Documents de travail

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Document de Travail n° 2022 – 03

Janvier 2022

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Reconsidering the interplay between endogenous growth and the Environmental Kuznets Curve

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January 21, 2022

Abstract

This paper develops a very simple model of endogenous growth *à la* Lucas (1988) in which a representative household has to choose between environmental preservation and human capital accumulation. After computing analytically all possible trajectories, we point out that one of them depicts an inverted U-shape relationship between human capital (production) and pollution (i.e. an Environmental Kuznets Curve). If the economy follows the EKC trajectory, then a steady state is reached in the long run, indicating the incompatibility between endogenous growth and the EKC. Moreover, this simple framework allows to compute explicitly the initial value of the control variable. It is then proved that the optimal trajectory is the balanced growth path, not the EKC. Finally, we show that endogenous growth is possible, whatever the effect of pollution on the marginal utility of consumption.

JEL codes: C61, O44

Keywords: Endogenous growth, environmental Kuznets curve, human capital

1 Introduction

It is well-known in the literature that positive externalities such as learning-by-doing (Romer, 1986) or human capital accumulation (Lucas, 1986) are able to lead to endogenous growth. However, as pointed out by Tahvonen and Kuuluvainen (1991) or by Michel and Rotillon (1995), since pollution is a negative externality, preservation of endogenous growth, in an environmental context, requires that the positive externality dominates the negative one.

Interestingly Michel and Rotillon (1995) have considered an endogenous growth model with learning by doing à la Romer (1986) where a pollution externality, coming from production, affects the household's utility. The originality of their approach is to consider a non-separable utility function while most of their predecessors have assumed separable preferences[1]. Discussing conditions

[1] See Keeler et al. (1971), Forster (1973) or Van Ewijk and Van Wijnbergen (1995) among others.

under which endogenous growth is preserved when the pollution externality is internalized (social planner), they observed that endogenous growth no longer exists in the long run when pollution reduces the marginal utility of consumption (*distaste effect*). The only configuration under which endogenous growth is preserved is when preferences depict a strong positive effect of pollution on marginal utility of consumption (*compensation effect*).

From an environmental ground, the possibility of endogenous growth in Michel and Rotillon (1995) is related to an unlimited increase of pollution. However, the literature on the Environmental Kuznets Curve (EKC hereafter) has pointed out the possibility of an inverted U-shape relationship between per capita income and pollution emissions. The rational for the decreasing branch of the EKC is intuitive: after a threshold income level, the willingness to pay for a clean environment starts to increase more rapidly than income (Kijima et al. 2010). The EKC hypothesis has been received a wide range of contributions, both from a theoretical and empirical grounds[2].

One can question the compatibility between the EKC and endogenous growth. In Michel and Rotillon (1995), the possible occurrence of an EKC is not observed while, as discussed before, the pollution externality challenges the existence of endogenous growth. In a seminal work, Stockey (1998) has considered an AK framework where pollution affects the household's utility in a separable way. The study of the planner solution reveals that there is no parametric space for which endogenous growth is preserved. This is explained by the fact that introduction of emissions standards concerning pollution reduces the rate of return on capital. In a sense, this result is related to the one pointed out by Michel and Rotillon (1995) since they have conclude that endogenous growth vanishes when preferences are separable. Moreover, Stockey (1998) shows that the optimal trajectory displays an inverted U-shape relationship between income and pollution (EKC), indicating the incompatibility between endogenous growth and the existence of an EKC. In the same spirit, Hartman and Kwon (2005) have followed Lucas (1988) by considering a growth framework with human capital accumulation. Pollution is assumed to come from production. Physical capital is an input of production and can also serves as depollution expenditures. As in Stockey (1998), pollution negatively affects the household's utility in a separable way and then, pollution is assumed to not affect the marginal utility of consumption. In their framework, conversely to Stockey (1998), endogenous growth is preserved in the long run such that, the more important is human capital in the production process, the higher is the long run growth rate. Moreover, Hartman and Kown (2005) observe also the possible occurrence of an EKC for a parametric space compatible with endogenous growth.

Even if, as Stockey (1998), Hartman and Kwon (2005) investigate the possible interplay between the EKC and endogenous growth, the comparison between their results and the ones obtained by Stockey (1998) is difficult. Indeed, the existence of an EKC in Hartman and Kwon (2005) comes from the possible shift

[2] Interested reader is referred to Dinda (2004) and Kijima et al. (2010), respectively for empirical and theoretical survey.

from (dirty) physical capital to (clean) human capital while there is only one type of (physical) capital in Stockey (1998).

