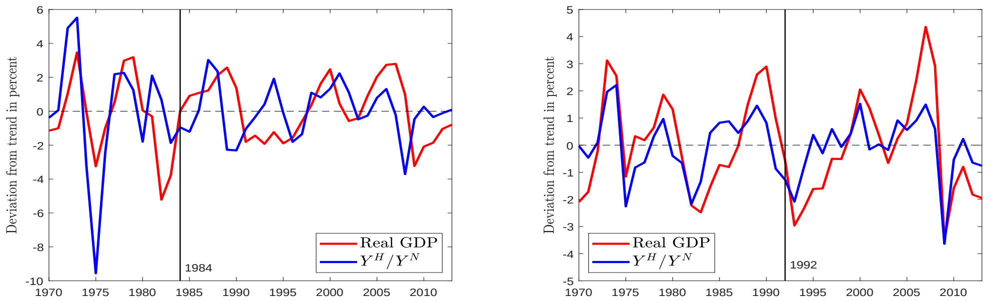
[15]Our empirical findings echo evidence by Hlatshwayo and Spence [2014] on U.S. data which reveals that tradable industries are the drivers of value added growth while employment growth originates from non-traded industries during expansions.

[16]In the Online Appendix A, we document evidence for the eighteen OECD countries and the United States which shows that the cyclical component of real GDP is (strongly) positively correlated with the ratio of traded to non-traded value added.



(a) United States (b) OECD (18 countries)

Figure 4: Real GDP and Traded relative to Non-Traded Hours Worked. Notes: Detrended (logged) real GDP and the detrended ratio of traded to non-traded hours worked are calculated as the difference between the actual series and the trend of time series. The trend is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ (as we use annual data) to the (logged) time series. Since we seek to investigate how market sectors are relatively affected by the stage of the business cycle, we abstract from the public sector and thus removed 'Community social and personal services' (which includes public services, health and education) from real GDP and non-traded hours worked. Sample: 18 OECD countries, 1970-2015, annual data.



(a) United States (b) OECD (18 countries)

Figure 5: Real GDP and Traded relative to Non-Traded Value Added. Notes: Detrended (logged) real GDP and the detrended ratio of traded to non-traded value added are calculated as the difference between the actual series and the trend of time series. The trend is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 100$ (as we use annual data) to the (logged) time series. Since we seek to investigate how market sectors are relatively affected by the stage of the business cycle, we abstract from the public sector and thus removed value added at constant prices of 'Community social and personal services' (which includes public services, health and education) from real GDP and non-traded hours value added. While we take the unweighted sum of time series in Fig. 5, we have alternatively used the working age population weighted sum of the eighteen OECD countries and it gives similar results. Sample: 18 OECD countries, 1970-2015, annual data.

the relative value added of tradables co-vary. When we turn the United States shown in Fig. 5(a), we also find a positive correlation but the link between the two variables has somewhat declined over time. Over 1970-2015, the correlation between the two variables stands at 0.40 for the United States.

B Sectoral Decomposition of Real GDP and Total Hours Worked

B.1 Sectoral Decomposition of Real GDP

We consider an open economy which produces domestic traded goods, denoted by a superscript H, and non-traded goods, denoted by a superscript N. The foreign-produced traded good is the numeraire and its price is normalized to 1. We consider an initial steady-state where prices are those at the base year so that initially real GDP, denoted by Y_R, and the value added share at constant prices, denoted by $\nu^{Y,j}$, collapses to nominal GDP (i.e., Y) and the value added share at current prices, respectively. Before moving forward, it is worth mentioning that whilst in the model and the quantitative analysis, we add a tilde when value added is inclusive of the technology utilization rate since we allow for endogenous utilization of existing technologies, we do not need to make this distinction in the data and Y_t^j refers to value added inclusive of technology improvement below.

Summing value added at constant prices across sectors gives real GDP:

$$Y_{R,t} = P^H Y_t^H + P^N Y_t^N, \tag{38}$$

where P^H and P^N stand for the price of home-produced traded goods and non-traded goods, respectively, which are kept fixed since we consider value added at constant prices.

Log-linearizing (38), and denoting the percentage deviation from initial steady-state by a hat leads to:

$$\hat{Y}_{R,t} = \nu^{Y,H}\hat{Y}_t^H + \nu^{Y,N}\hat{Y}_t^N, \tag{39}$$

where $\nu^{Y,j} = \frac{P^jY^j}{Y}$ is the value added share of home-produced traded goods evaluated at the initial steady-state. **Eq. (39) corresponds to eq. (1) in the main text**. We drop the time index below as long as it does not cause confusion.

Subtracting real GDP growth from both sides of (39) leads to the sum of the change in the value added share denoted by $d\nu_t^{Y,j}$:

$$0 = d\nu_t^{Y,H} + d\nu_t^{Y,N}. \tag{40}$$

The change in the value added share is computed as the excess (measured in ppt of GDP) of value added growth at constant prices in sector $j = H, N$ over real GDP growth:

$$d\nu_t^{Y,j} = \nu^{Y,j}\left(\hat{Y}_t^j - \hat{Y}_{R,t}\right). \tag{41}$$

Capital K^j can be freely reallocated across sectors while labor L^j is subject to mobility costs which creates a sectoral wage differential. We denote the capital rental cost by R and the wage rate in sector j by W^j (with $j = H, N$). Under assumption of perfect competition in product and input markets, factors of production are paid their marginal product in both sectors:

$$P^j\frac{\partial Y^j}{\partial L^j} = W^j, \tag{42a}$$

$$P^j\frac{\partial Y^j}{\partial K^j} = R. \tag{42b}$$

Assuming constant returns to scale in production and making use of (42), the log-linearized version of the production function reads:

$$\hat{Y}_t^j = \hat{\text{TFP}}_t^j + s_L^j\hat{L}_t^j + \left(1 - s_L^j\right)\hat{K}_t^j, \tag{43}$$

where s_L^j and TFP^j are the labor income share and total factor productivity in sector j, respectively, and $k^j \equiv K^j/L^j$ stands for the capital-labor ratio.

We derive below an expression of the deviation of real GDP relative to initial steady-state. Since we assume perfect capital mobility, the resource constraint for capital reads as follows $K = K^H + K^N$. Totally differentiating, multiplying both sides by the capital rental cost R, and dividing by GDP leads to:

$$\left(1 - s_L\right)\hat{K}_t = \nu^{Y,H}\left(1 - s_L^H\right)\hat{K}_t^H + \left(1 - \nu^{Y,H}\right)\left(1 - s_L^N\right)\hat{K}_t^N. \tag{44}$$

The same logic applies to labor except that we assume imperfect mobility of labor across sectors. In this case, the percentage deviation of total hours worked relative to its initial steady-state is defined as the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state, i.e., $\hat{L}_t = \alpha_L\hat{L}_t^H + (1 - \alpha_L)\hat{L}_t^N$, where $\alpha_L = \frac{W^HL^H}{WL}$ is the labor compensation share for tradables. Multiplying both sides by total compensation of employees, WL, and dividing by GDP leads to:

$$s_L\hat{L}_t = \nu^{Y,H}s_L^H\hat{L}_t^H + \left(1 - \nu^{Y,H}\right)s_L^N\hat{L}_t^N. \tag{45}$$

Plugging (43) into (39) and making use of (44)-(45) allows us to express the change in real GDP in terms of aggregate TFP changes and accumulation of inputs:

$$\hat{Y}_{R,t} = \hat{\text{TFP}}_t + s_L\hat{L}_t + \left(1 - s_L\right)\hat{K}_t, \tag{46}$$

where the percentage deviation of aggregate TFP relative to its initial steady-state is equal to the weighted sum of the percentage deviation of TFP relative to initial steady-state in the traded and the non-traded sector

$$\hat{\text{TFP}}_t = \nu^{Y,H}\hat{\text{TFP}}_t^H + \left(1 - \nu^{Y,H}\right)\hat{\text{TFP}}_t^N. \tag{47}$$

Considering the non-traded sector (i.e., setting $j = N$) and plugging (46) into (41) shows that the change in the value added share at constant prices of non-tradables can be brought about by a TFP growth differential, a labor and/or a capital inflow. Formally, the decomposition of the change in the value added share of the non-tradables reads:

$$d\nu_t^{Y,N} = \left(1 - \nu^{Y,H}\right)\left[\left(\hat{\text{TFP}}_t^N - \hat{\text{TFP}}_t\right) + \left(\hat{L}_t^N - \hat{L}_t\right) + \left(1 - s_L^N\right)\hat{k}_t^N - \left(1 - s_L\right)\hat{k}_t\right], \tag{48}$$

where $k^N \equiv K^N/L^N$ is the capital-labor ratio in the non traded sector, $k \equiv K/L$ is the aggregate capital-labor ratio, and we used the fact that changes in sectoral value added and aggregate real GDP, as described by (43) and (46), respectively, can be rewritten as follows:

$$\hat{Y}_t^j = \hat{\text{TFP}}_t^j + \hat{L}_t^j + \left(1 - s_L^j\right)\hat{k}_t^j, \tag{49a}$$

$$\hat{Y}_{R,t} = \hat{\text{TFP}}_t + \hat{L}_t + \left(1 - s_L\right)\hat{k}_t. \tag{49b}$$

Plugging the sectoral decomposition of the deviation of aggregate TFP relative to its initial steady-state described by eq. (47) into the change in the value added share of non-tradables at constant prices (48) leads to the decomposition of the change in the value added share of non-tradables which reads:

$$\begin{aligned} d\nu_t^{Y,N} &= -\left(1-\nu^{Y,H}\right)\nu^{Y,H}\left(\hat{\text{TFP}}_t^H - \hat{\text{TFP}}_t^N\right) + \left(1-\nu^{Y,H}\right)\Big[\left(\hat{L}_t^N - \hat{L}_t\right) \\ &\quad + \left(1-s_L^N\right)\hat{k}_t^N - \left(1-s_L\right)\hat{k}_t\Big]. \end{aligned} \tag{50}$$

B.2 Sectoral Decomposition of Hours Worked

In this section, we detail the steps of derivation of the relationship between the labor share of non-tradables and the responses of LISs. While Y_t^j refers to value added inclusive of technology improvement below, we make the distinction between the capital stock K_t^j and the capital stock inclusive of capital utilization $\tilde{K}_t^j$ by adding a tilde.

In an economy where labor is imperfectly mobile across sectors, the percentage deviation of total hours worked relative to its initial steady-state (i.e., $\hat{L}_t$) following a shock to government consumption is equal to the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state (i.e., $\hat{L}_t^j$):

$$\hat{L}_t = \alpha_L \hat{L}_t^H + \left(1-\alpha_L\right)\hat{L}_t^N. \tag{51}$$

where α_L $(1-\alpha_L)$ is the labor compensation share of tradables (non-tradables). **Eq. (51) corresponds to eq. (3) in the main text**. Note that we use interchangeably $\alpha_L = \alpha_L^H$ and $1-\alpha_L = \alpha_L^N$.

If we subtract the share of $\hat{L}_t$ received by each sector from the change in sectoral hours worked, we obtain the change in the labor share of sector j, denoted by $\nu^{L,j}$, which measures the contribution of the reallocation of labor across sectors to the change in hours worked in sector j:[17]

$$d\nu_t^{L,j} = \alpha_L^j\left(\hat{L}_t^j - \hat{L}_t\right) \qquad j = H, N. \tag{52}$$

The differential between the responses of sectoral and total hours worked on the RHS of eq. (52) can be viewed as the change in labor in sector j if L remained fixed and thus reflects higher employment in this sector resulting from the reallocation of labor.

Both the traded and non-traded sectors use physical capital (inclusive of capital utilization), $\tilde{K}^j = u^{K,j}K^j$, and labor, L^j, according to constant returns to scale production functions which are assumed to take a CES form:

$$Y_t^j = \left[\gamma^j\left(A_t^j L_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(B_t^j \tilde{K}_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}, \tag{53}$$

where γ^j and $1-\gamma^j$ are the weight of labor and capital in the production technology, σ^j is the elasticity of substitution between capital and labor in sector $j = H, N$, A^j and B^j are labor- and capital-augmenting efficiency. Both sectors face two cost components: a capital rental cost equal to R, and a labor cost equal to the wage rate, i.e., W^H in the traded sector and W^N in the non-traded sector.

Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given:

$$\max_{\tilde{K}_t^j, L_t^j} \Pi_t^j = \max_{K_t^j, L_t^j}\left\{P_t^j Y_t^j - W_t^j L_t^j - R_t \tilde{K}_t^j\right\}. \tag{54}$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P_t^j \gamma^j \left(A_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\left(L_t^j\right)^{-\frac{1}{\sigma^j}}\left(Y_t^j\right)^{\frac{1}{\sigma^j}} \equiv W_t^j, \tag{55a}$$

$$P_t^j\left(1-\gamma^j\right)\left(B_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\left(\tilde{k}_t^j\right)^{-\frac{1}{\sigma^j}}\left(y_t^j\right)^{\frac{1}{\sigma^j}} \equiv R_t, \tag{55b}$$

where we denote by $\tilde{k}_t^j \equiv \tilde{K}_t^j / L_t^j$ the capital-labor ratio for sector $j = H, N$, and $y_t^j \equiv Y_t^j / L_t^j$ value added per hours worked described by

$$y_t^j = \left[\gamma^j\left(A_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(B_t^j \tilde{k}_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}. \tag{56}$$

[17] While the two measures are equivalent in level, we differentiate between $\nu^{L,j}$ and α_L since the change in the labor share is calculated by keeping W^j/W constant.

Denoting the LIS in sector j by s_L^j, and pre-multiplying both sides of (55a) by L^j and dividing by value added at current prices in sector j, $P^j Y^j$ leads to the labor income share:

$$s_L^j = \gamma^j \left(\frac{A^j}{y^j}\right)^{\frac{\sigma^j - 1}{\sigma^j}}. \tag{57}$$

Multiplying both sides of (55b) by K^j and dividing by value added at current prices in sector j leads to the capital income share:

$$1 - s_L^j = \left(1 - \gamma^j\right) \left(\frac{B^j \tilde{k}^j}{y^j}\right)^{\frac{\sigma^j - 1}{\sigma^j}}. \tag{58}$$

Dividing eq. (57) by eq. (58), the ratio of the labor to the capital income share denoted by $S^j = \frac{s_L^j}{1 - s_L^j}$ reads as follows:

$$S_t^j = \frac{\gamma^j}{1 - \gamma^j} \left(\frac{B_t^j \tilde{K}_t^j}{A_t^j L_t^j}\right)^{\frac{1 - \sigma^j}{\sigma^j}}. \tag{59}$$

Let us denote:

$$\text{FBTC}^j = \left(\frac{B_t^j u_t^{K,j}}{A_t^j}\right)^{\frac{1 - \sigma^j}{\sigma^j}}, \tag{60a}$$

$$\text{FBTC}_{adjK}^j = \left(\frac{B_t^j}{A_t^j}\right)^{\frac{1 - \sigma^j}{\sigma^j}}. \tag{60b}$$

Using the definition of the LIS, the demand for labor by firms in sector j (55a) can be rewritten as follows:

$$s_{L,t}^j \frac{P_t^j Y_t^j}{L_t^j} = W_t^j. \tag{61}$$

Aggregating across sectors and dividing by GDP at current prices, i.e., $\sum_j s_{L,t}^j \frac{P_t^j Y_t^j}{Y_t} = \frac{W_t L_t}{Y_t}$, leads to aggregate demand for labor:

$$s_{L,t} \frac{P_t Y_t}{L_t} = W_t. \tag{62}$$

Dividing the demand for labor in sector j (61) by aggregate labor demand (62):

$$\frac{W_t^j}{W_t} \frac{L_t^j}{L_t} = \frac{s_{L,t}^j}{s_{L,t}} \omega_t^{Y,j}, \tag{63}$$

where $\omega_t^{Y,j} = \frac{P_t^j Y_t^j}{Y_t}$ stands for the value added share of sector j at current prices. Drawing on Horvath [2000], we generate imperfect mobility of labor ny assuming that sectoral hours worked are imperfect substitutes which gives rise to a labor share in sector j which is elastic to the relative wage:

$$\frac{L_t^j}{L_t} = \vartheta^j \left(\frac{W_t^j}{W_t}\right)^{\epsilon}, \tag{64}$$

where ϑ^j stands for the weight attached to labor supply in sector $j = H, N$ and ϵ is the elasticity of labor supply across sectors which captures the degree of labor mobility. Plugging labor supply to sector j (64) into (63), the equilibrium labor share in sector j reads as follows:

$$\frac{L_t^j}{L_t} = \left(\vartheta^j\right)^{\frac{1}{1+\epsilon}} \left(\frac{s_{L,t}^j}{s_{L,t}}\right)^{\frac{\epsilon}{1+\epsilon}} \left(\omega_t^{Y,j}\right)^{\frac{\epsilon}{1+\epsilon}}. \tag{65}$$

Eq. (65) corresponds to eq. (4) in the main text.

C Numerical Decomposition of the Value Added and Labor Shares of Non-Tradables

In this section, we detail the steps of the decomposition of the changes in the value added and labor share of non-tradables in order to compute numerically the contribution of technology to their responses.

Panel D of Table Table 3 decomposes the response of the sectoral LIS. Log-linearizing (22) and using the fact that $\hat{s}^j_L(t) = \hat{S}^j(t)\left(1 - s^j_L\right)$, shows that the response of the LIS in sector j is driven by capital deepening and FBTC:

$$ds^j_L(t) = s^j_L\left(1 - s^j_L\right)\frac{1-\sigma^j}{\sigma^j}\hat{\tilde{k}}^j(t) + s^j_L\left(1 - s^j_L\right)\hat{\text{FBTC}}^j_{adjK}, \tag{66}$$

where $\tilde{k}^j(t) = u^{K,j}(t)k^j(t)$ and $\text{FBTC}^j_{adjK} = \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$. The first term on the RHS captures the effect of capital deepening while the second term reflects the impact of utilization-adjusted-FBTC in sector j.

Panel B of Table 3 decomposes the response of labor share of non-tradables into three effects. By log-linearizing its equilibrium value described by eq. (4) and by making use of (66), the change in the labor share of non-tradables can be broken down into three components:

$$\begin{aligned} d\nu^{L,N}(t) &= \alpha_L\left(1-\alpha_L\right)\frac{\epsilon}{1+\epsilon}\frac{\hat{\omega}^{Y,N}(t)}{1-\omega^{Y,N}} \\ &+ \alpha_L\left(1-\alpha_L\right)\frac{\epsilon}{1+\epsilon}\left[\left(1-s^N_L\right)\left(\frac{1-\sigma^N}{\sigma^N}\right)\hat{\tilde{k}}^N(t) - \left(1-s^H_L\right)\left(\frac{1-\sigma^H}{\sigma^H}\right)\hat{\tilde{k}}^H(t)\right] \\ &+ \frac{\epsilon}{1+\epsilon}\left[\left(1-s^N_L\right)\hat{\text{FBTC}}^N_{adjK}(t) - \left(1-s^H_L\right)\hat{\text{FBTC}}^H_{adjK}(t)\right], \end{aligned} \tag{67}$$

The first term on the RHS of (67) measures the change in $\nu^{L,N}(t)$ driven by the rise in the value added share of non-tradables at current prices denoted by $\omega^{Y,N}(t)$. This term measures the change in $\nu^{L,N}(t)$ driven by the biasedness of the demand shock toward non-tradables which increases $\omega^{Y,N}(t)$. The second and third term on the RHS of eq. (67) measures the change in $\nu^{L,N}(t)$ driven by the rise in the non-traded relative to the traded LIS.

To calculate the contribution of each component k to the change in the labor share of non-tradables, we proceed as follows:

$$\text{Contribution of } k \text{ to } d\nu^{L,N}(t) = \frac{d\nu^{L,N}_{\text{k}}(t)}{\sum_k \left|d\nu^{L,N}_{\text{k}}(t)\right|},$$

where k is either $\omega^{Y,N}(t)$, the capital deepening differential between non-tradables and tradables, or the FBTC differential between non-tradables and tradables. Because each factor contributing to $d\nu^{L,N}(t)$ may exert either a positive or a negative impact on the labor share of non-tradables, we divide the contribution of each component to the sum of changes in $\nu^{L,N}(t)$ driven by all components, each change being expressed in absolute value terms.

To quantify the role of technology in driving the distribution of the government spending multiplier across sectors, we break down analytically the change in the value added share of non-tradables into three components. Using the fact that $d\nu^{Y,N}(t) = \left(1-\nu^{Y,H}\right)\nu^H\left(\hat{\tilde{Y}}^N(t) - \hat{\tilde{Y}}^H(t)\right)$, and $\hat{\tilde{Y}}^j(t) = \hat{\text{TFP}}^j(t) + \hat{L}^j(t) + \left(1-s^j_L\right)\hat{k}^j(t)$, the change in the value added share of non-tradables can be broken down into three components:

$$\begin{aligned} d\nu^{Y,N}(t) &= \nu^{Y,H}\left(1-\nu^{Y,H}\right)\left(\hat{\text{TFP}}^H(t) - \hat{\text{TFP}}^N(t)\right) + \nu^{Y,H}\left(1-\nu^{Y,H}\right)\left(\hat{L}^H(t) - \hat{L}^N(t)\right) \\ &+ \nu^{Y,H}\left(1-\nu^{Y,H}\right)\left[\left(1-s^H_L\right)\hat{k}^H(t) - \left(1-s^L_L\right)\hat{k}^N(t)\right]. \end{aligned} \tag{68}$$

The first term on the RHS of (68) measures the change in $\nu^{Y,N}(t)$ driven by the TFP differential. The second and the third term on the RHS of (68) captures the change in the value added share of non-tradables driven by labor and capital reallocation.

D Data Description for Empirical Analysis

Coverage: Our sample consists of a panel of 18 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), the United Kingdom (GBR) and the United States (USA). The baseline period is running from 1970 to 2015, except for Japan (1974-2015). Table 4 summarizes our dataset.

Sources: Our primary sources for sectoral data are the OECD and EU KLEMS databases. We use data from EU KLEMS ([2011], [2017]) March 2011 and July 2017 releases. The EU KLEMS

Table 4: Sample Range for Empirical and Numerical Analysis

Country	Code	Period	Obs.
Australia	(AUS)	1970 - 2015	46
Austria	(AUT)	1970 - 2015	46
Belgium	(BEL)	1970 - 2015	46
Canada	(CAN)	1970 - 2015	46
Denmark	(DNK)	1970 - 2015	46
Spain	(ESP)	1970 - 2015	46
Finland	(FIN)	1970 - 2015	46
France	(FRA)	1970 - 2015	46
Great Britain	(GBR)	1970 - 2015	46
Ireland	(IRL)	1970 - 2015	46
Italy	(ITA)	1970 - 2015	46
Japan	(JPN)	1974 - 2015	41
Korea	(KOR)	1970 - 2015	46
Netherlands	(NLD)	1970 - 2015	46
Norway	(NOR)	1970 - 2015	46
Portugal	(PRT)	1970 - 2015	46
Sweden	(SWE)	1970 - 2015	46
United States	(USA)	1970 - 2015	46
Total number of obs.			823
Main data sources		EU KLEMS & OECD STAN	

Notes: Column 'period' gives the first and last observation available. Obs. refers to the number of observations available for each country.

dataset covers all countries of our sample, with the exceptions of Canada and Norway. For these two countries, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD ([2011], [2017]). For both EU KLEMS and STAN databases, the March 2011 release provides data for eleven 1-digit ISIC-rev.3 industries over the period 1970-2007 while the July 2017 release provides data for thirteen 1-digit-rev.4 industries over the period 1995-2015.

The construction of time series for sectoral variables over the period 1970-2015 involves two steps. First, we identify tradable and non-tradable sectors. We adopt the classification proposed by De Gregorio et al. [1994]. Following Jensen and Kletzer [2006], we have updated this classification by treating the financial sector as a traded industry. We map the ISIC-rev.4 classification into the ISIC-rev.3 classification in accordance with the concordance Table 5. Once industries have been classified as traded or non-traded, for any macroeconomic variable X, its sectoral counterpart X^j for $j = H, N$ is constructed by adding the X_k of all sub-industries k classified in sector $j = H, N$ as follows $X^j = \sum_{k \in j} X_k$. Second, series for tradables and non-tradables variables from EU KLEMS [2011] and OECD [2011] databases (available over the period 1970-2007) are extended forward up to 2015 using annual growth rate estimated from EU KLEMS [2017] and OECD [2017] series (available over the period 1995-2015).

Table 5: Summary of Sectoral Classifications

Sector	ISIC-rev.4 Classification (sources: EU KLEMS [2017] and OECD ([2017]))		ISIC-rev.3 Classification (sources: EU KLEMS [2011] and OECD ([2011]))	
	Industry	Code	Industry	Code
Tradables (H)	Agriculture, Forestry and Fishing	A	Agriculture, Hunting, Forestry and Fishing	AtB
	Mining and Quarrying	B	Mining and Quarrying	C
	Total Manufacturing	C	Total Manufacturing	D
	Transport and Storage	H	Transport, Storage and Communication	I
	Information and Communication	J		
	Financial and Insurance Activities	K	Financial Intermediation	J
Non Tradables (N)	Electricity, Gas and Water Supply	D-E	Electricity, Gas and Water Supply	E
	Construction	F	Construction	F
	Wholesale and Retail Trade, Repair of Motor Vehicles and Motorcycles	G	Wholesale and Retail Trade	G
	Accommodation and Food Service Activities	I	Hotels and Restaurants	H
	Real Estate Activities	L	Real Estate, Renting and Business Services	K
	Professional, Scientific, Technical, Administrative and Support Service Activities	M-N		
	Community Social and Personal Services	O-U	Community Social and Personal Services	LtQ

Construction of sectoral variables. Once industries have been classified as traded or non-traded, we construct sectoral variables by taking time series from e EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. These two databases provide data, for each industry and year, on value added at current and constant prices, permitting the construction of sectoral deflators of value added, as well as details on labor compensation and hours worked data, allowing the construction of sectoral wage rates. Time and countries are indexed by subscripts i and t below while the sector is indexed by the superscript $j = H, N$.

All quantity variables are scaled by the working age population (15-64 years old). Source: OECD ALFS Database for the working age population (data coverage: 1970-2015). We describe below the construction for the sectoral data employed in the main text (mnemonics are given in parentheses):

- **Sectoral value added**, Y_{it}^{j}: sectoral value added at constant prices in sector $j = H, N$ (VA_QI). Series for sectoral value added in current (constant) prices are constructed by adding value added in current (constant) prices for all sub-industries k in sector $j = H, N$, i.e., $P_{it}^{j} Y_{it}^{j} = \sum_k P_{k,it}^{j} Y_{k,it}^{j}$ ($\bar{P}_{it}^{j} Y_{it}^{j} = \sum_k \bar{P}_{k,it}^{j} Y_{k,it}^{j}$ where the bar indicates that prices P^j are those of the base year), from which we construct price indices (or sectoral value added deflators), P_{it}^{j}. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Sectoral value added share**, $\nu_{it}^{Y,j}$, is constructed as the ratio of value added at constant prices in sector j to GDP at constant prices, i.e., $Y_{it}^{j}/(Y_{it}^{H} + Y_{it}^{N})$ for $j = H, N$.
- **Relative price of non-tradables**, P_{it}. Normalizing base year price indices $\bar{P}^j$ to 1, the relative price of non-tradables, P_{it}, is constructed as the ratio of the non-traded value added deflator to the traded value added deflator (i.e., $P_{it} = P_{it}^{N}/\mathsf{P}_{it}^{H}$). The sectoral value added deflator P_{it}^{j} for sector $j = H, N$ is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector j. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Terms of trade**, $\text{TOT}_{it} = P_{it}^{H}/P_{it}^{H,\star}$, is computed as the ratio of the traded value added deflator of the home country i, P_{it}^{H}, to the geometric average of the traded value added deflator of the seventeen trade partners of the corresponding country i, $P_{it}^{H,\star}$, the weight being equal to the share $\alpha_i^{M,k}$ of imports from the trade partner k. We use the traded value added deflator to approximate foreign prices as it corresponds to a value-added concept. The Direction of Trade Statistics (DOTS, IMF) gives the share of imports $\alpha_i^{M,k}$ of country i by trade partner k for all countries of our sample over 1970-2015. The traded value added deflator P_{it}^{H} is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector H. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) for P^H. Prices of foreign goods and services are calculated as follows: $P_{it}^{H,\star} = \Pi_{k\neq i} \left(P_t^{H,k}\right)^{\alpha_i^{M,k}}$. While the seventeen trade partners of a representative home country do not fully account for the totality of trade between country i and its trade partners $k \neq i$, it covers 58% of total trade on average for a representative OECD country of our sample. Source: Direction of Trade Statistics [2017]. Period: 1970-2015 for all countries except for Belgium (1997-2015).
- **Sectoral hours worked**, L_{it}^{j}, correspond to hours worked by persons engaged in sector j (H_EMP). Likewise sectoral value added, sectoral hours worked are constructed by adding hours worked for all sub-industries k in sector $j = H, N$, i.e., $L_{it}^{j} = \sum_k L_{k,it}^{j}$. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Sectoral labor share**, $\nu_{it}^{L,j}$, is constructed as the ratio of hours worked in sector j to total hours worked, i.e., $L_{it}^{j}/(L_{it}^{H} + L_{it}^{N})$ for $j = H, N$.
- **Sectoral nominal wage**, W_{it}^{j} is calculated as the ratio of the labor compensation (compensation of employees plus compensation of self-employed) in sector $j = H, N$ (LAB) to total hours worked by persons engaged (H_EMP) in that sector. ources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Relative wage**, W_{it}^{j}/W_{it}, is constructed as the ratio of the nominal wage in the sector j to the aggregate nominal wage W.
- **Labor income share (LIS)**, $s_{L,it}^{j}$, is constructed as the ratio of labor compensation (compensation of employees plus compensation of self-employed) in sector $j = H, N$ (LAB) to value added at current prices (VA) of that sector. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

We detail below the data construction for aggregate variables (mnemonics are in parentheses). For all variables, the reference period is running from 1970 to 2015:

- **Government spending**, G_{it}: government final consumption expenditure (CGV). Source: OECD Economic Outlook Database [2017].

- **Real gross domestic product**, $Y_{R,it}$, is the sum of traded and non-traded value added at constant prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

- **Total hours worked**, L_{it}, are total hours worked by persons engaged (H_EMP). By construction, total hours worked is the sum of traded and non-traded hours worked. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

- **Real consumption wage**, $W_{C,it} = W_{it}/P_{C,it}$, is constructed as the nominal aggregate wage divided by the consumer price index (CPI). Source: OECD Prices and Purchasing Power Parities Database [2017] for the consumer price index. The nominal aggregate wage is calculated by dividing labor compensation (LAB) by total hours worked by persons engaged (H_EMP). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

- **Aggregate total factor productivity**, TFP_{it}, is constructed as the Solow residual from constant-price domestic currency series of GDP, capital, LIS $s_{L,i}$, and total hours worked. In Appendix E, we detail the procedure to construct time series for the aggregate capital stock. The aggregate LIS, s_L, i, is the ratio of labor compensation (compensation of employees plus compensation of self-employed) (LAB) to GDP at current prices (VA) in sector averaged over the period 1970-2015 (except Japan: 1974-2015). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

E Construction of Time Series for Capital-Utilization-Adjusted TFP at a Sectoral Level from Imbs's [1999] Method

We construct time-varying capital utilization series using the procedure discussed in Imbs [1999] to construct our own series of utilization-adjusted TFP. We assume perfectly competitive factor and product markets. Both the traded and non-traded sectors use physical capital, K^j, and labor, L^j, according to constant returns to scale production functions which are assumed to take a CES form:

$$Y_t^j = \left[\gamma^j \left(A_t^j L_t^j\right)^{\frac{\sigma^j - 1}{\sigma^j}} + \left(1-\gamma^j\right)\left(B_t^j u_t^{K,j} K_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}. \tag{69}$$

We denote the capital utilization rate by $u_t^{K,j}$. Because more intensive capital use depreciates the capital more rapidly, we assume the following relationship between capital use and depreciation:

$$\delta_{K,t}^j = \delta_K \left(u_t^{K,j}\right)^{\phi_K}, \tag{70}$$

where δ_K is the capital depreciation rate and ϕ_K is the parameter which must be determined. At the steady-state, we have $u^{K,j} = 1$ and thus capital depreciation collapses to δ_K which is assumed to be symmetric across sectors. Firms also choose A^j and B^j along the technology frontier that we assume to be Cobb-Douglas:

$$Z_t^j = \left(A_t^j\right)^{s_{L,t}^j} \left(B_t^j\right)^{1-s_{L,t}^j}. \tag{71}$$

Note that both A^j and B^j in (69) include technology utilization. Thus in contrast to the model's notations, Y^j stands for value added at constant prices and thus is inclusive of technology utilization. While in the main text, we assume that the technology frontier (24) is CES and above we assume it is Cobb-Doublas, it leads to the same outcome, i.e., $\hat{Z}_t^j = s_L^j \hat{A}_t^j + \left(1 - s_L^j\right)\hat{B}_t^j$, see eq. (26).

Denoting the capital rental cost by $R_t = P_{J,t}\left(\delta_{K,t} + r^\star\right)$, and the labor cost by W_t^j, firms choose the capital stock, capital utilization and labor so as the maximize profit:

$$\Pi_t^j = P_t^j Y_t^j - W_t^j L_t^j - R_t K_t^j. \tag{72}$$

Profit maximization leads to first order conditions on K^j, $u^{K,j}$, L^j:

$$P_t^j \left(1-\gamma^j\right)\left(B_t^j u_t^{K,j}\right)^{\frac{\sigma^j-1}{\sigma^j}} \left(K_t^j\right)^{-\frac{1}{\sigma^j}} \left(Y_t^j\right)^{\frac{1}{\sigma^j}} = R_t, \tag{73a}$$

$$P_t^j \left(1-\gamma^j\right)\left(B_t^j K_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}} \left(u_t^{K,j}\right)^{-\frac{1}{\sigma^j}} \left(Y_t^j\right)^{\frac{1}{\sigma^j}} = P_{J,t}\delta_K \phi_K \left(u_t^{K,j}\right)^{\phi_K - 1} K^j, \tag{73b}$$

$$P_t^j \gamma^j \left(A_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}} \left(L_t^j\right)^{-\frac{1}{\sigma^j}} \left(Y_t^j\right)^{\frac{1}{\sigma^j}} = W_t^j. \tag{73c}$$

Multiplying both sides of the first equality by K^j and dividing by sectoral value added leads to the capital income share:

$$1-s^j_{L,t} = \left(1-\gamma^j\right)\left(\frac{B^j_t u^{K,j}_t K^j_t}{Y^j_t}\right)^{\frac{\sigma^j-1}{\sigma^j}}. \tag{74}$$

By using the definition of the capital income share above and inserting the expression for the capital rental cost, first-order conditions can be rewritten as follows:

$$\left(1-s^j_L\right)\frac{P^j_t Y^j_t}{P_{J,t}K^j_t} = \left(\delta_{K,t}+r^\star\right), \tag{75a}$$

$$\left(1-s^j_L\right)\frac{P^j_t Y^j_t}{P_{J,t}K^j_t} = \delta_{K,t}\phi_K, \tag{75b}$$

$$s^j_{L,t}\frac{P^j_t Y^j_t}{L^j_t} = W^j_t. \tag{75c}$$

Evaluating (75a) and (75b) at the steady-state and rearranging terms leads to:

$$\left(r^\star+\delta_K\right) = \delta_K\phi_K, \tag{76}$$

which allows us to pin down ϕ_K. We let the capital depreciation rate δ_K and the real interest rate $r^\star$ (long-run interest rate minus CPI inflation rate) vary across countries to compute ϕ_K.

In the line of Garofalo and Yamarik [2002], we use the value added share at current prices to allocate the aggregate capital stock to sector j:

$$K^j_t = \omega^{Y,j}_t K_t, \tag{77}$$

where K_t is the aggregate capital stock at constant prices and $\omega^{Y,j}_t = \frac{P^j_t Y^j_t}{P_t Y_{R,t}}$ is the value added share of sector $j = H, N$ at current prices. The methodology by Garofalo and Yamarik [2002] is based on the assumption of perfect mobility of capital across sectors and a small discrepancy in the LIS across sectors, i.e., $s^H_L \simeq s^N_L$. Inserting (77) into (75a)-(75b), first order conditions on K^j and $u^{K,j}$ now read as follows:

$$\left(1-s^j_{L,t}\right)\frac{P_t Y_{R,t}}{P_{J,t}K_t} = \left(\delta_{K,t}+r^\star\right), \tag{78a}$$

$$\left(1-s^j_{L,t}\right)\frac{P_t Y_{R,t}}{P_{J,t}K_t} = \delta_{K,t}\phi_K. \tag{78b}$$

Solving (78b) for $u^{K,j}_t$ leads to:

$$u^{K,j}_t = \left[\frac{\left(1-s^j_{L,t}\right)}{\delta_K\phi_K}\frac{P_t Y_{R,t}}{P_{J,t}K_t}\right]^{\frac{1}{\phi_K}}, \tag{79}$$

where $\phi_K = \frac{r^\star+\delta_K}{\delta_K}$ (see eq. (76)). Dropping the time index to denote the steady-state value, the capital utilization rate is:

$$u^{K,j} = \left[\frac{\left(1-s^j_L\right)}{\delta_K\phi_K}\frac{PY_R}{P_J K}\right]^{\frac{1}{\phi_K}}. \tag{80}$$

Dividing (79) by (80) leads to the capital utilization rate relative to its steady-state:

$$\frac{u^{K,j}_t}{u^{K,j}} = \left[\left(\frac{1-s^j_{L,t}}{1-s^j_L}\right)\frac{P_t Y_{R,t}}{PY_R}\frac{P_J K}{P_{J,t}K_t}\right]^{\frac{1}{\phi_K}}, \tag{81}$$

We denote total factor productivity in sector $j = H, N$ by TFP^j which is defined as follows:

$$\text{TFP}^j_t = \frac{Y^j_t}{\left[\gamma^j\left(L^j_t\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(K^j_t\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}}. \tag{82}$$

Log-linearizing (82), the Solow residual is:

$$T\hat{F}P^j_t = \hat{Y}^j_t - s^j_L\hat{L}^j_t - \left(1-s^j_L\right)\hat{K}^j_t. \tag{83}$$

Log-linearizing the production function (69) shows that the Solow residual can alternatively be decomposed into utilization-adjusted TFP and capital utilization correction:

$$\hat{\mathrm{TFP}}_t^j = \hat{Z}_t^j + \left(1 - s_L^j\right) \hat{u}_t^{K,j}, \tag{84}$$

where utilization-adjusted TFP denoted by Z^j is equal to:

$$\hat{Z}_t^j = s_L^j \hat{A}_t^j + \left(1 - s_L^j\right) \hat{B}_t^j. \tag{85}$$

Construction of time series for sectoral capital stock, K_t^j. To construct the series for the sectoral capital stock, we proceed as follows. We first construct time series for the aggregate capital stock for each country in our sample. To construct K_t, we adopt the perpetual inventory approach. The inputs necessary to construct the capital stock series are a i) capital stock at the beginning of the investment series, K_{1970}, ii) a value for the constant depreciation rate, δ_K, iii) real gross capital formation series, I_t. Real gross capital formation is obtained from OECD National Accounts Database [2017] (data in millions of national currency, constant prices). We construct the series for the capital stock using the law of motion for capital in the model:

$$K_{t+1} = I_t + (1 - \delta_K) K_t. \tag{86}$$

for $t = 1971, ..., 2015$. The value of δ_K is chosen to be consistent with the ratio of capital depreciation to GDP observed in the data and averaged over 1970-2015:

$$\frac{1}{46} \sum_{t=1970}^{2015} \frac{\delta_K P_{J,t} K_t}{Y_t} = \frac{CFC}{Y}, \tag{87}$$

where $P_{J,t}$ is the deflator of gross capital formation series, Y_t is GDP at current prices, and CFC/Y is the ratio of consumption of fixed capital at current prices to nominal GDP averaged over 1970-2015. Deflator of gross capital formation, GDP at current prices and consumption of fixed capital are taken from the OECD National Account Database [2017]. The second column of Table 6 shows the value of the capital depreciation rate obtained by using the formula (87). The capital depreciation rate averages to 5%.

To have data on the capital stock at the beginning of the investment series, we use the following formula:

$$K_{1970} = \frac{I_{1970}}{g_I + \delta_K}, \tag{88}$$

where I_{1970} corresponds to the real gross capital formation in the base year 1970, g_I is the average growth rate from 1970 to 2015 of the real gross capital formation series. The system of equations (86), (87) and (88) allows us to use data on investment to solve for the sequence of capital stocks and for the depreciation rate, δ_K. There are 47 unknowns: K_{1970}, δ_K, K_{1971}, ..., and K_{2015}, in 47 equations: 45 equations (86), where $t = 1971, ..., 2015$, (87), and (88). Solving this system of equations, we obtain the sequence of capital stocks and a calibrated value for depreciation, δ_K. Following Garofalo and Yamarik [2002], the gross capital stock is then allocated to traded and non-traded industries by using the sectoral value added share, see eq. (77).

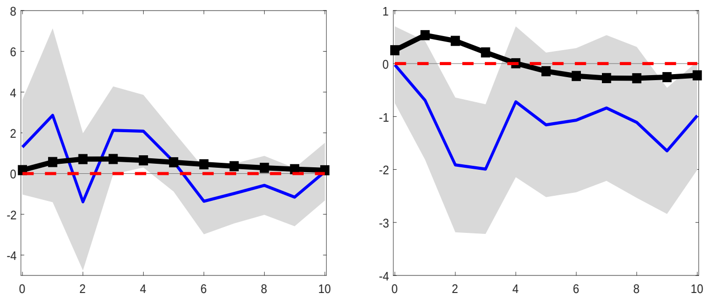
Construction of time series for sectoral TFPs. Sectoral TFPs, TFP_t^j, at time t are constructed as Solow residuals from constant-price (domestic currency) series of value added, Y_t^j, capital stock, K_t^j, and hours worked, L_t^j, by using eq. (83). The LIS in sector j, s_L^j, is the ratio labor compensation (compensation of employees plus compensation of self-employed) to nominal value added in sector $j = H, N$, averaged over the period 1970-2015 (except Japan: 1974-2015). Data for the series of constant price value added (VA_QI), current price value added (VA), hours worked (H_EMP) and labor compensation (LAB) are taken from the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

Construction of time series for real interest rate, $r^\star$. The real interest rate is computed as the real long-term interest rate which is the nominal interest rate on 10 years government bonds minus the rate of inflation which is the rate of change of the Consumption Price Index (CPI). Sources: OECD Economic Outlook Database [2017] for the long-term interest rate on government bonds and OECD Prices and Purchasing Power Parities Database [2017] for the CPI. Data coverage: 1970-2015 except for IRL (1990-2015) and KOR (1983-2015). The first column of Table 6 shows the value of the real interest rate which averages 3% over the period 1970-2015.

Construction of time series for capital utilization, $u_t^{K,j}$. To construct time series for the capital utilization rate, $u_t^{K,j}$, we proceed as follows. We use time series for the real interest rate, $r^\star$ and for the capital depreciation rate, δ_K to compute $\phi = \frac{r^\star + \delta_K}{\delta_K}$ (see eq. (76)). Once we have calculated ϕ for each country, we use time series for the LIS in sector j, $s_{L,t}^j$, GDP at current prices, $P_t Y_{R,t} = Y_t$, the deflator for investment, $P_{J,t}$, and times series for the aggregate capital stock, K_t

Table 6: Data on Real Interest Rate ($r^\star$) and Fixed Capital Depreciation Rate (δ_K)

Country	$r^\star$	δ_K
AUS	0.029	0.058
AUT	0.030	0.040
BEL	0.033	0.041
CAN	0.032	0.100
DNK	0.046	0.062
ESP	0.020	0.036
FIN	0.025	0.048
FRA	0.032	0.043
GBR	0.025	0.031
IRL	0.035	0.042
ITA	0.025	0.029
JPN	0.017	0.050
KOR	0.052	0.061
NLD	0.030	0.035
NOR	0.027	0.102
PRT	0.023	0.038
SWE	0.031	0.026
USA	0.026	0.069
OECD	0.030	0.050



(a) Traded Capital Utilization Rate (b) Non-Traded Capital Utilization Rate

Figure 6: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Capital Utilization Rate. Notes: Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC while the dashed red line shows predictions of a model with Cobb-Douglas production functions and abstracting from capital and technology utilization.

to compute time series for $u_t^{K,j}$ by using the formula (79). Fig. 6 plots empirical responses of the capital utilization rate for the traded and the non-traded sector shown in blue lines. Black lines with squares plots theoretical responses for $u_t^{K,H}$ and $u_t^{K,N}$. The confidence bounds indicate that none of the responses are statistically significant. The reason is that there exists a wide cross-country dispersion in the movement of the capital utilization rates across countries in terms of both direction and magnitude. As shown in Fig. 6(a), our model reproduces well the adynamic adjustment of the capital utilization rate for tradables while Fig. 6(b) indicates that the model tends to somewhat overstate the response of $u^{K,N}$, especially in the short-term.

Construction of time series for utilization-adjusted TFP, Z_t^j. According to (84), capital utilization-adjusted sectoral TFP expressed in percentage deviation relative to the steady-state reads:

$$\begin{aligned}\hat{Z}_t^j &= \hat{\text{TFP}}_t^j - \left(1 - s_L^j\right)\hat{u}_t^{K,j}, \\ \ln Z_t^j - \ln \bar{Z}_t^j &= \left(\ln \text{TFP}_t^j - \ln \bar{\text{TFP}}_t^j\right) - \left(1 - s_L^j\right)\left(\ln u_t^{K,j} - \ln \bar{u}_t^{K,j}\right). \end{aligned} \tag{89}$$

The percentage deviation of variable X_t from initial steady-state is denoted by $\hat{X}_t = \ln X_t - \ln \bar{X}_t$ where we let the steady-state varies over time; the time-varying trend $\ln \bar{X}_t$ is obtained by applying a HP filter with a smoothing parameter of 100 to logged time series. To compute $T\hat{F}P_t^j$, we take the log of TFP_t^j and subtract the trend component extracted from a HP filter applied to logged TFP_t^j, i.e., $\ln \text{TFP}_t^j - \ln \bar{\text{TFP}}_t^j$. The same logic applies to $u_t^{K,j}$. Once we have computed the percentage deviation $\ln Z_t^j - \ln \bar{Z}_t^j$, we reconstruct time series for $\ln Z_t^j$:

$$\ln Z_t^j = \left(\ln Z_t^j - \ln \bar{Z}_t^j\right) + \ln \bar{Z}_t^j. \tag{90}$$

The construction of time series of logged sectoral TFP, $\ln \text{TFP}_t^j$, capital utilization-adjusted sectoral TFP, $\ln Z_t^j$, is consistent with the movement of capital utilization along the business cycle.

F Construction of Non-Traded Demand Components

In this section, we detail the construction of time series for non-traded government consumption, G_t^N, non-traded consumption, C_t^N, and non-traded investment, J_t^N. We use the World Input-Output Databases ([2013], [2016]). The 2013 release provides data for eleven 1-digit ISIC-rev.3 industries over the period 1995-2011 while the 2016 release provides data for thirteen 1-digit-rev.4 industries over the period 2000-2014. As sectoral data are classified using identical ISIC revisions in both the EU KLEMS and WIOD datasets, we map the WIOD ISIC-rev.4 classification (the 2016 release) into the WIOD ISIC-rev.3 classification (the 2013 release) in accordance with the concordance Table 5. Consistent with the methodology we used to extend series taken from the EU KLEMS ([2011], [2017]), time series for traded and non-traded variables from the WIOD [2013] dataset (available over the period 1995-2011) are extended forward up to 2014 using annual growth rate estimated from WIOD [2016] series (available over the period 2000-2014). Coverage: 1995-2014 except for NOR (2000-2014).

To compute non-traded demand components, we have to overcome two difficulties. While the input-output WIOD dataset gives purchases of non-traded goods and services from the private sector, data also includes purchases of imported goods and services. Whereas consumption and investment expenditure can be split into traded and non-traded expenditure, this split does not exist for government spending for most of the countries in our sample. We detail below how we overcome the two aforementioned difficulties.

To begin with, the non traded and the home-produced traded goods markets must clear such that:

$$Y^N = C^N + J^N + G^N + X^N - M^N, \tag{91a}$$
$$Y^H = C^H + J^H + G^H + X^H - M^H, \tag{91b}$$

where Y^j is value added at constant prices in sector $j = H, N$, C^j consumption in good j, J^j investment in good j, G^j government consumption in good j and X^j stands for exports. Imports (by households, firms, and the government) in good j denoted by M^j can be broken into three components:

$$M^N = C^{N,F} + J^{N,F} + G^{N,F}, \tag{92a}$$
$$M^H = C^{H,F} + J^{H,F} + G^{H,F}, \tag{92b}$$

where $C^{H,F}$, $J^{H,F}$ and $G^{H,F}$ are foreign-produced traded good for consumption, investment and government spending respectively, and $C^{N,F}$, $J^{N,F}$ and $G^{N,F}$ denote consumption, investment and government spending domestic demand for non-traded goods produced by the rest of the world respectively. Next, each demand component C^j, J^j, G^j of sector $j = H, N$ can be split into a domestic demand for home-produced good (denoted by $C^{j,D}$, $J^{j,D}$, $G^{j,D}$) and a domestic demand for foreign-produced good (denoted by $C^{j,F}$, $J^{j,F}$, $G^{j,F}$) by the rest of the world. This decomposition yields the following identities:

$$C^N = C^{N,D} + C^{N,F}, \tag{93a}$$
$$J^N = J^{N,D} + J^{N,F}, \tag{93b}$$
$$G^N = G^{N,D} + G^{N,F}, \tag{93c}$$
$$C^H = C^{H,D} + C^{H,F}, \tag{93d}$$
$$J^H = J^{H,D} + J^{H,F}, \tag{93e}$$
$$G^H = G^{H,D} + G^{H,F}. \tag{93f}$$

We denote total imports by M which consist of imports of consumption goods by households and the government and imports of capital goods by firms:

$$M = M^N + M^H. \tag{94}$$

Total exports to the rest of the world include exports of non-traded and traded goods:

$$X = X^N + X^H. \tag{95}$$

Obviously, we are aware that non-traded goods are not subject to international trade but we use this terminology to avoid confusion between the model's annotations and the data.

Combining (91a) and (91b) and using (94)-(95) leads to the standard accounting identity between the sum of sectoral value added and final expenditure:

$$\begin{aligned} P^H Y^H + P^N Y^N &= P_C C + P_J J + G + P_X X - P_M M, \\ Y &= P_C C + P_J J + G + NX, \end{aligned} \tag{96}$$

where we normalize P_G to one in (96) to be consistent with the model's annotations. Dividing (96) by GDP implies that consumption expenditure, investment expenditure, government spending, and net exports as a share of GDP must sum to one:

$$1 = \omega_C + \omega_J + \omega_G + \omega_{NX}. \tag{97}$$

We focus first on components of government spending. We use the accounting identity (91a) to compute times series for G^N:

$$P^N G^N = P^N Y^N - P^N C^N - P^N J^N - P^N X^N + P^N M^N \tag{98}$$

We divide both sides by nominal GDP, i.e., $P^H Y^H + P^N Y^N = Y$. The LHS of eq. (98) divided by nominal GDP reads:

$$\begin{aligned} \frac{P^N G^N}{Y} &= \frac{P^N G^N}{G} \frac{G}{Y}, \\ &= \omega_{G^N} \omega_G. \end{aligned} \tag{99}$$

Making use of (98)-(99), we can calculate time series for ω_{G^N} as follows:

$$\omega_{G^N} = \frac{1}{\omega_G} \left[\frac{P^N Y^N}{Y} - \frac{P^N C^N - P^N J^N - P^N X^N + P^N M^N}{Y} \right]. \tag{100}$$

While in the model, we assume that non-traded industries do not trade with the rest of the world, the definition of a non-traded industry in the data is based on an arbitrary rule. Industries whose the sum of exports plus imports in percentage of GDP is lower than 20% are treated as non-tradables; since these industries trade, we have to split G^N into $G^{N,D}$ and $G^{N,F}$ so as to calculate time series for $\omega_{G^{N,D}}$. According to (92a), total imports of non-traded goods and services include imports by households, firms and the government, i.e., $M^N = C^{N,F} + J^{N,F} + G^{N,F}$. Thus, $G^{N,F} = M^N - C^{N,F} - J^{N,F}$, from which we get $\omega_{G^{N,F}} = G^{N,F}/G$. By using (93c), $G^{N,D}$ can be computed as $G^{N,D} = G^N - G^{N,F}$. This allows us to recover the share of non-traded government consumption which excludes imports: $\omega_{G^{N,D}} = \omega_{G^N} - \omega_{G^{N,F}}$. Next, government spending on foreign-produced traded goods $G^{H,F}$ can be calculated by using the definition of imports of final traded goods and services: $M^F = C^{H,F} + J^{H,F} + G^{H,F}$, where $C^{H,F}$ and $J^{H,F}$ are consumption and investment in home-produced traded goods. Rearranging the last equation give $G^{H,F} = M^H - C^{H,F} - J^{H,F}$. It follows that $\omega_{G^{H,F}} = G^{H,F}/G$. Once we have time series for $G^{N,D}$, $G^{N,F}$, $G^{H,F}$, we can recover time series for government spending in home-produced traded goods, $G^{H,D}$ by using the accounting identity which says that total government spending is equal to the sum of four components: $G = G^{N,D} + G^{N,F} + G^{H,D} + G^{H,F}$. Dividing both sides by G gives:

$$\begin{aligned} 1 &= \omega_{G^{N,D}} + \omega_{G^{N,F}} + \omega_{G^{H,F}} + \omega_{G^{H,D}}, \\ 1 &= \omega_{G^{N,D}} + \omega_{G^F} + \omega_{G^{H,D}}, \\ \omega_{G^{H,D}} &= 1 - \omega_{G^{N,D}} - \omega_{G^F}, \end{aligned} \tag{101}$$

where $\omega_{G^{N,F}} + \omega_{G^{H,F}} = \omega_{G^F}$ is the import content of government spending

Since data taken from WIOD allows to differentiate between domestic demand for home- and foreign-produced goods, we are able to construct time series for the home content of consumption and investment in traded goods as follows:

$$\alpha^H = \frac{P^H C^{H,D}}{P^T C^T} = \frac{\left(P^T C^T - C^{H,F}\right)}{P^T C^T}, \tag{102a}$$

$$\alpha_J^H = \frac{P^H J^{H,D}}{P_J^T J^T} = \frac{\left(P_J^T J^T - J^{H,F}\right)}{P_J^T J^T}. \tag{102b}$$

To compute time series for non-traded consumption, $C^{N,D}$, and non-traded investment, $J^{N,D}$, we make use of imports of final consumption and investment goods, and then we divide by total consumption and investment expenditure, respectively, to obtain their non-tradable content:

$$1 - \alpha_C = \frac{P^N C^{N,D}}{P_C C} = \frac{P^N \left(C^N - C^{N,F}\right)}{P_C C}, \tag{103a}$$

$$1 - \alpha_J = \frac{P^N J^{N,D}}{P_J J} = \frac{P^N \left(J^N - J^{N,F}\right)}{P_J J}. \tag{103b}$$

We obtain data on GDP and its demand components (consumption, investment, government spending, exports and imports) from the World Input-Output Databases ([2013], [2016]) for all years between 1995 and 2014 and all 1-digit ISIC rev.3 and rev.4 industries. Indexing the sector with a superscript $j = H, N$ and indexing the origin of demand of goods and services with a superscript $k = D, F$ where D refers to domestic demand of home-produced goods and services and F refers to domestic demand of foreign-produced goods and services, we provide below details about data construction:

- Consumption $C^{j,k}$ for $j = H, N$ and $k = D, F$: total consumption expenditure (at current prices) by households and by non-profit organizations serving households on good j produced by firms from country k. Data coverage: 1995-2014 except for NOR (2000-2014).
- Investment $I^{j,k}$ for $j = H, N$ and $k = D, F$: total gross fixed capital formation plus changes in inventories and valuables (at current prices) on good j produced by firms from country k. Data coverage: 1995-2014 except for NOR (2000-2014).
- Government spending $G^{j,k}$ for $j = H, N$ and $k = D, F$: total consumption expenditure (at current prices) by government on good j produced by firms from country k. Data coverage: 1995-2014 except for NOR (2000-2014).
- Exports $X^{j,k}$ for $j = H, N$ and $k = D, F$: total exports (at current prices) of final and intermediate good j produced by firms from country k. Data coverage: 1995-2014 except for NOR (2000-2014).
- Imports $M^{j,k}$ for $j = H, N$ and $k = D, F$: total imports (at current prices) of final and intermediate good j produced by firms from country k. Data coverage: 1995-2014 except for NOR (2000-2014).

Finally, when we use (98) to obtain the time series for G^N, the valuation of output Y^N and imports M^N include taxes and subsidies on products and trade and transport margins respectively. These adjustments are necessary to achieve consistency and to balance resources and uses.

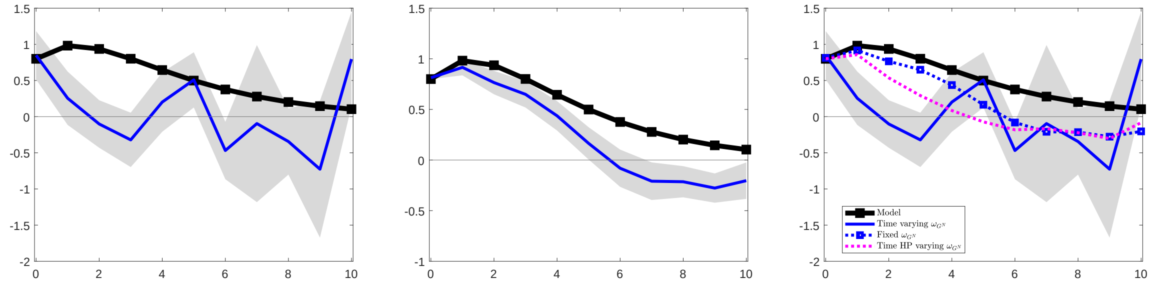
Response of non-traded government consumption to government spending shock. World Input-Output Databases ([2013], [2016]) allow us to get time series for total government spending with a breakdown of G_{it} by components $G_{it}^{j,D}$ and $G_{it}^{j,F}$ for $j = H, N$. All the obtained series are available at current prices which allows us to compute the non-tradable content of government consumption. To compute time series for non-traded government consumption at constant prices, we can take two routes. First, we can use time series for the non-tradable content of government spending, $\omega_{G^{N,D},it}$, obtained from WIOD dataset and then we construct time series for $G_{it}^{N,D}$ at constant prices by multiplying time series for real government final consumption expenditure, G_{it}, with the time-varying non-tradable content of government consumption, $\omega_{G^{N,D},it}$. Second, we can alternatively construct time series for G_{it}^N at constant prices by multiplying time series for real government final consumption expenditure, G_{it}, with $\omega_{G^{N,D},i}$ averaged over 1995-2014. This alternative is guided by the data as the non-tradable content of government spending is somewhat erratic. In Fig. 7, we plot empirical responses of non-traded government consumption at constant prices to an exogenous increase in aggregate government consumption by 1% of GDP shown in the blue line and contrast them with theoretical responses shown in black lines with squares. To compute the theoretical response, we proceed as follows. Denoting by ω_{G^N} the non-traded content of government spending, by ω_{G^H} the home component of the traded content of government spending, we have:

$$G(t) = \omega_{G^N} G(t) + \omega_{G^H} G(t) + \omega_{G^F} G(t), \tag{104}$$

where ω_{G^F} is the imported content of government spending. Using the dynamic adjustment of $dG(t)/Y$ described by eq. (31) and assuming that ω_{G^j} is fixed over time, the endogenous response of the content of government spending in good $j = H, F, N$ to an exogenous shock to aggregate government consumption reads:

$$\frac{dG^j(t)}{Y} = \omega_{G^j} \left[e^{-\xi t} - (1-g)\, e^{-\chi t} \right]. \tag{105}$$

As can be seen in Fig. 7(a) which plots the response of G^N (in real terms) constructed by using the time-varying non-traded content of government spending, the theoretical response shown in the black line with squares and derived from (105) accounts reasonably well for the empirical IRF. Since the empirical response of G^N to an exogenous shock to government consumption is erratic, we alternatively construct time series for G^N by assuming that ω_{G^N} is constant over time and corresponds to its average over 1995-2014. As shown in Fig. 7(b), the theoretical response derived from (105) replicates very well the empirical response the first four years and somewhat overstates the empirical response afterwards. However, the empirical response is not statistically significant after six years. In Fig. 7(c), we compare the model's prediction shown in the solid black line with



(a) Non-Traded Government Consumption: Time-Varying ω_{G^N}

(b) Non-Traded Government Consumption: ω_{G^N} Averaged 1970-2014

(c) Non-Traded Government Consumption: All Specifications

Figure 7: Empirical vs. Theoretical Responses of Non-Traded Government Consumption following a Shock to Aggregate Government Consumption. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP. In Fig. 7(a), we use time series for the non-traded content of government spending, $\omega_{G^N,it}$, obtained from WIOD dataset and then we construct time series for G^N_{it} at constant prices by multiplying time series for real government final consumption expenditure, G_{it}, with $\omega_{G^N,it}$. In Fig. 7(b), we estimate the response of G^N at constant prices whose time series are obtained by calculating the product of time series for real government final consumption expenditure, G_{it}, with the non-traded content of government consumption averaged over 1995-2014. In Fig. 7(c), we compare the model's prediction shown in the solid black line with squares with the empirical responses with time-varying and fixed ω_{G^N} shown in the solid blue line and the dotted blue line with squares, respectively. In the dotted magenta line line, we allow the non-traded content of government consumption to vary across time and smooth its adjustment by applying a HP filter to $\omega_{G^N,it}$ with a smoothing parameter of 100. Shaded areas indicate the 90 percent confidence bounds. The black line with squares shows the theoretical response of G^N. Sample: 18 OECD countries, 1970-2014 (except for NOR (2000-2014)), annual data.

squares with the empirical responses with time-varying and fixed ω_{G^N} shown in the dotted blue line with squares and the solid blue line, respectively. In the dotted magenta line line, we allow the non-traded content of government consumption to vary across time and smooth its adjustment by applying a HP filter to $\omega_{G^N,it}$ with a smoothing parameter of 100.

G Construction of Time Series for FBTC

In this section, we detail the methodology to construct time series for capital-utilization-adjusted-FBTC in sector $j = H, N$. We choose the initial steady-state in a model with Cobb-Douglas production functions as the normalization point. When we calibrate the model with Cobb-Douglas production functions to the data, the ratios we target are averaged values over 1970-2015.

The starting point is the ratio of the labor to the capital income share in sector j given by eq. (59) which can be solved for capital-utilization-adjusted-FBTC in sector j:

$$\text{FTBC}^j_{t,adjK} \equiv \left(\frac{B^j_t}{A^j_t}\right)^{\frac{1-\sigma^j}{\sigma^j}} = S^j_t \frac{1-\gamma^j}{\gamma^j} \left(k^j_t\right)^{-\frac{1-\sigma^j}{\sigma^j}} \left(u^{K,j}_t\right)^{-\frac{1-\sigma^j}{\sigma^j}}, \tag{106}$$

where $u^{K,j}_t$ is constructed by using the formula (79).

Since we normalize CES production functions so that the relative weight of labor and capital is consistent with the labor and capital income share in the data, solving for γ^j leads to:

$$\gamma^j = \left(\frac{\bar{A}^j}{\bar{y}^j}\right)^{\frac{1-\sigma^j}{\sigma^j}} \bar{s}^j_L, \tag{107a}$$

$$1-\gamma^j = \left(\frac{\bar{B}^j \bar{u}^{K,j} \bar{k}^j}{\bar{y}^j}\right)^{\frac{1-\sigma^j}{\sigma^j}} \left(1-\bar{s}^j_L\right). \tag{107b}$$

Dividing (107a) by (107b) leads to:

$$\bar{S}^j = \frac{\gamma^j}{1-\gamma^j} \left(\frac{\bar{B}^j \bar{u}^{K,j} \bar{k}^j}{\bar{A}^j}\right)^{\frac{1-\sigma^j}{\sigma^j}}, \tag{108}$$

where variables with a bar are averaged values of the corresponding variables over 1970-2015.

The methodology adopted to calculate γ^j amounts to using averaged values as the normalization point to compute time series for FBTC. Dividing (106) by (108) yields:

$$\left(\frac{B^j_t/\bar{B}^j}{A^j_t/\bar{A}^j}\right)^{\frac{1-\sigma^j}{\sigma^j}} = \frac{S^j_t}{\bar{S}^j} \left(\frac{k^j_t}{\bar{k}^j}\right)^{-\frac{1-\sigma^j}{\sigma^j}} \left(\frac{u^{K,j}_t}{\bar{u}^{K,j}}\right)^{-\frac{1-\sigma^j}{\sigma^j}}. \tag{109}$$

Eq. (109) corresponds to eq. (5) in the main text. To construct time series for $\text{FTBC}^j_{t,adjK}$, we plug estimates for the elasticity of substitution between capital and labor, σ^j, and time series for the ratio of the labor to the capital income share, S^j_t, the capital-labor ratio, k^j_t, and the capital utilization rate, $u^{K,j}_t$, in sector $j = H, N$. Next we divide yearly data by averaged values of the corresponding variable over 1970-2015. In Appendix L.3, we detail the empirical strategy to estimate the elasticity of substitution between capital and labor σ^j.

H Construction of the Unit Cost for Producing

In this section, we detail how we construct time series for the real unit cost for producing. Dividing (55a) by (55b) leads to a positive relationship between the relative cost of labor and the capital-labor ratio in sector j:

$$\frac{W^j}{R} = \frac{\gamma^j}{1-\gamma^j}\left(\frac{B^j}{A^j}\right)^{\frac{1-\sigma^j}{\sigma^j}}\left(\frac{\tilde{K}^j}{L^j}\right)^{\frac{1}{\sigma^j}}, \tag{110}$$

where $\tilde{K}^j = u^{K,j}K^j$. We manipulate (110) To to determine the conditional demands for both inputs:

$$\begin{aligned} L^j &= \tilde{K}^j\left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j}\left(\frac{B^j}{A^j}\right)^{1-\sigma^j}\left(\frac{W^j}{R}\right)^{-\sigma^j}, && (111a)\\ \tilde{K}^j &= L^j\left(\frac{1-\gamma^j}{\gamma^j}\right)^{\sigma^j}\left(\frac{B^j}{A^j}\right)^{\sigma^j-1}\left(\frac{W^j}{R}\right)^{\sigma^j}. && (111b)\end{aligned}$$

Inserting eq. (111a) (eq. (111a) resp.) in the CES production function (53) and solving for L^j ($\tilde{K}^j$ resp.) leads to the conditional demand for labor (capital resp.):

$$\begin{aligned} \gamma^j\left(A^jL^j\right)^{\frac{\sigma^j-1}{\sigma^j}} &= \left(Y^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j}{A^j}\right)^{1-\sigma^j}\left(X^j\right)^{-1}, && (112a)\\ \left(1-\gamma^j\right)\left(B^j\tilde{K}^j\right)^{\frac{\sigma^j-1}{\sigma^j}} &= \left(Y^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\left(\frac{R}{B^j}\right)^{\sigma^j}\left(X^j\right)^{\frac{\sigma^j}{1-\sigma^j}}, && (112b)\end{aligned}$$

where X^j is given by:

$$X^j = \left(\gamma^j\right)^{\sigma^j}\left(A^j\right)^{\sigma^j-1}\left(W^j\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}\left(B^j\right)^{\sigma^j-1}R^{1-\sigma^j}. \tag{113}$$

Total cost is equal to the sum of the labor and capital cost:

$$C^j = W^jL^j + R\tilde{K}^j. \tag{114}$$

Inserting conditional demand for inputs (112) into total cost (114), we find that C^j is homogenous of degree one with respect to value added:

$$C^j = c^jY^j, \quad \text{with} \quad c^j = \left(X^j\right)^{\frac{1}{1-\sigma^j}}, \tag{115}$$

where the unit cost for producing is:

$$c^j = \left[\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j}{A^j}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{R}{B^j}\right)^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}. \tag{116}$$

Because we investigate how firms adjust technology when the unit cost for producing is modified, we construct a technology adjusted unit cost for producing, denoted by UC^j:

$$\text{UC}^j = \left[\left(\gamma^j\right)^{\sigma^j}\left(W^j\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}R^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}, \tag{117}$$

where γ^j is described by eq. (107a). To ensure that $0 < \gamma^j < 1$, we normalize $\bar{A}^j = \bar{B}^j = \bar{y}^j = 1$ (y^j is a volume index and thus this assumption has no impact) so that $\gamma^j = s^j_L$ averaged over 1970-2015. To construct time series for UC^j, we insert our estimates of the elasticity of substitution between capital and labor, σ^j, shown in columns 18 and 19 of Table 7, and we plug time series for the wage rate in sector j and the capital rental rate. While we assume perfect mobility of capital, the capital rental rate in the traded sector may temporarily deviate from the capital rental rate in the non-traded sector in the data. The property of constant returns to scale in production implies that value added is exhausted by the payment of factors, i.e., $P^jY^j = W^jL^j + R^j\tilde{K}^j$. Solving the latter equality for the capital rental rate leads to $R^j = \frac{P^jY^j - W^jL^j}{u^{K,j}K^j}$ where P^jY^j is value added at current prices, W^jL^j is labor compensation, K^j is the stock of capital at constant prices, and $u^{K,j}$ the capital utilization rate in sector j (see eq. (79)).

Firms choose the optimal level of value added by equating sectoral prices to the ratio of the unit cost for producing to the capital-utilization-adjusted TFP:

$$P^j = \frac{\mathrm{UC}^j}{u^{Z,j}}. \tag{118}$$

Eq. (118) shows that in face of a rise the unit cost for producing UC^j, firms can increase prices, P^j, or can achieve some technology improvement (i.e., $u^{Z,j}$ increases), or both. Firms will increase the technology utilization rate by a larger amount in sectors/countries where the cost of adjusting technology is lower.

I Technology Frontier and FBTC

Following Caselli and Coleman [2006] and Caselli [2016], the menu of possible choices of production functions is represented by a set of possible (A^j, B^j) pairs. These pairs are chosen along the technology frontier which is assumed to take a CES form:

$$\left[\gamma_Z^j \left(A^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} + \left(1-\gamma_Z^j\right)\left(B^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}\right]^{\frac{\sigma_Z^j}{\sigma_Z^j-1}} \leq Z^j(t), \tag{119}$$

where $Z^j > 0$ is the height of the technology frontier, $0 < \gamma_Z^j < 1$ is the weight of labor efficiency in TFP and $\sigma_Z^j > 0$ corresponds to the elasticity of substitution between labor and capital efficiency. Totally differentiating (119) leads to

$$\begin{aligned} 0 &= \gamma_Z^j \left(A^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} \hat{A}^j(t) + \left(1-\gamma_Z^j\right)\left(B^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} \hat{B}^j(t), \\ \frac{\hat{B}^j(t)}{\hat{A}^j(t)} &= -\frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}}. \end{aligned} \tag{120}$$

Firms choose A^j and B^j along the technology frontier so that minimizes the unit cost function described by (116) subject to (119) which holds as an equality. Differentiating (116) w.r.t. A^j and B^j (while keeping W^j and R fixed) leads to:

$$\hat{c^j}(t) = -\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j}\left(c^j(t)\right)^{\sigma^j-1}\hat{A}^j(t) - \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{R(t)}{B^j(t)}\right)^{1-\sigma^j}\left(c^j(t)\right)^{\sigma^j-1}\hat{B}^j(t). \tag{121}$$

Setting (121) to zero and inserting (119), the cost minimization leads to the following optimal choice of technology:

$$\begin{aligned} \left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j}\left(\frac{W^j(t)}{R(t)}\right)^{1-\sigma^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{1-\sigma^j} &= \frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}}, \\ \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}\left(\frac{B^j(t)}{A^j(t)}\right)^{-\left(\frac{1-\sigma_Z^j}{\sigma_Z^j}\right)\frac{1}{\sigma^j}} &= \left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)^{\frac{1}{\sigma^j}}\left(\frac{1-\gamma^j}{\gamma^j}\right)\left(\frac{W^j(t)}{R(t)}\right)^{-\frac{1-\sigma^j}{\sigma^j}}, \\ \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}\left[1-\left(\frac{1-\sigma_Z^j}{1-\sigma^j}\right)\frac{1}{\sigma_Z^j}\right]} &= \left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)^{\frac{1}{\sigma^j}}\left(\frac{1-\gamma^j}{\gamma^j}\right)\left(\frac{W^j(t)}{R(t)}\right)^{-\frac{1-\sigma^j}{\sigma^j}}, \end{aligned} \tag{122}$$

where $\mathrm{FBCT}^j_{adjK} = \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$ is capital-utilization-adjusted-FBTC in sector j. According to eq. (122), when we let σ_Z^j tend toward one so that the technology frontier is Cobb-Douglas, relative capital efficiency $B^j(t)/A^j(t)$ in sector j is decreasing in the wage-to-capital-rental-rate ratio $W^j(t)/R(t)$. Thereby, in face of a rise in W^j/R, firms increase A^j and thus lower B^j/A^j. Intuitively, it is optimal for firms to bias factor efficiency toward the most expensive factor. However, when A^j and B^j are gross complements in technology production, the rise in A^j will require more units of

B^j (which may increase disproportionately) which may result in an increase in B^j/A^j if the gross complementarity between A^j and B^j is high enough, i.e.,

$$\frac{1-\sigma_Z^j}{\sigma_Z^j} > 1-\sigma^j. \tag{123}$$

When the inequality (123) holds, then a rise in $W^j(t)/R(t)$ may increase FBCT^j_{adjK} instead of decreasing it.

Denoting $d^j = \frac{1}{\sigma^j}\ln\left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)+\ln\left(\frac{1-\gamma^j}{\gamma^j}\right)$ and $\Omega^j(t) = \left(\frac{W^j(t)}{R(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$, and taking log of both sides of eq. (122) leads to:

$$\ln \text{FBTC}^j_{adjK}(t) = e^j + \left(\delta^j\right)^{-1}\ln\Omega^j(t), \tag{124}$$

where $e^j = \frac{d^j}{\delta^j}$ with $\delta^j = \left[\left(\frac{1-\sigma_Z^j}{1-\sigma^j}\right)\frac{1}{\sigma_Z^j}-1\right] \gtreqless 0$. Our objective is to estimate the responses of $\text{FBTC}^j_{adjK}(t)$ and $\Omega^j(t)$ to an increase in government consumption by 1% of GDP and to investigate whether the response of capital-utilization-adjusted FBTC in sector j moves in opposite direction relative to the response of the adjusted wage-to-capital-rental-rate ratio $\left(\delta^j\right)^{-1}\ln\Omega^j(t)$. When $\sigma_Z^j = 1$, δ^j collapses to minus one so that $\ln\text{FBTC}^j_{adjK}(t)$ and $\ln\Omega^j(t)$ should move in opposite direction. However, our estimates reveal that both co-move which suggest that σ_Z^j takes values much lower than one because δ^j turns out to be positive, which thus generates a positive relationship between $\ln\text{FBTC}^j_{adjK}(t)$ and $\left(\delta^j\right)^{-1}\ln\Omega^j(t)$. To explore empirically the relationship between capital-utilization-adjusted FBTC and the adjusted wage-to-capital-rental-rate ratio, we have to estimate the elasticity of substitution between labor- and capital-augmenting efficiency σ_Z^j to compute the value of δ^j in order to scale the response of $\Omega^j(t) = \left(\frac{W^j(t)}{R(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$.

To pin down the value of σ_Z^j, we proceed as follows. Using the fact that $\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j}\left(c^j(t)\right)^{\sigma^j-1} = s_L^j(t)$, eq. (121) can be rewritten as $-s_L^j\hat{A}^j(t)-\left(1-s_L^j\right)\hat{B}^j(t) = \hat{c}^j(t)$. Setting this equality to zero and inserting (120) leads to:

$$\frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}} = \frac{s_L^j(t)}{1-s_L^j(t)} \equiv S^j(t). \tag{125}$$

Eq. (125) corresponds to eq. (25) in the main text. Demand for inputs can be rewritten in terms of their respective cost in value added; for labor, we have $s_L^j(t) = \gamma^j\left(\frac{A^j(t)}{y^j(t)}\right)^{\frac{\sigma^j-1}{\sigma^j}}$. Applying the same logic for capital and denoting the ratio of labor to capital income share by $S^j(t) \equiv \frac{s_L^j(t)}{1-s_L^j(t)}$, we have:

$$S^j(t) \equiv \frac{s_L^j(t)}{1-s_L^j(t)} = \frac{\gamma^j}{1-\gamma^j}\left(\frac{B^j(t)u^{K,j}(t)K^j(t)^j(t)}{A^j(t)L^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}. \tag{126}$$

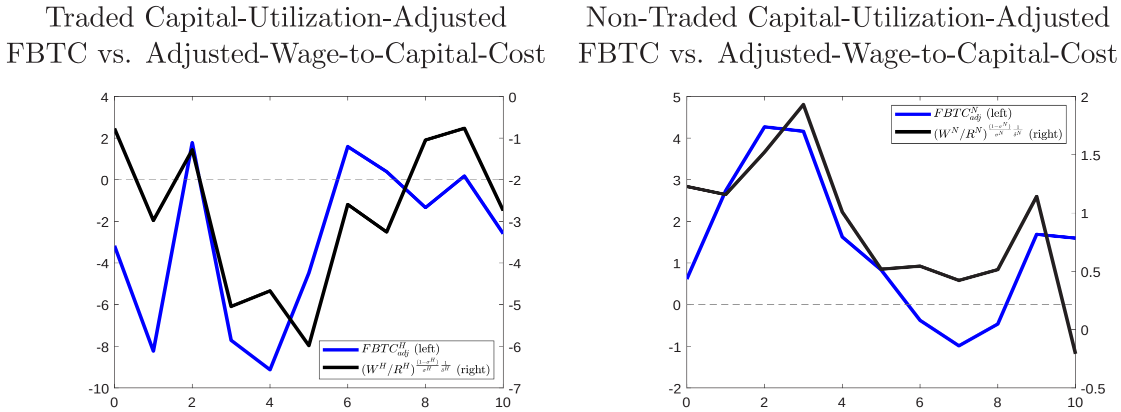
Making use of (125) to eliminate B^j/A^j from eq. (126) and solving leads to:

$$u^{K,j}(t)k^j(t) = \left(S^j(t)\right)^{\frac{\sigma^j}{1-\sigma^j}-\frac{\sigma_Z^j}{1-\sigma_Z^j}}\left(\frac{1-\gamma^j}{\gamma^j}\right)^{\frac{\sigma^j}{1-\sigma^j}}\left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)^{\frac{\sigma_Z^j}{1-\sigma_Z^j}}. \tag{127}$$

Denoting $f^j = \frac{\sigma^j}{1-\sigma^j}\ln\left(\frac{1-\gamma^j}{\gamma^j}\right)+\frac{\sigma_Z^j}{1-\sigma_Z^j}\ln\left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)$ and taking log of both sides of eq. (127) leads to:

$$\ln\left[u^{K,j}(t)k^j(t)\right] = f^j + \zeta^j\ln S^j(t). \tag{128}$$

where $\zeta^j = \left[\frac{\sigma^j}{1-\sigma^j}-\frac{\sigma_Z^j}{1-\sigma_Z^j}\right]$. We add error terms on the RHS of eq. (128) and run the regression of the logged capital-labor ratio inclusive of the capital utilization rate, i.e., $\ln\left[u^{K,j}(t)k^j(t)\right]$, on the logged ratio of labor to capital income share, $\ln S^j(t)$, by allowing for country fixed effects, time dummies and country-specific linear time trend. Since all variables display unit root process, we estimate cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000]. Panel data estimations return a value of $\zeta^H = -0.6$ for the traded sector and a value of $\zeta^N = -0.59$ for the non-traded sector. By using panel data estimations of the elasticity of substitution between capital and labor in the traded and non-traded sector, i.e., $\sigma^H = 0.638$ and $\sigma^N = 0.799$, we use the formula $\sigma_Z^j = \frac{\frac{\sigma^j}{1-\sigma^j}-\zeta^j}{1+\frac{\sigma^j}{1-\sigma^j}-\zeta^j}$ to infer the value



(a) $\text{FBTC}^H_{adjK}(t)$ against $\left(\frac{W^H(t)}{R^H(t)}\right)^{\left(\frac{1-\sigma^H}{\sigma^H}\right)\frac{1}{\delta^H}}$

(b) $\text{FBTC}^N_{adjK}(t)$ against $\left(\frac{W^N(t)}{R^N(t)}\right)^{\left(\frac{1-\sigma^N}{\sigma^N}\right)\frac{1}{\delta^N}}$

Figure 8: Utilization-Adjusted FBTC and Adjusted-Wage-to-Capital Cost following a Government Spending Shock. Notes: Fig. 8 explores the factors leading firms to adjust their technology following an exogenous increase in government consumption b 1% of GDP. Fig. 8(a) and Fig. 8(b) plot the response of capital-utilization-adjusted FBTC in sector j to the government spending shock shown in the blue line against the response of the adjusted-wage-to-capital-rental-rate ratio in sector j, i.e., $\Omega^j(t) = \left(\frac{W^j(t)}{R(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$, where the response of $\Omega^j(t)$ is augmented by the inverse of $\delta^j = \left[1 - \left(\frac{1-\sigma^j_Z}{1-\sigma^j}\right)\frac{1}{\sigma^j_Z}\right]$. The response of $\left(\delta^j\right)^{-1}\ln\Omega^j(t)$ is shown in the black line. Sample: 18 OECD countries, 1970-2015, annual data.

for the elasticity of substitution between labor- and capital-augmenting efficiency which is equal to $\sigma^H_Z = 0.65$ for the traded sector and $\sigma^N_Z = 0.72$ for the non-traded sector. The values for σ^j and σ^j_Z implies that the inequality (123) holds so that $\delta^j > 0$ for both the traded and the non-traded sector, i.e., $\delta^H = 0.515$ and $\delta^N = 0.905$. Because δ^j is positive, capital-utilization-adjusted-FBTC in sector j is positively related to the wage-to-capital-rental-rate ratio. Since government spending shocks are biased toward non-tradables and thus triggers a reallocation of labor towards the non-traded sector, the existence of labor mobility costs lead non-traded firms to pay higher wages to encourage workers to shift (i.e., W^N/R rises), while W^H/R falls in the traded sector. Whilst it is optimal for firms to bias factor efficiency toward the most expensive factor, the complementarity between labor- and capital-augmenting productivity leads to an increase in $\text{FBTC}^N_{adjK}(t)$ in the non-traded sector and a fall in $\text{FBTC}^H_{adjK}(t)$ in the traded sector.

To estimate the dynamic adjustment of capital-utilization-adjusted-FBTC in sector j, we construct time series by using (106) and generate the response of $\text{FBTC}^j_{adjK}(t)$ by using local projection where the shock is identified by running a VAR model including government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP. To construct $\Omega^j(t) = \left(\frac{W^j(t)}{R(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$, we divide the wage rate $W^j(t)$ by the capital rental rate in sector j calculated as follows $R^j(t) = \frac{P^j(t)Y^j(t) - W^j(t)L^j(t)}{u^{K,j}(t)K^j(t)}$ and we use estimates of σ^j shown in the last line of columns 18 and 19 of Table 7. Fig. 8(a) and Fig. 8(b) plot the response of capital-utilization adjusted FBTC in sector j to the government spending shock shown in the blue line against the response of the adjusted wage-to-capital-rental-rate ratio in sector j. As displayed by the black line in Fig. 8(a), the adjusted wage-to-capital-rental-rate ratio scaled by $\frac{1}{\delta^H}$ falls following a rise in government spending. If the technology frontier were of the Cobb-Douglas type, traded firms would increase capital- relative to labor-augmenting efficiency. However, our estimates reveal that the technology frontier is of the CES type with an elasticity of substitution between A^H and B^H smaller than one. The gross complementarity between A^H and B^H in technology is high enough to encourage firms to increase A^H relative to B^H and generate a fall in $\text{FBTC}^H_{adjK}(t)$ which reflects the fact that the traded sector biases technological change toward capital. As displayed by the black line in Fig. 8(b), the adjusted wage-to-capital-rental-rate ratio scaled by $\frac{1}{\delta^N}$ increases following a rise in government spending. While the higher wages lead non-traded firms to bias technological change toward the most expensive factor, say labor, the gross complementarity between A^N and B^N is high enough to generate an increase in the relative capital efficiency. Therefore, as displayed by the blue line in Fig. 8(b), technological change is biased toward labor labor as reflected into a rise in $\text{FBTC}^N_{adjK}(t)$. In both the traded and the non-traded sector, the correlation between the two impulse response functions is high as it stands at 0.6 in the traded sector and 0.7 in the non-traded sector.

J Isolating Empirically the Technology Channel: Sims and Zha [2006]

To further explore the role of technological change in driving the effects of a shock to government consumption, we adapt the Sims and Zha [2006] methodology to provide an attempt to answer the following question: what would the sectoral government spending multiplier be if the technology channel were shut down? The method proposed by Sims and Zha [2006], and Bachmann and Sims [2012] amounts to constructing the hypothetical sequence of the other shocks in the system so that the response of technological factors to a government spending shock is zero at all horizons.

More specifically, the responses of sectoral value added (and labor) to an exogenous increase in government spending can be broken down into two effects. First, a government spending shock has a direct impact on sectoral value added and sectoral hours worked by increasing the demand for the sectoral good. In addition to this standard mechanism, there is a second effect passing through technological change. Because a rise in government consumption puts upward pressure on the unit cost for producing, firms may decide to increase the efficiency in the use of inputs and to modify the mix of capital- and labor-augmenting efficiency. By increasing its sectoral TFP, technology improvements performed by one sector increase its output multiplier relative to the other sector. If firms increase capital- relative to labor-augmenting productivity in one sector, the technology of production becomes more intensive in labor if capital and labor are gross complements which increases the relative magnitude of the fiscal multiplier on labor of this sector. To study how important the response of technology is in the transmission of a government spending shock, we generate the dynamic adjustment of sectoral variables if technological factors were unresponsive to the fiscal shock and compare it to the actual response of sectoral variables.

Empirical strategy. In order to shed some light on the role of the technology channel for fiscal transmission and guide our quantitative analysis, we estimate the VAR model in panel format on annual data. We consider a structural model with $k = 2$ lags in the following form:

$$AZ_{i,t} = \sum_{k=1}^{2} B_k Z_{i,t-k} + \epsilon_{i,t}, \tag{129}$$

where subscripts i and t denote the country and the year, respectively, $Z_{i,t}$ is the vector of endogenous variables, A is a matrix that describes the contemporaneous relation among the variables collected in vector $Z_{i,t}$, B_k is a matrix of lag specific own- and cross-effects of variables on current observations, and the vector $\epsilon_{i,t}$ contains the structural disturbances which are uncorrelated with each other.

Because the VAR model cannot be estimated in its structural form, we pre-multiply (129) by A^{-1} which gives the reduced form of the VAR model:

$$Z_{i,t} = \sum_{k=1}^{2} A^{-1} B_k Z_{i,t-k} + e_{i,t}, \tag{130}$$

where $A^{-1}B_k$ and $e_{it} = A^{-1}\epsilon_{it}$ are estimated by using a panel OLS regression with country fixed effects and country specific linear trends. To identify the VAR model and recover the government spending shocks, we need assumptions on the matrix A as the reduced form of the VAR model that we estimate contains fewer parameters than the structural VAR model shown in eq. (129). Like Blanchard and Perotti [2002], we base the identification scheme on the assumption that discretionary government spending is subject to certain decision and implementation lags that prevent government spending from responding to current output developments. This amounts to a recursive identification with government spending shocks ordered first which implies that matrix A in eq. (129) is lower-triangular:

$$\begin{pmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix} \begin{pmatrix} g_{i,t} \\ \text{tech}_{i,t} \\ \text{sec}_{i,t} \end{pmatrix} = \sum_{k=1}^{2} B_k \begin{pmatrix} g_{i,t-k} \\ \text{tech}_{i,t-k} \\ \text{sec}_{i,t-k} \end{pmatrix} + \begin{pmatrix} \epsilon^G_{i,t} \\ \epsilon^Z_{i,t} \\ \epsilon^S_{i,t} \end{pmatrix}, \tag{131}$$

where the VAR model includes three variables, i.e., (logged) government consumption, (logged) technology variables, and (logged) sectoral variables where all quantities are divided by population.

The term a_{31} in the structural form (131) of the VAR model captures the direct or standard effect of government consumption on sectoral variables. The term a_{21} captures the effect of government consumption on technology variables and the term a_{32} captures the effect of technological change on sectoral variables. The multiplicative term $a_{21} \times a_{32}$ thus measures the technology channel of government spending on impact. This decomposition paves the way to isolate the technology channel in the data in the same way as we would do in a model. Our objective is to determine the hypothetical response of sectoral variables if technology remained unresponsive to the government spending shock at all forecast horizons. When we compare the actual response of the sectoral variable with its hypothetical response, we have a measure of the importance of the technology channel in the transmission of government spending shocks. To determine the hypothetical response of sectoral variables if the technology channel were shut down, we follow the same methodology as Bachmann and Sims [2012]. Intuitively, this method amounts to creating a sequence of technology shocks so as to zero out the response of technology to a rise in government spending. Given this sequence, we can compute the modified impulse response function of sectoral variables as if technology were unresponsive to government spending shocks.