The question we raise in the present paper is the possible coexistence of an EKC and endogenous growth in a very simple framework à la Lucas (1988) in which human capital is the only productive input and where pollution can affects marginal utility of consumption (non-separable utility). In this sense, we are close to both Stockey (1998) and Michel and Rotillon (1995). In this very simple framework, we propose to consider a representative household who arbitrates between the time spend to accumulate human capital and the time devoted to depollute. Intuitively, we expect to contemplate configurations in which endogenous growth is possible, particularly when the magnitude of the pollution effect on utility is small[3] or when the *compensation effect* is high[4]. Moreover, because of the utility function encompassing both the *distaste effect* and the *compensation effect*, we expect also to obtain parametric configurations for which the opportunity cost of depollution decreases, indicating that the representative household will substitute consumption by depollution, rendering possible the EKC. That is, while we expect to encounter both the EKC and endogenous growth, are these two phenomena compatible? If not, does the optimal trajectory follow the endogenous growth process or the EKC?

The very simple framework developed here renders possible the computation of all possible trajectories. In particular, we point out the possibility to obtain both the EKC and endogenous growth. However, they are incompatible: If the economy follows the balanced growth path, there is no room for the EKC. Conversely, if the economy follows the EKC, then, as in Stockey (1998), the economy reaches a steady state in the long run. Our very simple framework allows us to compute explicitly the optimal initial level of the control variable and we prove that, since pollution affects moderately the household's utility[5], it is always optimal to jump on the balanced growth path at the initial date and then, we prove that it is never optimal to follow the EKC trajectory in this case. This result complete the literature in two ways. First, it shows that conclusions of a simple Lucas (1988) framework with pollution are exactly opposite than the ones of an AK framework with pollution (Stockey, 1998): optimal endogenous growth and suboptimal EKC. Secondly, conversely to Michel and Rotillon (1995), endogenous growth is possible whatever the pollution effect on marginal utility of consumption. The explanation for those two interesting results comes from the fact that pollution affects moderately the household's utility. In this case, the positive externality induced by human capital accumulation dominates the negative effect of pollution, rendering optimal the endogenous growth. Since the EKC implies to reaches a steady state in the present paper, then, the EKC is not the optimal trajectory.

[3] In this case, the positive externality coming from human capital accumulation will probably dominates the negative externality induced by pollution.

[4] Such a configuration is the one in which Michel and Rotillon (1995) have observed endogenous growth in a framework à la Romer (1986) with pollution.

[5] The case where pollution highly affects the household's utility violates the transversality condition, and then, we are not able to say something in this case.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses all possible trajectories of the economy while section 4 evaluates the optimal one. Section 5 concludes the paper.

2 The model

We explore an environmental extension of the Lucas (1988) endogenous growth model. More precisely, each period, a representative household[6] is endowed by one unit of time. She chooses the share of this time devoted to accumulate human capital and the share devoted to preserve environmental quality. As in Ben-Porath (1967), to keep things as simple as possible, human capital is the only input of production. That is, human capital accumulation allows to increase future production/consumption and hence future utility level. However, production generates a pollution externality which negatively affects household's utility. Preferences are depicted by a non-separable isoelastic utility function, namely:

$$u\left(c,P\right)=\frac{\left(cP^{-\eta}\right)^{1-\varepsilon}}{1-\varepsilon} \tag{1}$$

Where c and P represent respectively the consumption and the pollution level. Moreover, as usual, $-1/\varepsilon<0$ depicts the intertemporal elasticity of substitution in consumption while $\eta>0$ captures the magnitude of the negative pollution effect on the household's utility. The function (1) is widely used in the literature[7] and encompasses the so-called *distaste effect* and *compensation effect* (Michel and Rotillon, 1995). Indeed, even if a higher pollution level always implies a lower utility, things are more ambiguous concerning the marginal utility of consumption. On the one hand, the household could enjoy to consume in a pleasant environment. In this case, a higher pollution level reduces the marginal utility of consumption implying the existence of a *distaste effect*. On the other hand, the household could also compensate the drop of utility due to a higher pollution level by increasing her consumption level. In this last case, pollution increases the marginal utility of consumption, implying the existence of a *compensation effect*. To the best of our knowledge, there is no empirical evidence allowing us to discriminate between those two effects and then, we choose to not restrict the parameter space at this step of the reasoning.

Let us explore the parametric configurations for which preferences describe a *distaste effect* or a *compensation effect*:

$$\frac{Pu_{cP}}{u_c}=\eta\left(\varepsilon-1\right)$$

Clearly, if $\varepsilon<1$, $u_{cP}<0$ and hence, preferences are driven by a *distaste effect*. Conversely, if $\varepsilon>1$, $u_{cP}>0$ and then, a *compensation effect* occurs. Following Lucas (1988), human capital evolves according to a linear process:

[6] We normalize the population size to the unity.
[7] See Bosi and Desmarchelier (2018) among others.

$$\dot{h} = B(1-l)h \tag{2}$$

where h represents the level of human capital, $B > 0$ is set to capture the efficacity of the learning process while $(1-l)$ represents the time devoted to accumulate human capital. Consequently, $l \in (0,1)$ depicts the time devoted to preserve the environment (abatement activities).

Following Ben-Porath (1967), human capital is assumed to be the only productive input, namely,

$$Y = Ah \tag{3}$$

Y and $A > 0$ represent respectively the production level and human capital productivity. As discussed earlier, the production process is dirty and generates a pollution externality. To respect the philosophy of a very simple framework, pollution P is viewed as a pure flow, namely:

$$P = \alpha \frac{Ah}{l} \tag{4}$$

$\alpha > 0$ capture both the environmental impact of production and the depollution efficiency. A low (high) value of α represents at the same time a small (high) environmental impact of production or a high (low) depollution efficiency. A higher time spend to abatement activities (i.e. a higher l) implies a lower pollution level P. Function (4) is similar to the one introduced by Fernandez et al. (2012). Finally, in the spirit of Ben-Porath (1967), there is no physical capital and hence, the whole production at time t is consumed at time t. More precisely,

$$Y = c \tag{5}$$

The only possibility to modify future consumption is to modulate the learning effort $(1-l)$ in order to modify the future level of human capital (see (2)).

The program faced by the omniscient representative agent (or by the central planner) is to choose the time path of $l \in (0,1)$ which maximizes the intertemporal utility:

$$\max_{l} \int_0^{+\infty} e^{-\theta t} u(c, P)\, dt \tag{6}$$

with respect to the law of motion of human capital (2) and considering (1), (3), (4) and (5) and with $l \in (0,1)$. Focusing on (6), $\theta > 0$ is the discount factor. The more important is θ, the more impatient the representative household is.

The rest of this paper consists to solve explicitly this program. In most cases in environmental economics, frameworks are too complicated to give closed form solutions. One of the objective of this paper is precisely to be able to compute all possible trajectories.

Proposition 1 *Assume that there exists an interior continuous solution* $(l,h) \in [0,1] \times [0,+\infty[$ *for the program (6). Then,* l *and* h *verify the following necessary conditions:*

$$\begin{aligned} \dot{l} &= B\left(\frac{1-\eta}{\eta}\right)l^2 + \frac{\theta - B(1-\eta)(1-\varepsilon)}{\eta(1-\varepsilon)-1} l \equiv \varphi(l) && (7) \\ \dot{h} &= B(1-l)h && (8) \end{aligned}$$

jointly with the transversality condition[8] $\lim_{t\to+\infty} e^{-\theta t}\lambda h = 0$.

Proof. See the Appendix. ■

Proposition 2 *Necessary conditions (7) and (8) are also sufficient if:*

$$\varepsilon < 1 \text{ jointly with } \eta \in \left]0, 1/2\right[\cup \left]1/(1-\varepsilon), +\infty\right[\tag{9}$$

or if:

$$\varepsilon > 1 \text{ jointly with } \eta \in \left]0, 1/2\right[\cup \left]\varepsilon/(\varepsilon-1), +\infty\right[\tag{10}$$

Proof. See Appendix. ■

At this step of the reasoning, we restrict the analysis to the parameter space described by (9) and (10).