VAR specifications. In the main text, we consider three variants of the VAR model:

- In the first variant, we consider a VAR model which includes (logged) government consumption, the (logged) ratio of traded to non-traded TFP, the (logged) value added share of non-tradables, i.e., $[g_{it}, \text{tfp}^H_{it} - \text{tfp}^N_{it}, \ln \nu^{Y,N}_{it}]$. Estimates are shown in the first column of Fig. 11.
- In the second variant, we consider a VAR model which includes (logged) government consumption, the (logged) ratio of non-traded to traded capital-income-hare-adjusted-FBTC, the (logged) ratio of non-traded to non-traded LIS, i.e., $[g_{it}, \left(1 - s^N_L\right) \text{fbtc}^N_{adjK,it} - \left(1 - s^H_L\right) \text{fbtc}^H_{adjK,it}, \ln\left(s^N_{L,it}/s^H_{L,it}\right)]$. Estimates are shown in the second column of Fig. 11.
- In the third variant, we consider a VAR model which includes (logged) government consumption, the (logged) ratio of non-traded to aggregate LIS, the (logged) adjusted labor share of non-tradables, i.e., $[g_{it}, \ln\left(s^N_{L,it}/s_{L,it}\right), \ln\left[\frac{L^N_{it}}{L_{it}}\left(\omega^{Y,N}_{it}\right)^{\frac{\epsilon_i}{1+\epsilon_i}}\right]]$ where s_L is the aggregate LIS, ϵ is the elasticity of labor supply across sectors whose estimated values are taken from column 17 of Table 7, $\omega^{Y,N}$ is the value added share of non-tradables at current prices. We augment the labor share of non-tradables with ϵ and $\omega^{Y,N}$, in accordance with eq. (4), in order to control the effects of international differences in labor mobility costs and in the biasedness of the demand shock toward non-tradables. Estimates are shown in the third column of Fig. 11.
- In the fourth variant, we estimate a VAR model including government consumption, utilization adjusted-FBTC in sector j, i.e., $\ln \text{FBTC}^j_{adjK,it}$, and the LIS in sector j. The results are not included in the main text for reasons of space. Estimates are shown in Fig. 10.

Results. We first consider on three VAR models where all variables are logged and quantities are divided by population. Within each VAR specification, government consumption is ordered first, technology is ordered second and sectoral variables are ordered third. Fig. 9 shows the dynamic effects of an exogenous increase in government consumption by 1% of GDP. The blue line displays the actual response of variables while the red line shows the hypothetical response of the same variable when we restrict government consumption not to move technology at any horizon. Whilst the first row shows the response of government consumption, the second and the third row shows the dynamic adjustment of technology and sectoral variables. To start with, as can be seen in the first row, the endogenous response of government consumption remains fairly unchanged whether technology is shut down or not.

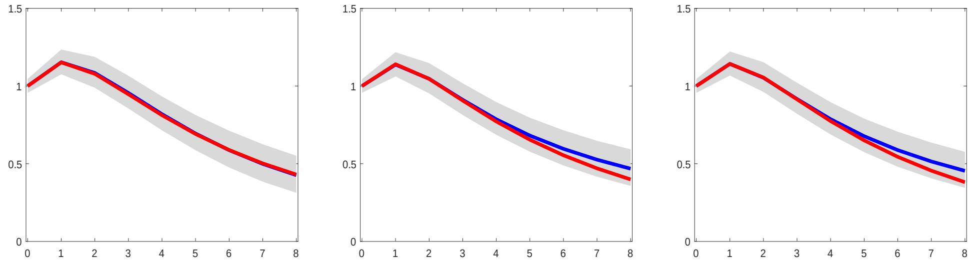
Value added share of non-tradables. In the first column of Fig. 9, we plot the response of the ratio of traded to non-traded TFP and the dynamic adjustment of the value added share of non-tradables to a shock to government consumption, in the middle and lower panel, respectively.[18] In Fig. 9(d), the blue line shows that traded TFP increases relative to non-traded TFP by 1% on average the first six years. As the productivity differential builds up, Fig. 9(g) reveals that the value added share of non-tradables does not increase significantly. In the red line in Fig. 9(d), we shut down the response of the relative productivity of tradables and as can be seen in Fig. 9(g), the hypothetical adjustment of the value added share of non-tradables is distinct from its actual response. Quantitatively, when we divide the present value of the cumulative change in $\nu^{Y,N}_t$ by the present value of the cumulative change in G_t over a six-year horizon, we find a rise in the value added share of non-tradables by 0.26 ppt of GDP as the result of the shift of labor and capital toward the non-traded sector when holding the response of $\text{TFP}^H_t/\text{TFP}^N_t$ constant. The rise in $\nu^{Y,N}_t$ is almost divided by a factor of three, as it amounts to 0.09 ppt of GDP, when we allow sectoral TFPs to react to the demand shock. In the latter case, the response of $\nu^{Y,N}_t$ is not statistically significant however.

The relative LIS of non-tradables. In the second column of Fig. 9, we plot the response of the differential in capital-utilization-adjusted-FBTC between non-tradables and tradables in the upper panel and the dynamic adjustment of the ratio of the non-traded to the traded LIS, $s^N_{L,it}/s^H_{L,it}$, in the lower panel.[19] As shown in the blue line in Fig. 9(h), $s^N_{L,it}/s^H_{L,it}$ increases significantly after two years and the rise in the relative LIS of non-tradables averages 1.73% the first six years. The

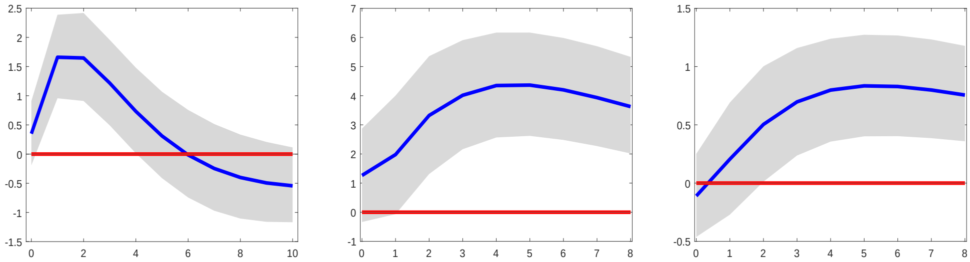
[18] We estimate a VAR model which includes government consumption, the ratio of traded to non-traded TFP and the value added share of non-tradables.

[19] In accordance with the decomposition of the labor share of non-tradables, see eq. (67), we scale sec-

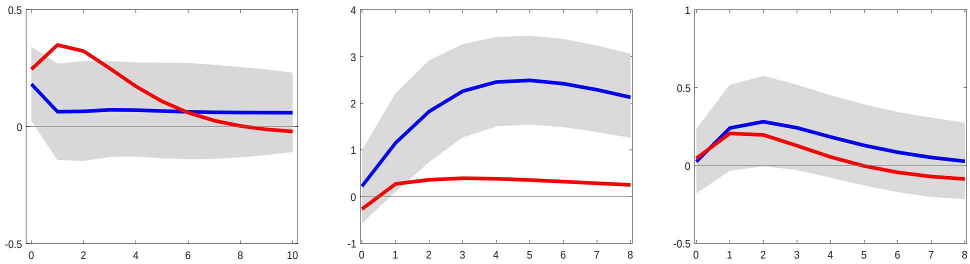
Value Added share of Non Tradables | Ratio of Non-Traded to Traded LIS | Labor Share of of Non Tradables



(a) Government Consumption (b) Government Consumption (c) Government Consumption



(d) Ratio of Traded to Non-Traded TFP (e) $\text{FBTC}^{N}_{adjK,t}/\text{FBTC}^{H}_{adjK,t}$ (f) Ratio of Non-Traded to Aggregate LIS



(g) Value Added Share of Non-Tradables (h) Ratio of Non-Traded to Traded LIS (i) Adjusted Labor Share of Non-Tradables

Figure 9: Dynamic Adjustment to Government Spending Shocks: Isolating the Technology Channel. Notes: Fig. 9 plots the dynamic adjustment of sectoral variables to an exogenous increase in government consumption by 1% of GDP by isolating the pure technology effect. We plot the actual dynamic adjustment of sectoral variables to a government spending shock in the blue line. The red line shows the hypothetical dynamic adjustment of sectoral variables if technology were unresponsive to the spending shock at all horizons. The first row displays the endogenous response of government consumption to the exogenous fiscal shock. The second row shows the actual dynamic adjustment of technology to a government spending shock in the blue line while the red line keeps the dynamic response of technology unchanged. In the third row, we plot the actual dynamic adjustment of sectoral variables to a government spending shock in the blue line while the red line shows the responses if technology were shut down. The first and the third column displays the dynamic response of the value added share and the labor share of non-tradables, respectively. The second column plots the dynamic response of the ratio of the non-traded to the traded LIS. Sample: 18 OECD countries, 1970-2015, annual data.

blue line in Fig. 9(e) reveals that the rise in the non-traded relative to the traded LIS is driven by the differential in FBTC. In Online Appendix J, see Fig. 10, we find empirically that technological change is biased toward labor in the non-traded sector and biased toward capital in the traded sector. When we shut down FBTC in the red line, the annual increase in the non-traded relative to the traded LIS averages 0.25% only, the rise in the non-traded LIS being driven by the the capital inflow which pushes k^N up.

The adjusted labor share of non-tradables. In the third column of Fig. 9, we quantify the reallocation of labor toward the non-traded sector driven by the response of the non-traded relative to aggregate LIS to a government spending shock. To isolate the pure effect of the movement in the LIS on the labor share of non-tradables, in accordance with (4), we adjust the share of non-traded hours worked in total hours worked with the value added share of non-tradables at current prices augmented with the elasticity of labor supply across sectors, i.e., $\frac{L^N_{it}}{L_{it}}\left(\omega^{Y,N}_{it}\right)^{-\frac{\epsilon_i}{1+\epsilon_i}}$.[20] The blue line in Fig. 9(i) shows the actual rise in the (adjusted) labor share of non-tradables while the red line displays its adjustment if the LIS were unresponsive to the government spending shock. As displayed by the blue line in Fig. 9(f), the shock to government consumption increases the non-traded LIS relative to the aggregate LIS which reflects the fact that the technology of production becomes more labor intensive. When we divide the present value of the cumulative change in $\nu^{L,N}_t$ by the present value of the cumulative change in G_t over a six-year horizon, we find a rise in the (adjusted) labor share of non-tradables by 0.11 ppt of total hours worked when we shut down the response of the non-traded relative to the aggregate LIS, and by 0.2 ppt of total hours worked when we allow $s^N_{L,it}/s_{L,it}$ to respond to the government spending shock. Since the bulk of the variation in LIS is driven by FBTC, our empirical findings reveal that technological change biased toward labor in the non-traded sector and biased toward capital in the traded sector almost doubles the reallocation of labor toward the non-traded sector.

Technology channel and sectoral LISs. The first row of Fig. 10 displays the dynamic adjustment of the non-traded and traded LIS while the second row shows the dynamic adjustment of capital-utilization-adjusted FBTC in the non-traded and traded sector. The blue line shows actual responses of variables while the red line shows responses of the variable when FBTC is shut down. As is clear from Fig. 10, the rise in the non-traded LIS is driven by technological change biased toward labor, as captured by an increase in $\ln \text{FBTC}^N_{adjK,it}$. Conversely, the decline in the traded LIS is brought about by technological change biased toward capital, as reflected into a fall in $\ln \text{FBTC}^H_{adjK,it}$.

K Fiscal Transmission and Technology: Cross-Country Differences

Our evidence in subsection 2.4 in the the main text reveals that a shock to government consumption leads traded firms to improve their technology and non-traded firms to increase capital- relative to labor-augmenting efficiency. In this subsection, we further explore the role of the technology channel following a shock to government consumption by considering cross-country differences. To conduct this analysis, we use a two-step estimation procedure as in section 2.2, except that we consider one country at a time and plot responses of sectoral variables against measures of technology in Fig. 11. In Fig. 11(a), we plot the change in the value added share of non-tradables against the TFP differential between tradables and non-tradables, while in Fig. 11(d) we plot the variation in the non-traded-goods-share of total hours worked against the capital-utilization-adjusted FBTC differential between non-tradables and tradables. Because $d\nu^{Y,N}_t$ and $d\nu^{L,N}_t$ determine the size of the government spending multiplier on non-traded value added and hours worked, respectively, in the first column we show the present value of the cumulative change of the corresponding variable divided by the present value of the cumulative change in government consumption, both computed over a six-year horizon.[21] In the second and third column Fig. 11, we focus on impact responses.

Value added share of non-tradables and relative TFP. Evidence in Fig. 11(a) reveals

toral FBTC by the capital income share, i.e., the blue line in Fig. 9(e) shows $\left(1-s^N_{L,i}\right)\hat{\text{FBTC}}^N_{adjK,it} - \left(1-s^H_{L,i}\right)\hat{\text{FBTC}}^H_{adjK,it}$

[20] Our sample includes eighteen OECD countries which differ in terms of labor mobility costs and intensity of the non-traded sector in the government spending shock. To control for cross-country differences in labor mobility and biasedness of the government spending shock toward non-tradables, we adjust the ratio of non-traded hours worked to total hours worked with $\left(\omega^{Y,N}_{it}\right)^{-\frac{\epsilon_i}{1+\epsilon_i}}$. In doing this, we ensure that we capture the rise in the labor share of non-tradables driven by an increase in s^N_L/s_L.

[21] In doing this, we control for cross-country differences in the adjustment of G_t. When we compute the present value of the cumulative change of a variable at a country level, we take the (country-specific) interest rate from the first column of Table 6.

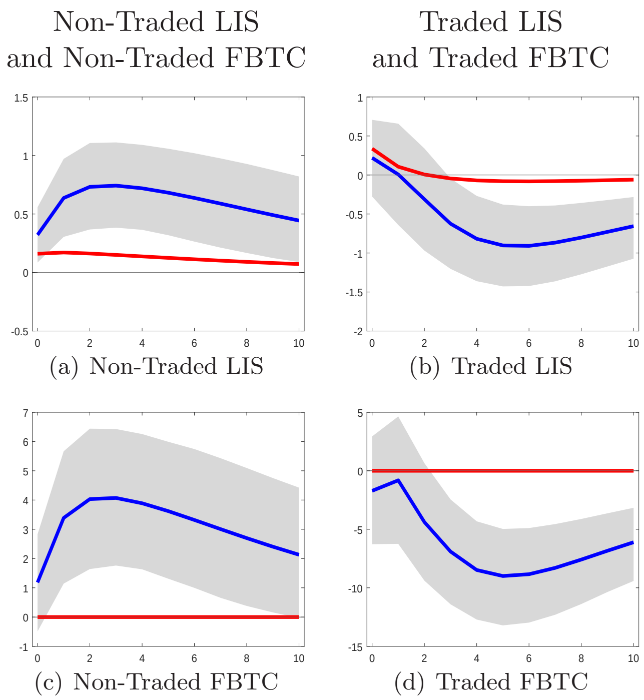


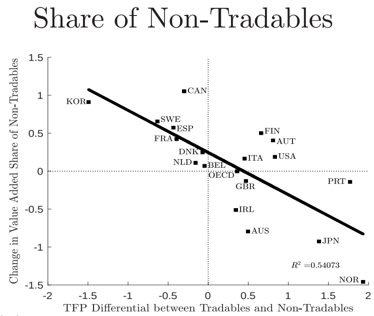
Figure 10: Dynamic Adjustment to Government Spending Shocks: Isolating the Technology Channel. Notes: Fig. 9 plots the dynamic adjustment of sectoral LISs to a 1% exogenous increase in government by isolating the pure technology effect. We estimate a VAR model including logged government consumption, logged utilization adjusted-FBTC in sector j, i.e., $\ln \text{FBTC}^{j}_{adjK,it}$, and the (logged) LIS in sector j. The blue line shows the actual dynamic adjustment when we let technological change responds to the government spending shock while the red line shows the hypothetical dynamic adjustment of variables if FBTC were unresponsive to the demand shock at all horizons. While the first row shows the responses of LISs, the second row shows the dynamic adjustment of utilization-adjusted-FBTC in sector j scaled by the capital income share in this sector. Sample: 18 OECD countries, 1970-2015, annual data.

that $\text{TFP}^H_t/\text{TFP}^N_t$ increases in two-third of the countries of our sample while traded TFP declines significantly relative to non-traded TFP in six countries, including Canada, France, Korea, Netherlands, Spain, Sweden.[22] The downward sloping trend line shows that the value added share of non-tradables increases less in countries where traded relative to non-traded TFP increases more since the TFP differential offsets the impact of the reallocation of labor triggered by the biasedness of the demand shock toward non-tradables (see eq. (48)).

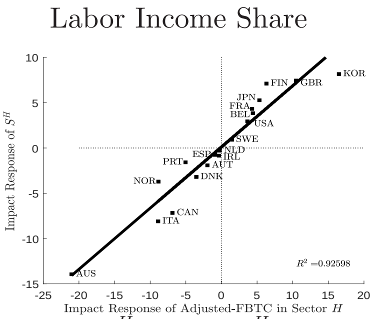
Unit cost and technology adjustment. The wide cross-country dispersion in the adjustment of the relative TFP of tradables shown in Fig. 11(a) suggests that technology decisions vary substantially between OECD economies. In the top panel of the third column of Fig. 11, we shed some light on the factor which leads firms to increase the efficiency in the use of inputs. In a model with flexible prices, firms equate their prices to the unit cost for producing divided by the capital-utilization-adjusted-TFP. In face of a higher unit cost for producing, sectors can increase prices or improve technology or both. Firms will decide to further increase the efficiency in the use of inputs as the cost of improving technology is lower. As detailed in Online Appendix H, we construct a measure of the unit cost for producing and we divide this measure by the value added deflator of the corresponding sector to control for price adjustments. In Fig. 11(c) and Fig. 11(f), we plot the response of the capital-utilization-adjusted-TFP on the vertical axis for tradables and non-tradables, respectively, against the change in the real unit cost on the horizontal axis. The trend line reveals a strong positive cross-country relationship between the decision to adjust technology and the real cost for producing which suggests that technology improvements are driven by a cost-minimization strategy. In accordance with our estimates, we model the decision to increase the utilization of available technology in the model (laid out in the next section) as a trade-off between the rise in output generated by enhanced productivity and the cost associated with a higher utilization rate of technology within each sector $j = H, N$. The adjustment costs associated with using more intensively available technology can be interpreted as the efforts and the costs caused by the reorganization of the production to increase the efficiency in the use of inputs.

Sectoral LIS and FBTC. In the second column of Fig. 11, we plot the responses of the ratio of labor to capital income share, $S^j_t = \frac{s^j_{L,t}}{1-s^j_{L,t}}$, against the adjustment of our measure of capital-

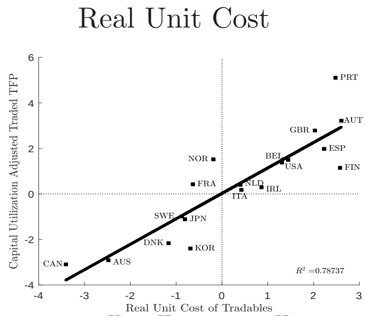
[22] While traded TFP also declines relative to non-traded TFP in Belgium and Denmark, this rise is not confirmed when we reconstruct the change in the sectoral TFP as follows $\hat{\text{TFP}}^j_t = \hat{Y}^j_t - \hat{L}^j_t - \left(1 - s^j_L\right)\hat{k}^j_t$ due to the uncertainty surrounding estimates at a country level.



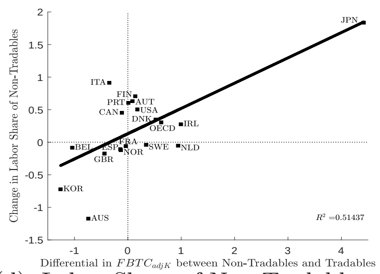
(a) Value Added Share of Non-Tradables vs. $\text{TFP}^H/\text{TFP}^N$



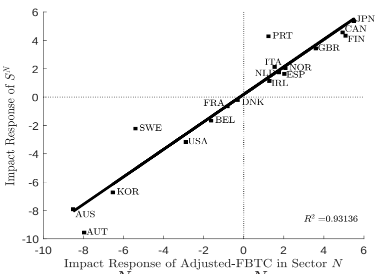
(b) S^H vs. FBTC^H_{adjK}



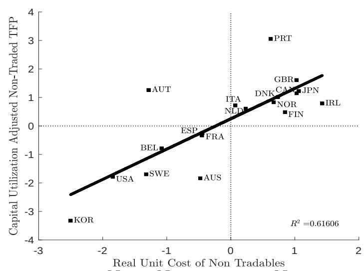
(c) UC^H/P^H vs. TFP^H_{adjK}



(d) Labor Share of Non-Tradables vs. FBTC differential between Non-Tradables and Tradables



(e) S^N vs. FBTC^N_{adjK}



(f) UC^N/P^N vs. TFP^N_{adjK}

Figure 11: Effects of Government Spending Shocks and Cross-Country Differences in Technology Adjustment. Notes: Fig. 11 plots responses to an exogenous increase in government consumption by 1% of GDP against measures of technological change. Fig. 11(a) plots the present value of the cumulative change in the value added share of non-tradables, $\nu_t^{Y,N}$ (vertical axis) against the present value of the cumulative change in the ratio of traded TFP to non-traded TFP (horizontal axis), both computed over a six-year horizon and divided by the present value of the cumulative change in government consumption. In accordance with eq. (48), the TFP differential is scaled by multiplying by $\left(1-\nu^{Y,H}\right)\nu^{Y,H}$. Fig. 11(d) plots the present value of the cumulative change in the share of non-traded hours worked in total hours worked (vertical axis) against the present value of the cumulative change in the differential in capital-utilization-adjusted FBTC between non-tradables and tradables (horizontal axis), both computed over a six-year horizon and divided by the present value of the cumulative change in government consumption. To construct time series for $\text{FTBC}^j_{t,adjK}$ for each country, we use eq. (5) and take estimates of the elasticity of substitution between capital and labor σ^j from columns 18 and 19 of Table 7. The response of $\text{FTBC}^j_{t,adjK}$ in sector j is adjusted with $1-s_L^j$ and the differential is scaled by $\alpha_L^H\alpha_L^N$, see eq. (67) in Online Appendix C. The second column of Fig. 11 plots impact responses of sectoral LISs (vertical axis) against the adjustment of capital-utilization adjusted FBTC (the horizontal axis). The third column of Fig. 11 plots impact responses of the ratio of the unit cost for producing to the value added deflator (vertical axis) against the adjustment of capital-utilization adjusted TFP (horizontal axis). The construction of the unit cost for producing is detailed in Online Appendix H. Sample: 18 OECD countries, 1970-2015, annual data.

utilization-adjusted-FBTC (see eq. (5)).[23] As can be seen in Fig. 11(b) for tradables and Fig. 11(e) for non-tradables, a rise in government spending increases the share of the value added paid to workers in half of the countries while the LIS falls in the rest of the sample. The trend line reveals that there exists a strong positive cross-country relationship between the variations of the LIS and capital-utilization-adjusted-FBTC. For example, countries which lie in the north-east of the scatter-plot experience an increase in the LIS driven by technological change biased toward labor. By contrast, for countries which lie in the south-west of the scatter-plot, technological change biased toward capital lowers the LIS.

Labor share of non-tradables and relative non-traded LIS. While the non-traded-goods-share of total hours worked increases as the result of the biasedness of the government spending shock toward non-tradables, the reallocation of labor toward the non-traded sector will be amplified if technological change makes non-traded production more labor intensive. This channel is captured a rise in $s^N_{L,t}/s_{L,t}$ in eq. (4). According to the evidence displayed by the second column of Fig. 11, the non-traded LIS increases relative to the aggregate LIS as long as technological change is biased toward labor in the non-traded sector and is biased toward capital in the traded sector. Fig. 11(d) shows that the change in the labor share of non-tradables is positively correlated with the differential in capital-utilization-adjusted FBTC between non-tradables and tradables. More specifically, in countries positioned in the north-east, technological change is more biased toward labor in the non-traded relative to the traded sector which amplifies the rise in L^N_t/L_t. Conversely, as can be seen in the south-west of the scatter-plot, when technological change is more biased toward labor (or less biased toward capital) in the traded sector, the labor share of non-tradables declines such as in Australia, Korea, Norway, Spain, the UK.

L Data for Calibration

L.1 Non-Tradable Content of GDP and its Demand Components

Table 7 shows the non-tradable content of GDP, consumption, investment, government spending, labor and labor compensation (columns 1 to 6). In addition, it gives information about the sectoral labor income shares (columns 11 and 12). The home content of consumption and investment expenditure in tradables and the home content of government spending are reported in columns 8 to 10. Column 7 shows the ratio of exports to GDP. Columns 11 and 12 shows the labor income share in the traded and non-traded sector. Columns 13 to 15 display the aggregate labor income share, investment-to-GDP ratio and government spending in % of GDP, respectively, for the whole economy. Our sample covers the 18 OECD countries mentioned in section B.1. The reference period for the calibration of labor variables is 1970-2015 while the reference period for demand components is 1995-2014 due to data availability, as detailed below. When we calibrate the model to a representative economy, we use the last line which shows the (unweighted) average of the corresponding variable.

Aggregate ratios. Columns 13 to 15 show the aggregate labor income share, s_L, the investment-to-GDP ratio, ω_J and government spending as a share of GDP, ω_G. The aggregate labor income share is calculated as the ratio of labor compensation (compensation of employees plus compensation of self-employed) to GDP at current prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2015 (1974-2015 for JPN). To calculate ω_J, we use time series for gross capital formation at current prices and GDP at current prices, both obtained from the OECD National Accounts Database [2017]. Data coverage: 1970-2015 for all countries. To calculate ω_G, we use time series for final consumption expenditure of general government (at current prices) and GDP (at current prices). Source: OECD National Accounts Database [2017]. Data coverage: 1970-2015 for all countries. We consider a steady-state where trade is initially balanced and we calculate the consumption-to-GDP ratio, ω_C by using the accounting identity between GDP and final expenditure:

$$\omega_C = 1 - \omega_J - \omega_G. \tag{132}$$

As displayed by the last line of Table 7, investment expenditure (see column 14) and government spending (see column 15) as a share of GDP average to 24% and 19%, respectively, while the aggregate labor income share averages to 66% (see column 13).

Non-traded demand components. Columns 2 to 4 show non-tradable content of consumption (i.e., $1-\alpha_C$), investment (i.e., $1-\alpha_J$), and government spending (i.e., ω_{G^N}), respectively. These demand components have been calculated by adopting the methodology described in eqs. (103a)-(103b), and eq. (100). Sources: World Input-Output Databases ([2013], [2016]). Data coverage:

[23] Since $\hat{S}^j_{it} = \frac{\hat{s}^j_{L,it}}{1-s^j_{L,i}}$ and thus the percentage deviation of the ratio of labor to capital income share relative to its initial steady-state is proportional to the percentage change in the LIS, $\hat{s}^j_{L,t}$, we refer interchangeably to the LIS or the ratio of factor income share as long as it does not cause confusion.

1995-2014 except for NOR (2000-2014). The non-tradable share of consumption, investment and government spending shown in column 2 to 4 of Table 7 averages to 56%, 69% and 80%, respectively.

In the empirical analysis, we use data from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases for constructing sectoral value added over the period running from 1970 to 2015. Since the demand components for non-tradables are computed over 1995-2014 by using the WIOD dataset, to ensure that the value added is equal to the sum of its demand components, we have calculated the non-tradable content of value added shown in column 1 of Table 7 as follows:

$$\begin{aligned} \omega^{Y,N} &= = \frac{P^N Y^N}{Y}, \\ &= \omega_C \left(1-\alpha_C\right) + \omega_J \left(1-\alpha_J\right) + \omega_{G^N}\omega_G, \end{aligned} \tag{133}$$

where $1-\alpha_C$ and $1-\alpha_J$ are the non-tradable content of consumption and investment expenditure shown in columns 2 and 3, ω_{G^N} is the non-tradable content of government spending shown in column 4, ω_C and ω_J are consumption- and investment-to-GDP ratios, and ω_G is government spending as a share of GDP.

Non-tradable content of hours worked and labor compensation. To calculate the non-tradable share of labor shown in column 5 and labor compensation shown in column 6, we split the eleven industries into traded and non-traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jensen and Kletzer [2006]. Details about data construction for sectoral output and sectoral labor are provided above. We calculate the non-tradable share of labor compensation as the ratio of labor compensation in the non-traded sector (i.e., $W^N L^N$) to overall labor compensation (i.e., WL). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2015 for all countries (except Japan: 1974-2015). The non-tradable content of labor and labor compensation, shown in columns 5 and 6 of Table 7, average to 62% and 63% respectively.

Home content of consumption and investment expenditure in tradables. Columns 8 to 9 of Table 7 show the home content of consumption and investment in tradables, denoted by α^H and α_J^H in the model. These shares are obtained from time series calculated by using the formulas (102a)-(102b). Sources: World Input-Output Databases ([2013], [2016]). Data coverage: 1995-2014 except for NOR (2000-2014). Column 10 shows the content of government spending in home-produced traded goods. Taking data from the WIOD dataset, time series for ω_{G^H} are constructed by using the formula (101). Data coverage: 1995-2014 except for NOR (2000-2014). As shown in the last line of columns 8 and 9, the home content of consumption and investment expenditure in traded goods averages to 66% and 43%, respectively, while the share of government spending in home-produced traded goods averages 19%. Since the non-tradable content of government spending averages 80% (see column 4), the import content of government spending is 1% only.

Since we set initial conditions so that the economy starts with balanced trade, export as a share of GDP, ω_X, shown in column 7 of Table 7 is endogenously determined by the import content of consumption, $1-\alpha^H$, investment expenditure, $1-\alpha_J^H$, and government spending, ω_{G^F}, along with the consumption-to-GDP ratio, ω_C, the investment-to-GDP ratio, ω_J, and government spending as a share of GDP, ω_G. More precisely, dividing the current account equation at the steady-state by GDP, Y, leads to an expression that allows us to calculate the GDP share of exports of final goods and services produced by the home country:

$$\omega_X = \frac{P^H X^H}{Y} = \omega_C \alpha_C \left(1-\alpha^H\right) + \omega_J \alpha_J \left(1-\alpha_J^H\right) + \omega_G \omega_{G^F}, \tag{134}$$

$\omega_{G^F} = 1 - \omega_{G^{N,D}} - \omega_{G^{H,D}}$. The last line of column 7 of Table 7 shows that the export to GDP ratio averages 13%.

Sectoral labor income shares. The labor income share for the traded and non-traded sector, denoted by s_L^H and s_L^N, respectively, are calculated as the ratio of labor compensation of sector j to value added of sector j at current prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2015 for all countries (except Japan: 1974-2015). As shown in columns 11 and 12 of Table 7, s_L^H and s_L^N averages to 63% and 69%, respectively.

Estimated elasticities. Columns from 16 to 20 of Table 7 display estimates of the elasticity of substitution between tradables and non-tradables in consumption, ϕ, the elasticity of labor supply across sectors, ϵ, the elasticity of substitution between capital and labor in the traded and the non-traded sector, i.e., σ^H and σ^N, respectively, the elasticity of exports w.r.t. the terms of trade, ϕ_X. In subsections L.2 and L.3, we detail the empirical strategy to estimate these parameters, except for the price elasticity of exports shown in column 20 of Table 7 whose estimates are taken from Imbs and Mejean [2015].

L.2 Estimates of ϵ and ϕ: Empirical Strategy

Table 9 shows our estimates of the elasticity of labor supply across sectors, ϵ, while Table 10 shows our estimates of the elasticity of substitution in consumption between traded and non-traded goods,

Table 7: Data to Calibrate the Two-Sector Model

Countries	Non-tradable share						Home share				Labor Share		Aggregate ratios			Elasticities					Interest
	GDP	Cons.	Inv.	Gov.	Labor	Lab. comp.	X^H	C^H	I^H	G^H	LIS^H	LIS^N	LIS	I/Y	G/Y	ϕ	ϵ	σ^H	σ^N	ϕ_X	r
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
AUS	0.62	0.58	0.76	0.54	0.65	0.64	0.09	0.76	0.49	0.44	0.58	0.67	0.64	0.27	0.18	0.45	0.41	0.55	0.67	1.22	0.029
AUT	0.64	0.56	0.60	0.91	0.61	0.61	0.18	0.56	0.42	0.05	0.68	0.68	0.68	0.25	0.18	1.27	1.10	0.94	1.43	1.77	0.030
BEL	0.65	0.53	0.63	0.97	0.65	0.63	0.22	0.44	0.20	0.00	0.66	0.67	0.67	0.23	0.22	1.22	0.60	0.74	1.10	1.81	0.033
CAN	0.62	0.49	0.66	0.95	0.67	0.65	0.14	0.68	0.31	0.05	0.54	0.63	0.60	0.22	0.21	0.55	0.39	1.09	0.90	2.53	0.032
DNK	0.68	0.60	0.73	0.83	0.66	0.67	0.16	0.49	0.22	0.15	0.64	0.70	0.68	0.21	0.25	1.04	0.28	0.57	1.17	n.a.	0.046
ESP	0.64	0.59	0.77	0.65	0.60	0.63	0.11	0.73	0.38	0.33	0.60	0.66	0.63	0.24	0.16	1.42	0.95	1.10	0.90	1.93	0.020
FIN	0.66	0.57	0.73	0.81	0.59	0.61	0.12	0.67	0.41	0.16	0.64	0.74	0.70	0.25	0.20	0.51	0.42	1.14	0.84	1.62	0.025
FRA	0.70	0.57	0.76	0.98	0.64	0.66	0.10	0.71	0.46	0.00	0.72	0.69	0.70	0.23	0.22	0.98	1.39	0.75	0.82	1.67	0.032
GBR	0.65	0.58	0.70	0.81	0.66	0.61	0.12	0.66	0.42	0.19	0.69	0.74	0.72	0.20	0.19	0.00	0.61	0.40	0.46	1.54	0.025
IRL	0.62	0.50	0.74	0.87	0.58	0.60	0.21	0.48	0.19	0.12	0.50	0.69	0.60	0.22	0.18	1.27	0.09	0.71	0.64	n.a.	0.035
ITA	0.64	0.55	0.64	0.98	0.58	0.58	0.09	0.79	0.60	0.01	0.73	0.67	0.70	0.22	0.18	0.31	1.65	0.86	0.50	1.60	0.025
JPN	0.67	0.66	0.71	0.61	0.61	0.63	0.04	0.85	0.82	0.39	0.60	0.66	0.64	0.29	0.16	0.88	0.79	0.94	0.90	1.46	0.017
KOR	0.56	0.55	0.67	0.31	0.49	0.52	0.09	0.81	0.59	0.67	0.72	0.82	0.77	0.31	0.12	0.59	2.27	0.43	0.80	1.35	0.052
NLD	0.66	0.52	0.69	0.97	0.67	0.67	0.18	0.55	0.25	0.00	0.61	0.74	0.69	0.22	0.22	0.83	0.22	1.08	0.61	n.a.	0.030
NOR	0.62	0.53	0.63	0.86	0.62	0.64	0.14	0.66	0.49	0.11	0.43	0.63	0.54	0.25	0.20	1.01	0.13	0.42	0.84	1.88	0.027
PRT	0.60	0.54	0.71	0.67	0.55	0.58	0.15	0.65	0.33	0.31	0.73	0.61	0.66	0.25	0.16	0.30	0.59	0.53	0.46	2.12	0.023
SWE	0.67	0.56	0.60	0.97	0.65	0.65	0.15	0.63	0.38	0.01	0.67	0.74	0.71	0.24	0.25	0.49	0.53	0.65	0.52	1.81	0.031
USA	0.67	0.69	0.61	0.63	0.70	0.66	0.06	0.83	0.68	0.37	0.62	0.62	0.62	0.22	0.16	0.78	2.44	0.71	1.10	1.16	0.026
OECD	0.64	0.56	0.69	0.80	0.62	0.63	0.13	0.66	0.43	0.18	0.63	0.69	0.66	0.24	0.19	0.77	0.83	0.64	0.80	1.70	0.030

Notes: Columns 1-6 show the GDP share of non-tradables, the non-tradable content of consumption, investment and government expenditure, the share of non-tradables in labor, and the non-tradable content of labor compensation. Column 7 gives the ratio of exports of final goods and services to GDP; columns 8 and 9 show the home share of consumption and investment expenditure in tradables and column 10 shows the content of government spending in home-produced traded goods; LIS^j stands for the labor income share in sector $j = H, N$ while LIS refers to the aggregate LIS; I/Y is the investment-to-GDP ratio and G/Y is government spending as a share of GDP; ϕ is the elasticity of substitution between traded and non-traded goods in consumption; ϵ is the elasticity of labor supply across sectors; σ^j is the elasticity of substitution between capital and labor in sector $j = H, N$; estimates of the elasticity of exports w.r.t. terms of trade, ϕ_X, are taken from Imbs and Mejean [2015]. The real interest rate is the real long-term interest rate calculated as the nominal interest rate on 10 years government bonds minus the rate of inflation which is the rate of change of the Consumption Price Index.

Table 8: Baseline Parameters (Representative OECD Economy)

Definition	Value CD	Value CES/Restricted	Reference
Period of time	year	year	data frequency
A.Preferences			
Subjective time discount rate, β	3%	3%	equal to the world interest rate
Intertemporal elasticity of substitution for consumption, σ_C	1	1	standard
Intertemporal elasticity of substitution for labor, σ_L	1	1	standard
Elasticity of substitution between C^T and C^N, ϕ	0.77	0.77	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Elasticity of substitution between J^T and J^N, ϕ_J	1	1	Bems [2008]
Elasticity of substitution between C^H and C^F, ρ	1.5	1.5	Backus, Kehoe and Kydland [1994]
Elasticity of substitution between J^H and J^F, ρ_J	1.5	1.5	Backus, Kehoe and Kydland [1994]
Elasticity of labor supply across sectors, ϵ	0.83	0.83	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
B.Non-tradable share			
Weight of consumption in non-traded goods, $1-\varphi$	0.54	0.54	set to target $1-\alpha_C = 56\%$ (United Nations, COICOP [2017])
Weight of labor supply to the non-traded sector, $1-\vartheta$	0.60	0.60	set to target $L^N/L = 62\%$ (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Weight of non-traded investment, $1-\iota$	0.69	0.69	set to target $1-\alpha_J = 69\%$ (OECD Input-Output database [2017])
Non-tradable content of government expenditure, ω_{G^N}	0.80	0.80	our estimates (Input-Output dataset, WIOD [2013])
C.Home share			
Weight of consumption in home traded goods, φ^H	0.73	0.73	set to target $\alpha^H = 66\%$ (United Nations, Comtrade [2017])
Weight of home traded investment, ι^H	0.52	0.52	set to target $\alpha_J^H = 43\%$ (United Nations, Comtrade [2017])
Home traded content of government expenditure, ω_{G^H}	0.18	0.18	our estimates (Input-Output dataset, WIOD [2013])
Export price elasticity, ϕ_X	1.7	1.7	Imbs and Mejean [2015]
C.Production			
Labor income share in the non-traded sector, θ^N	0.69	0.69	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Labor income share in the traded sector, θ^H	0.63	0.63	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Elasticity of of substitution between K^H and L^H, σ^H	1	0.64	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Elasticity of of substitution between K^N and L^N, σ^N	1	0.80	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
D.GDP demand components			
Physical capital depreciation rate, δ_K	7.8%	7.8%	set to target $\omega_J = 24\%$ (Source: OECD Economic Outlook Database)
Parameter governing capital adjustment cost, κ	17	17	set to match the elasticity I/K to Tobin's q (Eberly et al. [2008])
Government spending as a ratio of GDP, ω_G	19%	19%	our estimates (Source: OECD Economic Outlook Database)
E.Utilization adjustment costs			
Parameter governing capital utilization cost, ξ_2^H	0.27	∞	set to reproduce IRF for $u^{K,H}(t)$
Parameter governing capital utilization cost, ξ_2^N	0.03	∞	set to reproduce IRF for $u^{K,N}(t)$
Parameter governing technology utilization cost, χ_2^H	0.80	∞	set to reproduce IRF for $u^{Z,H}(t)$
Parameter governing technology utilization cost, χ_2^N	2.85	∞	set to reproduce IRF for $u^{Z,N}(t)$
F.Government Spending Shock			
Exogenous fiscal shock, g	1%	1%	To generate $dG(0)/Y = 1\%$
Persistence and shape of endogenous response of G, ξ	0.430	0.430	set to target $\int_0^5 [dG(\tau)/Y]\, e^{-r^\star \tau} d\tau = g'$ and $\dot{G}(1) = 0$
Persistence and shape of endogenous response of G, χ	0.439	0.439	set to target $\int_0^5 [dG(\tau)/Y]\, e^{-r^\star \tau} d\tau = g'$ and $\dot{G}(1) = 0$

Notes: 'CD' refers to the Cobb-Douglas model which is taken as the reference point; 'CES' refers to the baseline model where production functions are of the CES type with $\sigma^j < 1$ and $\xi_2^j < \infty$, $\chi_2^j < \infty$; 'restricted' refers to the restricted model with Cobb-Douglas production functions (i.e., $\sigma^j = 1$) and technological change is shut down by letting ξ_2^j and χ_2^j tend toward infinity. Note that we omit $\xi_1^j = R/P^j$ and $\chi_1^j = Y^j$ for reasons of space together with the twelve parameters' values ξ_X^j, χ_X^j, x^j with $X = A^j, B^j$ and $j = H, N$ to calibrate the dynamics of labor- and capital-augmenting technological change in the traded and non-traded sectors.

ϕ. We present our empirical strategy to estimate these two parameters. The derivation of equations we explore empirically is detailed in the technical appendix of the working paper by Bertinelli, Cardi and Restout [2020].

Elasticity of labor supply across sectors. Drawing on Horvath [2000], we derive a testable equation by combining optimal rules for labor supply and labor demand and estimate ϵ by running the regression of the worker inflow in sector $j = H, N$ of country i at time t arising from labor reallocation across sectors computed as $\hat{L}^j_{i,t} - \hat{L}_{i,t}$ on the relative labor's share percentage changes in sector j, $\hat{\beta}^j_{i,t}$:

$$\hat{L}^j_{i,t} - \hat{L}_{i,t} = f_i + f_t + \gamma_i \hat{\beta}^j_{i,t} + \nu^j_{i,t}, \tag{135}$$

where $\nu^j_{i,t}$ is an i.i.d. error term; country fixed effects are captured by country dummies, f_i, and common macroeconomic shocks by year dummies, f_t. The LHS term of (135) is calculated as the difference between changes (in percentage) in hours worked in sector j, $\hat{L}^j_{i,t}$, and in total hours worked, $\hat{L}_{i,t}$. The RHS term β^j corresponds to the fraction of labor's share of value added accumulating to labor in sector j. Denoting by $P^j_t Y^j_t$ value added at current prices in sector $j = H, N$ at time t, β^j_t is computed as $\frac{s^j_L P^j_t Y^j_t}{\sum_{j=H}^N s^j_L P^j_t Y^j_t}$ where s^j_L is the LIS in sector $j = H, N$ defined as the ratio of the compensation of employees to value added in the jth sector, averaged over the period 1970-2015. Because hours worked are aggregated by means of a CES function, percentage change in total hours worked, $\hat{L}_{i,t}$, is calculated as a weighted average of sectoral hours worked percentage changes, i.e., $\hat{L}_t = \sum_{j=H}^N \beta^j_{t-1} \hat{L}^j_t$. The parameter we are interested in, say the degree of substitutability of hours worked across sectors, is given by $\epsilon_i = \gamma_i/(1-\gamma_i)$. In the regressions that follow, the parameter γ_i is assumed to be different across countries when estimating ϵ_i for each economy ($\gamma_i \neq \gamma_{i'}$ for $i \neq i'$). To construct $\hat{L}^j$ and $\hat{\beta}^j$ we combine raw data on hours worked L^j, nominal value added $P^j Y^j$ and labor compensation $W^j L^j$. All required data are taken from the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. The sample includes the 18 OECD countries mentioned above over the period 1971-2015 (except for Japan: 1975-2015). Table 9 reports empirical estimates that are consistent with $\epsilon > 0$. All values are statistically significant at 10%, except for Norway. Overall, we find that ϵ ranges from a low of 0.023 for NOR to a high of 2.439 for USA. Since the estimated value for ϵ is not statistically significant for Norway, we run the same regression as in eq. (135) but use the output instead of value added to construct $\hat{\beta}^j$. We find a value of 0.13, as reported in column 17 of Table 7, and this estimated value is statistically significant.

Elasticity of substitution between traded and non-traded goods in consumption. To estimate the elasticity of substitution in consumption, ϕ, between traded and non-traded goods, we derive a testable equation by rearranging the optimal rule for optimal demand for non-traded goods, i.e., $C^N_t = (1-\varphi)\left(\frac{P^N_t}{P_{C,t}}\right)^{-\phi} C_t$, since time series for consumption in non-traded goods are too short. More specifically, we derive an expression for the non-tradable content of consumption expenditure by using the market clearing condition for non-tradables and construct time series for $1 - \alpha_{C,t}$ by using time series for non-traded value added and demand components of GDP while keeping the non-tradable content of investment and government expenditure fixed, in line with the evidence documented by Bems [2008] for the share of non-traded goods in investment and building on our own evidence for the non-tradable content of government spending. After verifying that the (logged) share of non-tradables and the (logged) ratio of non-traded prices to the consumption price index are both integrated of order one and cointegrated, we run the regression by adding country and time fixed effects by using a FMOLS estimator. We consider two variants, one including a country-specific time trend and one without the time trend. We provide more details below.

Multiplying both sides of $C^N_t = (1-\varphi)\left(\frac{P^N_t}{P_{C,t}}\right)^{-\phi} C_t$ by P^N/P_C leads to the non-tradable content of consumption expenditure:

$$1 - \alpha_{C,t} = \frac{P^N_t C^N_t}{P_{C,t} C_t} = (1-\varphi)\left(\frac{P^N_t}{P_{C,t}}\right)^{1-\phi}. \tag{136}$$

Because time series for non-traded consumption display a short time horizon for most of the countries of our sample while data for sectoral value added and GDP demand components are available for all of the countries of our sample over the period running from 1970 to 2015, we construct time series for the share of non-tradables by using the market clearing condition for non-tradables:

$$\frac{P^N_t C^N_t}{P_{C,t} C_t} = \frac{1}{\omega_{C,t}} \left[\frac{P^N_t Y^N_t}{Y_t} - (1-\alpha_J)\,\omega_{J,t} - \omega_{G^N}\omega_{G,t}\right]. \tag{137}$$

Since the time horizon is too short at a disaggregated level (for I^j and G^j) for most of the countries, we draw on the evidence documented by Bems [2008] which reveals that $1 - \alpha^J = \frac{P^N J^N}{P^J J}$ is constant over time; we further assume that $\frac{P^N G^N}{G} = \omega_{G^N}$ is constant as well in line with our evidence. We

Table 9: Estimates of Elasticity of Labor Supply across Sectors (ϵ)

Country	Elasticity of labor supply across Sectors (ϵ), eq. (135)
AUS	0.412^a (2.83)
AUT	1.102^b (2.49)
BEL	0.602^a (2.97)
CAN	0.388^a (3.42)
DNK	0.277^b (2.05)
ESP	0.948^a (3.08)
FIN	0.425^a (3.61)
FRA	1.389^b (2.36)
GBR	0.610^a (3.31)
IRL	0.090^b (2.22)
ITA	1.651^b (2.53)
JPN	0.793^a (2.94)
KOR	2.267^a (2.79)
NLD	0.218^c (1.73)
NOR	0.023 (0.62)
PRT	0.586^a (3.48)
SWE	0.527^a (3.53)
USA	2.439^c (1.79)
Countries	18
Observations	806
Data coverage	1971-2015
Country fixed effects	yes
Time dummies	yes
Time trend	no

Notes: ^{a}, ^{b} and ^{c} denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.

thus recover time series for the share of non-tradables by using time series for the non-traded value added at current prices, $P_t^N Y_t^N$, GDP at current prices, Y_t, consumption expenditure, gross fixed capital formation, I_t, government spending, G_t while keeping the non-tradable content of investment and government expenditure, $1-\alpha_J$, and ω_{G^N}, fixed.

Once we have constructed time series for $1-\alpha_{C,t} = \frac{P_t^N C_t^N}{P_{C,t} C_t}$ by using (137), we take the logarithm of both sides of (136) and run the regression of the logged share of non-tradables on the logged ratio of non-traded prices to the consumption price index:

$$\ln\left(1-\alpha_{C,it}\right) = f_i + f_t + \alpha_i\ .t + (1-\phi)\ln\left(P^N/P_C\right)_{it} + \mu_{it}, \tag{138}$$

where f_i captures the country fixed effects, f_t are time dummies, and μ_{it} are the i.i.d. error terms. Because parameter φ in (136) may display a trend over time, we add country-specific trends, as captured by $\alpha_i t$. It is worth mentioning that P^N is the value added deflator of non-tradables.

Data for non-traded value added at current prices, $P_t^N Y_t^N$ and GDP at current prices, Y_t, are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases (data coverage: 1970-2015 for all countries, except Japan: 1974-2015). To construct time series for consumption, investment and government expenditure as a percentage of nominal GDP, i.e., $\omega_{C,t}$, $\omega_{J,t}$ and $\omega_{G,t}$, respectively, we use data at current prices obtained from the OECD Economic Outlook [2017] Database (data coverage: 1970-2015). Sources, construction and data coverage of time series for the share of non-tradables in investment $(1-\alpha_J)$ and in government spending (ω_{G^N}) are described in depth in Section F (see eq. (100)); P^N is the value added deflator of non-tradables. Data are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases (data coverage: 1970-2015 for all countries, except for Japan: 1974-2015). Finally, data for the consumer price index $P_{C,t}$ are obtained from the OECD Prices and Purchasing Power Parities [2017] database (data coverage: 1970-2015).

Since both sides of (138) display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000], [2001]. FMOLS estimates of (138) are reported in Table 10. When we include a country-specific time trend, the vast majority (16 out of 18) of the FMOLS estimated coefficients are positive; yet, twelve out of seventeen are statistically significant, including AUS, AUT, CAN, DNK, ESP, FIN, IRL, ITA, JPN, KOR, NOR, USA. We thus also run the same regression as in eq. (138) by ignoring country-specific time trends. We replace inconsistent (i.e., negative or no statistically significant) estimates for ϕ when adding a country-specific time trend with those obtained when we excluded the country-specific time trend. Except for GBR for which estimates are negative in both cases, one out of the two regressions leads to consistent estimates for the elasticity of substitution. For the countries mentioned below, estimates for ϕ obtained with a time trend are replaced with those when we drop the time trend: $\phi = 1.221$ ($t = 1.68$) for BEL, $\phi = 0.978$ ($t = 2.10$) for FRA, $\phi = 0.826$ ($t = 6.25$) for NLD, $\phi = 0.299$ ($t = 2.61$) for PRT and $\phi = 0.487$ ($t = 2.49$) for SWE. For GBR, the estimated value is negative whether there is a time trend in the regression or not and thus we set ϕ to zero for the rest of the analysis for this country. Table 10 shows estimates for ϕ for each country. All values are statistically significant at 10%. Overall, we find that ϕ ranges from a low of 0.299 for PRT to a high of 1.417 for ESP.

L.3 Estimates of σ^j: Empirical strategy

To estimate the elasticity of substitution between capital and labor, σ^j, we draw on Antràs [2004]. We let labor- and capital-augmenting technological change grow at a constant rate:

$$A_t^j = A_0^j e^{a^j t}, \tag{139a}$$

$$B_t^j = B_0^j e^{b^j t}, \tag{139b}$$

where a^j and b^j denote the constant growth rate of labor- and capital-augmenting technical progress and A_0^j and B_0^j are initial levels of technology. Inserting first (139a) and (139b) into the demand for labor and capital, taking logarithm and rearranging gives:

$$\ln(Y_t^j/L_t^j) = \alpha_1 + \left(1-\sigma^j\right) a^j t + \sigma_j \ln(W_t^j/P_t^j), \tag{140a}$$

$$\ln(Y_t^j/K_t^j) = \alpha_2 + \left(1-\sigma^j\right) b^j t + \sigma_j \ln(R_t/P_t^j), \tag{140b}$$

where $\alpha_1 = \left[(1-\sigma^j)\ln A_0^j - \sigma^j \ln\gamma^j\right]$ and $\alpha_2 = \left[(1-\sigma^j)\ln B_0^j - \sigma^j \ln(1-\gamma^j)\right]$ are constants. Above equations describe firms' demand for labor and capital respectively.

We estimate the elasticity of substitution between capital and labor in sector $j = H, N$ from first-order conditions (140a)-(140b) in panel format on annual data. Adding an error term and

Table 10: Elasticity of Substitution between Tradables and Non-Tradables (ϕ)

Country	eq. (138)	Time trend
AUS	0.447^{b} (2.36)	yes
AUT	1.275^{a} (5.60)	yes
BEL	1.221^{c} (1.68)	no
CAN	0.546^{a} (3.71)	yes
DNK	1.039^{a} (2.72)	yes
ESP	1.417^{b} (2.54)	yes
FIN	0.509^{a} (2.82)	yes
FRA	0.978^{b} (2.10)	yes
GBR	0	
IRL	1.273^{a} (3.55)	yes
ITA	0.314^{a} (2.68)	yes
JPN	0.884^{a} (4.01)	yes
KOR	0.592^{b} (2.15)	yes
NLD	0.826^{a} (6.25)	no
NOR	1.006^{a} (4.72)	yes
PRT	0.299^{a} (2.61)	no
SWE	0.487^{b} (2.49)	no
USA	0.777^{a} (3.32)	yes
Countries	18	
Observations	824	
Data coverage	1970-2015	
Country fixed effects	yes	
Time dummies	yes	

Notes: ^{a}, ^{b} and ^{c} denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.

controlling for country fixed effects, we explore empirically the following equations:

$$\ln(Y_{it}^j/L_{it}^j) = \alpha_{1i} + \lambda_{1i}t + \sigma_i^j \ln(W_{it}^j/P_{it}^j) + u_{it}, \quad (141a)$$
$$\ln(Y_{it}^j/K_{it}^j) = \alpha_{2i} + \lambda_{2i}t + \sigma_i^j \ln(R_{it}/P_{it}^j) + v_{it}, \quad (141b)$$

where i and t index country and time and u_{it} and v_{it} are i.i.d. error terms. Country fixed effects are represented by dummies α_{1i} and α_{2i}, and country-specific trends are captured by λ_{1i} and λ_{2i}. Since all variables display unit root process, we estimate cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000].

Estimation of (141a) and (141b) requires data for each sector $j = H, N$ on sectoral value added at constant prices Y^j, sectoral hours worked L^j, sectoral capital stock K^j, sectoral value added deflator P^j, sectoral wage rate W^j and capital rental cost R. Data for sectoral value added Y^H and Y^N, hours worked L^H and L^N, value added price deflators P^H and P^N, and, nominal wages W^H and W^N are taken form the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. To construct the national stock of capital K, we use the perpetual inventory method with a fixed depreciation rate taken from Table 6 and the time series of constant prices investment from the OECD Economic Outlook [2017] Database. Next, following Garofalo and Yamarik [2002], the capital stock is allocated to traded and non-traded industries by using sectoral output shares. Finally, we measure the aggregate rental price of capital R as the ratio of capital income to capital stock. Capital income is derived as nominal value added minus labor compensation. For all aforementioned variables, the sample includes the 18 OECD countries over the period 1970-2015 (except for Japan: 1974-2015).

Employing Monte Carlo experiments, León-Ledesma et al. [2010] compare different approaches for estimating the elasticity of substitution between capital and labor (single equation based on FOCs, system, linear, non-linear and normalization). Their evidence suggests that estimates of both the elasticity of substitution and technical change are close to their true values when the FOC with respect to labor is used. While we take the demand for labor as our baseline model (i.e. eq. (141a)), Table 11 provides FMOLS estimates of σ^j for the demand of labor and capital. The bulk (32 out of 36) of the FMOLS estimated coefficients from eq. (141a) are positive and statistically significant. One estimated coefficient is negative (σ^H for IRL) while estimates of σ^H for Finland and Portugal and σ^N for Japan are positive but not statistically significant. To deal with this issue, we run again the same regression by dropping time dummies which gives consistent estimates for σ^H for Finland, Portugal and for σ^N for Japan. However, the estimate for σ^H is still negative for Ireland. As in Antràs [2004], we alternatively run the regression of the ratio of value added to capital stock at constant prices on the real capital cost R/P^j in sector j, i.e., eq. (141b). We then replace inconsistent estimates for σ^j obtained from labor demand with those obtained from the demand of capital. Columns 19-20 of Table 7 report estimates for σ^H and σ^N.

M Fiscal Transmission and Technology: More Numerical Results

In the main text, we focus on cumulative changes in value added and in labor together with their sectoral decomposition, see Table 3. In this section, in Table 12, we focus on sectoral TFPs and utilization-adjusted-TFPs in panel D and sectoral LISs in panel E and show points estimates obtained from local projections in columns 1 and 4 that we contrast with impact and cumulative effects at a six-year horizon from the baseline and restricted models.

The responses of technology to an exogenous shock to government consumption are shown in panel D. The first two rows of panel D reveal that the capital-utilization-adjusted TFP increases by 0.28% and 0.41%, respectively, in the traded and the non-traded sector, close to what we estimate empirically (i.e., 0.21% and 0.37%) on impact. Whilst technology improvements are similar across sectors on impact, the cumulative effect over a six-year horizon reveals that the increase in the efficiency in the use of inputs is much more pronounced in the traded than in the non-traded sector because the cost of adjusting technology is lower in the traded than in the non-traded sector.[24] The combined effect of a higher capital and technology utilization rate pushes up traded and non-traded TFP by

[24] Although the model understates the cumulative change in the capital-utilization-adjusted-traded TFP (4.35% against 6.04% in the data), the model reproduces well the cumulative change in traded value added (3.12 ppt against 2.86 ppt of GDP in the data).

Table 11: FMOLS Estimates of the Sectoral Elasticity of Substitution between Capital and Labor (σ^j)

Country	Tradables (σ^H)			Non-Tradables (σ^N)		
Dependent var.	$\ln(Y^H/K^H)$	$\ln(Y^H/L^H)$	$\ln(Y^H/L^H)$	$\ln(Y^N/K^N)$	$\ln(Y^N/L^N)$	$\ln(Y^N/L^N)$
Explanatory var.	$\ln(R/P^H)$	$\ln(W^H/P^H)$	$\ln(W^H/P^H)$	$\ln(R/P^N)$	$\ln(W^N/P^N)$	$\ln(W^N/P^N)$
AUS	0.339^a	0.550^a	0.417^a	0.490^a	0.669^a	0.522^a
	(2.62)	(7.19)	(3.34)	(3.38)	(12.19)	(5.64)
AUT	0.511^a	0.936^a	0.909^a	0.004	1.432^a	1.341^a
	(3.62)	(15.78)	(6.36)	(0.02)	(9.48)	(12.82)
BEL	0.079	0.739^a	0.839^a	−0.067	1.098^a	1.113^a
	(0.44)	(9.51)	(10.12)	(−0.83)	(8.42)	(7.42)
CAN	−0.028	1.091^a	0.510^a	0.867^a	0.902^a	0.669^a
	(−0.20)	(5.66)	(3.13)	(5.88)	(12.28)	(7.90)
DNK	−0.063	0.574^a	0.447^a	0.407^a	1.168^a	1.253^a
	(−0.46)	(5.04)	(5.05)	(7.13)	(6.64)	(7.10)
ESP	0.410^a	1.098^a	0.996^a	0.272^b	0.900^a	0.485^a
	(3.63)	(10.43)	(11.09)	(2.07)	(4.54)	(3.27)
FIN	0.105	0.100	1.141^a	0.409^a	0.841^a	0.847^a
	(0.60)	(0.34)	(3.24)	(6.51)	(8.36)	(8.77)
FRA	0.103	0.753^a	1.003^a	0.095^a	0.815^a	0.910^a
	(0.73)	(7.55)	(5.94)	(2.73)	(3.99)	(3.76)
GBR	−0.041	0.398^a	0.617^a	0.126	0.464^a	0.644^a
	(−0.24)	(5.32)	(8.04)	(1.24)	(3.49)	(3.33)
IRL	0.714^a	−0.017	−0.130	0.635^a	0.983^a	0.609^a
	(11.15)	(−0.09)	(−0.71)	(3.97)	(4.32)	(2.67)
ITA	0.512^a	0.860^a	0.892^a	0.333^b	0.503^a	0.246
	(2.65)	(11.90)	(9.28)	(2.08)	(4.15)	(1.37)
JPN	0.793^a	0.936^a	1.202^a	0.502^a	0.290	0.899^a
	(7.69)	(4.43)	(7.82)	(6.13)	(1.43)	(3.63)
KOR	0.279^a	0.432^a	0.636^a	0.513^a	0.795^a	0.827^a
	(5.45)	(5.85)	(7.57)	(12.42)	(6.83)	(6.85)
NLD	0.436^a	1.075^a	0.970^a	0.170^a	0.610^a	0.422^a
	(4.85)	(9.41)	(6.70)	(7.74)	(6.62)	(3.38)
NOR	0.797^a	0.421^a	0.664^a	0.674^a	0.840^a	0.588^a
	(4.36)	(3.07)	(3.98)	(12.70)	(6.57)	(5.24)
PRT	-0.066^b	0.175	0.525^b	0.308	0.455^a	0.495^a
	(−2.14)	(1.31)	(2.52)	(1.19)	(9.02)	(10.58)
SWE	0.086	0.648^a	0.604^a	0.054	0.519^a	0.116
	(0.34)	(14.52)	(8.36)	(0.61)	(4.54)	(0.53)
USA	−0.081	0.706^a	0.758^a	0.400^a	1.098^a	0.854^a
	(−0.58)	(3.51)	(10.31)	(5.47)	(7.90)	(4.83)
Whole sample	0.271^a	0.638^a	0.722^a	0.344^a	0.799^a	0.713^a
	(10.49)	(28.46)	(26.44)	(18.96)	(28.46)	(23.36)
Countries	18	18	18	18	18	18
Observations	824	824	824	824	824	824
Fixed effects	yes	yes	yes	yes	yes	yes
Time dummies	yes	yes	no	yes	yes	no
Time trend	yes	yes	yes	yes	yes	yes

Notes: ^{a}, ^{b} and ^{c} denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. Data coverage: 1970-2015.

Table 12: Impact and Cumulative Effects of an Increase in Government Consumption by 1% of GDP

	LP $t = 0$	Impact Responses		LP $t = 0..5$	Cumulative Responses	
	Data	CES-TECH	CD	Data	CES-TECH	CD
	(1)	(2)	(3)	(4)	(5)	(6)
D.Technology						
Traded technology utilization, $du^{Z,H}(t)$	0.21	0.28	0.00	6.04	4.35	0.00
Non-Traded technology utilization, $du^{Z,N}(t)$	0.37	0.41	0.00	2.15	1.97	0.00
Traded TFP, $d\text{TFP}^H(t)$	0.66	0.34	0.00	7.82	5.50	0.00
Non-Traded TFP, $d\text{TFP}^N(t)$	0.36	0.49	0.00	0.28	2.37	0.00
E.Redistributive effects						
Traded LIS, $ds^H_L(t)$	0.14	0.12	0.00	-2.23	-3.08	0.00
Decomposition						
Caused by capital deepening		-0.09			0.96	
Caused by FBTC		0.21			-4.04	
Non-traded LIS, $ds^N_L(t)$	0.25	0.25	0.00	3.07	2.99	0.00
Decomposition						
Caused by capital deepening		-0.04			-0.58	
Caused by FBTC		0.29			3.57	

Notes: Impact ($t = 0$) and cumulative ($t = 0...5$) effects of an exogenous temporary increase in government consumption by 1% of GDP. Panels D,E show the deviation in percentage relative to steady-state for technological change and LISs. Responses of sectoral LISs are measured in percent of value added of the corresponding sector. Columns 2 and 5 labelled 'CES-TECH' show predictions of the baseline model while columns 3 and 6 labelled 'CD' shows predictions of the restricted version of the model. In the restricted model, we impose $\sigma^j = 1$ so that production functions are Cobb-Douglas, let ξ^j_2, χ^j_2 tend toward infinity so that the capital and technology utilization rate collapses to one, and set ξ^j_A, χ^j_A, ξ^j_B, χ^j_B to zero so that labor- and capital-augmenting technological rate remains fixed. In columns 1 and 4, we report point estimates from the VAR model. We also report the reconstructed response of non-traded TFP and capital-utilization adjusted non-traded TFP in order to be consistent with macroeconomic identities. We reconstruct the empirical response of $\text{TFP}^N(t)$ and $Z^N(t)$ because we found a discrepancy between empirically estimated and reconstructed responses. To reconstruct the empirical responses of $\text{TFP}^N(t)$ and $Z^N(t)$, we use empirical responses of aggregate and traded TFP which are both statistically significant to recover the dynamic response of $\hat{\text{TFP}}^N(t)$ (by using eq. (47)) and we plug the latter together with the response of $\hat{u}^{K,N}(t)$ to recover $\hat{Z}^N(t)$. Inspection of Fig. 2(d) and Fig. 2(e) gives a sense of the discrepancy between the estimated vs. reconstructed response of capital-utilization adjusted non-traded TFP and non-traded TFP.

5.5% and 2.37% in cumulative terms, respectively.[25]

Panel E of Table 3 shows that our model reproduces well the adjustment in the sectoral LISs both on impact and over a six-year horizon. As shown in Online Appendix C, the response of the LIS in sector $j = H, N$ can be broken down into a capital deepening effect and a FBTC effect. When we shut down technological change, $s^j_L(t)$ is only affected through the capital deepening effect. Because both sectors experience a fall in $\tilde{k}^j(t)$ and $\sigma^j < 1$ (as evidence suggests), $s^j_L(t)$ declines in contradiction with our evidence.[26] Conversely, as long as we allow for time-varying FBTC, the ability of the model to reproduce the dynamics of sectoral LISs substantially increases. As shown in column 5, technological change biased toward capital in the traded sector generates a cumulative decline in $s^H_L(t)$ by -4.04% while technological change biased toward labor in the non-traded sector increases $s^N_L(t)$ by 3.57%. Because FBTC now shifts capital toward the traded sector which becomes more capital intensive, the traded LIS increases through the capital deepening channel by 0.96%. Conversely, the non-traded sector experiences a capital outflow which lowers $s^N_L(t)$ by -0.58% through the capital deepening channel. In both cases, the FBTC channel more than offsets the capital deepening channel so that the baseline model predicts a cumulative fall in the traded LIS by -3.08% (-2.28% in the data) and a cumulative increase in the non-traded LIS by 2.99% (3.07% in the data).

[25] The model overstates the rise in non-traded TFP as it somewhat overpredicts the increase in the non-traded capital utilization rate.

[26] A restricted model assuming CES production functions and abstracting from technological change predicts a fall in both $\tilde{k}^H(t)$ and $\tilde{k}^N(t)$ which generates a (present value) cumulative decline in $s^H_L(t)$ by -0.6% and in $s^N_L(t)$ by -0.12%, both computed over six-year horizon.

N More Numerical Results: Technology Channel across Restricted Versions of the Baseline Model

Since in the main text, we cannot show the impact effects for all variables together with cumulative responses for reasons of space, in this section, we provide more numerical results. We consider eight variants of the baseline model where we shut down one or several dimensions related to barriers to mobility (such as capital installation costs, labor mobility costs, endogenous terms of traded) and technology factors (such as capital and technology utilization rates, CES production functions and factor-biased technological change.

Table 13 shows impact effects while Table 14 displays cumulative effects over a six-years horizon. In columns 1-5, we consider small open economy model where home-produced and foreign-produced traded goods are perfect substitutes so that terms of trade are exogenous. In columns 1-4 we consider four variants of a model with perfect mobility of labor while in column 5 we assume imperfect mobility of labor across sectors. In column 1, we abstract from capital installation costs and shut down the technology channel. In column 2, we allow for capital installation costs and switch off the technology channel. In column 3, we allow for capital installation costs and endogenous technology utilization rate. In column 4, we we allow for capital installation costs, endogenous technology and capital utilization rates. In column 5, we assume imperfect mobility of labor across sectors and allow for technology channel. In columns 6-7, we allow for capital installation costs, imperfect substitutability between home- and foreign- produced traded goods and assume Cobb-Douglas production functions. In column 6, we allow for endogenous technology and capital utilization rates while in column 7, we shut down endogenous capital utilization rates. In column 8, we consider the baseline model with CES production functions, factor-biased technological change, and endogenous technology utilization rates except that we shut down the capital utilization rate.

O More Empirical Results and Robustness Checks

In this section, we conduct some robustness checks. Due to data availability, we use annual data for eleven 1-digit ISIC-rev.3 industries that we classify as tradables or non-tradables. At this level of disaggregation, the classification is somewhat ambiguous because some broad sectors are made-up of heterogenous sub-industries, a fraction being tradables and the remaining industries being non-tradables. Since we consider a sample of 18 OECD countries and a period running from 1970 to 2015, the classification of some sectors may vary across time and countries. Industries such as 'Transport and Communication', 'Finance Intermediation' classified as tradables, 'Hotels and Restaurants' classified as non-tradables display intermediate levels of tradedness which may vary considerably across countries but also across time. Subsection O.1 deals with this issue and conducts a robustness check to investigate the sensitivity of our empirical results to the classification of industries as tradables or non-tradables.

In the main text, we compute the labor income share in the lines of Gollin [2002], i.e., labor compensation is defined as the sum of compensation of employees plus compensation of self-employed. Since there exists alternative ways in constructing labor compensation, we explore empirically in subsection O.2 whether the results we document in the main text are robust to alternative measures of the labor income share.

Our dataset covers eleven industries which are classified as tradables or non tradables. The traded sector is made up of five industries and the non-traded sector of six industries. In subsection O.3, we conduct our empirical analysis at a more disaggregate level. The objective is twofold. First, we investigate whether all industries classified as tradables or non-tradables behave homogeneously or heterogeneously. Second, we explore empirically which industry drives the responses of broad sectors following a rise in government spending by 1% of GDP.

A close empirical analysis to ours is that performed by Cardi, Restout and Claeys (CRC henceforth) [2020] who investigate the sectoral and reallocation effects of an exogenous and temporary increase in government consumption. We compare our empirical findings shown

Table 13: Impact Effects of an Increase in Government Consumption by 1% of GDP in Restricted Versions of the Baseline Model

	CD: PML				CD: IML	CD: IML & TOT		CES: IML & TOT & FBTC	
	No CAC	CAC	$u^{Z,j}$	$u^{K,j}$	exo TOT	endo TOT	No $u^{K,j}$	No $u^{K,j}$	Bench
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A.Aggregate Multipliers									
Gov. spending, $dG(t)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Total hours worked, $dL(t)$	0.14	0.16	-0.15	0.30	0.60	0.96	0.69	0.88	0.97
Real GDP, $d\tilde{Y}_R(t)$	0.09	0.11	-0.55	0.40	0.72	1.36	0.60	0.85	1.07
B.Sectoral Labor									
Traded labor, $dL^H(t)$	-0.49	-0.62	-0.81	-0.65	-0.13	0.16	0.06	0.14	0.18
Non-traded labor, $dL^N(t)$	0.63	0.78	0.66	0.95	0.63	0.80	0.63	0.74	0.78
Labor share of non-tradables, $d\nu^{L,N}(t)$	0.53	0.67	0.80	0.83	0.29	0.16	0.19	0.16	0.16
C.Sectoral Value Added									
Traded VA, $dY^H(t)$	-0.52	-0.65	-1.57	-1.01	-0.27	0.21	-0.14	0.08	0.20
Non-traded VA, $dY^N(t)$	0.61	0.76	1.02	1.42	0.99	1.14	0.74	0.77	0.88
VA share of non-tradables, $dY^N(t)$	0.55	0.69	1.47	1.33	0.56	0.27	0.36	0.22	0.20
D.Factor Prices									
Traded wages, $dW^H(t)$	0.00	0.01	0.13	0.15	-0.20	0.05	0.17	0.20	0.17
Non-traded wages, $dW^N(t)$	0.00	0.01	0.13	0.15	0.58	0.69	0.59	0.82	0.86
Capital rental cost, $dR(t)$	0.00	-0.01	-0.22	-0.26	0.35	0.53	1.06	0.55	0.39
E.Relative Prices									
Relative price of non-tradables, $dP(t)$	0.00	0.00	0.02	0.02	0.51	0.42	0.24	0.41	0.47
Terms of trade, $dP^H(t)$	0.00	0.00	0.00	0.00	0.00	0.22	0.50	0.32	0.25
F.Current Account (% of GDP)									
Current account, $dCA(0)$	-0.62	-0.81	-0.97	-0.74	-0.34	-0.04	-0.08	-0.06	-0.05
G.Technology									
Traded capital utilization, $du^{K,H}(t)$	0.00	0.00	0.00	-0.76	-0.07	0.24	0.00	0.00	0.17
Non-Traded capital utilization, $du^{K,N}(t)$	0.00	0.00	0.00	1.28	0.74	1.21	0.00	0.00	0.25
Traded technology utilization, $du^{Z,H}(t)$	0.00	0.00	-2.27	-1.52	-0.49	0.29	-0.21	0.11	0.28
Non-Traded technology utilization, $du^{Z,N}(t)$	0.00	0.00	0.48	0.65	0.40	0.52	0.35	0.35	0.41
H.Redistributive effects									
Traded LIS, $ds_L^H(t)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.12
Non-traded LIS, $ds_L^N(t)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.25

Notes: This table shows impact effects on selected sectoral variables of a 1% temporary increase in government consumption in the baseline model (column 9) and in restricted versions of the model (columns 1-8). In columns 1-5, we consider a small open economy model where home-produced and foreign-produced traded goods are perfect substitutes so that terms of trade are exogenous. In columns 1-4 we consider four variants of a model with perfect mobility of labor while in column 5 we assume imperfect mobility of labor across sectors. In column 1, we abstract from capital installation costs and shut down the technology channel. In column 2, we allow for capital installation costs and switch off the technology channel. In column 3, we allow for capital installation costs and endogenous technology utilization rate. In column 4, we allow for capital installation costs, endogenous technology and capital utilization rates. In column 5, we assume imperfect mobility of labor across sectors and allow for the technology channel. In columns 6-7, we allow for capital installation costs, imperfect substitutability between home- and foreign- produced traded goods and assume Cobb-Douglas production functions. In column 6, we allow for endogenous technology and capital utilization rates while in column 7, we shut down capital utilization rates. In column 8, we consider the baseline model with CES production functions, factor-biased technological change, and endogenous technology utilization rates except that we shut down the capital utilization rate.

Table 14: Cumulative Effects of an Increase in Government Consumption by 1% of GDP in Restricted Versions of the Baseline Model

	CD: PML				CD: IML	CD: IML & TOT		CES: IML & TOT & FBTC	
	No CAC	CAC	$u^{Z,j}$	$u^{K,j}$	exo TOT	endo TOT	No $u^{K,j}$	No $u^{K,j}$	Bench
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A.Aggregate Multipliers									
Gov. spending, $dG(t)$	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46	5.46
Total hours worked, $dL(t)$	0.77	0.91	-0.56	2.04	3.28	5.11	3.62	4.80	5.61
Real GDP, $d\tilde{Y}_R(t)$	0.46	0.63	-2.56	3.00	3.94	7.18	3.01	5.14	7.36
B.Sectoral Labor									
Traded labor, $dL^H(t)$	-3.31	-3.27	-4.30	-3.24	-0.64	0.86	0.28	0.12	0.69
Non-traded labor, $dL^N(t)$	4.15	4.25	3.79	5.31	3.44	4.25	3.34	4.68	4.92
Labor share of non-tradables, $d\nu^{L,N}(t)$	3.61	3.66	4.46	4.40	1.58	0.88	1.00	1.51	1.26
C.Sectoral Value Added									
Traded VA, $dY^H(t)$	-3.52	-3.45	-8.26	-4.86	-1.40	1.10	-0.84	1.61	3.12
Non-traded VA, $dY^N(t)$	4.04	4.15	5.75	7.89	5.35	6.08	3.85	3.54	4.24
VA share of non-tradables, $d\nu^{Y,N}(t)$	3.73	3.75	8.06	6.94	2.97	1.48	1.96	0.24	-0.41
D.Current Account									
Current Account, $dCA(t)$	-4.80	-4.80	-5.82	-4.18	-1.96	-0.19	-0.42	-0.09	-0.02
E.Technology									
Traded technology utilization, $du^{Z,H}(t)$	0.00	0.00	-12.14	-7.55	-2.57	1.50	-1.27	2.26	4.35
Non-Traded technology utilization, $du^{Z,N}(t)$	0.00	0.00	2.71	3.60	2.17	2.79	1.81	1.62	1.97
F.Redistributive effects									
Traded LIS, $ds_L^H(t)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.18	-3.08
Non-traded LIS, $ds_L^N(t)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.92	2.99

Notes: This table shows cumulative effects on selected sectoral variables of a 1% temporary increase in government consumption in the baseline model (column 9) and in restricted versions of the model (columns 1-8). In columns 1-5, we consider a small open economy model where home-produced and foreign-produced traded goods are perfect substitutes so that terms of trade are exogenous. In columns 1-4 we consider four variants of a model with perfect mobility of labor while in column 5 we assume imperfect mobility of labor across sectors. In column 1, we abstract from capital installation costs and shut down the technology channel. In column 2, we allow for capital installation costs and switch off the technology channel. In column 3, we allow for capital installation costs and endogenous technology utilization rate. In column 4, we allow for capital installation costs, endogenous technology and capital utilization rates. In column 5, we assume imperfect mobility of labor across sectors and allow for the technology channel. In columns 6-7, we allow for capital installation costs, imperfect substitutability between home- and foreign- produced traded goods and assume Cobb-Douglas production functions. In column 6, we allow for endogenous technology and capital utilization rates while in column 7, we shut down capital utilization rates. In column 8, we consider the baseline model with CES production functions, factor-biased technological change, and endogenous technology utilization rates except that we shut down the capital utilization rate.

in Fig. 1 with CRC [2020] estimates. In contrast to CRC [2020], we identify shocks to government consumption once for all by estimating one unique VAR model instead of different VAR models which can potentially pickup different structural government spending shocks and we trace the dynamic effects by adopting the local projection method, all variables responding to the same government spending shock. We consider a sample of 18 OECD countries over 1970-2015 instead of 16 OECD countries over 1970-2007. In subsection O.4, we report significant differences between our own findings and estimates documented by CRC [2020] and show that both the sample and the empirical strategy matters.[27]

Finally, since we split the gross capital stock into traded and non-traded industries by using sectoral valued added shares, in subsection O.5, we conduct a robustness check by taking time series for sectoral capital stock from KLEMS which are available for a limited number of countries.

O.1 Classification of Industries as Tradables vs. Non-Tradables

This section explores the robustness of our findings to the classification of the eleven 1-digit ISIC-rev.3 industries as tradables or non tradables.

Following De Gregorio et al. [1994], we define the tradability of an industry by constructing its openness to international trade given by the ratio of total trade (imports + exports) to gross output. Data for trade and output are provided by the World Input-Output Databases ([2013], [2016]). Table 15 gives the openness ratio (averaged over 1995-2014) for each industry in all countries of our sample. Unsurprisingly, "Agriculture, Hunting, Forestry and Fishing", "Mining and Quarrying", "Total Manufacturing" and "Transport, Storage and Communication" exhibit high openness ratios (0.54 in average if "Mining and Quarrying", due to its relatively low weight in GDP, is not considered). These four sectors are consequently classified as tradables. At the opposite, "Electricity, Gas and Water Supply", "Construction", "Wholesale and Retail Trade" and "Community Social and Personal Services" are considered as non tradables since the openness ratio in this group of industries is low (0.07 in average). For the three remaining industries "Hotels and Restaurants", "Financial Intermediation", "Real Estate, Renting and Business Services" the results are less clearcut since the average openness ratio amounts to 0.18 which is halfway between the two aforementioned averages. In the benchmark classification, we adopt the standard classification of De Gregorio et al. [1994] by treating "Real Estate, Renting and Business Services" and "Hotels and Restaurants" as non traded industry. Given the dramatic increase in financial openness that OECD countries have experienced since the end of the eighties, we allocate "Financial Intermediation" to the traded sector. This choice is also consistent with the classification of Jensen and Kletzer [2006] who categorize "Finance and Insurance" as tradable. They use locational Gini coefficients to measure the geographical concentration of different sectors and classify sectors with a Gini coefficient below 0.1 as non-tradable and all others as tradable (the authors classify activities that are traded domestically as potentially tradable internationally).

We conduct below a sensitivity analysis with respect to the three industries ("Real Estate, Renting and Business Services", "Hotels and Restaurants" and "Financial Intermediation") which display some ambiguity in terms of tradedness to ensure that the benchmark classification does not drive the results. In order to address this issue, we re-estimate the dynamic responses to a government spending shock for the main variables of interest using local projections for different classifications in which one of the three above industries initially marked as tradable (non tradable resp.) is classified as non tradable (tradable resp.), all other industries staying in their original sector. In doing so, the classification of only one industry is altered, allowing us to see if the results are sensitive to the inclusion of a particular industry in the traded or the non traded sector.

As an additional robustness check, we also exclude the industry "Community Social and Personal Services" from the non-tradable industries' set. This robustness analysis is based

[27] CRC [2020] document evidence pointing at a significant increase in the value added and the labor share of non-tradables. We find that the value added share of non-tradables is unresponsive and show that CRC's [2020] finding stems from using a different sample and adopting a one step VAR approach which lead technological change and labor income shares to be unresponsive.

Table 15: Openness Ratios per Industry: 1995-2014 Averages

	Agri.	Minig	Manuf.	Elect.	Const.	Trade	Hotels	Trans.	Finance	Real Est.	Public
AUS	0.242	0.721	0.643	0.007	0.005	0.025	0.255	0.247	0.054	0.051	0.054
AUT	0.344	2.070	1.152	0.178	0.075	0.135	0.241	0.491	0.302	0.221	0.043
BEL	1.198	13.374	1.414	0.739	0.067	0.186	0.389	0.536	0.265	0.251	0.042
CAN	0.304	0.821	0.966	0.098	0.002	0.030	0.338	0.211	0.169	0.121	0.038
DNK	0.470	0.786	1.329	0.214	0.014	0.092	0.021	0.832	0.138	0.131	0.027
ESP	0.386	4.699	0.680	0.021	0.003	0.044	0.008	0.206	0.130	0.149	0.022
FIN	0.228	2.899	0.796	0.117	0.006	0.094	0.131	0.280	0.153	0.256	0.021
FRA	0.280	3.632	0.815	0.049	0.004	0.048	0.001	0.224	0.068	0.070	0.014
GBR	0.360	0.853	0.958	0.017	0.010	0.024	0.148	0.209	0.233	0.147	0.041
IRL	0.298	1.384	1.127	0.154	0.013	0.652	0.772	0.285	1.014	0.988	0.049
ITA	0.300	4.130	0.603	0.041	0.013	0.087	0.035	0.150	0.095	0.082	0.012
JPN	0.158	3.923	0.293	0.004	0.000	0.067	0.021	0.159	0.034	0.020	0.005
KOR	0.175	18.603	0.507	0.012	0.003	0.213	0.029	0.388	0.071	0.114	0.052
NLD	0.988	1.496	1.259	0.082	0.076	0.106	0.011	0.562	0.245	0.405	0.114
NOR	0.391	0.837	0.995	0.146	0.024	0.097	0.009	0.354	0.146	0.143	0.058
PRT	0.351	2.954	0.880	0.050	0.005	0.067	0.105	0.378	0.125	0.114	0.026
SWE	0.294	2.263	0.969	0.119	0.020	0.163	0.019	0.392	0.274	0.256	0.026
USA	0.207	0.541	0.428	0.012	0.001	0.055	0.003	0.109	0.066	0.052	0.008
OECD	0.388	3.666	0.879	0.114	0.019	0.121	0.141	0.334	0.199	0.198	0.036
H/N	H	H	H	N	N	N	N	H	H	N	N

Notes: the complete designations for each industry are as follows (EU KLEMS codes are given in parentheses). "Agri.": "Agriculture, Hunting, Forestry and Fishing" (AtB), "Minig": "Mining and Quarrying" (C), "Manuf.": "Total Manufacturing" (D), "Elect.": "Electricity, Gas and Water Supply" (E), "Const.": "Construction" (F), "Trade": "Wholesale and Retail Trade" (G), "Hotels": "Hotels and Restaurants" (H), "Trans.": "Transport, Storage and Communication" (I), "Finance": "Financial Intermediation" (J), "Real Est.": "Real Estate, Renting and Business Services" (K), "Public": "Community Social and Personal Services" (LtQ). The openness ratio is the ratio of total trade (imports + exports) to gross output (source: World Input-Output Databases ([2013], [2016]).

on the presumption that among the industries provided by the EU KLEMS and STAN databases, this industry is government-dominated. While this exercise is interesting on its own as it allows us to explore the size of the impact of a government spending shock on the business sector, we also purge for the potential and automatic link between non-traded value added and public spending because government purchases (to the extent that the government is the primary purchaser of goods from this industry) account for a significant part of non-traded value added.[28] The baseline and the four alternative classifications considered in this exercise are shown in Table 16. The last line provides the matching between the color line (when displaying IRFs below) and the classification between tradables and non tradables.

Table 16: Robustness check: Classification of Industries as Tradables or Non Tradables

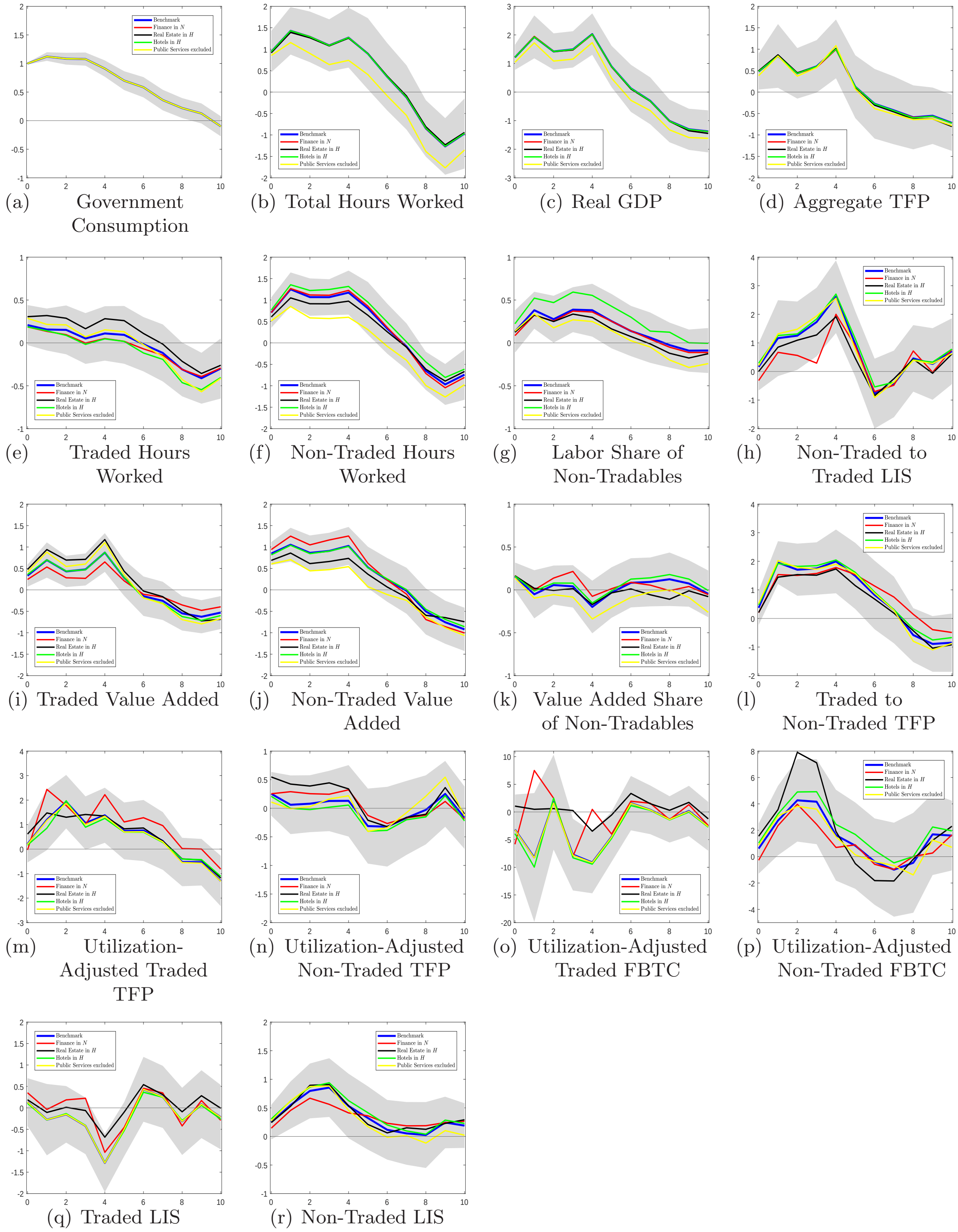
	KLEMS	Classification				
	code	Baseline	#1	#2	#3	#4
Agriculture, Hunting, Forestry and Fishing	AtB	H	H	H	H	H
Mining and Quarrying	C	H	H	H	H	H
Total Manufacturing	D	H	H	H	H	H
Electricity, Gas and Water Supply	E	N	N	N	N	N
Construction	F	N	N	N	N	N
Wholesale and Retail Trade	G	N	N	N	N	N
Hotels and Restaurants	H	N	N	N	**H**	N
Transport, Storage and Communication	I	H	H	H	H	H
Financial Intermediation	J	H	**N**	H	H	H
Real Estate, Renting and Business Services	K	N	N	**H**	N	N
Community Social and Personal Services	LtQ	N	N	N	N	**neither H or N**
Color line in Figure 12		blue	red	black	green	yellow

Notes: H stands for the Traded sector and N for the Non traded sector.