The system (7)-(8) gives the dynamics of the economy. It is interesting to observe that equation (7) is completely independent of equation (8). Moreover, equation (7) is a differential equation of the Bernoulli type and then, is analytically solvable. Before going further, remark that (7) possesses two steady states: a trivial one, namely l^*, and a non-trivial one, namely $\bar{l}$. More precisely:

$$\begin{aligned} l^* &= 0 \\ \bar{l} &= \frac{\eta}{B(\eta-1)} \frac{\theta - B(1-\eta)(1-\varepsilon)}{(1-\varepsilon)\eta - 1} \end{aligned} \tag{11}$$

$\bar{l}$ exists if and only if $\bar{l} \in (0,1)$. The following proposition studies under which conditions this is true, taking into account of (9) and (10).

Proposition 3 *Taking into account restrictions (9) and (10), $\bar{l} \in (0,1)$ in three cases:*

(1) If $\eta < 1/2$, two cases arise:

(1.1) If $\varepsilon < 1$, $\bar{l} \in (0,1)$ if and only if:

$$\theta\left(\frac{\eta}{1-\eta}\right) < B < \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

(1.2) If $\varepsilon > 1$, $\bar{l} \in (0,1)$ if and only if:

$$B > \theta\left(\frac{\eta}{1-\eta}\right)$$

(2) If $\eta > \varepsilon/(\varepsilon-1)$ jointly with $\varepsilon > 1$, $\bar{l} \in (0,1)$ if and only if:

$$B > \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

[8] λ is the Lagrangian multiplier.

Proof. See the Appendix. ■

The three cases exposed in proposition 3 are consistent with sufficiency conditions obtained in proposition 2. In what follows, we restrict the analysis to those three cases. The following proposition studies the stability of each steady states.

Proposition 4 *Let us consider the three cases presented in proposition 3. In cases (1.1) and (1.2) l^* is stable while $\bar{l}$ is unstable. Conversely, in case (2), l^* is unstable while $\bar{l}$ is stable.*

Proof. See the Appendix. ■

From the previous proposition, it follows that, $\forall l_0 \neq l^*$, l will converge to $\bar{l}$ in case (2). However, in cases (1.1) and (1.2), if $l_0 < \bar{l}$, l will decrease from l_0 to 0 (i.e. to l^*). If $l_0 > \bar{l}$, l will increase from l_0 to 1. More precisely, if l converges to $\bar{l}$ or to l^* then, the economy reaches a balanced growth path in the long run. The growth rate is respectively given by $\bar{\gamma}$ and γ^* such that:

$$\begin{aligned}\frac{\dot{h}}{h} &= \frac{\dot{Y}}{Y} = \frac{\dot{c}}{c} = B\left(1-\bar{l}\right) \equiv \bar{\gamma} \\ \frac{\dot{h}}{h} &= \frac{\dot{Y}}{Y} = \frac{\dot{c}}{c} = B \equiv \gamma^*\end{aligned}$$

Clearly, $\gamma^* > \bar{\gamma}$ but when the growth rate is given by $\bar{\gamma}$, the pollution is controlled since $\bar{l} > 0$. Conversely, when $l \to 0$, the growth rate is maximal (i.e. given by γ^*), but pollution is uncontrolled in the long run: the environment is sacrificed to ensure the highest possible growth rate. The opposite situation appears if $l = 1$ in the long run. In this case, this is the growth rate which is sacrificed to preserve the environmental quality. Indeed, when $l = 1$, the economy reaches a steady state since $\dot{h}/h = \dot{Y}/Y = \dot{c}/c = 0$. This qualitative analysis of all possible trajectories will be completed by analytical results in the next section.

3 Trajectories

As discussed before, equation (7) is a differential equation of the Bernoulli type and then, is fully solvable. Using usual technics, it follows that:

$$l(t) = \frac{l_0 e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}}{1-\frac{l_0}{\bar{l}}\left(1-e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}\right)} \tag{12}$$

As pointed out by proposition 4, we observe from (12) that, since $\eta > \varepsilon/(\varepsilon-1) > 1$ (Proposition 3, case (2)),

$$\lim_{t\to+\infty} l(t) = \bar{l}$$

Moreover, since $\eta < 1/2$ (Proposition 3, cases (1.1) and (1.2)), if $l_0 < \bar{l}$, then:

$$\lim_{t \to +\infty} l(t) = 0$$

If $l_0 > \bar{l}$, things are more complicated. Indeed, since by definition $l(t) \in (0,1)$, $l(t)$ has to increases until $l(t) = 1$ after which, $l(t)$ has to remain equal to 1 forever. Let t^* be the date such that $\forall t < t^*$, $l(t) < 1$ while $\forall t \geq t^*$, $l(t) = 1$. Focusing on (12):

$$t^* = \frac{\eta}{B(1-\eta)\bar{l}} \ln\left[\left(\frac{1-\bar{l}}{l_0-\bar{l}}\right) l_0\right] \equiv t^*(l_0)$$