Fig. 12 shows the responses of variables of interest to an exogenous increase in government consumption by one percent of GDP. The solid blue line shows results for the baseline classification while the responses for the alternative classifications are shown in the four colored lines. In each panel, the shaded area corresponds to the 90% confidence bounds for

[28]This exercise has been conducted by Cardi et al. [2020] and Beetsma and Giuliodori [2011], among others, in order to deal with the potential endogeneity of government purchases with respect to output.

the baseline.



(a) Government Consumption
(b) Total Hours Worked
(c) Real GDP
(d) Aggregate TFP
(e) Traded Hours Worked
(f) Non-Traded Hours Worked
(g) Labor Share of Non-Tradables
(h) Non-Traded to Traded LIS
(i) Traded Value Added
(j) Non-Traded Value Added
(k) Value Added Share of Non-Tradables
(l) Traded to Non-Traded TFP
(m) Utilization-Adjusted Traded TFP
(n) Utilization-Adjusted Non-Traded TFP
(o) Utilization-Adjusted Traded FBTC
(p) Utilization-Adjusted Non-Traded FBTC
(q) Traded LIS
(r) Non-Traded LIS

Figure 12: Sensitivity of the Effects of a Government Spending Shock to the Classification of Industries as Tradable or Non-Tradable. Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The green line and the black line show results when "Hotels and Restaurants" and "Real Estate, Renting and Business Services" are treated as tradables, respectively. The red line shows results when "Financial Intermediation" is classified as non-tradables. The yellow line displays results when "Community Social and Personal Services" is not considered. Sample: 18 OECD countries, 1970-2015, annual data.

The first row of Fig. 12 reports the effects of an exogenous increase in government consumption by 1% of GDP on the main aggregate variables, namely, government spending, total hours, real GDP and TFP. For government spending and TFP, the responses are remarkably similar a cross the baseline and alternative classifications. We can notice that the expansionary effect of an exogenous increase in government consumption on total hours is mitigated when the public sector is excluded (classification #4 and the yellow line) but the shape of the dynamic adjustment of the two variables is similar to the benchmark classification and the alternative IRFs lie within the confidence bounds of the baseline classification. The second and third row of Fig. 12 contrast the responses of sectoral hours worked (L^j), sectoral value added (Y^j), the value added share of non-tradables ($\nu^{Y,N}$), the labor share of non-tradables ($\nu^{L,N}$), the ratio of non-traded to traded LIS, i.e., s_L^N/s_L^H, the ratio of traded to non-traded TFP, i.e., TFP^H/TFP^N for the baseline classification with those obtained for alternative classifications of industries as tradables or non-tradables. Alternative responses are fairly close to those estimated for the baseline classification as they lie within the confidence interval (for the baseline classification) for all the selected horizons (8 years). With regard to the responses of utilization-adjusted TFPs shown in the fourth row, the dynamic adjustment of $Z^j(t)$ displays a similar pattern across the baseline and alternative classifications: utilization-adjusted TFP increases in the traded sector while it is essentially unchanged in the non-traded sector. The response of utilization-adjusted FBTC in sector N is similar across all classifications. The dynamic adjustment of the utilization-adjusted FBTC in sector H displays some differences across the baseline and the four alternative classifications: the decline is less pronounced when the industry "Real Estate, Renting and Business Services" is treated as tradables (classification #2 and the black line) and the IRF is more erratic when the public sector is excluded (classification #4 and the yellow line). However, in both cases, the IRF lies well within the confidence interval for almost all the selected horizons. In addition, in the last row of Fig. 12, we investigate whether our conclusion for redistributive effects (i.e., for sectoral labor income shares) is robust to the classification of industries. Across all scenarios, LISs in both sectors exhibit a similar dynamic adjustment following an increase in government spending. One can notice that the discrepancy in the estimated effect between the benchmark and the alternative classifications is not statistically significant. In conclusion, our main findings hold and remain unsensitive to the classification of one specific industry as tradable or non-tradable. In this regard, the specific treatment of "Hotels and Restaurants", "Real Estate, Renting and Business Services", "Financial Intermediation" or "Community Social and Personal Services" does not drive the results.

O.2 Alternative Measures of the Labor Income Share

When exploring empirically the effects of an exogenous increase in government spending on the labor income share, an issue is the way the share of labor in total income is constructed. Gollin [2002] pointed out that the treatment of self-employment income affects the measurement of the LIS. In particular, it is unclear how the income of proprietors (self-employed) should be allocated to labor income or to capital revenue. In the main text, our preferred measure (called benchmark *bench* hereafter) is to treat all the income of self-employed as labor income. Although this choice overstates the measure of the LIS, it has the virtue of being simple and transparent. Moreover data involved in the construction of this calculation of the LIS are comparable across industries and directly available for all countries of our sample. Specifically, the LIS in sector $j = H, N$ is constructed as follows:

$$s_L^{j,bench} = \frac{W^j_{empl} L^j_{empl} + Inc^j_{self}}{P^j Y^j}, \tag{142}$$

where $W^j_{empl} L^j_{empl}$ is the labor compensation of employees, Inc_{self} is total income of self-employed and $P^j Y^j$ is the valued added at current prices in sector j. Note that labor compensation of employees includes total labor costs: wages, salaries and all other costs of employing labor which are born by the employer whilst Inc_{self} comprises both labor and capital income components, noted $W^j_{self} L^j_{self}$ and $R^j_{self} K^j_{self}$ respectively such that $Inc^j_{self} = W^j_{self} L^j_{self} + R^j_{self} K^j_{self}$.

As a first alternative measure of the LIS, we use only employees compensation as a measure of labor income. This LIS measure, denoted by $s_L^{j,1}$, is constructed as follows:

$$s_L^{j,1} = \frac{W^j_{empl} L^j_{empl}}{P^j Y^j}. \tag{143}$$

The measure described by eq. (143) omits the income of the self-employed, i.e. this income being entirely counted as capital income.

As a second alternative measure, we split self-employed income into capital and labor income based on the assumption that the labor income of the self-employed has the same mix of labor and capital income as the rest of the economy. In other words, total labor compensation comprises labor compensation of employees, $W^j_{empl}L^j_{empl}$, and the self-employed income scaled by the LIS of employees only, i.e. $Inc^j_{self} \times s^{j,1}_L$. With this adjustment, the LIS, denoted by $s^{j,2}_L$, is constructed as follows:

$$s^{j,2}_L = \frac{W^j_{empl}L^j_{empl} + Inc^j_{self} \times s^{j,1}_L}{P^jY^j}. \tag{144}$$

The third alternative to compute the LIS relies upon the assumption that self-employed earn the same hourly compensation as employees. Thus, we use the hourly wage earned by employees W^j_{empl} as a shadow price of labor of self-employed workers. The LIS, denoted by $s^{j,3}_L$, is constructed as follows:

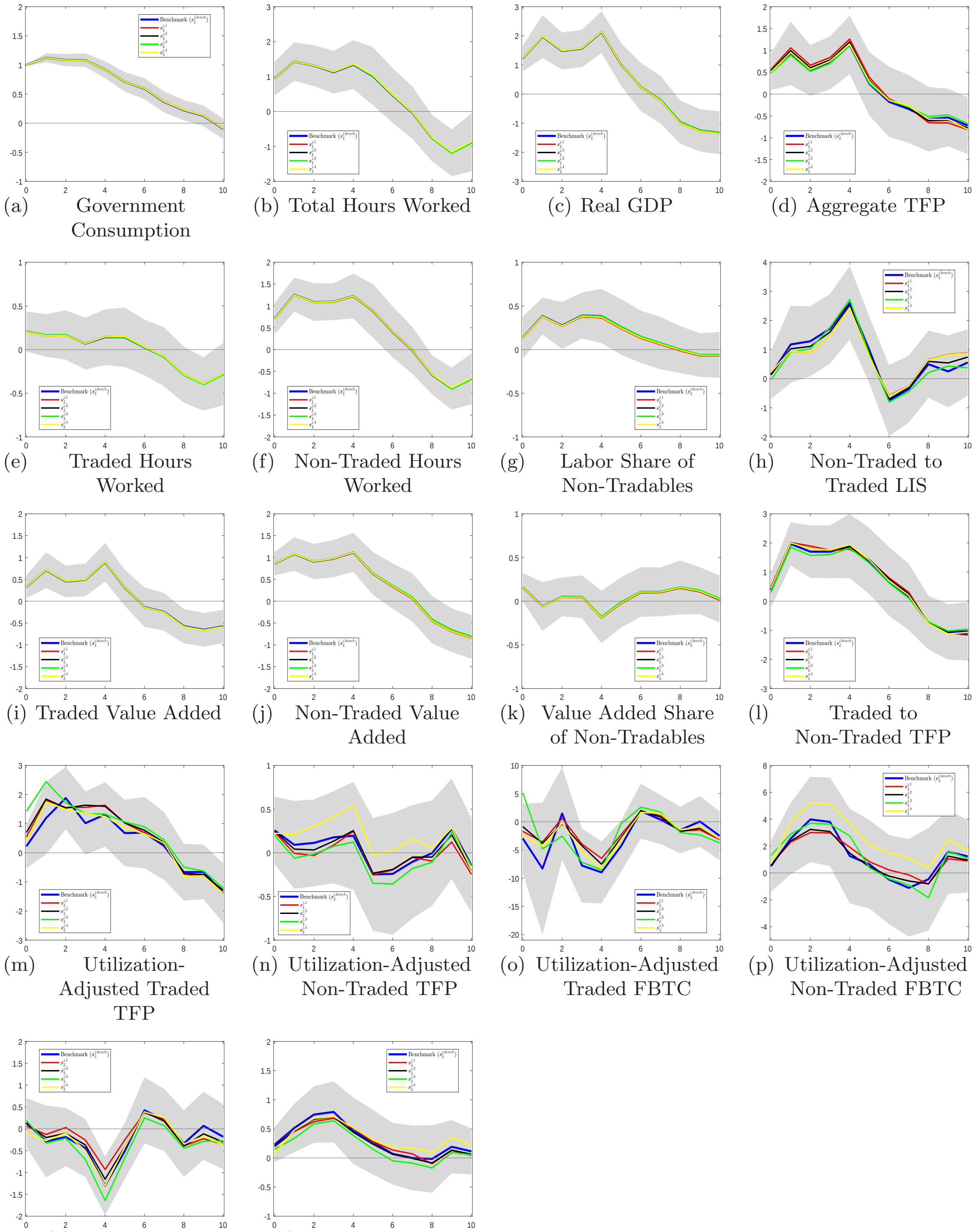
$$s^{j,3}_L = \frac{W^j_{empl} \times (L^j_{empl} + L^j_{self})}{P^jY^j}. \tag{145}$$

Finally, and following Bridgman [2018], the labor income share is adjusted for capital depreciation. In that case, the LIS, denoted by $s^{j,4}_L$, is constructed as follows:

$$s^{j,4}_L = \frac{W^j_{empl}L^j_{empl}}{P^jY^j - CFC^j} \tag{146}$$

where CFC^j is the Consumption of Fixed Capital (current prices) in sector j. In the benchmark and the first three alternatives, the "gross" labor income share treats depreciation as a return to capital. By contrast, the construction $s^{j,4}_L$ is a "net" income distribution indicator as it adjusts value added for depreciation.

In Fig. 13 we display the results of this sensitivity analysis with respect to the construction of the labor income share. We measure the effects to an exogenous increase in government spending by one percent of GDP on aggregate and sectoral variables of interest by contrasting the impulse response functions of the variables when the LIS is measured as either $s^{j,bench}_L$ (blue line), or $s^{j,1}_L$ (red line), or $s^{j,2}_L$ (black line), or $s^{j,3}_L$ (green line), or $s^{j,4}_L$ (yellow line). In addition, the last row of the figure presents the IRFs of the labor income share $j = H, N$. Fig. 13 demonstrates that all the measures of s^j_L being compared imply essentially identical IRFs to an increase in government spending. For a large set of variables (G, L, TFP, Y_R, L^H, L^N, Y^H, Y^N, L^N/L, Y^N/Y, s^N_L/s^H_L and TFP^H/TFP^N) the IRFs for the five specifications are qualitatively and quantitatively similar, if not identical. With regard to the responses of utilization-adjusted TFP and FBTC, the IRFs obtained from the use of four alternative measures of s^j_L display the same dynamic adjustment and are well within the confidence interval (for the benchmark specification $s^{j,bench}_L$) for all horizons. Finally, the responses of the labor income share s^j_L in both sectors for the four specifications are also very close to the IRF obtained in the benchmark. Overall, our main findings are robust and unsensitive to the method adopted to construct the labor income share.



(a) Government Consumption
(b) Total Hours Worked
(c) Real GDP
(d) Aggregate TFP
(e) Traded Hours Worked
(f) Non-Traded Hours Worked
(g) Labor Share of Non-Tradables
(h) Non-Traded to Traded LIS
(i) Traded Value Added
(j) Non-Traded Value Added
(k) Value Added Share of Non-Tradables
(l) Traded to Non-Traded TFP
(m) Utilization-Adjusted Traded TFP
(n) Utilization-Adjusted Non-Traded TFP
(o) Utilization-Adjusted Traded FBTC
(p) Utilization-Adjusted Non-Traded FBTC
(q) Traded LIS
(r) Non-Traded LIS

Figure 13: Sensitivity of the Effects of an Unanticipated Government Spending Shock to the Construction of the LIS. Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification ($s_L^j = s_L^{j,bench}$) are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red (black, green and yellow resp.) line reports results when $s_L^j = s_L^{j,1}$ ($s_L^j = s_L^{j,2}$, $s_L^j = s_L^{j,3}$ and $s_L^j = s_L^{j,4}$ resp.). Sample: 18 OECD countries, 1970-2015, annual data.

O.3 Effects of the Government Spending Shock at Industry Level

Empirical analysis at a disaggregate sectoral level. Our dataset covers eleven industries which are classified as tradables or non-tradables. The traded sector is made up of five industries and the non-traded sector of six industries. To conduct a decomposition of the sectoral effects at a sub-sector level, we estimate the responses of sub-sectors to the same identified government spending shock by adopting the two-step approach detailed in the main text. More specifically, indexing countries with i, time with t, sectors with j, and sub-sectors with k, we first identify the shock to government consumption by estimating the VAR model which includes aggregate variables: $[g_{i,t}, y^{k,j}_{i,t}, l^j_{i,t}, w^j_{C,i,t}, \text{tfp}^A_{i,t}]$ where low-case letters indicate that the variable is logged (all quantities are divided by the working age population) and next we estimate the dynamic effects by using the Jordà's [2005] single-equation method. The local projection method amounts to running a series of regression of each variable of interest on a structural identified shock for each horizon $h = 0, 1, 2, ...$:

$$x^{k,j}_{i,t+h} = \alpha^{k,j}_{i,h} + \alpha^{k,j}_{t,h} + \beta^{k,j}_{i,h}.t + \psi^{k,j}_h(L)\, z_{i,t-1} + \gamma^{k,j}_h .\epsilon^G_{i,t} + \nu^{k,j}_{i,t+h}, \tag{147}$$

where $x^{k,j} = L^{k,j}, L^{k,j}/L, Y^{k,j}, Y^{k,j}/Y_R, \text{TFP}^{k,j}$ (variables are logged). To express the results in meaningful units, i.e., in GDP units or total hours worked units, we multiply the responses of value added at constant prices and value added share at constant prices of sub-sector k by its share in GDP (at current prices), i.e., $\omega^{Y,k} = \frac{P^{k,j}Y^{k,j}}{PY_R}$ where Y_R is real GDP, and we multiply the responses of hours worked and labor share of sub-sector k by its labor compensation share, i.e., $\alpha^{L,k} = \frac{W^{k,j}L^{k,j}}{WL}$. We detail below the mapping between the responses of broad sector's variables and responses of variables in sub-sector k of one broad sector j.

The response of $\ln L^{k,j}$ to a shock to government consumption is the percentage deviation of hours worked in sub-sector $k \in j$ relative to initial steady-state: $\ln L^{k,j}_t - \ln L^{k,j} \simeq \frac{dL^{k,j}_t}{L^{k,j}} = \hat{L}^{k,H}_t$ where $L^{k,j}$ is the initial steady-state. We assume that hours worked of the broad sector is an aggregate of sub-sector hours worked which are imperfect substitutes. Therefore, the response of hours worked in the broad sector $\hat{L}^j_t$ is a weighted average of the responses of hours worked $\frac{W^{k,j}L^{k,j}}{W^jL^j}\hat{L}^{k,j}_t$ where $\frac{W^{k,j}L^{k,j}}{W^jL^j}$ is the share of labor compensation of sub-sector k in labor compensation of the broad sector j:

$$\begin{aligned}
\hat{L}^j_t &= \sum_{k\in j} \frac{W^{k,j}L^{k,j}}{W^jL^j}\hat{L}^{k,j}_t, \\
\frac{W^jL^j}{WL}\hat{L}^j_t &= \sum_{k\in j} \frac{W^{k,j}L^{k,j}}{WL}\hat{L}^j_t, \\
\alpha^{L,j}\hat{L}^j_t &= \sum_{k\in j} \alpha^{L,k}\hat{L}^{k,j}_t,
\end{aligned} \tag{148}$$

where $\sum_j \sum_k \alpha^{L,k} = 1$. Above equation breaks down the response of hours worked in broad sector j into the responses of hours worked in sub-sectors $k \in j$ weighted by their labor compensation share $\alpha^{L,k} = \frac{W^{k,j}L^{k,j}}{WL}$ averaged over 1970-2015. In multiplying $\hat{L}^{k,j}_t$ by $\alpha^{L,k}$, we express the response of hours worked in sub-sector $k \in j$ in percentage point of total hours worked.

We turn to the normalization of the response of value added at constant prices in sub-sector k. The value added at constant prices of sector j is a weighted average of value added of sub-sector $k \in j$, i.e., $P^jY^j_t = \sum_{k\in j} P^{k,j}Y^{k,j}_t$ where prices are those at the base year. Log-linearizing $P^jY^j_t = \sum_{k\in j} P^{k,j}Y^{k,j}_t$ in the neighborhood of the steady-state leads to:

$$
\begin{aligned}
P^j Y^j \frac{dY_t^j}{Y^j} &= \sum_{k \in j} P^{k,j} Y^{k,j} \frac{dY_t^{k,j}}{Y^{k,j}}, \\
\frac{P^j Y^j}{PY_R} \hat{Y}_t^j &= \sum_{k \in j} \frac{P^{k,j} Y^{k,j}}{PY_R} \hat{Y}_t^{k,j}, \\
\omega^{Y,j} \hat{Y}_t^j &= \sum_{k \in j} \omega^{Y,k} \hat{Y}_t^{k,j},
\end{aligned}
\tag{149}
$$

where $\omega^{Y,k} = \frac{P^{k,j}Y^{k,j}}{PY_R}$ averaged over 1970-2015 is the value added share at current prices of sub-sector $k \in j$ which collapses (at the initial steady-state) to the value added share at constant prices as prices at the base year are prices at the initial steady-state. Note that $\sum_j \sum_{k \in j} \omega^{Y,k} = 1$. In multiplying the response of value added at constant prices in sub-sector $k \in j$ by its value added share $\omega^{Y,k,j}$, we express the response of value added at constant prices in sub-sector $k \in j$ in percentage point of GDP.

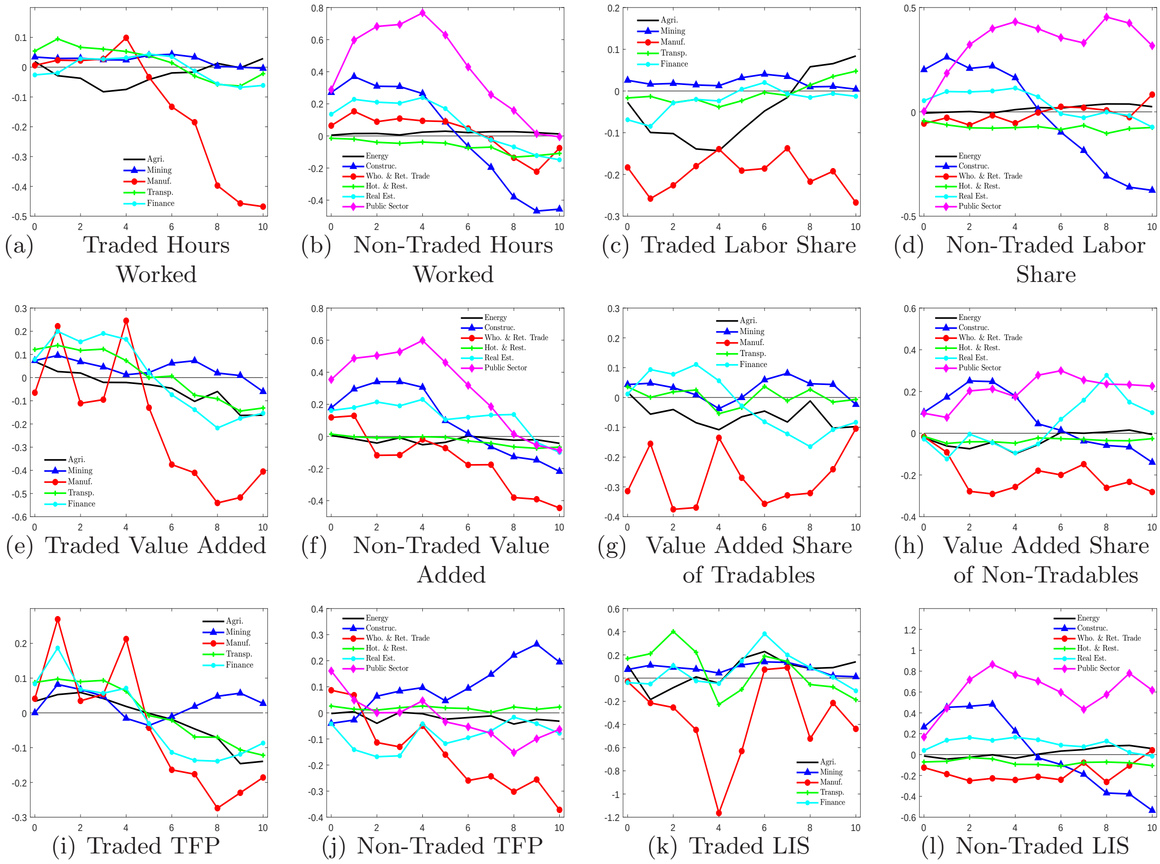
The response of TFP in the broad sector j is a weighted average of responses $\text{TFP}_t^{k,j}$ of TFP in sub-sector $k \in j$ where the weight collapses to the value added share of sub-sector k:

$$
\begin{aligned}
\text{TFP}_t^j &= \sum_{k \in j} \frac{P^{k,j} Y^{k,j}}{P^j Y^j} \hat{\text{TFP}}_t^{k,j}, \\
\frac{P^j Y^j}{PY_R} \text{TFP}_t^j &= \sum_{k \in j} \frac{P^{k,j} Y^{k,j}}{PY_R} \hat{\text{TFP}}_t^{k,j}, \\
\omega^{Y,j} \text{TFP}_t^j &= \sum_{k \in j} \omega^{Y,k} \hat{\text{TFP}}_t^{k,j},
\end{aligned}
\tag{150}
$$

where $\omega^{Y,k} = \frac{P^{k,j}Y^{k,j}}{PY_R}$ averaged over 1970-2015.

Empirical results. The first and third columns of Fig. 14 show results for sub-sectors classified in the traded sector. Overall, all traded industries behave as the broad traded sector. More specifically, as shown in Fig. 14(a), hours worked increase slightly in the short-run in traded sub-sectors. As can be seen in Fig. 14(c), the labor share falls in all traded industries, except in Mining, while Manufacturing contributes the most to the decline in the traded goods-sector share of total hours worked. As shown in Fig. 14(e), value added at constant prices increase in all traded industries in the short-run. Fig. 14(g) shows that Manufacturing experiences the greatest decline in its value added share which somewhat balance out with the increase experienced by industries such as Finance, Mining, Transport and Communication. Importantly, Fig. 14(i) shows that all traded industries experience an increase in their TFP. This result lends some credence to our our classification of traded industries and also reveals that the rise in TFP in the traded sector is driven by a rise in TFP within each sub-sector.

The second and fourth columns show results for sub-sectors classified in the non-traded sector. As shown in Fig. 14(b), except for 'Hotels and Restaurants', 'Electricity, Gas, Water Supply', all non-traded sub-sectors experience a significant rise in hours worked. As can be seen in Fig. 14(d), the significant rise in the labor share of non-tradables is driven by the 'Community Social and Personal Services' (i.e., the public sector which also includes health and education services) and next by 'Construction' together with 'Real Estate, Renting and Business Services'. Fig. 14(f) reveals that value added at constant prices increases in most of non-traded industries although the responses are somewhat muted in 'Hotels and Restaurants' and 'Electricity, Gas, Water Supply'. Fig. 14(h) shows that the value added share of non-tradables increases in the public sector and Construction while it declines in all remaining non-traded industries, especially in 'Wholesale and Retail Trade'. Fig. 14(j) reveals that the responses of non-traded TFP are muted across all non-traded industries, in accordance with its response at the broad sector level. Finally, Fig. 14(k) shows that the responses of the LIS in traded sub-sectors are fairly muted except for the LIS of 'Manufacturing' which declines significantly. When we turn to the non-traded



(a) Traded Hours Worked (b) Non-Traded Hours Worked (c) Traded Labor Share (d) Non-Traded Labor Share

(e) Traded Value Added (f) Non-Traded Value Added (g) Value Added Share of Tradables (h) Value Added Share of Non-Tradables

(i) Traded TFP (j) Non-Traded TFP (k) Traded LIS (l) Non-Traded LIS

Figure 14: Effects of Exogenous Government Spending Shock on Sub-Sectors Notes: Because the traded and non-traded sector are made up of industries, we conduct a decomposition of the sectoral effects at a sub-sector level following a an exogenous increase in government consumption final expenditure by 1% of GDP. To quantify the contribution of each industry to the change in the sectoral variable of the corresponding broad sector, we estimate the responses of each sub-sector variable to the identified government spending shock by using the Jordà's [2005] single-equation method. To express the results in meaningful units, i.e., in GDP units or total hours worked units, we multiply the responses of value added at constant prices and value added share at constant prices of sub-sector k by its share in GDP (at current prices), and we multiply the responses of hours worked and labor share of sub-sector k by its labor compensation share. The first/third columns show results for sub-sectors classified in the traded sector. The black line shows results for 'Agriculture', the blue line with triangles for 'Mining and Quarrying', the red line with circles for 'Manufacturing', the green line with a plus for 'Transport and Communication', and the cyan line with a circle for 'Financial Intermediation'. The second/fourth columns show results for sub-sectors classified in the non-traded sector. The black line shows results for 'Electricity, Gas and Water Supply', the blue line with triangles for 'Construction', the red line with circles for 'Wholesale and Retail Trade', the green line with a plus for 'Hotels and Restaurants', the cyan line with a circle for 'Real Estate, Renting, and Business Services', and the line in magenta with diamond for 'Community Social and Personal Services'. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs). Sample: 18 OECD countries, 1970-2015, annual data.

sector, Fig. 14(l) shows that the rise in the non-traded LIS is driven by increases of the LIS in 'Community Social and Personal Services' (i.e., the public sector which also includes health and education services), 'Construction' together with 'Real Estate, Renting and Business Services'.

O.4 Comparison with CRC's [2020] Empirical Findings: Sample and Empirical Strategy

In this subsection, we run a robustness check with regard to the sample/period and the empirical strategy. A close empirical analysis to ours is that performed by Cardi, Restout and Claeys (henceforth CRC) [2020]. Like CRC [2020], we estimate a panel VAR on annual data and investigate the effects of a government spending shock (identified by adopting Blanchard and Perotti's [2002] method) on both traded and non-traded value added and hours worked. Yet, our empirical analysis differs in three major respects:

- **Sample**. First, regarding the sample, we use a panel of 18 OECD economies over 1970-2015 while CRC [2020] consider a sample of 16 OECD economies over 1970-2007. Note that we consider the sixteen OECD countries included in the CRC's

[2020] sample plus Korea and Portugal.

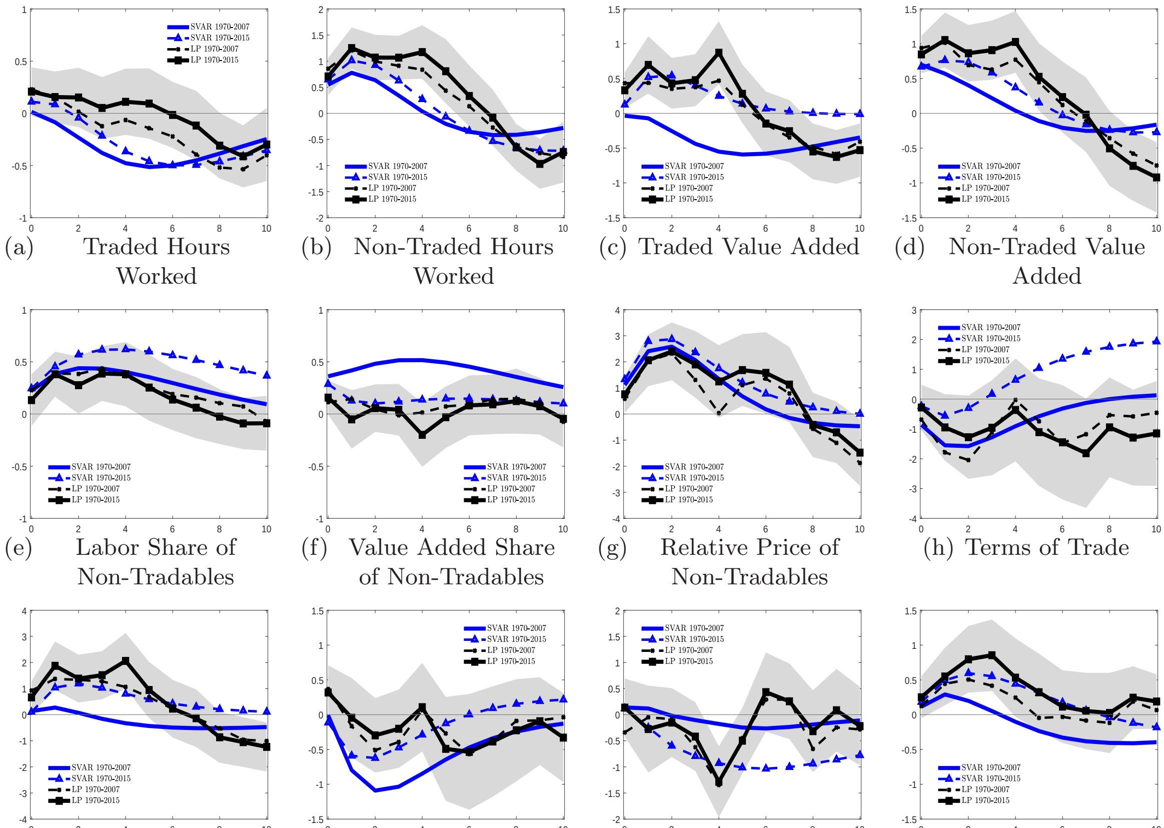
- **Empirical strategy**. Second, our empirical strategy differs along one major dimension. CRC [2020] estimate the effects of a shock to government consumption on sectoral variables by estimating a VAR model including government consumption (ordered first), sectoral value added at constant prices, sectoral hours worked, sectoral wage rate or a VAR model where sectoral variables are divided by their aggregate counterpart. In contrast to CRC [2020], we adopt a two-step approach where we identify the structural shock by using a Cholesky decomposition in which government spending is ordered before the other variables and trace out the dynamic effects by using Jordà's [2005] projection method. The advantage of our two-step approach is twofold. First, we estimate one unique VAR model and thus identify one unique structural shock. Second, the local project method has the advantage that it does not impose the dynamic restrictions implicitly embedded in VARs and can accommodate non-linearities in the response function.

- **Variables**. Third, CRC [2020] restrict their attention to the effects of a government spending shock on hours worked and value added while in the present paper, we explore empirically the impact of a rise in government consumption on both labor income shares and technological change as measured by TFP, capital-utilization adjusted TFP, capital-utilization adjusted FBTC. Importantly, we show that the endogenous response of technological change to a shock to government consumption can rationalize the differences in sectoral fiscal multipliers.

CRC [2020] estimate different VAR models while we estimate one unique VAR model which includes government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP where all quantities are divided by the working age population and all variables are logged. CRC [2020] explore the size of sectoral fiscal multipliers empirically by estimating a VAR model, $\left[g_{it}, y_{it}^{j}, l_{it}^{j}, w_{C,it}^{j}\right]$, which includes government consumption, value added at constant prices in sector j, hours worked in sector j, the real consumption wage in sector j, where all sectoral quantities are divided by population and all variables are logged. To estimate the change in the value added share of sector j and the response of the labor share in sector j, CRC [2020] estimate a VAR model where sectoral quantities are divided by their aggregate counterpart. While CRC [2020] do not estimate the effects of a rise in government spending on sectoral TFP, we also run a VAR model which includes government consumption (ordered first), traded TFP, non-traded TFP to investigate the extent to which the sample and the method matter in determining the response of technology. As mentioned above, our two-step method has the advantage to estimate the dynamic adjustment of sectoral variables to one unique government spending shock by adopting the local projection method which imposes fewer dynamic restrictions that those implicitly embedded in VARs.

In Fig. 15, we compare our empirical findings with those obtained when using the same sample (i.e., 16 OECD countries over 1970-2007) and/or adopting the same methodology as CRC [2020]. In the black line with squares, we report the results obtained in the main text; more specifically, we adopt the two-step approach detailed in length in the main text (i.e., we identify one unique government spending shock and trace out the dynamic responses of variables by using the local projection method) and consider a sample of 18 OECD countries over 1970-2015. The dashed black line with stars shows results when we re-estimate the effects of a shock to government consumption by considering 16 OECD countries over the period 1970-2007, as considered by CRC [2020], and still adopt the two-step approach. Our conclusions remain unchanged. The most notable quantitative difference is the response of the non-traded LIS shown in Fig. 15(l) which reveals that the increase in s_L^N is twice as less as in the baseline scenario.

The solid blue line shows results of [2020] when we consider a sample of 16 OECD countries over the period 1970-2007 and estimate the government spending shock and the response of sectoral variables by considering different VAR models and thus potentially identifying different structural shocks, i.e., some VAR models could potentially identify a



(a) Traded Hours Worked (b) Non-Traded Hours Worked (c) Traded Value Added (d) Non-Traded Value Added

(e) Labor Share of Non-Tradables (f) Value Added Share of Non-Tradables (g) Relative Price of Non-Tradables (h) Terms of Trade

(i) Traded TFP (j) Non-Traded TFP (k) Traded LIS (l) Non-Traded LIS

Figure 15: Sectoral Effects of a Shock to Government Consumption: Robustness Check<u>Notes</u>: The solid black line with squares shows the response of sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP. Shaded areas indicate the 90 percent confidence bounds. While in the baseline scenario shown in the black line with squares, the period is running from 1970 to 2015 and the sample includes 18 OECD countries, the dashed black line shows the effects before the Great Recession, i.e., over the period 1970-2007 and for 16 OECD countries. In both cases, we estimate the dynamic responses to a shock to government consumption by adopting a two-step method. In the first step, the government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, real GDP, total hours worked, the real consumption wage, and aggregate TFP. In the second step, we estimate the effects by using Jordà's [2005] single-equation method. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, relative prices). The blue lines show the effects of shock to government consumption when we identify and generate empirical IRFs by running a VAR model which includes sectoral variables. The solid blue line shows the sectoral effects when estimating the VAR model over 1970-2007 for 16 OECD countries while the dashed blue line with triangles shows estimates when we estimate the same VAR model over 1970-2015 for 18 OECD countries. The solid blue line shows results obtained by Cardi, Restout and Claeys [2020]. Sample: 18 OECD countries, 1970-2015, annual data.

shock to government spending which is more biased toward non-tradables. To investigate the extent of the role of the sample in driving the empirical findings, we show results in the dashed blue line with triangles when we consider a sample of 18 OECD countries over 1970-2015. As shown in the first row of Fig. 15, adopting a VAR methodology instead of local projection method tends to mitigate the rise in hours worked and in value added at constant prices. As shown in the solid blue line in Fig. 15(f), a shock to government consumption has a strong and significant impact on the value added share which suggests that the shock is strongly biased toward non-tradables. We can notice that traded TFP remains unchanged while non-traded TFP declines as can be seen in Fig. 15(i) and Fig. 15(j), respectively, when considering 16 OECD countries over 1970-2007 (see the solid blue line). The fall in non-traded TFP is not large enough to overturn the positive impact on $\nu^{Y,N}(t)$ driven by the biasedness of the government spending shock toward non-tradables.

In contrast, when we consider the sample of 18 OECD countries over 1970-2015, the rise in the value added share of non-tradables vanishes, as shown in the dashed blue line. The reason is that as shown in Fig. 15(i), traded TFP increases which neutralizes the impact of the biasedness of the spending shock toward non-tradables. Because traded TFP increases less when adopting the VAR methodology (see both the solid and dashed blue lines in Fig. 15(i)), the terms of trade depreciate less in the medium-run as can be seen in Fig. 15(h). As displayed by the blue lines in Fig. 15(g) the relative price of non-tradables appreciates more when we estimate the dynamic effects by using a VAR instead of local projection which suggests that the VAR methodology picks up government spending shocks which are more strongly biased toward non-tradables.

The labor share of non-tradables increases significantly in the VAR approach, regardless of the sample, as can be seen in the blue lines in Fig. 15(e). We can notice that the rise in $\nu^{L,N}(t)$ is more pronounced when we consider the sample of 18 OECD countries over 1970-2015, as can be seen in the dashed blue line. The reason is that the responses of LISs in the traded and non-traded sector shown in Fig. 15(k) and Fig. 15(l) are muted at any horizon when considering 16 OECD countries over 1970-2007 (see the slid blue line) while considering a sample of 18 OECD countries over 1970-2015 leads s_L^H to decline and s_L^N to increase significantly, thus amplifying the rise in the demand for labor in the non-traded relative to the traded sector.

In conclusion, we can notice some important differences with the empirical findings reported by CRC [2020]. First, we find that the empirical strategy adopted by CRC [2020] tends to lead the authors to pick up a shock to government consumption which is more strongly biased toward non-tradables, thus leading the value added share of non-tradables to increase more. Second, the sample also plays a role. It appears that both traded TFP and the non-traded labor income share increase less when we consider a sample of 16 OECD countries over period 1970-2007 than in a sample of 18 OECD countries over 1970-2015. Third, both the VAR method and the sample tend to generate a fall in non-traded TFP and understate the rise in traded TFP which can rationalize the smaller increase in both traded and non-traded value added. Fourth, it stands out that the non-traded LIS increases much less when we consider a sample of 16 OECD countries over 1970-2007 instead of a sample of 18 OECD countries over 1970-2015. Because non-traded firms do not bias technological change toward labor, non-traded hours worked increase much less in CRC [2020].

O.5 Robustness Check to the Construction of Sectoral Physical Capital Time Series

In the main text, due to data availability, we construct time series for sectoral capital by computing the overall capital stock by adopting the perpetual inventory approach and then by splitting the gross capital stock into traded and non-traded industries by using sectoral valued added shares. In this Appendix, we investigate whether the effects on TFP and FBTC we estimate empirically are not driven by our assumption about the construction of time series for sectoral capital stock. To conduct this robustness check, we take time series for sectoral capital stock from EU KLEMS [2011], [2017] databases and contrast below empirical responses when sectoral capital stocks are measured by adopting the Garofalo and Yamarik's [2002] methodology (our benchmark) with those obtained by using sectoral data on K^j provided by EU KLEMS [2011], [2017] databases. Due to data availability, our results in the latter case include a sample of thirteen OECD countries which provide time series on sectoral capital of reasonable length. To be consistent, our benchmark also includes these thirteen countries only.

The methodology by Garofalo and Yamarik's [2002] is based on the assumption of perfect mobility of capital across sectors and a small discrepancy in the LIS across sectors, i.e., $s_L^H \simeq s_L^N$. The assumption of perfect capital mobility implies that the marginal revenue product of capital must equalize across sectors:

$$P_t^H \left(1 - s_L^H\right) \frac{Y_t^H}{K_t^H} = P_t^N \left(1 - s_L^N\right) \frac{Y_t^N}{K_t^N}. \tag{151}$$

Using the resource constraint for capital, $K = K^H + K^N$, dividing the numerator and the denominator in the LHS of (151) by GDP, Y, and denoting by $\omega_t^{Y,j} = \frac{P_t^j Y_t^j}{Y_t}$ the share of value added of sector j in GDP at current prices (at time t), eq. (151) can be solved for the K^H/K:

$$\frac{K_t^H}{K_t} = \frac{\omega_t^{Y,H}\left(1 - s_L^H\right)}{\left(1 - s_L^N\right)\left(1 - \omega_t^{Y,H}\right) + \left(1 - s_L^H\right)\omega_t^{Y,H}}. \tag{152}$$

Assuming that $s_L^H \simeq s_L^N$ leads to the rule we apply to split the aggregate stock of capital into tradables and non tradables:

$$\frac{K_t^H}{K_t} = \omega_t^{Y,H}. \tag{153}$$

In the baseline, we adopt the methodology of Garofalo and Yamarik [2002] to split the national gross capital stock into traded and non-traded industries by using sectoral value added shares at current prices. Let $\omega^{Y,j}$ be the share of sector j's value added (at current prices) $P^j Y^j$ for $j = H, N$ in overall output (at current prices) $Y \equiv P^H Y^H + P^N Y^N$, the allocation of the national capital stock to sector j is given by the rule:

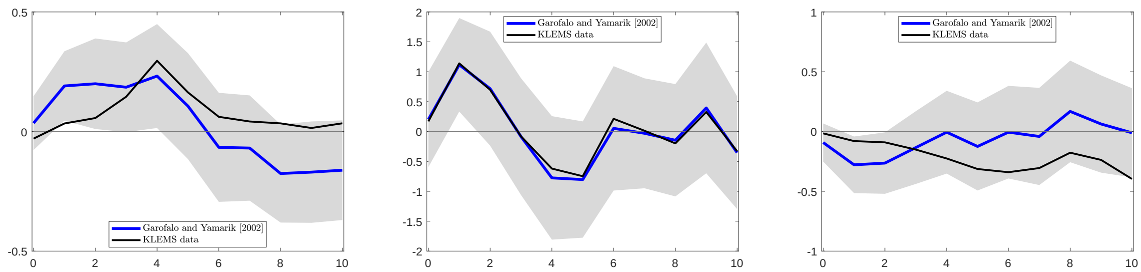
$$K_{GY}^j = \omega^{Y,j} K = \frac{P^j Y^j}{Y} K, \tag{154}$$

where we denote the sectoral stock of capital obtained with the decomposition by Garofalo and Yamarik [2002] by K_{GY}^j. National capital stocks are estimated from the perpetual inventory approach. Following Garofalo and Yamarik [2002], the gross capital stock is then allocated to traded and non-traded industries by using sectoral value added shares according to eq. (154). Once the series for K_{GY}^j are obtained, we can construct the sectoral capital-labor ratios, $k_{GY}^j = K_{GY}^j / L^j$, sectoral capital utilization rates, $u_{GY}^{K,j}$, sectoral utilization-adjusted-TFPs, Z_{GY}^j, and sectoral utilization-adjusted-FBTC (see eq. (5) in the main text).

As a robustness check, we alternatively take capital stock series from the EU KLEMS [2011] and [2017] databases which provide disaggregated capital stock data (at constant prices) at the 1-digit ISIC-rev.3 level for up to 11 industries, but only for thirteen countries of our sample which include Australia, Canada, Denmark, Finland, Italy, Spain, the United Kingdom, the Netherlands over the entire period 1970-2015, plus Austria (1976-2015), France (1978-2015), Japan (1973-2006), Korea (1970-2014). In efforts to have time series of a reasonable length, we exclude Belgium (1995-2015) and Sweden (1993-2015) because the period is too short while Ireland, Norway, and Portugal do not provide disaggregated capital stock series.

In Fig. 16, we compare the responses of technlogical change and capital-labor ratios when we split the national gross capital stock into tradables and non-tradables by using the sectoral value added share with those obtained from the alternative approach where we take data on sectoral capital from KLEMS [2011], [2017] databases. We estimate the effects of a 1% increase in government consumption on variables that might be affected by our assumption and contrast the IRFs when the sectoral capital stock is measured by adopting the methodology by Garofalo and Yamarik [2002] (blue line) with those when the sectoral capital stock is obtained directly from KLEMS (black line). For comparison purposes and

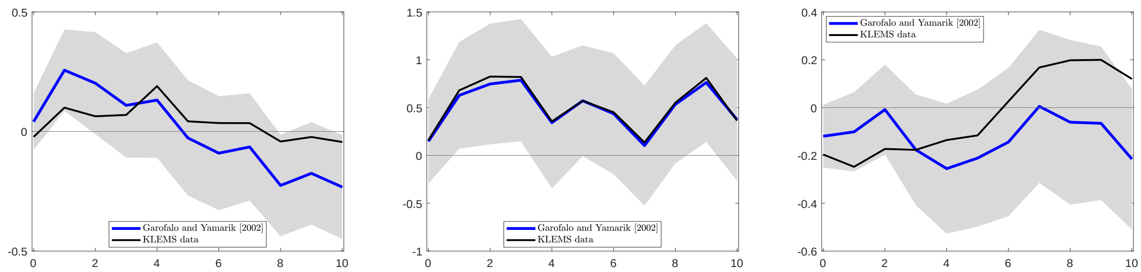
$\text{TFP}^H/\text{TFP}^N$ $\qquad$ FBTC^j_{adjK} $\qquad$ $k^j = K^j/L^j$



(a) Ratio of Traded to Non-Traded TFP

(b) Traded Utilization-Adjusted FBTC

(c) Traded Capital-Labor Ratio, k^H



(d) Ratio of Utilization-Adjusted TFP, $\text{TFP}^H_{adjK}/\text{TFP}^N_{adjK}$

(e) Non-Traded Utilization-Adjusted FBTC

(f) Non-Traded Capital-Labor Ratio, k^N

Figure 16: Effects of a Government Spending Shock on Technology and Capital-Labor Ratios. Notes: Effects of a 1% temporary increase government consumption. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend. Results for baseline specification (i.e., we use the method of Garofalo and Yamarik [2002] to construct the sectoral capital stocks K^H and K^N) are displayed by blue lines with shaded area indicating 90 percent confidence bounds. The black line reports results when we use the EU KLEMS [2011] and [2017] databases to construct sectoral capital stocks series K^j. Sample: 13 OECD countries, 1970-2015, annual data.

to ensure consistency, we compare the results for same sample, i.e. the restricted sample that includes 13 OECD countries over the period 1970-2015.

As shown in the last column of Fig. 16, the responses of capital-labor ratios for the two methods are both qualitatively and quantitatively similar since the solid black line lies within the original confidence bounds of those obtained when K^j is constructed with the use of the methodology of Garofalo and Yamarik [2002]. In particular, one can observe that the discrepancy in the results is small and not statistically significant at conventional level. Importantly, the ratio of traded TFP to non-traded TFP (first column) and utilization-adjusted FBTC (second column) are not affected by our assumption underlying the construction of time series of sectoral capital stock. In conclusion, our main findings are robust and unsensitive to the way the sectoral capital stocks are constructed in the data.

P Effects of Government Spending Shocks: Empirical Strategy and Robustness Checks

The main obstacle in empirical fiscal policy analysis is to identify exogenous and unexpected fiscal events because changes in aggregate output can contemporaneously influence government spending. To extract exogenous variations in government spending unrelated to contemporaneous changes in aggregate output, we assume that government spending does not respond to changes in output. This assumption relies on the idea that policy-makers need time to learn about the economic stance, decide on, approve, and implement changes in fiscal policy. Blanchard and Perotti [2002] and subsequent studies by Auerbach and Gorodnichenko [2012], Ilzetzki, Mendoza, and Végh [2013], Miyamoto, Nguyen and Sergeyev [2018] and others have used this identification assumption. Since there are some delays inherent to the legislative system, this is a natural assumption when using quarterly data. However, this argument may not necessarily be true when using annual data since some adjustment could be possible within the year. We conduct below a number of robustness checks to investigate whether our empirical strategy is subject to endogeneity issues.

Another concern is related to the presence of anticipation effects. As argued by Ramey [2011], Blanchard and Perotti's [2002] approach to identifying government spending shocks in VAR models may lead to incorrect timing of the identified fiscal shocks. If the fiscal shock is anticipated in advance, agents may have modified their decisions before the rise in government spending actually materializes. Consequently, when the fiscal shock is anticipated, and thus VAR approach captures the shocks too late, it misses the initial changes in variables that occur as soon as the news is learned. Subsection P.4 conducts an investigation of the potential presence of anticipation effects by using a dataset constructed by Born, Juessen and Müller [2013] which contains one year-ahead OECD forecasts for government spending.

In Online Appendix P.6, we compare our results by adopting the two-step method with those when we adopt the one-step approach like in Ramey and Zubairy [2018]. In Online Appendix P.7, we compare the dynamic responses when we use local projections with the dynamic effects when IRFs are estimated through VAR estimates.

In running these robustness checks, we investigate whether all of conclusions in the main text hold. Following a government spending shock:

- both real GDP and hours increase significantly
- non-traded labor increases disproportionately as labor shifts away from the traded sector
- real GDP growth is uniformly distributed between the traded and the non-traded sector
- technology improvements are concentrated in traded industries
- technological change is biased toward labor in non-traded industries and biased toward capital in traded industries.

Below, we point out empirical facts which do not hold. When all empirical facts hold, we just indicate that our empirical strategy is robust to the empirical test to avoid a repetition.

P.1 Dealing with Potential Endogeneity Issue: Quarterly vs. Annual Data

In the main text, we identify fiscal shocks by adopting the SVAR methodology pioneered by Blanchard and Perotti [2002]. The identification scheme proposed by Blanchard and Perotti [2002] is based on the assumption that government spending does not respond contemporaneously to current output developments due to delays between current output observation and the implementation of fiscal measures. The advantage of the Blanchard and Perotti's approach over the narrative approach is that it can be implemented for a large set of countries. The potential problem is that Blanchard and Perotti's argument is not necessarily true when using annual data, as some adjustment could be possible within the year. Below we assess the legitimacy of our identifying restriction by estimating our model using quarterly data rather than annual data, we also implicitly control for news effects as agents often react quickly to news about government spending, as pointed out by Ramey and Zubairy [2018].

Method. We use quarterly data and assume that government spending does not respond within the quarter to the other variables included in the VAR model. This assumption is in the spirit of Blanchard and Perotti [2002]. Like in the main text, we adopt a two-step method where we estimate the baseline VAR model which includes aggregate variables to identify the government spending and then we estimate the dynamic effects by means of local projections. Since our objective is to contrast the dynamic effects we obtain when we identify the shock on annual data with the impulse responses when we identify the shock on the basis of quarterly data, we annualize the estimated shock in the latter case. Due to limited data availability, we estimate a baseline VAR model which includes (logged) government consumption, real GDP and total employment, all variables being divided by the working force population. When estimating the VAR model on quarterly data, we set the number of lags to 8.

Data Source. We take quarterly data from OECD Economic Outlook [2017]. All variables are per capita. Data coverage: from 1970q1 to 2015q4 for 17 OECD countries, i.e., the eighteen countries of our sample except for Ireland. We have considered three alternative VAR models over 1970q1-2015q4: a) $[g_{it}, y_{R,it}, \text{Employment}_{it}, w_{L,it} - p_{C,it}]$ where W_L is the wage per person employed (sample: 14 countries, i.e., the baseline sample less (IRL, KOR, NOR and PRT), b) $[g_{it}, y_{R,it}, \text{hours worked}_{it}]$ (sample: 11 countries, i.e., baseline sample less IRL, AUT, CAN, FIN, KOR, NOR and PRT) $[g_{it}, y_{R,it}, \text{hours worked}_{it}, w_{h,it} - p_{C,it}]$ where W_h is the wage per hour worked (sample: 11 countries, i.e., the baseline sample less IRL, AUT, CAN, FIN, KOR, NOR and PRT).

Results. In order to make our baseline results comparable with those when we estimate the VAR model on quarterly data, we have re-estimated our baseline results when we identify the government spending shock by estimating $z_{it} = [g_{it}, y_{it}, l_{it}]$ where l_{it} is employment for 17 OECD countries (instead of eighteen). The solid blue line in Fig. 17 displays the baseline results for a government spending shock identified on annual data, while the solid black line displays the results when the government spending shock is identified on quarterly data. All of our conclusions hold; we can notice that the government spending shock displays more persistence and is somewhat more pronounced when identified on quarterly data.

P.2 Dealing with a Potential Endogeneity Issue: Instrumenting Government Spending by Using Military Expenditure

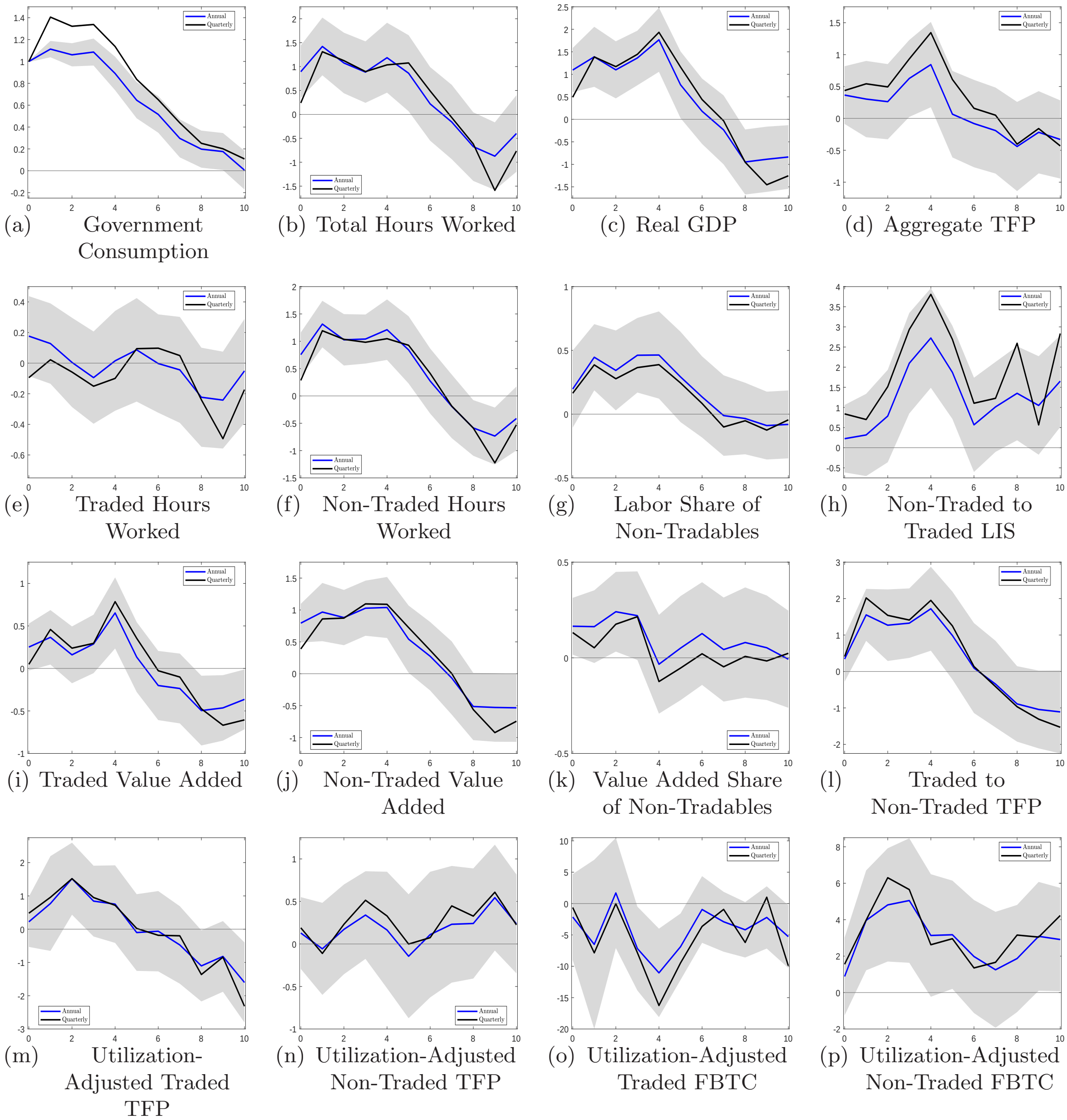
An alternative estimation strategy to Blanchard and Perotti [2002] identification of government spending shocks has been employed by Ramey and Shapiro [1998], Hall [2009], Barro and Redlick [2011], Ramey [2011], Ramey and Zubairy [2018]. Ramey and Shapiro [1998] who consider a narrative approach that amounts to considering major political events which led to large military buildups associated with significant increases in government spending. The advantage of the narrative approach over alternatives is that political events are arguably exogenous (with respect to economic conditions) and thus identified government spending shocks are not subject to the potential endogeneity problem. Because narratively-identified military spending shocks are not available for most of the countries in our sample, we further address the potential issue of endogneity by following Miyamoto et al. [2019] who identify government spending shocks using exogenous variation in military spending.

Data Source. Military expenditure data are taken from the Stockholm International Peace Research Institute (SIPRI). Time series are available at an annual frequency and . Data Coverage: Time series are available for the 18 OECD countries of our sample over 1970-2015. All variables are per capita. The correlation between government consumption and military spending is high at 0.975. While both series are highly correlated, military spending accounts for 2.2% of GDP, the United States having the highest share at 4.8% of GDP.

Method. The identification of government spending shocks comes from the assumption that military spending is exogenous to the state of the economy. The reason is that use changes in military spending are often large and less likely to be driven by countercyclical reasons. In our estimation, government consumption is instrumented by military spending. By adding country fixed effects, time dummies, country-specific linear time trend, we run the regression of (logged) government consumption on current values of variables included in the SVAR of the main text plus past values of (logged) government consumption (we allow for two lags) and the current value of military spending g_{it}^M:

$$g_{it} = \delta_i + \delta_t + \zeta_i t + \sum_{k=1,2} \alpha_k g_{i,t-k} + \text{control}_{it} + \eta_M g_{it}^M + \epsilon_{i,t+h}, \tag{155}$$

where control_{it} are two lags on logged on real GDP, logged total hours worked, logged real consumption wage, logged aggregate TFP, and logged military spending. Once we have



(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP

(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS

(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP

(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 17: Addressing Endogeneity Issue: Identification on Quarterly vs. Annual Data. Notes: The solid blue line and solid black line show the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP when we identify the government spending shock by using Blanchard and Perotti [2002] identification method. In the blue line, we identify government spending shocks on annual data and in the solid black line, we identify the government spending shock on quarterly data. Shaded areas indicate the 90 percent confidence bounds. We estimate the dynamic effects by using Jordà's [2005] single-equation method. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Sample for annal data : 17 OECD countries, 1970-2015. Sample for quarterly data : 17 OECD countries, 1970q1-2015q4

estimated (155), we construct predicted values of government consumption, i.e., $\bar{g}_{i,t}$. In the second step, we estimate the dynamic effects by using Jordà's [2005] local projections method where we plug instrumented government consumption and the same controls as in the first step:

$$x_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h}t + \psi_h (L) z_{i,t-1} + \gamma_h \bar{g}_{i,t} + \eta_{i,t+h}, \tag{156}$$

where x is the logarithm of the variable of interest, β_h measures the response of variable x at horizon $h = 0, 1, 2..10$, $\alpha_{i,h}$ are country fixed effects, $\alpha_{t,h}$ are time dummies, and we include country-specific linear time trends; z is a vector of control variables (i.e., past values of government spending and of the variable of interest), $\psi_h (L)$ is a polynomial (of order two) in the lag operator and $\bar{g}_{i,t}$ is instrumented government consumption. Eq. 156 can be estimated separately for each horizon h by OLS.

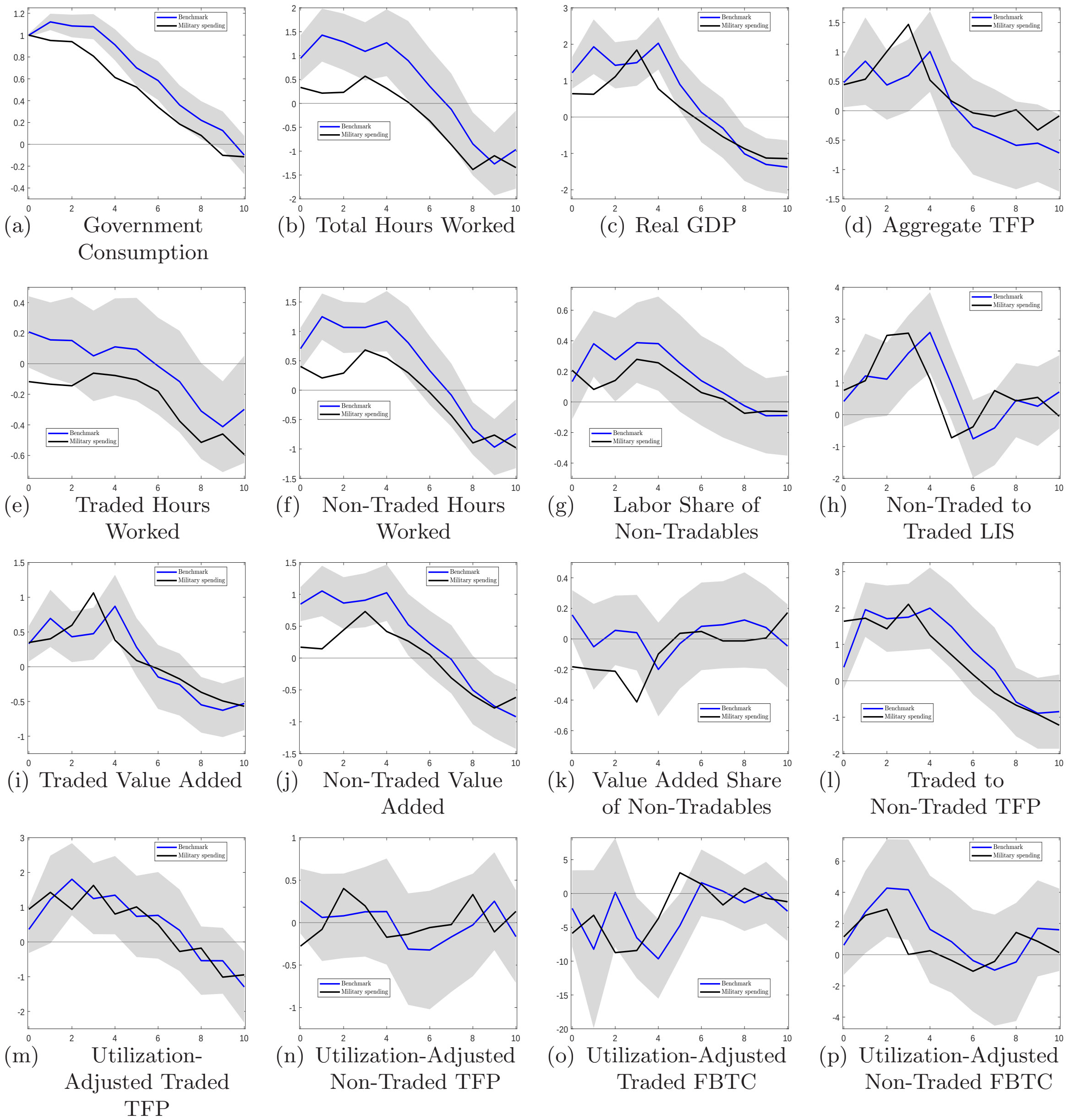
Results. Fig. 18 contrasts the dynamic effects of a shock to government consumption based on Blanchard and Perotti [2002] identification and displayed by the solid blue line with the results shown in the solid black line where we adopt a two-stage least squares (2SLS) method and instrument government consumption by using military spending and estimate dynamic effects by using local projections like Miyamoto et al. [2019]. Almost all of our conclusions hold except that when government spending is instrumented by military spending, the value added share of tradables at constant prices significantly increases.[29] As we pointed in the main text, identifying government spending shocks by using military spending tends to 'select' shocks which are biased toward traded goods, see e.g., Ramey and Shapiro [1998] who show that defense spending increases disproportionately traded labor and output. Because the government spending shock is biased toward traded goods, labor does not significantly shift toward non-traded industries. Aggregate and traded TFP tends to increase more in the short-run. This result is in line with evidence documented by Antolin-Diaz and Surico [2022] who find that military spending has sizable effects on long-run growth as it shifts the composition of public spending towards R&D which both boosts innovation and productivity.

P.3 Why are Government Spending Shocks Identified on Annual Data Exogenous?

The Blanchard-Perotti identification of government spending shocks is based on the assumption that government spending does not react contemporaneously to other variables included in the VAR model. The conduct of fiscal policy suggests that there exists some delays to learn about the shock, to take a decision (delays inherent to the legislative system) and to implement the fiscal package. These lags justify the absence of response of government spending to the current economic developments. So identification requires that public purchases cannot respond to output developments within the same period. Instead, spending is assumed to respond to past growth developments as well as expectations about economic activity formed one period in advance.

The Blanchard-Perotti identification is based on decision and implementation lags and thus is more likely to hold when we use quarterly data. Due to limited availability of sectoral data at a quarterly frequency for most of OECD countries, we use annual data. To identify government spending shocks, we thus assume that government consumption does not react to real GDP and total hours worked within the year. Our identifying assumption could be violated if fiscal policy reacts automatically to aggregate macroeconomic conditions or if fiscal policy contains contains discretionary elements. However, since we consider government spending net of transfers, automatic stabilizers (which operate through taxes and transfers) should not pose a problem. A second potential problem is discretionary fiscal policy action in response to current economic developments. The budget process, i.e., budget formulation and execution, typically follows a strict calendar and if there is discretionary expenditure within the year, it might involve a shift of expenditure across items, i.e., purchases initially targeted toward certain items are shifted toward other items.

[29] We have also computed the confidence bounds for military spending shocks. Results are available from the authors upon request.



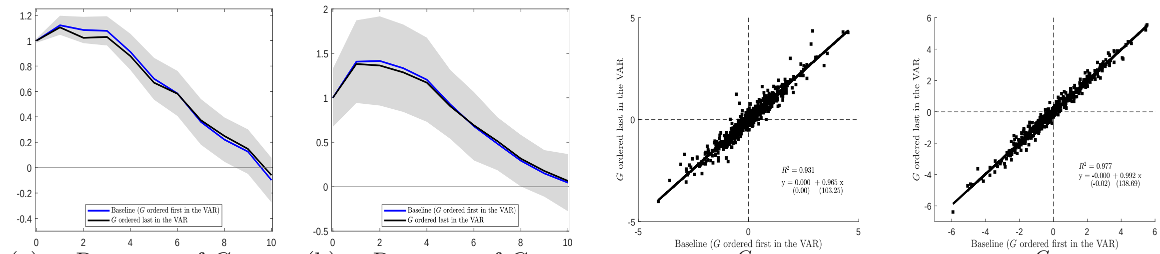
(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP

(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS

(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP

(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 18: Addressing Endogeneity Issue: Identification on Military Spending vs. Government Consumption. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP when we identify the government spending shock by using Blanchard and Perotti [2002] identification method. The solid black lines shows results when we adopt a 2SLS method where we instrument government consumption by using military spending and estimate dynamic effects by using local projections like Miyamoto et al. [2019]. Shaded areas indicate the 90 percent confidence bounds. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Sample: 18 OECD countries, 1970-2015, annual data.



(a) Response of G_{it} when G_{it} is Ordered First (blue line) vs. G_{it} is Ordered Last: Annual Data

(b) Response of G_{it} when G_{it} is Ordered First (blue line) vs. G_{it} is Ordered Last: Quarterly Data

(c) Plot of ϵ_{it}^{G} when G_{it} is Ordered First (blue line) vs. $\epsilon_{it}^{G,\prime}$ when G_{it} is Ordered Last: Annual Data

(d) Plot of ϵ_{it}^{G} when G_{it} is Ordered First (blue line) vs. $\epsilon_{it}^{G,\prime}$ when G_{it} is Ordered Last: Quarterly Data

Figure 19: Impulse Response Functions for Government Spending when G is Ordered First vs. Last in the VAR Model. Notes: The first two columns of Fig. 19 contrast the empirical response of G to an exogenous government spending shock depending on whether G is ordered first or last in the VAR model. The solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds when government consumption is ordered first in the VAR model. The black line reports results for a VAR model in which G is ordered last. The last two columns plot identified structural government spending shocks when government consumption is ordered first against structural government spending shocks when government consumption is ordered last. Sample for annual data: 18 OECD countries, 1970-2015. Sample for quarterly data: 11 OECD countries, 1970q1-2015q4.

To support our identification of government spending shocks based on annual data, we have identified government spending shocks by using quarterly data in section P.1. While sectoral data are not available at a quarterly frequency, our two-step method lends nicely to robustness checks as we identify the government spending shocks by estimating a VAR model in the fist step which involves aggregate variables. When we use quarterly data, we find no significant differences with the effects we estimate on annual data. An alternative identification assumption is based on the use of military expenditure which are supposed to be exogenous to the state of the economy, see Miyamoto et al. [2019]. We do detect significant differences except that shocks to government consumption instrumented by military spending are more biased toward traded goods because as pointed out by Ramey and Shapiro [1998], military expenditure are concentrated toward a few manufacturing industries.

To further deal with the potential endogeneity problem, like Beetsma and Giuliodori [2011] and Brückner and Pappa [2012], we order government purchases last in the VAR model and thus let G respond to all variables included in the VAR model. If the endogenous response of G to an exogenous fiscal shock is similar to that when the VAR is estimated by ordering G first, then we can be confident that the endogeneity problem is mitigated as G is not or at least little responsive to output shocks. The first two columns of Fig. 19 contrast the empirical response of G to an exogenous government spending shock depending on whether G is ordered first (solid blue line) or last in the VAR model (solid black line). The endogenous response of G lies within the confidence bounds of the primary VAR model where G is ordered first whether we use annual or quarterly data. A rationale to this finding lies in the fact that we consider government spending net of transfers (which eliminates the impact of automatic stabilizers) and the official timetable for the budget process mitigates the presence of discretionary fiscal policy within the same period.

The last two columns of Fig. 19 plot identified structural government spending shocks when government consumption is ordered first (horizontal axis) against structural government spending shocks when government consumption is ordered last (vertical axis). Fig. 19(c) contrasts identified government spending shocks on annual data and Fig. 19(d) contrasts identified government spending shocks on quarterly data. While the correlation is higher for quarterly data ($R^2 = 0.977$) than for for annual data ($R^2 = 0.931$), in both cases, we find a high correlation between the two time series.

P.4 Controlling for Potential Anticipation Effects

We now address one potential concern related to the anticipation effects. When the fiscal shock is anticipated, and the VAR approach captures the shocks too late, it misses the initial changes in variables that occur as soon as the news is learned. We conduct below an investigation of the potential presence of anticipation effects, using two alternative measures of forecasts for government spending. We also re-estimate the effects of a government spending shock by controlling for anticipation effects. The first measure is provided by Born, Juessen and Müller [2013] and stems from the OECD forecasts, while the second is taken from a dataset constructed by Fioramanti et al. [2016] where forecasts are performed by the European Commission. We use two alternative datasets as the former contains observations from 1986 to 2014 for all countries, while the latter provides a longer time horizon but for a restricted set of countries.

Drawing on previous studies, we conduct three robustness exercises to explore the potential implications of anticipations effects:

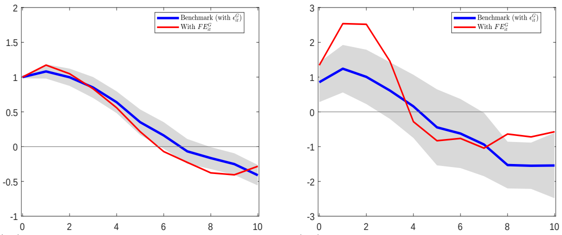
- First, we replace in the local-projection regressions the SVAR identified government spending shock ϵ_{it}^G with the forecast error FE_{it}^G computed as the difference between actual series and forecast series of the government spending growth rate: $FE_{it}^G = \Delta g_{it} - for_{it}^G$, where Δg_{it} is the actual government consumption growth and for_{it}^G is the previous period's forecast. The idea is simply to purge actual government spending growth of what professional forecasters project spending growth to be.

- A second way to deal with the potentially anticipated government spending shocks is to augment the SVAR specification we used to estimate the identified government spending shock ϵ_{it}^G with the forecasts for government spending growth for_{it}^G. This allows us to identify the unanticipated shock to government spending in the presence of fiscal foresight. Drawing on Beetsma and Giuliodori [2011], we estimate two VAR models: 1) we extend our baseline model with government spending growth forecast for_{it}^G we used earlier and the vector of variables in the VAR now reads $[g_{it}, for_{it}^G, y_{R,it}, l_{it}, w_{C,it}, TFP_{it}]$ implying that for_{it}^G is treated as an endogenous variable and 2) we augment the baseline VAR model with for_{it}^G as an exogenous variable, i.e. ϵ_{it}^G is identified in a VARX model in which the vector of endogenous variables is $[g_{it}, y_{R,it}, l_{it}, w_{C,it}, TFP_{it}]$ and for_{it}^G is an exogenous variable. The first approach is attractive because it accounts automatically for any effects that expectations might have on the others aggregate variables and for the determinants of the expectations themselves. However, this method increases the number of estimated parameters within the VAR structure. Given the data limitations on the variable for_{it}^G, we complement the first approach 1) with the second exercise to get more parsimonious VAR models.

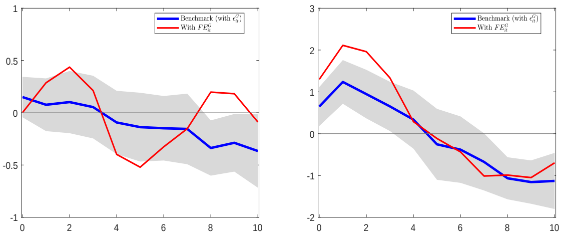
- The third robustness test repeats the previous exercise by considering an alternative measure of forecasts: the forecast for the budget balance-GDP ratio which we denote by for_{it}^{bbr}. The year-ahead forecasts are taken from the Commission's Fall forecasts, see Fioramanti et al. [2016] for details of construction of for_{it}^{bbr}.

In the following, we conduct an investigation of the potential presence of anticipation effects by performing the three robustness exercises mentioned above. To perform the first two robustness tests, we use a dataset constructed by Born, Juessen and Müller [2013] that contains time series for forecasts for government spending growth from the OECD.[30]. The data availability for the variable for_{it}^G is: AUS (1997-2014), BEL (1999-2014), CAN (1986-2014), DNK (1997-2010), ESP (1997-2014), FIN (1997-2014), FRA (1986-2014), GBR (1986-2014), IRL (1997-2014), ITA (1986-2014), JPN (1986-2014), KOR (1998-2014), NLD (1997-2014), NOR (1997-2014), PRT (1997-2014), SWE (1997-2014) and USA (1986-2014). No data are available for Austria. Regarding the general government balance to GDP ratio forecast (one year ahead) used in the third exercise, time series are available for AUT (1995-2014), BEL (1971-2014), DNK (1977-2014), ESP (1987-2014), FIN (1995-2014), FRA (1970-2014), GBR (1974-2014), IRL (1974-2014), ITA (1970-2014), NLD (1970-2014), PRT (1987-2014) and SWE (1995-2014). Note that, the European Commission provide forecasts for the budget balance-GDP ratio only for European countries. Accordingly, non-European

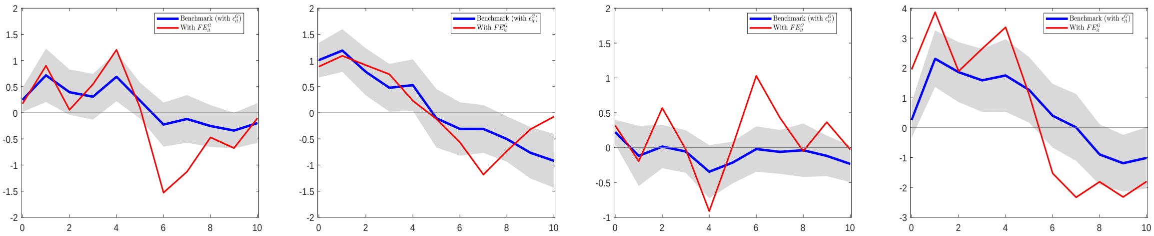
[30] We thank Gernot Müller for providing this dataset to us.



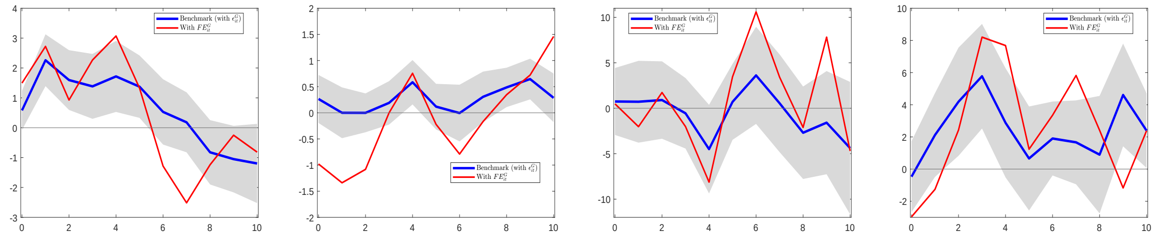
(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP



(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS



(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP



(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 20: Effects of an Unanticipated Government Spending Shock: Controlling for Anticipation Effects with Forecast Errors (FE^G_{it}). Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification (i.e. the government spending shock ϵ^G_{it} is identified in a SVAR model) are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red line shows results when the forecast error FE^G_{it} is used as a measure of the government spending shock in local projections. Sample: 17 OECD countries (AUS, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, NOR, PRT, SWE and USA), 1986-2014, annual data (367 observations).

countries of our panel (AUS, CAN, JPN, KOR, NOR and USA) are not considered in the third robustness test.

Figure 20 shows IRFs when the SVAR identified government spending shock ϵ^G_{it} is replaced by the forecast error FE^G_{it} in the local-projection regressions. The blue line reports the results for the baseline case (i.e. the fiscal shock is ϵ^G_{it}), while the red line displays the results when the fiscal shock considered is FE^G_{it}. In both cases, the local-projection regressions are estimated over the period running from 1986 to 2014 to obtain comparable results. Overall, we find that the response of the vast majority of variables, with the notable exception of the utilization-adjusted TFP in the non-traded sector, is consistent with the baseline. We can notice some quantitative differences however. When using the forecast error FE^G_{it} as a fiscal shock, the rise in both total hours worked and non-traded hours worked is more pronounced. The response of the labor share of non-tradables, $\nu^{L,N}$, positive and statistically significant, also increases more than in the baseline case. Thus the larger increase in non-traded hours worked in driven by the greater rise in total hours worked and also the higher reallocation of labor toward the non-traded sector. Like in the baseline, the value added share of non-tradables, $\nu^{Y,N}$, remains unresponsive to the fiscal shock whether it is measured with ϵ_{it}^{G} or FE_{it}^{G}. One may be concerned with the negative response at impact of the utilization-adjusted TFP in the non-traded sector when we use the forecast error FE_{it}^{G} as a measure of the fiscal shock (-0.98), however this fall is not statistically significant. Finally, we may also note that, for utilization-adjusted TFP in the traded sector and also for traded and non-traded FBTC, the responses lie within the confidence interval for almost all horizons.

As mentioned above, in the second exercise, the forecasts for government spending growth for_{it}^{G} are used directly to control for anticipations effects. For that purpose, the baseline VAR model that allows us to identify structural fiscal shocks is modified by allowing the latter variable to enter the vector of variables as an endogenous variable (the VAR case in the sequel) or as an exogenous variable (the VARX case). Fig. 21 shows results for the baseline (for_{it}^{G} is not considered) together with the VAR and VARX cases to control for potential fiscal foresight. Overall, it turns out that differences are moderate and anticipation effects thus play a limited quantitative role in the dynamic adjustment to a government spending shock. The impulse response functions for the two alternatives are, qualitatively, similar to those under the baseline shown in the blue line. Quantitatively, despite some differences, for almost all variables and all horizons considered, the IRFs are within the confidence interval.

The final exercise we consider amounts to repeating the previous analysis by replacing the forecast variable for_{it}^{G} with the forecast of the budget balance-GDP ratio for_{it}^{bbr}. Fig. 22 plots the estimated impulse response for this robustness test. Overall, the results are qualitatively and quantitatively similar to those obtained in the baseline and thus do not deserve further comments.

P.5 Controlling for Potential Influence of Monetary Policy: Rossi and Zubairy [2011]

The estimation of the VAR model and the identification of exogenous government spending shocks is based in the assumption that structural shocks are orthogonal. Therefore, structural government spending shocks are orthogonal to monetary policy shocks. While fiscal and monetary policy shocks are orthogonal, a shock to government consumption may give rise to a monetary policy response, reflected by a change in the nominal interest rate, and this this monetary policy response could have a feedback effect on aggregate and/or sectoral variables.

Since we construct our VAR model in line with the main features of our model with flexible prices and an exogenous world interest rate, we do not include the interest rate in the VAR model. Several papers such as Ramey's [2011] include the three-month T-bill rate to control for monetary policy, in accordance with the recommendation of Rossi and Zubairy [2011]. The authors explore the following question (among others): How Does the Inclusion of Monetary Policy Affect Our Understanding of U.S. Fiscal Policy? They run a VAR model which includes the government spending and additional controls and they identify government spending shocks by running the VAR model with and without the federal funds rate. They identify the government spending shock via a Cholesky decomposition where government spending is ordered first. The authors detect differences in the identified structural government spending only during the period 1973-1980 which is period of high inflation.

Three papers investigating fiscal transmission in open economy include the interest rate in the VAR model and fail to find a significant response in the interest rate. Corsetti et al. [2012] investigate the responses of eight variables of interest, including the nominal short-term interest rate and do not find any statistically significant change in the interest rate (see Figure 1, page 546). Ilzetzki et al. [2013] fail to detect any significant response of the interest rate (see figure 5, page 249) either in a fixed or in a floating exchange rate regime. Born, Juessen, and Müller [2013] detect only a slight change in the interest rate in the short run in a fixed exchange rate regime.

To control for monetary policy, we include the short-term interest rate based on three-

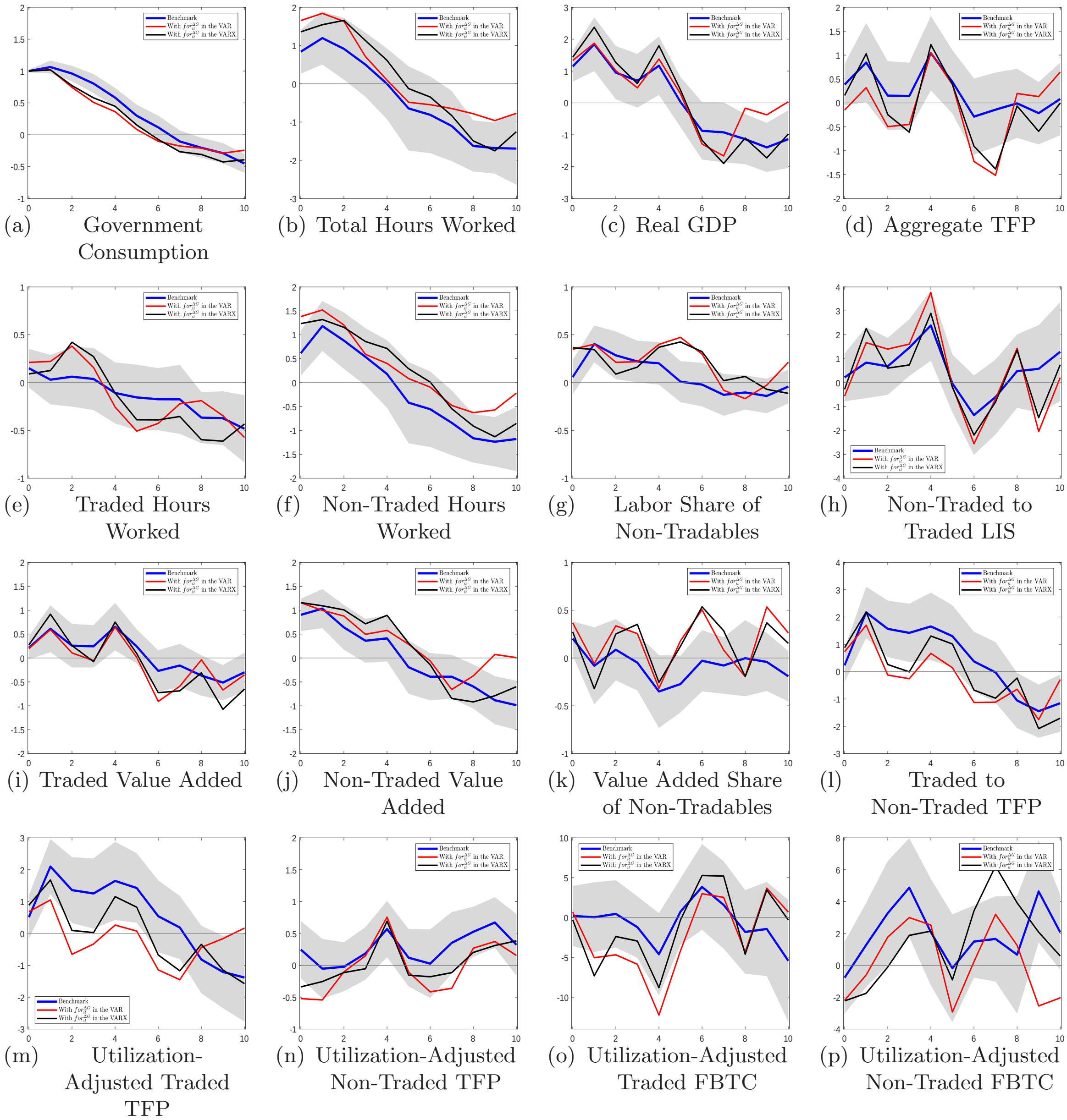
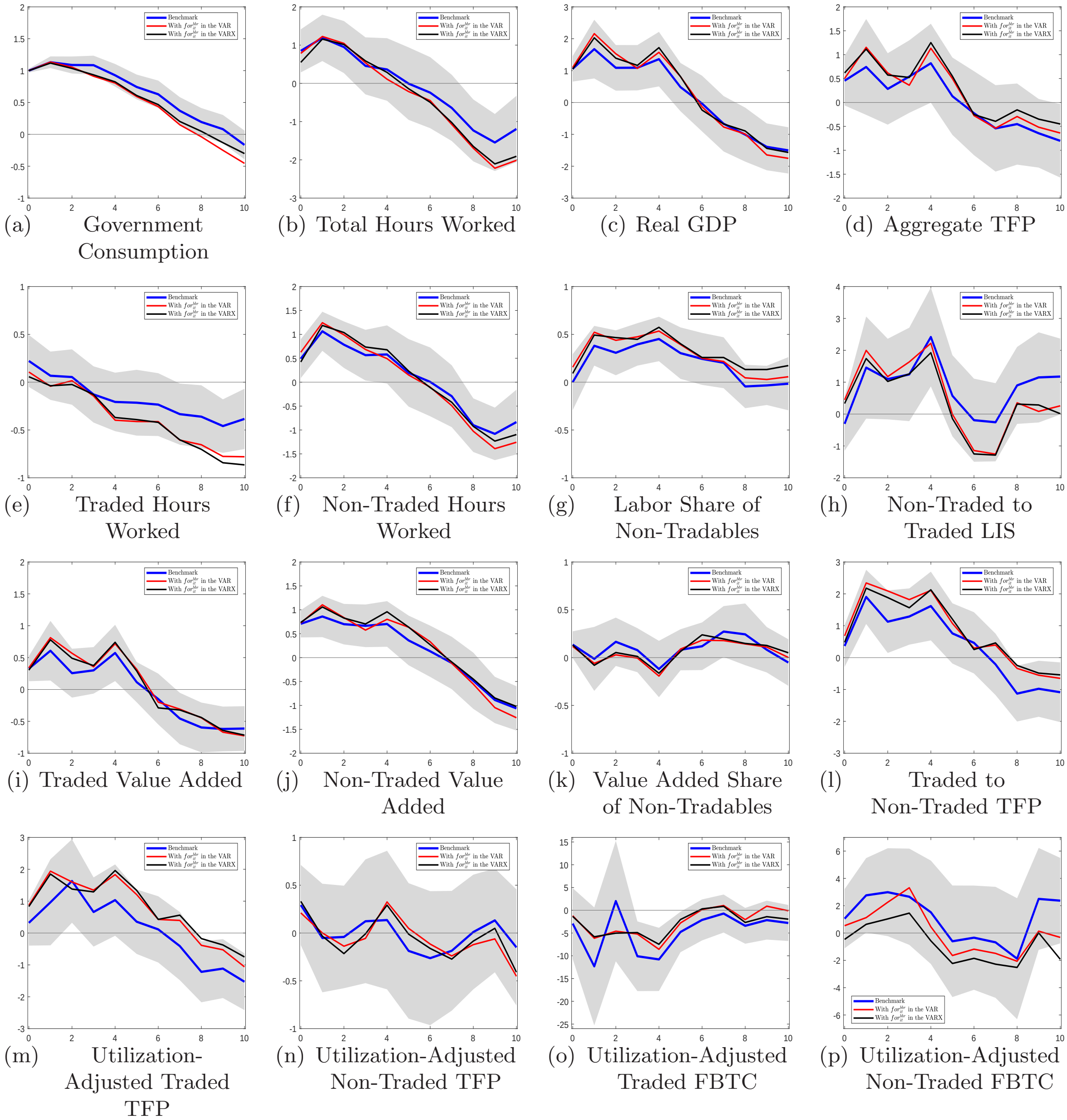


Figure 21: Effects of an Unanticipated Government Spending Shock: Controlling for Anticipation Effects with the Forecast for Government Spending Growth (for_{it}^{G}). Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red line shows results when the identified spending shock ϵ_{it}^{G} is estimated in the baseline VAR model augmented with the forecast for government spending growth for_{it}^{G}. The black line shows results when the identified spending shock ϵ_{it}^{G} is estimated in a VARX model that includes the forecast for government spending growth for_{it}^{G} as an exogenous variable. Sample: 17 OECD countries (AUS, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, NOR, PRT, SWE and USA), 1986-2014, annual data (367 observations).



(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP

(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS

(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP

(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 22: Effects of an Unanticipated Government Spending Shock: Controlling for Anticipation Effects with the Budget Balance-GDP Ratio (for^{bbr}_{it}). Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red line shows results when the identified spending shock ϵ^G_{it} is estimated in the baseline VAR model augmented with the forecast for the budget balance-GDP ratio for^{bbr}_{it}. The black line shows results when the identified spending shock ϵ^G_{it} is estimated in a VARX model that includes the forecast for the budget balance-GDP ratio for^{bbr}_{it} as an exogenous variable. Sample: 12 OECD countries (AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, NLD, PRT and SWE), 1970-2014, annual data (415 observations).

Table 17: How Does the Inclusion of Monetary Policy Affect Government Spending Shocks?

Variable	ϵ^{G} (bench)	$\epsilon^{G,M}$ (with r_{it})	$\epsilon^{G,M}$ (with r_{it}^{C})	$\epsilon^{G,M}$ (with $r_{it}^{S,\star}$)
ϵ^{G} (bench)	1.000	0.979	0.980	0.979
$\epsilon^{G,M}$ (with r_{it})		1.000	0.998	0.999
$\epsilon^{G,M}$ (with r_{it}^{C})			1.000	0.997
$\epsilon^{G,M}$ (with $r_{it}^{S,\star}$)				1.000

month money market rates taken from OECD Economic Outlook Database. The VAR model we estimate to extract the structural government spending shock thus includes government final consumption expenditure, real GDP, total hours worked, the real consumption wage, aggregate total factor productivity and the interest rate; we consider three alternative measures of the interest rate: the nominal interest rate, r_{it}, the real interest rate calculating by subtracting the rate of change of the CPI from the nominal interest rate, denoted by r_{it}^{C}, and the nominal interest rate deflated by the price of foreign goods which is the numeraire in our model and thus we subtract the rate of change of the weighted average of the traded value added deflators of trade partners of the country i from the nominal interest rate, denoted by $r_{it}^{S,\star}$.

Data coverage: the period is running over 1970-2015 except for Denmark (1980-2015), Spain 1977-2015), Great Britain (1978-2015), Ireland (1990-2015), Italy (1971-2015), Korea (1991-2015), Sweden (1982-2015).

Likewise Rossi and Zubairy [2011], we estimate the VAR model with and without the interest rate, and compare identified structural government spending shocks when we control for monetary policy, $\epsilon_{it}^{G,M}$, with the structural government spending shocks when we do not control for monetary policy, ϵ_{it}^{G}. The correlations shown in Table 17 reveal that controlling for monetary policy does not affect our identification of government spending shocks. One reason to this is that the forecast error variance decomposition conducted by Rossi and Zubairy [2011] indicates that monetary policy shocks contribute to short-run business cycle fluctuations and the fact that we use annual data would significantly mitigate the impact of monetary policy.

Once we have identified structural government spending shocks, we estimate the dynamic responses of aggregate and sectoral variables along with the impact on our three alternative measures of the interest rate which control for monetary policy. The results displayed by Fig. 23 reveal that controlling for monetary policy by including the short-run interest rate in the VAR model does not affect the dynamic effects of a government spending shock. Our evidence shown in Fig. 23(q), 23(r), and 23(s) indicate that both the short-term nominal and real interest rates remain unresponsive to the government spending shock.

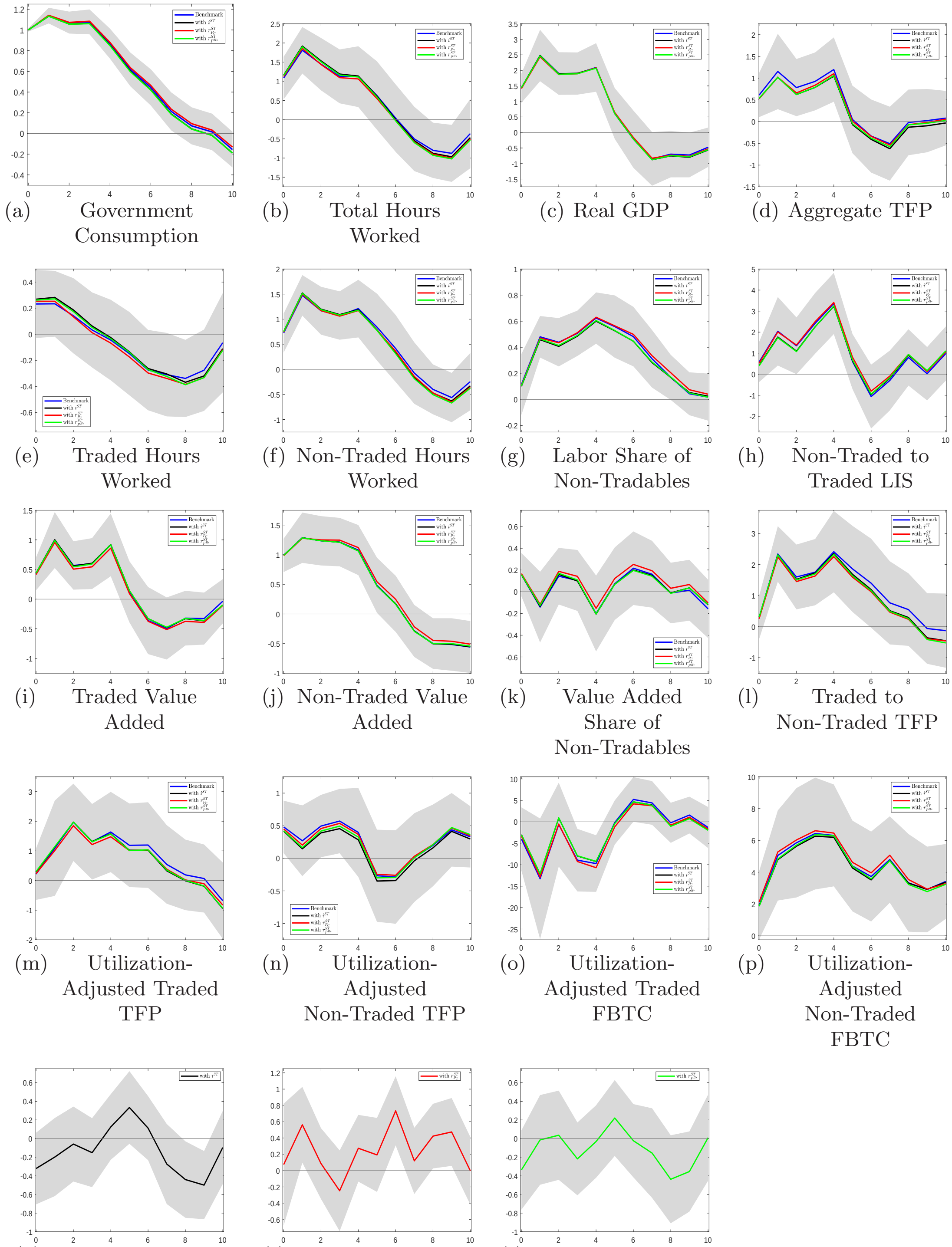
Overall, the evidence point out of a limited role of monetary policy when identifying structural government spending shocks.

P.6 Robustness Checks: One-Step vs. Two-Step Method

In the main text, we adopt a two-step approach where we first identify the exogenous shock to government consumption by estimating a panel SVAR which includes (logged) government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP, and assume that government consumption does not react within the same year to other variables included in the VAR model. Once we have identified the government spending shock, we estimate the dynamic effects by using the local projection method which simply requires estimation of a series of regressions for each horizon h for each variable of interest on the identified shock:

$$x_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h}t + \psi_h(L)\, z_{i,t-1} + \gamma_h \text{Shock}_{it} + \eta_{i,t+h}. \tag{157}$$

In the main text, baseline control variables collected in z, includes past values of government spending and of the variable of interest.



(a) Government Consumption
(b) Total Hours Worked
(c) Real GDP
(d) Aggregate TFP
(e) Traded Hours Worked
(f) Non-Traded Hours Worked
(g) Labor Share of Non-Tradables
(h) Non-Traded to Traded LIS
(i) Traded Value Added
(j) Non-Traded Value Added
(k) Value Added Share of Non-Tradables
(l) Traded to Non-Traded TFP
(m) Utilization-Adjusted Traded TFP
(n) Utilization-Adjusted Non-Traded TFP
(o) Utilization-Adjusted Traded FBTC
(p) Utilization-Adjusted Non-Traded FBTC
(q) Short-Term Nominal Interest Rate, r_{it}
(r) Short-Term Real Interest Rate (adjusted withg CPI), r_{it}^{C}
(s) Short-Term Real Interest Rate (adjusted with import prices), $r_{it}^{S,\star}$

Figure 23: Controlling for Monetary Policy: Identification by Including the Short-Term Interest Rate. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP when we identify the government spending shock by using Blanchard and Perotti [2002] identification method when we do not control for monetary policy. The black line shows results when we we control for monetary policy by including the short-term nominal interest rate in the VAR model. The red line shows results when we we control for monetary policy by including the short-term real interest rate in the VAR model calculated by using the CPI. The green line shows results when we we control for monetary policy by including the short-term real interest rate in the VAR model calculated by using foreign prices calculated as an import-share weighted average of trade partners' traded value added deflators. Shaded areas indicate the 90 percent confidence bounds in the original VAR. We estimate the dynamic effects by using Jordà's [2005] single-equation method. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Sample: 1970-2015 except for Denmark (1980-2015), Spain 1977-2015), Great Britain (1978-2015), Ireland (1990-2015), Italy (1971-2015), Korea (1991-2015), Sweden (1982-2015).

Ramey and Zubairy [2018] adopt a one-step approach (to ensure a straightforward comparison of results with estimates from defense news variables) by replacing the shock with current government spending in the single-equation method.

The objective of our robustness test below is threefold. First, we contrast the dynamic effects when we consider the two-step method with those obtained when we consider the one step method of Ramey and Zubairy [2018]. Second, we include in the second step the same controls as in the first step which allows us to investigate empirically whether adding the same controls in the Jordà single-equation as in the VAR model modifies the results.

Third, we compare the confidence bounds associated with point estimates obtained in the two-step approach with the confidence interval associated with point estimates obtained in the one-step approach. The reason is that in the two-step method, we estimate a VAR model which allows us to recover the structural government spending shocks and we plug these estimated error terms which possess a certain variance in the Jordà single-equation and estimate the dynamic effects of the shock on variables of interest. When we plug the shock into the Jordà single-equation, we do not consider the variance of the shock and thus to have a sense of the extent to which ignoring the variance of the shock might affect the confidence bounds of point estimates for the dynamic effects, we compare the confidence bound in the two-step approach with the confidence bounds in the one-step approach where we run the regression of variables of interest on current government spending (see below).

Literature adopting the two-step method. Several papers have adopted the two-step method, including Bernardini et al. [2020], Corsetti et al. [2012], Liu [2022], Miyamoto et al. [2018]. While Corsetti et al. [2012] and Miyamoto et al. [2018] identify the unexpected innovations in government spending by estimating a fiscal rule in the first approach to recover the error terms, our empirical strategy is closer to that adopted by Liu [2022] who considers the standard Cholesky decomposition proposed by Blanchard and Perotti [2002] to identify shocks to government consumption. The residuals estimated in the first step, ϵ_G, are the unexpected government spending changes orthogonal to the expected component of government spending and information so they are government spending shocks. In the second step, we estimate the dynamic effects by using local projections like Miyamoto et al. [2018] and Liu [2022].

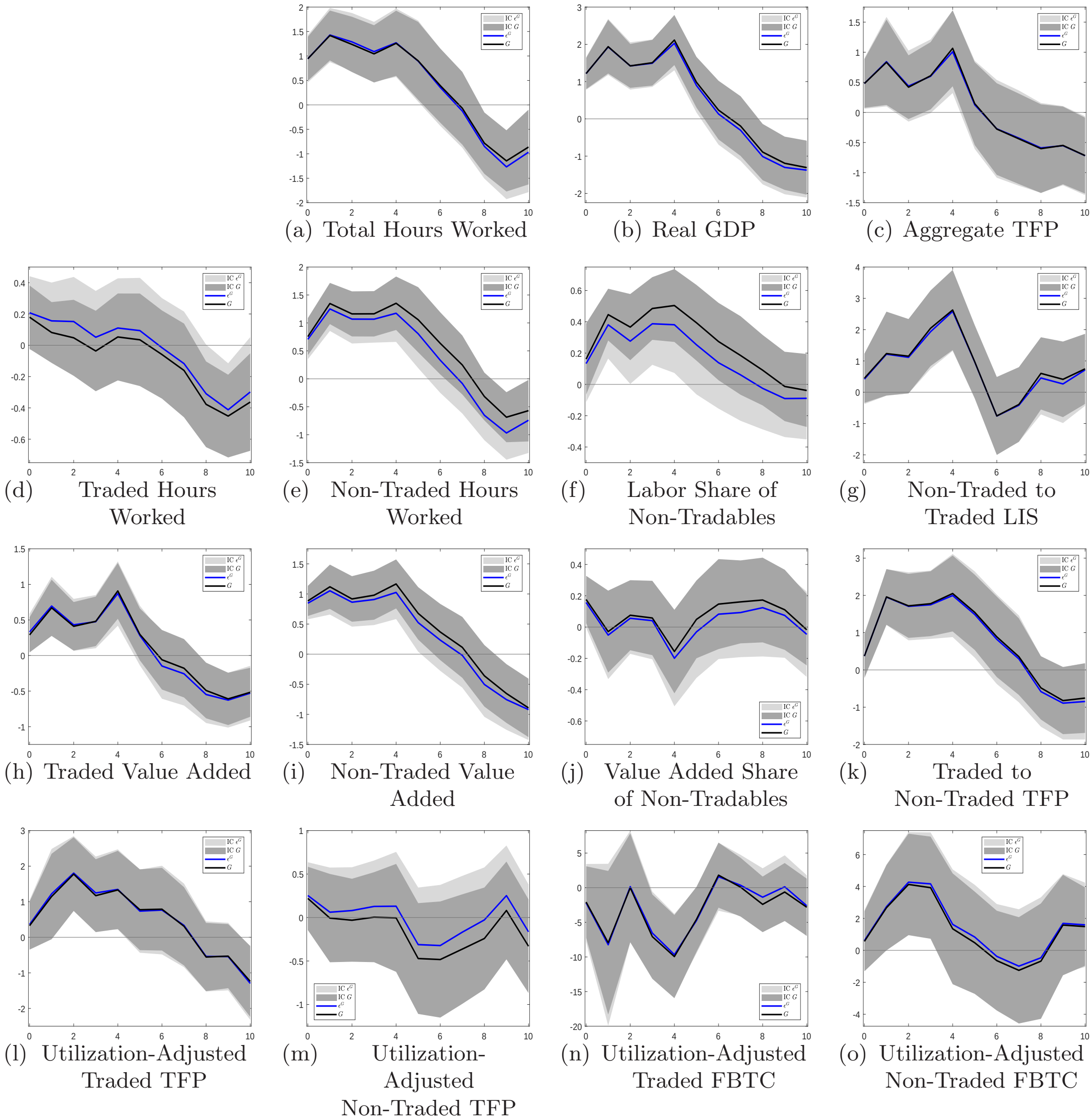
Robustness of the two-step method. Because there has been a growing tendency to estimate regression equations in which constructed variables appear, Pagan [1984] provides a complete treatment of the econometric problems arising when generated variables appear in a regression equation. In particular, the author explores the situation when generated residuals are used as regressors. It is found that the coefficient and its standard error from an OLS program would be a consistent estimator of the true coefficient and the true standard error for the coefficient of the unanticipated variable. In other words, running a regression with structural government spending shocks recovered from the estimation of government spending over a set of regressors is equivalent to running a regression where the regressor is government consumption itself.

Method. When Ramey and Zubairy [2018] employ the Blanchard-Perotti identification, the shock is simply given by current government spending, and controls includes lagged measures of GDP and government spending:

$$x_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h} t + \psi_h (L) z_{i,t-1} + \gamma_h g_{it} + \eta_{i,t+h}. \tag{158}$$

As pointed out by Ramey and Zubairy [2018], estimating equation (158) is equivalent to the Blanchard-Perotti structural VAR (SVAR) identification as it includes as control variables government consumption and real GDP where both variables are lagged.

Results. The solid blue line in Fig. 24 shows the responses of variables in the baseline case where we adopt the two-step method. We contrast these responses with those shown in the solid black line where we adopt a one-step approach which is equivalent to the Blanchard-Perotti structural VAR identification. Shaded areas in light grey indicate the 90 percent confidence bounds for the point estimate from our two-step method. Shaded areas in dark grey indicate the 90 percent confidence bounds when we adopt the one-step approach like Ramey and Zubairy [2018]. First, for all variables, we find that there are no marked differences between the two IRFs as point estimates are almost identical. Second,



(a) Total Hours Worked (b) Real GDP (c) Aggregate TFP

(d) Traded Hours Worked (e) Non-Traded Hours Worked (f) Labor Share of Non-Tradables (g) Non-Traded to Traded LIS

(h) Traded Value Added (i) Non-Traded Value Added (j) Value Added Share of Non-Tradables (k) Traded to Non-Traded TFP

(l) Utilization-Adjusted Traded TFP (m) Utilization-Adjusted Non-Traded TFP (n) Utilization-Adjusted Traded FBTC (o) Utilization-Adjusted Non-Traded FBTC

Figure 24: One Step vs. Two-Step Method. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP when we adopt a two-step method where we identify the shock by adopting Blanchard-Perotti approach and next estimate dynamic effects by using Local Projections. Shaded areas in light grey indicate the 90 percent confidence bounds. The solid black lines shows results when we adopt a one-step approach like Ramey and Zubairy [2018] who employ the Blanchard-Perotti identification with the shock being simply given by current government spending, with the set of controls that includes lagged measures of government spending, real GDP, total hours worked, the real consumption wage and aggregate total factor productivity. Shaded areas in dark grey indicate the 90 percent confidence bounds. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Sample: 18 OECD countries, 1970-2015, annual data.

there are no marked differences in the confidence bounds between the two-step and the one-step method which confirms the consistency and accuracy of the two-step method.

P.7 Robustness Checks: LP vs. SVAR

In the main text, we recover structural government spending shocks by estimating a SVAR and then we estimate the responses by using local projections. An alternative option would be to consider a one-step SVAR methodology where one estimate simultaneously the structural shock and generate the dynamic effects of the exogenous shock to government consumption. The advantage of the two-step method is twofold. The first advantage of the two-step method is that all variables of interest respond to the same shock which is extracted one for all while when we estimate different SVAR, we might fear that we estimate different structural shocks. However, our robustness check below shows that the shock is identical across all VAR models. The second advantage over the standard VAR approach is that the Jordà's [2005] local projection method we use in the second step reduces the number of coefficients to estimate which might improve the accuracy of estimates and also does not impose the dynamic restrictions implicitly embedded in VARs and can accommodate non-linearities in the response function. The flip side of the coin is that by imposing fewer restrictions, impulse responses obtained by using the local projection method are rather erratic.

The solid blue line in Fig. 25 shows the responses of variables in the baseline case where we adopt the two-step method. We contrast these responses with those shown in the solid black line where we adopt a one-step approach where we estimate a SVAR and adopt a Cholesky decomposition in the lines of Blanchard-Perotti structural VAR identification. By and large, point estimates are very similar between the two approaches. We may notice some differences however. While qualitatively, the responses are very similar as they lie within the confidence bounds associated with our two-step method, we may notice some differences. In particular, the endogenous response of G to the exogenous fiscal shock displays more persistence in the SVAR but the difference shows up only after eight years.

Q Alternative Measures of Technology and Determinants of Technology Adjustment

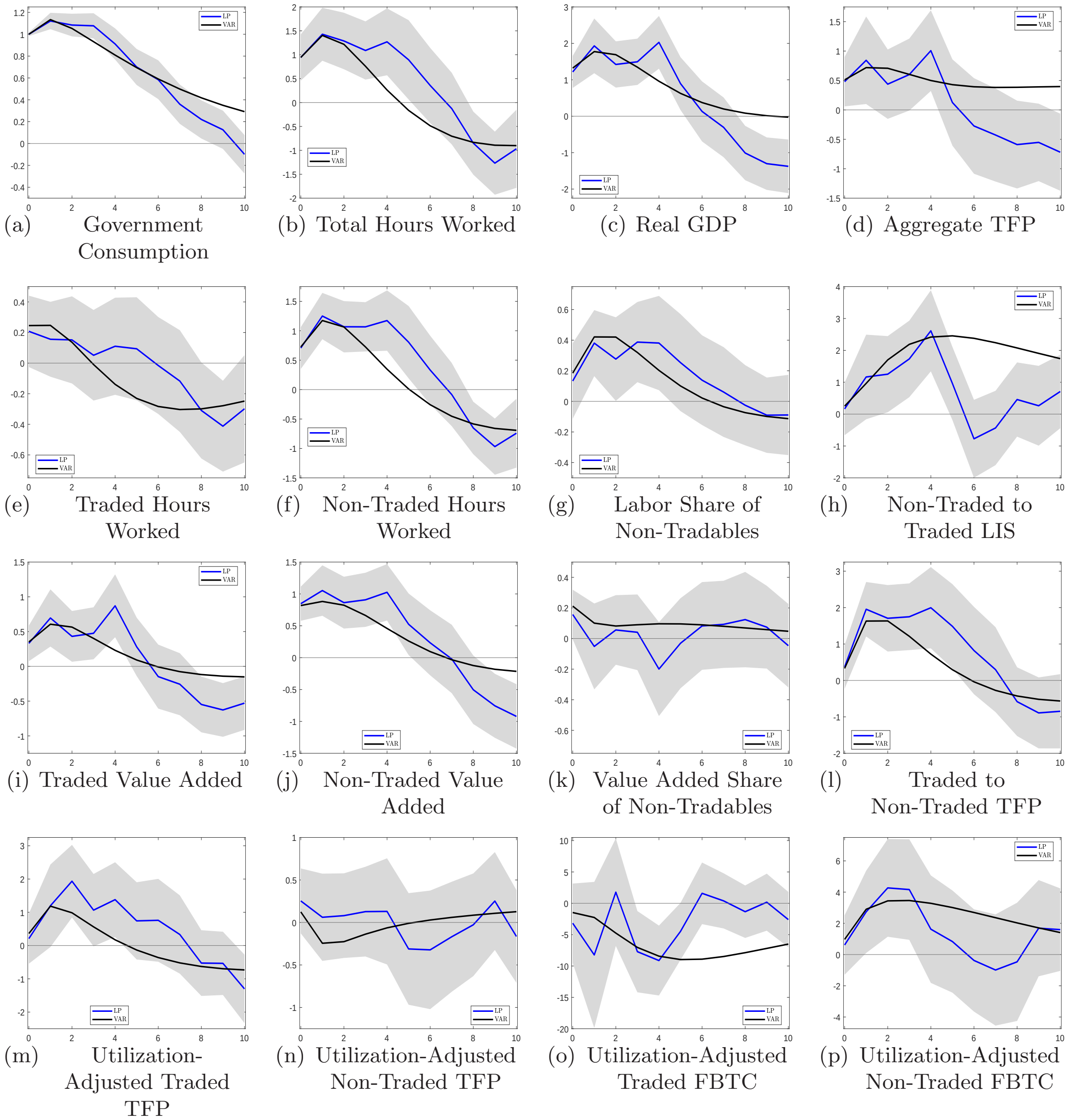
In the main text we investigate the dynamic effects of a shock to aggregate government consumption on technology variables. To measure technology, in line with the recommendation of Basu, Fernald and Kimball (BFK henceforth) [2006], we adjust aggregate and sectoral TFPs with the utilization rate. Because time series for utilization-adjusted TFP are only available for the United States at an aggregate level, we have constructed time series for the capital utilization rate for the 18 OECD countries of our sample and at a sectoral level by adopting the methodology proposed by Imbs [1999].

To check whether our purified measure of efficiency reflects technology, we conduct below a robustness check where we use alternative measures to ours and we also propose a set of factors that can rationalize our findings. Note that in contrast to existing methods which 'purify' TFP measure from variations in the utilization rate, our method has two advantages over others: first, we are able to construct time series at a sectoral level in line with our classification T/N for our sample of eighteen OECD countries over 1970-2015 and second we adapt the existing methodology to CES production functions where the labor income share is variable over time.

In subsection Q.1, we contrast our results by adapting the construction of utilization-adjusted-TFP proposed by Imbs [1999] with the evidence when we use the utilization-adjusted series of total factor productivity (TFP) constructed by Fernald [2014] that has become the main measure for adjusted TFP for the United States. In section Q.2, we use the OECD measure of TFP which controls ofr the quality of capital assets. In section Q.3, we apply the methodology pioneered by Basu [1996]. Finally, in section Q.4, we explore empirically the effects of a government spending shock on investment in R&D and capital stock of R&D, and we put forward some potential determinants of variations in the utilization-adjusted TFP we estimate empirically.

Q.1 Fernald [2014] Utilization-Adjusted TFP on Annual and Quarterly data

Basu et al. [2006] propose an alternative methodology to Imbs [1999] for adjusting Solow residuals with variations in factor utilization. In this subsection, we contrast our results by adapting the construction of utilization-adjusted-TFP proposed by Imbs [1999] with the evidence when we use the utilization-adjusted series of total factor productivity constructed by Fernald [2014] that has become the main measure for adjusted TFP for the United States. The measure of technology proposed by Fernald is thinner than ours because instead of taking the Solow residual, the author uses a measure of TFP which controls for the composition of labor and capital heterogeneity, i.e., his dataset weights different inputs



(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP

(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS

(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP

(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 25: One-Step SVAR vs. Two-Step Method. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP when we adopt a two-step method where we identify the shock by adopting Blanchard-Perotti identification assumption and next estimate dynamic effects by using Local Projections. The solid black lines shows results when we adopt a one-step approach where we estimate a SVAR and employ the Blanchard-Perotti identification to recover the shock and use the same VAR to generate impulse responses. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Sample: 18 OECD countries, 1970-2015, annual data.

using actual or estimated relative factor prices to control for these differences in implied marginal products. In addition, Fernald [2014] takes into account both capital utilization and labor efforts to construct the time series for utilization-adjusted aggregate TFP.

To explore empirically whether both approaches lead to similar or different results, we use the utilization-adjusted series of total factor productivity (TFP) constructed by Fernald [2014]. Because this measure is available for the U.S. only, we have re-estimated the effects of a shock to government consumption on aggregate TFP for this country. Since time series for the utilization-adjusted TFP are available on a quarterly basis, we have estimated the VAR model on quarterly data. More specifically, we estimate two VAR models which include both government consumption, real GDP, total worked, the real consumption wage for the United States. We also include either Fernald [2014] measure of aggregate TFP or utilization-adjusted aggregate TFP. We adopt the Blanchard-Perotti identification scheme and assume that G_t does not respond to the other variables included in the VAR model. Once we have identified the structural shock to government consumption on quarterly data, we annualize the shocks and estimate the dynamic adjustment by using local projections. This exercise on annual data may be viewed as tentative as there is substantial uncertainty surrounding point estimates given the relatively small number of observations available per country (i.e., 46 observations for the US) because sectoral data are only available at a yearly frequency.

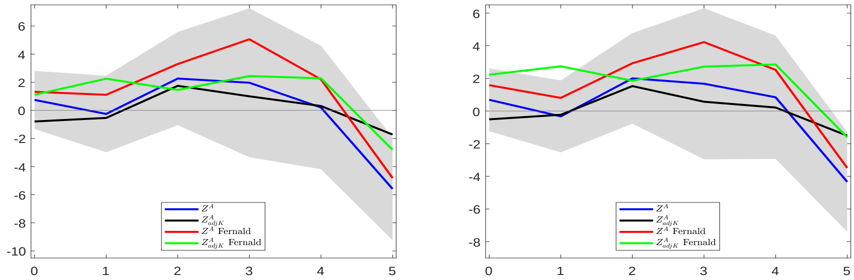
Source. OECD [2017], for government consumption, real GDP, total worked, the real consumption wage for the United States, and Fernald [2014] for the aggregate TFP or utilization-adjusted aggregate TFP. Frequency: either annual (1970-2015) or quarterly data (1970q1-2015q4).

In Fig. 26(a) and Fig. 26(b), we contrast the responses of our measure of technology with the measure by Fernald [2014], adjusted or not with factor utilization. The four measures include the Solow residual we have calculated for the United States by using EU KLEMS (blue line), the aggregate TFP measured by Fernald [2014] (red line), our utilization-adjusted TFP measure based on Imbs [1999] (black line) and the utilization-adjusted TFP measure based on Fernald [2014] (green line).

As it stands out, the effects of a temporary rise in government consumption are similar whether we calculate the Solow residual (blue line) by using time series from EU KLEMS [2011], [2017], or we take directly Fernald's [2014] TFP series (red line). The solid black line shows the response of aggregate TFP when we adjust the Solow residual with the capital utilization rate constructed by adapting the methodology proposed by Imbs [1999] to CES production functions at a sectoral level. The solid green line shows the response of Fernald's [2014] TFP adjusted with inputs' utilization. Overall, whether we consider the Solow residual or a thinner measure and whether we adjust TFP with capital utilization only or with both capital utilization and worker efforts, technology improves in the short-run and estimated responses of alternative measures of technology lie within the 90% confidence bounds associated with the point estimate obtained for our aggregate Solow residual.

We many notice some differences however. The difference between the response in TFP and utilization-adjusted TFP is insignificant in both cases, i.e., between Fernald's [2014] TFP and utilization-adjusted TFP, and between the Solow residual and the capital-utilization-adjusted Solow residual. Therefore, the difference between ours and Fernald's measure of technology is driven by the measure of TFP which is adjusted with the quality of inputs. When we control for the compositions of capital assets by using the OECD measure of TFP, we find very similar results with the Solow residual however.

Fig. 26 shows that the improvement in TFP is more pronounced by using Fernald's [2014] measure while our measure based on the Solow residual generates a smaller technology improvement. There are two potentials reasons to these differences. First, the measure of TFP used by Fernald controls for the quality of inputs. Second, we calculate the Solow residual for the whole economy, i.e., including both the market and public sectors while Fernald data covers the U.S. business sector only.



(a) Technology Responses when Fernald's [2014] TFP is Ordered Last in the VAR

(b) Utilization-Adjusted-Aggregate-TFP on Quarterly Data: Measure from Fernald [2014]

Figure 26: Technology Responses when Fernald's [2014] utilization-adjusted-TFP is Ordered Last in the VAR Notes: In Fig. 26(a) and Fig. 26(b), we estimate a VAR on quarterly data where the technology variable ordered last is the aggregate TFP or utilization-adjusted TFP by Fernald [2014], respectively, and we generate the responses of aggregate TFP (blue line) and utilization-adjusted TFP (black line) based on our measure and contrast them with the responses of Fernald's [2014] aggregate TFP (red line) and utilization-adjusted TFP (green line). Sample: United States 1970q1-2015q4.

Q.2 OECD [2021] TFP adjusted with Capital Quality

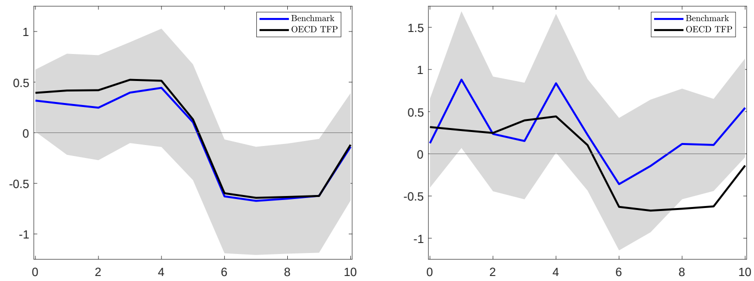
While evidence for the United States from the Fernald's measure of technology corroborates our measure of efficiency, we use a different measure provided by the OECD [2021]. The OECD measure of TFP is based on the construction of time series for capital services which are computed separately for eight non-residential fixed assets, including computer hardware, telecommunications equipment, transport equipment, other machinery and equipment and weapons systems, non-residential construction, computer software and databases, R&D and other intellectual property products. The volume index of total capital services is computed by aggregating the volume change of capital services of all individual assets using a Törnqvist index that applies asset specific user cost shares as weights.

Source. OECD [2021] Measuring Productivity. Data are available for 18 OECD countries over 1985-2015 except for Austria (1996-2015), South Korea (1990-2015) and Norway (1990-2015).

The disadvantage of time series of TFP constructed by the OECD [2021] is that they are not adjusted with the utilization rate but the advantage is that they take into account the quality of capital and thus the measure of OECD TFP is a purified measure of capital services. Because we include a measure of aggregate TFP in the VAR model estimated in the first step to recover the structural government spending shocks, we check whether the measure of aggregate TFP we use in the main text, i.e., the Solow residual by using the time series from EU KLEMS [2011], [2017], and the OECD [2021] measure of aggregate TFP which controls for the composition of capital, give similar results. As can be seen in Fig. 27(a), our shock (blue line) and the shock recovered by consider the OECD measure of efficiency produces a rise in aggregate TFP which has the same shape in both cases. To be consistent, we have estimated the VAR model over the same period, i.e., 1985-2015. In Fig. 27(b), we contrast the response of our measure of aggregate TFP (included in the VAR model to recover the government spending shock) displayed by the blue line with the response of the OECD [2021] measure of aggregate TFP which controls for the composition of capital displayed by the black line. In both cases, the government spending leads to a rise in aggregate TFP. It is worth mentioning that the limited horizon for estimations (85-15 instead of 70-15) increases the uncertainty of the point estimate.

Q.3 Utilization-Adjusted TFP: Basu [1996] vs. Imbs [1999]

So far, we have shown empirically that our measure of aggregate TFP is robust to controlling for capital composition quality and our robustness checks also reveal that our measure of technology where we adjust aggregate TFP with capital utilization is robust to alternative measure of utilization-adjusted TFP for the United States where Fernald [2014] controls for labor effort and average workweek of capital.



(a) Sensitivity of G-Shock to TFP Measure: Our Measure of Agggregate TFP vs. OECD [2021] Measure

(b) Response of Aggregate TFP: Our Mesure (blue line) vs. OECD [2021] (black line) Measure

Figure 27: Empirical Responses of our Measure of Aggregate TFP vs. OECD [2021] Measure to a Shock to Government Consumption. Notes: The solid blue line in the left panel, i.e., in Fig. 27(a), shows the response of aggregate TFP based on the Solow residual calculated by using the time series from EU KLEMS [2011], [2017]. To generate the response, we have estimated the standard VAR model which includes our measure of aggregate TFP ordered last. In the black line, we show the response of our measure of aggregate TFP when we estimate a VAR model which includes the OECD [2021] measure of aggregate TFP. In Fig. 27(b), we contrast the response of our measure of aggregate TFP when the latter is included in the VAR model to estimate the shock with the response of a OECD [2021] measure of aggregate TFP adjusted with the composition of capital. Sample: 18 OECD countries, 1985-2015, annual data.

Because time series for utilization-adjusted TFP at a sectoral level are not available for the countries in our sample over 1970-2015, we conduct a third robustness check where we construct time series of utilization-adjusted TFP measure at a sectoral level for all OECD countries by adopting the methodology developed by Basu [1996] and we compare the responses of utilization-adjusted TFP based on Basu [1996] methodology with the responses of utilization-adjusted TFP based on Imbs [1999] approach.

Detailed steps of derivation of the utilization rate in Basu [1996] approach It is useful to detail the steps of derivation of the capacity utilization rate by Basu [1996] as it shows that the methodology is completely different from ours. The advantage of Basu [1996] over Imbs [1999] approach is that we control for unobserved changes in both capital utilization and in the intensity of work effort by using an ingenious and simple assumption based on the fact that intermediate inputs is a convenient indicator of cyclical factor utilization because its input does not have an extra effort or intensity dimension. Therefore, we can infer increasing extraction from capital and labor services by firms from materials use as firms need more material to produce more. Variations in the use of intermediate inputs relative to measured capital and labor are an index of unmeasured capital and labor input.

Both the traded and non-traded sectors use physical capital inclusive of capital utilization, $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$, and labor inclusive of workers' efforts, $\tilde{L}^j(t) = u^{L,j}(t)L^j(t)$, according to constant returns to scale production functions which are assumed to take a CES form:

$$Y_t^j = \left[\gamma^j \left(u_t^{L,j} L_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(u_t^{K,j} K_t^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}, \tag{159}$$

where γ^j and $1-\gamma^j$ are the weight of labor and capital in the production technology, σ^j is the elasticity of substitution between capital and labor in sector $j = H, N$. Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to $R(t)$, and a labor cost equal to the wage rate $W^j(t)$.

Aggregate output denoted by Q_t^j is an aggregate of value added Y_t^j and intermediate inputs M_t^j:

$$Q_t^j = Z_t^j \left[\xi^j \left(Y_t^j\right)^{\frac{\sigma_M^j-1}{\sigma_M^j}} + \left(1-\xi^j\right)\left(M_t^j\right)^{\frac{\sigma_M^j-1}{\sigma_M^j}}\right]^{\frac{\sigma_M^j}{\sigma_M^j-1}}, \tag{160}$$

where σ_M^j is the elasticity of substitution between value added and intermediate inputs. We denote the unit cost for production value added by $c_t^j = P_{Y,t}^j$ where P^Y is the value added deflator since the goods market is perfectly competitive and we denote the aggregate price index of intermediate inputs by $P^j_{M,t}$. Both sectors are perfectly competitive and maximize profits by taking prices as given. Denoting the gross output deflator by P^j_Q, firms choose value added and intermediate inputs so as to maximize:

$$\max_{Y^j,M^j} \Pi^j_Q = \max_{Y^j,M^j} \left\{ P^j_Q Q^j - P^j_Y Y^j - P^j_M M^j \right\}. \tag{161}$$

First-order conditions lead to optimal demand for value added and intermediate inputs:

$$P^j_Q \xi^j \left(Y^j\right)^{-\frac{1}{\sigma^j_M}} \left(Q^j\right)^{\frac{1}{\sigma^j_M}} \equiv P^j_Y, \tag{162a}$$

$$P^j_Q \left(1-\xi^j\right) \left(M^j\right)^{-\frac{1}{\sigma^j_M}} \left(Q^j\right)^{\frac{1}{\sigma^j_M}} \equiv P^j_M. \tag{162b}$$

Dividing the demand for value added by the demand for intermediate inputs leads to:

$$\frac{Y^j}{M^j} = \left(\frac{\xi^j}{1-\xi^j}\right)^{\sigma^j_M} \left(\frac{P^j_Y}{P^j_M}\right)^{-\sigma^j_M}. \tag{163}$$

Log-linearizing (163) gives:

$$\hat{Y}^j - \hat{M}^j = -\sigma^j_M \left(\hat{P}^j_Y - \hat{P}^j_M\right). \tag{164}$$

Log-linearizing the production function for gross output (160) leads to:

$$\begin{aligned} \hat{Q}^j &= \hat{Z}^j + \alpha^j_Y \hat{Y}^j + \left(1-\alpha^j_Y\right)\hat{M}^j, \\ &= \hat{M}^j + \alpha^j_Y \left(\hat{Y}^j - \hat{M}^j\right), \\ &= \hat{Z}^j + \hat{M}^j - \alpha^j_Y \sigma^j_M \left(\hat{P}^j_Y - \hat{P}^j_M\right), \end{aligned} \tag{165}$$

were $\alpha^j_Y = \frac{P^j_Y Y^j}{P^j_Q Q^j}$.

Log-linearizing the production function for value added (159) leads to:

$$\hat{Y}^j = s^j_L \left(\hat{u}^{L,j} + \hat{L}^j\right) + \left(1-s^j_L\right)\left(\hat{u}^{K,j} + \hat{K}^j\right), \tag{166}$$

where $s^j_L = \frac{W^j L^j}{P^j Y^j}$.

Plugging (166) into the first line of (165) leads to:

$$\hat{Q}^j = \hat{Z}^j + \alpha^j_Y \left\{ s^j_L \left(\hat{u}^{L,j} + \hat{L}^j\right) + \left(1-s^j_L\right)\left(\hat{u}^{K,j} + \hat{K}^j\right)\right\} + \left(1-\alpha^j_Y\right)\hat{M}^j. \tag{167}$$

Equating (167) to the last line of (165) allows us to derive an expression for the capacity utilization rate:

$$\begin{aligned} \hat{u}^j_Y &= s^j_L \hat{u}^{L,j} + \left(1-s^j_L\right)\hat{u}^{K,j}, \\ &= \hat{M}^j - s^j_L \hat{L}^j - \left(1-s^j_L\right)\hat{K}^j - \sigma^j_M \left(\hat{P}^j_Y - \hat{P}^j_M\right). \end{aligned} \tag{168}$$

Assuming that $\sigma^j_M = 0$ implies that the capacity utilization rate can be calculated as follows:

$$\hat{u}^j_Y = \hat{M}^j - s^j_L \hat{L}^j - \left(1-s^j_L\right)\hat{K}^j, \tag{169}$$

where M^j are intermediate inputs (i.e., intermediate consumption) at constant prices, L^j hours worked, K^j the capital stock at constant prices, s^j_L is the LIS.

We use (169) to measure the intensity in the use of capital and labor at a sectoral level (i.e., for each industry) and adjust the Solow residual with this measure to construct time series for the utilization-adjusted TFP in sector $j = H, N$:

$$\hat{Z}^j = \hat{\mathrm{TFP}}^j - \hat{u}^j_Y. \tag{170}$$

Source: Time series for intermediate inputs at constant prices are taken from EU KLEMS. Data coverage: 1970-2015 for 18 OECD countries except for AUS, CAN, ESP, GBR, IRL, KOR and NOR where time series are available only over 1970-2007.

Results. In Fig. 28, we contrast the response of utilization-adjusted TFP based on the Solow residual adjusted with the capital utilization rate constructed by adopting Imbs [1999] method (displayed by the solid blue line) with the response of TFP based on the Solow residual adjusted with the capacity utilization rate in eq. (170) which is based on Basu [1996] method (displayed by the solid black line). Note that when we adopt the Basu's [1996] method, we construct time series of utilization-adjusted TFP at a sectoral level by assuming $\sigma_M^j = 0$. A shock to government consumption increases utilization-adjusted aggregate TFP regardless of whether the elasticity σ_M^j between intermediate inputs and value added is zero or 1.

The methods by Imbs [1999] and Basu [1996] are very different since the first method is based on the assumption that an increase in the rate of utilization of capital accelerates the depreciation of capital goods, while the second method is based on the observation that intermediate inputs do not have an extra effort or intensity dimension. As pointed out by Basu [1996], by this logic, changes in the input of materials relative to measured capital and labor are an index of unmeasured capital and labor input, so we can estimate the degree to which the procyclicality of productivity is driven by variable utilization. The first row of Fig. 28 shows that a government spending shock leads to a significant technology improvement in the traded sector and at the level of the whole economy while technology remains unchanged in the non-traded sector. Importantly, the responses of the measure of technology based on Basu [1996] lie within the confidence bounds of the point estimate the measure of technology based on Imbs [1999]. While the method of Basu controls for both capital utilization and worker efforts, the evidence displayed by Fig. 28 reveals that controlling for capital utilization only leads to similar results. More specifically, as shown in the second row, both Imbs and Basu's measures of the intensity in the use of production capacity are very similar in the traded sector while the measure based on Basu shows a discrepancy for the non-traded sector as it predicts an increae in $u^{Y,N}$ while our measure slightly decreases. In fact, our model produces an increase in non-traded utilization which tends to mimic that of Basu. However, the slight difference in the utilization rate does not produce significant differences in the utilization-adjusted non-traded TFP. Our measure of technology based on Imbs [1999] methodology is thus robust to alternative measures of technology proposed by Basu [1996] or Fernald [2014].

Q.4 Technology Utilization Rate and Determinants of Sectoral Technology Adjustments

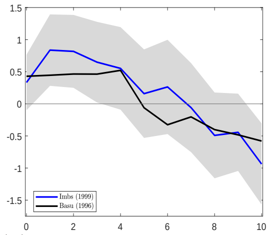
In this subsection, we clarify the concept of technology utilization rate and propose some potential determinants of cumulative changes in utilization-adjusted sectoral TFP.

Changes in utilization-adjusted sectoral TFP and the concept of technology utilization rate

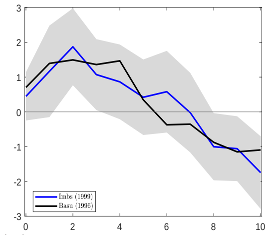
Variations in utilization-adjusted TFP, Z^j, can be driven by a change in the stock of knowledge, $\bar{Z}^j$, or in the rate of utilization of the stock of ideas, $u^{Z,j}$, or both;

$$Z^j(t) = u^{Z,j}\bar{Z}j. \tag{171}$$

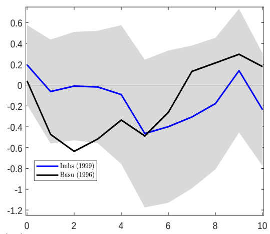
Endogenous decisions about intangible assets and the technology are isomorphic to the decisions of the capital stock and the capital utilization rate. The model we have in mind is a model where households would decide about both aggregate R& D investment and the stock of intangible assets where the installation of new ideas would be subject to 'installation' costs like physical capital. We could think of installation costs of ideas as reflecting marketing and/or advertising costs. Building on VAR evidence which reveals that a shock to government consumption has no long-run effect on utilization-adjusted TFP, we interpret technology improvement as the result of a higher utilization in the stock of knowledge rather than a rise in innovation. Because a positive variation in utilization-adjusted TFP reflects a better efficiency in transforming inputs into outputs and our evidence suggests that



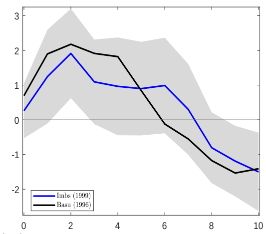
(a) Utilization-Adjusted Aggregate TFP, Z^A



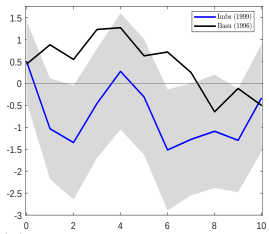
(b) Utilization-Adjusted Traded TFP, Z^H



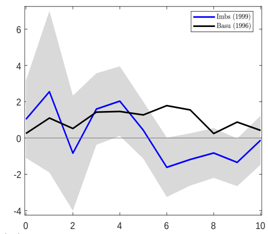
(c) Utilization-Adjusted Non-Traded TFP, Z^N



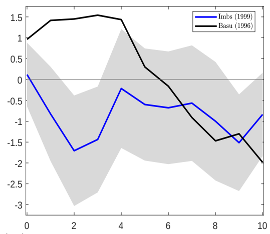
(d) Traded relative to Non-Traded Utilization-Adjusted TFP, Z^H/Z^N



(e) Aggregate Utilization Rate, $u^{Y,A}$ (black line) vs. $u^{K,A}$ (blue line)



(f) Traded Utilization Rate, $u^{Y,H}$ (black line) vs. $u^{K,H}$ (blue line)



(g) Non-Traded Utilization Rate, $u^{Y,N}$ (black line) vs. $u^{K,N}$ (blue line)

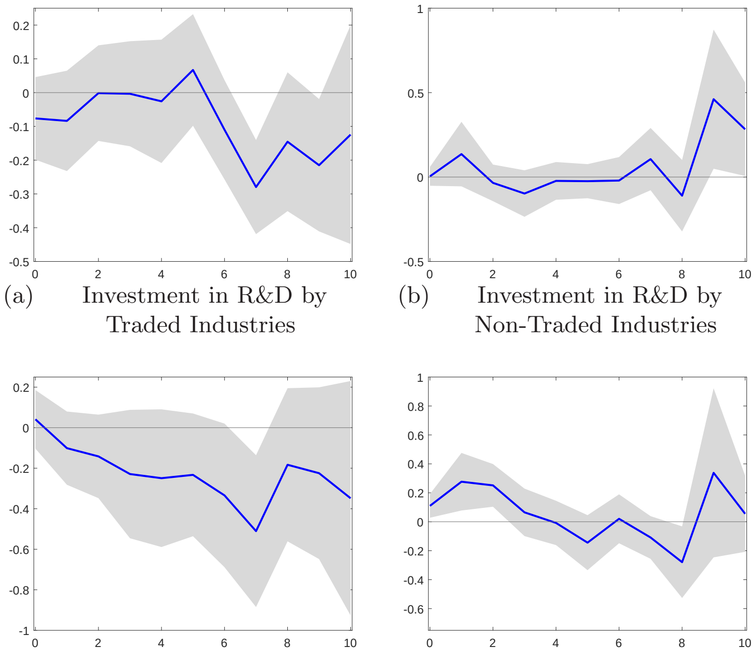
Figure 28: Empirical Responses of utilization-adjusted TFP to a Shock to Government Consumption: Basu [1996] vs. Imbs [1999]. Notes: The solid blue line shows the response of utilization-adjusted TFP based on the Solow residual adjusted with the capital utilization rate constructed by adopting Imbs [1999] method. The solid black line shows the response of TFP based on the Solow residual adjusted with the capacity utilization rate in eq. (170) which is based on Basu [1996] method. To generate the responses, we have estimated the standard VAR model which includes the measure of the Solow residual ordered last. The black line shows responses of technology factors when $\sigma_M^j = 0$ (i.e., intermediate inputs are Edgeworth complements in production). Results for the case $\sigma_M^j = 1$ are very similar if not identical. Sample: 18 OECD countries, 1970-2015, annual data.

innovation is not responsible for improved efficiency after a government spending shock, the technology improvements we detect capture a better firm organization to meet higher demand while offsetting the upward pressure on production costs.

In order to provide more evidence about our interpretation of variations in utilization-adjusted sectoral TFP as the result of changes in technology utilization rates, we investigate empirically the impact of a government spending shock on R&D expenditure and the stock of knowledge at a sectoral level.

Source. We take data from EU KLEMS, Stehrer et al. [2019], which includes time series for gross fixed capital formation (GFCF) in volume in research and development (mnemonic Iq_RD) and time series for the capital stock in research and development, volume 2010 reference prices (mnemonic Kq_RD). Data coverage for GFCF in R&D: 12 countries (AUT, DNK, ESP, FIN, FRA, GBR, ITA, JPN, NLD, PRT, SWE and USA) over 1995-2015, annual data. Data coverage for GFCF in R&D: 12 countries (AUT, BEL, DNK, ESP, FIN, FRA, GBR, ITA, JPN, NLD, SWE and USA) over 1995-2015, annual data. The difference between the two samples is that Portugal has data for investment in R&D only while Belgium has data for the capital stock in R&D only.

Results. Before discussing results about the responses of investment in R&D and the stock of knowledge, we first re-estimate the responses of utilization-adjusted TFP for the whole economy, the traded sector and the non-traded sector, respectively. The first row of Fig. 30 shows results for the twelve countries for which the data for investment and the capital stock of R&D are available while the second row shows responses for the eighteen countries of our sample. For both the restricted (i.e., 12 countries) and the whole sample (i.e., 18 countries), we estimate the responses over a limited period of time 1995-2015. Given the small amount of observations, we lose some accuracy in the estimations and thus there is large uncertainty around point estimates as reflected in large confidence bounds. Importantly, even with a limited number of observations, we find that a government spending shock increases significantly utilization-adjusted traded TFP in the short-run while utilization-adjusted non-traded TFP remains unchanged.



(a) Investment in R&D by Traded Industries

(b) Investment in R&D by Non-Traded Industries

(c) Stock of Knowledge in Traded Industries

(d) Stock of Knowledge in Non-Traded Industries

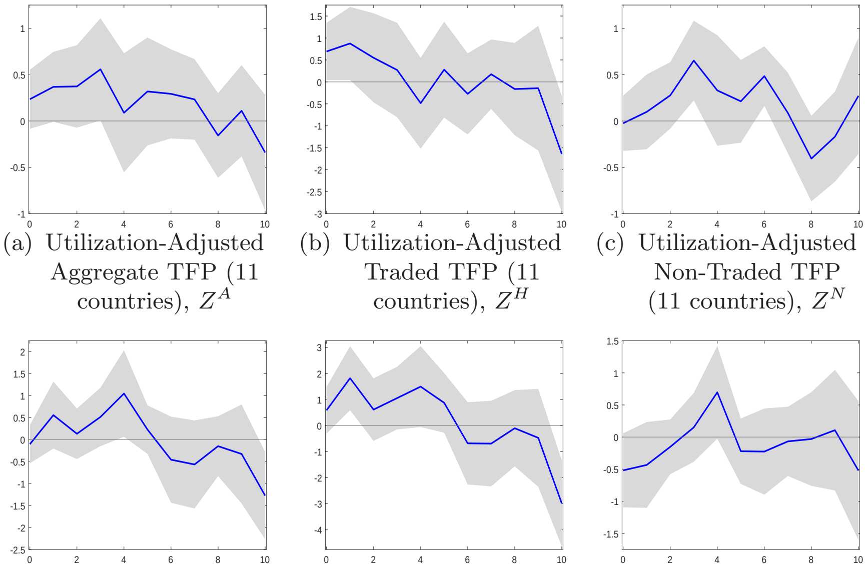
Figure 29: Empirical Responses of Investment in R&D and Stock of Knowledge to a Shock to Government Consumption. Notes: The blue line in the first row of Fig. 29 shows the responses of investment in R&D at constant prices in the traded and the non-traded. The blue line in the second row of Fig. 29 shows the responses of the stock of knowledge at constant prices in the traded and the non-traded. Source: EU KLEMS, Stehrer et al. [2019]. Sample: 12 countries (AUT, DNK, ESP, FIN, FRA, GBR, ITA, JPN, NLD, SWE and USA plus BEL or PRT depending on whether we consider K_RD or I_RD) over 1995-2015, annual data.

The blue line in the first row of Fig. 29 shows the responses of investment in R&D in the traded and the non-traded sector. Evidence reveals that traded and non-traded firms do not increase their investment in R&D following a rise in government consumption. The blue line in the second row of Fig. 29 shows the responses of the stock of knowledge in R&D in the traded and the non-traded sector. We can notice than the response of the stock of knowledge in the traded sector is not significant while it merely increases in the non-traded industries the first two years. Overall, our evidence shows that technology improvements concentrated in traded industries are not driven by a rise in innovation as the stock of knowledge remains unchanged in traded industries and thus instead should reflect a rise in the intensity in the use of the stock of ideas. We show below the variations in the rate of utilization of the stock of ideas are driven by the ability of the firm to increase efficiency to meet higher demand.

Potential determinants of technology adjustment: Cross-country analysis

While we leave a thorough analysis of the determinants of technology improvements following a government spending shock to future research, we take advantage of our panel data dimension to suggest some explanatory factors. Our interpretation of efficiency gains we estimate following a government spending shock is based on the internal organization of firms which is a mediating factor through which demand conditions affect technology adjustment.

Since we interpret the technology utilization rate as the capacity of the firm of making productivity gains when production must be increased to meet higher demand, firms' characteristics related to capital and/or skilled labor intensity and/or R&D intensity should influence this ability to adapt to market changes. More specifically, the evidence documented by the OECD [2016] reveals that more capital intensive sectors like Manufacturing are also more intensive in routine tasks which are repetitive and can be automated. Thus the reorganization of the chain of production for efficiency purposes should be less costly in capital intensive firms/industries. By contrast, abstract tasks are non-routine cognitive tasks involving analytical skills and/or interpersonal skills and often these tasks are combined along a fragmented chain of production, see e.g., Costinot [2009] who formalizes this



(a) Utilization-Adjusted Aggregate TFP (11 countries), Z^A

(b) Utilization-Adjusted Traded TFP (11 countries), Z^H

(c) Utilization-Adjusted Non-Traded TFP (11 countries), Z^N

(d) Utilization-Adjusted Aggregate TFP (18 countries), Z^A

(e) Utilization-Adjusted Traded TFP (18 countries), Z^H

(f) Utilization-Adjusted Non-Traded TFP (18 countries), Z^N

Figure 30: Empirical Responses of utilization-adjusted TFP to a Shock to Government Consumption over 1995-2015. Notes: The solid blue line shows the response of TFP based on the Solow residual adjusted with the capital utilization rate. To generate the response, we have extracted the government spending shock by estimating the VAR model and then estimated dynamic responses to the government spending shock by using local projections. Sample: 11 or 18 OECD countries, 1995-2015, annual data. Note that we consider 11 countries instead of 12 because BEL has data K_RD only while PRT has data only for I_RD.

idea. Industries intensive in high-skilled labor or in R&D tend to produce complex goods (e.g., aircraft; computer, electronic, and optical products; pharmaceuticals) or services (e.g., business services such as economics consultancy) and this fragmented chain of production is less prone to swift adaptation to demand changes due to the inherent complementarity of tasks along the production chain and coordination costs (see e.g., Costinot [2009]).

In line with our assumption, the first row of Fig. 31 reveals that efficiency gains measured by the cumulative change in utilization-adjusted traded TFP divided by the cumulative change in G_t over a six-year horizon is increasing in capital intensity and decreasing in high (and medium) skill intensity among traded industries. For example, the traded sector in France and Sweden is highly intensive in labor and is also highly intensive in high skilled labor and evidence shows that the cost of adjusting technology is too pronounced in traded industries in these two countries. In contrast, in Ireland and Norway, traded industries are capital intensive, or in Italy and Spain where traded industries display a low intensity in high- or medium-skill labor, it is less costly to improve overall efficiency when production must be adjusted to meet higher demand.

Source. Time series about high- (denoted by the superscript S), medium- (denoted by the superscript M), and low-skilled labor (denoted by the superscript U) are taken from EU KLEMS Database, Timmer et al. [2008]. Data are available for all countries except Norway. The baseline period is running from 1970 to 2015 but is different and shorter for several countries as indicated in braces for the corresponding countries: Australia (1982-2005), Austria (1980-2015), Belgium (1980-2015), Canada (1970-2005), Denmark (1980-2015), Finland (1970-2015), France (2008-2015), Ireland (2008-2015), Italy (1970-2015), Japan (1973-2015), the Netherlands (1979-2015), Portugal (2008-2015), Spain (1980-2015), Sweden (2008-2015), the United Kingdom (1970-2015), and the United States (1970-2005). We calculate the share of labor compensation in industry j for skilled labor as the ratio of the sum of labor compensation of high- and medium-skilled labor to total labor compensation in sector j, i.e., $s^j_S = \frac{W^{S,j}S^j + W^{M,j}M^j}{W^j L^j}$. To calculate the intensity of industry j in skilled labor, we multiply the share of labor compensation is skilled labor by the labor income share, i.e., $s^j_S \times s^j_L$, to ensure a consistency with the measure of capital intensity which is expressed as a percentage of value added.

To measure the intensity of the traded and non-traded sectors in the stock of knowledge, we take data from EU KLEMS, Stehrer et al. [2019], see above for data coverage. The second row of Fig. 31 shows that the cumulative change in utilization-adjusted TFP divided by the cumulative change in G_t over a six-year horizon is strongly negatively correlated with the intensity of industries in the stock of knowledge in both the traded and the non-traded sectors. For both sectors, Japan and Spain display lower intensity in the stock of knowledge and have incentives to improve technology. By contrast Sweden for the traded sector and Denmark for the non-traded sector are both highly intensive in the stock of knowledge and display a significant cost of improving technology to meet higher demand.

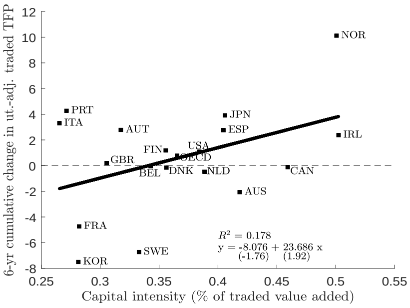
Aside from capital, skill, and knowledge intensity of sectors which influence the ability of the firm to make efficiency gains by reorganizing the organization of the production to meet higher demand, Bloom et al. [2012] and Aghion et al. [2021] show that social capital proxied by trust is a key determinant of efficiency by affecting the organization of firms. The intuition is that higher trust allows the CEO to delegate more decisions, and more decentralized power provides firms with the necessary flexibility needed to respond to changes in demand conditions. By using the World Values Survey [2020] Index which is available for 13 countries of our sample over the period 1981-2015 we find that in countries where the trust index takes higher values, traded firms/industries tend to improve technology to meet higher demand.

Potential determinants of technology adjustment: Split-Sample.

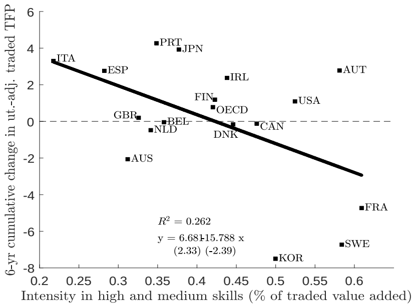
According to the predictions discussed above, we expect traded/non-traded industries to be more prone to improve technology if industries are relatively more intensive in physical capital, and less intensive in skilled labor or in intangible assets. To test this hypothesis, Fig. 32 and Fig. 33 plot technology multipliers defined as cumulative responses of utilization-adjusted aggregate TFP over a 10-year horizon divided by cumulative responses of government consumption following a 1% increase in government consumption. We perform a split-sample analysis on the basis of the median for three dimensions of factor intensity. We consider the intensity of traded and non-traded industries in tangible assets (i.e., physical capital), in skilled labor and in intangible assets. Table 18 details the composition of the sample for each factor intensity index.

Capital and skilled-labor intensity: Split-sample. In Fig. 32, we focus on two dimensions which are captured by the capital income share in traded and non-traded industries, and the skilled labor income share in traded and non-traded industries. The blue line in the first row of Fig. 32 plots the technology multiplier for the first group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively more intensive in physical capital and skilled labor. The black line in the second row of Fig. 32 plots the technology multiplier for the second group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively less intensive in physical capital and in skilled labor than the first group. In line with our predictions, as shown in the first two columns of Fig. 32, the technology multiplier is significantly positive in traded and non-traded industries which are relatively more capital intensive (see the blue line in the first row) while the technology multiplier is not statistically different from zero in relatively less capital intensive industries. The last two columns of Fig. 32 show that the technology multiplier is significantly positive only in industries which are relatively less intensive in skilled labor (see the black line in the second row).

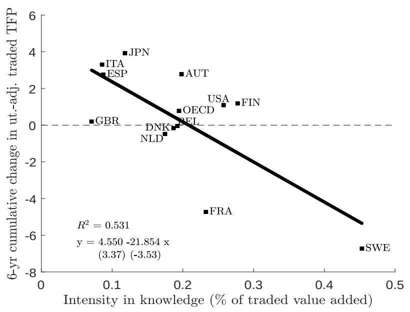
R&D and knowledge intensity: Split-sample. In Fig. 33, we plot technology multipliers for two groups of countries. We perform a simple split-sample analysis based on the median of the sample for the intensity of traded and non-traded industries in R&D (i.e., ratio of R&D investment to sectoral value added) and in intangible assets (i.e., ratio of R&D capital to sectoral value added). The blue line in the first row of Fig. 33 plots the technology multiplier for the first group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively more intensive in R&D and knowledge. The black line in the second row of Fig. 33 plots the technology multiplier for the second group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively less intensive in R&D and knowledge than the first group. In accordance with our hypothesis, the technology multipliers are significantly positive in industries which are relatively less intensive in R&D and/or in intangible assets



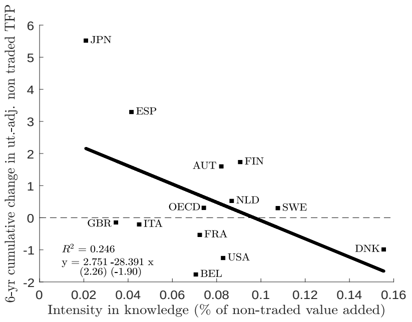
(a) Six-Year-Horizon Technology Multiplier of Tradables over a Six-Year Horizon vs. Capital Intensity



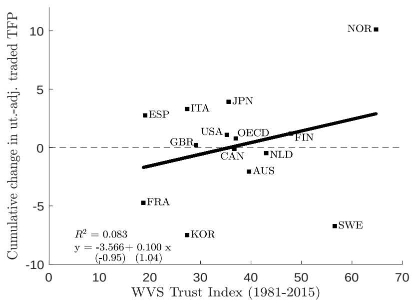
(b) Six-Year-Horizon Technology Multiplier of Tradables vs. High& Medium Workers Skills Intensity



(c) Six-Year-Horizon Technology Multiplier of Tradables vs. R&D Intensity



(d) Six-Year-Horizon Technology Multiplier of Non-Tradables vs. R&D Intensity



(e) Six-Year-Horizon Technology Multiplier of Tradables vs. Trust

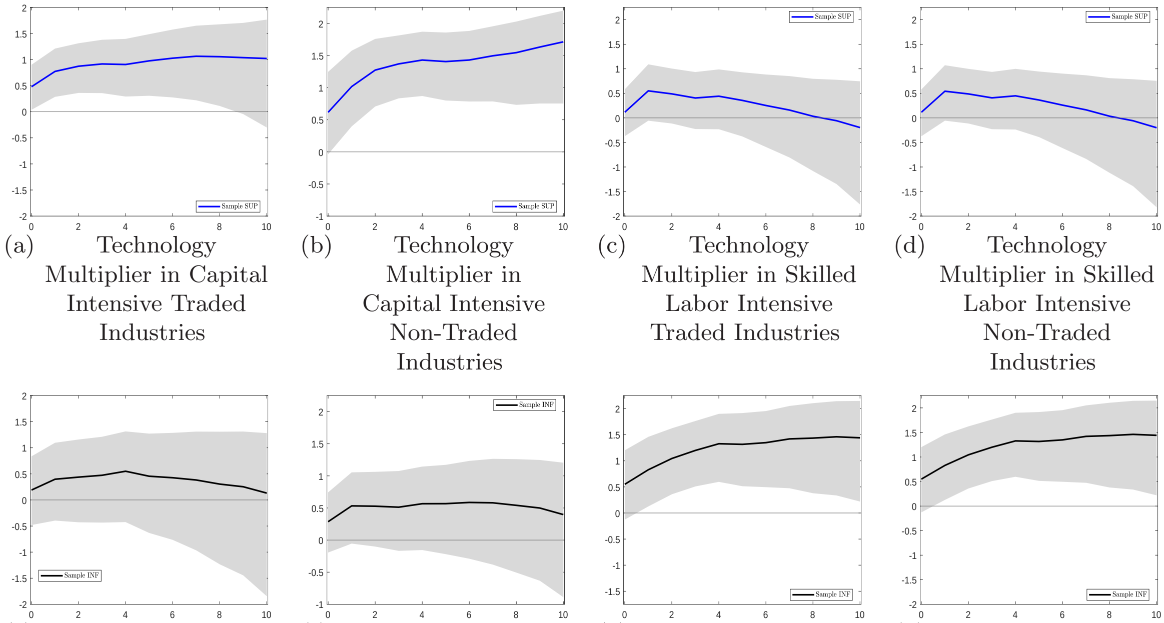
Figure 31: Potential Determinants of Six-Year-Horizon Technology Multiplier of Tradables and Non-Tradables: A Cross-Country Analysis. Notes: Fig. 31(a) and Fig. 31(b) plot six-year-horizon technology multiplier against the intensity of value added in tangible assets (i.e., in physical capital), as captured by the capital income share, and in high and medium worker skills, as captured by the share of high and medium skills in labor compensation times the labor income share (to express the labor cost in % of value added). Fig. 31(b) and Fig. 31(c) plot the six-year-horizon technology multiplier for tradables and non-tradables, respectively, against the intensity of value added in R&D. Fig. 31(e) plots the six-year-horizon technology multiplier for tradables against the measure of trust taken from WVS [2020]. The technology multiplier is calculated as the ratio of the cumulative change in utilization-adjusted TFP for each country over a six-year horizon (i.e., $t = 0...5$) to the cumulative change in government consumption over a six-year horizon for the corresponding country, and both cumulative changes are expressed in present value terms. Source for the stock of knowledge: EU KLEMS, Stehrer et al. [2019]. Sample: 12 countries (AUT, DNK, ESP, FIN, FRA, GBR, ITA, JPN, NLD, SWE and USA) over 1995-2015, annual data.

Capital intensity

$1 - s_L^H = \frac{R^H K^H}{P^H Y^H}$ $\quad$ $1 - s_L^N = \frac{R^N K^N}{P^N Y^N}$

Skill intensity

$s_S^H s_L^H = \frac{W^{S,H} S^H}{P^H Y^H}$ $\quad$ $s_S^N s_L^N = \frac{W^{S,N} S^N}{P^N Y^N}$



(a) Technology Multiplier in Capital Intensive Traded Industries

(b) Technology Multiplier in Capital Intensive Non-Traded Industries

(c) Technology Multiplier in Skilled Labor Intensive Traded Industries

(d) Technology Multiplier in Skilled Labor Intensive Non-Traded Industries

(e) Technology Multiplier in Less Capital Intensive Traded Industries

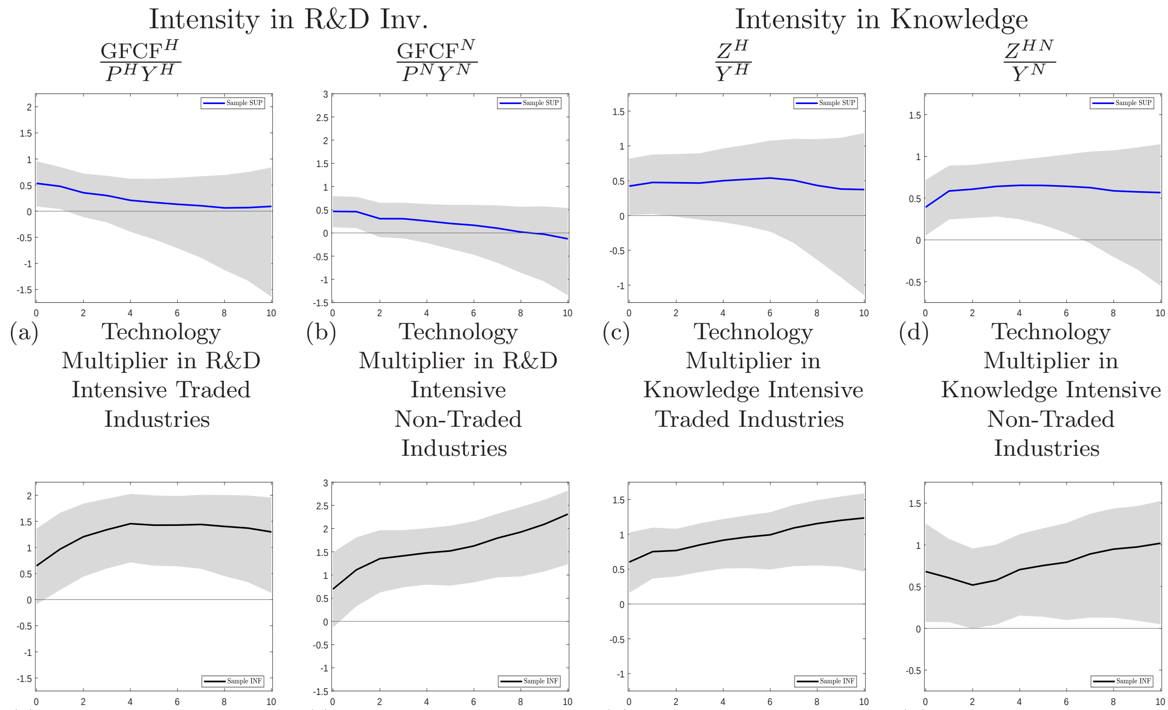
(f) Technology Multiplier in Less Capital Intensive Non-Traded Industries

(g) Technology Multiplier in Less Skilled Labor Intensive Traded Industries

(h) Technology Multiplier in Less Skilled Labor Intensive Non-Traded Industries

Figure 32: Determinants of Technology Multipliers vs. Capital and Skilled Labor Intensity: Split-Sample Analysis. Notes: In Fig. 32, we plot technology multipliers defined as cumulative responses of utilization-adjusted aggregate TFP over a 10-year horizon divided by cumulative responses of government consumption following a 1% increase in government consumption. We perform a simple split-sample analysis based on the median of the sample for three dimensions of factor intensity. We consider the intensity of traded and non-traded industries in tangible assets (i.e., physical capital), in skilled labor and in intangible assets. In Fig. 32, we focus on two dimensions which are captured by the capital income share in traded and non-traded industries, and the skilled labor income share in traded and non-traded industries. The blue line in the first row of Fig. 32 plots the technology multiplier for the first group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively more intensive in physical capital and skilled labor. The black line in the second row of Fig. 32 plots the technology multiplier for the second group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively less intensive in physical capital and skilled labor than the first group. Sample: 18 OECD countries, 1970-2015, annual data.

(obviously both measures are strongly positively correlated). By contrast, as shown in the first row, the group of countries where traded and non-traded industries are relatively more intensive in R&D and/or in intangible assets display a technology multiplier which is not statistically different from zero.



(a) Technology Multiplier in R&D Intensive Traded Industries
(b) Technology Multiplier in R&D Intensive Non-Traded Industries
(c) Technology Multiplier in Knowledge Intensive Traded Industries
(d) Technology Multiplier in Knowledge Intensive Non-Traded Industries
(e) Technology Multiplier in Less R&D Intensive Traded Industries
(f) Technology Multiplier in Less R&D Intensive Non-Traded Industries
(g) Technology Multiplier in Less Knowledge Intensive Traded Industries
(h) Technology Multiplier in Less Knowledge Intensive Non-Traded Industries

Figure 33: Determinants of Technology Multipliers vs. R&D and Knowledge Intensity: Split-Sample Analysis. Notes: In Fig. 33, we plot technology multipliers defined as cumulative responses of utilization-adjusted aggregate TFP over a 10-year horizon divided by cumulative responses of government consumption following a 1% increase in government consumption. We perform a simple split-sample analysis based on the median of the sample for the intensity of traded and non-traded industries in R&D (i.e., ratio of R&D investment to sectoral value added) and in intangible assets (i.e., ratio of R&D capital to sectoral value added). The blue line in the first row of Fig. 33 plots the technology multiplier for the first group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively more intensive in R&D and knowledge. The black line in the second row of Fig. 33 plots the technology multiplier for the second group of countries where the traded (columns 1 and 3) and the non-traded (columns 2 and 4) sectors are relatively less intensive in R&D and knowledge than the first group. Sample: 18 OECD countries, 1970-2015, annual data.

Table 18: Composition of subsamples

Variable	All sample		Subsample SUP			Subsample INF		
	Number	Mean	Number	Mean	Composition	N	Mean	Composition
$1-s_L^H$	18	0.365	9	0.424	AUS CAN DNK ESP IRL JPN NLD NOR USA	9	0.306	AUT BEL FIN FRA GBR ITA KOR PRT SWE
$1-s_L^N$	19	0.311	9	0.348	AUS AUT BEL CAN ESP ITA JPN PRT USA	9	0.273	DNK FIN FRA GBR IRL KOR NLD NOR SWE
$sS^H \times s_L^H$	17	0.420	9	0.509	AUT CAN DNK FIN FRA IRL KOR SWE USA	8	0.320	AUS BEL ESP GBR ITA JPN NLD PRT
$sS^N \times s_L^N$	17	0.500	9	0.593	AUT CAN DNK FIN FRA IRL KOR SWE USA	8	0.396	AUS BEL ESP GBR ITA JPN NLD PRT
$K^H_{R\&D}/Y^H$	12	0.195	6	0.269	AUT BEL FIN FRA SWE USA	6	0.121	DNK ESP GBR ITA JPN NLD
$K^N_{R\&D}/Y^N$	12	0.074	6	0.101	AUT DNK FIN NLD SWE USA	6	0.047	BEL ESP FRA GBR ITA JPN
$I^H_{R\&D}/Y^H$	12	0.045	6	0.064	DNK FIN FRA JPN SWE USA	6	0.025	AUT ESP GBR ITA NLD PRT
$I^N_{R\&D}/Y^N$	12	0.016	6	0.020	DNK FIN GBR NLD SWE USA	6	0.013	AUT ESP FRA ITA JPN PRT
Trust Index	13	36.99	6	48.10	AUS CAN FIN NLD NOR SWE	7	27.46	ESP FRA GBR ITA JPN KOR USA

R Sector-Specific Government Spending Shocks

In the main text we explore the aggregate and sectoral effects of a shock to aggregate government consumption which is spent on non-traded goods, and also on both home- and foreign-produced traded goods. Because our study investigates the distribution across sectors of real GDP and labor after a rise in government consumption, our analysis has the advantage to be comparable with other existing papers which identify an aggregate government spending shock. In this regard, we adopt the same approach as Bouakez et al. [2022a] who estimate the effects of an aggregate change in government consumption on sectoral value added growth.

Recently, several papers have focused on sectoral government consumption shocks, e.g., Cox et al. [2020] and Bouakez et al. [2022b]. By using a calibration based on granular U.S. data, Cox et al. [2020] and Bouakez et al. [2022b] investigate numerically the conditions and thus the sectors that will produce the maximum aggregate output effects. Differently, Nekarda and Ramey [2011] estimate empirically the sectoral output multiplier, i.e., the rise in sectoral output caused by a rise in public demand for its produced good. Our analysis differs in two respects. First, we are interested into the actual distribution of real GDP growth, labor growth and aggregate TFP growth across sectors following a government spending shock and we rationalize the cross-sector allocation we observe on the basis of the intensity of sectors in government consumption, barriers to factors' mobility (capital adjustment costs, labor mobility costs and imperfect substitutability between home- and foreign-produced traded goods), and technology factors (capital and technology utilization adjustment costs, and sectoral factor-biased technological change). Second, in line with our main objective, we identify a shock to aggregate government consumption and estimates empirically the responses of sectoral value added, labor and technology to the aggregate shock.

We summarize below our main findings:

- Our evidence reveals that: i) government spending components move together and thus are not independent from each other and should not be studied separately if we want to rationalize the size of the government spending multiplier in the data, ii) the sum of the effects of a shock to sectoral components of government consumption collapses to the effects of a shock to aggregate government consumption.

- Our numerical results reveal that a shock to non-traded government consumption maximizes output and labor effects when we shutdown the technology channel while a shock to traded government consumption maximizes output and labor effects when we let technology respond to the government spending shock; when we normalize sectoral government shocks by their share in total government consumption expenditure, we find that the sum of their effects collapses to the effects driven by a shock to aggregate government consumption.

We also clarify two important aspects of the investigation of sectoral demand shocks:

- The investigation of the effects of shocks to sectoral government spending theoretically is relevant if the objective is to analyze which sectoral government purchases maximize output and labor effects. However, this analysis is misleading if we seek to quantify the size of both sectoral and aggregate government spending multipliers because it would ignore the impact of the effects of the reallocation of productive resources across sectors and also the general equilibrium effects caused by the variations of the government purchases from other sectors.

- We also show in section R.4 that the analysis conducted by Nekarda and Ramey [2011] who estimate empirically the effects of changes in sectoral government purchases on output and labor of the corresponding sector, amounts to adopting the Blanchard and Perotti [2002] identification and estimating the fiscal multiplier on impact.

R.1 Sector-Specific Biasedness of Government Spending Shocks: Definitions and Condition

Because government spending on imported goods represents an insignificant share of total government consumption, we aggregate government consumption on both home- and foreign-produced traded goods so that aggregate government consumption spent on non-traded and traded goods:

$$G(t) = P^N G^N(t) + P^T G^T(t). \tag{172}$$

Denoting by ω_{G^j} the share of good j in government consumption, we have $G(t) = \omega_{G^N} G(t) + \omega_{G^T} G(t)$; differentiating while assuming that ω_{G^j} is constant over time in line with our evidence, a rise in government spending is split into non-tradables and tradables in accordance with their respective shares:

$$dG(t) = \omega_{G^N} dG(t) + \omega_{G^T} dG(t). \tag{173}$$

Based on our estimates documented later, we do not make the distinction between the share of good j in government spending and the intensity of the government spending shock in this good j, since the latter collapses to the former.

Our objective is to determine under which condition the government spending shock is biased toward one sector-specific good. For this purpose, we use the equality between value added and its final use, i.e., $Y^j = E^j + G^j$ where G^j stands for government purchases of good j and E^j is private demand. Differentiating $Y^j = E^j + G^j$ while keeping private demand fixed and making use of (173), we find that the fiscal multiplier of sector j is increasing in the intensity of this sector in the government spending shock, i.e., $\nu^{Y,j}\hat{Y}^j(t) = \omega_{G^j}\left(\frac{dG(t)}{Y}\right)$.

Keeping aggregate private demand fixed, we have $\hat{Y}_R(t) = \frac{dG(t)}{Y}$ so that the sectoral multiplier collapses to $\nu^{Y,j}\hat{Y}^j(t) = \nu^{Y,j}\frac{dG(t)}{Y}$. As mentioned in the main text, the value added share of sector j at constant prices is a sufficient statistic to determine whether a demand shock is uniformly distributed across sectors, i.e., in accordance with their share in GDP. To see this, the change in the value added share of sector j is given by:

$$\nu^{Y,j}\hat{Y}^j(t) = \nu^{Y,j}\hat{Y}_R(t) + d\nu^{Y,j}(t). \tag{174}$$

When each sector receives a share of the demand shock which is proportional to its share in GDP, the value added share remains unchanged, i.e., $d\nu^{Y,j}(t) = 0$, so that real GDP growth is uniformly distributed across sectors, i.e., $\nu^{Y,j}\hat{Y}^j(t) = \nu^{Y,j}\hat{Y}_R(t) = \nu^{Y,j}\frac{dG(t)}{Y}$.

Subtracting the change in value added in sector $j = H, N$ when the value added share remains unchanged, i.e., $\nu^{Y,j}\hat{Y}^j(t) = \nu^{Y,j}\frac{dG(t)}{Y}$, from the change in value added in sector $j = H, N$ driven by the actual intensity of sector j in the government spending shock, i.e., $\nu^{Y,j}\hat{Y}^j(t) = \omega_{G^j}\left(\frac{dG(t)}{Y}\right)$, leads to a measure of the sector-biasedness of the demand shock:

$$d\nu^{Y,j}(t) = \left(\omega_{G^j} - \nu^{Y,j}\right)\left(\frac{dG(t)}{Y}\right), \tag{175}$$

where we have used the fact that $d\nu^{Y,j}(t) = \nu^{Y,j}\left(\hat{Y}^j(t) - \hat{Y}_R(t)\right) = \omega_{G^j}\left(\frac{dG(t)}{Y}\right) - \nu^{Y,j}\frac{dG(t)}{Y}$. According to (175), a government spending shock is said to be biased toward sector $j = H, N$ if the intensity of sector j in the government spending shock is larger than the share of this sector in GDP, i.e., if $\omega_{G^j} > \nu^{Y,j}$.

R.2 Shocks to G^N and G^T: Evidence

Two sources are available to construct time series for sectoral government consumption:

- The World Input-Output Databases ([2013], [2016]) provide time series in domestic currency units for each demand component by sector. To determine the intensity of each sector in government spending, we had to treat imports in a consistent way non-tradable expenditure includes some imports (there is a low proportion but a share of both non-traded consumption and non-traded investment expenditure is imported in the data) although in the model we abstract from these imports. In carrying out this analysis, we inferred sectoral government consumption by using market clearing conditions and private demand components, see Section F. The disadvantage of using this dataset is that the sum of G^T plus G^N might be different from government final consumption expenditure time series provided by the OECD.

Data coverage: WIOD provides time series for all countries in our sample over 1995-2014 except for Norway for which time series are available over 2000-2014. We find that on average, 80% of government spending is spent on non-traded goods and 20% is spent on traded goods.

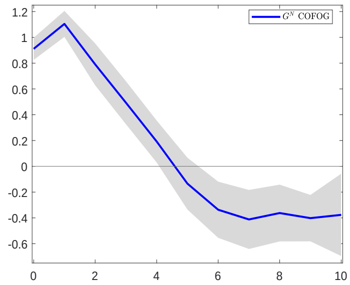
- One alternative to WIOD is the COFOG [2017] database from the OECD which provides a breakdown of government expenditure by function. The disadvantage of this dataset is that it does not provide information about the sales of goods or services of an industry to the government. It just gives the purpose of the government service and thus provides information about the industry which benefits government spending. For example, when the government purchases goods and services to provide education services, it increases the value added of the industry 'education' by the same amount than government services. Since this industry is non-tradables, the counterpart is government consumption on non-tradables. See Cardi, Restout and Claeys [2020] for more details. The advantage of this database over WIOD is that the sum of G^T and G^N is very close to government final consumption expenditure time series provided by the OECD that we use to identify shocks to government consumption in section 2 of the main text.

  Data coverage: COFOG allows us to construct time series (in domestic currency units) over 1995-2015 for 16 OECD countries as data are not available for Canada and they are too short for Japan. We find that on average, 91% of government consumption is spent on non-traded goods and 9% is spent on traded goods. The cross-country correlation between ω^{G^N} from COFOG and the intensity of the non-traded sector in government consumption from WIOD stands at 0.36. Note that with COFOG, international differences in ω_{G^N} are small.

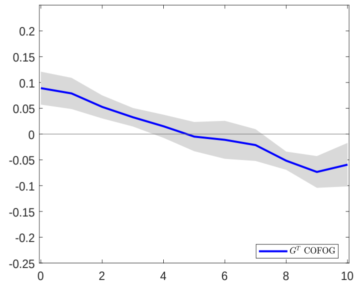
Because WIOD provides data for demand components, it is more accurate than COFOG which gives information about the purpose of the government transaction only. By contrast, time series from WIOD display some significant variations over time while time series from COFOG are not characterized by abrupt changes. Therefore we use WIOD data to calculate the intensity of a sector in government spending and calibrate our model while we use COFOG dataset to run a time series analysis. In both cases, we find that time series are very close to the time series from OECD for government final consumption expenditure.

A Shock to government consumption increases both traded and non-traded government spending. In Fig. 34, we plot the dynamic responses of the traded and non-traded components of government consumption to an exogenous increase in government consumption by 1% of GDP. The first major result is that a government spending shock lead to a significant increase in both components and each component increases by the same amount as its share in government consumption, i.e., at 0.91 for G^N and 0.09 for G^T since we use COFOG for the time series analysis. Note that the low persistence in the rise in government spending components comes from the limited time horizon, i.e., 1995-2015 instead of 1970-2015.