It is interesting to remark that $\eta < 1/2$ jointly with $\bar{l} < l_0 < 1$ ensures that $t^* > 0$. Moreover,

$$t'^*(l_0) = -\frac{\eta}{B(1-\eta)} \frac{1}{l_0\left(l_0-\bar{l}\right)} < 0$$

Since $l_0 > \bar{l}$ jointly with $\eta < 1/2$, it follows that $t'^*(l_0) < 0$. Interpretation is obvious: the higher is l_0, the sooner $l(t) = 1$.

Considering (12), it is possible to solve the differential equation (8), that is:

$$h(t) = h_0 e^{Bt}\left[1 + \frac{l_0}{\bar{l}}\left(e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t} - 1\right)\right]^{\frac{\eta}{1-\eta}}$$

Considering (8), if $l(t) = 1$ (that is, $t \geq t^*$ such that $l_0 > \bar{l}$ in cases (1.1) and (1.2) of proposition 3), $\dot{h}/h = 0$ and then, the economy reaches a steady state given by $h(t^*)$ such that:

$$h(t^*) = h_0 l_0^{\frac{\eta}{(1-\eta)\bar{l}}}\left(\frac{l_0-\bar{l}}{1-\bar{l}}\right)^{\frac{\eta(\bar{l}-1)}{(1-\eta)\bar{l}}}$$

The previous discussion is summarized in the following proposition.

Proposition 5 *Following (12), it appears that:*
(1) If $\eta > \varepsilon/(\varepsilon-1) > 1$ (Proposition 3, case (2)),

$$l(t) = \frac{l_0 e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}}{1 - \frac{l_0}{\bar{l}}\left(1 - e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}\right)} \tag{13}$$

$$h(t) = h_0 e^{Bt}\left[1 + \frac{l_0}{\bar{l}}\left(e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t} - 1\right)\right]^{\frac{\eta}{1-\eta}} \tag{14}$$

(2) If $\eta < 1/2$ (Proposition 3, cases (1.1) and (1.2)), two cases arise:

(2.1) If $l_0 < \bar{l}$,

$$
\begin{aligned}
l(t) &= \frac{l_0 e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}}{1-\frac{l_0}{\bar{l}}\left(1-e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}\right)} && (15)\\
h(t) &= h_0 e^{Bt}\left[1+\frac{l_0}{\bar{l}}\left(e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}-1\right)\right]^{\frac{\eta}{1-\eta}} && (16)
\end{aligned}
$$

(2.2) If $l_0 > \bar{l}$,

$$
\begin{aligned}
l(t) &= \frac{l_0 e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}}{1-\frac{l_0}{\bar{l}}\left(1-e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}\right)},\ \forall t < t^* && (17)\\
l(t) &= 1,\ \forall t \geq t^*
\end{aligned}
$$

and

$$
\begin{aligned}
h(t) &= h_0 e^{Bt}\left[1+\frac{l_0}{\bar{l}}\left(e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t}-1\right)\right]^{\frac{\eta}{1-\eta}},\ \forall t < t^*\\
h(t) &= h_0 l_0^{\frac{\eta}{(1-\eta)\bar{l}}}\left(\frac{l_0-\bar{l}}{1-\bar{l}}\right)^{\frac{\eta(\bar{l}-1)}{(1-\eta)\bar{l}}},\ \forall t \geq t^*
\end{aligned}
$$

The previous proposition allows to complete the discussion proposed after proposition 4 by giving the explicit expressions of trajectories in each configurations. The challenge now is to determine the initial value of the control variable, namely l_0, when $\eta > \varepsilon/(\varepsilon-1) > 1$ and when $\eta < 1/2$. But before that, let us point out an interesting possibility: the existence of an EKC. Despite the extreme simplicity of this economy and the monotonicity of all possible trajectories exposed within proposition 5, nothing exclude a possible non-monotonicity of pollution. The next proposition gives conditions under which the EKC is possible.