Sectoral components of government spending co-move. We now move a step further by identifying a shock to sectoral government consumption and by investigating if the second component reacts to the identified shock. We estimate a VAR in the first step which includes non-traded (traded) government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP. In the second step we use local projections to estimate the dynamic response of G^T (G^N) to the sectoral demand shock. Fig. 35(a) shows that the response of G^T is not significant and thereby a shock to non-traded government consumption does not cause any changes in other spending components. By contrast, as can be seen in Fig. 35(b), a shock to G^T leads to an increase in G^N. The latter

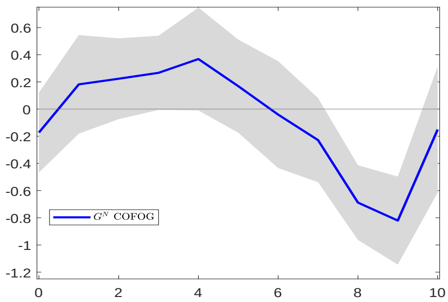


(a) Response of G^N to a Shock to G

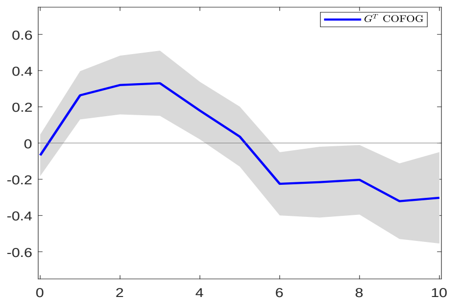


(b) Response of G^T to a Shock to G)

Figure 34: Effects of Shock To Government Consumption on Traded and Non-Traded Government Spending Components<u>Notes</u>: The solid blue line shows the response of non-traded and traded components of government consumption to an exogenous increase in government final consumption expenditure by 1% of GDP. We adopt a two-step method where we identify the shock by adopting Blanchard-Perotti identification assumption and next estimate dynamic effects by using Local Projections. Source: COFOG [2017] database from the OECD Sample: 16 OECD countries, 1995-2015, annual data.



(a) Response of G^T to a Shock to G^N



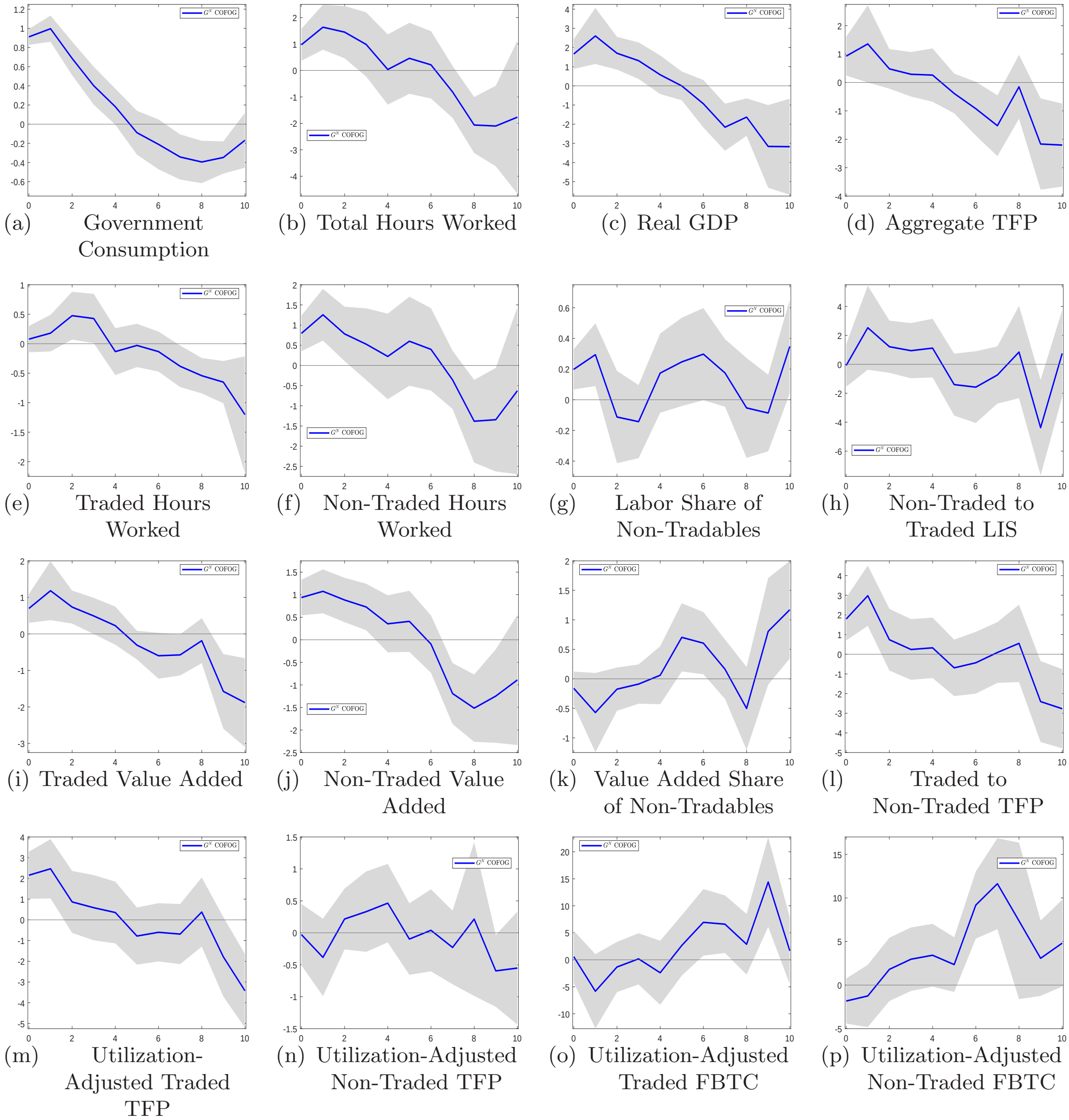
(b) Response of G^N to a Shock to G^T)

Figure 35: Effects of a Shock To Non-Traded (Traded) Government Consumption on Traded (Non-Traded) Government Spending Components<u>Notes</u>: The solid blue line shows the response of traded (non-traded) components of government consumption to an exogenous increase in non-traded (traded) government consumption. We adopt a two-step method where we identify the shock by adopting Blanchard-Perotti identification assumption and next estimate dynamic effects by using Local Projections. We estimate a VAR in the first step which includes non-traded (traded) government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP. Source: COFOG [2017] database from the OECD Sample: 16 OECD countries, 1995-2015, annual data.

finding suggests that government spending components co-move and should be considered together instead of separately.

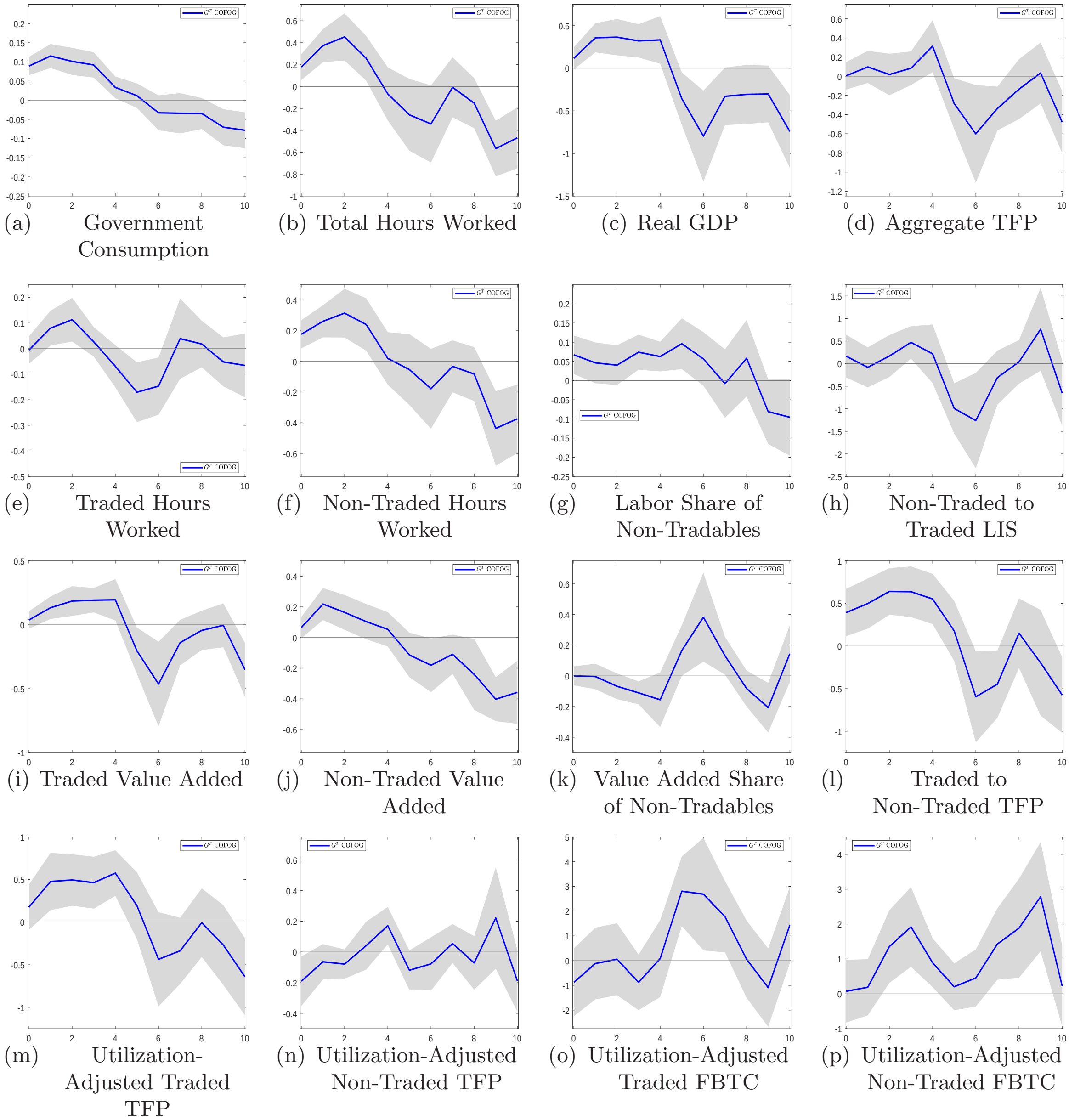
The sum of effects of sectoral government spending shocks collapses to the effects of an aggregate government consumption shock. In Fig. 36 and Fig. 37, we investigate the dynamic effects of a shock to the non-traded and traded components of government consumption. We adopt the two-step method where we estimate a VAR model which includes government consumption on the non-traded/traded good, G^j with $j = N, T$, real GDP, total hours worked, the real consumption wage, and aggregate TFP.

The blue line shows dynamic responses to a shock to G^N and a shock to G^T, respectively. Before discussing the results, it is worth mentioning that the time horizon is limited, i.e., 1995-2015 instead of 1970-2015 in the main text. Inspection of Fig. 36 reveals that a shock to G^N produces similar effects to those estimated in the main text. Interestingly, while after a shock to G^N, government consumption is fully spent on non-traded goods, the value added share of non-tradables remains unresponsive to a shock to G^N. The reason is that technology improvement is concentrated in traded industries. As can be seen in Fig. 37, a shock to G^T also produces similar effects to those estimated after a rise in G. This finding is more surprising as an increase in government consumption which is fully spent on traded goods should reallocate labor toward traded industries and leads to a fall in the labor share of non-tradables. The explanation is that as shown in Fig. 35(b), a shock to G^T leads to a rise in G^N; therefore the two components are complementary and a shock to G^T is not orthogonal to non-traded government consumption.



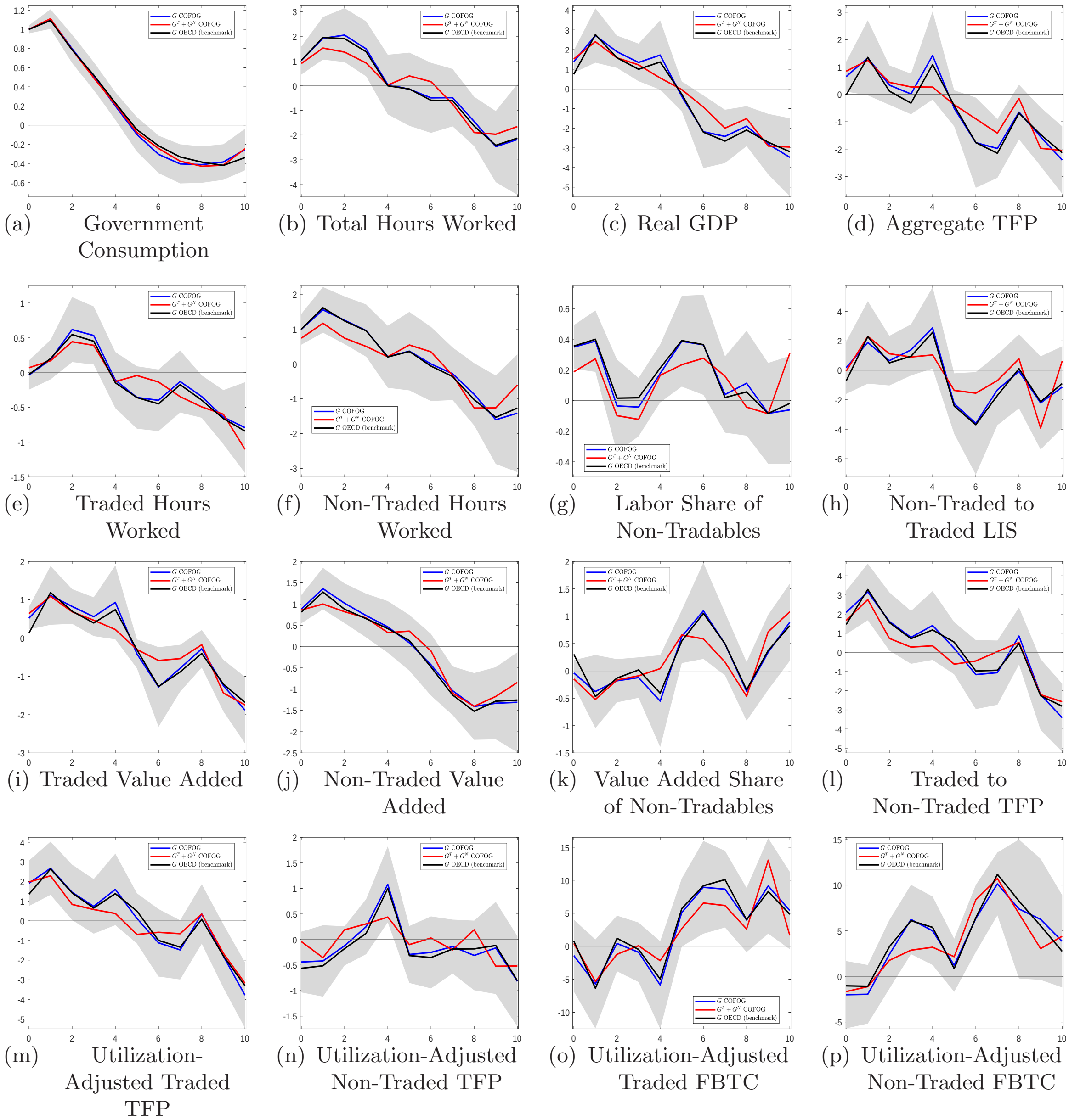
(a) Government Consumption
(b) Total Hours Worked
(c) Real GDP
(d) Aggregate TFP
(e) Traded Hours Worked
(f) Non-Traded Hours Worked
(g) Labor Share of Non-Tradables
(h) Non-Traded to Traded LIS
(i) Traded Value Added
(j) Non-Traded Value Added
(k) Value Added Share of Non-Tradables
(l) Traded to Non-Traded TFP
(m) Utilization-Adjusted Traded TFP
(n) Utilization-Adjusted Non-Traded TFP
(o) Utilization-Adjusted Traded FBTC
(p) Utilization-Adjusted Non-Traded FBTC

Figure 36: Effects of a Shock to Non-Traded Government Consumption G^N. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in the non-traded component of government final consumption expenditure by 0.91% of GDP when we adopt a two-step method where we identify the shock by adopting Blanchard-Perotti identification assumption and next estimate dynamic effects by using Local Projections. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Source: COFOG. Sample: 16 OECD countries, 1995-2015 (except Australia, 1998-2015), annual data.



(a) Government Consumption
(b) Total Hours Worked
(c) Real GDP
(d) Aggregate TFP
(e) Traded Hours Worked
(f) Non-Traded Hours Worked
(g) Labor Share of Non-Tradables
(h) Non-Traded to Traded LIS
(i) Traded Value Added
(j) Non-Traded Value Added
(k) Value Added Share of Non-Tradables
(l) Traded to Non-Traded TFP
(m) Utilization-Adjusted Traded TFP
(n) Utilization-Adjusted Non-Traded TFP
(o) Utilization-Adjusted Traded FBTC
(p) Utilization-Adjusted Non-Traded FBTC

Figure 37: Effects of a Shock to Traded Government Consumption G^T. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in the traded component of government final consumption expenditure by 0.09% of GDP when we adopt a two-step method where we identify the shock by adopting Blanchard-Perotti identification assumption and next estimate dynamic effects by using Local Projections. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Source: COFOG. Sample: 16 OECD countries, 1995-2015 (except Australia, 1998-2015), annual data.



(a) Government Consumption (b) Total Hours Worked (c) Real GDP (d) Aggregate TFP

(e) Traded Hours Worked (f) Non-Traded Hours Worked (g) Labor Share of Non-Tradables (h) Non-Traded to Traded LIS

(i) Traded Value Added (j) Non-Traded Value Added (k) Value Added Share of Non-Tradables (l) Traded to Non-Traded TFP

(m) Utilization-Adjusted Traded TFP (n) Utilization-Adjusted Non-Traded TFP (o) Utilization-Adjusted Traded FBTC (p) Utilization-Adjusted Non-Traded FBTC

Figure 38: Effects of a Shock to G vs. Sum of Effects of Shocks to G^T and G^N. Notes: The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in aggregate government final consumption expenditure taken from COFOG [2017] database. The solid black line shows the response of aggregate and sectoral variables to an exogenous increase in aggregate government final consumption expenditure taken from OECD [2017] database. The solid red line shows the sum of the effects of a shock to G^N and a shock to G^T taken from COFOG [2017] database. In each case, we adopt a two-step method where we identify the shock by adopting the Blanchard-Perotti identification assumption and next estimate dynamic effects by using Local Projections. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBTC). Source: COFOG. Sample: 16 OECD countries, 1995-2015 (except Australia, 1998-2015), annual data.

In Fig. 38, we compare the dynamic effects of a shock to government consumption shown in the blue line with the sum of the effects of a shock to G^N and a shock to G^T shown in the red line. Because time series for government consumption are constructed by summing the traded and non-traded components, i.e., G^T and G^N, taken from COFOG database, we also compare the results displayed by the blue line with the results shown in the black line where we use the time series for government consumption provided by the OECD. The conclusion that emerges is that a shock to aggregate government consumption produces the same effects as those driven by the sum of the effects of shocks to its components.

R.3 Shocks to G^N and G^T: Theoretical Predictions

While in the main text, we quantify the aggregate and sectoral effects of a rise in government consumption which is spent on both non-traded and traded goods, we estimate numerically the aggregate and sectoral effects of a shock to one component of government consumption and shut down the second component.

In Table 19, we consider an increase in government consumption which is either fully spent on non-traded goods, i.e., $dG(t) = dG^N(t)$, see columns 1-2 and 4-5, or fully spent on traded goods, i.e., $dG(t) = dG^T(t)$, see columns 3-4 and 7-8. Columns 1-4 shows impact responses while columns 5-8 display cumulative responses. Panel A shows the responses of total hours worked and real GDP along with the change in government consumption. When we shut down technology, we find that a shock to government consumption which is fully spent on non-traded goods produces larger labor and real GDP effects than if government consumption were fully spent on traded goods. Intuitively, a government spending shock biased toward non-traded goods appreciates the relative price of non-traded goods $P(t)$ (see the fist row of panel D) and encourages non-traded firms to pay higher wages to encourages workers (who experience mobility costs) to shift away from the traded sector (see the last row of panel B). Because the non-tradable content of overall labor compensation is two-third, the aggregate wage increases which amplifies the rise in labor supply. When the shock is fully biased toward traded goods, labor shifts toward the traded sector. Because the tradable content of labor compensation is one-third, the aggregate wage index increases less than if G were fully spent on G^N, thus resulting in lower cumulative effects.

In Table 19 we consider a rise in either G^N or G^T so that $G(t)$ increases by 1% of GDP. In Table 20 we also consider a shock to G^j keeping unchanged the other component of government consumption but each sectoral demand shock is now assumed to increase $G(t)$ in the same proportion as the share of G^j in G which stands at 0.80% for G^N and 0.20% for G^T. The normalization of the rise in each component of $G(t)$ by its share in government spending implies that summing the effects of sectoral demand shocks generates the same effects as those caused by the aggregate demand shock where $G(t)$ increases by 1% of GDP, i.e., $\frac{dG(t)}{Y} = \omega_{G^N} \frac{dG(t)}{Y} + (1 - \omega_{G^N}) \frac{dG(t)}{Y}$.[31]

Once we allow for the technology channel, as displayed by columns 5 and 8 for cumulative effects of Table 20 which disentangles the aggregate demand shock into two sectoral demand shocks, we find that a shock to G^T produces larger labor and real GDP multipliers that those following a shock to G^N because traded industries display a low cost of adjustment for technology.

Taking stock of the effects of shocks to sector-specific components of $G(t)$. Two major conclusions emerge from decomposing the aggregate demand shock into sectoral demand shocks. In columns 6 and 8 of Table 20, we quantify the cumulative effects of a shock to G^N and a shock to G^T, respectively, by shutting down the technology channel. Results shown in panel A reveals that both labor ($2.76/4.48 = 0.62$) and real GDP multipliers ($1.77/4.48 = 0.40$) after a shock to G^N are larger than those after a shock to G^T. Intuitively, a government spending shock spent on non-traded goods leads non-traded firms to pay

[31] Summing the response of $L(t)$ and $Y_R(t)$ (second and third row of panel A of Table 20) shown in columns 1 and 3 implies that total hours worked and real GDP increases by 0.97% and 0.76%. The aggregate effects are somewhat different from those shown in Table 3 because our assumption $\beta = r^\star$ results in the joint determination of the steady-state (which is non-linear) and the (linearized) dynamics. Non-linearities imply that the sum of two smaller sectoral demand shocks does not add up to the aggregate demand shock, especially when we allow for endogenous technological change.

Table 19: Impact and Cumulative Effects of an Increase in Government Consumption by 1% of GDP in the Baseline Model: Shock to G^N (keeping G^T fixed) vs. Shock to G^T (keeping G^N fixed)

	Impact Responses				Cumulative Responses			
	G^N Shock		G^T Shock		G^N Shock		G^T Shock	
	Tech	No Tech	Tech	No Tech	Tech	No Tech	Tech	No Tech
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A.Aggregate Multipliers								
Gov. spending, $dG(t)$	1.00	1.00	1.00	1.00	5.46	5.46	5.46	5.46
Total hours worked, $dL(t)$	0.91	0.64	1.19	0.46	5.32	3.37	6.78	2.42
Real GDP, $d\tilde{Y}_R(t)$	0.86	0.43	1.93	0.30	6.19	2.15	12.07	1.61
B.Sectoral Labor								
Traded labor, $dL^H(t)$	-0.01	-0.02	0.93	0.38	-0.34	-0.13	4.82	2.09
Non-traded labor, $dL^N(t)$	0.91	0.66	0.27	0.07	5.65	3.50	1.96	0.32
Labor share of non-tradables, $d\nu^{L,N}(t)$	0.32	0.24	-0.51	-0.22	2.19	1.30	-2.45	-1.25
C.Sectoral Value Added								
Traded VA, $d\tilde{Y}^H(t)$	-0.22	-0.16	1.88	0.41	0.80	-0.91	12.43	2.25
Non-traded VA, $d\tilde{Y}^N(t)$	1.08	0.59	0.06	-0.11	5.39	3.07	-0.36	-0.65
Non-traded VA share, $d\tilde{\nu}^{Y,N}(t)$	0.54	0.32	-1.16	-0.30	1.48	1.71	-7.98	-1.66
D.Relative Prices								
Relative Price of Non-Tradables, $dP(t)$	0.60	0.79	-0.09	-0.81	5.58	4.24	1.73	-4.52
Terms of trade, $dP^H(t)$	0.23	0.33	0.32	1.04	-0.32	1.79	-0.04	5.58
E.Technology								
Traded technology utilization, $du^{Z,H}(t)$	-0.30	0.00	2.61	0.00	1.13	0.00	17.27	0.00
Non-Traded technology utilization, $du^{Z,N}(t)$	0.50	0.00	0.03	0.00	2.51	0.00	-0.17	0.00
F.Redistributive effects								
Traded LIS, $ds_L^H(t)$	0.11	-0.14	0.14	0.01	-3.13	-0.76	-2.89	0.04
Non-traded LIS, $ds_L^N(t)$	0.26	-0.01	0.23	-0.05	3.01	-0.09	2.88	-0.26

Notes: Columns 1-4 show impact effects of a temporary increase in government consumption and columns 5-8 show the present discounted value cumulative effects. In all scenarios, we simulate the baseline model with capital adjustment costs, imperfect mobility of labor, endogenous terms of trade and CES production functions. In columns 1-2 and 5-6, we assume that the rise in government consumption is fully spent on non-traded goods, i.e., $dG(t) = dG^N(t)$, while in columns 3-4 and 7-8, we assume that the rise in government consumption is fully spent on traded goods, i.e., $dG(t) = dG^T(t)$. Note that government consumption in traded goods includes both home- and foreign-produced traded goods, i.e., $G^T = P^H G^H + G^F$. In odd columns, we allow for endogenous capital and technology utilization rate together with factor-biased technological change while in even columns, we shut-down technology.

Table 20: Normalized Shocks to G^N (keeping G^T fixed) vs. G^T (keeping G^N fixed)

	Impact Responses				Cumulative Responses			
	G^N Shock		G^T Shock		G^N Shock		G^T Shock	
	Tech	No Tech	Tech	No Tech	Tech	No Tech	Tech	No Tech
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Non-traded gov., $dG^N(t)$	0.80	0.80	0.00	0.00	4.48	4.48	0.00	0.00
Traded gov., $dG^T(t)$	0.00	0.00	0.20	0.20	0.00	0.00	1.14	1.14
A.Aggregate Multipliers								
Gov. spending, $dG(t)$	0.80	0.80	0.20	0.20	4.48	4.48	1.14	1.14
Total hours worked, $dL(t)$	0.73	0.51	0.24	0.09	4.44	2.76	1.76	0.50
Real GDP, $d\tilde{Y}_R(t)$	0.63	0.34	0.13	0.06	5.07	1.77	2.50	0.34
B.Sectoral Labor								
Traded labor, $dL^H(t)$	-0.00	-0.01	0.20	0.08	-0.31	-0.10	0.87	0.44
Non-traded labor, $dL^N(t)$	0.73	0.53	0.04	0.02	4.75	2.86	0.89	0.07
Labor share of non-tradables, $d\nu^{L,N}(t)$	0.25	0.19	-0.12	-0.05	1.86	1.04	-0.26	-0.27
C.Sectoral Value Added								
Traded VA, $d\tilde{Y}^H(t)$	-0.19	-0.13	0.34	0.08	0.99	-0.73	4.05	0.47
Non-traded VA, $d\tilde{Y}^N(t)$	0.81	0.47	-0.20	-0.02	4.08	2.50	-1.55	-0.14
Non-traded VA share, $d\tilde{\nu}^{Y,N}(t)$	0.42	0.25	-0.29	-0.06	0.88	1.36	-3.13	-0.35
D.Relative Prices								
Relative Price of Non-Tradables, $dP(t)$	0.48	0.62	-0.02	-0.17	4.99	3.42	2.19	-0.97
Terms of trade, $dP^H(t)$	0.20	0.26	0.11	0.21	-0.51	1.47	-1.08	1.19
E.Technology								
Traded technology utilization, $du^{Z,H}(t)$	-0.26	0.00	0.47	0.00	1.40	0.00	5.67	0.00
Non-Traded technology utilization, $du^{Z,N}(t)$	0.38	0.00	-0.09	0.00	1.90	0.00	-0.72	0.00
F.Redistributive effects								
Traded LIS, $ds_L^H(t)$	0.12	-0.11	0.16	0.00	-3.07	-0.62	-2.84	0.01
Non-traded LIS, $ds_L^N(t)$	0.26	-0.01	0.24	-0.01	3.00	-0.08	2.94	-0.05

Notes: Columns 1-4 show impact effects of a temporary increase in government consumption and columns 5-8 show the present discounted value cumulative effects. In all scenarios, we simulate the baseline model with capital adjustment costs, imperfect mobility of labor, endogenous terms of trade and CES production functions. In columns 1-2 and 5-6, we assume that the rise in government consumption is fully spent on non-traded goods, i.e., $dG(t) = dG^N(t)$, while in columns 3-4 and 7-8, we assume that the rise in government consumption is fully spent on traded goods, i.e., $dG(t) = dG^T(t)$. In contrast to Table 19, we scale the rise in sectoral government consumption by their share in government consumption, at 0.8 for G^N and 0.2 for G^T. This scaling implies that the sum of the effects of a rise in G^N and G^T should collapse to the effects of a rise in government consumption G. However, because we re-scale the shocks which vary in size, the sum of the effects of shocks to sectoral government consumption is not exactly equal to the effects of aggregate government spending due to non-linearities in computing the stead-state changes because our model with $\beta = r^\star$ implies that the steady-state is jointly determined with the dynamics. In odd columns, we allow for endogenous capital and technology utilization rate together with factor-biased technological change while in even columns, we shut-down technology.

higher wages to encourage workers to shift (see panel B) as they experience switching costs. Labor mobility costs result in upward pressure on aggregate wages which leads agents to supply more labor. This effect is considerably smaller after a shock to G^T as the traded sector accounts for one-third only of overall labor compensation. This conclusion is reversed when we allow for the technology channel. Results displayed by columns 5 and 7 reveal that both labor ($1.76/1.14 = 1.54$) and real GDP multipliers ($2.50/1.14 = 2.19$) after a shock to G^T are larger than those after a shock to G^N. Intuitively, to meet higher demand, the traded sector finds it optimal to improve technology so as to curb the upward pressure on production costs. Because the technology adjustment cost is smaller in the traded than in the non-traded sector, the rise in traded TFP increases real GDP growth directly and also indirectly by increasing wages and thus encouraging workers to supply more labor.

R.4 Shedding some Light on the Empirical Strategy by Nekarda and Ramey [2011]

Nekarda and Ramey [2011] aim at estimating the effects of a change in government purchases in one industry on output and labor of this industry. The objective of this subsection is twofold. First, we show that the empirical strategy amounts to adopting Blanchard-Perotti identification although by including sectoral output or sectoral employment instead of real GDP or total hours worked. Second, we show that the assumptions set in their empirical strategy collapse to the assumptions we set in our own setting (both in the empirical part and in calibrating the model).

Pre-requisites. The government purchases goods and services to all industries of the economy. We consider two broad sectoral goods: traded (which includes both home- and foreign-produced traded goods), G^T, and non-traded goods, G^N. Denoting base year prices in sector j by dropping the time index, i.e., P^j, government final consumption expenditure G_t is equal to the sum of purchases of traded and non-traded goods:

$$G_t = P^T G_t^T + P^N G_t^N. \tag{176}$$

Differentiating the above equation leads to:

$$dG_t = P^T dG_t^T + P^N dG_t^N. \tag{177}$$

We assume that the share of government purchases in good j in total government purchases, $\omega_{G^j} = \frac{P^j G^j}{G}$, is constant over time so that $G_t = \sum_j \omega_{G^j} G_t$ where $\sum_j \omega_{G^j} = 1$. Differentiating $G_t = \sum_j \omega_{G^j} G_t$ leads to:

$$dG_t = \omega_{G^T} dG_t + \omega_{G^N} dG_t. \tag{178}$$

Combining (177) with (179) implies that the change in government purchases in good j is a function of total government purchases as in Nekarda and Ramey [2011] (see equation 8, page 43)

$$P^j \frac{dG_t^j}{Y} = \omega_{G^j} \frac{dG_t}{Y}, \tag{179}$$

where Y is GDP while Nekarda and Ramey [2011] divide both sides by sectoral output. As stressed by the authors, by considering a fixed share ω_{G^j} instead of a time-varying share, they escape from a potential endogeneity issue between the change in sectoral government purchases $P^j \frac{dG_t^j}{Y}$ and the change in sectoral output or sectoral hours. To see it, we use the market clearing condition for good j which says that value added has a private and public final demand components, i.e., $Y_t^j = E_t^j + G_t^j$. Differentiate the market clearing condition by keeping the private sector demand component leads to $dY_t^j = dG_t^j$. Multiplying both sides by P^j and dividing by GDP shows that the cumulative change of sectoral value added in ppt of GDP, i.e., $\nu^{Y,j} \hat{Y}_t^j = \frac{dY_t^j}{Y}$, is equal to the change in government purchases of good j in ppt of GDP when we keep private sector demand components fixed.

Nekarda and Ramey [2011] empirical strategy is equivalent to Blanchard and Perotti identification. The authors regress the rate of change of the variable of interest in sector j (or the change in the logarithm of the variable of interest) on the change in sectoral government purchases in percentage of sales of sector j. For clarification purposes, it is helpful to revisit the empirical strategy of the authors by adopting the Jordà [2005] single-equation method. Denoting the log of variables in low-case letters, i.e., $x^j_{i,t} = \log Y^j_{i,t}$ and $g_{i,t} = \log G_{i,t}$, the dynamic effects of higher government purchases in good j can be estimated from running a series of regressions below:

$$x^j_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h} t + \psi_h (L) z_{i,t-1} + \gamma_h g^j_{i,t} + \eta_{i,t+h}, \tag{180}$$

where $\alpha_{i,h}$ are country fixed effects, $\alpha_{t,h}$ are time dummies, and we include country-specific linear time trends; x is the logarithm of the variable of interest, z is a vector of control variables (i.e., past values of government spending and of the variable of interest), $\psi_h (L)$ is a polynomial (of order two) in the lag operator and $g^j_{i,t}$ is the logged government spending in sector j.

By rearranging equation 9 (page 46) considered by Nekarda and Ramey [2011], it can be shown that this equation amounts to running the regression of the logarithm of the variable of interest on government consumption in sector j with lagged values on both the variable of interest and government consumption in sector j and this equation 9 collapses to eq. (180) with $h = 0$. So in short, Nekarda and Ramey [2011] estimate the sectoral fiscal multiplier on impact. Since the authors assume that government purchases of good j are a fixed proportion of total government purchases, i.e., $G^j_{i,t} = \omega_{G^j} G_{i,t}$, taking log of both sides and using the fact that ω_{G^j} is fixed and thus is absorbed by fixed effects, eq. (180) can be rewritten as follows:

$$x^j_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h} t + \psi_h (L) z_{i,t-1} + \gamma_h g_{i,t} + \eta_{i,t+h}, \tag{181}$$

where $g_{i,t} = \log G_{i,t}$. Ramey and Zubairy [2018] stress that employing the Blanchard-Perotti identification by estimating a VAR model which includes $g_{i,t}$ (ordered first), and $x^j_{i,t}$ amounts to estimating eq. (181) since the set of controls, z, includes lagged measures of the variable of interest and government spending (see also our section P.6). Thus estimating (181) is equivalent to the Blanchard-Perotti structural VAR (SVAR) identification. The coefficient γ_h gives the response of x at time $t + h$ to the shock at time t.

S Semi-Small Open Economy Model

This Appendix puts forward an open economy version of the neoclassical model with tradables and non-tradables, imperfect mobility of labor across sectors, capital adjustment costs and endogenous terms of trade. This section illustrates in detail the steps we follow in solving this model. We assume that production functions take a Cobb-Douglas form since this economy is the reference model for our calibration as we normalize CES productions by assuming that the initial steady state of the Cobb-Douglas economy is the normalization point.

Households supply labor, L, and must decide on the allocation of total hours worked between the traded sector, L^H, and the non-traded sector, L^N. They consume both traded, C^T, and non-traded goods, C^N. Traded goods are a composite of home-produced traded goods, C^H, and foreign-produced foreign (i.e., imported) goods, C^F. Households also choose investment which is produced using inputs of the traded, J^T, and the non-traded good, J^N. As for consumption, input of the traded good is a composite of home-produced traded goods, J^H, and foreign imported goods, J^F. The numeraire is the foreign good whose price, P^F, is thus normalized to one. While households choose the utilization of the stock of physical and intangible capital, firms decide about the mix of labor- and capital-augmenting productivity.

S.1 Households

At each instant of time, the representative household consumes traded and non-traded goods denoted by C^T and C^N, respectively, which are aggregated by means of a CES function:

$$C = \left[\varphi^{\frac{1}{\phi}} \left(C^T\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(C^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{182}$$

where $0 < \varphi < 1$ is the weight of the traded good in the overall consumption bundle and ϕ corresponds to the elasticity of substitution between traded goods and non-traded goods. The index C^T is defined as a CES aggregator of home-produced traded goods, C^H, and foreign-produced traded goods, C^F:

$$C^T = \left[\left(\varphi^H\right)^{\frac{1}{\rho}} \left(C^H\right)^{\frac{\rho-1}{\rho}} + (1-\varphi_H)^{\frac{1}{\rho}} \left(C^F\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{183}$$

where $0 < \varphi_H < 1$ is the weight of the home-produced traded good in the overall traded consumption bundle and ρ corresponds to the elasticity of substitution between home-produced traded goods goods and foreign-produced traded goods.

As in De Cordoba and Kehoe [2000], the investment good is produced using inputs of the traded good and the non-traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

$$J = \left[\iota^{\frac{1}{\phi_J}} \left(J^T\right)^{\frac{\phi_J-1}{\phi_J}} + (1-\iota)^{\frac{1}{\phi_J}} \left(J^N\right)^{\frac{\phi_J-1}{\phi_J}}\right]^{\frac{\phi_J}{\phi_J-1}}, \tag{184}$$

where ι is the weight of the investment traded input $(0 < \iota < 1)$ and ϕ_J corresponds to the elasticity of substitution in investment between traded and non-traded inputs. The index J^T is defined as a CES aggregator of home-produced traded inputs, J^H, and foreign-produced traded inputs, J^F:

$$J^T = \left[(\iota_H)^{\frac{1}{\rho_J}} \left(J^H\right)^{\frac{\rho_J-1}{\rho_J}} + (1-\iota_H)^{\frac{1}{\rho_J}} \left(J^F\right)^{\frac{\rho_J-1}{\rho_J}}\right]^{\frac{\rho_J}{\rho_J-1}}, \tag{185}$$

where $0 < \iota_H < 1$ is the weight of the home-produced traded in input in the overall traded investment bundle and ρ_J corresponds to the elasticity of substitution between home- and foreign-produced traded inputs.

Following Horvath [2000], we assume that hours worked in the traded and the non-traded sectors are aggregated by means of a CES function:

$$L = \left[\vartheta^{-1/\epsilon}\left(L^H\right)^{\frac{\epsilon+1}{\epsilon}} + (1-\vartheta)^{-1/\epsilon}\left(L^N\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}}, \tag{186}$$

where $0 < \vartheta < 1$ is the weight of labor supply to the traded sector in the labor index $L(.)$ and ϵ measures the ease with which hours worked can be substituted for each other and thereby captures the degree of labor mobility across sectors.

The representative agent is endowed with one unit of time, supplies a fraction $L(t)$ as labor, and consumes the remainder $1 - L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^\infty \left\{\frac{1}{1-\frac{1}{\sigma_C}} C(t)^{1-\frac{1}{\sigma_C}} - \frac{\gamma}{1+\frac{1}{\sigma_L}} L(t)^{1+\frac{1}{\sigma_L}}\right\} e^{-\beta t}\mathrm{d}t, \tag{187}$$

where $\beta > 0$ is the discount rate, $\sigma_C > 0$ the intertemporal elasticity of substitution for consumption, and $\sigma_L > 0$ the Frisch elasticity of (aggregate) labor supply.

We assume that the households own the physical capital stock and choose the level of capital utilization $u^{K,j}(t)$. The households also own the intangible stock of capital $\bar{Z}^j(t)$ and choose the level of utilization of existing technology $u^{Z,j}(t)$, i.e., $Z^j(t) = u^{Z,j}(t)\bar{Z}^j$. We further assume that the technology utilization rate is Hicks-neutral. In the sequel, we normalize the stock of knowledge, $\bar{Z}^j$, to one as we abstract from endogenous choices on the stock of knowledge. Households lease capital services (the product of utilization and physical capital) to firms in sector j at rental rate $R(t)$ augmented with the technology utilization rate, i.e., $R(t)u^{Z,j}(t)$. Thus capital income received by households reads $\sum_j u^{Z,j}(t)R(t)u^{K,j}(t)K^j(t)$. Households supply labor services to firms in sector j at a wage rate $W^j(t)$ augmented with $u^{Z,j}(t)$, i.e., $u^{Z,j}(t)W^j(t)$. Thus labor income received by households reads $\sum_j u^{Z,j}(t)W^j(t)L^j(t)$. In addition, households accumulate internationally traded bonds, $N(t)$, that yield net interest rate earnings of $r^\star N(t)$. Denoting lump-sum taxes by $T(t)$, households' flow budget constraint states that real disposable income can be saved by accumulating traded bonds, consumed, $P_C(t)C(t)$, invested, $P_J(t)J(t)$, and covers the capital and technology utilization costs:

$$\begin{aligned}
\dot{N}(t) &= r^\star N(t) + \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1-\alpha_K(t))\,u^{K,N}(t)u^{Z,N}(t)\right] R(t)K(t) \\
&+ \left[\alpha_L(t)u^{Z,H}(t) + (1-\alpha_L(t))\,u^{Z,N}(t)\right] W(t)L(t) - T(t) - P_C(t)C(t) - P_J(t)J(t) \\
&- P^H(t)C^{K,H}(t)\alpha_K(t)K(t) - P^N(t)C^{K,N}(t)\left(1-\alpha_K(t)\right)K(t) \\
&- P^H(t)C^{Z,H}(t) - P^N(t)C^{Z,N}(t),
\end{aligned} \tag{188}$$

where we denote the share of traded capital in the aggregate capital stock by $\alpha_K(t) = K^H(t)/K(t)$ and the labor compensation share of tradables by $\alpha_L(t) = \frac{W^H(t)L^H(t)}{W(t)L(t)}$ defined below.

The role of the capital utilization rate is to mitigate the effect of a rise in the capital cost. Symmetrically, the role of the technology utilization rate is to dampen the effects of increased costs of factors of production. We let the function $C^{K,j}(t)$ and $C^{Z,j}(t)$ denote the adjustment costs associated with the choice of capital and technology utilization rates which are increasing and convex functions of utilization rates $u^{K,j}(t)$ and $u^{Z,j}(t)$:

$$C^{K,j}(t) = \xi_1^j\left(u^{K,j}(t) - 1\right) + \frac{\xi_2^j}{2}\left(u^{K,j}(t) - 1\right)^2, \tag{189a}$$

$$C^{Z,j}(t) = \chi_1^j\left(u^{Z,j}(t) - 1\right) + \frac{\chi_2^j}{2}\left(u^{Z,j}(t) - 1\right)^2, \tag{189b}$$

where $\xi_2^j > 0$, χ_2^j are free parameters; as $\xi_2^j \to \infty$, $\chi_2^j \to \infty$, utilization is fixed at unity; ξ_1^j, χ_1^j must be restricted so that the optimality conditions are consistent with the normalization of steady state utilization of 1.

Capital accumulation evolves as follows:

$$\dot{K}(t) = I(t) - \delta_K K(t), \tag{190}$$

where I is investment and $0 \leq \delta_K < 1$ is a fixed depreciation rate. We assume that capital accumulation is subject to increasing and convex cost of net investment:

$$J(t) = I(t) + \Psi\left(I(t), K(t)\right) K(t), \tag{191}$$

where $\Psi\left(.\right)$ is increasing (i.e., $\Psi'(.) > 0$), convex (i.e., $\Psi''(.) > 0$), is equal to zero at δ_K (i.e., $\Psi(\delta_K) = 0$), and has first partial derivative equal to zero as well at δ_K (i.e., $\Psi'(\delta_K) = 0$). We suppose the following functional form for the adjustment cost function:

$$\Psi\left(I(t), K(t)\right) = \frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)^2. \tag{192}$$

Using (192), partial derivatives of total investment expenditure are:

$$\frac{\partial J(t)}{\partial I(t)} = 1 + \kappa\left(\frac{I(t)}{K(t)} - \delta_K\right), \tag{193a}$$

$$\frac{\partial J(t)}{\partial K(t)} = -\frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)\left(\frac{I(t)}{K(t)} + \delta_K\right). \tag{193b}$$

Denoting the co-state variables associated with (188) and (190) by λ and Q', respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$\left(C(t)\right)^{-\frac{1}{\sigma_C}} = P_C(t)\lambda(t), \tag{194a}$$

$$\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}} = \lambda(t)\left[\alpha_L(t)u^{Z,H}(t) + \left(1 - \alpha_L(t)\right)u^{Z,N}(t)\right]W(t), \tag{194b}$$

$$Q(t) = P_J(t)\left[1 + \kappa\left(\frac{I(t)}{K(t)} - \delta_K\right)\right], \tag{194c}$$

$$\dot{\lambda}(t) = \lambda\left(\beta - r^\star\right), \tag{194d}$$

$$\begin{aligned}\dot{Q}(t) = \left(r^\star + \delta_K\right)Q(t) - \Big\{ & \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + \left(1 - \alpha_K(t)\right)u^{K,N}(t)u^{Z,N}(t)\right]R(t) \\ & - P^H(t)C^{K,H}(t)\alpha_K(t) - P^N(t)C^{K,N}(t)\left(1 - \alpha_K(t)\right) + P_J(t)\frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)\left(\frac{I(t)}{K(t)} + \delta_K\right)\Big\},\end{aligned} \tag{194e}$$

$$R(t)u^{Z,H}(t) = P^H(t)\left[\xi_1^H + \xi_2^H\left(u^{K,H}(t) - 1\right)\right], \tag{194f}$$

$$R(t)u^{Z,N}(t) = P^N(t)\left[\xi_1^N + \xi_2^N\left(u^{K,N}(t) - 1\right)\right], \tag{194g}$$

$$R(t)u^{K,H}(t)K^H(t) + W^H(t)L^H(t) = P^H(t)\left[\chi_1^H + \chi_2^H\left(u^{Z,H}(t) - 1\right)\right], \tag{194h}$$

$$R(t)u^{K,N}(t)K^N(t) + W^N(t)L^N(t) = P^N(t)\left[\chi_1^N + \chi_2^N\left(u^{Z,N}(t) - 1\right)\right], \tag{194i}$$

and the transversality conditions $\lim_{t\to\infty} \bar{\lambda}N(t)e^{-\beta t} = 0$ and $\lim_{t\to\infty} Q(t)K(t)e^{-\beta t} = 0$; to derive (194c) and (194e), we used the fact that $Q(t) = Q'(t)/\lambda(t)$.

Given the above consumption indices, we can derive appropriate price indices. With respect to the general consumption index, we obtain the consumption-based price index P_C:

$$P_C = \left[\varphi\left(P^T\right)^{1-\phi} + (1 - \varphi)\left(P^N\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}, \tag{195}$$

where the price index for traded goods is:

$$P^T = \left[\varphi_H\left(P^H\right)^{1-\rho} + (1 - \varphi_H)\right]^{\frac{1}{1-\rho}}. \tag{196}$$

Given the consumption-based price index (195), the representative household has the following demand of traded and non-traded goods:

$$C^T = \varphi\left(\frac{P^T}{P_C}\right)^{-\phi} C, \tag{197a}$$

$$C^N = (1 - \varphi)\left(\frac{P^N}{P_C}\right)^{-\phi} C. \tag{197b}$$

Given the price indices (195) and (196), the representative household has the following demand of home-produced traded goods and foreign-produced traded goods:

$$C^H = \varphi \left(\frac{P^T}{P_C}\right)^{-\phi} \varphi_H \left(\frac{P^H}{P^T}\right)^{-\rho} C, \tag{198a}$$

$$C^F = \varphi \left(\frac{P^T}{P_C}\right)^{-\phi} (1-\varphi_H) \left(\frac{1}{P_T}\right)^{-\rho} C. \tag{198b}$$

As will be useful later, the percentage change in the consumption price index is a weighted average of percentage changes in the price of traded and non-traded goods in terms of foreign goods:

$$\hat{P}_C = \alpha_C \hat{P}^T + (1-\alpha_C)\,\hat{P}^N, \tag{199a}$$

$$\hat{P}^T = \alpha_H \hat{P}^H, \tag{199b}$$

where α_C is the tradable content of overall consumption expenditure and α^H is the home-produced goods content of consumption expenditure on traded goods:

$$\alpha_C = \varphi \left(\frac{P^T}{P_C}\right)^{1-\phi}, \tag{200a}$$

$$1-\alpha_C = (1-\varphi) \left(\frac{P^N}{P_C}\right)^{1-\phi}, \tag{200b}$$

$$\alpha^H = \varphi_H \left(\frac{P^H}{P^T}\right)^{1-\rho}, \tag{200c}$$

$$1-\alpha^H = (1-\varphi_H) \left(\frac{1}{P^T}\right)^{1-\rho}. \tag{200d}$$

Given the CES aggregator functions above, we can derive the appropriate price indices for investment. With respect to the general investment index, we obtain the investment-based price index P_J:

$$P_J = \left[\iota \left(P_J^T\right)^{1-\phi_J} + (1-\iota)\left(P^N\right)^{1-\phi_J}\right]^{\frac{1}{1-\phi_J}}, \tag{201}$$

where the price index for traded goods is:

$$P_J^T = \left[\iota^H \left(P^H\right)^{1-\rho_J} + \left(1-\iota^H\right)\right]^{\frac{1}{1-\rho_J}}. \tag{202}$$

Given the investment-based price index (201), we can derive the demand for inputs of the traded good and the non-traded good:

$$J^T = \iota \left(\frac{P_J^T}{P_J}\right)^{-\phi_J} J, \tag{203a}$$

$$J^N = (1-\iota) \left(\frac{P^N}{P_J}\right)^{-\phi_J} J. \tag{203b}$$

Given the price indices (201) and (202), we can derive the demand for inputs of home-produced traded goods and foreign-produced traded goods:

$$J^H = \iota \left(\frac{P_J^T}{P_J}\right)^{-\phi_J} \iota^H \left(\frac{P^H}{P_J^T}\right)^{-\rho_J} J, \tag{204a}$$

$$J^F = \iota \left(\frac{P_J^T}{P_J}\right)^{-\phi_J} \left(1-\iota^H\right) \left(\frac{1}{P_J^T}\right)^{-\rho_J} J. \tag{204b}$$

As will be useful later, the percentage change in the investment price index is a weighted average of percentage changes in the price of traded and non-traded inputs in terms of foreign inputs:

$$\hat{P}_J = \alpha_J \hat{P}_J^T + (1-\alpha_J)\,\hat{P}^N, \tag{205a}$$

$$\hat{P}_J^T = \alpha_J^H \hat{P}^H, \tag{205b}$$

where α_J is the tradable content of overall investment expenditure and α_J^H is the home-produced goods content of investment expenditure on traded goods:

$$\alpha_J = \iota \left(\frac{P_J^T}{P_J}\right)^{1-\phi_J}, \tag{206a}$$

$$1 - \alpha_J = (1-\iota)\left(\frac{P^N}{P_J}\right)^{1-\phi_J}, \tag{206b}$$

$$\alpha_J^H = \iota^H \left(\frac{P^H}{P_J^T}\right)^{1-\rho_J}, \tag{206c}$$

$$1 - \alpha_J^H = \left(1-\iota^H\right)\left(\frac{1}{P_J^T}\right)^{1-\rho_J}. \tag{206d}$$

Before deriving the allocation of hours worked across sectors, it is convenient to rewrite the optimal decision for aggregate labor supply described by eq. (194b). As shall be useful, we denote sectoral wages including technology utilization rates with a tilde, i.e., $\tilde{W}^j(t) = u^{Z,j}(t)W^j(t)$. Multiplying both sides of (194b) by $L(t)$ and denoting by $\tilde{W}(t)$ the aggregate wage index inclusive of technology utilization leads to:

$$\begin{aligned}
\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}+1} &= \lambda(t)\left[\alpha_L(t)u^{Z,H}(t) + \left(1-\alpha_L(t)\right)u^{Z,N}(t)\right]W(t)L(t), \\
\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}+1} &= \lambda(t)\left[W^H(t)u^{Z,H}(t)L^H(t) + W^N(t)u^{Z,N}(t)L^N(t)\right], \\
\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}+1} &= \lambda(t)\left[\tilde{W}^H(t)L^H(t) + \tilde{W}^N(t)L^N(t)\right],
\end{aligned}$$

where we used the definition of the labor compensation share of tradables and non-tradables, i.e., $\alpha_L(t)W(t)L(t) = W^H(t)L^H(t)$ and $\left(1-\alpha_L(t)\right)W(t)L(t) = W^N(t)L^N(t)$. Dividing both sides of the above equation by $L(t)$ and using the definition of the aggregate wage index which includes technology utilization rates enables us to rewrite eq. (194b) as follows:

$$\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}} = \lambda(t)\tilde{W}(t). \tag{207}$$

The aggregate wage index, $\tilde{W}(t)$, associated with the labor index defined above (186) is:

$$\tilde{W}(t) = \left[\vartheta\left(\tilde{W}^H(t)\right)^{\epsilon+1} + (1-\vartheta)\left(\tilde{W}^N(t)\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}}, \tag{208}$$

where $\tilde{W}^H(t) = u^{Z,H}(t)W^H(t)$ and $\tilde{W}^N = u^{Z,N}(t)W^N(t)$ are wages paid in the traded and the non-traded sectors, respectively.

Given the aggregate wage index, we can derive the allocation of aggregate labor supply to the traded and the non-traded sector:

$$L^H = \vartheta\left(\frac{\tilde{W}^H(t)}{\tilde{W}(t)}\right)^{\epsilon} L(t), \tag{209a}$$

$$L^N = (1-\vartheta)\left(\frac{\tilde{W}^N(t)}{\tilde{W}(t)}\right)^{\epsilon} L(t). \tag{209b}$$

As will be useful later, log-linearizing the aggregate wage index in the neighborhood of the initial steady-state leads to:

$$\hat{\tilde{W}}(t) = \alpha_L\hat{\tilde{W}}^H(t) + \left(1-\alpha_L\right)\hat{\tilde{W}}^N(t), \tag{210}$$

where α_L is the tradable content of aggregate labor compensation:

$$\alpha_L = \vartheta\left(\frac{W^H}{W}\right)^{1+\epsilon}, \tag{211a}$$

$$1-\alpha_L = (1-\vartheta)\left(\frac{W^N}{W}\right)^{1+\epsilon}. \tag{211b}$$

Note that because we log-linearize in the neighborhood of the steady-state, the labor compensation share, $\tilde{\alpha}_L$, inclusive of the technology utilization rate collapses to the technology utilization adjusted labor compensation share, α_L.

S.2 Firms

We denote the value added in sector $j = H, N$ by Y^j. When we add a tilde, it means that value added is inclusive of the technology utilization rate. Both the traded and non-traded sectors use physical capital inclusive of capital utilization, $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$, and labor, L^j, according to constant returns to scale production functions which are assumed to take a Cobb-Douglas form. We allow for labor- and capital-augmenting productivity denoted by $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. We assume that factor-augmenting productivity has a symmetric time-varying component denoted by $u^{Z,j}(t)$ such that $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$ and $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$. Since factor-augmenting productivity has no impact when considering Cobb-Douglas production function, we assume $\bar{Z}^j = \left(A^j\right)^{\theta^j}\left(B^j\right)^{1-\theta^j}$ and as mentioned above, we normalize $\bar{Z}^j$ to one. The production function of sector j reads as follows:

$$\begin{aligned} \tilde{Y}^j(t) &= u^{Z,j}(t)Y^j(t), \\ &= u^{Z,j}(t)\left(L^j(t)\right)^{\theta^j}\left(\tilde{K}^j(t)\right)^{1-\theta^j}, \end{aligned} \tag{212}$$

where θ^j is the labor income share in sector j.

Firms face two cost components: a capital rental cost equal to $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$, and a labor cost equal to the wage rate $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$, both inclusive of technology utilization. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given:

$$\max_{\tilde{K}^j(t),L^j(t)} \Pi^j(t) = \max_{\tilde{K}^j(t),L^j(t)} \left\{P^j(t)\tilde{Y}^j(t) - \tilde{W}^j(t)L^j(t) - \tilde{R}^j(t)\tilde{K}^j(t)\right\}. \tag{213}$$

The first order conditions of the firm problem are:

$$P^j(t)\theta^j u^{Z,j}(t)\left(\tilde{k}^j(t)\right)^{1-\theta^j} \equiv \tilde{W}^j(t), \tag{214a}$$

$$P^j(t)\left(1-\theta^j\right)u^{Z,j}(t)\left(\tilde{k}^j(t)\right)^{-\theta^j} = \tilde{R}^j(t), \tag{214b}$$

where $\tilde{k}^j(t) = \frac{\tilde{K}^j(t)}{L^j(t)}$ is the capital-labor ratio in sector j. By using the definition of $\tilde{W}^j(t)$ and $\tilde{R}^j(t)$, the technology utilization rate vanishes from first-order conditions. Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P^H(t)\left(1-\theta^H\right)\left(u^{K,H}(t)k^H(t)\right)^{-\theta^H} = P^N(t)\left(1-\theta^N\right)\left(u^{K,N}(t)k^N(t)\right)^{-\theta^N} \equiv R(t), \tag{215a}$$

$$P^H(t)\theta^H\left(u^{K,H}(t)k^H(t)\right)^{1-\theta^H} \equiv W^H(t), \tag{215b}$$

$$P^N(t)\theta^N\left(u^{K,N}(t)k^N(t)\right)^{1-\theta^N} \equiv W^N(t). \tag{215c}$$

The resource constraint for capital is:

$$K^H(t) + K^N(t) = K(t). \tag{216}$$

S.3 Solving the Model

Consumption and Labor

Before linearizing, we have to determine short-run solutions. First-order conditions (194a) and (194b) can be solved for consumption and aggregate labor supply which of course must hold at any point of time:

$$C = C\left(\bar{\lambda}, P^N, P^H\right), \quad L = L\left(\bar{\lambda}, \tilde{W}^H, \tilde{W}^N\right), \tag{217}$$

with partial derivatives given by

$$\hat{C} = -\sigma_C\hat{\bar{\lambda}} - \sigma_C\alpha_C\alpha^H\hat{P}^H - \sigma_C\left(1-\alpha_C\right)\hat{P}^N, \tag{218a}$$

$$\hat{L} = \sigma_L\hat{\bar{\lambda}} + \sigma_L\left(1-\alpha_L\right)\hat{\tilde{W}}^N + \sigma_L\alpha_L\hat{\tilde{W}}^H, \tag{218b}$$

where we have used (199) and (210).

Inserting first the solution for consumption (217) into (197b), (198a), (198b) enables us to solve for C^N, C^H, and C^F:

$$C^N = C^N\left(\bar{\lambda}, P^N, P^H\right), \quad C^H = C^H\left(\bar{\lambda}, P^N, P^H\right), \quad C^F = C^F\left(\bar{\lambda}, P^N, P^H\right), \tag{219}$$

with partial derivatives given by

$$\begin{aligned}
\hat{C}^N &= -\phi \hat{P}^N + (\phi - \sigma_C)\hat{P}_C - \sigma_C \hat{\bar{\lambda}}, \\
&= -\left[\alpha_C \phi + (1-\alpha_C)\sigma_C\right]\hat{P}^N + (\phi - \sigma_C)\alpha_C \alpha^H \hat{P}^H - \sigma_C \hat{\bar{\lambda}}, && (220a)\\
\hat{C}^H &= -\left[\rho\left(1-\alpha^H\right) + \phi(1-\alpha_C)\alpha^H + \sigma_C \alpha_C \alpha^H\right]\hat{P}^H + (1-\alpha_C)(\phi-\sigma_C)\hat{P}^N - \sigma_C \hat{\bar{\lambda}}, && (220b)\\
\hat{C}^F &= \alpha^H\left[\rho - \phi(1-\alpha_C) - \sigma_C \alpha_C\right]\hat{P}^H + (1-\alpha_C)(\phi-\sigma_C)\hat{P}^N - \sigma_C \hat{\bar{\lambda}}. && (220c)
\end{aligned}$$

Inserting first the solution for labor (217) into (209a)-(209b) allows us to solve for L^H and L^N:

$$L^H = L^H\left(\bar{\lambda}, W^H, W^N, u^{Z,H}, u^{Z,N}\right), \quad L^N = L^N\left(\bar{\lambda}, W^H, W^N, u^{Z,H}, u^{Z,N}\right), \tag{221}$$

with partial derivatives given by:

$$\begin{aligned}
\hat{L}^H &= \left[\epsilon(1-\alpha_L) + \sigma_L \alpha_L\right]\left(\hat{W}^H + \hat{u}^{Z,H}\right) - (1-\alpha_L)(\epsilon - \sigma_L)\left(\hat{W}^N + \hat{u}^{Z,N}\right) \\
&\quad + \sigma_L \hat{\bar{\lambda}}, && (222a)\\
\hat{L}^N &= \left[\epsilon \alpha_L + \sigma_L(1-\alpha_L)\right]\left(\hat{W}^N + \hat{u}^{Z,N}\right) - \alpha_L(\epsilon - \sigma_L)\left(\hat{W}^H + \hat{u}^{Z,H}\right) \\
&\quad + \sigma_L \hat{\bar{\lambda}}. && (222b)
\end{aligned}$$

Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for L^H and L^N given by (221) into the resource constraint for capital (216), the system of four equations consisting of (215a)-(215c) together with (216) can be solved for sectoral wages W^j and sectoral capital-labor ratios k^j. Log-differentiating (215a)-(215c) together with (216) yields in matrix form:

$$\begin{pmatrix}
-\theta^H & \theta^N & 0 & 0 \\
\left(1-\theta^H\right) & 0 & -1 & 0 \\
0 & \left(1-\theta^N\right) & 0 & -1 \\
\alpha_K & 1-\alpha_K & \Psi_{W^H} & \Psi_{W^N}
\end{pmatrix}
\begin{pmatrix}
\hat{k}^H \\ \hat{k}^N \\ \hat{W}^H \\ \hat{W}^N
\end{pmatrix}
= \begin{pmatrix}
\hat{P}^N - \hat{P}^H + \theta^H \hat{u}^{K,H} - \theta^N \hat{u}^{K,N} \\
-\hat{P}^H - \left(1-\theta^H\right)\hat{u}^{K,H} \\
-\hat{P}^N - \left(1-\theta^N\right)\hat{u}^{K,N} \\
\hat{K} - \Psi_{\bar{\lambda}}\hat{\bar{\lambda}} - \sum_j \Psi_{u^{Z,j}}\hat{u}^{Z,j}
\end{pmatrix}, \tag{223}$$

where we set:

$$\begin{aligned}
\Psi_{W^j} W^j &= \alpha_K \frac{L^H_{W^j} W^j}{L^H} + (1-\alpha_K)\frac{L^N_{W^j} W^j}{L^N}, && (224a)\\
\Psi_{u^{Z,j}} u^{Z,j} &= \alpha_K \frac{L^H_{u^{Z,j}} u^{Z,j}}{L^H} + (1-\alpha_K)\frac{L^N_{u^{Z,j}} u^{Z,j}}{L^N}, && (224b)\\
\Psi_{\bar{\lambda}} \bar{\lambda} &= \alpha_K \sigma_L + (1-\alpha_K)\sigma_L = \sigma_L. && (224c)
\end{aligned}$$

where $u^{Z,j} = 1$ at $\alpha_K \equiv K^H/K$ stands for the share of traded capital in the aggregate stock of physical capital.

The short-run solutions for sectoral wages and capital-labor ratios are:

$$\begin{aligned}
W^j &= W^j\left(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right), && (225a)\\
k^j &= k^j\left(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). && (225b)
\end{aligned}$$

Inserting first sectoral wages (225a), sectoral hours worked (221) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods, P^N, the terms of trade, capital and technology utilization rates:

$$L^j = L^j\left(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \tag{226}$$

Log-linearizing the production function (212), i.e., $Y^j = \left(L^j\right)^{\theta^j}\left(\tilde{K}^j\right)^{1-\theta^j}$ where $\tilde{K}^j = u^{K,j}K^j$, using the fact that $k^j = K^j/L^j$, leads to:

$$\hat{Y}^j = \hat{L}^j + \left(1-\theta^j\right)\left(\hat{k}^j + \hat{u}^{K,j}\right). \tag{227}$$

Plugging solutions for sectoral hours worked (226) and sectoral capital-labor ratios (225b) enables us to solve for technology utilization adjusted sectoral value added:

$$Y^j = Y^j\left(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \tag{228}$$

By using the fact that $K^j = k^j L^j$ and inserting solutions for k^j and L^j described by (225b) and (226) enables us to solve for the sectoral capital stocks:

$$K^j = K^j\left(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \tag{229}$$

Capital and Technology Utilization Rates, $u^{K,j}(t)$ and $u^{Z,j}(t)$

Inserting firm's optimal decisions for capital (214b), i.e., $P^j(t)\left(1-\theta^j\right)u^{Z,j}(t)\left(\tilde{k}^j(t)\right)^{-\theta^j} = R(t)u^{Z,j}(t)$ into optimal choices for capital utilization (194f)-(194g), and invoking the Euler theorem which leads to $W^H L^H + Ru^{K,H}K^H = P^H Y^H$ to rewrite optimal choices for technology utilization (194h)-(194i), we have:

$$\left(1-\theta^H\right)u^{Z,H}(t)\left(u^{K,H}(t)k^H(t)\right)^{-\theta^H} = \xi_1^H + \xi_2^H\left(u^{K,H}(t)-1\right), \tag{230a}$$
$$R(t)u^{Z,N}(t) = P^N(t)\left[\xi_1^N + \xi_2^N\left(u^{K,N}(t)-1\right)\right], \tag{230b}$$
$$Y^H(t) = \chi_1^H + \chi_2^H\left(u^{Z,H}(t)-1\right)], \tag{230c}$$
$$Y^N(t) = \chi_1^N + \chi_2^N\left(u^{Z,N}(t)-1\right), \tag{230d}$$

Log-linearizing optimal decisions on capital and technology utilization rates described by (230a)-(230d) leads to in a matrix form:

$$\begin{pmatrix} \left[\frac{\xi_2^H}{\xi_1^H} + \theta^H + \theta^H\frac{k^H_{u^{K,H}}}{k^H}\right] & \theta^H\frac{k^H_{u^{K,N}}}{k^H} & \left[\theta^H\frac{k^H_{u^{Z,H}}}{k^H} - 1\right] & \theta^H\frac{k^H_{u^{Z,N}}}{k^H} \\ \theta^N\frac{k^N_{u^{K,H}}}{k^N} & \left[\frac{\xi_2^N}{\xi_1^N} + \theta^N + \theta^N\frac{k^N_{u^{K,N}}}{k^N}\right] & \theta^N\frac{k^N_{u^{Z,H}}}{k^N} & \left[\theta^N\frac{k^N_{u^{Z,N}}}{k^N} - 1\right] \\ -\frac{Y^H_{u^{K,H}}}{Y^H} & -\frac{Y^H_{u^{K,N}}}{Y^H} & \left[\frac{\chi_2^H}{\chi_1^H} - \frac{Y^H_{u^{Z,H}}}{Y^H}\right] & -\frac{Y^H_{u^{Z,N}}}{Y^H} \\ -\frac{Y^N_{u^{K,H}}}{Y^N} & -\frac{Y^N_{u^{K,N}}}{Y^N} & -\frac{Y^N_{u^{Z,H}}}{Y^N} & \left[\frac{\chi_2^N}{\chi_1^N} - \frac{Y^N_{u^{Z,N}}}{Y^N}\right] \end{pmatrix} \begin{pmatrix} \hat{u}^{K,H} \\ \hat{u}^{K,N} \\ \hat{u}^{Z,H} \\ \hat{u}^{Z,N} \end{pmatrix}$$
$$= \begin{pmatrix} -\theta^H\frac{k^H_K}{k^H}dK - \theta^H\frac{k^H_{P^H}}{k^H}dP^H - \theta^H\frac{k^H_{P^N}}{k^H}dP^N - \theta^H\frac{k^H_{\bar{\lambda}}}{k^H}d\bar{\lambda} \\ -\theta^N\frac{k^N_K}{k^N}dK - \theta^N\frac{k^N_{P^H}}{k^N}dP^H - \theta^N\frac{k^N_{P^N}}{k^N}dP^N - \theta^N\frac{k^N_{\bar{\lambda}}}{k^N}d\bar{\lambda} \\ \frac{Y^H_K}{Y^H}dK + \frac{Y^H_{P^H}}{Y^H}dP^H + \frac{Y^H_{P^N}}{Y^H}dP^N + \frac{Y^H_{\bar{\lambda}}}{Y^H}d\bar{\lambda} \\ \frac{Y^N_K}{Y^N}dK + \frac{Y^N_{P^H}}{Y^N}dP^H + \frac{Y^N_{P^N}}{Y^N}dP^N + \frac{Y^N_{\bar{\lambda}}}{Y^N}d\bar{\lambda} \end{pmatrix}. \tag{231}$$

The short-run solutions for capital and technology utilization rates are:

$$u^{K,j} = u^{K,j}\left(\bar{\lambda}, K, P^N, P^H\right), \tag{232a}$$
$$u^{Z,j} = u^{Z,j}\left(\bar{\lambda}, K, P^N, P^H\right). \tag{232b}$$

Intermediate Solutions for k^j, W^j, L^j, Y^j, K^j

Plugging back solutions for capital and technology utilization rates (232a)-(232b) into (225a), (225b), (226), (228), (229) leads to intermediate solutions for sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, and sectoral capital stocks:

$$W^j, k^j, L^j, Y^j, K^j\left(\bar{\lambda}, K, P^N, P^H\right). \tag{233}$$

Optimal Investment Decision, I/K

Eq. (206d) can be solved for the investment rate:

$$\frac{I}{K} = v\left(\frac{Q}{P_I\left(P^T, P^N\right)}\right) + \delta_K, \tag{234}$$

where

$$v\left(.\right) = \frac{1}{\kappa}\left(\frac{Q}{P_J} - 1\right), \tag{235}$$

with

$$v_Q = \frac{\partial v(.)}{\partial Q} = \frac{1}{\kappa}\frac{1}{P_J} > 0, \tag{236a}$$

$$v_{P^H} = \frac{\partial v(.)}{\partial P^H} = -\frac{1}{\kappa}\frac{Q}{P_J}\frac{\alpha_J \alpha_J^H}{P^H} < 0, \tag{236b}$$

$$v_{P^N} = \frac{\partial v(.)}{\partial P^N} = -\frac{1}{\kappa}\frac{Q}{P_J}\frac{\left(1-\alpha_J\right)}{P^N} < 0. \tag{236c}$$

Inserting (234) into (191), investment including capital installation costs can be rewritten as follows:

$$\begin{aligned} J &= K\left[\frac{I}{K} + \frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)^2\right], \\ &= K\left[v(.) + \delta_K + \frac{\kappa}{2}\left(v(.)\right)^2\right]. \end{aligned} \tag{237}$$

Eq. (237) can be solved for investment including capital installation costs:

$$J = J\left(K, Q, P^N, P^H\right), \tag{238}$$

where

$$J_K = \frac{\partial J}{\partial K} = \frac{J}{K}, \tag{239a}$$

$$J_X = \frac{\partial J}{\partial X} = K v_X\left(1 + \kappa v(.)\right) > 0, \tag{239b}$$

with $X = Q, P^H, P^N$.

Substituting (238) into (203b), (211a), and (211b) allows us to solve for the demand for non-traded, home-produced traded, and foreign-produced traded inputs:

$$J^N = J^N\left(K, Q, P^N, P^H\right), \quad J^H = J^H\left(K, Q, P^N, P^H\right), \quad J^F = J^F\left(K, Q, P^N, P^H\right), \tag{240}$$

with partial derivatives given by

$$\begin{aligned}
\hat{J}^N &= -\alpha_J\phi_J\hat{P}^N + \phi_J\alpha_J\alpha_J^H\hat{P}^H + \hat{J}, \\
&= \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\hat{Q} - \left[\alpha_J\phi_J + \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}(1-\alpha_J)\right]\hat{P}^N \\
&+ \alpha_J\alpha_J^H\left[\phi_J - \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\right]\hat{P}^H + \hat{K}, \qquad (241a) \\
\hat{J}^H &= -\left[\rho_J\left(1-\alpha_J^H\right) + \alpha_J^H\phi_J(1-\alpha_J)\right]\hat{P}^H + \phi_J(1-\alpha_J)\hat{P}^N + \hat{J}, \\
&= -\left\{\left[\rho_J\left(1-\alpha_J^H\right) + \alpha_J^H\phi_J(1-\alpha_J)\right] + \alpha_J\alpha_J^H\frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\right\}\hat{P}^H \\
&+ (1-\alpha_J)\left[\phi_J - \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\right]\hat{P}^N + \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\hat{Q} + \hat{K}, \qquad (241b) \\
\hat{J}^F &= \alpha_J^H\left[\rho_J - \phi_J(1-\alpha_J)\right]\hat{P}^H + \phi_J(1-\alpha_J)\hat{P}^N + \hat{J}, \\
&= \alpha_J^H\left\{\left[\rho_J - \phi_J(1-\alpha_J)\right] - \alpha_J\frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\right\}\hat{P}^H \\
&+ (1-\alpha_J)\left[\phi_J - \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\right]\hat{P}^N + \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\hat{Q} + \hat{K}, \qquad (241c)
\end{aligned}$$

where use has been made of (239), i.e.,

$$\begin{aligned}
\hat{J} &= \hat{K} + \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\hat{Q} - \frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}(1-\alpha_J)\hat{P}^N \\
&\quad -\alpha_J\alpha_J^H\frac{QK}{P_J J}\frac{(1+\kappa v(.))}{\kappa}\hat{P}^H.
\end{aligned}$$

S.4 Market Clearing Conditions

Finally, we have to solve for the relative price of non-traded goods and the terms of trade.

Market Clearing Condition for Non-Tradables

The role of the price of non-tradables in terms of foreign-produced traded goods is to clear the non-traded goods market:

$$u^{Z,N}(t)Y^N(t) = C^N(t) + G^N(t) + J^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t). \qquad (242)$$

Inserting solutions for C^N, J^N, Y^N given by (219), (220a), (215), respectively, the non-traded goods market clearing condition (242) can be rewritten as follows:

$$\begin{aligned}
&u^{Z,N}\left(\bar{\lambda}, K, P^N, P^H\right)Y^N\left(\bar{\lambda}, K, P^N, P^H\right) = C^N\left(\bar{\lambda}, P^N, P^H\right) + G^N \\
&+J^N\left(K, Q, P^N, P^H\right) + C^{K,N}\left[u^{K,N}\left(\bar{\lambda}, K, P^N, P^H\right)\right]K^N\left(\bar{\lambda}, K, P^N, P^H\right) \\
&+C^{Z,N}\left[u^{Z,N}\left(\bar{\lambda}, K, P^N, P^H\right)\right]. \qquad (243)
\end{aligned}$$

Linearizing (243) leads to:

$$Y^N du^{Z,N}(t) + dY^N(t) = dC^N(t) + dG^N(t) + dJ^N(t) + K^N\xi_1^N du^{K,N}(t) + \chi_1^N du^{Z,N}(t), \qquad (244)$$

where the terms $Y^N du^{Z,N}(t)$ and $\chi_1^N du^{Z,N}(t)$ cancel out because eq. (230d) evaluated at the steady-state implies $Y^N = \chi_1^N$.

Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced goods in terms of foreign-produced goods or the terms of trade is to clear the home-produced traded goods market:

$$u^{Z,H}(t)Y^H(t) = C^H(t) + G^H(t) + J^H(t) + X^H(t) + C^{K,H}(t)K^H(t) + C^{Z,H}(t), \qquad (245)$$

where X^H stands for exports which are negatively related to the terms of trade:

$$X^H = \varphi_X\left(P^H\right)^{-\phi_X}, \qquad (246)$$

where ϕ_X is the elasticity of exports with respect to the terms of trade.

Inserting solutions for C^H, J^H, Y^H given by (219), (220a), (215), respectively, the traded goods market clearing condition (245) can be rewritten as follows:

$$\begin{aligned}
&u^{Z,H}\left(\bar{\lambda}, K, P^N, P^H\right) Y^H\left(\bar{\lambda}, K, P^N, P^H\right) = C^H\left(\bar{\lambda}, P^N, P^H\right) + G^H \\
&+J^H\left(K, Q, P^N, P^H\right) + C^{K,H}\left[u^{K,H}\left(\bar{\lambda}, K, P^N, P^H\right)\right] K^H\left(\bar{\lambda}, K, P^N, P^H\right) \\
&+C^{Z,H}\left[u^{Z,H}\left(\bar{\lambda}, K, P^N, P^H\right)\right]. \qquad (247)
\end{aligned}$$

Linearizing (247) leads to:

$$Y^H du^{Z,H}(t)+dY^H(t) = dC^H(t)+dG^H(t)+dJ^H(t)+dX^H(t)+K^H\xi_1^H du^{K,H}(t)+\chi_1^H du^{Z,H}(t), \qquad (248)$$

where the terms $Y^H du^{Z,H}(t)$ and $\chi_1^H du^{Z,H}(t)$ cancel out because eq. (230c) evaluated at the steady-state implies $Y^H = \chi_1^H$.

Sectoral Government Spending

We assume that the rise in government consumption is broken into non-traded and traded goods, and into home- and foreign-produced traded goods in accordance with their respective shares, $\omega_{G^N} = P^N G^N/G$ and $\omega_{G^H} = \frac{P^H G^H}{G}$. Formally, we have:

$$dG(t) = \omega_{G^N} dG(t) + \omega_{G^H} dG(t) + \omega_{G^F} dG(t). \qquad (249)$$

Linearizing the allocation of total government consumption across sectoral goods (measured at constant prices) leads to:

$$dG(t) = P^N dG^N(t) + P^H dG^H(t) + dG^F(t). \qquad (250)$$

Eqs. (249)-(250) can be solved for sectoral government consumption:

$$G^N, G^H, G^F(G(t)), \qquad (251)$$

where partial derivatives are given by

$$G_G^N = \frac{\partial G^N}{\partial G} = \frac{\omega_{G^N}}{P^N}, \qquad (252a)$$

$$G_G^H = \frac{\partial G^H}{\partial G} = \frac{\omega_{G^H}}{P^H}, \qquad (252b)$$

$$G_G^F = \frac{\partial G^F}{\partial G} = \omega_{G^F}. \qquad (252c)$$

Solving for Relative Prices

As shall be useful below, we write out a number of useful notations:

$$\Psi_{P^N}^N = Y_{P^N}^N - C_{P^N}^N - J_{P^N}^N - K^N\xi_1^N u_{P^N}^{K,N}, \qquad (253a)$$

$$\Psi_{P^H}^N = Y_{P^H}^N - C_{P^H}^N - J_{P^H}^N - K^N\xi_1^N u_{P^H}^{K,N}, \qquad (253b)$$

$$\Psi_K^N = Y_K^N - J_K^N - K^N\xi_1^N u_K^{K,N}, \qquad (253c)$$

$$\Psi_\lambda^N = Y_\lambda^N - C_\lambda^N - K^N\xi_1^N u_\lambda^{K,N}, \qquad (253d)$$

$$\Psi_{P^N}^H = Y_{P^N}^H - C_{P^N}^H - J_{P^N}^H - K^H\xi_1^H u_{P^N}^{K,H}, \qquad (253e)$$

$$\Psi_{P^H}^H = Y_{P^H}^H - C_{P^H}^H - J_{P^H}^H - X_{P^H}^H - K^H\xi_1^H u_{P^H}^{K,H}, \qquad (253f)$$

$$\Psi_K^H = Y_K^H - J_K^H - K^H\xi_1^H u_K^{K,H}, \qquad (253g)$$

$$\Psi_\lambda^H = Y_\lambda^H - C_\lambda^H - K^H\xi_1^H u_\lambda^{K,H}. \qquad (253h)$$

Linearized versions of market clearing conditions described by eq. (244) and eq. (248) can be rewritten in a matrix form:

$$\begin{aligned}
&\begin{pmatrix} \Psi_{P^N}^N & \Psi_{P^H}^N \\ \Psi_{P^N}^H & \Psi_{P^H}^H \end{pmatrix} \begin{pmatrix} dP^N \\ dP^H \end{pmatrix} \\
&= \begin{pmatrix} -\Psi_K^N dK + J_Q^N dQ + G_G^N dG - \Psi_\lambda^N d\bar{\lambda} \\ -\Psi_K^H dK + J_Q^H dQ + G_G^H dG - \Psi_\lambda^H d\bar{\lambda} \end{pmatrix}. \qquad (254)
\end{aligned}$$

The short-run solutions for non-traded and home-produced traded good prices are:

$$P^N = P^N\left(\bar{\lambda}, K, Q, G\right), \qquad (255a)$$

$$P^H = P^H\left(\bar{\lambda}, K, Q, G\right). \qquad (255b)$$

S.5 Solving the Model

In our model, there is one state variable, namely the capital stock K, and one control variable, namely the shadow price of the capital stock Q. To solve the model, we have to express all variables in terms of state and control variables. Plugging back solutions for the relative price of non-tradables (255a) and the terms of trade (255b) into (219), (240), (233), capital and technology utilization rates (232a)-(232b) leads to solutions for sectoral consumption, sectoral inputs for capital goods, sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, sectoral capital stocks:

$$C^j, J^j, W^j, k^j, L^j, Y^j, K^j, v, u^{K,j}, u^{Z,j}\left(\bar{\lambda}, K, Q, G\right). \tag{256}$$

The technology-utilization-adjusted-return on domestic capital is:

$$R(t) = P^H(t)\left(1-\theta^H\right)\left(u^{K,H}(t)k^H(t)\right)^{-\theta^H}. \tag{257}$$

Log-linearizing (257) in the neighborhood of the initial steady-state leads to:

$$\hat{R}(t) = \hat{P}^H(t) - \theta^H\left(\hat{u}^{K,H}(t) + \hat{k}^H(t)\right). \tag{258}$$

Inserting the solution for the terms of trade, P^H, the capital utilization rate, $u^{K,H}$, and the capital-labor ratio k^H described by by (256), eq. (257) can be solved for the return on domestic capital:

$$R = R\left(\bar{\lambda}, K, Q, G\right). \tag{259}$$

Remembering that the non-traded input J^N used to produce the capital good is described by $(1-\iota)\left(\frac{P^N}{P_J}\right)^{-\phi_J} J$ (see eq. (203b)) with $J = I + \frac{\kappa}{2}\left(\frac{I}{K}-\delta_K\right)^2 K$, using the fact that $J^N = Y^N - C^N - G^N$ and inserting $I = \dot{K} + \delta_K$, the capital accumulation equation reads as follows:

$$\dot{K}(t) = \frac{u^{Z,N}(t)Y^N(t) - C^N(t) - G^N(t) - C^{K,N}(t)K^N(t) - C^{Z,N}(t)}{(1-\iota)\left(\frac{P^N(t)}{P_J(t)}\right)^{-\phi_J}} - \delta_K K(t) - \frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)^2 K(t). \tag{260}$$

We drop the time index below so as to write equations in a more compact form. Inserting first solutions for non-traded output, consumption in non-tradables, demand for non-traded input, non-traded capital and technology utilization rates described by eq. (256) together with optimal investment decision (239a) into the physical capital accumulation equation (260), and plugging the short-run solution for the return on domestic capital (259) into the dynamic equation for the shadow value of capital stock (194e), the dynamic system reads as follows:

$$\begin{aligned}
\dot{K} &\equiv \Upsilon\left(K, Q, G\right), \\
&= \frac{Y^N\left(K,Q,G\right) - C^N\left(K,Q,G\right) - G^N\left(G\right) - C^{K,N}\left[u^{K,N}\left(K,Q,G\right)\right]K^N - C^{Z,N}\left[u^{Z,N}\left(K,Q,G\right)\right]}{(1-\iota)\left[\frac{P^N(K,Q,G)}{P_J(K,Q,G)}\right]^{-\phi_J}} \\
&\quad -\delta_K K - \frac{K}{2\kappa}\left[\frac{Q}{P_J\left(K,Q,G\right)} - 1\right]^2, \qquad (261a) \\
\dot{Q} &\equiv \Sigma\left(K,Q,G\right) \\
&= \left(r^\star + \delta_K\right)Q - \left[\frac{R\left(K,Q,G\right)}{K}\sum_{j=H,N} u^{K,j}\left(K,Q,G\right)u^{Z,j}\left(K,Q,G\right)K^j\left(K,Q,G\right)\right. \\
&\quad - \sum_{j=H,N} P^j\left(K,Q,G\right)C^{K,j}\left[u^{K,j}\left(K,Q,G\right)\right]\frac{K^j\left(K,Q,G\right)}{K} \\
&\quad \left. + P_J\left[P^H\left(.\right), P^N\left(.\right)\right]\frac{\kappa}{2}v(.)\left(v(.) + 2\delta_K\right)\right]. \qquad (261b)
\end{aligned}$$

To ease the linearization, it is useful to break down the capital accumulation into two components:

$$\hat{K} = J - \delta_K K - \frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)^2 K. \tag{262}$$

The first component is J. Using the fact that $J = \frac{J^N}{(1-\iota)\left(\frac{P^N}{P_J}\right)^{-\phi_J}}$ and log-linearizing gives:

$$\hat{J} = \hat{J}^N + \phi_J \alpha_J \hat{P}^N - \phi_J \alpha_J \alpha_J^H \hat{P}^H \tag{263}$$

where we used the fact that $\hat{P}_J = \alpha_J \alpha_J^H \hat{P}^H + (1-\alpha_J)\hat{P}^N$. Using (262) and the fact that $J^N = Y^N - C^N - G^N - C^{K,N}K^N - C^{Z,N}$, linearizing (262) in the neighborhood of the steady-state gives:

$$\begin{aligned} \dot{K} &= \frac{J}{J^N}\left[dY^N(t) - dC^N(t) - dG^N(t) - K^N \xi_1^N du^{K,N}(t)\right] + \phi_J \frac{J}{P^N}\alpha_J dP^N(t) \\ &- \phi_J \frac{J}{P^H}\alpha_J \alpha_J^H dP^H(t) - \delta_K dK(t), \end{aligned} \tag{264}$$

where $J = I = \delta_K K$ in the long-run and we used the fact that $Y^N du^{Z,N}(t)$ and $\chi_1^N du^{Z,N}(t)$ cancel out.

Let us denote by Υ_K, Υ_Q, and Υ_G the partial derivatives evaluated at the steady-state of the capital accumulation equation w.r.t. K, Q, and G, respectively. Using (256) and (264), these elements of of the linearized capital accumulation equation are given by:

$$\Upsilon_K \equiv \frac{\partial \dot{K}}{\partial K} = \frac{J}{J^N}\left(Y_K^N - C_K^N - K^N \xi_1^N u_K^{K,N}\right) + \alpha_J \phi_J J\left(\frac{P_K^N}{P^N} - \alpha_J^H \frac{P_K^H}{P^H}\right) - \delta_K \gtrless 0, \tag{265a}$$

$$\Upsilon_Q \equiv \frac{\partial \dot{K}}{\partial Q} = \frac{J}{J^N}\left(Y_Q^N - C_Q^N - K^N \xi_1^N u_Q^{K,N}\right) + \alpha_J \phi_J J\left(\frac{P_Q^N}{P^N} - \alpha_J^H \frac{P_Q^H}{P^H}\right) > 0, \tag{265b}$$

$$\Upsilon_G \equiv \frac{\partial \dot{K}}{\partial G} = \frac{J}{J^N}\left(Y_G^N - G_G^N - C_G^N - K^N \xi_1^N u_G^{K,N}\right) + \alpha_J \phi_J J\left(\frac{P_G^N}{P^N} - \alpha_J^H \frac{P_G^H}{P^H}\right), \tag{265c}$$

where $J = \delta_K K$ in the long run.