Proposition 6 *(Environmental Kuznets Curve) Focus on case (2.2) of proposition 5, let:*

$$
\tilde{l} \equiv \frac{\eta(\theta+B\varepsilon)}{B(\eta(\varepsilon-1)+1)} \text{ and } \tilde{t} \equiv \frac{\eta}{B(\eta-1)\bar{l}}\ln\left(\frac{\tilde{l}}{l_0}\frac{l_0-\bar{l}}{\tilde{l}-\bar{l}}\right)
$$

and assume that $\bar{l} < l_0 < \tilde{l}$, then:
- When $t < \tilde{t}$, both P and h increases through time.
- When $\tilde{t} < t < t^*$, P decreases over time while h increases.
This EKC trajectory leads to a steady state where $l = 1$ at $t = t^*$ and then, meet the transversality condition $\lim_{t\to+\infty} e^{-\theta t}\lambda h = 0$.

Proof. See the Appendix. ■

Dinda (2004) defines the EKC as an "inverted-U-shaped relationship between different pollutants and per capita income". Proposition 6 describes an inverted-U-shaped relationship between pollution P and human capital. However, in the present economy, population is normalized to the unity ($N = 1$) while $Y = Ah$. That is, an inverted-U-shaped relationship between pollution P and per capita income (Y/N) is strictly equivalent to an inverted-U-shaped relationship between pollution P and h.

The EKC trajectory clearly leads to a steady state which exclude any possible compatibility with endogenous growth. In this sense, we recover a closely related result that the one pointed out by Stockey (1998). However, in Stockey (1998), there is no parametric configuration for which endogenous growth is possible. Conversely, in the present paper, endogenous growth is a possibility (see case (1) in proposition 5 or case (2) with $l_0 = \bar{l}$). To study if we recover exactly the optimal trajectory of Stockey (1998) in a Lucas (1988) framework, we have to compute the optimal l_0. This is the purpose of the next section.

4 Optimality

The previous section has described all the possible scenarios and has pointed out in particular the possible occurrence of an EKC, a trajectory leading to a steady state as in Stockey (1998). However, to be able to precisely describe the optimal trajectory followed by this economy, it is necessary to discuss the optimal value of l_0. The strategy followed is the one proposed recently by Borisov et al. (2020). The idea is to compute the intertemporal welfare function (6) by considering $l(t)$ and $h(t)$ obtained within proposition 5. After that, we simply have to maximize the intertemporal welfare with respect to l_0.

Focus on case (1) of proposition 5. The welfare is given by:

$$W_1 \equiv \int_0^{+\infty} e^{-\theta t} u(c, P)\, dt = \frac{\omega \Delta}{1-\varepsilon} l_0^{\eta(1-\varepsilon)} h_0^{(1-\eta)(1-\varepsilon)}$$

with,

$$\begin{aligned} \omega &\equiv \left(A(\alpha A)^{-\eta}\right)^{1-\varepsilon} > 0 \\ \Delta &\equiv \int_0^{+\infty} e^{\frac{B(\eta-1)}{\eta}\bar{l}t} dt > 0 \end{aligned}$$

Clearly,

$$\frac{\partial W_1}{\partial l_0} = \eta \omega \Delta l_0^{\eta(1-\varepsilon)-1} h_0^{(1-\eta)(1-\varepsilon)} > 0 \tag{18}$$

The relation (18) implies that it is optimal to set $l_0 = 1$. However, if $l_0 = 1$, (13), (14) and (22) imply that:

$$\lim_{t\to+\infty} e^{-\theta t}\lambda h = \lim_{t\to+\infty} \frac{\omega\eta}{B} h_0^{(1-\eta)(1-\varepsilon)} \left[1 + \frac{1}{\bar{l}}\left(e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t} - 1\right)\right] = +\infty$$

That is, the transversality condition is violated. That is, we are not able to say anything in case (1) of proposition 5, that is, in case where $\eta > \varepsilon/(\varepsilon-1) > 1$.