Let us denote by Σ_K, Σ_Q, and Σ_G the partial derivatives evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t. K, Q, and G, respectively:

$$\begin{aligned} \Sigma_K \equiv \frac{\partial \dot{Q}}{\partial K} &= -\Bigg\{R_K - \frac{R}{K} + \frac{R}{K}\sum_{j=H,N}\left[K^j u_K^{K,j} + K^j u_K^{Z,j}\right] \\ &+ \frac{R}{K}\left[K_K^H + K_K^N\right] - \sum_{j=H,N}\frac{P^j K^j}{K}\xi_1^j u_K^{K,j} \\ &+ P_J \kappa v_K \delta_K\Bigg\} > 0, \end{aligned} \tag{266a}$$

$$\begin{aligned} \Sigma_Q \equiv \frac{\partial \dot{Q}}{\partial Q} &= (r^\star + \delta_K) - \Bigg\{R_Q + \frac{R}{K}\sum_{j=H,N}\left[K^j u_Q^{K,j} + K^j u_Q^{Z,j}\right] \\ &+ \frac{R}{K}\left(K_Q^H + K_Q^N\right) - \sum_{j=H,N}\frac{P^j K^j}{K}\xi_1^j u_Q^{K,j} \\ &+ P_J \kappa v_Q \delta_K\Bigg\} > 0, \end{aligned} \tag{266b}$$

$$\begin{aligned} \Sigma_G \equiv \frac{\partial \dot{Q}}{\partial G} &= -\Bigg\{R_G + \frac{R}{K}\sum_{j=H,N}\left[K^j u_G^{K,j} + K^j u_G^{Z,j}\right] \\ &+ \frac{R}{K}\left(K_G^H + K_G^N\right) - \sum_{j=H,N}\frac{P^j K^j}{K}\xi_1^j u_G^{K,j} \\ &+ P_J \kappa v_G \delta_K\Bigg\} > 0. \end{aligned} \tag{266c}$$

Assuming that the saddle-path stability condition is fulfilled, and denoting the negative eigenvalue by ν_1 and the positive eigenvalue by ν_2, the general solutions for K and Q are:

$$K(t) - K = D_1 e^{\nu_1 t} + D_2 e^{\nu_2 t}, \quad Q(t) - Q = \omega_2^1 D_1 e^{\nu_1 t} + \omega_2^2 D_2 e^{\nu_2 t}, \tag{267}$$

where K_0 is the initial capital stock and $\left(1, \omega_2^i\right)'$ is the eigenvector associated with eigenvalue ν_i:

$$\omega_2^i = \frac{\nu_i - \Upsilon_K}{\Upsilon_Q}. \tag{268}$$

Because $\nu_1 < 0$, $\Upsilon_K > 0$ and $\Upsilon_Q > 0$, we have $\omega_2^1 < 0$, regardless of sectoral capital intensities, which implies that the shadow value of investment and the stock physical capital move in opposite direction along a stable path (i.e., $D_2 = 0$).

S.6 Current Account Equation and Intertemporal Solvency Condition

To determine the current account equation, we use the following identities and properties:

$$P_C(t)C(t) = P^H(t)C^H(t) + C^F(t) + P^N(t)C^N(t), \tag{269a}$$

$$P_J(t)J(t) = P^H(t)J^H(t) + J^F(t) + P^N(t)J^N(t), \tag{269b}$$

$$T(t) = G(t) = P^H(t)G^H(t) + G^F(t) + P^N(t)G^N(t), \tag{269c}$$

$$\tilde{W}(t)L(t) + \tilde{R}(t)\tilde{K}(t) = \sum_{j=H,N} u^{Z,j}(t)\left(W^j(t)L^j(t) + R^j(t)\tilde{K}^j(t)\right) = \sum_{j=H,N} P^j(t)\tilde{Y}^j(t), \tag{269d}$$

where (269d) follows from Euler theorem. Using (269d), inserting (269a)-(269c) into (188) and invoking market clearing conditions for non-traded goods (242) and home-produced traded goods (245) yields:

$$\begin{aligned}\dot{N}(t) &= r^\star N(t) + P^H(t)\left(\tilde{Y}^H(t) - C^H(t) - G^H(t) - J^H(t) - C^{K,H}(t)K^H(t) - C^{Z,H}(t)\right) \\ &\quad - \left(C^F(t) + J^F(t) + G^F(t)\right), \\ &= r^\star N(t) + P^H(t)X^H(t) - M^F(t),\end{aligned} \tag{270}$$

where $X^H = Y^H - C^H - G^H - J^H - C^{K,H}K^H - C^{Z,H}$ stands for exports of home-produced traded goods and we denote imports of foreign consumption and investment goods by M^F:

$$M^F(t) = C^F(t) + G^F(t) + J^F(t). \tag{271}$$

Inserting (256) into (270) and the solution for P^H described by eq. (255b) into $X^H = X^H\left(P^H\right)$ leads to:

$$\begin{aligned}\dot{N}(t) &\equiv r^\star N(t) + \Xi\left(K(t), Q(t), G(t)\right), \\ &= r^\star N(t) + P^H\left(K(t), Q(t), G(t)\right)X^H\left[P^H\left(K(t), Q(t), G(t)\right)\right] - M^F\left(K(t), Q(t), G(t)\right)\end{aligned} \tag{272}$$

Let us denote by Ξ_K, Ξ_Q, and Ξ_{Z^j} the partial derivatives evaluated at the steady-state of the dynamic equation for the current account w.r.t. K, Q, and Z^j, respectively:

$$\Xi_K \equiv \frac{\partial \dot{N}}{\partial K} = \left(1 - \phi_X\right)X^H P_K^H - M_K^F, \tag{273a}$$

$$\Xi_Q \equiv \frac{\partial \dot{N}}{\partial Q} = \left(1 - \phi_X\right)X^H P_Q^H - M_Q^F, \tag{273b}$$

$$\Xi_G \equiv \frac{\partial \dot{N}}{\partial G} = \left(1 - \phi_X\right)X^H P_G^H - M_G^F. \tag{273c}$$

where we used the fact that $P^H X^H = \varphi_X\left(P^H\right)^{1-\phi_X}$ (see eq. (246)).

Linearizing (272) in the neighborhood of the steady-state, making use of (273a) and (273b), inserting solutions for $K(t)$ and $Q(t)$ given by (267) and solving yields the general solution for the net foreign asset position:

$$N(t) = N + \left[(N_0 - N) - \Psi_1 D_1 - \Psi_2 D_2\right]e^{r^\star t} + \Psi_1 D_1 e^{\nu_1 t} + \Psi_2 D_2 e^{\nu_2 t}, \tag{274}$$

where N_0 is the initial stock of traded bonds and we set

$$
\begin{aligned}
E_i &= \Xi_K + \Xi_Q \omega_2^i, && (275a)\\
\Psi_i &= \frac{E_i}{\nu_i - r^\star}. && (275b)
\end{aligned}
$$

Invoking the transversality condition leads to the linearized version of the nations's intertemporal solvency condition:

$$
N - N_0 = \Psi_1 \left(K - K_0\right), \tag{276}
$$

where K_0 is the initial stock of physical capital.

S.7 Derivation of the Accumulation Equation of Non Human Wealth

The stock of financial wealth $A(t)$ is equal to $N(t) + Q(t)K(t)$; differentiating w.r.t. time, i.e., $\dot{A}(t) = \dot{N}(t) + \dot{Q}(t)K(t) + Q(t)\dot{K}(t)$, plugging the dynamic equation for the marginal value of capital (194e), inserting the accumulation equations for physical capital (190) and for traded bonds (188), yields the accumulation equation for the stock of financial wealth or the dynamic equation for private savings:

$$
\dot{A}(t) = r^\star A(t) + \sum_{j=H,N} u^{Z,j}(t) W^j(t) L^j(t) - T(t) - P_C(t)C(t) - \sum_{j=H,N} P^j(t) C^{Z,j}(t), \tag{277}
$$

where we assume that the government levies lump-sum taxes, T, to finance purchases of foreign-produced, home-produced traded goods and non-traded goods, i.e., $T = G = G^F + P^H G^H + P^N G^N$.

Inserting short-run solutions for the relative price of non-tradables (255a) and the terms of trade (255b) into $G = G^F + P^H G^H + P^N G^N$ and (195) allows us to solve government spending and the consumption price index:

$$
\begin{aligned}
G &= G\left(K, Q, G, \bar{\lambda}\right), && (278a)\\
P_C &= P_C\left(K, Q, G, \bar{\lambda}\right), && (278b)
\end{aligned}
$$

where partial derivatives are $G_X = P^H_X G^H + P^N_X G^N$ with $X = K, Q, G$ and

$$
\frac{\partial P_C}{\partial X} = \alpha_C \alpha^H \frac{P_C}{P^H} P^H_X + \left(1 - \alpha_C\right) \frac{P_C}{P^N} P^N_X, \tag{279}
$$

where $X = K, Q, G$.

Inserting first short-run solutions for consumption and labor together with solutions for technology utilization rates given by eq. (256), substituting solutions for government spending and the consumption price index described by (278a)-(278b) leads to:

$$
\begin{aligned}
\dot{A} &\equiv r^\star A + \Lambda\left(K, Q, G\right),\\
&= r^\star A + \sum_{j=H,N} u^{Z,j}\left(K, Q, G\right) W^j\left(K, Q, G\right) L^j\left(K, Q, G\right) - G\left(K, Q, G\right)\\
&- P_C\left(K, Q, G\right) C\left(K, Q, G\right) - \sum_{j=H,N} P^j\left(K, Q, G\right) C^{Z,j}\left(K, Q, G\right). && (280)
\end{aligned}
$$

Let us denote by Λ_K, Λ_Q, and Λ_G the partial derivatives evaluated at the steady-state of the dynamic equation for the non human wealth w.r.t. K, Q, and G, respectively, which are given by:

$$\begin{aligned}
\Lambda_K &\equiv \frac{\partial \dot{A}}{\partial K} = \sum_{j=H,N} \left(W^j_K L^j + W^j L^j_K + W^j L^j u^{Z,j}_K \right) - G_K - \left(\frac{\partial P_C}{\partial K} C + P_C C_K \right) \\
&- \sum_{j=H,N} P^j \xi^j_1 u^{Z,j}_K, \qquad (281a) \\
\Lambda_Q &\equiv \frac{\partial \dot{A}}{\partial Q} = \sum_{j=H,N} \left(W^j_Q L^j + W^j L^j_Q + W^j L^j u^{Z,j}_Q \right) - G_Q - \left(\frac{\partial P_C}{\partial Q} C + P_C C_Q \right) \\
&- \sum_{j=H,N} P^j \xi^j_1 u^{Z,j}_Q, \qquad (281b) \\
\Lambda_G &\equiv \frac{\partial \dot{A}}{\partial G} = \sum_{j=H,N} \left(W^j_G L^j + W^j L^j_G + W^j L^j u^{Z,j}_G \right) - G_G - \left(\frac{\partial P_C}{\partial G} C + P_C C_G \right) \\
&- \sum_{j=H,N} P^j \xi^j_1 u^{Z,j}_G. \qquad (281c)
\end{aligned}$$

Linearizing (280) in the neighborhood of the steady-state, making use of (281a) and (281b), inserting solutions for $K(t)$ and $Q(t)$ given by (267) and solving yields the general solution for the stock of non human wealth:

$$A(t) = A + \left[(A_0 - A) - \Delta_1 D_1 - \Delta_2 D_2\right] e^{r^\star t} + \Delta_1 D_1 e^{\nu_1 t} + \Delta_2 D_2 e^{\nu_2 t}, \qquad (282)$$

where A_0 is the initial stock of financial wealth and we set

$$\begin{aligned}
M_i &= \Lambda_K + \Lambda_Q \omega^i_2, \qquad (283a) \\
\Delta_i &= \frac{M_i}{\nu_i - r^\star}. \qquad (283b)
\end{aligned}$$

The linearized version of the representative household's intertemporal solvency condition is:

$$A - A_0 = \Delta_1 \left(K - K_0\right), \qquad (284)$$

where A_0 is the initial stock of non human wealth.

T Semi-Small Open Economy Model with CES Production Functions

This section extends the model laid out in section S to CES production functions and factor biased technological change. Since first order conditions from households' maximization problem detailed in subsection S.1 remain identical, we do not repeat them and emphasize the main changes caused by the assumption of CES production functions.

T.1 Firms

We denote technology adjusted value added in sector $j = H, N$ by Y^j. When we add a tilde, it means that value added is inclusive of the technology utilization rate, i.e., $\tilde{Y}^j(t) = u^Z(t) Y^j(t)$. We allow for labor- and capital-augmenting productivity denoted by $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. We allow for labor- and capital-augmenting efficiency denoted by $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. We assume that factor-augmenting productivity has a symmetric time-varying component denoted by $u^{Z,j}(t)$ such that $\tilde{A}^j(t) = u^{Z,j}(t) A^j(t)$ and $\tilde{B}^j(t) = u^{Z,j}(t) B^j(t)$. Both the traded and non-traded sectors use physical capital inclusive of capital utilization, $\tilde{K}^j(t) = u^{K,j}(t) K^j(t)$, and labor, L^j, according to constant returns to scale production functions which are assumed to take a CES form:

$$\tilde{Y}^j(t) = \left[\gamma^j \left(\tilde{A}^j(t) L^j(t) \right)^{\frac{\sigma^j - 1}{\sigma^j}} + \left(1 - \gamma^j\right) \left(\tilde{B}^j(t) \tilde{K}^j(t) \right)^{\frac{\sigma^j - 1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j - 1}}, \qquad (285)$$

where γ^j and $1-\gamma^j$ are the weight of labor and capital in the production technology, σ^j is the elasticity of substitution between capital and labor in sector $j = H, N$. Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$, and a labor cost equal to the wage rate $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$, both inclusive of technology utilization.

First-Order Conditions

Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$, and a labor cost equal to the wage rate $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$, both inclusive of technology utilization. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given:

$$\begin{aligned}\max_{\tilde{K}^j,L^j} \tilde{\Pi}^j &= \max_{\tilde{K}^j,L^j} \left\{P^j\tilde{Y}^j - \tilde{W}^jL^j - \tilde{R}^j\tilde{K}^j\right\}, \\ &= \max_{\tilde{K}^j,L^j} u^{Z,j}(t)\left\{P^jY^j - W^jL^j - R\tilde{K}^j\right\}, \\ &= \max_{\tilde{K}^j,L^j} u^{Z,j}\Pi^j, \end{aligned} \tag{286}$$

where technology-utilization-adjusted CES production function reads:

$$Y^j(t) = \left[\gamma^j\left(A^j(t)L^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(B^j(t)\tilde{K}^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}. \tag{287}$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P^H\left(1-\gamma^H\right)\left(B^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(u^{K,H}k^H\right)^{-\frac{1}{\sigma^H}}\left(y^H\right)^{\frac{1}{\sigma^H}} = P^N\left(1-\gamma^N\right)\left(B^N\right)^{\frac{\sigma^N-1}{\sigma^N}}\left(u^{K,N}k^N\right)^{-\frac{1}{\sigma^N}}\left(y^N\right)^{\frac{1}{\sigma^N}} \equiv R, \tag{288a}$$

$$P^H\gamma^H\left(A^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(L^H\right)^{-\frac{1}{\sigma^H}}\left(Y^H\right)^{\frac{1}{\sigma^H}} \equiv W^H, \tag{288b}$$

$$P^N\gamma^N\left(A^N\right)^{\frac{\sigma^N-1}{\sigma^N}}\left(L^N\right)^{-\frac{1}{\sigma^N}}\left(Y^N\right)^{\frac{1}{\sigma^N}} \equiv W^N, \tag{288c}$$

where we denote by $k^j \equiv K^j/L^j$ the capital-labor ratio for sector $j = H, N$, and $y^j \equiv Y^j/L^j$ value added per hours worked described by

$$y^j = \left[\gamma^j\left(A^j\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(B^ju^{K,j}k^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}. \tag{289}$$

The resource constraint for capital is:

$$K^H + K^N = K. \tag{290}$$

Some Useful Results

Multiplying both sides of (288b)-(288c) by L^j and dividing by sectoral value added leads to the labor income share:

$$s_L^j = \gamma^j\left(\frac{A^j}{y^j}\right)^{\frac{\sigma^j-1}{\sigma^j}}. \tag{291}$$

Multiplying both sides of (288a) by K^j and dividing by sectoral value added leads to the capital income share:

$$1 - s_L^j = \left(1-\gamma^j\right)\left(\frac{B^ju^{K,j}k^j}{y^j}\right)^{\frac{\sigma^j-1}{\sigma^j}}. \tag{292}$$

Dividing eq. (291) by eq. (292), the ratio of the labor to the capital income share denoted by $S^j = \frac{s_L^j}{1-s_L^j}$ reads as follows:

$$S^j = \frac{\gamma^j}{1-\gamma^j}\left(\frac{B^ju^{K,j}K^j}{A^jL^j}\right)^{\frac{1-\sigma^j}{\sigma^j}}. \tag{293}$$

Unit Cost for Producing

Dividing (288b)-(288c) by (288a) leads to a positive relationship between the wage-to-capital-rental-rate ratio and the capital-labor ratio in sector j:

$$\frac{W^j}{R} = \frac{\gamma^j}{1-\gamma^j} \left(\frac{B^j}{A^j}\right)^{\frac{1-\sigma^j}{\sigma^j}} \left(\frac{\tilde{K}^j}{L^j}\right)^{\frac{1}{\sigma^j}}. \tag{294}$$

To determine the conditional demands for both inputs, we solve (294) for hours worked and next for capital:

$$L^j = \tilde{K}^j \left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j} \left(\frac{B^j}{A^j}\right)^{1-\sigma^j} \left(\frac{W^j}{R}\right)^{-\sigma^j}, \tag{295a}$$

$$\tilde{K}^j = L^j \left(\frac{1-\gamma^j}{\gamma^j}\right)^{\sigma^j} \left(\frac{B^j}{A^j}\right)^{\sigma^j-1} \left(\frac{W^j}{R}\right)^{\sigma^j}. \tag{295b}$$

Eq. (295a) can be rewritten as follows:

$$\gamma^j \left(A^j L^j\right)^{\frac{\sigma^j-1}{\sigma^j}} = \left(\gamma^j\right)^{\sigma^j} \left(1-\gamma^j\right)^{-\sigma^j} \left(\frac{W^j}{A^j}\right)^{1-\sigma^j} \left(\frac{R}{B^j}\right)^{\sigma^j-1} \left(1-\gamma^j\right) \left(B^j \tilde{K}^j\right)^{\frac{\sigma^j-1}{\sigma^j}}.$$

Plugging the above equation into (287) leads to:

$$\left(1-\gamma^j\right) \left(B^j \tilde{K}^j\right)^{\frac{\sigma^j-1}{\sigma^j}} = \left(Y^j\right)^{\frac{\sigma^j-1}{\sigma^j}} \left(X^j\right)^{-1} \left(1-\gamma^j\right)^{\sigma^j} \left(\frac{R}{B^j}\right)^{1-\sigma^j} \tag{296}$$

where we set

$$X^j = \left(\gamma^j\right)^{\sigma^j} \left(A^j\right)^{\sigma^j-1} \left(\frac{W^j}{A^j}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j} \left(\frac{R}{B^j}\right)^{1-\sigma^j}. \tag{297}$$

Eq. (295b) can be rewritten as follows:

$$\left(1-\gamma^j\right) \left(B^j \tilde{K}^j\right)^{\frac{\sigma^j-1}{\sigma^j}} = \left(\gamma^j\right)^{-\sigma^j} \left(1-\gamma^j\right)^{\sigma^j} \left(\frac{R}{B^j}\right)^{1-\sigma^j} \left(\frac{W^j}{A^j}\right)^{\sigma^j-1} \gamma^j \left(A^j L^j\right)^{\frac{\sigma^j-1}{\sigma^j}}.$$

Plugging the above equation into (287) leads to:

$$\gamma^j \left(A^j L^j\right)^{\frac{\sigma^j-1}{\sigma^j}} = \left(Y^j\right)^{\frac{\sigma^j-1}{\sigma^j}} \left(X^j\right)^{-1} \left(\gamma^j\right)^{\sigma^j} \left(A^j\right)^{\sigma^j-1} \left(\frac{W^j}{A^j}\right)^{1-\sigma^j}, \tag{298}$$

where X^j is given by (297).

Eq. (298) can be solved for the conditional demand for labor and eq. (296) can be solved for the conditional demand for capital (inclusive of capital utilization):

$$L^j = Y^j \left(A^j\right)^{\sigma^j-1} \left(\frac{\gamma^j}{W^j}\right)^{\sigma} \left(X^j\right)^{\frac{\sigma^j}{1-\sigma^j}}, \tag{299a}$$

$$\tilde{K}^j = Y^j \left(B^j\right)^{\sigma^j-1} \left(\frac{1-\gamma^j}{R}\right)^{\sigma^j} \left(X^j\right)^{\frac{\sigma^j}{1-\sigma^j}}, \tag{299b}$$

where X^j is given by (296).

Total cost is equal to the sum of the labor and capital cost:

$$\text{Cost}^j = W^j L^j + R\tilde{K}^j. \tag{300}$$

Inserting conditional demand for inputs (312) into total cost (300) leads to:

$$\begin{aligned} \text{Cost}^j &= Y^j \left(X^j\right)^{\frac{\sigma^j}{1-\sigma^j}} \left(\gamma^j\right)^{\sigma^j} \left(A^j\right)^{\sigma^j-1} \left(\frac{W^j}{A^j}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j} \left(\frac{R}{B^j}\right)^{1-\sigma^j}, \\ &= Y^j \left(X^j\right)^{\frac{1}{1-\sigma^j}}. \end{aligned}$$

The above equation shows that Cost^j is homogenous of degree one with respect to the level of production

$$\text{Cost}^j = c^j Y^j, \quad \text{with} \quad c^j = \left(X^j\right)^{\frac{1}{1-\sigma^j}}, \tag{301}$$

where

$$c^j \equiv \left[\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j}{A^j}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{R}{B^j}\right)^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}. \tag{302}$$

When we include technology utilization, eqs. (300)-(301) can be rewritten as follows:

$$\begin{aligned}\tilde{\text{Cost}}^j &= \tilde{W}^j L^j + \tilde{R}^j \tilde{K}^j, \\ &= c^j \tilde{Y}^j,\end{aligned} \tag{303}$$

where the unit cost for producing, denoted by c^j, inclusive of the technology utilization rate reads:

$$c^j = \left[\left(\gamma^j\right)^{\sigma^j}\left(\frac{\tilde{W}^j}{\tilde{A}^j}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{\tilde{R}^j}{\tilde{B}^j}\right)^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}, \tag{304}$$

where $\tilde{W}^j = u^{Z,j} W^j$, $\tilde{R}^j = u^{Z,j} R$, $\tilde{A}^j = u^{Z,j} A^j$, $\tilde{B}^j = u^{Z,j} B^j$. Because the unit cost for producing is homogeneous of degree one, denoting the technology-utilization-adjusted unit cost by UC^j enables us to rewrite total cost described by eq. (303) as follows:

$$\tilde{\text{Cost}}^j = \frac{\text{UC}^j}{u^{Z,j}} \tilde{Y}^j, \tag{305}$$

where

$$\text{UC}^j = \left[\left(\gamma^j\right)^{\sigma^j}\left(\frac{\tilde{W}^j}{A^j}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{\tilde{R}^j}{B^j}\right)^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}. \tag{306}$$

Using the fact that $\left(c^j\right)^{1-\sigma^j} = X^j$, conditional demand for labor (312) can be rewritten as $L^j = Y^j \left(\gamma^j\right)^{\sigma^j} \left(\frac{W^j}{A^j}\right)^{1-\sigma^j} \left(c^j\right)^{\sigma^j}$ which gives the labor share denoted by s_L^j:

$$s_L^j = \frac{W^j L^j}{P^j Y^j} = \left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j}{A^j}\right)^{1-\sigma^j}\left(c^j\right)^{\sigma^j-1}, \tag{307a}$$

$$1 - s_L^j = \frac{R\tilde{K}^j}{P^j Y^j} = \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{R}{B^j}\right)^{1-\sigma^j}\left(c^j\right)^{\sigma^j-1}, \tag{307b}$$

where we used the fact that $P^j = c^j$.

T.2 Short-Run Solutions

Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for L^H and L^N given by (221) into the resource constraint for capital (290), the system of four equations consisting of (288a)-(288c) together with (290) can be solved for sectoral wages W^j and sectoral capital-labor ratios k^j. Log-differentiating (288a)-(288c) together with (290) yields in matrix form:

$$\begin{pmatrix} -\left(\frac{s_L^H}{\sigma^H}\right) & \left(\frac{s_L^N}{\sigma^N}\right) & 0 & 0 \\ \left(\frac{1-s_L^H}{\sigma^H}\right) & 0 & -1 & 0 \\ 0 & \left(\frac{1-s_L^N}{\sigma^N}\right) & 0 & -1 \\ \frac{K^H}{K} & \frac{K^N}{K} & \Psi_{W^H} & \Psi_{W^N} \end{pmatrix} \begin{pmatrix} \hat{k}^H \\ \hat{k}^N \\ \hat{W}^H \\ \hat{W}^N \end{pmatrix}$$

$$= \begin{pmatrix} \hat{P}^N - \hat{P}^H - \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\hat{B}^H + \left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\hat{B}^N - \left(\frac{s_L^H}{\sigma^H}\right)\hat{A}^H + \left(\frac{s_L^N}{\sigma^N}\right)\hat{A}^N + \frac{s_L^H}{\sigma^H}\hat{u}^{K,H} - \frac{s_L^N}{\sigma^N}\hat{u}^{K,N} \\ -\hat{P}^H - \left[\frac{\left(\sigma^H-1\right)+s_L^H}{\sigma^H}\right]\hat{A}^H - \left(\frac{1-s_L^H}{\sigma^H}\right)\hat{B}^H - \left(\frac{1-s_L^H}{\sigma^H}\right)\hat{u}^{K,H} \\ -\hat{P}^N - \left[\frac{\left(\sigma^N-1\right)+s_L^N}{\sigma^N}\right]\hat{A}^N - \left(\frac{1-s_L^N}{\sigma^N}\right)\hat{B}^N - \left(\frac{1-s_L^N}{\sigma^N}\right)\hat{u}^{K,N} \\ \hat{K} - \Psi_{\bar{\lambda}}\hat{\bar{\lambda}} - \sum_j \Psi_{u^{Z,j}}\hat{u}^{Z,j} \end{pmatrix} \tag{308}$$

where we set:

$$\Psi_{W^j} = \frac{K^H}{K}\frac{L^H_{W^j}W^j}{L^H} + \frac{K^N}{K}\frac{L^N_{W^j}W^j}{L^N}, \quad (309a)$$

$$\Psi_{u^{Z,j}} = \frac{K^H}{K}\frac{L^H_{u^{Zj}}u^{Z,j}}{L^H} + \frac{K^N}{K}\frac{L^N_{u^{Z,j}}u^{Z,j}}{L^N}, \quad (309b)$$

$$\Psi_{\bar{\lambda}} = \frac{K^H}{K}\sigma_L + \frac{K^N}{K}\sigma_L = \sigma_L. \quad (309c)$$

The short-run solutions for sectoral wages and capital-labor ratios are:

$$W^j = W^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right), \quad (310a)$$

$$k^j = k^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \quad (310b)$$

Inserting first sectoral wages (310), sectoral hours worked (221) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods, P^N, and the terms of trade:

$$L^j = L^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \quad (311)$$

Totally differentiating output per hours worked (289) leads to:

$$\hat{y}^j = s^j_L \hat{A}^j + \left(1 - s^j_L\right)\left[\hat{B}^j + \hat{u}^{K,j} + \hat{k}^j\right], \quad (312)$$

where s^j_L and $1 - s^j_L$ are the labor and capital income share, respectively, described by eqs. (291)-(292). Plugging solutions for sectoral capital-labor ratios (310) into (312) allows us to solve for sectoral value added per hours worked:

$$y^j = y^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \quad (313)$$

where for example the change in technology utilization adjusted value added per hour worked for tradables reads

$$\begin{aligned} dy^H = & \left[\frac{y^H}{A^H}s^H_L + \frac{y^H}{k^H}\left(1 - s^H_L\right)k^H_{A^H}\right]dA^H \\ + & \left[\frac{y^H}{B^H}s^H_L + \frac{y^H}{k^H}\left(1 - s^H_L\right)k^H_{B^H}\right]dB^H \\ + & \left[\frac{y^H}{u^{K,H}}s^H_L + \frac{y^H}{k^H}\left(1 - s^H_L\right)k^H_{u^{K,H}}\right]du^{K,H} \\ + & \frac{y^H}{k^H}\left(1 - s^H_L\right)dk^H. \end{aligned}$$

Using the fact that $Y^j = y^j L^j$, inserting solutions for y^j (313) and L^j (311), and differentiating, one obtains the solutions for sectoral value added:

$$Y^j = Y^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \quad (314)$$

Using the fact that $K^j = k^j L^j$, inserting solutions for k^j (310b) and L^j (311), differentiating, one obtains the solutions for the sectoral capital stock:

$$K^j = K^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}\right). \quad (315)$$

Capital and Technology Utilization Rates, $u^{K,j}(t)$ and $u^{Z,j}(t)$

Inserting firm's optimal decisions for capital (288a), i.e., $P^j\left(1 - \gamma^j\right)\left(B^j\right)^{\frac{\sigma^j - 1}{\sigma^j}}\left(u^{K,j}k^j\right)^{-\frac{1}{\sigma^j}}\left(y^j\right)^{\frac{1}{\sigma^j}} = R$ into optimal choices for capital utilization (194f)-(194g), and invoking the Euler theorem which leads to $W^H L^H + Ru^{K,H}K^H = P^H Y^H$ to rewrite optimal choices for technology utilization (194h)-(194i), we have:

$$\frac{R(t)u^{Z,H}(t)}{P^H(t)} = \left(1-\gamma^H\right) u^{Z,H}(t) \left(B^H(t)\right)^{\frac{\sigma^H-1}{\sigma^H}} \left(u^{K,H}(t)k^H(t)\right)^{-\frac{1}{\sigma^H}} \left(y^H(t)\right)^{\frac{1}{\sigma^H}} = \xi_1^H + \xi_2^H \left(u^{K,H}(t)-1\right), \tag{316a}$$

$$\frac{R(t)u^{Z,N}(t)}{P^N(t)} = \left(1-\gamma^N\right) \left(B^N(t)\right)^{\frac{\sigma^N-1}{\sigma^N}} \left(u^{K,N}(t)k^N(t)\right)^{-\frac{1}{\sigma^N}} \left(y^N(t)\right)^{\frac{1}{\sigma^N}} = \left[\xi_1^N + \xi_2^N \left(u^{K,N}(t)-1\right)\right], \tag{316b}$$

$$Y^H(t) = \chi_1^H + \chi_2^H \left(u^{Z,H}(t)-1\right)], \tag{316c}$$

$$Y^N(t) = \chi_1^N + \chi_2^N \left(u^{Z,N}(t)-1\right). \tag{316d}$$

Log-linearizing optimal decisions on capital and technology utilization rates described by (316a)-(316d) leads to in a matrix form:

$$\begin{pmatrix} \left[\left(\frac{\xi_2^H}{\xi_1^H} + \frac{s_L^H}{\sigma^H}\right) + \frac{s_L^H}{\sigma^H}\frac{k^H_{u^{K,H}}}{k^H}\right] & \frac{s_L^H}{\sigma^H}\frac{k^H_{u^{K,N}}}{k^H} & \left[\frac{s_L^H}{\sigma^H}\frac{k^H_{u^{Z,H}}}{k^H} - 1\right] & \frac{s_L^H}{\sigma^H}\frac{k^H_{u^{Z,N}}}{k^H} \\ \frac{s_L^N}{\sigma^N}\frac{k^N_{u^{K,H}}}{k^N} & \left[\left(\frac{\xi_2^N}{\xi_1^N} + \frac{s_L^N}{\sigma^N}\right) + \frac{s_L^N}{\sigma^N}\frac{k^N_{u^{K,N}}}{k^N}\right] & \frac{s_L^N}{\sigma^N}\frac{k^N_{u^{Z,H}}}{k^N} & \left[\frac{s_L^N}{\sigma^N}\frac{k^N_{u^{Z,N}}}{k^N} - 1\right] \\ -\frac{Y^H_{u^{K,H}}}{Y^H} & -\frac{Y^H_{u^{K,N}}}{Y^H} & \left[\frac{\chi_2^H}{\chi_1^H} - \frac{Y^H_{u^{Z,H}}}{Y^H}\right] & -\frac{Y^H_{u^{Z,N}}}{Y^H} \\ -\frac{Y^N_{u^{K,H}}}{Y^N} & -\frac{Y^N_{u^{K,N}}}{Y^N} & -\frac{Y^N_{u^{Z,H}}}{Y^N} & \left[\frac{\chi_2^N}{\chi_1^N} - \frac{Y^N_{u^{Z,N}}}{Y^N}\right] \end{pmatrix} \begin{pmatrix} \hat{u}^{K,H} \\ \hat{u}^{K,N} \\ \hat{u}^{Z,H} \\ \hat{u}^{Z,N} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{s_L^H}{\sigma^H}\frac{k^H_{X^H}}{k^H}dX^H + \frac{s_L^H}{\sigma^H}\left[\frac{1}{A^H} - \frac{k^H_{A^H}}{k^H}\right]dA^H + \left[\left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\frac{1}{B^H} - \frac{s_L^H}{\sigma^H}\frac{k^H_{B^H}}{k^H}\right]dB^H \\ -\frac{s_L^N}{\sigma^N}\frac{k^N_{X^N}}{k^N}dX^N + \frac{s_L^N}{\sigma^N}\left[\frac{1}{A^N} - \frac{k^N_{A^N}}{k^N}\right]dA^N + \left[\left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\frac{1}{B^N} - \frac{s_L^N}{\sigma^N}\frac{k^N_{B^N}}{k^N}\right]dB^N \\ \frac{Y^H_{X^H}}{Y^H}dX^H + \frac{Y^H_{A^H}}{Y^H}dA^H + \frac{Y^H_{B^H}}{Y^H}dB^H \\ \frac{Y^N_{X^N}}{Y^N}dX^N + \frac{Y^N_{A^N}}{Y^N}dA^N + \frac{Y^N_{B^N}}{Y^N}dB^N \end{pmatrix}, \qquad (31$$

where $X^H = P^H, P^N, K, \bar{\lambda}, A^N, B^N$ and $X^N = P^H, P^N, K, \bar{\lambda}, A^H, B^H$.

The short-run solutions for capital and technology utilization rates are:

$$u^{K,j} = u^{K,j}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right), \tag{318a}$$

$$u^{Z,j} = u^{Z,j}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right). \tag{318b}$$

Intermediate Solutions for k^j, W^j, L^j, Y^j, K^j

Plugging back solutions for capital and technology utilization rates (318a)-(318b) into (310a), (310b), (311), (313), (314), (315) leads to intermediate solutions for sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, and sectoral capital stocks:

$$W^j, k^j, L^j, Y^j, K^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right). \tag{319}$$

Market Clearing Condition for Non-Tradables

The role of the price of non-tradables in terms of foreign-produced traded goods is to clear the non-traded goods market:

$$u^{Z,N}(t)Y^N(t) = C^N(t) + G^N(t) + J^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t). \tag{320}$$

Inserting solutions for C^N, J^N, Y^N given by (219), i.e., $C^N\left(\bar{\lambda}, P^N, P^H\right)$, (240), i.e., $J^N = J^N\left(K, Q, P^N, P^H\right)$, (319), i.e., $Y^N = Y^N\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right)$, the non-traded goods market clearing condition (320) can be rewritten as follows:

$$\begin{aligned} & u^{Z,N}Y^N\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right) = C^N\left(\bar{\lambda}, P^N, P^H\right) + G^N + J^N\left(K, Q, P^N, P^H\right) \\ + \quad & C^{K,N}\left[u^{K,N}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right)\right] K^N\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right) \\ & + C^{Z,N}\left(u^{Z,N}\right). \end{aligned} \tag{321}$$

Linearizing (321) leads to:

$$Y^N du^{Z,N}(t) + dY^N(t) = dC^N(t) + dG^N(t) + dJ^N(t) + K^N\xi_1^N du^{K,N}(t) + \chi_1^N du^{Z,N}(t), \tag{322}$$

where the terms $Y^N du^{Z,N}(t)$ and $\chi_1^N du^{Z,N}(t)$ cancel out because eq. (230d) evaluated at the steady-state implies $Y^N = \chi_1^N$.

Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced traded goods in terms of foreign-produced traded goods or the terms of trade is to clear the home-produced traded goods market:

$$u^{Z,H}(t)Y^H(t) = C^H(t) + G^H(t) + J^H(t) + X^H(t) + C^{K,H}(t)K^H(t) + C^{Z,H}(t), \tag{323}$$

where X^H stands for exports which are negatively related to the terms of trade:

$$X^H = \varphi_X \left(P^H\right)^{-\phi_X}, \tag{324}$$

where ϕ_X is the elasticity of exports with respect to the terms of trade.

Inserting solutions for C^H, J^H, Y^H given by (219), (220a), (319), respectively, the traded goods market clearing condition (323) can be rewritten as follows:

$$\begin{aligned} & u^{Z,H}Y^H\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right) = C^H\left(\bar{\lambda}, P^N, P^H\right) + G^H + J^H\left(K, Q, P^N, P^H\right) \\ + \; & C^{K,H}\left[u^{K,H}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right)\right] K^H\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right)) \\ & + C^{Z,H}\left(u^{Z,H}\right). \end{aligned} \tag{325}$$

Linearizing (325) leads to:

$$Y^H du^{Z,H}(t) + dY^H(t) = dC^H(t) + dG^H(t) + dJ^H(t) + dX^H(t) + K^H \xi_1^H du^{K,H}(t) + \chi_1^H du^{Z,H}(t), \tag{326}$$

where the terms $Y^H du^{Z,H}(t)$ and $\chi_1^H du^{Z,H}(t)$ cancel out because eq. (230c) evaluated at the steady-state implies $Y^H = \chi_1^H$.

Return on domestic capital

The return on domestic capital is:

$$R = P^H\left(1-\gamma^H\right)\left(B^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(u^{K,H}k^H\right)^{-\frac{1}{\sigma^H}}\left(y^H\right)^{\frac{1}{\sigma^H}}. \tag{327}$$

Differentiating (327) and making use of (312) leads to:

$$\hat{R} = \hat{P}^H - \frac{s_L^H}{\sigma^H}\left(\hat{k}^H + \hat{u}^{K,H}\right) + \frac{s_L^H}{\sigma^H}\hat{A}^H + \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\hat{B}^H. \tag{328}$$

Inserting the short-run static solutions for the capital-labor ratio k^H and the capital utilization rate (333), eq. (327) can be solved for the return on domestic capital:

$$R = R\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right). \tag{329}$$

Solving for Relative Prices

As shall be useful below, we write out a number of useful notations:

$$\Psi^N_{P^N} = Y^N_{P^N} - C^N_{P^N} - J^N_{P^N} - K^N\xi_1^N u^{K,N}_{P^N}, \tag{330a}$$
$$\Psi^N_{P^H} = Y^N_{P^H} - C^N_{P^H} - J^N_{P^H} - K^N\xi_1^N u^{K,N}_{P^H}, \tag{330b}$$
$$\Psi^N_{K} = Y^N_{K} - J^N_{K} - K^N\xi_1^N u^{K,N}_{K}, \tag{330c}$$
$$\Psi^N_{A^j} = Y^N_{A^j} - K^N\xi_1^N u^{K,N}_{A^j}, \tag{330d}$$
$$\Psi^N_{B^j} = Y^N_{B^j} - K^N\xi_1^N u^{K,N}_{B^j}, \tag{330e}$$
$$\Psi^N_{\lambda} = Y^N_{\lambda} - C^N_{\lambda} - -K^N\xi_1^N u^{K,N}_{\lambda}, \tag{330f}$$
$$\Psi^H_{P^N} = Y^H_{P^N} - C^H_{P^N} - J^H_{P^N} - K^H\xi_1^H u^{K,H}_{P^N}, \tag{330g}$$
$$\Psi^H_{P^H} = Y^H_{P^H} - C^H_{P^H} - J^H_{P^H} - X^H_{P^H} - K^H\xi_1^H u^{K,H}_{P^H}, \tag{330h}$$
$$\Psi^H_{K} = Y^H_{K} - J^H_{K} - K^H\xi_1^H u^{K,H}_{K}, \tag{330i}$$
$$\Psi^H_{A^j} = Y^H_{A^j} - K^H\xi_1^H u^{K,H}_{A^j}, \tag{330j}$$
$$\Psi^H_{B^j} = Y^H_{B^j} - K^H\xi_1^H u^{K,H}_{B^j}, \tag{330k}$$
$$\Psi^H_{\lambda} = Y^H_{\lambda} - C^H_{\lambda} - K^H\xi_1^H u^{K,H}_{\lambda}. \tag{330l}$$

Linearized versions of market clearing conditions described by eq. (322) and eq. (326) can be rewritten in a matrix form:

$$
\begin{pmatrix} \Psi^N_{P^N} & \Psi^N_{P^H} \\ \Psi^H_{P^N} & \Psi^H_{P^H} \end{pmatrix} \begin{pmatrix} dP^N \\ dP^H \end{pmatrix}
$$

$$
= \begin{pmatrix} -\Psi^N_K dK + J^N_Q dQ + G^N_G dG - \sum_{j=H}^N \Psi^N_{A^j} dA^j - \sum_{j=H}^N \Psi^N_{B^j} dB^j - \Psi^N_\lambda d\bar{\lambda} \\ -\Psi^H_K dK + J^H_Q dQ + G^H_G dG - \sum_{j=H}^N \Psi^H_{A^j} dA^j - \sum_{j=H}^N \Psi^H_{B^j} dB^j - \Psi^H_\lambda d\bar{\lambda} \end{pmatrix} \tag{331}
$$

The short-run solutions for capital and technology utilization rates are:

$$
P^N = P^N \left(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N\right), \tag{332a}
$$

$$
P^H = P^H \left(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N\right). \tag{332b}
$$

T.3 Solving the Model

In our model, there are five state variables, namely the capital stock K, labor-augmenting productivity, A^H, A^N, capital-augmenting productivity, B^H, B^N, and one control variable, namely the shadow price of the capital stock Q. To solve the model, we have to express all variables in terms of state and control variables. Plugging back solutions for the relative price of non-tradables (332a) and the terms of trade (332b) into consumption in sectoral goods (219), investment inputs (240), sectoral output (319), capital and technology utilization rates (318a)-(318b) leads to solutions for sectoral consumption, sectoral inputs for capital goods, sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, sectoral capital stocks, return on capital:

$$
C^j, J^j, W^j, k^j, L^j, Y^j, K^j, v, u^{K,j}, u^{Z,j}, R\left(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N\right). \tag{333}
$$

Remembering that the non-traded input J^N used to produce the capital good is described by $(1-\iota)\left(\frac{P^N}{P_J}\right)^{-\phi_J} J$ (see eq. (203b)) with $J = I + \frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)^2 K$, using the fact that $J^N = Y^N - C^N - G^N$ and inserting $I = \dot{K} + \delta_K$, the capital accumulation equation reads as follows:

$$
\dot{K} = \frac{u^{Z,N} Y^N - C^N - G^N - C^{K,N} K^N - C^{Z,N}}{(1-\iota)\left(\frac{P^N}{P_J}\right)^{-\phi_J}} - \delta_K K - \frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)^2 K. \tag{334}
$$

Inserting first solutions for non-traded output, consumption in non-tradables, demand for non-traded input, non-traded capital and technology utilization rates described by eq. (333) together with optimal investment decision (239a) into the physical capital accumulation equation (334), and plugging the short-run solution for the return on domestic capital (329) into the dynamic equation for the shadow value of capital stock (194e), the dynamic system reads as follows:

$$
\begin{aligned}
\dot{K} &\equiv \Upsilon\left(K, Q, G, A^j, B^j\right), \\
&= \frac{Y^N\left(K, Q, G, A^j, B^j\right) - C^N\left(K, Q, G, A^j, B^j\right) - G^N(G)}{(1-\iota)\left[\frac{P^N(K,Q,G,A^j,B^j)}{P_J(K,Q,G,A^j,B^j)}\right]^{-\phi_J}} \\
&\quad \frac{-C^{K,N}\left[u^{K,N}\left(K, Q, G, A^j, B^j\right)\right]K^N - C^{Z,N}\left[u^{Z,N}\left(K, Q, G, A^j, B^j\right)\right]}{(1-\iota)\left[\frac{P^N(K,Q,G,A^j,B^j)}{P_J(K,Q,G,A^j,B^j)}\right]^{-\phi_J}} \\
&\quad -\delta_K K - \frac{K}{2\kappa}\left[\frac{Q}{P_J\left(K, Q, G, A^j, B^j\right)} - 1\right]^2, \qquad (335a) \\
\dot{Q} &\equiv \Sigma\left(K, Q, G, A^j, B^j\right), \\
&= \left(r^\star + \delta_K\right)Q - \left[\frac{R\left(K, Q, G, A^j, B^j\right)}{K}\right. \\
&\times \sum_{j=H,N} u^{K,j}\left(K, Q, G, A^j, B^j\right)u^{Z,j}\left(K, Q, G, A^j, B^j\right)K^j\left(K, Q, G, A^j, B^j\right) \\
&\quad - \sum_{j=H,N} P^j\left(K, Q, G, A^j, B^j\right)C^{K,j}\left[u^{K,j}\left(K, Q, G, A^j, B^j\right)\right]\frac{K^j\left(K, Q, G, A^j, B^j\right)}{K} \qquad (335b) \\
&\quad \left. + P_J\left[P^H(.), P^N(.)\right]\frac{\kappa}{2}v(.)\left(v(.) + 2\delta_K\right)\right]. \qquad (335c)
\end{aligned}
$$

Let us denote by Υ_X, the partial derivative evaluated at the steady-state of the capital accumulation equation w.r.t. $X = K, Q, G, A^j, B^j$. Partial derivatives evaluated at the steady-state are described by (265a)-(265c) together with:

$$
\Upsilon_{A^j} \equiv \frac{\partial \dot{K}}{\partial A^j} = \frac{J}{J^N}\left(Y^N_{A^j} - C^N_{A^j} - K^N\xi^N_1 u^{K,N}_{A^j}\right) + \alpha_J\phi_J J\left(\frac{P^N_{A^j}}{P^N} - \alpha^H_J\frac{P^H_{A^j}}{P^H}\right), \qquad (336a)
$$

$$
\Upsilon_{B^j} \equiv \frac{\partial \dot{K}}{\partial B^j} = \frac{J}{J^N}\left(Y^N_{B^j} - C^N_{B^j} - K^N\xi^N_1 u^{K,N}_{B^j}\right) + \alpha_J\phi_J J\left(\frac{P^N_{B^j}}{P^N} - \alpha^H_J\frac{P^H_{B^j}}{P^H}\right), \qquad (336b)
$$

Let us denote by Σ_X, the partial derivative evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t. $X = K, Q, G, A^j, B^j$. Partial derivatives evaluated at the steady-state are described by (266a)-(266c) together with:

$$
\begin{aligned}
\Sigma_{A^j} \equiv \frac{\partial \dot{Q}}{\partial A^j} &= -\left\{R_{A^j} + \frac{R}{K}\sum_{j=H,N}\left[K^j u^{K,j}_{A^j} + K^j u^{Z,j}_{A^j}\right]\right. \\
&\quad + \frac{R}{K}\left(K^H_{A^j} + K^N_{A^j}\right) - \sum_{j=H,N}\frac{P^j K^j}{K}\xi^j_1 u^{K,j}_{A^j} \\
&\quad \left. + P_J\kappa v_{A^j}\delta_K\right\} > 0. \qquad (337a) \\
\Sigma_{B^j} \equiv \frac{\partial \dot{Q}}{\partial B^j} &= -\left\{R_{B^j} + \frac{R}{K}\sum_{j=H,N}\left[K^j u^{K,j}_{B^j} + K^j u^{Z,j}_{B^j}\right]\right. \\
&\quad + \frac{R}{K}\left(K^H_{B^j} + K^N_{B^j}\right) - \sum_{j=H,N}\frac{P^j K^j}{K}\xi^j_1 u^{K,j}_{B^j} \\
&\quad \left. + P_J\kappa v_{B^j}\delta_K\right\} > 0. \qquad (337b)
\end{aligned}
$$

T.4 Current Account Equation and Intertemporal Solvency Condition

Following the same steps as in subsection S.6, the current account reads as:

$$
\dot{N}(t) = r^\star N(t) + P^H(t)X^H(t) - M^F(t), \qquad (338)
$$

where $X^H = Y^H - C^H - G^H - J^H - C^{K,H} K^H - C^{Z,H}$ stands for exports of home goods and we denote by M^F imports of foreign consumption and investment goods:

$$M^F(t) = C^F(t) + G^F(t) + J^F(t). \tag{339}$$

Inserting (319) into (338) and the solution for P^H described by eq. (332b) into $X^H = X^H\left(P^H\right)$ leads to:

$$\begin{aligned}
\dot{N} &\equiv r^\star N + \Xi\left(K, Q, G, A^H, A^N, B^H, B^N\right), \\
&= r^\star N + P^H\left(K, Q, G, A^H, A^N, B^H, B^N\right) X^H\left(K, Q, G, A^H, A^N, B^H, B^N\right) \\
&\quad -M^F\left(K, Q, G, A^H, A^N, B^H, B^N\right).
\end{aligned} \tag{340}$$

Let us denote by Ξ_X the partial derivative evaluated at the steady-state of the dynamic equation for the current account w.r.t. to $X = K, Q, G, A^j, B^j$. Partial derivatives evaluated at the steady-state are described by (273a)-(273c) together with:

$$\Xi_{A^j} \equiv \frac{\partial \dot{N}}{\partial A^j} = (1-\phi_X)\, X^H P^H_{A^j} - M^F_{A^j}, \tag{341a}$$

$$\Xi_{B^j} \equiv \frac{\partial \dot{N}}{\partial B^j} = (1-\phi_X)\, X^H P^H_{B^j} - M^F_{B^j}. \tag{341b}$$

T.5 The Technology Frontier

Since we relax the assumption of Hicks-neutral technological change, we have to relate changes in labor- and capital-augmenting efficiency, i.e., $\hat{\tilde{A}}^j(t)$ and $\hat{\tilde{B}}^j(t)$, respectively, to the percentage deviation of capital-utilization-adjusted TFP in sector j, i.e., $\hat{Z}^j(t)$, in order to be consistent with our empirical strategy. A natural way to map $\tilde{A}^j$ and $\tilde{B}^j$ into Z^j is to assume that besides optimally choosing factor inputs, firms also optimally choose the technology of production function. Following Caselli and Coleman [2006] and Caselli [2016], the menu of possible choices of production functions is represented by a set of possible $(\tilde{A}^j, \tilde{B}^j)$ pairs. We assume that firms in sector j choose labor and capital efficiency along the technology frontier which is assumed to take a CES form:

$$\left[\gamma_Z^j \left(u^{Z,j}(t) A^j(t)\right)^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} + \left(1-\gamma_Z^j\right)\left(u^{Z,j}(t) B^j(t)\right)^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}}\right]^{\frac{\sigma_Z^j}{\sigma_Z^j - 1}} \leq Z^j(t), \tag{342}$$

where $Z^j > 0$ is the height of the technology frontier, $0 < \gamma_Z^j < 1$ is the weight of labor efficiency in TFP and $\sigma_Z^j > 0$ corresponds to the elasticity of substitution between labor and capital efficiency. Using the fact that $Z^j(t) = u^{Z,j}(t)\bar{Z}^j$ and totally differentiating (342) leads to

$$\begin{aligned}
0 &= \gamma_Z^j \left(A^j(t)\right)^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \hat{A}^j(t) + \left(1-\gamma_Z^j\right)\left(B^j(t)\right)^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \hat{B}^j(t), \\
\frac{\hat{B}^j(t)}{\hat{A}^j(t)} &= -\frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}}.
\end{aligned} \tag{343}$$

Eq. (343) measures the number of capital-augmenting efficiency units the firm must give up to increase labor-augmenting productivity by one unit.

Firms choose A^j and B^j along the technology frontier so as to minimize the unit cost function (302) which we repeat for convenience:

$$c^j \equiv \left[\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j} + \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{R(t)}{B^j(t)}\right)^{1-\sigma^j}\right]^{\frac{1}{1-\sigma^j}}, \tag{344}$$

subject to (342) which holds as an equality. Differentiating (116) w.r.t. $A^j(t)$ and $B^j(t)$ (while keeping W^j and R fixed) and setting the expression to zero leads to:

$$
\begin{aligned}
\hat{c}^j(t) &= -\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j}\left(c^j(t)\right)^{\sigma^j-1}\hat{A}^j(t) - \left(1-\gamma^j\right)^{\sigma^j}\left(\frac{R(t)}{B^j(t)}\right)^{1-\sigma^j}\left(c^j(t)\right)^{\sigma^j-1}\hat{B}^j(t), \\
\frac{\hat{B}^j(t)}{\hat{A}^j(t)} &= -\left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j}\left(\frac{W^j(t)}{R(t)}\right)^{1-\sigma^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{1-\sigma^j}. \qquad (345)
\end{aligned}
$$

Eq. (345) measures the number of capital-augmenting efficiency units the firm must give up following an increase in labor-augmenting productivity by one unit to keep the unit cost for producing unchanged.

Performing the cost minimization of (344) subject to (342) amounts to equating (343) with (345) which leads to the following optimal choice of technology:

$$
\begin{aligned}
\left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j}\left(\frac{W^j(t)}{R(t)}\right)^{1-\sigma^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{1-\sigma^j} &= \frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}}, \\
\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}\left(\frac{B^j(t)}{A^j(t)}\right)^{-\left(\frac{1-\sigma_Z^j}{\sigma_Z^j}\right)\frac{1}{\sigma^j}} &= \left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)^{\frac{1}{\sigma^j}}\left(\frac{1-\gamma^j}{\gamma^j}\right)\left(\frac{W^j(t)}{R(t)}\right)^{-\frac{1-\sigma^j}{\sigma^j}} \qquad (346)
\end{aligned}
$$

Eq. (346) states that it is optimal for firms to bias factor efficiency toward the most expensive factor as long as $\frac{1-\sigma_Z^j}{\sigma_Z^j} < 1-\sigma^j$.

Using the fact that $\left(\gamma^j\right)^{\sigma^j}\left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j}\left(c^j(t)\right)^{\sigma^j-1} = s_L^j(t)$ (see eq. (307a)), eq. (345) can be rewritten as $-s_L^j\hat{A}^j(t) - \left(1-s_L^j\right)\hat{B}^j(t) = -\hat{c}^j(t)$. Setting this equality to zero and making use of (343) leads to:

$$
\frac{\gamma_Z^j}{1-\gamma_Z^j}\left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}} = \frac{s_L^j(t)}{1-s_L^j(t)} \equiv S^j(t). \qquad (347)
$$

Eq. (347) can be solved for $s_L^j(t)$:

$$
\begin{aligned}
s_L^j(t) &= \frac{\gamma_Z^j\left(A^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}}{\gamma_Z^j\left(A^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} + \left(1-\gamma_Z^j\right)\left(B^j(t)\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}}, \\
&= \gamma_Z^j\left(\frac{A^j(t)}{\bar{Z}^j}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}, \qquad (348)
\end{aligned}
$$

where we made use of (342) to obtain the last line and we used the fact that $Z^j(t) = u^{Z,j}(t)\bar{Z}^j$.

Log-linearizing (342) in the neighborhood of the initial steady-state and making use of eq. (348) leads to:

$$
\begin{aligned}
0 &= \gamma_Z^j\left(\frac{A^j}{\bar{Z}^j}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}\hat{A}^j(t) + \left(1-\gamma_Z^j\right)\left(\frac{B^j}{\bar{Z}^j}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}\hat{B}^j(t), \\
0 &= s_L^j\hat{A}^j(t) + \left(1-s_L^j\right)\hat{B}^j(t), \qquad (349)
\end{aligned}
$$

where we used the fact that $\bar{Z}^j$ is constant.

Log-linearizing (293) in the neighborhood of the initial steady-state leads to:

$$
\hat{B}^j(t) - \hat{A}^j(t) = \left(\frac{\sigma^j}{1-\sigma^j}\right)\hat{S}^j - \left(\hat{u}^{K,j}(t) + \hat{k}^j(t)\right). \qquad (350)
$$

The system which comprises eq. (349) and eq. (350) can be solved for the percentage deviation of factor-augmenting efficiency relative to the initial steady-state:

$$\hat{A}^j(t) = -\left(1 - s_L^j\right)\left[\left(\frac{\sigma^j}{1-\sigma^j}\right)\hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t)\right], \tag{351a}$$

$$\hat{B}^j(t) = s_L^j\left[\left(\frac{\sigma^j}{1-\sigma^j}\right)\hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t)\right]. \tag{351b}$$

Eq. (351a) and eq. (351b) correspond to eq. (37a) and eq. (37b) in the main text.

U Solving for Temporary Government Spending Shocks

U.1 Setting the Dynamics of Government Shock and Factor-Augmenting Efficiency

Because the endogenous response of government spending to an exogenous fiscal shock is hump-shaped, we assume that government consumption as a percentage of GDP evolves according to the following dynamic equation:

$$\frac{dG(t)}{Y} = e^{-\xi t} - (1-g)\, e^{-\chi t}, \tag{352}$$

where $dG(t) = G(t) - G$ is the deviation of government consumption relative to the initial steady-state, $g > 0$ parametrizes the magnitude of the exogenous fiscal shock, $\xi > 0$ and $\chi > 0$ are (positive) parameters which are set in order to capture the non-monotonic endogenous response of $G(t)$. We assume that the rise in government consumption is split between non-traded and traded goods and government consumption in traded goods is split between government consumption in home-produced traded goods and foreign-produced traded goods. Denoting the non-tradable content of government spending by ω_{G^N} and the home traded goods content of government spending by $\omega_{G^H} = \frac{P^H G^H}{G}$, formally we have:

$$\frac{dG(t)}{Y} = \omega_{G^N}\frac{dG(t)}{Y} + \omega_{G^H}\frac{dG(t)}{Y} + \omega_{G^F}\frac{dG(t)}{Y}, \tag{353}$$

where $\omega_{G^F} = \frac{G^F}{G}$ is the import content of government spending. In line with the evidence we document in Appendix F, ω_{G^N} refers to the non-tradable content of government consumption as well as the intensity of the government spending shock in non-traded goods.

We further specify a dynamic adjustment for $A^j(t)$ and $B^j(t)$:

$$\frac{dA^j(t)}{A^j} = e^{-\xi_A^j t} - \left(1 - a^j\right) e^{-\chi_A^j t}, \tag{354a}$$

$$\frac{dB^j(t)}{B^j} = e^{-\xi_B^j t} - \left(1 - b^j\right) e^{-\chi_B^j t}, \tag{354b}$$

where a^j (b^j) parameterizes the impact response of labor- (capital-) augmenting technological change; $\xi_A^j > 0$ ($\xi_B^j > 0$) and $\chi_A^j > 0$ ($\chi_B^j > 0$) are (positive) parameters which are set in order to reproduce the dynamic adjustment of labor-augmenting (capital-augmenting) technological change.

U.2 Solving for Temporary Government Spending Shocks

Linearizing (335a)-(335c) in the neighborhood of the steady-state, we get in a matrix form:

$$\begin{pmatrix} \dot{K}(t) \\ \dot{Q}(t) \end{pmatrix} = \begin{pmatrix} \Upsilon_K & \Upsilon_Q \\ \Sigma_K & \Sigma_Q \end{pmatrix}\begin{pmatrix} dK(t) \\ dQ(t) \end{pmatrix} + \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^N \Upsilon_{A^j} dA^j(t) + \sum_{j=H}^N \Upsilon_{B^j} dB^j(t) \\ \Sigma_G dG(t) + \sum_{j=H}^N \Sigma_{A^j} dA^j(t) + \sum_{j=H}^N \Sigma_{B^j} dB^j(t) \end{pmatrix}, \tag{355}$$

where the coefficients of the Jacobian matrix are partial derivatives evaluated at the steady-state, e.g., $\Upsilon_X = \frac{\partial \Upsilon}{\partial X}$ with $X = K, Q$, and the direct effects of an exogenous change in government spending on K and Q are described by $\Upsilon_G = \frac{\partial \Upsilon}{\partial G}$ and $\Sigma_G = \frac{\partial \Sigma}{\partial G}$, also evaluated at the steady-state.

As shall be useful below to write the solutions in a compact form, let us define the following terms:

$$\Phi_1^G = \left[\left(\Upsilon_K - \nu_2 \right) \Upsilon_G + \Upsilon_Q \Sigma_G \right], \tag{356a}$$
$$\Phi_2^G = \left[\left(\Upsilon_K - \nu_1 \right) \Upsilon_G + \Upsilon_Q \Sigma_G \right], \tag{356b}$$
$$\Phi_1^{A^j} = \left[\left(\Upsilon_K - \nu_2 \right) \Upsilon_{A^j} + \Upsilon_Q \Sigma_{A^j} \right], \tag{356c}$$
$$\Phi_2^{A^j} = \left[\left(\Upsilon_K - \nu_1 \right) \Upsilon_{A^j} + \Upsilon_Q \Sigma_{A^j} \right], \tag{356d}$$
$$\Phi_1^{B^j} = \left[\left(\Upsilon_K - \nu_2 \right) \Upsilon_{B^j} + \Upsilon_Q \Sigma_{B^j} \right], \tag{356e}$$
$$\Phi_2^{B^j} = \left[\left(\Upsilon_K - \nu_1 \right) \Upsilon_{B^j} + \Upsilon_Q \Sigma_{B^j} \right]. \tag{356f}$$

We denote by $V = \left(V^1, V^2 \right)$ the matrix of eigenvectors with $V^{i,\prime} = \left(1, \omega_2^i \right)$ and we denote by V^{-1} the inverse matrix of V. Let us define:

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \equiv V^{-1} \begin{pmatrix} dK(t) \\ dQ(t) \end{pmatrix}. \tag{357}$$

Differentiating w.r.t. time, one obtains:

$$\begin{aligned} \begin{pmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{pmatrix} &= \begin{pmatrix} \nu_1 & 0 \\ 0 & \nu_2 \end{pmatrix} \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} + V^{-1} \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^N \Upsilon_{A^j} dA^j(t) + \sum_{j=H}^N \Upsilon_{B^j} dB^j(t) \\ \Sigma_G dG(t) + \sum_{j=H}^N \Sigma_{A^j} dA^j(t) + \sum_{j=H}^N \Sigma_{B^j} dB^j(t) \end{pmatrix}, \\ &= \begin{pmatrix} \nu_1 X_1(t) \\ \nu_2 X_2(t) \end{pmatrix} + \frac{1}{\nu_1 - \nu_2} \begin{pmatrix} \Phi_1^G dG(t) + \sum_{j=H}^N \Phi_1^{A^j} dA^j(t) + \sum_{j=H}^N \Phi_1^{B^j} dB^j(t) \\ -\Phi_2^G dG(t) - \sum_{j=H}^N \Phi_2^{A^j} dA^j(t) - \sum_{j=H}^N \Phi_2^{B^j} dB^j(t) \end{pmatrix}. \end{aligned} \tag{358}$$

As will be useful below, in order to express solutions in a compact form, we set:

$$\Gamma_1^G = -\frac{\Phi_1^G Y}{\nu_1 - \nu_2} \frac{1}{\nu_1 + \xi}, \tag{359a}$$
$$\Gamma_2^G = -\frac{\Phi_2^G Y}{\nu_1 - \nu_2} \frac{1}{\nu_2 + \xi}, \tag{359b}$$
$$\Theta_1^G = (1-g) \frac{\nu_1 + \xi}{\nu_1 + \chi}, \tag{359c}$$
$$\Theta_2^G = (1-g) \frac{\nu_2 + \xi}{\nu_2 + \chi}. \tag{359d}$$
$$\tag{359e}$$

and for $X^j = A^j, B^j$:

$$\Gamma_1^{X^j} = -\frac{\Phi_1^{X^j} X^j}{\nu_1 - \nu_2} \frac{1}{\nu_1 + \xi_X^j}, \tag{360a}$$
$$\Gamma_2^{X^j} = -\frac{\Phi_2^{X^j} X^j}{\nu_1 - \nu_2} \frac{1}{\nu_2 + \xi_X^j}, \tag{360b}$$
$$\Theta_1^{X^j} = \left(1 - x^j \right) \frac{\nu_1 + \xi}{\nu_1 + \chi_X^j}, \tag{360c}$$
$$\Theta_2^{X^j} = \left(1 - x^j \right) \frac{\nu_2 + \xi}{\nu_2 + \chi_X^j}, \tag{360d}$$
$$\tag{360e}$$

where $x^j = a^j, b^j$.

Solving for $X_1(t)$ gives:

$$\begin{aligned}
X_1(t) &= e^{\nu_1 t}\left\{X_1(0) + \frac{\Phi_1^G}{\nu_1-\nu_2}\int_0^t dG(\tau)e^{-\nu_1\tau}d\tau + \sum_{X^j}\frac{\Phi_1^{X^j}}{\nu_1-\nu_2}\int_0^t dX^j(\tau)e^{-\nu_1\tau}d\tau\right\}, \\
&= e^{\nu_1 t}\Bigg\{X_1(0) + \frac{\Phi_1^G Y}{\nu_1-\nu_2}\int_0^t\left[e^{-(\xi+\nu_1)\tau} - (1-g)\,e^{-(\chi+\nu_1)\tau}\right]d\tau \\
&+ \sum_{X^j}\frac{\Phi_1^{X^j}X^j}{\nu_1-\nu_2}\int_0^t\left[e^{-\left(\xi_X^j+\nu_1\right)\tau} - \left(1-x^j\right)e^{-\left(\chi_X^j+\nu_1\right)\tau}\right]d\tau\Bigg\}, \\
&= e^{\nu_1 t}X_1(0) + \frac{\Phi_1^G Y}{\nu_1-\nu_2}\left[\left(\frac{e^{\nu_1 t}-e^{-\xi t}}{\nu_1+\xi}\right) - (1-g)\left(\frac{e^{\nu_1 t}-e^{-\chi t}}{\nu_1+\chi}\right)\right] \\
&+ \sum_{X^j}\frac{\Phi_1^{X^j}X^j}{\nu_1-\nu_2}\left[\left(\frac{e^{\nu_1 t}-e^{-\xi_X^j t}}{\nu_1+\xi_X^j}\right) - \left(1-x^j\right)\left(\frac{e^{\nu_1 t}-e^{-\chi_X^j t}}{\nu_1+\chi_X^j}\right)\right], \\
&= e^{\nu_1 t}\left[X_1(0) - \Gamma_1^G\left(1-\Theta_1^G\right) - \sum_{X^j}\Gamma_1^{X^j}\left(1-\Theta_1^{X^j}\right)\right] \\
&+ \Gamma_1^G\left(e^{-\xi t} - \Theta_1^G e^{-\chi t}\right) + \sum_{X^j}\Gamma_1^{X^j}\left(e^{-\xi_X^j t} - \Theta_1^{X^j}e^{-\chi_X^j t}\right),
\end{aligned} \tag{361}$$

where Γ_1^G and Θ_1^G are given by (359a) and (359c), respectively, and $\Gamma_1^{X^j}$ and $\Theta_1^{X^j}$ are given by (360a) and (360c), respectively.

Solving for $X_2(t)$ gives:

$$X_2(t) = e^{\nu_2 t}\left\{X_2(0) - \frac{\Phi_2^G}{\nu_1-\nu_2}\int_0^t dG(\tau)e^{-\nu_2\tau}d\tau - \sum_{X^j}\frac{\Phi_2^{X^j}}{\nu_1-\nu_2}\int_0^t dX^j(\tau)e^{-\nu_2\tau}d\tau\right\}. \tag{362}$$

Because $\nu_2 > 0$, for the solution to converge to the steady-state, the term in brackets must be nil when we let t tend toward infinity:

$$\begin{aligned}
X_2(0) &= \frac{\Phi_2^G Y}{\nu_1-\nu_2}\left[\frac{1}{\xi+\nu_2} - (1-g)\frac{1}{\chi+\nu_2}\right] + \sum_{X^j}\frac{\Phi_2^{X^j}X^j}{\nu_1-\nu_2}\left[\frac{1}{\xi_X^j+\nu_2} - \left(1-x^j\right)\frac{1}{\chi_X^j+\nu_2}\right], \\
&= -\Gamma_2^G\left(1-\Theta_2^G\right) - \sum_{X^j}\Gamma_2^{X^j}\left(1-\Theta_2^{X^j}\right),
\end{aligned} \tag{363}$$

where Γ_2^G and Θ_2^G are given by (359b) and (359d), respectively, and $\Gamma_2^{X^j}$ and $\Theta_2^{X^j}$ are given by (360b) and (360d), respectively.

Inserting first $X_2(0)$, the 'stable' solution for $X_2(t)$, i.e., consistent with convergence toward the steady-state when t tends toward infinity, is thus given by:

$$\begin{aligned}
X_2(t) &= e^{\nu_2 t}\Bigg\{\frac{\Phi_2^G Y}{\nu_1-\nu_2}\left[\frac{e^{-(\xi+\nu_2)t}}{\xi+\nu_2} - (1-g)\frac{e^{-(\chi+\nu_2)t}}{\chi+\nu_2}\right] \\
&+ \sum_{X^j}\frac{\Phi_2^{X^j}X^j}{\nu_1-\nu_2}\left[\frac{e^{-\left(\xi_X^j+\nu_2\right)t}}{\xi_X^j+\nu_2} - \left(1-x^j\right)\frac{e^{-\left(\chi_X^j+\nu_2\right)t}}{\chi_X^j+\nu_2}\right]\Bigg\}, \\
&= -\Gamma_2^G\left(e^{-\xi t} - \Theta_2^G e^{-\chi t}\right) - \sum_{X^j}\Gamma_2^{X^j}\left(e^{-\xi_X^j t} - \Theta_2^{X^j}e^{-\chi_X^j t}\right).
\end{aligned} \tag{364}$$

Using the definition of $X_i(t)$ (with $i = 1, 2$) given by (357), we can recover the solutions for $K(t)$ and $Q(t)$:

$$K(t) - \tilde{K} = X_1(t) + X_2(t), \tag{365a}$$

$$Q(t) - \tilde{Q} = \omega_2^1 X_1(t) + \omega_2^2 X_2(t). \tag{365b}$$

Setting $t=0$ into (365a) gives $X_1(0)=(K(0)-K)-X_2(0)$ where $X_2(0)$ is described by eq. (363); the solution (361) for $X_1(t)$ can be rewritten as follows:

$$\begin{aligned}
X_1(t) &= e^{\nu_1 t}\Bigg[(K(0)-K)+\Gamma_2^G\left(1-\Theta_2^G\right)-\Gamma_1^G\left(1-\Theta_1^G\right)+\sum_{X^j}\Gamma_2^{X^j}\left(1-\Theta_2^{X^j}\right)-\sum_{X^j}\Gamma_1^{X^j}\left(1-\Theta_1^{X^j}\right)\Bigg] \\
&+ \Gamma_1^G\left(e^{-\xi t}-\Theta_1^G e^{-\chi t}\right)+\sum_{X^j}\Gamma_1^{X^j}\left(e^{-\xi_X^j t}-\Theta_1^{X^j}e^{-\chi_X^j t}\right). \qquad (366)
\end{aligned}$$

Linearizing the current account equation (338) around the steady-state:

$$\begin{aligned}
\dot{N}(t) &= r^\star dN(t)+\Xi_K dK(t)+\Xi_Q dQ(t)+\Xi_G dG(t)+\sum_{X^j}\Xi_{X^j}dX^j(t), \\
&= \left(\Xi_K+\Xi_Q\omega_2^1\right)X_1(t)+\left(\Xi_K+\Xi_Q\omega_2^2\right)X_2(t) \\
&+ \Xi_G Y\left[e^{-\xi t}-(1-g)\,e^{-\chi t}\right]+\sum_{X^j}X^j\left[e^{-\xi_X^j t}-\left(1-x^j\right)e^{-\chi_X^j t}\right]. \qquad (367)
\end{aligned}$$

Setting $N_1=\Xi_K+\Xi_Q\omega_2^1$, $N_2=\Xi_K+\Xi_Q\omega_2^2$, inserting solutions for $K(t)$ and $Q(t)$ given by (365), solving and invoking the transversality condition, yields the solution for traded bonds:

$$\begin{aligned}
dN(t) &= e^{r^\star t}\left(N_0-N\right)+\frac{\omega_N^1}{\nu_1-r^\star}\left(e^{r^\star t}-e^{\nu_1 t}\right) \\
&+ \frac{N_1\Gamma_1^G}{\xi+r^\star}\left[\left(e^{r^\star t}-e^{-\xi t}\right)-\Theta_1^{G,\prime}\left(e^{r^\star t}-e^{-\chi t}\right)\right] \\
&+ \sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j+r^\star}\left[\left(e^{r^\star t}-e^{-\xi_X^j t}\right)-\Theta_1^{X^j,\prime}\left(e^{r^\star t}-e^{-\chi_X^j t}\right)\right] \\
&- \frac{N_2\Gamma_2^G}{\xi+r^\star}\left[\left(e^{r^\star t}-e^{-\xi t}\right)-\Theta_2^{G,\prime}\left(e^{r^\star t}-e^{-\chi t}\right)\right] \\
&- \sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j+r^\star}\left[\left(e^{r^\star t}-e^{-\xi_X^j t}\right)-\Theta_2^{X^j,\prime}\left(e^{r^\star t}-e^{-\chi_X^j t}\right)\right] \\
&+ \frac{\Xi_G Y}{\xi+r^\star}\left[\left(e^{r^\star t}-e^{-\xi t}\right)-\Theta^{G,\prime}\left(e^{r^\star t}-e^{-\chi t}\right)\right] \\
&+ \sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j+r^\star}\left[\left(e^{r^\star t}-e^{-\xi_X^j t}\right)-\Theta^{X^j,\prime}\left(e^{r^\star t}-e^{-\chi_X^j t}\right)\right],
\end{aligned}$$

where

$$\begin{aligned}
\omega_N^1 &= N_1\Bigg[(K_0-K)+\Gamma_2^G\left(1-\Theta_2^G\right)-\Gamma_1^G\left(1-\Theta_1^G\right) \\
&+ \sum_{X^j}\Gamma_2^{X^j}\left(1-\Theta_2^{X^j}\right)-\sum_{X^j}\Gamma_1^{X^j}\left(1-\Theta_1^{X^j}\right)\Bigg]. \qquad (368)
\end{aligned}$$

and

$$\Theta^{G,\prime}=(1-g)\,\frac{\xi+r^\star}{\chi+r^\star}, \qquad (369a)$$

$$\Theta_1^{G,\prime}=\Theta_1^G\frac{\xi+r^\star}{\chi+r^\star}, \qquad (369b)$$

$$\Theta_2^{G,\prime}=\Theta_2^G\frac{\xi+r^\star}{\chi+r^\star}, \qquad (369c)$$

$$\Theta^{X^j,\prime}=\left(1-x^j\right)\frac{\xi_X^j+r^\star}{\chi_X^j+r^\star}, \qquad (369d)$$

$$\Theta_1^{X^j,\prime}=\Theta_1^{X^j}\frac{\xi_X^j+r^\star}{\chi_X^j+r^\star}, \qquad (369e)$$

$$\Theta_2^{X^j,\prime}=\Theta_2^{X^j}\frac{\xi_X^j+r^\star}{\chi_X^j+r^\star}. \qquad (369f)$$

By rearranging terms, we get

$$
\begin{aligned}
dN(t) &= e^{r^\star t}\Bigg[(N_0-N)\frac{\omega_N^1}{\nu_1-r^\star}+\frac{N_1\Gamma_1^G}{\xi+r^\star}\left(1-\Theta_1^{G,\prime}\right)+\sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j+r^\star}\left(1-\Theta_1^{X^j,\prime}\right)\\
&- \frac{N_2\Gamma_2^G}{\xi+r^\star}\left(1-\Theta_2^{G,\prime}\right)-\sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j+r^\star}\left(1-\Theta_2^{X^j,\prime}\right)\\
&+ \frac{\Xi_G Y}{\xi+r^\star}\left(1-\Theta_2^{G,\prime}\right)+\sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j+r^\star}\left(1-\Theta_2^{X^j,\prime}\right)\Bigg]\\
&- \frac{\omega_N^1}{\nu_1-r^\star}e^{\nu_1 t}-\frac{N_1\Gamma_1^G}{\xi+r^\star}\left(e^{-\xi t}-\Theta_1^{G,\prime}e^{-\chi t}\right)-\sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j+r^\star}\left(e^{-\xi_X^j t}-\Theta_1^{G,\prime}e^{-\chi_X^j t}\right)\\
&+ \frac{N_2\Gamma_2^G}{\xi+r^\star}\left(e^{-\xi t}-\Theta_2^{G,\prime}e^{-\chi t}\right)+\sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j+r^\star}\left(e^{-\xi_X^j t}-\Theta_2^{G,\prime}e^{-\chi_X^j t}\right)\\
&- \frac{\Xi_G Y}{\xi+r^\star}\left(e^{-\xi t}-\Theta^{G,\prime}e^{-\chi t}\right)-\sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j+r^\star}\left(e^{-\xi_X^j t}-\Theta_2^{G,\prime}e^{-\chi_X^j t}\right).
\end{aligned}
$$

Invoking the transversality condition, to ultimately remain solvent, the open economy must satisfy the following condition:

$$
(N_0-N)+\frac{\omega_N^1}{\nu_1-r^\star}+\frac{\omega_N^{2,G}}{\xi+r^\star}+\sum_{X^j}\frac{\omega_N^{2,X^j}}{\xi_X^j+r^\star}=0, \tag{370}
$$

where

$$
\omega_N^{2,G} = N_1\Gamma_1^G\left(1-\Theta_1^{G,\prime}\right)-N_2\Gamma_2^G\left(1-\Theta_2^{G,\prime}\right)+\Xi_G Y\left(1-\Theta^{G,\prime}\right), \tag{371a}
$$

$$
\omega_N^{2,X^j} = N_1\Gamma_1^{X^j}\left(1-\Theta_1^{X^j,\prime}\right)-N_2\Gamma_2^{X^j}\left(1-\Theta_2^{X^j,\prime}\right)+\Xi_{X^j}X^j\left(1-\Theta^{X^j,\prime}\right). \tag{371b}
$$

The convergent path for the net foreign asset position is:

$$
\begin{aligned}
dN(t) &= \frac{\omega_N^1}{\nu_1-r^\star}e^{\nu_1 t}-\frac{N_1\Gamma_1^G}{\xi+r^\star}\left(e^{-\xi t}-\Theta_1^{G,\prime}e^{-\chi t}\right)-\sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j+r^\star}\left(e^{-\xi_X^j t}-\Theta_1^{G,\prime}e^{-\chi_X^j t}\right)\\
&+ \frac{N_2\Gamma_2^G}{\xi+r^\star}\left(e^{-\xi t}-\Theta_2^{G,\prime}e^{-\chi t}\right)+\sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j+r^\star}\left(e^{-\xi_X^j t}-\Theta_2^{G,\prime}e^{-\chi_X^j t}\right)\\
&- \frac{\Xi_G Y}{\xi+r^\star}\left(e^{-\xi t}-\Theta^{G,\prime}e^{-\chi t}\right)-\sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j+r^\star}\left(e^{-\xi_X^j t}-\Theta_2^{G,\prime}e^{-\chi_X^j t}\right).
\end{aligned} \tag{372}
$$

V Semi-Small Open Economy Model with Public Debt and Distortionary Taxation

This section extends the model with CES production functions laid out in section T by introducing distortionary taxation. We will emphasize the main changes caused by the introduction of taxation.

In order to avoid confusion:

- K is the stock of physical capital;
- QK is the shadow value of the stock of physical capital;
- D is the stock of (traded) bonds issued by the government;
- B is the stock of traded bonds;
- $N = B - D$ is the net foreign asset position;

- $A = QK + N$ is the national non human wealth equal to the shadow value of the stock of physical capital plus the net foreign asset position which gives national savings $\dot{A}$;
- $\mathcal{A} = A + D = QK + N + D$ is non human wealth held by households which gives private savings $\dot{\mathcal{A}}$.

V.1 Households

Household's flow budget constraint states that real disposable income (on the RHS of the equation below) can be saved by accumulating traded bonds, consumed, $P_C(t)C(t)$, or invested, $P_J(t)J(t)$:

$$\begin{aligned} & \dot{N}(t) + P_C(t)\left(1+\tau^C(t)\right)C(t) + P_J(t)J(t) + P^H(t)C^{K,H}(t)\alpha_K(t)K(t) + P^N(t)C^{K,N}(t)\left(1-\alpha_K(t)\right)K(t) \\ + \; & P^H(t)C^{Z,H}(t) + P^N(t)C^{Z,N}(t) = \left[\alpha_L(t)u^{Z,H}(t) + \left(1-\alpha_L(t)\right)u^{Z,N}(t)\right]W(t)\left(1-\tau^L(t)\right)L(t) \\ + \; & \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + \left(1-\alpha_K(t)\right)u^{K,N}(t)u^{Z,N}(t)\right]R(t)K(t) + r^{\star}N(t), \end{aligned} \tag{373}$$

where we denote the share of traded capital in the aggregate capital stock by $\alpha_K(t) = K^H(t)/K(t)$ and the labor compensation share of tradables by $\alpha_L(t) = \frac{W^H(t)L^H(t)}{W(t)L(t)}$. In contrast to the budget constraint (14) in the main text, lump-sum taxes are replaced with labor and consumption taxation. More specifically, we denoted the consumption tax rate by $\tau^C(t)$ and the tax rate on labor income by $\tau^L(t)$.

Denoting the co-state variables associated with (373) and (190) by λ and Q', respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$(C(t))^{-\frac{1}{\sigma_C}} = P_C(t)\left(1+\tau^C(t)\right)\lambda(t), \tag{374a}$$

$$\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}} = \lambda(t)\left[\alpha_L(t)u^{Z,H}(t) + \left(1-\alpha_L(t)\right)u^{Z,N}(t)\right]W(t)\left(1-\tau^L(t)\right), \tag{374b}$$

$$Q(t) = P_J(t)\left[1+\kappa\left(\frac{I(t)}{K(t)} - \delta_K\right)\right], \tag{374c}$$

$$\dot{\lambda}(t) = \lambda\left(\beta - r^{\star}\right), \tag{374d}$$

$$\begin{aligned} \dot{Q}(t) = \left(r^{\star}+\delta_K\right)Q(t) - \Big\{ & \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + \left(1-\alpha_K(t)\right)u^{K,N}(t)u^{Z,N}(t)\right]R(t) \\ & -P^H(t)C^{K,H}(t)\alpha_K(t) - P^N(t)C^{K,N}(t)\left(1-\alpha_K(t)\right) + P_J(t)\frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)\left(\frac{I(t)}{K(t)} + \delta_K\right)\Big\}, \end{aligned} \tag{374e}$$

$$R(t)u^{Z,H}(t) = P^H(t)\left[\xi_1^H + \xi_2^H\left(u^{K,H}(t) - 1\right)\right], \tag{374f}$$

$$R(t)u^{Z,N}(t) = P^N(t)\left[\xi_1^N + \xi_2^N\left(u^{K,N}(t) - 1\right)\right], \tag{374g}$$

$$R(t)u^{K,H}(t)K^H(t) + W^H(t)\left(1-\tau^L(t)\right)L^H(t) = P^H(t)\left[\chi_1^H + \chi_2^H\left(u^{Z,H}(t) - 1\right)\right], \tag{374h}$$

$$R(t)u^{K,N}(t)K^N(t) + W^N(t)\left(1-\tau^L(t)\right)L^N(t) = P^N(t)\left[\chi_1^N + \chi_2^N\left(u^{Z,N}(t) - 1\right)\right], \tag{374i}$$

and the transversality conditions $\lim_{t\to\infty}\bar{\lambda}N(t)e^{-\beta t} = 0$ and $\lim_{t\to\infty}Q(t)K(t)e^{-\beta t} = 0$; to derive (374c) and (374e), we used the fact that $Q(t) = Q'(t)/\lambda(t)$.

Given the price indices (195) and (196), the representative household has the following demand of home-produced traded goods and foreign-produced traded goods:

$$C^N = (1-\varphi)\left(\frac{P^N}{P_C}\right)^{-\phi}C, \tag{375a}$$

$$C^H = \varphi\left(\frac{P^T}{P_C}\right)^{-\phi}\varphi_H\left(\frac{P^H}{P^T}\right)^{-\rho}C, \tag{375b}$$

$$C^F = \varphi\left(\frac{P^T}{P_C}\right)^{-\phi}\left(1-\varphi_H\right)\left(\frac{1}{P_T}\right)^{-\rho}C. \tag{375c}$$

Before deriving the allocation of hours worked across sectors, it is convenient to rewrite the optimal decision for aggregate labor supply described by eq. (374b). As shall be useful, we denote sectoral wages including technology utilization rates with a tilde, i.e.,

$\tilde{W}^j(t) = u^{Z,j}(t)W^j(t)$. Multiplying both sides of (374b) by $L(t)$ and denoting by $\tilde{W}(t)$ the aggregate wage index inclusive of technology utilization leads to:

$$\begin{aligned}
\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}+1} &= \lambda(t)\left[\alpha_L(t)u^{Z,H}(t) + \left(1-\alpha_L(t)\right)u^{Z,N}(t)\right]W(t)\left(1-\tau^L(t)\right)L(t), \\
\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}+1} &= \lambda(t)\left[W^H(t)u^{Z,H}(t)L^H(t) + W^N(t)u^{Z,N}(t)L^N(t)\right]\left(1-\tau^L(t)\right), \\
\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}+1} &= \lambda(t)\left[\tilde{W}^H(t)L^H(t) + \tilde{W}^N(t)L^N(t)\right]\left(1-\tau^L(t)\right),
\end{aligned}$$

where we used the definition of the labor compensation share of tradables and non-tradables, i.e., $\alpha_L(t)W(t)L(t) = W^H(t)L^H(t)$ and $\left(1-\alpha_L(t)\right)W(t)L(t) = W^N(t)L^N(t)$. Dividing both sides of the above equation by $L(t)$ and using the definition of the aggregate wage index which includes technology utilization rates enables us to rewrite eq. (374b) as follows:

$$\gamma\left(L(t)\right)^{\frac{1}{\sigma_L}} = \lambda(t)\tilde{W}(t)\left(1-\tau^L(t)\right). \tag{376}$$

Given the aggregate wage index (208), we can derive the allocation of aggregate labor supply to the traded and the non-traded sector:

$$L^H = \vartheta\left(\frac{\tilde{W}^H(t)}{\tilde{W}(t)}\right)^{\epsilon}L(t), \tag{377a}$$

$$L^N = (1-\vartheta)\left(\frac{\tilde{W}^N(t)}{\tilde{W}(t)}\right)^{\epsilon}L(t). \tag{377b}$$

V.2 Firms

First-Order Conditions

We denote technology adjusted value added in sector $j = H, N$ by Y^j. When we add a tilde, it means that value added is inclusive of the technology utilization rate, i.e., $\tilde{Y}^j(t) = u^Z(t)Y^j(t)$. We allow for labor- and capital-augmenting productivity denoted by $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. We allow for labor- and capital-augmenting efficiency denoted by $\tilde{A}^j(t)$ and $\tilde{B}^j(t)$. We assume that factor-augmenting productivity has a symmetric time-varying component denoted by $u^{Z,j}(t)$ such that $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$ and $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$. Both the traded and non-traded sectors use physical capital inclusive of capital utilization, $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$, and labor, L^j.

Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$, and a labor cost equal to the wage rate $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$, both inclusive of technology utilization. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given:

$$\begin{aligned}
\max_{\tilde{K}^j, L^j}\tilde{\Pi}^j &= \max_{\tilde{K}^j, L^j}\left\{P^j\tilde{Y}^j - \tilde{W}^jL^j - \tilde{R}^j\tilde{K}^j\right\}, \\
&= \max_{\tilde{K}^j, L^j}u^{Z,j}(t)\left\{P^jY^j - W^jL^j - R\tilde{K}^j\right\}, \\
&= \max_{\tilde{K}^j, L^j}u^{Z,j}\Pi^j,
\end{aligned} \tag{378}$$

where technology-utilization-adjusted CES production function reads:

$$Y^j(t) = \left[\gamma^j\left(A^j(t)L^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}} + \left(1-\gamma^j\right)\left(B^j(t)\tilde{K}^j(t)\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}. \tag{379}$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P^H\left(1-\gamma^H\right)\left(B^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(u^{K,H}k^H\right)^{-\frac{1}{\sigma^H}}\left(y^H\right)^{\frac{1}{\sigma^H}}=P^N\left(1-\gamma^N\right)\left(B^N\right)^{\frac{\sigma^N-1}{\sigma^N}}\left(u^{K,N}k^N\right)^{-\frac{1}{\sigma^N}}\left(y^N\right)^{\frac{1}{\sigma^N}}\equiv R, \tag{380a}$$

$$P^H\gamma^H\left(A^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(L^H\right)^{-\frac{1}{\sigma^H}}\left(Y^H\right)^{\frac{1}{\sigma^H}}\equiv W^H, \tag{380b}$$

$$P^N\gamma^N\left(A^N\right)^{\frac{\sigma^N-1}{\sigma^N}}\left(L^N\right)^{-\frac{1}{\sigma^N}}\left(Y^N\right)^{\frac{1}{\sigma^N}}\equiv W^N, \tag{380c}$$

where we denote by $k^j\equiv K^j/L^j$ the capital-labor ratio for sector $j=H,N$, and $y^j\equiv Y^j/L^j$ value added per hours worked described by

$$y^j=\left[\gamma^j\left(A^j\right)^{\frac{\sigma^j-1}{\sigma^j}}+\left(1-\gamma^j\right)\left(B^j u^{K,j}k^j\right)^{\frac{\sigma^j-1}{\sigma^j}}\right]^{\frac{\sigma^j}{\sigma^j-1}}. \tag{381}$$

The resource constraint for capital is:

$$K^H+K^N=K. \tag{382}$$

V.3 Short-Run Solutions

Solving for Sectoral Consumption and Sectoral Hours Worked

First-order conditions (374a) and (374b) can be solved for consumption and aggregate labor supply which of course must hold at any point of time:

$$C=C\left(\bar{\lambda},P^N,P^H,\tau^C\right),\quad L=L\left(\bar{\lambda},\tilde{W}^H,\tilde{W}^N,\tau^L\right), \tag{383}$$

with partial derivatives given by

$$\hat{C}=-\sigma_C\hat{\bar{\lambda}}-\sigma_C\alpha_C\alpha^H\hat{P}^H-\sigma_C\left(1-\alpha_C\right)\hat{P}^N-\sigma_C(1\hat{+}\tau^C), \tag{384a}$$

$$\hat{L}=\sigma_L\hat{\bar{\lambda}}+\sigma_L\left(1-\alpha_L\right)\hat{\tilde{W}}^N+\sigma_L\alpha_L\hat{\tilde{W}}^H+\sigma_L(1\hat{-}\tau^L), \tag{384b}$$

where we have used (199) and (210).