Focus on case (2.1) of proposition 5. The welfare is given by:

$$W_{21} \equiv \int_0^{+\infty} e^{-\theta t} u(c,P)\,dt = \frac{\omega}{1-\varepsilon}\frac{\eta}{B(1-\eta)\bar{l}} l_0^{\eta(1-\varepsilon)} h_0^{(1-\eta)(1-\varepsilon)}$$

It follows, that:

$$\frac{\partial W_{21}}{\partial l_0} = \frac{\omega\eta^2}{B(1-\eta)\bar{l}} l_0^{\eta(1-\varepsilon)-1} h_0^{(1-\eta)(1-\varepsilon)} > 0$$

That is, it is optimal to choose the highest possible l_0 since $l_0 < \bar{l}$. To have a complete picture, we have to discuss the case where $l_0 > \bar{l}$, namely, case (2.2) of proposition 5. The welfare is given by:

$$W_{22} \equiv \int_0^{+\infty} e^{-\theta t} u(c,P)\,dt = \int_0^{t^*} e^{-\theta t} u(c,P)\,dt + \int_{t^*}^{+\infty} e^{-\theta t} u(c,P)\,dt$$

Interestingly,

$$\begin{aligned}
\int_0^{t^*} e^{-\theta t} u(c,P)\,dt &= \frac{\omega}{1-\varepsilon}\frac{\eta}{B(1-\eta)} l_0^{\eta(1-\varepsilon)-1} h_0^{(1-\eta)(1-\varepsilon)} \left(\frac{1-l_0}{1-\bar{l}}\right) \\
\int_{t^*}^{+\infty} e^{-\theta t} u(c,P)\,dt &= \frac{1}{\theta}\frac{\omega}{1-\varepsilon} h_0^{(1-\eta)(1-\varepsilon)} l_0^{\eta(1-\varepsilon)-1} \left(\frac{l_0-\bar{l}}{1-\bar{l}}\right)
\end{aligned}$$

and then,

$$W_{22} \equiv \int_0^{+\infty} e^{-\theta t} u(c,P)\,dt = \frac{\omega}{1-\varepsilon} l_0^{\eta(1-\varepsilon)-1} h_0^{(1-\eta)(1-\varepsilon)} \frac{\eta(1-\varepsilon) + (1-(1-\varepsilon)\eta) l_0}{\theta}$$

it follows that:

$$\frac{\partial W_{22}}{\partial l_0} = \frac{\omega}{\theta} l_0^{\eta(1-\varepsilon)-2} h_0^{(1-\eta)(1-\varepsilon)} \eta (1-l_0)\left[\eta(1-\varepsilon) - 1\right] < 0$$

That is, when $l_0 > \bar{l}$, it is optimal to choose the lowest possible l_0. Conversely, we have also observed that, since $l_0 < \bar{l}$, it is optimal to choose the highest possible l_0. This indicate that the best choice is $l_0 = \bar{l}$, but to be able to conclude, we have to check the continuity of (6) at $\bar{l}$ and interestingly:

$$\lim_{l_0 \to \bar{l}} W_{21} = \frac{\omega}{1-\varepsilon} \frac{\eta}{B(1-\eta)} \bar{l}^{\eta(1-\varepsilon)-1} h_0^{(1-\eta)(1-\varepsilon)} = \lim_{l_0 \to \bar{l}} W_{22}$$

It follows that, when $\eta < 1/2$ (Proposition 5 case (2)), it is optimal to set $l_0 = \bar{l}$. Focusing on (15), (16) and (22):

$$\lim_{t \to +\infty} e^{-\theta t} \lambda h = \lim_{t \to +\infty} \frac{\omega \eta}{B} \bar{l}^{\eta(1-\varepsilon)-1} h_0^{(1-\eta)(1-\varepsilon)} e^{B\left(\frac{\eta-1}{\eta}\right)\bar{l}t} = 0$$

The transversality condition is satisfied in this case. The previous discussion lead to the following proposition.

Proposition 7 *Assume $\eta < 1/2$ such that:*
(1) If $\varepsilon < 1$,

$$\theta\left(\frac{\eta}{1-\eta}\right) < B < \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

(2) If $\varepsilon > 1$,

$$B > \theta\left(\frac{\eta}{1-\eta}\right)$$

then, the optimal trajectory for this economy is given by:

$$\begin{aligned} l(t) &= l_0 = \bar{l} \\ h(t) &= h_0 e^{B\left(1-\bar{l}\right)t} \end{aligned}$$