Inserting first the solution for consumption (383) into (375a), (375b), (375c) enables us to solve for C^N, C^H, and C^F:

$$C^N=C^N\left(\bar{\lambda},P^N,P^H,\tau^C\right),\quad C^H=C^H\left(\bar{\lambda},P^N,P^H,\tau^C\right),\quad C^F=C^F\left(\bar{\lambda},P^N,P^H,\tau^C\right), \tag{385}$$

with partial derivatives given by

$$\begin{aligned}
\hat{C}^N &= -\phi\hat{P}^N+\left(\phi-\sigma_C\right)\hat{P}_C-\sigma_C\hat{\bar{\lambda}},\\
&= -\left[\alpha_C\phi+\left(1-\alpha_C\right)\sigma_C\right]\hat{P}^N+\left(\phi-\sigma_C\right)\alpha_C\alpha^H\hat{P}^H-\sigma_C\hat{\bar{\lambda}}-\sigma_C(1\hat{+}\tau^C), && (386a)\\
\hat{C}^H &= -\left[\rho\left(1-\alpha^H\right)+\phi\left(1-\alpha_C\right)\alpha^H+\sigma_C\alpha_C\alpha^H\right]\hat{P}^H+\left(1-\alpha_C\right)\left(\phi-\sigma_C\right)\hat{P}^N-\sigma_C\hat{\bar{\lambda}}-\sigma_C(1\hat{+}\tau^C), && (386b)\\
\hat{C}^F &= \alpha^H\left[\rho-\phi\left(1-\alpha_C\right)-\sigma_C\alpha_C\right]\hat{P}^H+\left(1-\alpha_C\right)\left(\phi-\sigma_C\right)\hat{P}^N-\sigma_C\hat{\bar{\lambda}}-\sigma_C(1\hat{+}\tau^C). && (386c)
\end{aligned}$$

Inserting first the solution for labor (383) into (377a)-(377b) allows us to solve for L^H and L^N:

$$L^H=L^H\left(\bar{\lambda},W^H,W^N,u^{Z,H},u^{Z,N},\tau^L\right),\quad L^N=L^N\left(\bar{\lambda},W^H,W^N,u^{Z,H},u^{Z,N},\tau^L\right), \tag{387}$$

with partial derivatives given by:

$$\begin{aligned}
\hat{L}^H &= \left[\epsilon\left(1-\alpha_L\right)+\sigma_L\alpha_L\right]\left(\hat{W}^H+\hat{u}^{Z,H}\right)-\left(1-\alpha_L\right)\left(\epsilon-\sigma_L\right)\left(\hat{W}^N+\hat{u}^{Z,N}\right)\\
&\quad +\sigma_L\hat{\bar{\lambda}}+\sigma_L(1\hat{-}\tau^L), && (388a)\\
\hat{L}^N &= \left[\epsilon\alpha_L+\sigma_L\left(1-\alpha_L\right)\right]\left(\hat{W}^N+\hat{u}^{Z,N}\right)-\alpha_L\left(\epsilon-\sigma_L\right)\left(\hat{W}^H+\hat{u}^{Z,H}\right)\\
&\quad +\sigma_L\hat{\bar{\lambda}}+\sigma_L(1\hat{-}\tau^L). && (388b)
\end{aligned}$$

Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for L^H and L^N given by (387) into the resource constraint for capital (382), the system of four equations consisting of (380a)-(380c) together with (382) can be solved for sectoral wages W^j and sectoral capital-labor ratios k^j. Log-differentiating (380a)-(380c) together with (382) yields in matrix form:

$$\begin{pmatrix} -\left(\frac{s_L^H}{\sigma^H}\right) & \left(\frac{s_L^N}{\sigma^N}\right) & 0 & 0 \\ \left(\frac{1-s_L^H}{\sigma^H}\right) & 0 & -1 & 0 \\ 0 & \left(\frac{1-s_L^N}{\sigma^N}\right) & 0 & -1 \\ \frac{K^H}{K} & \frac{K^N}{K} & \Psi_{W^H} & \Psi_{W^N} \end{pmatrix} \begin{pmatrix} \hat{k}^H \\ \hat{k}^N \\ \hat{W}^H \\ \hat{W}^N \end{pmatrix}$$

$$= \begin{pmatrix} \hat{P}^N - \hat{P}^H - \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\hat{B}^H + \left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\hat{B}^N - \left(\frac{s_L^H}{\sigma^H}\right)\hat{A}^H + \left(\frac{s_L^N}{\sigma^N}\right)\hat{A}^N + \frac{s_L^H}{\sigma^H}\hat{u}^{K,H} - \frac{s_L^N}{\sigma^N}\hat{u}^{K,N} \\ -\hat{P}^H - \left[\frac{(\sigma^H-1)+s_L^H}{\sigma^H}\right]\hat{A}^H - \left(\frac{1-s_L^H}{\sigma^H}\right)\hat{B}^H - \left(\frac{1-s_L^H}{\sigma^H}\right)\hat{u}^{K,H} \\ -\hat{P}^N - \left[\frac{(\sigma^N-1)+s_L^N}{\sigma^N}\right]\hat{A}^N - \left(\frac{1-s_L^N}{\sigma^N}\right)\hat{B}^N - \left(\frac{1-s_L^N}{\sigma^N}\right)\hat{u}^{K,N} \\ \hat{K} - \Psi_{\bar{\lambda}}\hat{\bar{\lambda}} - \sum_j \Psi_{u^{Z,j}}\hat{u}^{Z,j} - \Psi_{\tau^L}\frac{d\tau^L}{1-\tau^L} \end{pmatrix} \quad (389)$$

where we set:

$$\begin{aligned} \Psi_{\tau^L} &= \frac{K^H}{K}\frac{L^H_{\tau^L}\left(1-\tau^L\right)}{L^H} + \frac{K^N}{K}\frac{L^N_{\tau^L}\left(1-\tau^L\right)}{L^N}, \\ &= -\frac{K^H}{K}\sigma_L - \frac{K^N}{K}\sigma_L = -\sigma_L. \end{aligned} \quad (390)$$

The short-run solutions for sectoral wages and capital-labor ratios are:

$$W^j = W^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, \tau^L\right), \quad (391a)$$

$$k^j = k^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, \tau^L\right). \quad (391b)$$

Inserting first sectoral wages (391), sectoral hours worked (387) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods, P^N, the terms of trade, P^H, and the labor tax rate, τ^L:

$$L^j = L^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, \tau^L\right). \quad (392)$$

Totally differentiating output per hours worked (381) leads to:

$$\hat{y}^j = s_L^j \hat{A}^j + \left(1 - s_L^j\right)\left[\hat{B}^j + \hat{u}^{K,j} + \hat{k}^j\right], \quad (393)$$

where s_L^j and $1 - s_L^j$ are the labor and capital income share, respectively, described by eqs. (291)-(292). Plugging solutions for sectoral capital-labor ratios (391) into (393) allows us to solve for sectoral value added per hours worked:

$$y^j = y^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, \tau^L\right). \quad (394)$$

where for example the change in technology utilization adjusted value added per hour worked for tradables reads

$$\begin{aligned} dy^H &= \left[\frac{y^H}{A^H}s_L^H + \frac{y^H}{k^H}\left(1-s_L^H\right)k^H_{A^H}\right]dA^H \\ &+ \left[\frac{y^H}{B^H}s_L^H + \frac{y^H}{k^H}\left(1-s_L^H\right)k^H_{B^H}\right]dB^H \\ &+ \left[\frac{y^H}{u^{K,H}}s_L^H + \frac{y^H}{k^H}\left(1-s_L^H\right)k^H_{u^{K,H}}\right]du^{K,H} \\ &+ \frac{y^H}{k^H}\left(1-s_L^H\right)dk^H. \end{aligned}$$

Using the fact that $Y^j = y^j L^j$, inserting solutions for y^j (394) and L^j (392), and differentiating, one obtains the solutions for sectoral value added:

$$Y^j = Y^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, \tau^L\right). \tag{395}$$

Using the fact that $K^j = k^j L^j$, inserting solutions for k^j (391b) and L^j (392), differentiating, one obtains the solutions for the sectoral capital stock:

$$K^j = K^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, \tau^L\right). \tag{396}$$

Capital and Technology Utilization Rates, $u^{K,j}(t)$ and $u^{Z,j}(t)$

Optimal choices for capital utilization are described by (316a)-(316b). Using the fact that $W^H L^H + R u^{K,H} K^H = P^H Y^H$, (374h)-(374i), we have:

$$\begin{aligned}
P^j(t)Y^j(t) - \tau^L(t)W^{j(t)}L^j(t) - \tau^L(t)W^j(t)L^j(t) &= P^j(t)\left[\chi_1^j + \chi_2^j\left(u^{Z,j}(t)-1\right)\right], \\
P^j(t)Y^j(t)\left[1-\tau^L s_L^j(t)\right] &= P^j(t)\left[\chi_1^j + \chi_2^j\left(u^{Z,j}(t)-1\right)\right], \\
Y^j(t)\left[1-\tau^L s_L^j(t)\right] &= \chi_1^j + \chi_2^j\left(u^{Z,j}(t)-1\right). \qquad (397)
\end{aligned}$$

For the traded and the non-traded sector, the endogenous technology rate decision reads:

$$Y^H(t)\left[1-\tau^L(t)s_L^H(t)\right] = \chi_1^H + \chi_2^H\left(u^{Z,H}(t)-1\right)], \tag{398a}$$
$$Y^N(t)\left[1-\tau^L(t)s_L^N(t)\right] = \chi_1^N + \chi_2^N\left(u^{Z,N}(t)-1\right). \tag{398b}$$

Log-linearizing optimal decisions on capital and technology utilization rates described by (316a)-(316b) and (398a)-(398b) leads to in a matrix form:

$$\begin{pmatrix}
\left[\left(\frac{\xi_2^H}{\xi_1^H}+\frac{s_L^H}{\sigma^H}\right)+\frac{s_L^H}{\sigma^H}\frac{k^H_{u^{K,H}}}{k^H}\right] & \frac{s_L^H}{\sigma^H}\frac{k^H_{u^{K,N}}}{k^H} & \left[\frac{s_L^H}{\sigma^H}\frac{k^H_{u^{Z,H}}}{k^H}-1\right] & \frac{s_L^H}{\sigma^H}\frac{k^H_{u^{Z,N}}}{k^H} \\
\frac{s_L^N}{\sigma^N}\frac{k^N_{u^{K,H}}}{k^N} & \left[\left(\frac{\xi_2^N}{\xi_1^N}+\frac{s_L^N}{\sigma^N}\right)+\frac{s_L^N}{\sigma^N}\frac{k^N_{u^{K,N}}}{k^N}\right] & \frac{s_L^N}{\sigma^N}\frac{k^N_{u^{Z,H}}}{k^N} & \left[\frac{s_L^N}{\sigma^N}\frac{k^N_{u^{Z,N}}}{k^N}-1\right] \\
-\left[d^H Y^H_{u^{K,H}} - Y^H\tau^L\frac{\partial s_L^H}{\partial u^{K,H}}\right] & -\left[d^H Y^H_{u^{K,N}} - Y^H\tau^L\frac{\partial s_L^H}{\partial u^{K,N}}\right] & \left\{\chi_2^H - \left[d^H Y^H_{u^{Z,H}} - Y^H\tau^L\frac{\partial s_L^H}{\partial u^{Z,H}}\right]\right\} & -\left[d^H Y^H_{u^{Z,N}} - Y^H\tau^L\frac{\partial s_L^H}{\partial u^{Z,N}}\right] \\
-\left[d^N Y^N_{u^{K,H}} - Y^N\tau^L\frac{\partial s_L^N}{\partial u^{K,H}}\right] & -\left[d^N Y^N_{u^{K,N}} - Y^N\tau^L\frac{\partial s_L^N}{\partial u^{K,N}}\right] & -\left[d^N Y^N_{u^{Z,H}} - Y^N\tau^L\frac{\partial s_L^N}{\partial u^{Z,H}}\right] & \left\{\chi_2^N - \left[d^N Y^N_{u^{Z,N}} - Y^N\tau^L\frac{\partial s_L^N}{\partial u^{Z,N}}\right]\right\}
\end{pmatrix}$$

$$\times \begin{pmatrix} \hat{u}^{K,H} \\ \hat{u}^{K,N} \\ \hat{u}^{Z,H} \\ \hat{u}^{Z,N} \end{pmatrix} = \begin{pmatrix}
-\frac{s_L^H}{\sigma^H}\frac{k^H_{X^H}}{k^H}dX^H + \frac{s_L^H}{\sigma^H}\left[\frac{1}{A^H} - \frac{k^H_{A^H}}{k^H}\right]dA^H + \left[\left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\frac{1}{B^H} - \frac{s_L^H}{\sigma^H}\frac{k^H_{B^H}}{k^H}\right]dB^H \\
-\frac{s_L^N}{\sigma^N}\frac{k^N_{X^N}}{k^N}dX^N + \frac{s_L^N}{\sigma^N}\left[\frac{1}{A^N} - \frac{k^N_{A^N}}{k^N}\right]dA^N + \left[\left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\frac{1}{B^N} - \frac{s_L^N}{\sigma^N}\frac{k^N_{B^N}}{k^N}\right]dB^N \\
\left[\left(1-\tau^L s_L^H\right)Y^H_{X^H} - Y^H\tau^L\frac{\partial s_L^H}{\partial X^H}\right]dX^H + \left[\left(1-\tau^L s_L^H\right)Y^H_{\tau^L} - Y^H\tau^L\frac{\partial s_L^H}{\partial \tau^L} - Y^H s_L^H\right]d\tau^L \\
\left[\left(1-\tau^L s_L^N\right)Y^N_{X^N} - Y^N\tau^L\frac{\partial s_L^N}{\partial X^N}\right]dX^N + \left[\left(1-\tau^L s_L^N\right)Y^N_{\tau^L} - Y^N\tau^L\frac{\partial s_L^N}{\partial \tau^L} - Y^N s_L^N\right]d\tau^L
\end{pmatrix},$$

where we set $d^j = 1-\tau^L s_L^j$, $X^H = P^H, P^N, K, \bar{\lambda}, A^N, B^N$ and $X^N = P^H, P^N, K, \bar{\lambda}, A^H, B^H$.

The short-run solutions for capital and technology utilization rates are:

$$u^{K,j} = u^{K,j}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right), \tag{400a}$$
$$u^{Z,j} = u^{Z,j}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right). \tag{400b}$$

Intermediate Solutions for k^j, W^j, L^j, Y^j, K^j

Plugging back solutions for capital and technology utilization rates (400a)-(400b) into (391a), (391b), (392), (394), (395), (396) leads to intermediate solutions for sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, and sectoral capital stocks:

$$W^j, k^j, L^j, Y^j, K^j\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right). \tag{401}$$

Market Clearing Condition for Non-Tradables

The role of the price of non-tradables in terms of foreign-produced traded goods is to clear the non-traded goods market:

$$u^{Z,N}(t)Y^N(t) = C^N(t) + G^N(t) + J^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t). \tag{402}$$

Inserting solutions for C^N, J^N, Y^N given by (385), i.e., $C^N\left(\bar{\lambda}, P^N, P^H, \tau^C\right)$, (240), i.e., $J^N = J^N\left(K, Q, P^N, P^H\right)$, (401), i.e., $Y^N = Y^N\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right)$, the non-traded goods market clearing condition (402) can be rewritten as follows:

$$\begin{aligned} & u^{Z,N} Y^N\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right) = C^N\left(\bar{\lambda}, P^N, P^H, \tau^C\right) + G^N + J^N\left(K, Q, P^N, P^H\right) \\ + & C^{K,N}\left[u^{K,N}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right)\right] K^N \\ & + C^{Z,N}\left(u^{Z,N}\right). \end{aligned} \tag{403}$$

Linearizing (403) leads to:

$$Y^N du^{Z,N}(t) + dY^N(t) = dC^N(t) + dG^N(t) + dJ^N(t) + K^N \xi_1^N du^{K,N}(t) + \chi_1^N du^{Z,N}(t), \tag{404}$$

where $\chi_1^N = Y^N\left(1 - \tau^L s_L^N\right)$; see eq. (398b) evaluated at the steady-state.

Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced traded goods in terms of foreign-produced traded goods or the terms of trade is to clear the home-produced traded goods market:

$$u^{Z,H}(t) Y^H(t) = C^H(t) + G^H(t) + J^H(t) + X^H(t) + C^{K,H}(t) K^H(t) + C^{Z,H}(t), \tag{405}$$

where X^H stands for exports which are negatively related to the terms of trade:

$$X^H = \varphi_X \left(P^H\right)^{-\phi_X}, \tag{406}$$

where ϕ_X is the elasticity of exports with respect to the terms of trade.

Inserting solutions for C^H, J^H, Y^H given by (219), (220a), (401), respectively, the traded goods market clearing condition (405) can be rewritten as follows:

$$\begin{aligned} & u^{Z,H} Y^H\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right) = C^H\left(\bar{\lambda}, P^N, P^H\right) + G^H + J^H\left(K, Q, P^N, P^H\right) \\ + & C^{K,H}\left[u^{K,H}\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right)\right] K^H\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N\right)) \\ & + C^{Z,H}\left(u^{Z,H}\right). \end{aligned} \tag{407}$$

Linearizing (407) leads to:

$$Y^H du^{Z,H}(t) + dY^H(t) = dC^H(t) + dG^H(t) + dJ^H(t) + dX^H(t) + K^H \xi_1^H du^{K,H}(t) + \chi_1^H du^{Z,H}(t), \tag{408}$$

where $\chi_1^H = Y^H\left(1 - \tau^L s_L^H\right)$; see eq. (398a) evaluated at the steady-state.

Return on domestic capital

The return on domestic capital is:

$$R = P^H\left(1 - \gamma^H\right)\left(B^H\right)^{\frac{\sigma^H - 1}{\sigma^H}}\left(u^{K,H} k^H\right)^{-\frac{1}{\sigma^H}}\left(y^H\right)^{\frac{1}{\sigma^H}}. \tag{409}$$

Differentiating (409) and making use of (393) leads to:

$$\hat{R} = \hat{P}^H - \frac{s_L^H}{\sigma^H}\left(\hat{k}^H + \hat{u}^{K,H}\right) + \frac{s_L^H}{\sigma^H}\hat{A}^N + \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\hat{B}^H. \tag{410}$$

Inserting the short-run static solutions for the capital-labor ratio k^H and the capital utilization rate (415), eq. (409) can be solved for the return on domestic capital:

$$R = R\left(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, \tau^L\right). \tag{411}$$

Solving for Relative Prices

As shall be useful below, we write out a number of useful notations:

$$\Psi^N_{P^N} = Y^N_{P^N} + \left(Y^N - \chi^N_1\right) u^{Z,N}_{P^N} - C^N_{P^N} - J^N_{P^N} - K^N \xi^N_1 u^{K,N}_{P^N}, \tag{412a}$$
$$\Psi^N_{P^H} = Y^N_{P^H} + \left(Y^N - \chi^N_1\right) u^{Z,N}_{P^H} - C^N_{P^H} - J^N_{P^H} - K^N \xi^N_1 u^{K,N}_{P^H}, \tag{412b}$$
$$\Psi^N_{K} = Y^N_{K} + \left(Y^N - \chi^N_1\right) u^{Z,N}_{K} - J^N_{K} - K^N \xi^N_1 u^{K,N}_{K}, \tag{412c}$$
$$\Psi^N_{A^j} = Y^N_{A^j} + \left(Y^N - \chi^N_1\right) u^{Z,N}_{A^j} - K^N \xi^N_1 u^{K,N}_{A^j}, \tag{412d}$$
$$\Psi^N_{B^j} = Y^N_{B^j} + \left(Y^N - \chi^N_1\right) u^{Z,N}_{B^j} - K^N \xi^N_1 u^{K,N}_{B^j}, \tag{412e}$$
$$\Psi^N_{\lambda} = Y^N_{\lambda} - C^N_{\lambda} - -K^N \xi^N_1 u^{K,N}_{\lambda}, \tag{412f}$$
$$\Psi^N_{\tau^L} = Y^N_{\tau^L} + \left(Y^N - \chi^N_1\right) u^{Z,N}_{\tau^L} - K^N \xi^N_1 u^{K,N}_{\tau^L}, \tag{412g}$$
$$\Psi^H_{P^N} = Y^H_{P^N} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{P^N} - C^H_{P^N} - J^H_{P^N} - K^H \xi^H_1 u^{K,H}_{P^N}, \tag{412h}$$
$$\Psi^H_{P^H} = Y^H_{P^H} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{P^H} - C^H_{P^H} - J^H_{P^H} - X^H_{P^H} - K^H \xi^H_1 u^{K,H}_{P^H}, \tag{412i}$$
$$\Psi^H_{K} = Y^H_{K} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{K} - J^H_{K} - K^H \xi^H_1 u^{K,H}_{K}, \tag{412j}$$
$$\Psi^H_{A^j} = Y^H_{A^j} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{A^j} - K^H \xi^H_1 u^{K,H}_{A^j}, \tag{412k}$$
$$\Psi^H_{B^j} = Y^H_{B^j} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{B^j} - K^H \xi^H_1 u^{K,H}_{B^j}, \tag{412l}$$
$$\Psi^H_{\lambda} = Y^H_{\lambda} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{\lambda} - C^H_{\lambda} - K^H \xi^H_1 u^{K,H}_{\lambda}, \tag{412m}$$
$$\Psi^H_{\tau^L} = Y^H_{\tau^L} + \left(Y^H - \chi^H_1\right) u^{Z,H}_{\tau^L} - K^N \xi^N_1 u^{K,N}_{\tau^L}. \tag{412n}$$

Linearized versions of market clearing conditions described by eq. (404) and eq. (408) can be rewritten in a matrix form:

$$\begin{pmatrix} \Psi^N_{P^N} & \Psi^N_{P^H} \\ \Psi^H_{P^N} & \Psi^H_{P^H} \end{pmatrix} \begin{pmatrix} dP^N \\ dP^H \end{pmatrix}$$
$$= \begin{pmatrix} -\Psi^N_K dK + J^N_Q dQ + G^N_G dG + C^N_{\tau^C} d\tau^C - \sum_{j=H}^{N} \Psi^N_{A^j} dA^j - - \sum_{j=H}^{N} \Psi^N_{B^j} dB^j - \Psi^N_{\lambda} d\bar{\lambda} - \Psi^N_{\tau^L} d\tau^L \\ -\Psi^H_K dK + J^H_Q dQ + G^H_G dG + C^H_{\tau^C} d\tau^C - \sum_{j=H}^{N} \Psi^H_{A^j} dA^j - \sum_{j=H}^{N} \Psi^H_{B^j} dB^j - \Psi^H_{\lambda} d\bar{\lambda} - \Psi^H_{\tau^L} d\tau^L \end{pmatrix} \tag{413}$$

The short-run solutions for capital and technology utilization rates are:

$$P^N = P^N \left(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N, \tau^L, \tau^C\right), \tag{414a}$$
$$P^H = P^H \left(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N, \tau^L, \tau^C\right). \tag{414b}$$

V.4 Solving the Model

In our model, there is five state variables, namely the capital stock K, labor-augmenting productivity, A^H, A^N, Capital-augmenting productivity, B^H, B^N, and one control variable, namely the shadow price of the capital stock Q. To solve the model, we have to express all variables in terms of state and control variables. Plugging back solutions for the relative price of non-tradables (414a) and the terms of trade (414b) into consumption in sectoral goods (385), investment inputs (240), sectoral output (401), capital and technology utilization rates (400a)-(400b) leads to solutions for sectoral consumption, sectoral inputs for capital goods, sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, sectoral capital stocks, return on capital:

$$C^j, J^j, W^j, k^j, L^j, Y^j, K^j, v, u^{K,j}, u^{Z,j}, R\left(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N, \tau^L\right). \tag{415}$$

Remembering that the non-traded input J^N used to produce the capital good is described by $(1-\iota)\left(\frac{P^N}{P_J}\right)^{-\phi_J} J$ (see eq. (203b)) with $J = I + \frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)^2 K$, using the fact that $J^N = Y^N - C^N - G^N$ and inserting $I = \dot{K} + \delta_K$, the capital accumulation equation reads as follows:

$$\dot{K} = \frac{u^{Z,N} Y^N - C^N - G^N - C^{K,N} K^N - C^{Z,N}}{(1-\iota)\left(\frac{P^N}{P_J}\right)^{-\phi_J}} - \delta_K K - \frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)^2 K. \tag{416}$$

Inserting first solutions for non-traded output, consumption in non-tradables, demand for non-traded input, non-traded capital and technology utilization rates described by eq. (415) together with optimal investment decision (239a) into the physical capital accumulation equation (416), and plugging the short-run solution for the return on domestic capital (411) into the dynamic equation for the shadow value of capital stock (194e), the dynamic system reads as follows:

$$
\begin{aligned}
\dot{K} &\equiv \Upsilon\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right), \\
&= \frac{Y^N\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right) - C^N\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right) - G^N(G)}{(1-\iota)\left[\frac{P^N(K,Q,G,A^j,B^j,\tau^L,\tau^C)}{P_J(K,Q,G,A^j,B^j,\tau^L,\tau^C)}\right]^{-\phi_J}} \\
&\quad \frac{-C^{K,N}\left[u^{K,N}\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right)\right]K^N - C^{Z,N}\left[u^{Z,N}\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right)\right]}{(1-\iota)\left[\frac{P^N(K,Q,G,A^j,B^j,\tau^L,\tau^C)}{P_J(K,Q,G,A^j,B^j,\tau^L,\tau^C)}\right]^{-\phi_J}} \\
&\quad -\delta_K K - \frac{K}{2\kappa}\left[\frac{Q}{P_J} - 1\right]^2, \qquad (417a) \\
\dot{Q} &\equiv \Sigma\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right), \\
&= \left(r^\star + \delta_K\right)Q - \left[\frac{R\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right)}{K}\right. \\
&\times \sum_{j=H,N} u^{K,j}\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right) u^{Z,j}\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right) K^j\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right) \\
&\quad - \sum_{j=H,N} P^j\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right) C^{K,j}\left[u^{K,j}\left(K, Q, G, A^j, B^j\right)\right] \frac{K^j\left(K, Q, G, A^j, B^j, \tau^L, \tau^C\right)}{K} \qquad (417b) \\
&\quad \left. + P_J\left[P^H(.), P^N(.)\right] \frac{\kappa}{2} v(.)\left(v(.) + 2\delta_K\right)\right]. \qquad (417c)
\end{aligned}
$$

Let us denote by Υ_X, the partial derivative evaluated at the steady-state of the capital accumulation equation w.r.t. $X = K, Q, G, A^j, B^j, \tau^L, \tau^C$. Partial derivatives evaluated at the steady-state are described by:

$$
\begin{aligned}
\Upsilon_K &\equiv \frac{\partial \dot{K}}{\partial K} = \frac{J}{J^N}\left(Y^N_K + \left(Y^N - \chi^N_1\right)u^{Z,N}_K - C^N_K - K^N \xi^N_1 u^{K,N}_K\right) + \alpha_J \phi_J J\left(\frac{P^N_K}{P^N} - \alpha^H_J \frac{P^H_K}{P^H}\right) - \delta_K \lessgtr 0, \qquad (418a) \\
\Upsilon_{X^H} &\equiv \frac{\partial \dot{K}}{\partial X^H} = \frac{J}{J^N}\left(Y^N_K + \left(Y^N - \chi^N_1\right)u^{Z,N}_{X^H} - C^N_{X^H} - K^N \xi^N_1 u^{K,N}_{X^H}\right) + \alpha_J \phi_J J\left(\frac{P^N_{X^H}}{P^N} - \alpha^H_J \frac{P^H_{X^H}}{P^H}\right), \qquad (418b)
\end{aligned}
$$

(418c)

where $X^H = Q, A^H, B^H, A^N, B^N, \tau^L, \tau^C$.

Let us denote by Σ_X, the partial derivative evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t. $X = K, Q, G, A^j, B^j, \tau^L, \tau^C$. Partial derivatives evaluated at the steady-state are described by

$$
\begin{aligned}
\Sigma_Q &\equiv \frac{\partial \dot{Q}}{\partial Q} = \left(r^\star + \delta_K\right) - \left\{R_Q + \frac{R}{K}\sum_{j=H,N}\left[K^j u^{K,j}_Q + K^j u^{Z,j}_Q\right]\right. \\
&\quad + \frac{R}{K}\left(K^H_Q + K^N_Q\right) - \sum_{j=H,N}\frac{P^j K^j}{K}\xi^j_1 u^{K,j}_Q \\
&\quad \left. + P_J \kappa v_Q \delta_K\right\} > 0, \qquad (419a) \\
\Sigma_{X^N} &\equiv \frac{\partial \dot{Q}}{\partial A^j} = -\left\{R_{X^N} + \frac{R}{K}\sum_{j=H,N}\left[K^j u^{K,j}_{X^N} + K^j u^{Z,j}_{X^N}\right]\right. \\
&\quad + \frac{R}{K}\left(K^H_{X^N} + K^N_{X^N}\right) - \sum_{j=H,N}\frac{P^j K^j}{K}\xi^j_1 u^{K,j}_{X^N} \\
&\quad \left. + P_J \kappa v_{X^N} \delta_K\right\} > 0, \qquad (419b)
\end{aligned}
$$

where $X^N = K, A^H, B^H, A^N, B^N, \tau^L, \tau^C$.

V.5 Setting the Dynamics of Tax Rates

The government issues traded bonds, D, in order to finance the excess of interest payments, $r^\star D$, government spending, and transfers, $Z(t)$, over taxes, $T(t)$:

$$\dot{D}(t) = r^\star D(t) + G(t) + Z - T(t) \tag{420}$$

where we assume that the government raises taxes on labor and consumption:

$$T^L(t) = \tau^L(t) W(t) L(t), \tag{421a}$$

$$T^C(t) = \tau^C(t) P_C(t) C(t), \tag{421b}$$

$$T(t) = T^L(t) + T^C(t), \tag{421c}$$

with τ^L the wage tax levied on households' wage income and τ^C the consumption tax.

Because our open economy displays hysteresis effects, a temporary increase in government consumption has permanent steady-state effects. Whilst the long-run effects of a temporary increase in government consumption are mitigated, we assume that transfers in eq. (420) adjusts in the long-run so as to ensure that public debet remains unchanged. Setting $\dot{D}(t) = 0$ into (420) and dividing by nominal GDP leads to:

$$\frac{Z}{Y} = \tau^L s_L + \tau^C \omega_C - \omega_G - r^\star \omega_D. \tag{422}$$

In line with the evidence, we set $\tau^L = 0.27$, $\tau^C = 0.19$, $\omega_G = 0.19$ (see column 15 of Table 7). We target $s_L = 0.66$ (see column 13 of Table 7) and $\omega_C = 0.57$. We set once for all the stead-state value of transfers Z so that Z/Y is consistent with a ratio of public debt to GDP of 60%, i.e., $\omega_D = 0.6$. It gives $Z/Y = 7.85\%$ of GDP.

Like Gali, Lopez-Salido and Vallès [2007], we assume a fiscal policy rule of the (linearized) form:

$$dT(t) = \phi_D dD(t) + \phi_G dG(t), \tag{423}$$

where $dT(t) = T(t) - T$, $dD(t) = D(t) - D$, and $dG(t) = G(t) - G$. Linearizing first the government budget constraint (420), inserting the fiscal rule (423) and collecting terms yields:

$$\begin{aligned} \dot{D}t &= r^\star dD(t) + dG(t) - dT(t), \\ &= \left(r^\star - \phi_D\right) dD(t) + \left(1 - \phi_G\right) dG(t). \end{aligned} \tag{424}$$

Inserting the dynamic equation for $\frac{dG(t)}{Y}$ given by eq. (352) into (424) and solving the differential equation leads to:

$$\frac{dD(t)}{Y} = \left[\frac{(D_0 - D)}{Y} + \Theta^D\right] e^{-\delta t} - \left[\Theta_1^D e^{-\xi t} - (1-g)\,\Theta_2^D e^{-\chi t}\right], \tag{425}$$

where we set

$$\Theta^D = \left(1 - \phi_G\right) \left[\frac{1}{\xi + r^\star - \phi_D} - \frac{(1-g)}{\chi + r^\star - \phi_D}\right], \tag{426a}$$

$$\Theta_1^D = \frac{\left(1 - \phi_G\right)}{\xi + r^\star - \phi_D}, \tag{426b}$$

$$\Theta_2^D = \frac{\left(1 - \phi_G\right)(1-g)}{\chi + r^\star - \phi_D}, \tag{426c}$$

$$\delta_D = \phi_D - r^\star. \tag{426d}$$

Note that there is a high uncertainty with regards to the parameters ϕ_D and ϕ_G. We assume that $\phi_D > r^\star$ so that $\delta_D > 0$.

Inserting (425) into (423) along with the dynamic equation for government spending (352) leads to the temporal path for taxes in percentage point of GDP:

$$\begin{aligned}
\frac{dT(t)}{Y} &= \phi_D \left[\frac{(D_0 - D)}{Y} + \Theta^D \right] e^{-\delta t} - \phi_D \left[\Theta_1 e^{-\xi t} - \Theta_2 e^{-\chi t} \right] + \phi_G \left[e^{-\xi t} - (1-g) \, e^{-\chi t} \right], \\
&= \Omega^D e^{-\delta t} - \left[\Omega_1^D e^{-\xi t} - (1-g) \, \Omega_2^D e^{-\chi t} \right], \qquad (427)
\end{aligned}$$

where we set

$$\Omega_D = \phi_D \left[\frac{(D_0 - D)}{Y} + \Theta^D \right], \qquad (428a)$$

$$\Omega_1^D = \phi_D \Theta_1 - \phi_G, \qquad (428b)$$

$$\Omega_2^D = \phi_D \Theta_2 - \phi_G. \qquad (428c)$$

We further assume that the labor tax rate evolves according to the following law of motion:

$$d\tau^L(t) = e^{-\xi_L t} - (1 - t_L) \, e^{-\chi_L t}. \qquad (429)$$

We choose t_L so as to reproduce the initial change in the labor tax rate following a rise in government spending. We set $t_L = 0$ since distortionary labor taxation does not respond to the government spending shock on impact. We also choose ξ_L and χ_L to reproduce the shape and the persistence of the response of the labor tax rate.

Whilst we assume that the labor tax rate is exogenous, we assume that the consumption tax rate adjusts so as to be consistent with the dynamic adjustment of taxes as a percentage of GDP as described by eq. (427). More specifically, differentiating (421a)-(422) leads to:

$$\frac{dT(t)}{Y} = s_L d\tau^L(t) + \omega_C d\tau^C(t), \qquad (430)$$

where we assume that the aggregate LIS, s_L, and the consumption-to-GDP ratio, ω_C, are fixed over time. Plugging the dynamic adjustment of $dT(t)/Y$ described by eq. (427) and the law of motion of $d\tau^L(t)$ described by eq. (429), the consumption tax rate must evolve as follows as as to be consistent with the dynamic path of public debt:

$$\begin{aligned}
d\tau^C(t) &= \frac{1}{\omega_C} \Big\{ \Omega^D e^{-\delta t} - \left[\Omega_1^D e^{-\xi t} - (1-g) \, \Omega_2^D e^{-\chi t} \right] \\
&\quad - s_L \left[e^{-\xi_L t} - (1 - t_L) \, e^{-\chi_L t} \right] \Big\}. \qquad (431)
\end{aligned}$$

V.6 Solving for Temporary Government Spending Shocks with Distortionary Taxation

Linearizing (417a)-(417c) in the neighborhood of the steady-state, we get in a matrix form:

$$\begin{aligned}
\begin{pmatrix} \dot{K}(t) \\ \dot{Q}(t) \end{pmatrix} &= \begin{pmatrix} \Upsilon_K & \Upsilon_Q \\ \Sigma_K & \Sigma_Q \end{pmatrix} \begin{pmatrix} dK(t) \\ dQ(t) \end{pmatrix} \\
&+ \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^N \Upsilon_{A^j} dA^j(t) + \sum_{j=H}^N \Upsilon_{B^j} dB^j(t) + \sum_{k=L,C} \Upsilon_{\tau^k} d\tau^k(t) \\ \Sigma_G dG(t) + \sum_{j=H}^N \Sigma_{A^j} dA^j(t) + \sum_{j=H}^N \Sigma_{B^j} dB^j(t) + \sum_{k=L,C} \Sigma_{\tau^k} d\tau^k(t) \end{pmatrix}, \qquad (432)
\end{aligned}$$

where the coefficients of the Jacobian matrix are partial derivatives evaluated at the steady-state, e.g., $\Upsilon_X = \frac{\partial \Upsilon}{\partial X}$ with $X = K, Q$, and the direct effects of an exogenous change in government spending on K and Q are described by $\Upsilon_G = \frac{\partial \Upsilon}{\partial G}$ and $\Sigma_G = \frac{\partial \Sigma}{\partial G}$, also evaluated at the steady-state.

As shall be useful below to write the solutions in a compact form, in addition to Φ_i^G, Φ_i^{Aj}, Φ_i^{Bj} with $i = 1, 2$ described by (356), let us define the following terms:

$$\begin{aligned}
\Phi_1^{TL} &= \left[(\Upsilon_K - \nu_2) \, \Upsilon_{\tau^L} + \Upsilon_Q \Sigma_{\tau^L} \right], && (433a) \\
\Phi_2^{TL} &= \left[(\Upsilon_K - \nu_1) \, \Upsilon_{\tau^L} + \Upsilon_Q \Sigma_{\tau^L} \right], && (433b) \\
\Phi_1^{TC} &= \left[(\Upsilon_K - \nu_2) \, \Upsilon_{\tau^C} + \Upsilon_Q \Sigma_{\tau^C} \right], && (433c) \\
\Phi_2^{TC} &= \left[(\Upsilon_K - \nu_1) \, \Upsilon_{\tau^C} + \Upsilon_Q \Sigma_{\tau^C} \right]. && (433d)
\end{aligned}$$

We denote by $V = \left(V^1, V^2\right)$ the matrix of eigenvectors with $V^{i,\prime} = \left(1, \omega_2^i\right)$ and we denote by V^{-1} the inverse matrix of V. Let us define:

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \equiv V^{-1} \begin{pmatrix} dK(t) \\ dQ(t) \end{pmatrix}. \tag{434}$$

Differentiating w.r.t. time, one obtains:

$$\begin{aligned}
\begin{pmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{pmatrix} &= \begin{pmatrix} \nu_1 & 0 \\ 0 & \nu_2 \end{pmatrix} \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \\
&+ V^{-1} \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^{N} \Upsilon_{A^j} dA^j(t) + \sum_{j=H}^{N} \Upsilon_{B^j} dB^j(t) + \sum_{k=L,C} \Upsilon_{\tau^k} d\tau^k(t) \\ \Sigma_G dG(t) + \sum_{j=H}^{N} \Sigma_{A^j} dA^j(t) + \sum_{j=H}^{N} \Sigma_{B^j} dB^j(t) + \sum_{k=L,C} \Sigma_{\tau^k} d\tau^k(t) \end{pmatrix}, \\
&= \begin{pmatrix} \nu_1 X_1(t) \\ \nu_2 X_2(t) \end{pmatrix} \\
&+ \frac{1}{\nu_1 - \nu_2} \begin{pmatrix} \Phi_1^G dG(t) + \sum_{j=H}^{N} \Phi_1^{A^j} dA^j(t) + \sum_{j=H}^{N} \Phi_1^{B^j} dB^j(t) + \sum_{k=L,C}^{N} \Phi_1^{Tk} d\tau^k(t) \\ -\Phi_2^G dG(t) - \sum_{j=H}^{N} \Phi_2^{A^j} dA^j(t) - \sum_{j=H}^{N} \Phi_2^{B^j} dB^j(t) - \sum_{k=L,C}^{N} \Phi_2^{Tk} d\tau^k(t) \end{pmatrix}
\end{aligned} \tag{435}$$

In addition to the terms defined by eqs. (436) and (437a), as will be useful below, in order to express solutions in a compact form, we set:

$$\Gamma_1^{TL} = -\frac{\Phi_1^{TL}}{\nu_1 - \nu_2} \frac{1}{\nu_1 + \xi_L}, \tag{436a}$$

$$\Gamma_2^{TL} = -\frac{\Phi_2^{TL}}{\nu_1 - \nu_2} \frac{1}{\nu_2 + \xi_L}, \tag{436b}$$

$$\Theta_1^{TL} = (1 - t_L) \frac{\nu_1 + \xi_L}{\nu_1 + \chi_L}, \tag{436c}$$

$$\Theta_2^{TL} = (1 - t_L) \frac{\nu_2 + \xi_L}{\nu_2 + \chi_L}. \tag{436d}$$

(436e)

and for τC:

$$\Gamma_1^{TCD} = -\frac{\Phi_1^{TC}}{\nu_1 - \nu_2} \frac{\Omega^D}{\omega_C} \frac{1}{\nu_1 + \delta_D}, \tag{437a}$$

$$\Gamma_2^{TCD} = -\frac{\Phi_2^{TC}}{\nu_1 - \nu_2} \frac{\Omega^D}{\omega_C} \frac{1}{\nu_2 + \delta_D}, \tag{437b}$$

$$\Gamma_1^{TCG} = -\frac{\Phi_1^{TC}}{\nu_1 - \nu_2} \frac{1}{\omega_C} \frac{1}{\nu_1 + \xi}, \tag{437c}$$

$$\Gamma_2^{TCG} = -\frac{\Phi_2^{TC}}{\nu_1 - \nu_2} \frac{1}{\omega_C} \frac{1}{\nu_2 + \xi}, \tag{437d}$$

$$\Gamma_1^{TCL} = -\frac{\Phi_1^{TC}}{\nu_1 - \nu_2} \frac{s_L}{\omega_C} \frac{1}{\nu_1 + \xi_L}, \tag{437e}$$

$$\Gamma_2^{TCL} = -\frac{\Phi_2^{TC}}{\nu_1 - \nu_2} \frac{1}{\omega_C} \frac{1}{\nu_2 + \xi_L}, \tag{437f}$$

where $x^j = a^j, b^j$.

Solving for $X_1(t)$ gives:

$$
\begin{aligned}
X_1(t) &= e^{\nu_1 t}\left\{X_1(0) + \frac{\Phi_1^G}{\nu_1-\nu_2}\int_0^t dG(\tau)e^{-\nu_1\tau}d\tau + \sum_{X^j}\frac{\Phi_1^{X^j}}{\nu_1-\nu_2}\int_0^t dX^j(\tau)e^{-\nu_1\tau}d\tau\right\} \\
&+ e^{\nu_1 t}\left\{\frac{\Phi_1^{TL}}{\nu_1-\nu_2}\int_0^t d\tau^L(\tau)e^{-\nu_1\tau}d\tau + \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\int_0^t d\tau^C(\tau)e^{-\nu_1\tau}d\tau\right\}, \qquad (438) \\
&= e^{\nu_1 t}\Bigg\{X_1(0) + \frac{\Phi_1^G Y}{\nu_1-\nu_2}\int_0^t \left[e^{-(\xi+\nu_1)\tau} - (1-g)\,e^{-(\chi+\nu_1)\tau}\right]d\tau \\
&+ \sum_{X^j}\frac{\Phi_1^{X^j}X^j}{\nu_1-\nu_2}\int_0^t \left[e^{-(\xi_X^j+\nu_1)\tau} - \left(1-x^j\right)e^{-(\chi_X^j+\nu_1)\tau}\right]d\tau\Bigg\} \\
&+ \frac{\Phi_1^{TL}}{\nu_1-\nu_2}\int_0^t \left[e^{-(\xi_L+\nu_1)\tau} - (1-t_L)\,e^{-(\chi_L+\nu_1)\tau}\right]d\tau \\
&+ \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\frac{\Omega^D}{\omega_C}\int_0^t e^{-(\delta_D+\nu_1)\tau}d\tau \\
&- \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\frac{1}{\omega_C}\int_0^t \left[\Omega_1^D e^{-(\xi+\nu_1)\tau} - (1-g)\,\Omega_2^D e^{-(\chi+\nu_1)\tau}\right]d\tau \\
&- \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\frac{s_L}{\omega_C}\int_0^t \left[e^{-(\xi_L+\nu_1)\tau} - (1-t_L)\,e^{-(\chi_L+\nu_1)\tau}\right]d\tau, \\
&= e^{\nu_1 t}X_1(0) + \frac{\Phi_1^G Y}{\nu_1-\nu_2}\left[\left(\frac{e^{\nu_1 t}-e^{-\xi t}}{\nu_1+\xi}\right) - (1-g)\left(\frac{e^{\nu_1 t}-e^{-\chi t}}{\nu_1+\chi}\right)\right] \\
&+ \sum_{X^j}\frac{\Phi_1^{X^j}X^j}{\nu_1-\nu_2}\left[\left(\frac{e^{\nu_1 t}-e^{-\xi_X^j t}}{\nu_1+\xi_X^j}\right) - \left(1-x^j\right)\left(\frac{e^{\nu_1 t}-e^{-\chi_X^j t}}{\nu_1+\chi_X^j}\right)\right] \\
&+ \frac{\Phi_1^{TL}}{\nu_1-\nu_2}\left[\left(\frac{e^{\nu_1 t}-e^{-\xi_L t}}{\nu_1+\xi_L}\right) - (1-t_L)\left(\frac{e^{\nu_1 t}-e^{-\chi_L t}}{\nu_1+\chi_L}\right)\right] \\
&+ \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\frac{\Omega^D}{\omega_C}\left(\frac{e^{\nu_1 t}-e^{-\delta_D t}}{\nu_1+\delta_D}\right) \\
&- \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\frac{1}{\omega_C}\left[\Omega_1^D\left(\frac{e^{\nu_1 t}-e^{-\xi t}}{\nu_1+\xi}\right) - (1-g)\,\Omega_2^D\left(\frac{e^{\nu_1 t}-e^{-\chi t}}{\nu_1+\chi}\right)\right] \\
&- \frac{\Phi_1^{TC}}{\nu_1-\nu_2}\frac{s_L}{\omega_C}\left[\left(\frac{e^{\nu_1 t}-e^{-\xi_L t}}{\nu_1+\xi_L}\right) - (1-t_L)\left(\frac{e^{\nu_1 t}-e^{-\chi_L t}}{\nu_1+\chi_L}\right)\right], \\
&= e^{\nu_1 t}\Bigg[X_1(0) - \Gamma_1^G\left(1-\Theta_1^G\right) - \sum_{X^j}\Gamma_1^{X^j}\left(1-\Theta_1^{X^j}\right) - \left(\Gamma_1^{TL}-\Gamma_1^{TCL}\right)\left(1-\Theta_1^{TL}\right) - \Gamma_1^{TCD} \\
&+ \Gamma_1^{TCG}\left(\Omega_1^D - \Theta_1^G\Omega_2^D\right)\Bigg] + \Gamma_1^G\left(e^{-\xi t} - \Theta_1^G e^{-\chi t}\right) + \sum_{X^j}\Gamma_1^{X^j}\left(e^{-\xi_X^j t} - \Theta_1^{X^j}e^{-\chi_X^j t}\right) \\
&+ \left(\Gamma_1^{TL}-\Gamma_1^{TCL}\right)\left(e^{-\xi_L t} - \Theta_1^{TL}e^{-\chi_L t}\right) + \Gamma_1^{TCD}e^{-\delta_D t} - \Gamma_1^{TCG}\left(\Omega_1^D e^{-\xi t} - \Theta_1^G\Omega_2^D e^{-\chi t}\right), \qquad (439)
\end{aligned}
$$

where Γ_1^{TL} and Θ_1^{TL} are given by (436a) and (436c), respectively, and Γ_1^{TCD}, Γ_1^{TCD}, Γ_1^{TCL} are given by (437a), (437c), (437e), respectively.

Solving for $X_2(t)$ gives:

$$
\begin{aligned}
X_2(t) &= e^{\nu_2 t}\left\{X_2(0) - \frac{\Phi_2^G}{\nu_1-\nu_2}\int_0^t dG(\tau)e^{-\nu_2\tau}d\tau - \sum_{X^j}\frac{\Phi_2^{X^j}}{\nu_1-\nu_2}\int_0^t dX^j(\tau)e^{-\nu_2\tau}d\tau\right\} \\
&- \frac{\Phi_2^{TL}}{\nu_1-\nu_2}\int_0^t d\tau^L(\tau)e^{-\nu_2\tau}d\tau - \frac{\Phi_2^{TC}}{\nu_1-\nu_2}\int_0^t d\tau^C(\tau)e^{-\nu_2\tau}d\tau. \qquad (440)
\end{aligned}
$$

Because $\nu_2 > 0$, for the solution to converge to the steady-state, the term in brackets must be nil when we let t tend toward infinity:

$$\begin{aligned}
X_2(0) &= \frac{\Phi_2^G Y}{\nu_1-\nu_2}\left[\frac{1}{\xi+\nu_2}-(1-g)\frac{1}{\chi+\nu_2}\right]+\sum_{X^j}\frac{\Phi_2^{X^j}X^j}{\nu_1-\nu_2}\left[\frac{1}{\xi_X^j+\nu_2}-\left(1-x^j\right)\frac{1}{\chi_X^j+\nu_2}\right] \\
&+ \frac{\Phi_2^{TL}}{\nu_1-\nu_2}\left[\frac{1}{\xi_L+\nu_2}-(1-t_L)\frac{1}{\chi_L+\nu_2}\right]+\frac{\Phi_2^{TC}}{\nu_1-\nu_2}\frac{\Omega_D}{\omega_C}\frac{1}{\delta_D+\nu_2} \\
&- \frac{\Phi_2^{TC}}{\nu_1-\nu_2}\frac{1}{\omega_C}\left[\Omega_1^D\frac{1}{\xi+\nu_2}-(1-g)\,\Omega_2^D\frac{1}{\chi+\nu_2}\right]-\frac{\Phi_2^{TC}}{\nu_1-\nu_2}\frac{s_L}{\omega_C}\left[\frac{1}{\xi_L+\nu_2}-(1-t_L)\frac{1}{\chi_L+\nu_2}\right], \\
&= -\Gamma_2^G\left(1-\Theta_2^G\right)-\sum_{X^j}\Gamma_2^{X^j}\left(1-\Theta_2^{X^j}\right)-\left(\Gamma_2^{TL}-\Gamma_2^{TCL}\right)\left(1-\Theta_2^{TL}\right)-\Gamma_2^{TCD} \\
&- \Gamma_2^{TCG}\left(\Omega_1^D-\Theta_2^G\Omega_2^D\right),
\end{aligned} \tag{441}$$

where Γ_2^{TL} and Θ_2^{TL} are given by (436b) and (436d), respectively, and Γ_2^{TCD}, Γ_2^{TCD}, Γ_2^{TCL} are given by (437b), (437d), (437f), respectively.

Inserting first $X_2(0)$, the 'stable' solution for $X_2(t)$, i.e., consistent with convergence toward the steady-state when t tends toward infinity, is thus given by:

$$\begin{aligned}
X_2(t) &= e^{\nu_2 t}\Bigg\{\frac{\Phi_2^G Y}{\nu_1-\nu_2}\left[\frac{e^{-(\xi+\nu_2)t}}{\xi+\nu_2}-(1-g)\frac{e^{-(\chi+\nu_2)t}}{\chi+\nu_2}\right] \\
&+ \sum_{X^j}\frac{\Phi_2^{X^j}X^j}{\nu_1-\nu_2}\left[\frac{e^{-\left(\xi_X^j+\nu_2\right)t}}{\xi_X^j+\nu_2}-\left(1-x^j\right)\frac{e^{-\left(\chi_X^j+\nu_2\right)t}}{\chi_X^j+\nu_2}\right], \\
&+ \frac{\Phi_2^{TL}}{\nu_1-\nu_2}\left[\frac{e^{-(\xi_L+\nu_2)t}}{\xi_L+\nu_2}-(1-t_L)\frac{e^{-(\chi_L+\nu_2)t}}{\chi_L+\nu_2}\right] \\
&+ \frac{\Phi_2^{TC}}{\nu_1-\nu_2}\frac{\Omega^D}{\omega_C}\frac{e^{-(\delta_D+\nu_2)t}}{\delta_D+\nu_2}-\frac{\Phi_2^{TC}}{\nu_1-\nu_2}\frac{1}{\omega_C}\left[\Omega_1^D\frac{e^{-(\xi+\nu_2)t}}{\xi+\nu_2}-(1-g)\,\Omega_2^D\frac{e^{-(\chi+\nu_2)t}}{\chi+\nu_2}\right] \\
&- \frac{\Phi_2^{TC}}{\nu_1-\nu_2}\frac{s_L}{\omega_C}\left[\frac{e^{-(\xi_L+\nu_2)t}}{\xi_L+\nu_2}-(1-t_L)\frac{e^{-(\chi_L+\nu_2)t}}{\chi_L+\nu_2}\right]\Bigg\}, \\
&= -\Gamma_2^G\left(e^{-\xi t}-\Theta_2^G e^{-\chi t}\right)-\sum_{X^j}\Gamma_2^{X^j}\left(e^{-\xi_X^j t}-\Theta_2^{X^j}e^{-\chi_X^j t}\right)-\left(\Gamma_2^{TL}-\Gamma_2^{TCL}\right)\left(e^{-\xi_L t}-\Theta_2^{TL}e^{-\chi_L t}\right) \\
&- \Gamma_2^{TCD}e^{-\delta_D t}+\Gamma_2^{TCG}\left(\Omega_1^D e^{-\xi t}-\Theta_2^G\Omega_2^D e^{-\chi t}\right).
\end{aligned} \tag{442}$$

Using the definition of $X_i(t)$ (with $i=1,2$) given by (434), we can recover the solutions for $K(t)$ and $Q(t)$:

$$K(t)-\tilde{K}=X_1(t)+X_2(t), \tag{443a}$$

$$Q(t)-\tilde{Q}=\omega_2^1 X_1(t)+\omega_2^2 X_2(t). \tag{443b}$$

Setting $t=0$ into (443a) gives $X_1(0)=(K(0)-K)-X_2(0)$ where $X_2(0)$ is described by eq. (441); the solution (439) for $X_1(t)$ can be rewritten as follows:

$$\begin{aligned}
X_1(t) &= e^{\nu_1 t}X_{11}+\Gamma_1^G\left(e^{-\xi t}-\Theta_1^G e^{-\chi t}\right)+\sum_{X^j}\Gamma_1^{X^j}\left(e^{-\xi_X^j t}-\Theta_1^{X^j}e^{-\chi_X^j t}\right) \\
&+ \left(\Gamma_1^{TL}-\Gamma_1^{TCL}\right)\left(e^{-\xi_L t}-\Theta_1^{TL}e^{-\chi_L t}\right)+\Gamma_1^{TCD}e^{-\delta_D t}-\Gamma_1^{TCG}\left(\Omega_1^D e^{-\xi t}-\Theta_1^G\Omega_1^D e^{-\chi t}\right),
\end{aligned} \tag{444}$$

where we set

$$\begin{aligned}
X_{11} &= X_1(0)-\Gamma_1^G\left(1-\Theta_1^G\right)-\sum_{X^j}\Gamma_1^{X^j}\left(1-\Theta_1^{X^j}\right)-\left(\Gamma_1^{TL}-\Gamma_1^{TCL}\right)\left(1-\Theta_1^{TL}\right)-\Gamma_1^{TCD} \\
&+ \Gamma_1^{TCG}\left(\Omega_1^D-\Theta_1^G\Omega_2^D\right).
\end{aligned} \tag{445}$$

Linearizing the current account equation (338) around the steady-state:

$$\begin{aligned}
\dot{N}(t) &= r^{\star} dN(t) + \Xi_K dK(t) + \Xi_Q dQ(t) + \Xi_G dG(t) + \sum_{X^j} \Xi_{X^j} dX^j(t) + \sum_{k=L,C} \Xi_{\tau^k} d\tau^k(t), \\
&= \left(\Xi_K + \Xi_Q \omega_2^1\right) X_1(t) + \left(\Xi_K + \Xi_Q \omega_2^2\right) X_2(t) \\
&+ \Xi_G Y \left[e^{-\xi t} - (1-g)\, e^{-\chi t}\right] + \sum_{X^j} X^j \left[e^{-\xi_X^j t} - \left(1 - x^j\right) e^{-\chi_X^j t}\right] + \Xi_{\tau^L} \left[e^{-\xi_L t} - (1 - t_L)\, e^{-\chi_L t}\right] \\
&+ \frac{\Xi_{\tau^C}}{\omega_C} \left\{\Omega^D e^{-\delta_D t} - \left[\Omega_1^D e^{-\xi t} - (1-g)\, \Omega_2^D e^{-\chi t}\right] - s_L \left[e^{-\xi_L t} - (1 - t_L)\, e^{-\chi_L t}\right]\right\}. \qquad (446)
\end{aligned}$$

Setting $N_1 = \Xi_K + \Xi_Q \omega_2^1$, $N_2 = \Xi_K + \Xi_Q \omega_2^2$, inserting solutions for $K(t)$ and $Q(t)$ given by (443), solving and invoking the transversality condition, yields the solution for traded bonds:

$$\begin{aligned}
dN(t) &= e^{r^\star t} (N_0 - N) + \frac{\omega_N^1}{\nu_1 - r^\star} \left(e^{r^\star t} - e^{\nu_1 t}\right) \\
&+ \frac{N_1 \Gamma_1^G}{\xi + r^\star} \left[\left(e^{r^\star t} - e^{-\xi t}\right) - \Theta_1^{G,\prime} \left(e^{r^\star t} - e^{-\chi t}\right)\right] - \frac{N_2 \Gamma_2^G}{\xi + r^\star} \left[\left(e^{r^\star t} - e^{-\xi t}\right) - \Theta_2^{G,\prime} \left(e^{r^\star t} - e^{-\chi t}\right)\right] \\
&+ \sum_{X^j} \frac{N_1 \Gamma_1^{X^j}}{\xi_X^j + r^\star} \left[\left(e^{r^\star t} - e^{-\xi_X^j t}\right) - \Theta_1^{X^j,\prime} \left(e^{r^\star t} - e^{-\chi_X^j t}\right)\right] \\
&- \sum_{X^j} \frac{N_2 \Gamma_2^{X^j}}{\xi_X^j + r^\star} \left[\left(e^{r^\star t} - e^{-\xi_X^j t}\right) - \Theta_2^{X^j,\prime} \left(e^{r^\star t} - e^{-\chi_X^j t}\right)\right] \\
&+ \frac{N_1 \left(\Gamma_1^{TL} - \Gamma_1^{TCL}\right)}{\xi_L + r^\star} \left[\left(e^{r^\star t} - e^{-\xi_L t}\right) - \Theta_1^{TL,\prime} \left(e^{r^\star t} - e^{-\chi_L t}\right)\right] \\
&- \frac{N_2 \left(\Gamma_2^{TL} - \Gamma_2^{TCL}\right)}{\xi_L + r^\star} \left[\left(e^{r^\star t} - e^{-\xi_L t}\right) - \Theta_2^{TL,\prime} \left(e^{r^\star t} - e^{-\chi_L t}\right)\right] \\
&+ \frac{N_1 \Gamma_1^{TCD}}{\delta_D + r^\star} \left(e^{r^\star t} - e^{-\delta_D t}\right) - \frac{N_2 \Gamma_2^{TCD}}{\delta_D + r^\star} \left(e^{r^\star t} - e^{-\delta_D t}\right) \\
&+ \frac{N_1 \Gamma_1^{TCG}}{\xi + r^\star} \left[\Omega_1^D \left(e^{r^\star t} - e^{-\xi t}\right) - \Theta_1^{G,\prime} \Omega_2^G \left(e^{r^\star t} - e^{-\chi t}\right)\right] \\
&- \frac{N_2 \Gamma_2^{TCG}}{\xi + r^\star} \left[\Omega_1^D \left(e^{r^\star t} - e^{-\xi t}\right) - \Theta_2^{G,\prime} \Omega_2^G \left(e^{r^\star t} - e^{-\chi t}\right)\right] \\
&+ \frac{\Xi_G Y}{\xi + r^\star} \left[\left(e^{r^\star t} - e^{-\xi t}\right) - \Theta^{G,\prime} \left(e^{r^\star t} - e^{-\chi t}\right)\right] \\
&+ \sum_{X^j} \frac{\Xi_{X^j} X^j}{\xi_X^j + r^\star} \left[\left(e^{r^\star t} - e^{-\xi_X^j t}\right) - \Theta^{X^j,\prime} \left(e^{r^\star t} - e^{-\chi_X^j t}\right)\right], \\
&+ \left(\Xi_{\tau^L} - \Xi_{\tau^C} \frac{s_L}{\omega_C}\right) \frac{1}{\xi_L + r^\star} \left[\left(e^{r^\star t} - e^{-\xi_L t}\right) - \Theta^{TL,\prime} \left(e^{r^\star t} - e^{-\chi_L t}\right)\right] \\
&+ \Xi_{\tau^C} \frac{\Omega^D}{\omega_C} \frac{1}{\delta_D + r^\star} \left(e^{r^\star t} - e^{-\delta_D t}\right), \\
&- \frac{\Xi_{\tau^C}}{\omega_C} \frac{1}{\xi + r^\star} \left[\Omega_1^D \left(e^{r^\star t} - e^{-\xi t}\right) - \Theta^{G,\prime} \Omega_2^D \left(e^{r^\star t} - e^{-\chi t}\right)\right],
\end{aligned}$$

where $\omega_N^1 = X_{11}$ and

$$\Theta^{G,\prime} = (1-g)\,\frac{\xi + r^\star}{\chi + r^\star}, \tag{447a}$$

$$\Theta_1^{G,\prime} = \Theta_1^G\,\frac{\xi + r^\star}{\chi + r^\star}, \tag{447b}$$

$$\Theta_2^{G,\prime} = \Theta_2^G\,\frac{\xi + r^\star}{\chi + r^\star}, \tag{447c}$$

$$\Theta^{X^j,\prime} = \left(1-x^j\right)\frac{\xi_X^j + r^\star}{\chi_X^j + r^\star}, \tag{447d}$$

$$\Theta_1^{X^j,\prime} = \Theta_1^{X^j}\,\frac{\xi_X^j + r^\star}{\chi_X^j + r^\star}, \tag{447e}$$

$$\Theta_2^{X^j,\prime} = \Theta_2^{X^j}\,\frac{\xi_X^j + r^\star}{\chi_X^j + r^\star}. \tag{447f}$$

$$\Theta^{TL,\prime} = (1-t_L)\,\frac{\xi_L + r^\star}{\chi_L + r^\star}, \tag{447g}$$

$$\Theta_1^{TL,\prime} = \Theta_1^{TL}\,\frac{\xi_L + r^\star}{\chi_L + r^\star}, \tag{447h}$$

$$\Theta_2^{TL,\prime} = \Theta_2^{TL}\,\frac{\xi_L + r^\star}{\chi_L + r^\star}, \tag{447i}$$

By rearranging terms, we get

$$\begin{aligned}
dN(t) \;=\; & e^{r^\star t}\Bigg[(N_0 - N) - \frac{\omega_N^1}{\nu_1 - r^\star} + \frac{N_1\Gamma_1^G}{\xi + r^\star}\left(1-\Theta_1^{G,\prime}\right) - \frac{N_2\Gamma_2^G}{\xi + r^\star}\left(1-\Theta_2^{G,\prime}\right) + \frac{\Xi_G Y}{\xi + r^\star}\left(1-\Theta^{G,\prime}\right) \\
+ & \sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j + r^\star}\left(1-\Theta_1^{X^j,\prime}\right) - \sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j + r^\star}\left(1-\Theta_2^{X^j,\prime}\right) + \sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j + r^\star}\left(1-\Theta_2^{X^j,\prime}\right) \\
+ & \frac{N_1\left(\Gamma_1^{TL} - \Gamma_1^{TCL}\right)}{\xi_L + r^\star}\left(1-\Theta_1^{TL,\prime}\right) - \frac{N_2\left(\Gamma_2^{TL} - \Gamma_2^{TCL}\right)}{\xi_L + r^\star}\left(1-\Theta_2^{TL,\prime}\right) \\
+ & \left(\Xi_{\tau^L} - \Xi_{\tau^C}\frac{s_L}{\omega_C}\right)\frac{1}{\xi_L + r^\star}\left(1-\Theta_2^{TL,\prime}\right) + \frac{N_1\Gamma_1^{TCD}}{\delta_D + r^\star} - \frac{N_2\Gamma_2^{TCD}}{\delta_D + r^\star} + \Xi_{\tau^C}\frac{\Omega^D}{\omega_C} \\
+ & \frac{N_1\Gamma_1^{TCLG}}{\xi + r^\star}\left(\Omega_1^D - \Theta_1^{G,\prime}\Omega_2^D\right) - \frac{N_2\Gamma_2^{TCG}}{\xi + r^\star}\left(\Omega_1^D - \Theta_2^{G,\prime}\Omega_2^D\right) + \frac{\Xi_{\tau^C}}{\omega_C}\frac{1}{\xi + r^\star}\left(\Omega_1^D - \Theta^{G,\prime}\Omega_2^D\right)\Bigg] \\
+ & \frac{\omega_N^1}{\nu_1 - r^\star}e^{\nu_1 t} - \frac{N_1\Gamma_1^G}{\xi + r^\star}\left(e^{-\xi t} - \Theta_1^{G,\prime}e^{-\chi t}\right) - \sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j + r^\star}\left(e^{-\xi_X^j t} - \Theta_1^{G,\prime}e^{-\chi_X^j t}\right) \\
+ & \frac{N_2\Gamma_2^G}{\xi + r^\star}\left(e^{-\xi t} - \Theta_2^{G,\prime}e^{-\chi t}\right) + \sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j + r^\star}\left(e^{-\xi_X^j t} - \Theta_2^{G,\prime}e^{-\chi_X^j t}\right) \\
- & \frac{\Xi_G Y}{\xi + r^\star}\left(e^{-\xi t} - \Theta^{G,\prime}e^{-\chi t}\right) - \sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j + r^\star}\left(e^{-\xi_X^j t} - \Theta_2^{G,\prime}e^{-\chi_X^j t}\right) \\
- & \frac{N_1\left(\Gamma_1^{TL} - \Gamma_1^{TCL}\right)}{\xi_L + r^\star}\left(e^{-\xi_L t} - \Theta_1^{TL,\prime}e^{-\chi_L t}\right) + \frac{N_2\left(\Gamma_2^{TL} - \Gamma_2^{TCL}\right)}{\xi_L + r^\star}\left(e^{-\xi_L t} - \Theta_2^{TL,\prime}e^{-\chi_L t}\right) \\
- & \frac{N_1\Gamma_1^{TLD}}{\delta_D + r^\star}e^{-\delta_D t} + \frac{N_{12}\Gamma_1^{TLD}}{\delta_D + r^\star}e^{-\delta_D t} - \Xi_{\tau^C}\frac{\Omega^D}{\omega_C}\frac{1}{\delta_D + r^\star}e^{-\delta_D t} \\
- & \frac{N_1\Gamma_1^{TCG}}{\xi + r^\star}\left(\Omega_1^D e^{-\xi t} - \Theta_1^{G,\prime}\Omega_2^D e^{-\chi t}\right) + \frac{N_2\Gamma_2^{TCG}}{\xi + r^\star}\left(\Omega_1^D e^{-\xi t} - \Theta_2^{G,\prime}\Omega_2^D e^{-\chi t}\right) \\
- & \left(\Xi_{\tau^L} - \Xi_{\tau^C}\frac{s_L}{\omega_C}\right)\frac{1}{\xi_L + r^\star}\left(1-\Theta^{TL,\prime}\right) + \frac{\Xi_{\tau^C}}{\omega_C}\frac{1}{\xi + r^\star}\left(\Omega_1^D e^{-\xi t} - \Theta^{G,\prime}\Omega_2^D e^{-\chi t}\right).
\end{aligned}$$

Invoking the transversality condition, to ultimately remain solvent, the open economy must satisfy the following condition:

$$(N_0 - N) - \frac{\omega_N^1}{\nu_1 - r^\star} + \frac{\omega_N^G}{\xi + r^\star} + \sum_{X^j}\frac{\omega_N^{X^j}}{\xi_X^j + r^\star} + \frac{\omega_N^{TL}}{\xi_L + r^\star} + \frac{\omega_N^{TCD}}{\delta_D + r^\star} + \frac{\omega_N^{TCG}}{\xi + r^\star} = 0, \tag{448}$$

where

$$\omega_N^G = N_1\Gamma_1^G\left(1-\Theta_1^{G,\prime}\right) - N_2\Gamma_2^G\left(1-\Theta_2^{G,\prime}\right) + \Xi_G Y\left(1-\Theta^{G,\prime}\right), \quad (449a)$$

$$\omega_N^{X^j} = N_1\Gamma_1^{X^j}\left(1-\Theta_1^{X^j,\prime}\right) - N_2\Gamma_2^{X^j}\left(1-\Theta_2^{X^j,\prime}\right) + \Xi_{X^j}X^j\left(1-\Theta^{X^j,\prime}\right). \quad (449b)$$

$$\omega_N^{TL} = N_1\left(\Gamma_1^{TL}-\Gamma_1^{TCL}\right)\left(1-\Theta_1^{TL,\prime}\right) - N_2\left(\Gamma_2^{TL}-\Gamma_2^{TCL}\right)\left(1-\Theta_2^{TL,\prime}\right) + \left(\Xi_{\tau^L} - \Xi_{\tau^C}\frac{s_L}{\omega_C}\right)\left(1-\Theta^{TL,\prime}\right), \quad (449c)$$

$$N_1\Gamma_1^{TCG}\left(\Omega_1^D - \Theta_1^{G,\prime}\Omega_2^D\right) - N_2\Gamma_2^{TCG}\left(\Omega_1^D - \Theta_2^{G,\prime}\Omega_2^D\right) + \frac{\Xi_{\tau^C}}{\omega_C}\left(\Omega_1^D - \Theta^{G,\prime}\Omega_2^D\right), \quad (449d)$$

$$\omega_N^{TCD} = N_1\Gamma_1^{TCD} - N_2\Gamma_2^{TCD} + \Xi_{\tau^C}\frac{\Omega^D}{\omega_C}. \quad (449e)$$

The convergent path for the net foreign asset position is:

$$\begin{aligned}
dN(t) &= \frac{\omega_N^1}{\nu_1 - r^\star}e^{\nu_1 t} - \frac{N_1\Gamma_1^G}{\xi + r^\star}\left(e^{-\xi t} - \Theta_1^{G,\prime}e^{-\chi t}\right) - \sum_{X^j}\frac{N_1\Gamma_1^{X^j}}{\xi_X^j + r^\star}\left(e^{-\xi_X^j t} - \Theta_1^{G,\prime}e^{-\chi_X^j t}\right) \\
&+ \frac{N_2\Gamma_2^G}{\xi + r^\star}\left(e^{-\xi t} - \Theta_2^{G,\prime}e^{-\chi t}\right) + \sum_{X^j}\frac{N_2\Gamma_2^{X^j}}{\xi_X^j + r^\star}\left(e^{-\xi_X^j t} - \Theta_2^{G,\prime}e^{-\chi_X^j t}\right) \\
&- \frac{\Xi_G Y}{\xi + r^\star}\left(e^{-\xi t} - \Theta^{G,\prime}e^{-\chi t}\right) - \sum_{X^j}\frac{\Xi_{X^j}X^j}{\xi_X^j + r^\star}\left(e^{-\xi_X^j t} - \Theta_2^{G,\prime}e^{-\chi_X^j t}\right) \\
&- \frac{N_1\left(\Gamma_1^{TL} - \Gamma_1^{TCL}\right)}{\xi_L + r^\star}\left(e^{-\xi_L t} - \Theta_1^{TL,\prime}e^{-\chi_L t}\right) + \frac{N_2\left(\Gamma_2^{TL} - \Gamma_2^{TCL}\right)}{\xi_L + r^\star}\left(e^{-\xi_L t} - \Theta_2^{TL,\prime}e^{-\chi_L t}\right) \\
&- \frac{N_1\Gamma_1^{TLD}}{\delta_D + r^\star}e^{-\delta_D t} + \frac{N_{12}\Gamma_1^{TLD}}{\delta_D + r^\star}e^{-\delta_D t} - \Xi_{\tau^C}\frac{\Omega^D}{\omega_C}\frac{1}{\delta_D + r^\star}e^{-\delta_D t} \\
&- \frac{N_1\Gamma_1^{TCG}}{\xi + r^\star}\left(\Omega_1^D e^{-\xi t} - \Theta_1^{G,\prime}\Omega_2^D e^{-\chi t}\right) + \frac{N_2\Gamma_2^{TCG}}{\xi + r^\star}\left(\Omega_1^D e^{-\xi t} - \Theta_2^{G,\prime}\Omega_2^D e^{-\chi t}\right) \\
&- \left(\Xi_{\tau^L} - \Xi_{\tau^C}\frac{s_L}{\omega_C}\right)\frac{1}{\xi_L + r^\star}\left(1 - \Theta^{TL,\prime}\right) + \frac{\Xi_{\tau^C}}{\omega_C}\frac{1}{\xi + r^\star}\left(\Omega_1^D e^{-\xi t} - \Theta^{G,\prime}\Omega_2^D e^{-\chi t}\right). \quad (450)
\end{aligned}$$

V.7 Lump-Sum Tax vs. Public Debt and Distortationary Tax

To calibrate the model with public debt and distortionary taxation, we proceed as follows. As detailed in section V.5, we assume that the persistent but temporary increase in government consumption is financed by an increase in public debt whilst taxation responds positively (but slowly) to higher government expenditure and the accumulation of public debt. Obviously, the dynamic adjustment of public debt fulfills the intertemporal budget constraint of the government as we impose the transversality condition $\lim_{t\to\infty} D(t)e^{-r^\star t} = 0$. We further assume an exogenous dynamic process for labor taxation, $\tau^L(t)$, whilst consumption taxation adjusts endogenously to ensure that the dynamic path for total taxes is consistent with the intertemporal budget constraint.

Estimating the dynamic adjustment of taxation and public debt to an exogenous shock to government consumption. To calibrate the model to the data, we set $\tau^C = 0.191$ and $\tau^L = 0.273$. Source: Both labor and consumption tax rates are provided by Mc Daniel [2007] who average labor tax for all OECD countries of our sample. Time series for labor tax and consumption tax rates cover the period 1970-2015 for all countries of our sample. The labor tax is the average tax rate on household income plus average payroll tax rate paid by employer and employee.

To calibrate the law of motion of tax rates, we first estimate the response of tax rates. We estimate the dynamic responses of tax rates on labor and consumption to the identified government spending shock by using the Jordà's [2005] single-equation method, see section 2.2. As can be seen in the solid blue lines of the first row of Fig. 39, a temporary increase in government consumption is financed by both higher taxation, see Fig. 39(a), and the accumulation of public debt, see Fig. 39(b). The solid blue line in the second row of Fig. 39 shows that taxation remains unchanged on impact and slowly increases for labor

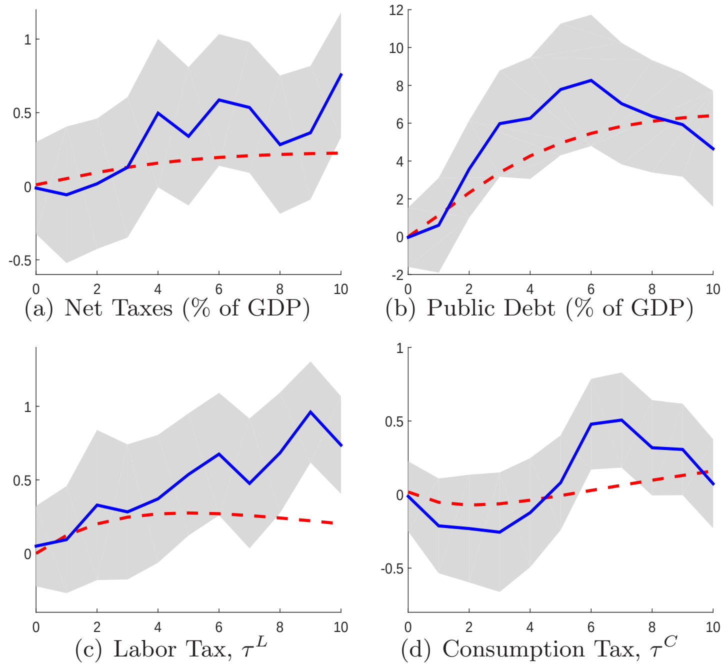


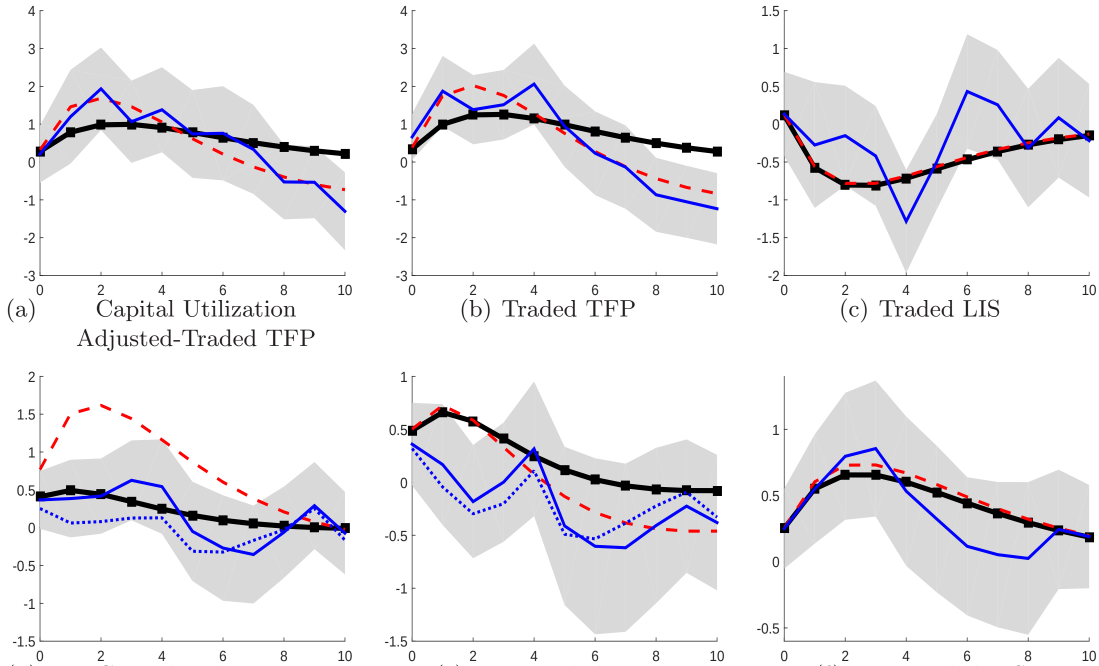
Figure 39: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Distortionary Taxation and Public Debt. Notes: Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds. The dashed red line shows results for the beseline model with distortionary taxation (i.e., labor and consumption) taxation.

whilst it declines first for consumption tax and then increases. By using a sample of EU countries, Lambertini and Proebsting [2022] also documents evidence pointing at a decline in consumption taxation in the short run following a rise in government consumption taxation.

Calibrating the model to the data. We start with the exogenous law of motion for labor taxation described by eq. (429), i.e., $d\tau^L(t) = e^{-\xi_L t} - (1 - t_L)\, e^{-\chi_L t}$. We choose values for exogenous parameters so as to reproduce the response of the labor tax τ^L. As can be seen in Fig. 39(c), the labor tax is unresponsive on impact and thus we set $t_L = 0$. We choose values for ξ_L and χ_L so to reproduce the shape of the dynamic adjustment of the labor tax rate. Because the rise in τ^L is persistent, we choose $\xi_L = 0.200$ and $\chi_L = 0.2015$. These values lead the model shown in the dashed red lines to understate the rise in the labor tax beyond $t = 4$ years. We have chosen to understate the rise in τ^L on purpose as higher values of τ^L would require a disproportionate decline in the consumption tax rate.

With regards to total taxation described by (423), i.e., $dT(t) = \phi_D dD(t) + \phi_G dG(t)$, we choose $\phi_D = 0.035$ and $\phi_G = 0.0301$. As shown in eq. (425), these values together with the values which govern the law of motion of government consumption ξ, χ, and g determine the dynamic adjustment of public debt. A quick inspection of Fig. 39(b) shows that these values lead the model shown in the dashed red lines to reproduce very well the dynamics of public debt. According to (430), i.e., $d\tau^C(t) = \frac{1}{\omega_C}\left[\frac{dT(t)}{Y} - s_L d\tau^L(t)\right]$, the adjustment of τ^C is determined by the adjustment of labor taxation and the public debt (see also eq. (431)). As can be seen in Fig. 39(d), the model can generate the slight decline in the short run and the increase in the medium-run.

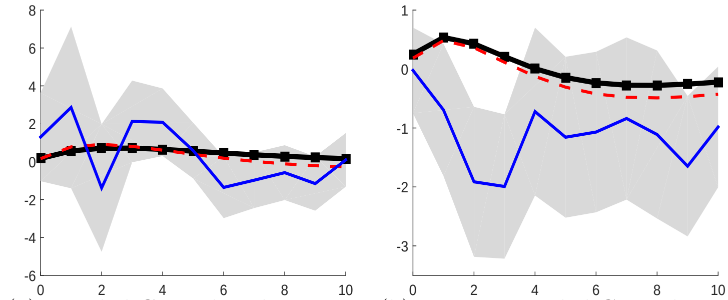
We assume that the calibration remains identical to the baseline calibration detailed in section 4.1. The calibration of parameters which govern the dynamics of the capital and technology utilization rate must be adjusted as utilization rates respond endogenously to change in the return of capital and in value added. Because tax rates have a negative impact on the demand of inputs which strongly affect the non-traded sector which is more intensive in labor, we maintained the value for ξ_2^H at 0.27 (which is thus unchanged with respect to the baseline scenario) and a value for ξ_2^N of 0.01 (instead of 0.03 in the baseline scenario) for the parameters governing the cost of adjustment of the capital utilization rate for the traded and the non-traded sector. With regards to the parameter that governs the cost of adjustment of the technology utilization rate, we choose a value for χ_2^H of 0.4



(a) Capital Utilization Adjusted-Traded TFP (b) Traded TFP (c) Traded LIS

(d) Capital Utilization Adjusted-Non-Traded TFP (e) Non-Traded TFP (f) Non-Traded LIS

Figure 40: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock with Distortionary Taxation: Technology Effects. Notes: Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds. The solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC. Whilst in the baseline scenario we assume that taxes are lump-sum, the dashed red line shows results for the same model when we allow for distortionary (i.e., labor and consumption) taxation.



(a) Traded Capital Utilization Rate (b) Non-Traded Capital Utilization Rate

Figure 41: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Capital Utilization Rate. Notes: Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds. The solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC. Whilst in the baseline scenario we assume that taxes are lump-sum, the dashed red line shows results for the same model when we allow for distortionary (i.e., labor and consumption) taxation.

for the traded sector (instead of 0.8) and a value for χ_2^N of 1.4 for the non-traded sector (instead of 2.85). As displayed by Fig. 40 and Fig. 41, these values allow the model with distortionary taxation to reproduce well the dynamics of the sectoral LIS, sectoral TFPs and utilization-adjusted-TFPs, capital utilization rates, except for the utilization-adjusted-TFP in the non-traded sector. We have to overstate the increased use of technology in the non-traded sector otherwise the negative impact of labor taxation would lead the model to substantially understate the expansionary effect of the government spending shock on the non-traded sector.

Results: Lump-sum vs. distortionary taxation. The solid black line with squares in Fig. 42 shows the predictions of the baseline model where taxes are lump-sum. Dashed red lines display results for the same model but allowing for distortionary taxation. By and large, the conclusions are identical to those stressed in the main text. More specifically, Fig. 42 shows that adding distortionary taxation does not change qualitatively the dynamics. The discrepancy between a model assuming lump-sum taxes and a model assuming

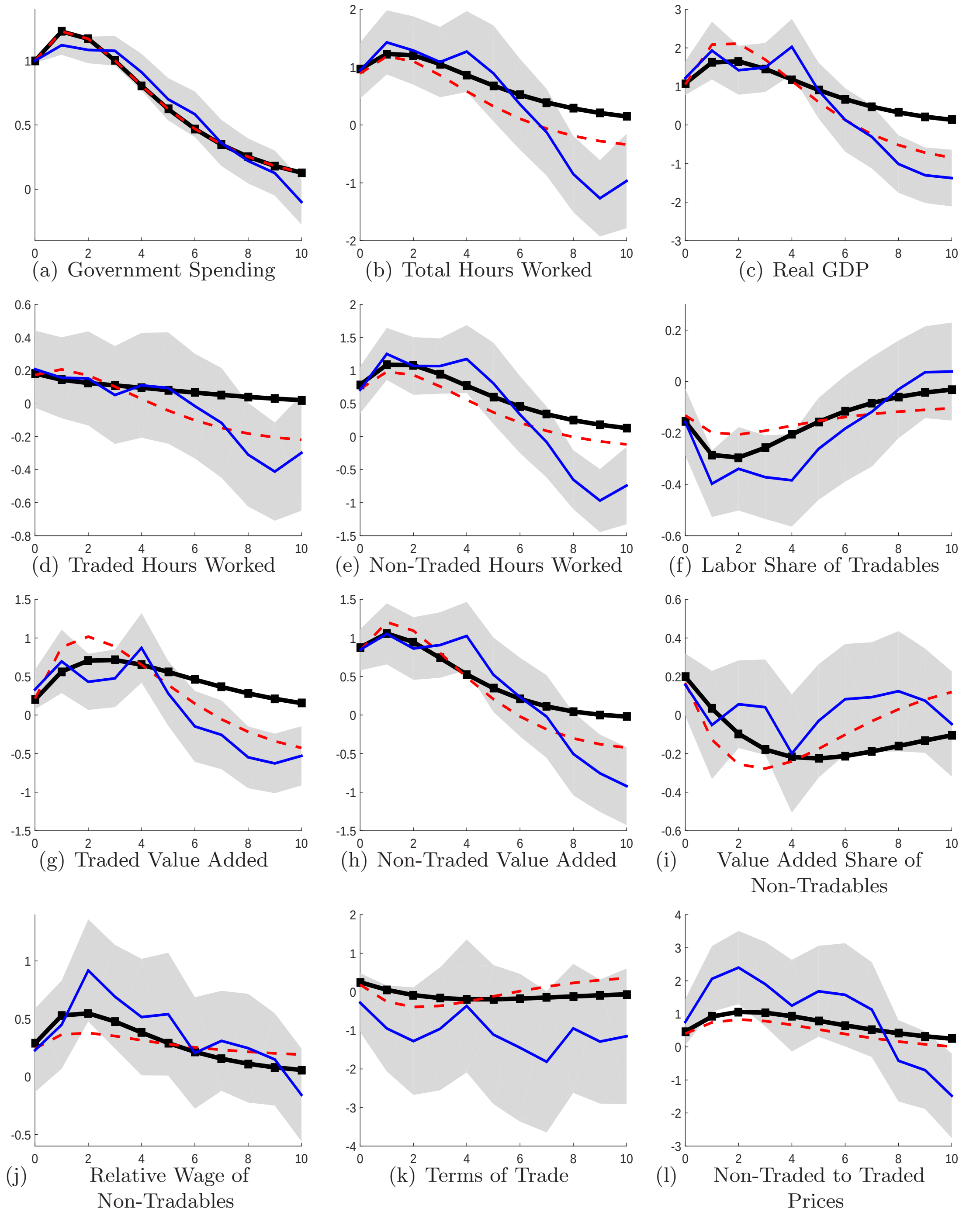


Figure 42: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock with Distortionary Taxatiion: Labor and Output Effects. Notes: Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds. The solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC. Whilst in the baseline scenario we assume that taxes are lump-sum, the dashed red line shows results for the same model when we allow for distortionary (i.e., labor and consumption) taxation.

distortionary taxation is that the latter generates a decline in total hours worked and in particular in real GDP after seven years, in line with the evidence. After seven years, the labor tax is 0.3 percentage point higher than its initial trend which exerts a significant negative impact on total hours worked and real GDP. It is worth mentioning that the model with distortionary taxation does a better job in replicating some variables in levels such as sectoral value added and sectoral hours worked but the model with lump-sum taxes reproduces relatively better the responses of both the labor share and the relative wage of tradables.

W Semi-Small Open Economy Model: Shimer [2009] Preferences and Imperfect Mobility of Capital

So far, we have considered a semi-small open economy where non-traded and home-produced traded goods are produced by means of CES production functions. Investment is subject to capital installation costs. There is imperfect mobility of labor across sectors as workers experience mobility costs. Home- and foreign-produced traded goods are imperfect substitutes. The last three ingredients generate barriers to mobility. The second set of factors is related to technology. More specifically, we assume that households supply capital services and choose the rate of capital utilization along with the stock of capital. They also choose the intensity in the use of the stock of knowledge. The allocation of capital and labor between traded and non-traded firms is driven by firm's maximization. Firms also choose a mix of capital and labor along the technology frontier which is sector-specific which gives rise to factor-biased technological change.

The objective of this section is to test the robustness of the theoretical results with respect to the baseline model's assumptions:

- In the main text, we consider MaCurdy [1981] preferences. We consider below a more general class of preferences by assuming non-separable preferences between consumption and labor in the lines of Shimer [2009].
- In the main text, we assume that capital can move freely across sectors. We investigate below the impact of allowing imperfect mobility of capital across sectors.

W.1 Non-Separability in Utility between Consumption and Leisure: Shimer [2009] Preferences

We consider a semi-small open economy with CES production functions which is identical to that laid out in section S.1 for households, except that we allow for non-separability in consumption and leisure in preferences. With regard to firms' decisions described in section T.1, the production side remains unchanged. We do not repeat the main elements of the model and emphasize the main changes caused by the assumption of non-separable preferences.

In the main text, we assume that preferences are separable in consumption and leisure. We relax this assumption and allow for consumption and leisure to be substitutes. In particular, this more general specification implies that consumption can be affected by the aggregate wage rate while labor supply can now be influenced by relative prices. As previously, the household's period utility function is increasing in his/her consumption C and decreasing in his/her labor supply L, with functional form (see Shimer [2009]):

$$\Lambda \equiv \frac{C^{1-\sigma}V(L)^{\sigma}-1}{1-\sigma}, \quad \text{if} \quad \sigma \neq 1, \quad V(L) \equiv \left(1+(\sigma-1)\gamma\frac{\sigma_L}{1+\sigma_L}L^{\frac{1+\sigma_L}{\sigma_L}}\right) \tag{451}$$

and

$$\Lambda \equiv \log C - \gamma\frac{\sigma_L}{1+\sigma_L}L^{\frac{1+\sigma_L}{\sigma_L}}, \quad \text{if} \quad \sigma = 1. \tag{452}$$

These preferences are characterized by two crucial parameters: σ_L is the Frisch elasticity of labor supply, and $\sigma > 0$ determines the substitutability between consumption and leisure; it is worthwhile noticing that if $\sigma > 1$, the marginal utility of consumption is increasing in hours worked. Importantly, such preferences imply that the Frisch elasticity of labor supply is constant.

As shall be useful below, we write down the partial derivatives of (451):

$$V_L = (\sigma-1)\gamma L^{\frac{1}{\sigma_L}}, \tag{453a}$$

$$\Lambda_C = C^{-\sigma}V(L)^{\sigma}, \tag{453b}$$

$$\Lambda_L = -C^{1-\sigma}\sigma V(L)^{\sigma-1}\gamma L^{\frac{1}{\sigma_L}}, \tag{453c}$$

$$\Lambda_{CL} = -\frac{\Lambda_L(\sigma-1)}{C}, \tag{453d}$$

where $\Lambda_C = \frac{\partial \Lambda}{\partial C}$. According to eq. (453d), the marginal utility of consumption is increasing in labor supply as long as $\sigma > 1$, i.e., if consumption and leisure are gross substitutes.

The first order conditions for firms are not modified and thus we focus on FOC for the representative household. The representative household chooses $C(t)$ and $L(t)$ so as to maximize his/her lifetime utility with an instantaneous utility given by (451) subject to (188), (190), and (189), and (191). While FOC (194c)-(194i) remain unchanged, the remaining first-order conditions characterizing the representative household's optimal plans read:

$$C^{-\sigma} V(L)^{\sigma} = P_C \lambda, \tag{454a}$$
$$C^{1-\sigma} \sigma \gamma L^{1/\sigma_L} V(L)^{\sigma-1} = W \lambda. \tag{454b}$$

The ratio of the marginal disutility of labor to the marginal utility of consumption is the marginal rate of substitution of consumption for leisure:

$$\sigma \frac{C V_L}{V(L)} = \frac{W}{P_C}. \tag{455}$$

Totally differentiating (455) and using the fact that $V(\hat{L}) = \frac{V_L L}{V(L)} \hat{L}$ and $\hat{V_L} = \frac{1}{\sigma_L} \hat{L}$ leads to:

$$\begin{aligned} \hat{C} &= -\left[\frac{1}{\sigma_L} - \frac{V_L L}{V(L)}\right] \hat{L} + \hat{W} - \hat{P}_C, \\ &= \left[\frac{V(L)-1}{V(L)} \frac{1+\sigma_L}{\sigma_L} - \frac{1}{\sigma_L}\right] \hat{L} + \hat{W} - \hat{P}_C. \end{aligned} \tag{456}$$

When $\sigma > 1$, we have $\frac{V_L L}{V(L)} > 0$ and C and L can potentially co-move. Consumption is increasing in aggregate wages which go up due to imperfect mobility of labor. When prices are fully flexible, the numerical analysis shows that consumption slightly increases instead of declining. Because traded goods can be imported while non-traded goods must be produced by domestic firms, the combined effect of the slight increase in consumption and the rise in government consumption which is biased toward non-traded goods amplifies the appreciation in the relative price of non-tradables. The shift of productive resources toward the non-traded sector is thus more pronounced. As we shall see, when non-traded prices are sticky, the consumption price index hardly changes in the short-run so that consumption might significantly increase after a government spending shock. It thus amplifies the demand boom in the non-traded sector.

W.2 Imperfect Mobility of Capital across Sectors

We consider a semi-small open economy with CES production functions which is identical to that laid out in section S.1 for households, except that we allow for imperfect substitutability between traded and non-traded physical capital in addition to non-separable preferences. With regard to firms' decisions described in section T.1, the production side remains unchanged except for the capital rental cost will vary across sectors. We do not repeat the main elements of the model and emphasize the main changes caused by imperfect mobility of capital.

Main changes

Like labor, we generate imperfect capital mobility by assuming that traded and non-traded capital stock are imperfect substitutes (from the point of view of households). Following Horvath [2000], we assume that capital in the traded and the non-traded sectors are aggregated by means of a CES function:

$$K = \left[\vartheta_K^{-1/\epsilon_K} \left(K^H\right)^{\frac{\epsilon_K+1}{\epsilon_K}} + (1-\vartheta_K)^{-1/\epsilon_K} \left(K^N\right)^{\frac{\epsilon_K+1}{\epsilon}}\right]^{\frac{\epsilon_K}{\epsilon_K+1}}, \tag{457}$$

where $0 < \vartheta_K < 1$ is the weight of capital supply to the traded sector in the aggregate capital index $K(.)$ and ϵ_K measures the ease with which sectoral capital can be substituted for each other and thereby captures the degree of capital mobility across sectors.

Households lease capital services (the product of utilization and physical capital) to firms in sector j at rental rate $R^j(t)$ augmented with the technology utilization rate, i.e., $R^j(t)u^{Z,j}(t)$. Thus capital income received by households reads $\sum_j u^{Z,j}(t)R^j(t)u^{K,j}(t)K^j(t)$. We denote the share of traded capital income in total capital income by $\alpha_K = \frac{R^H K^H}{RK}$ and thus $1 - \alpha_K = \frac{R^N K^N}{RK}$ is the non-tradable share of capital income. Because traded and non-traded capital offers a different return, we have to denote the share of traded capital in aggregate capital stock by $\nu^{K,H} = \frac{K^H}{K}$ and the share of non-traded capital in aggregate capital stock by $\nu^{K,N} = \frac{K^N}{K}$. The household budget constraint is:

$$
\begin{aligned}
\dot{N}(t) &= r^{\star}N(t) + \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1-\alpha_K(t))\,u^{K,N}(t)u^{Z,N}(t)\right] R(t)K(t) \\
&+ \left[\alpha_L(t)u^{Z,H}(t) + (1-\alpha_L(t))\,u^{Z,N}(t)\right] W(t)L(t) - T(t) - P_C(t)C(t) - P_J(t)J(t) \\
&- P^H(t)C^{K,H}(t)\nu^{K,H}(t)K(t) - P^N(t)C^{K,N}(t)\nu^{K,N}(t)K(t) \\
&- P^H(t)C^{Z,H}(t) - P^N(t)C^{Z,N}(t).
\end{aligned}
\tag{458}
$$

Denoting the co-state variables associated with (458) and (190) by λ and Q', respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$\Lambda_C = P_C(t)\lambda(t), \tag{459a}$$

$$-\Lambda_L = \lambda(t)\left[\alpha_L(t)u^{Z,H}(t) + (1-\alpha_L(t))\,u^{Z,N}(t)\right] W(t), \tag{459b}$$

$$Q(t) = P_J(t)\left[1 + \kappa\left(\frac{I(t)}{K(t)} - \delta_K\right)\right], \tag{459c}$$

$$\dot{\lambda}(t) = \lambda\left(\beta - r^{\star}\right), \tag{459d}$$

$$
\begin{aligned}
\dot{Q}(t) = (r^{\star} + \delta_K)\,Q(t) - \Bigg\{ & \frac{1}{K(t)}\left[R^H(t)u^{Z,H}(t)u^{K,H}(t)K^H(t) + R^N(t)u^{Z,N}(t)u^{K,N}(t)K^N(t)\right] \\
& -P^H(t)C^{K,H}(t)\nu^{K,H}(t) - P^N(t)C^{K,N}(t)\nu^{K,N}(t) + P_J(t)\frac{\kappa}{2}\left(\frac{I(t)}{K(t)} - \delta_K\right)\left(\frac{I(t)}{K(t)} + \delta_K\right)\Bigg\},
\end{aligned}
\tag{459e}
$$

$$\frac{R^H(t)u^{Z,H}(t)}{P^H(t)} = \left[\xi_1^H + \xi_2^H\left(u^{K,H}(t) - 1\right)\right], \tag{459f}$$

$$\frac{R^N(t)u^{Z,N}(t)}{P^N(t)} = \left[\xi_1^N + \xi_2^N\left(u^{K,N}(t) - 1\right)\right], \tag{459g}$$

$$\frac{R^H(t)u^{K,H}(t)K^H(t) + W^H(t)L^H(t)}{P^H(t)} = \left[\chi_1^H + \chi_2^H\left(u^{Z,H}(t) - 1\right)\right], \tag{459h}$$

$$\frac{R^N(t)u^{K,N}(t)K^N(t) + W^N(t)L^N(t)}{P^N(t)} = \left[\chi_1^N + \chi_2^N\left(u^{Z,N}(t) - 1\right)\right], \tag{459i}$$

and the transversality conditions $\lim_{t\to\infty}\bar{\lambda}N(t)e^{-\beta t} = 0$ and $\lim_{t\to\infty}Q(t)K(t)e^{-\beta t} = 0$; to derive (459c) and (459e), we used the fact that $Q(t) = Q'(t)/\lambda(t)$.

Using the fact that $\left[\alpha_L(t)u^{Z,H}(t) + (1-\alpha_L(t))\,u^{Z,N}(t)\right] W(t) = \tilde{W}(t)$ and from Euler Theorem $P^H(t)Y^H(t) = R^H(t)u^{K,H}(t)K^H(t) + W^H(t)L^H(t)$, eqs. (459b), (459h) and (459i) can be rewritten as follows:

$$-\Lambda_L = \lambda\tilde{W}(t), \tag{460a}$$

$$Y^H(t) = \chi_1^H + \chi_2^H\left(u^{Z,H}(t) - 1\right), \tag{460b}$$

$$Y^N(t) = \chi_1^N + \chi_2^N\left(u^{Z,N}(t) - 1\right). \tag{460c}$$

The aggregate capital rental rate, $\tilde{R}(t)$, associated with the capital aggregator function defined above (457) is:

$$\tilde{R}(t) = \left[\vartheta_K\left(\tilde{R}^H(t)\right)^{\epsilon_K+1} + (1-\vartheta_K)\left(\tilde{R}^N(t)\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon_K+1}}, \tag{461}$$

where $\tilde{R}^H(t) = u^{Z,H}(t)R^H(t)$ and $\tilde{R}^N = u^{Z,N}(t)R^N(t)$ are capital rental rates in the traded and non-traded sector, respectively.

Given the aggregate capital rental rate, we can derive the allocation of aggregate capital supply to the traded and the non-traded sector:

$$K^H = \vartheta_K \left(\frac{\tilde{R}^H(t)}{\tilde{R}(t)}\right)^{\epsilon} K(t), \tag{462a}$$

$$K^N = (1-\vartheta_K) \left(\frac{\tilde{R}^N(t)}{\tilde{R}(t)}\right)^{\epsilon} K(t). \tag{462b}$$

As will be useful later, log-linearizing the aggregate capital rental rate index in the neighborhood of the initial steady-state leads to:

$$\hat{\tilde{R}}(t) = \alpha_K \hat{\tilde{R}}^H(t) + (1-\alpha_K)\, \hat{\tilde{R}}^N(t), \tag{463}$$

where α_K is the tradable content of aggregate capital income (defined above):

$$\alpha_K = \vartheta \left(\frac{R^H}{R}\right)^{1+\epsilon}, \tag{464a}$$

$$1-\alpha_K = (1-\vartheta) \left(\frac{R^N}{R}\right)^{1+\epsilon}. \tag{464b}$$

Note that because we log-linearize in the neighborhood of the steady-state, the capital compensation share, $\tilde{\alpha}_K$, inclusive of the technology utilization rate collapses to the technology utilization adjusted capital compensation share, α_K.

Firms

Firms face two cost components: a capital rental cost equal to $\tilde{R}^j(t) = R^j(t)u^{Z,j}(t)$, and a labor cost equal to the wage rate $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$, both inclusive of technology utilization. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given:

$$\max_{\tilde{K}^j(t), L^j(t)} \Pi^j(t) = \max_{\tilde{K}^j(t), L^j(t)} u^{Z,j}(t) \left\{P^j(t)Y^j(t) - W^j(t)L^j(t) - R^j(t)\tilde{K}^j(t)\right\}, \tag{465}$$

where $\tilde{K}^j = u^{K,j}K^j$ and Y^j is given by eq. (287).

The first order conditions of the firm problem are:

$$P^H \left(1-\gamma^H\right) \left(B^H\right)^{\frac{\sigma^H-1}{\sigma^H}} \left(u^{K,H}K^H\right)^{-\frac{1}{\sigma^H}} \left(Y^H\right)^{\frac{1}{\sigma^H}} = R^H, \tag{466a}$$

$$P^N \left(1-\gamma^N\right) \left(B^N\right)^{\frac{\sigma^N-1}{\sigma^N}} \left(u^{K,N}K^N\right)^{-\frac{1}{\sigma^N}} \left(Y^N\right)^{\frac{1}{\sigma^N}} \equiv R^N, \tag{466b}$$

$$P^H \gamma^H \left(A^H\right)^{\frac{\sigma^H-1}{\sigma^H}} \left(L^H\right)^{-\frac{1}{\sigma^H}} \left(Y^H\right)^{\frac{1}{\sigma^H}} \equiv W^H, \tag{466c}$$

$$P^N \gamma^N \left(A^N\right)^{\frac{\sigma^N-1}{\sigma^N}} \left(L^N\right)^{-\frac{1}{\sigma^N}} \left(Y^N\right)^{\frac{1}{\sigma^N}} \equiv W^N. \tag{466d}$$

Solving the model

First-order conditions (459a) and (459b) can be solved for consumption and aggregate labor supply which of course must hold at any point of time:

$$C = C\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right), \quad L = L\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right), \tag{467}$$

with partial derivatives given by

$$\frac{\Lambda_{CC}}{\Lambda_C}\hat{C} + \frac{\Lambda_{CL}}{\Lambda_C}\hat{L} = \hat{\bar{\lambda}} + \alpha_C \alpha^H \hat{P}^H + (1-\alpha_C)\, \hat{P}^N, \tag{468a}$$

$$\frac{\Lambda_{LC}}{\Lambda_L}\hat{C} + \frac{\Lambda_{LL}}{\Lambda_L}\hat{L} = \hat{\bar{\lambda}} + \hat{\tilde{W}}, \tag{468b}$$

where $\hat{\tilde{W}} = \alpha_L \left(\hat{W}^H + \hat{u}^{Z,H}\right) + (1-\alpha_L)\left(\hat{W}^H + \hat{u}^{Z,H}\right)$.

Inserting first the solution for consumption (467) into (197b), (198a), (198b) enables us to solve for C^N, C^H, and C^F:

$$C^N = C^N\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right), \quad C^H = C^H\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right), \quad C^F = C^F\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right) \tag{469}$$

with partial derivatives given by

$$\hat{C}^N = \alpha_C\phi\left[-\hat{P}^N + \alpha^H\hat{P}^H\right] + \hat{C}, \tag{470a}$$

$$\hat{C}^H = (1-\alpha_C)\phi\hat{P}^N - \left[\rho\left(1-\alpha^H\right) + \phi\alpha^H(1-\alpha_C)\right]\hat{P}^H + \hat{C}, \tag{470b}$$

$$\hat{C}^F = (1-\alpha_C)\phi\hat{P}^N + \alpha^H\left[\rho - \phi(1-\alpha_C)\right]\hat{P}^H + \hat{C}. \tag{470c}$$

Eqs. (462a)-(462b) can be solved for K^H and K^N:

$$K^H = K^H\left(K, \tilde{R}^H, \tilde{R}^N\right), \quad K^N = K^N\left(K, \tilde{R}^H, \tilde{R}^N\right), \tag{471}$$

where partial derivatives are given by:

$$\hat{K}^H = \epsilon_K(1-\alpha_K)\left[\hat{R}^H + \hat{u}^{Z,H}\right] - \epsilon_K(1-\alpha_K)\left[\hat{R}^N + \hat{u}^{Z,N}\right] + \hat{K}, \tag{472a}$$

$$\hat{K}^N = \epsilon_K\alpha_K\left[\hat{R}^N + \hat{u}^{Z,N}\right] - \epsilon_K\alpha_K\left[\hat{R}^H + \hat{u}^{Z,H}\right] + \hat{K}. \tag{472b}$$

Inserting first the solution for aggregate labor supply (467), eqs. (209a)-(209b) can be solved for L^H and L^N:

$$\hat{L}^H = (1-\alpha_L)\epsilon\left(\hat{W}^H + \hat{u}^{Z,H}\right) - (1-\alpha_L)\epsilon\left(\hat{W}^N + \hat{u}^{Z,N}\right) + \hat{L}, \tag{473a}$$

$$\hat{L}^N = \alpha_L\epsilon\left(\hat{W}^N + \hat{u}^{Z,N}\right) - \alpha_L\epsilon\left(\hat{W}^H + \hat{u}^{Z,H}\right) + \hat{L}. \tag{473b}$$

The implicit functions theorem implies:

$$L^H = L^H\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right), \quad L^N = L^N\left(\bar{\lambda}, P^N, P^H, \tilde{W}^H, \tilde{W}^N\right). \tag{474}$$

Totally differentiating (466a)-(466d) leads to:

$$-\left(\frac{1-s_L^H}{\sigma^H}\right)\hat{L}^H + \left(\frac{1-s_L^H}{\sigma^H}\right)\hat{K}^H - \hat{W}^H = -\hat{P}^H\left[\frac{\sigma^H-1}{\sigma^H} + \frac{s_L^H}{\sigma^H}\right]\hat{A}^H - \left(\frac{1-s_L^H}{\sigma^H}\right)\left[\hat{B}^H + \hat{u}^{K,H}\right], \tag{475a}$$

$$-\left(\frac{1-s_L^N}{\sigma^N}\right)\hat{L}^N + \left(\frac{1-s_L^N}{\sigma^N}\right)\hat{K}^N - \hat{W}^N = -\hat{P}^N\left[\frac{\sigma^N-1}{\sigma^N} + \frac{s_L^N}{\sigma^N}\right]\hat{A}^N - \left(\frac{1-s_L^N}{\sigma^N}\right)\left[\hat{B}^N + \hat{u}^{K,N}\right], \tag{475b}$$

$$\left(\frac{s_L^H}{\sigma^H}\right)\hat{L}^H - \left(\frac{s_L^H}{\sigma^H}\right)\hat{K}^H - \hat{R}^H = -\hat{P}^H - \left(\frac{s_L^H}{\sigma^H}\right)\hat{A}^H - \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\hat{B}^H + \left(\frac{s_L^H}{\sigma^H}\right)\hat{u}^{K,H}, \tag{475c}$$

$$\left(\frac{s_L^N}{\sigma^N}\right)\hat{L}^N - \left(\frac{s_L^N}{\sigma^N}\right)\hat{K}^N - \hat{R}^N = -\hat{P}^N - \left(\frac{s_L^N}{\sigma^N}\right)\hat{A}^N - \left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\hat{B}^N + \left(\frac{s_L^N}{\sigma^N}\right)\hat{u}^{K,N}. \tag{475d}$$

Inserting solutions for L^H and L^N given by eq. (474) and solutions for K^H and K^N given by eq. (471) into (475a)-(475d) allow us to solve the demand for labor and capital in the traded and the non-traded sector for sectoral wage rates and sectoral capital rental rates:

$$W^H, W^N, R^H, R^N\left(P^N, P^H, K, u^{K,H}, u^N, u^{Z,H}, u^{Z,N}, A^H, B^H, A^N, B^N, \bar{\lambda}\right). \tag{476}$$

Plugging back the solutions for wages and capital rental rates into solutions for capital, labor, production functions, and consumption leads to:

$$L^j, K^j, Y^j, C^g\left(P^N, P^H, K, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}, A^H, B^H, A^N, B^N, \bar{\lambda}\right), \tag{477}$$

where $j = H, N$, $g = F, H, N$.

Plugging the demand for capital (466a)-(466b) in sector $j = H, N$ into the decisions about capital utilization rates (459f)-(459g), and totally differentiating together with decisions about technology utilization rates (459h)-(459i) leads to:

$$\left[\frac{\xi_2^H}{\xi_1^H} + \frac{s_L^H}{\sigma^H}\right] \hat{u}^{K,H} - \hat{u}^{Z,H} + \frac{s_L^H}{\sigma^H}\left(\hat{K}^H - \hat{L}^H\right) = \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right)\hat{B}^H + \left(\frac{s_L^H}{\sigma^H}\right)\hat{A}^H, \quad (478a)$$

$$\left[\frac{\xi_2^N}{\xi_1^N} + \frac{s_L^N}{\sigma^N}\right] \hat{u}^{K,H} - \hat{u}^{Z,N} + \frac{s_L^N}{\sigma^N}\left(\hat{K}^N - \hat{L}^N\right) = \left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\hat{B}^N + \left(\frac{s_L^N}{\sigma^N}\right)\hat{A}^N, \quad (478b)$$

$$\frac{\chi_2^H}{\chi_1^H}\hat{u}^{Z,H} = \hat{Y}^H, \quad (478c)$$

$$\frac{\chi_2^N}{\chi_1^N}\hat{u}^{Z,N} = \hat{Y}^N. \quad (478d)$$

Inserting first solutions for L^H, L^N, K^H, K^N, Y^H, Y^N, and invoking the implicit functions theorem leads to:

$$u^{K,j}, u^{Z,j}\left(P^N, P^H, K, A^H, B^H, A^N, B^N, \bar{\lambda}\right). \quad (479)$$

Plugging back solutions for capital and technology utilization rates into (479) imply

$$L^j, K^j, Y^j, C^g\left(P^N, P^H, K, A^H, B^H, A^N, B^N, \bar{\lambda}\right), \quad (480)$$

and plugging solutions into the market clearing conditions for the home-produced and non-traded goods, i.e., (323) and (320) allow us to solve for terms of trade and non-traded good prices:

$$P^H, P^N\left(K, A^H, B^H, A^N, B^N, G, \bar{\lambda}\right). \quad (481)$$

Inserting (481) into (479) and (480) leads to:

$$L^j, K^j, Y^j, C^g, u^{K,j}, u^{Z,j}\left(K, A^H, B^H, A^N, B^N, G, \bar{\lambda}\right). \quad (482)$$

The rest of the steps are similar to those described in section U.

X Semi-Small Open Economy Model with Sticky Prices

In this section, we extend our model by introducing nominal price rigidities on non-traded goods. We allow for non-separable preferences and assume imperfect mobility of capital across sectors. We do not repeat the main elements of the model and emphasize the main changes caused by the assumption of sticky prices.

We propose a New Keynesian closed economy model with heterogeneous good producers building on Farhi and Werning [2016] and Kaplan, Moll, and Violante [2018]. Like Farhi and Werning [2017] we consider a two-sector open economy with sticky prices in the non-traded sector. In the lines of of Kaplan et al. [2018], we allow for capital accumulation and generate sticky prices by assuming quadratic adjustment costs. Time is continuous.

There are five agents: households, the government, intermediate good firms, retailers and final goods producers. While inflation of tradables is mitigated following a government spending shock, inflation of non-tradables becomes significant in the medium-run only as inflation of non-tradables builds up over time, thus suggesting the presence of sticky prices in the short-run as they remain almost unchanged the first year. To allow for sticky prices in the non-traded sector, we assume that there are imperfectly competitive intermediate good producers in the non-traded sector which produce differentiated goods which are sold at (flexible) prices M^N to retailers. Monopolistically competitive retailers purchase input goods from the input good firms, differentiate them and sell them to final good producers. Each retailer i chooses the sales price to maximize profits subject to price adjustment costs as in Rotemberg [1982], taking as given the demand curve and the price of input goods M^N. Adjustment costs are assumed to be quadratic in the rate of price change and to be proportional to value added in the non-traded sector:

$$\Theta^N\left(\frac{\dot{P}_i^N}{P_i^N}\right) = \frac{\theta}{2}\left(\frac{\dot{P}_i^N}{P_i^N}\right)^2 P^N Y^N, \quad (483)$$

where $\frac{\dot{P}_i^N}{P_i^N}$ stands for the individual price inflation π_i^N; $\theta > 0$ determines the degree of price stickiness in the non-traded sector. Existence of quadratic costs generate profits in the retail sector, $\Pi_i^{N,R}$. While the government provides a subsidy τ^N to retailers so as to reduce the price over the marginal cost to one, the subsidy is financed by means of a lump-sump tax T^N which is transferred to the households lump sum. Finally, a competitive representative final goods producer aggregates a continuum of output produced by retailers.

The small open economy takes as given the world interest rate. Like Chodorow-Reich et al. [2021], we consider an open economy with a fixed exchange rate regime which has removed all capital controls so that the domestic interest rate collapses to the world interest rate. While this assumption avoids adding too much complexity because the Taylor rule collapses to $r = r^\star$, this ensures that the baseline model is obtained when we let the parameter of the price adjustment cost function be zero.

X.1 Households

While households supply labor and capital services, they also choose the capital and technology utilization rates. They are the owners of retailers and thus receive the profit $\Pi_i^{N,R}$ which will be detailed later:

$$\begin{aligned}
\dot{N}(t) &= r^\star N(t) + \left[\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1-\alpha_K(t))\, u^{K,N}(t)u^{Z,N}(t)\right] R(t)K(t) + \int_0^1 \Pi_i^{N,R}(t)di \\
&+ \left[\alpha_L(t)u^{Z,H}(t) + (1-\alpha_L(t))\, u^{Z,N}(t)\right] W(t)L(t) - T(t) - P_C(t)C(t) - P_J(t)J(t) \\
&- P^H(t)C^{K,H}(t)\nu^{K,H}(t)K(t) - P^N(t)C^{K,N}(t)\nu^{K,N}(t)K(t) \\
&- P^H(t)C^{Z,H}(t) - P^N(t)C^{Z,N}(t). \qquad (484)
\end{aligned}$$

Households maximize their lifetime utility where instantaneous utility is assumed to be non-separable in consumption and leisure (451) (i.e., we consider Shimer [2009] preferences) subject to the budget constraint (484). First-order conditions are described by the set of equations (459a)-(459i).

X.2 Home-Produced Traded Good Firms: Flexible Terms of Trade

Firms in the traded sector faces two cost components: a capital rental rate $\tilde{R}^H(t)$ and a wage rate $\tilde{W}^H(t)$:

$$\max_{\tilde{K}^H(t),L^H(t)} \Pi_I^H(t) = \max_{\tilde{K}^H(t),L^H(t)} u^{Z,H}(t)\left\{P^H(t)Y^H(t) - W^H(t)L^H(t) - R^H(t)\tilde{K}^H(t)\right\}. \qquad (485)$$

The first order conditions of the firm problem in the traded sector are:

$$P^H\left(1-\gamma^H\right)\left(B^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(u^{K,H}K^H\right)^{-\frac{1}{\sigma^H}}\left(Y^H\right)^{\frac{1}{\sigma^H}} = R^H, \qquad (486a)$$

$$P^H\gamma^H\left(A^H\right)^{\frac{\sigma^H-1}{\sigma^H}}\left(L^H\right)^{-\frac{1}{\sigma^H}}\left(Y^H\right)^{\frac{1}{\sigma^H}} \equiv W^H. \qquad (486b)$$

X.3 Final and Intermediate Non-Traded Good Producers

We assume that within the non-traded sector, there are a large number of intermediate good producers which produce differentiated varieties and thus are imperfectly competitive.

Final Non-Traded Good Firms

The final non-traded output, Y^N, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of sectoral goods:

$$Y^N = \left[\int_0^1 \left(X_i^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}i\right]^{\frac{\omega}{\omega-1}}, \qquad (487)$$

where $\omega > 0$ represents the elasticity of substitution between any two different varieties and X_i^N stands for intermediate consumption of ith-variety (with $i \in (0,1)$). The final good producers behave competitively, and the households use the final good for both consumption and investment.

Denoting by P^N and P_i^N the price of the final good in the non-traded sector and the price of the ith variety of the intermediate good, respectively, the profit of the final good producer reads:

$$\Pi_F^N = P^N \left[\int_0^1 \left(X_i^N\right)^{\frac{\omega-1}{\omega}} \mathrm{d}i\right]^{\frac{\omega}{\omega}} - \int_0^1 P_i^N X_i^N \mathrm{d}i. \tag{488}$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$X_i^N = \left(\frac{P_i^N}{P^N}\right)^{-\omega} Y^N, \tag{489}$$

and the price of the final output is given by:

$$P^N = \left(\int_0^1 \left(P_i^N\right)^{1-\omega} \mathrm{d}i\right)^{\frac{1}{1-\omega}}, \tag{490}$$

where P_i^N is the price of variety i in sector j and P^N is the price of the final good in sector $j = H, N$. According to eq. (489), the price-elasticity of output of the ith variety within the non-traded sector is:

$$-\frac{\partial X_i^N}{\partial P_i^N}\frac{P_i^N}{X_i^N} = \omega. \tag{491}$$

Intermediate Goods Firms

Each intermediate good producer faces two cost components: a capital rental cost equal to $\tilde{R}^N(t) = R^N(t)u^{Z,N}(t)$, and a labor cost equal to the wage rate $\tilde{W}^N(t) = W^N(t)u^{Z,N}(t)$, both inclusive of technology utilization. Intermediate good producers choose capital services and labor by taking prices as given:

$$\max_{\tilde{K}^N(t),L^N(t)} \Pi_I^N(t) = \max_{\tilde{K}^N(t),L^N(t)} u^{Z,N}(t)\left\{M^N(t)Y^N(t) - W^N(t)L^N(t) - R^N(t)\tilde{K}^N(t)\right\}, \tag{492}$$

where $\tilde{K}^N = u^{K,N}K^N$ and Y^N is given by eq. (287).

The first-order conditions of the firm problem are:

$$M^N\left(1-\gamma^N\right)\left(B^N\right)^{\frac{\sigma^N-1}{\sigma^N}}\left(u^{K,N}K^N\right)^{-\frac{1}{\sigma^N}}\left(Y^N\right)^{\frac{1}{\sigma^N}} \equiv R^N, \tag{493a}$$

$$M^N\gamma^N\left(A^N\right)^{\frac{\sigma^N-1}{\sigma^N}}\left(L^N\right)^{-\frac{1}{\sigma^N}}\left(Y^N\right)^{\frac{1}{\sigma^N}} \equiv W^N. \tag{493b}$$

X.4 Retailers and Price Stickiness

We assume that the monopolistic competition occurs at the retail level. The retailers purchase input goods from intermediate good producers, differentiate them and sell them to final good producers. Each retailer chooses the sales price P_i^N to maximize profits subject to price adjustment costs as they differentiate and sell them to final good producers. Retailers experience quadratic costs in adjusting type-i good variety and thus are the source of sticky prices: the price P_i^N is therefore a state variable. Each retailer i in the non-traded sector charges a price P_i^N to maximize profits subject to price adjustment costs à la Rotemberg [1982], taking as given the demand curve for type-i good variety and the aggregate price index in the non-traded sector P^N. The adjustment costs are assumed to be quadratic in the rate of change of non-traded prices and are assumed to be proportional to value added in non-traded sector:

$$\Theta^N\left(\frac{\dot{P}_i^N}{P_i^N}\right) = \frac{\theta}{2}\left(\frac{\dot{P}_i^N}{P_i^N}\right)^2 P^N Y^N, \tag{494}$$

where $\theta > 0$ the individual wage inflation is $\pi_i^N = \frac{\dot{P}_i^N}{P_i^N}$; θ determines the degree of price stickiness in the non-traded sector. We assume that retailers receive a proportional constant subsidy on type-i good variety, τ^N, setting the steady-state markup to one. This subsidy is financed by a lump sum tax on retailers T^N.

Each retailed maximizes the expected profit stream discounted at the real rate $r^N(s) = r^\star - \pi^N(s)$, i.e.,

$$
\begin{aligned}
&\max_{\dot{P}_i^N, P_i^N} \frac{\Pi_i^N(t)}{P^N(t)}, \\
&\max_{\dot{P}_i^N, P_i^N} \int_0^\infty e^{-\int_0^t r^N(s)ds} \left[\frac{P_i^N\left(1+\tau^N\right) - M^N}{P^N} X_i^N - \frac{\theta}{2}\left(\frac{\dot{P}_i^N}{P_i^N}\right)^2 Y^N\right], \quad (495)
\end{aligned}
$$

subject to $\dot{P}_i^N(t) = \pi_i^N(t) P_i^N(t)$. Note that in line with the current practice, we divide the profit by the price index. The control variable is $\dot{P}_i^N(t)$ and the state variable is $P_i^N(t)$. To solve the optimization problem, we set up the current-value Hamiltonian for the i-th retailers (R) in the non-traded sector (N):

$$
\begin{aligned}
\mathcal{H}_i^{R,N} &= \frac{P_i^N}{P^N}\left(1+\tau^N\right)\left(\frac{P_i^N}{P^N}\right)^{-\omega} Y^N - \frac{M^N}{P^N}\left(\frac{P_i^N}{P^N}\right)^{-\omega} Y^N - \frac{\theta}{2}\left(\frac{\dot{P}_i^N}{P_i^N}\right)^2 Y^N + \Lambda_i^N \dot{P}_i^N, \\
&= \left(\frac{P_i^N}{P^N}\right)^{1-\omega}\left(1+\tau^N\right) Y^N - \frac{M^N}{\left(P^N\right)^{1-\omega}}\left(P_i^N\right)^{-\omega} Y^N - \frac{\theta}{2}\left(\frac{\dot{P}_i^N}{P_i^N}\right)^2 Y^N + \Lambda_i^N \dot{P}_i^N, \quad (496)
\end{aligned}
$$

where we have inserted $X_i^N = \left(\frac{P_i^N}{P^N}\right)^{-\omega} Y^N$ (see eq. (489)). First-order conditions read:

$$
\frac{\partial \mathcal{H}_i^{R,N}}{\partial \dot{P}_i^N} = 0, \qquad \theta \frac{\pi_i^N}{P_i^N} = \Lambda_i^N, \quad (497a)
$$

$$
\begin{aligned}
\frac{\partial \mathcal{H}_i^{R,N}}{\partial P_i^N} &= \left(r^\star - \pi^N\right)\Lambda_i^N - \dot{\Lambda}_i^N, \\
\frac{(1-\omega)\left(P_i^N\right)^{-\omega}}{\left(P^N\right)^{1-\omega}}\left(1+\tau^N\right) Y^N + \frac{M^N}{\left(P^N\right)^{1-\omega}}\omega\left(P_i^N\right)^{-\omega-1} Y^N + \theta\frac{\left(\dot{P}_i^N\right)^2}{\left(P_i^N\right)^3} Y^N & \\
= \left(r^\star - \pi^N\right)\Lambda_i^N - \dot{\Lambda}_i^N, & \\
\frac{(1-\omega)\left(1+\tau^N\right) Y^N}{P^N} + \frac{M^N \omega Y^N}{\left(P^N\right)^2} + \theta\frac{\left(\pi^N\right)^2}{P^N} Y^N & \\
= \left(r^\star - \pi^N\right)\theta\frac{\pi^N}{P^N} Y^N - \theta\frac{\dot{\pi}^N}{P^N} Y^N - \theta\frac{\pi^N}{P^N}\dot{L}^j + \theta\frac{\pi^N}{P^N}\frac{\dot{W}^j}{P^N} Y^N, & \\
\frac{(1-\omega)\left(1+\tau^N\right)}{\theta} + \frac{M^N}{\theta}\frac{\omega}{P^N} + \left(\pi^N\right)^2 = \left(r^\star - \pi^N\right)\pi^N - \dot{\pi}^N - \pi^N\frac{\dot{L}^j}{Y^N} + \left(\pi^N\right)^2, & \\
\dot{\pi}^N + \frac{\omega}{\theta}\left[\frac{M^N}{P^N} - \left(\frac{\omega-1}{\omega}\right)\left(1+\tau^N\right)\right] = \pi^N\left[r^\star - \pi^N - \frac{\dot{L}^j}{Y^N}\right], & \\
\dot{\pi}^N + \frac{\omega}{\theta}\left[\frac{M^N}{P^N} - 1\right] = \pi^N\left[r^\star - \pi^N - \frac{\dot{Y}^N}{Y^N}\right], & \quad (497b)
\end{aligned}
$$

where we assume a symmetric situation to get the second line of the second first-order condition, i.e., $P_i^N = P^N$, and we have inserted (497a) which has also been differentiated w.r.t. time:

$$
\dot{\Lambda}_i^N = \theta\frac{\dot{\pi}^N}{P^N} Y^N + \theta\frac{\pi^N}{P^N}\dot{Y}^N - \theta\frac{\pi^N}{P^N}\frac{\dot{P}^N}{P^N} Y^N.
$$

To get the last line, we assume that the government sets the revenue subsidy τ^N so that $\left(\frac{\omega-1}{\omega}\right)\left(1+\tau^N\right) = 1$, i.e.,

$$
\tau^N = \frac{1}{\omega-1} > 0. \quad (498)
$$

This subsidy τ^N is financed by a lump sum tax on retailers T^N which is transferred to the households lump sum. We drop the subindex i because we consider a symmetric situation. The total profit of retailers, net of the lump sum tax, is:

$$\int_0^1 \Pi_i^{R,N} di = \Pi^N = \left(P^N - M^N\right) X_i^N - \frac{\theta}{2} \left(\frac{\dot{P}_i^N}{P_i^N}\right)^2 P^N Y^N. \qquad (499)$$

where $X_i^N = X^N = Y^N$ in a symmetric steady-state.

X.5 Solving the Model

Totally differentiating (486a)-(486b), (493a)-(493b) and inserting solutions for L^H and L^N given by eq. (474) and solutions for K^H and K^N given by eq. (471) into (475a)-(475d) allow us to solve the demand for labor and capital in the traded and the non-traded sector for sectoral wage rates and sectoral capital rental rates:

$$W^H, W^N, R^H, R^N \left(M^N, P^H, K, P^N, u^{K,H}, u^N, u^{Z,H}, u^{Z,N}, A^H, B^H, A^N, B^N, \bar{\lambda}\right). \qquad (500)$$

Plugging the demand for capital (493a) in the non-traded sector into the decision about capital utilization rate (459g), and totally differentiating together with decisions about technology utilization rates (459i) leads to:

$$\left[\frac{\xi_2^N}{\xi_1^N} + \frac{s_L^N}{\sigma^N}\right] \hat{u}^{K,H} - \hat{u}^{Z,N} + \frac{s_L^N}{\sigma^N}\left(\hat{K}^N - \hat{L}^N\right) = \left(\frac{\sigma^N - s_L^N}{\sigma^N}\right)\hat{B}^N + \left(\frac{s_L^N}{\sigma^N}\right)\hat{A}^N, \qquad (501a)$$

$$\frac{\chi_2^N}{\chi_1^N} \hat{u}^{Z,N} = \hat{Y}^N, \qquad (501b)$$

where we do not repeat the log-linearized versions of the optimal decisions for the capital and technology utilization rates for the traded sector described by eqs (478a) and (478c). Inserting first solutions for L^j, K^j, and Y^j, and invoking the implicit functions theorem leads to:

$$u^{K,j}, u^{Z,j} \left(M^N, P^H, K, P^N, A^H, B^H, A^N, B^N, \bar{\lambda}\right). \qquad (502)$$

Plugging back solutions for capital and technology utilization rates into (500) imply

$$L^j, K^j, Y^j, C^g \left(M^N, P^H, K, P^N, A^H, B^H, A^N, B^N, \bar{\lambda}\right), \qquad (503)$$

Inserting appropriate solutions, the non-traded goods market clearing condition (320) can be rewritten as follows:

$$\begin{aligned} & u^{Z,N} Y^N \left(M^N, P^H, K, P^N, A^H, B^H, A^N, B^N, \bar{\lambda}\right) = C^N \left(\bar{\lambda}, P^N, P^H\right) + G^N(G) + J^N \left(K, Q, P^N, P^H\right) \\ + \; & C^{K,N} \left[u^{K,N} \left(M^N, P^H, K, P^N, A^H, B^H, A^N, B^N, \bar{\lambda}\right)\right] K^N \\ & + C^{Z,N} \left(u^{Z,N}\right). \end{aligned} \qquad (504)$$

Linearizing (504) leads to:

$$Y^N du^{Z,N}(t) + dY^N(t) = dC^N(t) + dG^N(t) + dJ^N(t) + K^N \xi_1^N du^{K,N}(t) + \chi_1^N du^{Z,N}(t). \qquad (505)$$

Inserting appropriate solutions, the traded goods market clearing condition (323) can be rewritten as follows:

$$\begin{aligned} & u^{Z,H} Y^H \left(M^N, P^H, K, P^N, A^H, B^H, A^N, B^N, \bar{\lambda}\right) = C^H \left(\bar{\lambda}, P^N, P^H\right) + G^H + J^H \left(K, Q, P^N, P^H\right) \\ + \; & C^{K,H} \left[u^{K,H} \left(M^N, P^H, K, P^N, A^H, B^H, A^N, B^N, \bar{\lambda}\right)\right] K^H \\ & + C^{Z,H} \left(u^{Z,H}\right). \end{aligned} \qquad (506)$$

Linearizing (506) leads to:

$$Y^H du^{Z,H}(t) + dY^H(t) = dC^H(t) + dG^H(t) + dJ^H(t) + dX^H(t) + K^H \xi_1^H du^{K,H}(t) + \chi_1^H du^{Z,H}(t), \qquad (507)$$

The market clearing conditions for the home-produced and non-traded goods, i.e., (504) and (506) allow us to solve for terms of trade and non-traded intermediate good prices:

$$P^H, M^N\left(K, P^N A^H, B^H, A^N, B^N, G, \bar{\lambda}\right). \tag{508}$$

Inserting (508) into (502) and (503) leads to:

$$L^j, K^j, Y^j, C^g, u^{K,j}, u^{Z,j}\left(K, P^N, A^H, B^H, A^N, B^N, G, \bar{\lambda}\right). \tag{509}$$

The dynamic system comprises four dynamic equations:

$$\dot{K} = \frac{Y^N - C^N - G^N - C^{K,N}\left(u^{K,N}\right)K^N - C^{Z,N}\left(u^{Z,N}\right)}{(1-\iota)\left[\frac{P^N}{P_J}\right]^{-\phi_J}} - \delta_K K - \frac{K}{2\kappa}\left[\frac{Q}{P_J} - 1\right]^2, \tag{510a}$$

$$\dot{Q} = (r^\star + \delta_K) Q - \left\{\frac{1}{K}\left[R^H u^{Z,H} u^{K,H} K^H + R^N u^{Z,N} u^{K,N} K^N\right]\right.$$
$$\left.-P^H C^{K,H}\nu^{K,H} - P^N C^{K,N}\nu^{K,N} + P_J\frac{\kappa}{2}\left(\frac{I}{K} - \delta_K\right)\left(\frac{I}{K} + \delta_K\right)\right\}, \tag{510b}$$

$$\dot{P}^N = \pi^N P^N, \tag{510c}$$

$$\dot{\pi}^N = \pi^N\left[r^\star - \pi^N - \frac{\dot{Y}^N}{Y^N}\right] - \frac{\omega}{\theta}\left[\frac{M^N}{P^N} - 1\right] \tag{510d}$$

where $Y^N, C^N, J^N, u^{K,N}, u^{Z,N}, M^N\left(K, Q, P^N, A^H, B^H, A^N, B^N, G\right)$ and $G^N = G^N(G)$. The dynamic system can be rewritten in a compact form:

$$\dot{K} = \Upsilon\left(K, Q, P^N, A^H, B^H, A^N, B^N, G\right), \tag{511a}$$

$$\dot{Q} = \Sigma\left(K, Q, P^N, A^H, B^H, A^N, B^N, G\right), \tag{511b}$$

$$\dot{P}^N = \pi^N P^N, \tag{511c}$$

$$\dot{\pi}^N = \Pi\left(K, Q, P^N, A^H, B^H, A^N, B^N, G\right). \tag{511d}$$

Linearizing (511a)-(511d) in the neighborhood of the steady-state, we get in a matrix form:

$$\begin{pmatrix} \dot{K}(t) \\ \dot{Q}(t) \\ \dot{P}^N(t) \\ \dot{\pi}^N(t) \end{pmatrix} = \begin{pmatrix} \Upsilon_K & \Upsilon_Q & \Upsilon_{P^N} & 0 \\ \Sigma_K & \Sigma_Q & \Sigma_{P^N} & 0 \\ 0 & 0 & 0 & \pi^N \\ \Pi_K & \Pi_Q & \Pi_{P^N} & r^\star \end{pmatrix}\begin{pmatrix} dK(t) \\ dQ(t) \\ dP^N(t) \\ d\pi^N(t) \end{pmatrix}$$
$$+ \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^N \Upsilon_{A^j} dA^j(t) + \sum_{j=H}^N \Upsilon_{B^j} dB^j(t) \\ \Sigma_G dG(t) + \sum_{j=H}^N \Sigma_{A^j} dA^j(t) + \sum_{j=H}^N \Sigma_{B^j} dB^j(t) \\ 0 \\ \Pi_G dG(t) + \sum_{j=H}^N \Pi_{A^j} dA^j(t) + \sum_{j=H}^N \Pi_{B^j} dB^j(t) \end{pmatrix}, \tag{512}$$

where the coefficients of the Jacobian matrix are partial derivatives evaluated at the steady-state.

We define auxiliary variables $\dot{X}(t) = \Lambda X(t) + V^{-1}\Gamma S(t)$ where $S(t)$ is the vector of shocks and Γ is a matrix which collects the effects of shocks on the dynamics, and Λ is the matrix of eigenvalues with $\nu_1, \nu_2 < 0$ and $\nu_3, \nu_4 > 0$ on its diagonal, and V is the matrix of eigenvectors. We define $V^{-1}\Gamma = U$ which is a matrix which has the same size as the matrix of shocks.

The solutions are:

$$X_1(t) = X_{11}e^{\nu_1 t} + \sum_{X^j} \Delta_1^{X^j} \left[e^{-\xi_X^j t} - \left(1 - x^j\right) e^{-\chi_X^j t}\right] + \Delta_1^G \left[e^{-\xi t} - \left(1 - x^j\right) e^{-\chi t}\right], \quad (513a)$$

$$X_2(t) = X_{21}e^{\nu_2 t} + \sum_{X^j} \Delta_2^{X^j} \left[e^{-\xi_X^j t} - \left(1 - x^j\right) e^{-\chi_X^j t}\right] + \Delta_2^G \left[e^{-\xi t} - \left(1 - x^j\right) e^{-\chi t}\right], \quad (513b)$$

$$X_3(t) = -\sum_{X^j} \Delta_3^{X^j} \left[e^{-\xi_X^j t} - \left(1 - x^j\right) e^{-\chi_X^j t}\right] - \Delta_3^G \left[e^{-\xi t} - \left(1 - x^j\right) e^{-\chi t}\right], \quad (513c)$$

$$X_4(t) = -\sum_{X^j} \Delta_4^{X^j} \left[e^{-\xi_X^j t} - \left(1 - x^j\right) e^{-\chi_X^j t}\right] - \Delta_4^G \left[e^{-\xi t} - \left(1 - x^j\right) e^{-\chi t}\right], \quad (513d)$$

where $X^j = A^j, B^j$ (with $j = H, N$) and

$$X_{11} = X_1(0) - \sum_{X^j} \Delta_1^{X^j} \left(1 - \Theta_1^{X^j}\right) - \Delta_1^G \left(1 - \Theta_1^G\right), \quad (514a)$$

$$X_{21} = X_2(0) - \sum_{X^j} \Delta_2^{X^j} \left(1 - \Theta_2^{X^j}\right) - \Delta_2^G \left(1 - \Theta_2^G\right), \quad (514b)$$

$$\Delta_1^{X^j} = -\frac{u_{1x}X^j}{\nu_1 + \xi_X^j}, \quad \Delta_2^{X^j} = -\frac{u_{2x}X^j}{\nu_2 + \xi_X^j}, \quad (514c)$$

$$\Delta_1^G = -\frac{u_{15}Y}{\nu_1 + \xi}, \quad \Delta_2^G = -\frac{u_{25}Y}{\nu_2 + \xi}, \quad (514d)$$

$$\Delta_3^{X^j} = \frac{u_{3x}X^j}{\nu_3 + \xi_X^j}, \quad \Delta_4^{X^j} = \frac{u_{4x}X^j}{\nu_4 + \xi_X^j}, \quad (514e)$$

$$\Delta_3^G = \frac{u_{35}Y}{\nu_3 + \xi}, \quad \Delta_4^G = \frac{u_{45}Y}{\nu_4 + \xi}, \quad (514f)$$

where $X^j = A^j, B^j$ with $j = H, N$ and $x = 1, 2, 3, 4$.

Using the fact the definition of $Y(t) = VX(t)$, the solutions for capital, the shadow price of capital, non-traded prices and inflation of non-tradables read:

$$K(t) - K = \sum_{i=1}^{4} X_i(t), \quad (515a)$$

$$Q(t) - Q = \sum_{i=1}^{4} \omega_2^i X_i(t), \quad (515b)$$

$$P^N(t) - P^N = \sum_{i=1}^{4} \omega_3^i X_i(t), \quad (515c)$$

$$\pi^N(t) - \pi^N = \sum_{i=1}^{4} \omega_4^i X_i(t), \quad (515d)$$

where $\pi^N = 0$ at the steady-state. Setting $t = 0$ into the solutions of state variables (515a) and (515c) leads to $K(0) - K = \sum_{i=1}^4 X_i(0)$ and $P^N(0) - P^N = \sum_{i=1}^4 \omega_3^i X_i(0)$. Solutions for $X_1(0)$ and $X_2(0)$ are:

$$X_1(0) = \frac{\omega_3^2(K(0) - K) - (P^N(0) - P^N) - X_3(0)\left(\omega_3^2 - \omega_3^3\right) - X_4(0)\left(\omega_3^2 - \omega_3^4\right)}{\omega_3^2 - \omega_3^1}, \quad (516a)$$

$$X_1(0) = \frac{(P^N(0) - P^N) - \omega_3^1(K(0) - K) + X_3(0)\left(\omega_3^1 - \omega_3^3\right) + X_4(0)\left(\omega_3^1 - \omega_3^4\right)}{\omega_3^2 - \omega_3^1}. \quad (516b)$$

Y Robustness Analysis: Preferences, Barriers to Capital Mobility and Price Stickiness

In section W and X, we have laid out three variants of the baseline model. First, in the baseline model, we assume that preferences are additively separably in consumption and leisure. In section W.1, we consider a more general class of preferences in the lines of Shimer [2009] where the marginal utility of consumption is increasing in labor when consumption and leisure are gross substitutes. While in the baseline model, we assume perfect mobility of capital, we allow for barriers to capital mobility in section W.2. As shown by Bilbiie [2011], Christiano et al. [2011], a RBC model produces a government consumption multiplier lower than one because private consumption falls while a model with sticky prices can produce a government consumption multiplier larger than one as long as consumption and leisure are gross substitutes because under this assumption, private consumption can increase. In section X, like Farhi and Werning [2016], [2017], we allow for non-traded goods' sticky prices and like Kaplan, Moll, and Violante [2018], we assume Rotemberg [1982] price adjustment costs.

We simulate three variants of our model: i) augmented with non-separable utility in consumption and leisure in the lines of Shimer [2009], ii) augmented with imperfect mobility of capital, iii) augmented with Shimer [2009] preferences and sticky prices for non-traded goods. The latter scenario is interesting as the complementarity between consumption and labor together with non-traded goods' sticky prices produces a significant increase in private consumption that allows us to test the extent to which a model with sticky prices but without endogenous technology can account for the fiscal multipliers we estimate empirically.

Y.1 Calibration

In this section, we explore quantitatively the role of each ingredient for fiscal transmission in a model with two sectors. To conduct this quantitative analysis, we calibrate the model as follows:

- When we allow for non-separability in preferences between consumption and leisure in the lines of Shimer [2009], the parameter σ in eq. (451) collapses to the coefficient of relative risk aversion and also determines the substitutability between consumption and leisure. When numerically exploring the implications of non-separability in preferences between consumption and leisure, we set the substitutability between consumption and leisure captured by σ to 2, which is a standard value when adopting this class of preferences, see e.g., Shimer [2009], keeping unchanged the baseline calibration discussed in section 4.1-4.2, in particular we maintain $\sigma_L = 1$. It is worth mentioning with the class of preferences shown in eq. (451), the IES reduces to $1/2 = 0.5$.

- Like labor, we generate imperfect mobility of capital by assuming that traded and non-traded capital are imperfect substitutes from the household's point of view. While time series for labor are directly available at an industry level, time series for the capital stock are not available for all countries of our sample and over the whole period 1970-2015. As detailed in the main text, we construct the capital stock by using the perpetual inventory method and split the capital stock into the traded and non-traded sector by using the value added share of sector $j = H, N$, like Garofalo and Yamarik [2002]. While the capital stock by industry is available for a few countries over a sufficient long period of time, the second issue is the calculation of the return on capital $\tilde{R}^j$. In contrast to wages which are directly observable, we have to recourse to assumptions to calculate the capital rental rate at a sectoral level. We have calculated the return on capital in the traded and the non-traded sector by using the fact that value added at current prices in sector j is exhausted by the payment of factors of production, i.e., $P^j\tilde{Y}^j = \tilde{R}^j\tilde{K}^j + \tilde{W}^jL^j$, which implies that $\tilde{R}^j = \frac{P^j\tilde{Y}^j - \tilde{W}^jL^j}{\tilde{K}^j}$. By using this formula, we have calculated the capital rental rate which allows us to calculate the tradable content of capital income, i.e., α_K. As shown in the last row of Table 21, the tradable content of capital income averaged 0.43 over 1970-2015. Like for labor, we set the elasticity of substitution between traded and non-traded capital ϵ_K to 0.8 and we choose a value for ϑ_K (see eq. (457)) so as to target a tradable content of capital income of 0.43. We set ϑ_K to 0.405.

- In the third variant of our open economy model with tradables and non-tradables, we

Table 21: Tradable Content of Capital Income (1970-2015, 18 OECD countries)

Country	Code	Period	α_K
Australia	(AUS)	1970 - 2015	0.45
Austria	(AUT)	1970 - 2015	0.38
Belgium	(BEL)	1970 - 2015	0.38
Canada	(CAN)	1970 - 2015	0.44
Denmark	(DNK)	1970 - 2015	0.38
Spain	(ESP)	1970 - 2015	0.44
Finland	(FIN)	1970 - 2015	0.50
France	(FRA)	1970 - 2015	0.32
Great Britain	(GBR)	1970 - 2015	0.44
Ireland	(IRL)	1970 - 2015	0.59
Italy	(ITA)	1970 - 2015	0.34
Japan	(JPN)	1974 - 2015	0.44
Korea	(KOR)	1970 - 2015	0.61
Netherlands	(NLD)	1970 - 2015	0.47
Norway	(NOR)	1970 - 2015	0.56
Portugal	(PRT)	1970 - 2015	0.29
Sweden	(SWE)	1970 - 2015	0.43
United States	(USA)	1970 - 2015	0.33
OECD	Mean	1970 - 2015	0.43

Notes: Column 'period' gives the first and last observation available. $\alpha_K = \frac{R^H K^H}{RK}$ is the tradable content of capital income averaged over 1970-2015. Data source: EU KLEMS and OECD STAN.

choose the same values as Kaplan et al. [2018]: we choose a value of 10 for the elasticity of substitution ω between intermediate goods for final goods producers, implying a steady-state markup of 11%. We set θ in the price adjustment cost function to 100, so that the slope of the Phillips curve is $\omega/\theta = 0.1$.

Y.2 Sensitivity w.r.t. Preferences, Barriers to Capital Mobility and Price Stickiness: Numerical results

In Fig. 43, we contrast the predictions of the baseline model shown in solid black lines with squares with the results of three variants of the baseline model. The dashed red line shows predictions of the baseline model augmented with non-separable preferences in the lines of Shimer [2009] where our parametrization implies that consumption and leisure are substitutes. The dotted magenta line shows predictions of the baseline model augmented with non-separability in preferences between consumption and leisure together with imperfect mobility of capital. The dashed-dotted green line with diamonds shows the predictions of the baseline model augmented with sticky prices in the non-traded sector together with non-separability in utility between consumption and leisure (while we assume perfect mobility of capital across sectors). As shown by Bilbiie [2011], Christiano et al. [2011], the combination of non-separability in preferences where the marginal utility of consumption is increasing in labor and sticky prices ensures that private consumption can rise (conditional on taxes being lump-sum) following a temporary increase in government consumption.

Because we want to see the impact of price stickiness on the magnitude of aggregate and sectoral government spending multipliers, we shut down the technology channel in the sticky prices variant, i.e., we abstract from capital and technology utilization and factor-biased technological change. The reason is that because a model with sticky prices and non-separability in utility leads to a significant rise in private consumption, it produces large aggregate multipliers and we want to see if a model with sticky prices could account for our evidence when we shut down the technology channel. We show below that a model with sticky prices and Shimer [2009] preferences cannot replicate the evidence because it dramatically biases the demand shock toward non-traded goods.

Non-separability in utility between consumption and leisure. We start with the first variant of our model where we move from additive separable preferences to preferences where the marginal utility of consumption is increasing in total hours worked. This scenario is shown in dashed red lines in Fig. 43. To see formally the impact of preferences on the behavior of private consumption, it is useful to re-arrange the log-linearized version of the optimal decision for consumption (see eq. (453b)) as follows:

$$\hat{C}(t) = -\frac{\Lambda_{CL}}{\Lambda_{CC}}\hat{L}(t) + \frac{1}{\Lambda_{CC}}\left[\hat{\lambda} + \hat{P}_C(t)\right]. \tag{517}$$

Because we set $\sigma = 2 > 1$ into (451), consumption and leisure are gross substitutes and therefore $\Lambda_{CL} > 0$ (see eq. (453d)). Because utility is concave in consumption, i.e., $\Lambda_{CC} < 0$, an increase in labor supply leads agents to consume more, all else equal. This positive relationship paves the way for an increase in private consumption. Because our model allows for endogenous technological change which exerts a downward pressure on the marginal utility of wealth $\bar{\lambda}$, our model can potentially produce an increase in private consumption even in a model with flexible prices.

The responses in Fig. 45 below show that private consumption indeed slightly increases following a government spending shock. Because traded goods can be imported while non-traded goods must be produced by domestic firms, the slight increase in consumption leads the open economy to borrow more from abroad. The larger current account deficit allows the open economy to reallocate productive resources toward the non-traded sector, see the dashed red line in Fig. 43(f) and Fig. 43(i). The shift of productive resources away from the traded sector caused by the current account deficit reduces the rise in traded value added (see Fig. 43(g)) and thus the incentive for traded firms to improve technology, see Fig. 44(a). In sum, while real GDP growth and labor growth are similar to those obtained in the baseline, their distribution is more biased toward non-traded industries with non-separable preferences.

Imperfect mobility of capital across sectors. In the magenta dotted line in Fig. 43, we show results when we augment the baseline model with non-separable preferences in consumption and leisure and allow for imperfect mobility of capital across sectors. As can be seen in Fig. 43(f) and 43(i), adding barriers to capital mobility has a negative impact on the traded sector. The reason is as follows. In a model where we shut down technology, the traded sector experiences both a capital and a labor outflow. When we allow for factor-biased technological change in a model with labor mobility costs only, because non-traded firms bias technological change toward labor and traded firms bias technological change toward capital, the former amplifies the labor inflow in the non-traded sector while this sector now experiences a capital outflow as capital is not subject to mobility costs. When we allow for capital mobility costs, the shift of capital toward the traded sector is significantly reduced. Traded value added increases much less, as shown in Fig. 43(g) which in turn mitigates the incentives to improve technology, see Fig. 44(a). Because the traded sector is the engine of technological change following a government spending shock, smaller technology improvements by traded firms result in a smaller real GDP growth, see Fig. 43(c).

Sticky prices in the non-traded sector. In the dashed-dotted green line with diamonds in Fig. 43 and 44, we show results when we augment the baseline model with sticky prices in the non-traded sector and also allow for non-separability in utility between consumption and leisure. As shown in Fig. 43(m) and Fig. 43(o), non-traded wages move significantly upward while non-traded prices do not respond significantly on impact and gradually increase over time, thus suggesting the existence of price stickiness in the non-traded sector. To isolate the 'pure' effect of price stickiness, we shut down the technology channel. As shown in Fig. 43(o), non-traded prices do not change on impact and only increase gradually.

The combined effect of non-separability in utility in consumption and leisure and price stickiness leads households to raise their consumption (even when we shut down technological change), see Fig. 45 below. While traded goods can be imported, the non-traded good must be produced by domestic firms. To meet higher demand for non-traded goods, more labor and capital must shift toward the non-traded sector (we shut down factor-biased technological change), see Fig. 43(f) and 43(i). Because labor is subject to mobility costs while capital can move freely across sectors, the traded sector experiences a large capital outflow which results in a fall in traded value added, see Fig. 43(g). By contrast, the demand boom for non-tradables caused by the current account deficit and sticky prices for non-traded goods causes a reallocation of productive resources toward the non-traded sector which amplifies the rise in non-traded value added, see Fig. 43(h). The relative price of non-traded intermediate goods appreciates less relative to home-produced traded goods, see Fig. 43(l).

In sum, the combination of non-separability in utility in consumption and leisure together with sticky prices generates an increase in private consumption. However, the demand boom for non-traded goods causes a reallocation of productive resources that results in a decline in traded value added, in contradiction with our evidence.

Y.3 Private consumption, CPI and the real consumption wage: Numerical results

One strand of the existing literature related to fiscal transmission investigates the conditions under which private consumption increases following a government spending shock. By reducing the present value of the after-tax income stream and thus producing a negative wealth effect, a government spending shock generates a pronounced decline in private consumption in a standard real business cycle model, see Baxter and King [1993]. Bilbiie [2011] and Christiano et al. [2011] have shown that the combined effect of non-separable preferences where consumption and leisure are substitutes and sticky prices can overturn the negative wealth effect and produces an increase in private consumption. The former ingredient implies that the marginal utility of consumption is increasing in hours worked while the latter element implies that a government shock has a strong expansionary effect on labor demand. Jørgensen and Ravn [2022] have recently shown that the technology channel in a sticky price model can also produce an increase in private consumption. Intuitively, if technology improvement is large enough, the marginal cost falls which results in a negative response of prices that lead the monetary authorities to cut interest rates which increase private consumption.

In Fig. 45, we show three scenarios. In the dashed-dotted black line with diamonds, we augment the baseline model with non-separable utility in consumption and leisure. The baseline model with flexible prices can produce a slight increase in private consumption but the magnitude falls below what we estimate empirically. The baseline model can generate an increase in $C(t)$ even with flexible prices because we allow for labor mobility costs and endogenous technological change in a two-sector model. Since the non-traded sector is highly intensive in the government spending shock, labor shifts toward non-traded industries. Because workers experience mobility costs, non-traded firms must pay higher wages to encourage workers to shift which put upward pressure on aggregate wages. Improvement in technology in the traded sector further increases aggregate wages. With non-separable preferences, the combined effect of labor mobility costs and endogenous technology amplifies the rise in labor supply which results in a slight increase in private consumption instead of a decline since consumption and labor co-moves with $\sigma > 2$.

When we allow for sticky prices in the non-traded sector, as displayed by the dotted red line with crosses in Fig. 45, we find that the positive response of private consumption to the government spending shock is amplified. Intuitively, sticky prices leads to a demand boom for non-traded goods which leads firms in this sector to recruit more. Because the non-tradable content of labor compensation is two-third, higher non-traded wages amplifies the rise in the aggregate wage and leads agents to supply more labor. The complementarity between consumption and hours worked generates a strong positive response of private consumption. As shown in the dashed-dotted green line with diamonds, the response of private consumption to the government spending shock is amplified and lies within the confidence bounds once we allow for the technology channel. In accordance with Zeev and Pappa [2015], Caldara and Kamps [2017], Ferrara et al. [2021] who find that a government spending shock is inflationary, a model where technological change is concentrated in traded industries and non-traded prices are sticky can produce simultaneously an increase in $C(t)$ together with a rise in the CPI following a government spending shock.

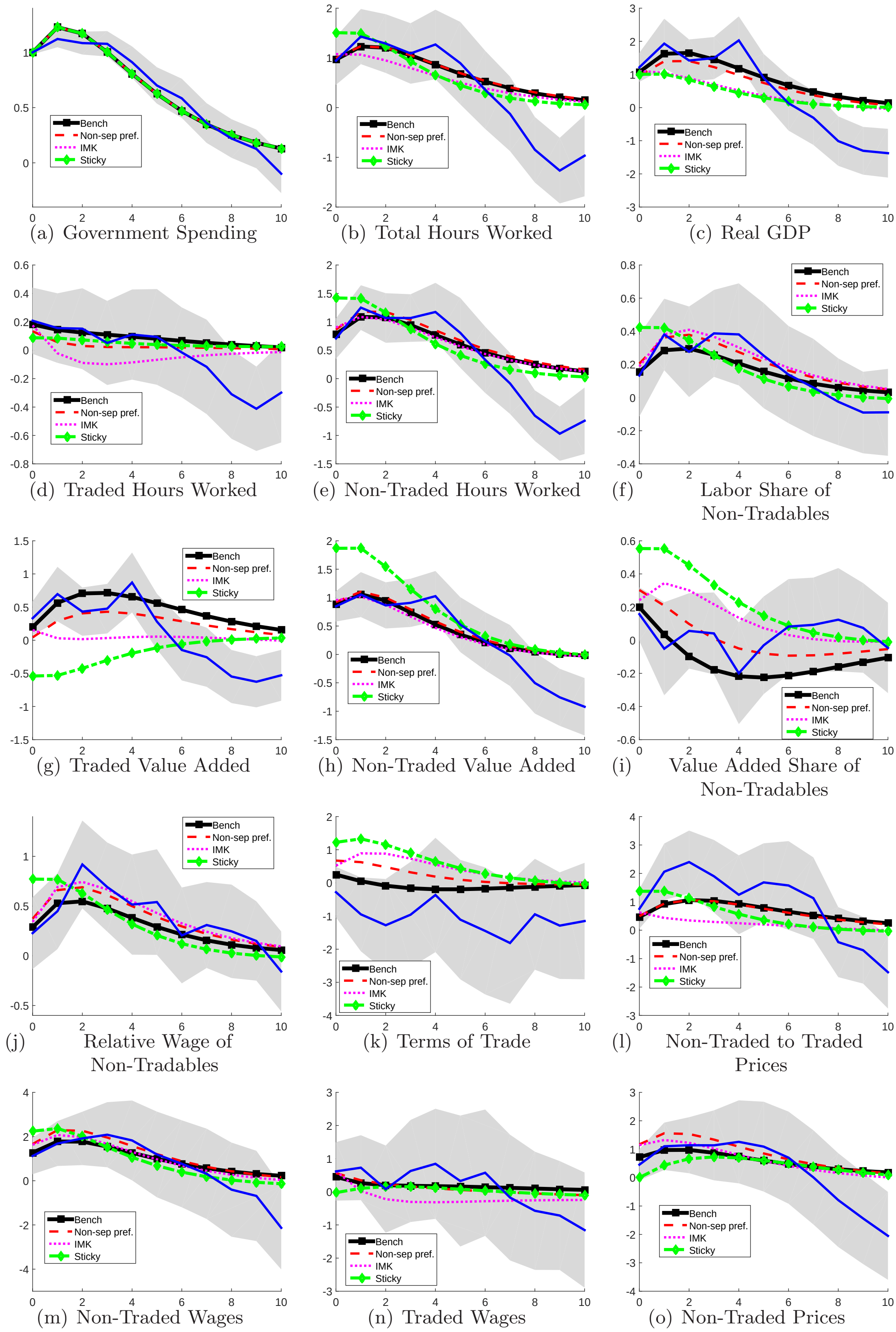
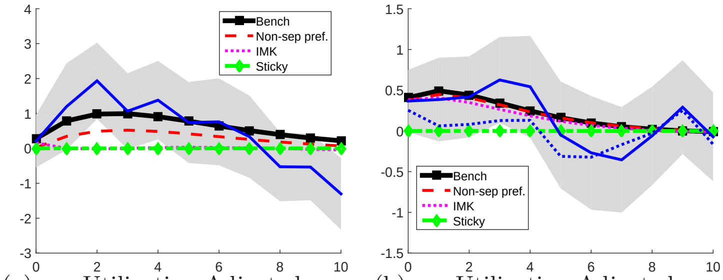
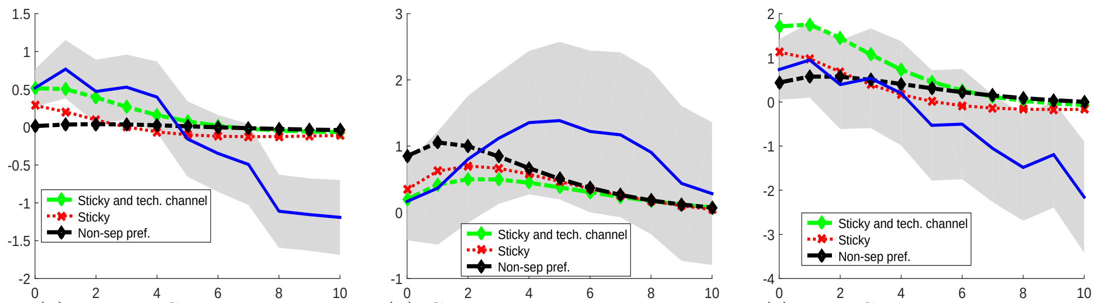


Figure 43: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Labor and Output Effects. Notes: Solid blue line displays point estimate from local projections with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC. The dashed red line shows predictions of the baseline model augmented with non-separability in preferences between consumption and leisure in the lines of Shimer [2009]. The dotted magenta line shows predictions of the baseline model augmented with non-separability in preferences between consumption and leisure together with imperfect mobility of capital. The dashed-dotted green line with diamonds shows the predictions of the baseline model where we allow for sticky prices in the non-traded sector and shut down the technology channel, i.e., abstracting from capital and technology utilization and factor-biased technological change



(a) Utilization-Adjusted Traded TFP (b) Utilization-Adjusted Non-Traded TFP

Figure 44: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Technology Effects. Notes: Solid blue line displays point estimate from local projections with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC. The dashed red line shows predictions of the baseline model augmented with non-separability in preferences between consumption and leisure in the lines of Shimer [2009]. The dotted magenta line shows predictions of the baseline model augmented with non-separability in preferences between consumption and leisure together with imperfect mobility of capital. The dashed-dotted green line with diamonds shows the predictions of the baseline model where we allow for sticky prices in the non-traded sector and shut down the technology channel, i.e., abstracting from capital and technology utilization and factor-biased technological change



(a) Private Consumption (b) Consumption Price Index (c) Real Consumption Wage

Figure 45: Responses of Consumption, CPI and Real Consumption Wage Following Unanticipated Government Spending Shock. Notes: Solid blue line displays point estimate from local projections with shaded areas indicating 90% confidence bounds; the dashed-dotted green line with diamonds displays model predictions of the baseline model where we allow for sticky prices in the non-traded sector, non-separability in utility between consumption and leisure in the lines of Shimer [2009]. In the dotted red line with crosses, we consider the same model with sticky prices but we shut down the technology channel. In the dashed-dotted black line with diamonds, we augment the baseline model with non-separable Shimer [2009] preferences only.

References

Acemoglu, Daron (2003) Labor- and Capital-Augmenting Technical Change. *Journal of the European Economic Association*, 1(1), pages 1-37.

Alvarez-Cuadrado, Francisco, Ngo Van Long and Markus Poschke (2018) Capital-Labor Substitution, Structural Change and the Labor income Share. *Journal of Economic Dynamics and Control*, 87, pages 206-231.

Antràs Pol (2004) Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *The B.E. Journal of Macroeconomics*, vol. 4(1), pages 1-36.

Auerbach, Alan J. and Yuriy Gorodnichenko (2012) Measuring the Output Responses to Fiscal Policy. *American Economic Journal: Economic Policy* 4, pp. 1-27.

Bachmann, Rüdiger and Eric R. Sims (2012) Confidence and the Transmission of Government Spending Shocks. *Journal of Monetary Economics*, 59(3), pp. 235-249.

Barro, Robert J., and Charles J. Redlick (2011) Macroeconomic Effects From Government Purchases and Taxes. *The Quarterly Journal of Economics*, 126(1), pages 51-102.

Basu Susanto, John G. Fernald and Miles S. Kimball (2006) Are Technology Improvements Contractionary? *American Economic Review*, 96 (5), 1418-1448.

Baxter, Marianne and Robert G. King (1993) Fiscal Policy in General Equilibrium. *American Economic Review*, 83(3), pp. 315-334.

Beetsma, Roel, Massimo Giuliodori, and Franck Klaassen (2008) The Effects of Public Spending Shocks on Trade Balances and Budget Deficits in the European Union. *Journal of the European Economic Association* 6(2-3), pp. 414-423.

Beetsma, Roel and Massimo Giuliodori (2011) The Effects of Government Purchases Shocks: Review and Estimates for the EU. *Economic Journal*,121(550), pp. 4-32.

Benati, Luca (2008) The "Great Moderation" in the United Kingdom. *Journal of Money, Credit and Banking*, 40(1), pp. 121-147.

Benigno, Gianluca, Luca Fornaro, and Martin Wolf (2020) The Global Financial Resource Curse. *Staff Reports* 915, Federal Reserve Bank of New York.

Bertinelli L., O. Cardi and R. Restout (2020) Relative Productiviy and Search Unemployment in an Open Economy. *Journal of Economic Dynamics and Control*, 117.

Bilbiie, Florin O. (2011) Nonseparable Preferences, Frisch Labor Supply, and the Consumption Multiplier of Government Spending: One Solution to a Fiscal Policy Puzzle *Journal of Money, Credit and Banking*, 43(1), pp. 221-251.

Bloom, Nicholas and Raffaella Sadun (2012) The Organization of Firms Across Countries. *The Quarterly Journal of Economics*, 127(4), pages 1663-1705.

Born, Benjamin, Falko Juessen, and Gernot J. Müller (2013) Exchange Rate Regimes and Fiscal Multipliers. *Journal of Economic Dynamics and Control*, 37(2), pp. 446-465.

Breitung Jörg (2000) The Local Power of Some Unit Root Tests for Panel Data, in B. Baltagi (ed.), Advances in Econometrics, vol. 15: Nonstationary Panels, Panel Cointegration, and Dynamic Panels, Amsterdam: JAI Press, 161-178.

Bridgman, Benjamin (2018) Is Labor's Loss Capital's Gain? Gross Versus Net Labor Shares. *Macroeconomic Dynamics*, vol. 22(8), 2070-2087.

Brinca, Pedro, Hans A. Holter, Per Krusell, and Laurence Malafry, (2016) Fiscal Multipliers in the 21st Century. *Journal of Monetary Economics*, 77(C), pp. 53-69.

Brückner, Markus, and Evi Pappa (2012) Fiscal Expansions, Unemployment, And Labor Force Participation: Theory And Evidence. *International Economic Review*, 53(4), pages 1205-1228.

Burstein, Ariel T., Joao C. Neves and Sergio Rebelo (2004) Investment Prices and Exchange Rates: Some Basic Facts. *Journal of the European Economic Association* 2(2-3), pp. 302-309.

Caldara, Dario, and Christophe Kamps, 2017. The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers. *Review of Economic Studies*, 84(3), pp. 1015-1040.

Cashin, Paul and John C. McDermott (2003) Intertemporal Substitution and Terms-of-Trade Shocks. *Review of International Economics*, vol. 11(4), 604-618.

Chirinko, Robert S., and Debdulal Mallick (2017) The Substitution Elasticity, Factor Shares, and the Low-Frequency Panel Model. *American Economic Journal: Macroeconomics*, 9(4), pp 225-253.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011) When is the Government Spending Multiplier is Large? *Journal of Political Economy*, 119(1), pp. 78-121.

De Cordoba, Gonzalo Fernandez and Timothy J. Kehoe (2000) Capital Flows and Real Exchange Rate Fluctuations Following Spain's Entry into the European Community. *Journal of International Economics*, 51(1), 49-78.

De Ferra, Sergio (2018) Sovereign Debt Crises, Fiscal Austerity and Corporate Default. Mimeo.

Farhi, Emmanuel, and Ivan Werning (2016) Fiscal Multipliers: Liquidity Traps and Currency Unions Handbook of Macroeconomics, 2, pages 2417-2492.

Farhi, Emmanuel, and Ivan Werning (2017) Fiscal Unions. *American Economic Review*, 107(12), pp. 3788-3834.

Fioramanti, Marco, Laura González Cabanillas, Bjorn Roelstraete and Salvador Adrian Ferrandis Vallterra (2016) European Commission's Forecasts Accuracy Revisited: Statistical Properties and Possible Causes of Forecast Errors. *European Economy Discussion Papers*, n°027.

Fornaro, Luca (2018) International Debt Deleveraging. *Journal of the European Economic Association*, 16(5), pp. 1394-1432.

Galí, Jordi, J. David López-Salido and J. Vallés (2007) Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association* 5, pp. 227-270.

Garìn, Julio, Michael Pries and Eric Sims (2018) The Relative Importance of Aggregate and Sectoral Shocks and the Changing Nature of Economic Fluctuations. *American Economic Journal: Macroeconomics*, 10(1), 119-148.

Garofalo, Gasper A., and Steven Yamarik (2002) Regional Convergence: Evidence From A New State-By-State Capital Stock Series. *Review of Economics and Statistics*, 84(2), pp. 316-323.

Gollin, Douglas (2002) Getting Income Shares Right. *Journal of Political Economy*n vol. 110(2), pages 458-474.

González Cabanillas, Laura, and Eric Ruscher (2016) The Great Moderation in the Euro Area: What Role have Macroeconomic Policies played? *European Economy Discussion Papers*, n°331.

Guajardo, Jaime, Daniel Leigh, and Andrea Pescatori (2014) Expansionary Austerity? International Evidence. *Journal of the European Economic Association*, 12(4), pp. 949-968.

Hall, Robert E., and Charles I. Jones (1999) Why Do Some Countries Produce So Much More Output Per Worker Than Others? *The Quarterly Journal of Economics*, 114(1), pp. 83-116.

Hadri, Kaddour (2000) Testing for Unit Roots in Heterogeneous Panel Data. *Econometrics Journal*, 3, pp. 148-161.

Hall, Robert E. (2009) By How Much Does GDP Rise If the Government Buys More Output? *Brookings Papers on Economic Activity* Fall, pp. 183-231.

Herrendorf, Berthold, Christopher Herrington, and Ákos Valentinyi (2015) Sectoral Technology and Structural Transformation. *American Economic Journal: Macroeconomics*, 7(4), pp. 104-133.

Horvath, Michael (2000) Sectoral Shocks and Aggregate Fluctuations. *Journal of Monetary Economics*, 45, pp. 69-106.

Ilzetzki, Ethan, Enrique G., Mendoza and Carlos A. Végh, (2013) How big (small?) are fiscal multipliers. *Journal of Monetary Economics*, 60(2), pp. 239-254.

Im, Kyung So, Hashem M. Pesaran and Yongcheol Shin (2003) Testing for Unit Roots in Heterogeneous Panels. *Journal of Econometrics*, 115, pp. 53-74.

IMF DOTS: Direction of Trade Statistics (2017).

Inglehart, R., Haerpfer, C., Moreno, A., Welzel, C., Kizilova, K., Diez-Medrano J., M. Lagos, P. Norris, E. Ponarin, and B. Puranen et al. (eds.) (2020) World Values Survey: All Rounds - Country-Pooled Datafile. Madrid, Spain and Vienna, Austria: JD Systems Institute & WVSA Secretariat [Version: `http://www.worldvaluessurvey.org/WVSDocumentationWVL.jsp`].

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. Monetary Policy according to HANK. *American Economic Review*, 108(3), 697-743.

Karabarbounis, Loukas, and Brent Neiman (2014) The Global Decline of the Labor Share. *The Quarterly Journal of Economics*, 129(1), pages 61-103.

Karamysheva Madina, and Anton Skrobotov (2022) Do We Reject Restrictions Identifying Fiscal Shocks? Identification Based on non-Gaussian Innovations. *Journal of Economic Dynamics and Control*, forthcoming.

Kilian, Lutz and Helmut Lütkepohl (2017) Structural Vector Autoregressive Analysis. Cambridge University Press.

Klump, Rainer, Peter McAdam and Alpo Willman (2007) Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach. *Review of Economics and Statistics*, 89(1), pp. 183-92.

Kwiatkowski Denis, Peter C.B. Phillips, Peter Schmidt and Yongcheol Shin (1992) Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root. *Journal of Econometrics*, 54, pp. 159-178.

Lee, Donghoon and Kenneth I. Wolpin (2006) Intersectoral Labor Mobility and the Growth of the Service Sector. *Econometrica*, 74, pp. 1-46.

León-Ledesma, Miguel A., Peter McAdam and Alpo Willman (2010) Identifying the Elasticity of Substitution with Biased Technical Change. *American Economic Review*, 100(4), pp. 1330-1357.

Levin Andrew, Chien-Fu Lin and Chia-Shang James Chu (2002) Unit Root Test in Panel Data: Asymptotic and Finite Sample Properties. *Journal of Econometrics*, 108, pp. 1-24.

MaCurdy, Thomas (1981) An Empirical Model of Labor Supply in a Life-Cycle Setting. *Journal of Political Economy*, 89 (6), pp. 1059-1085.

Maddala, Gangadharrao S. and Shaowen Wu (1999) A Comparative Study of Unit Root Tests with Panel Data and a New Test. *Oxford Bulletin of Economics and Statistics*, 61, pp. 631-652.

McDaniel C (2007) Average Tax Rates on Consumption, Investment, Labor and Capital in the OECD 1950-2003. Mimeo. `www.caramcdaniel.com/mcdaniel_tax_2007.pdf`

Mendoza, Enrique, G. (1995) The Terms of Trade, Real Exchange Rate and Economic Fluctuations. *International Economic Review*, 36, pp. 101-137.

Mertens Karel and Morten O. Ravn (2011) Technology-Hours Redux: Tax Changes and the Measurement of Technology Shocks. NBER Chapters, in: NBER International Seminar on Macroeconomics 2010, pages 41-76 National Bureau of Economic Research, Inc.

Miyamoto, Wataru, Thuy Lan Nguyen and Dmitriy Sergeyev (2018) Government Spending Multipliers under the Zero Lower Bound: Evidence from Japan. *American Economic Journal: Macroeconomics*, 10(3), pages 247-277.

Miyamoto, Wataru, Thuy Lan Nguyen, Viacheslav Sheremirov (2019) The Effects of Government Spending on Real Exchange Rates: Evidence from Military Spending Panel Data. *Journal of International Economics*, 106(C), pages 143-164.

Nekarda, Christopher J., and Valerie A. Ramey (2011) Industry Evidence on the Effects of Government Spending. *American Economic Journal: Macroeconomics*, 3(1), pages 36-59.

Oberfield, Ezra, and Devesh Raval (2021) Micro Data and Macro Technology. *Econometrica*, 89(2), pages 703-732.

Organization for Economic Cooperation and Development (2011), *Structural Analysis Database*, OECD, Paris.

Organization for Economic Cooperation and Development (2017), *Structural Analysis Database*, OECD, Paris.

Organization for Economic Cooperation and Development (2017), *Annual National Accounts*, OECD, Paris.

Organization for Economic Cooperation and Development (2017), *Economic Outlook Database*, OECD, Paris.

Organization for Economic Cooperation and Development (2017), *Input-Output Database*, OECD, Paris.

Organization for Economic Cooperation and Development (2017), *National Accounts: General Government Accounts*, Government Expenditure by Function (COFOG), OECD, Paris.

Organization for Economic Cooperation and Development (2017), *Prices and Purchasing Power Parities Database*, OECD, Paris.

Organization for Economic Cooperation and Development (2021), *OECD Productivity Statistics Database*, OECD, Paris.

OECD Global Value Chains (GVCs), Jobs and Routine Content of Occupations.

Pagan, Adrian (1984) Econometric Issues in the Analysis of Regressions with Generated Regressors. *International Economic Review*, 25(1), pp. 221-247.

Pappa, Evi, and Nadav Ben Zeev (2015) Multipliers of Unexpected Increases in Defense Spending: An Empirical Investigation. *Journal of Economic Dynamics and Control*, vol. 57(C), pp. 205-226.

Pedroni, Peter (1999) Critical Values for Cointegration Tests in Heterogeneous Panels with Multiple Regressors. *Oxford Bulletin of Economics and Statistics*, 61, pp. 653-670.

Pedroni, Peter (2000) Fully Modified OLS for Heterogeneous Cointegrated Panels. *Advances in Econometrics*, vol. 15, edited in B. Baltagi: Nonstationary Panels, Panel Cointegration and Dynamic Panels, 93-130.

Pedroni, Peter (2001) Purchasing Power Parity Tests in Cointegrated Panels. *The Review of Economics and Statistics*, 83(4), pp. 727-731.

Pedroni, Peter (2004) Panel Cointegration: Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the PPP Hypothesis. *Econometric Theory*, 20, pp. 597-625.

Pesaran, Hashem M., Yongcheol Shin and Ron P. Smith (1999) Pooled Mean Group Estimation of Dynamic Heterogeneous Panels.*Journal of the American Statistical Association*, 94, pp. 621-634.

Piton, Sophie (2017) Economic Integration and the Non-Tradable Sector: The European Experience. Mimeo.

Ramey, Valerie A. (2011) Identifying Government Spending Shocks: It's All in the Timing *Quarterly Journal of Economics*, 126(1), pp. 1-50.

Rossi, Barbara and Sarah Zubairy (2011) What Is the Importance of Monetary and Fiscal Shocks in Explaining U.S. Macroeconomic Fluctuations? *Journal of Money, Credit and Banking*, 43(6), pages 1247-270.

Rotemberg, Julio J (1982) Sticky Prices in the United States. *Journal of Political Economy*, 90(6), pp. 1187-1211.

Stehrer, Robert, Alexandra Bykova, Kirsten Jäger, Oliver Reiter and Monika Schwarzhappel (2019) Industry Level Growth and Productivity Data with Special Focus on Intangible Assets. Statistical Report No. 8.

Stockman Alan C. and Linda L. Tesar (1995) Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. *American Economic Review* 85(1), pp. 168-185.

Timmer, Marcel, Mary O'Mahony, and Bart Van Ark (2008) The EU KLEMS Growth and Productivity Accounts: An Overview, University of Groningen and University of Birmingham; downloadable at `www.euklems.net`

Turnosvsky, Stephen J. (1997) International Macroeconomic Dynamics. MIT press.

United Nations Comtrade Database (2017) Classification of Broad Economic Categories.

United Nations Statistics Division (2017) Classification of individual consumption by purpose.

WIOD: Construction and Applications (2013). World Input-Output Database.

WIOD: Construction and Applications (2016). World Input-Output Database.