Along the optimal trajectory,

$$\frac{\dot{h}}{h} = \frac{\dot{Y}}{Y} = \frac{\dot{c}}{c} = \frac{\dot{P}}{P} = B\left(1-\bar{l}\right)$$

Among all possible trajectories described in the previous section, the last proposition concludes that the optimal trajectory consists to jump on the balanced growth path, that is, to choose $l_0 = \bar{l}$. Even if, as in Stockey (1998), an EKC trajectory leading to a steady state is possible, conversely to Stockey (1998), this trajectory is not optimal. There are two explanations for which our result sharply differs from the one pointed out by Stockey (1998). First of all, introduction of emissions standards concerning pollution in Stockey (1998) reduces the rate of return on capital, excluding the possible occurrence of endogenous growth. In our framework à la Lucas (1988), endogenous growth comes from the linearity of human capital accumulation (8) and this property is not affected by pollution which explains why endogenous growth is possible. Moreover, remark that the only case where it is possible to describe the optimal behavior of the economy is when $\eta < 1/2$. That is, a case where the pollution effect on utility is moderate. In this sense, we recover the intuition of Tahvonen and Kuuluvainen (1991), endogenous growth requires that the positive externality (linearity of the human capital accumulation) dominates the negative externality (moderate pollution effect on utility). Finally, it is interesting to observe that endogenous growth is possible and optimal whatever the pollution effect on marginal utility of consumption ($\varepsilon \gtrless 1$) while in a model à la Romer (1986), Michel and Rotillon (1995) have stressed that endogenous growth requires a (strong) *compensation effect* (i.e. $\varepsilon > 1$). In their paper, the utility function is not specified and then, it is difficult to modulate the effect of the negative externality. In our model, as discussed just before, the magnitude of the negative externality is driven by η. If $\eta = 0$, pollution no longer affects the economy and then, we recover Lucas (1988) which implies optimality of endogenous growth in the long run. By continuity, it is not surprising to observe also optimal endogenous growth for low values of η, namely $\eta < 1/2$, even if preferences are driven by a *distaste effect*.

5 Conclusion

The following paper has developed a simple environmental extension of the Lucas (1998) framework. More precisely, a representative household arbitrates between the time devoted to accumulate human capital and the time devoted to depollute. Considering human capital as the only productive input, we have computed all possible trajectories for this simple economy. One of them depicts an inverted U-shape relationship between income and pollution, indicating the possible existence of an EKC. After discussing the optimal initial value for the control variable, we conclude that the optimal trajectory follows exactly the balanced growth path, excluding the EKC. This conclusion completes the literature in two respects. First, it proposes a completely reverted conclusion than the one obtained by Stockey (1998) since in her paper, there is no room for endogenous growth while the optimal trajectory follows an EKC and leads to a steady state. One of the main explanation for this surprising outcome is that introduction of pollution in the AK framework in Stockey (1998) reduces the rate of return on capital, excluding endogenous growth in the long run. Conversely, the linearity of the process of human capital accumulation in our framework à la Lucas (1988) is not affected by pollution and then, endogenous growth is preserved in the long run. The second interesting result is that endogenous growth is possible in our, whatever the pollution effect on the marginal utility of consumption. This result is surprising since Michel and Rotillon (1995) have stressed that endogenous growth requires a strong positive pollution effect on marginal utility of consumption (compensation effect) in a model à la Romer (1986). The main explanation for this second surprising outcome of our model is that the optimal trajectory is obtained in the special case where pollution moderately affects the household's utility, indicating that the negative externality (pollution) is dominated by the positive externality (human capital accumulation).

6 Appendix

Proof of proposition 1

First of all, considering jointly (1), (3), (4) and (5), it appears that:

$$u(c,P)=\omega\frac{l^{\eta(1-\varepsilon)}h^{(1-\eta)(1-\varepsilon)}}{1-\varepsilon}$$

where $\omega\equiv\left(A(\alpha A)^{-\eta}\right)^{1-\varepsilon}>0$.

To solve the program (6), we apply the Pontryagin's maximum principle. In particular, we follow Seierstad and Sydsaeter (1987, Theorem 12, p. 234)[9]. The current value Hamiltonian is given by:

$$H=\omega\frac{l^{\eta(1-\varepsilon)}h^{(1-\eta)(1-\varepsilon)}}{1-\varepsilon}+\lambda B(1-l)h$$

λ is the Lagrangian multiplier. First order conditions writes:

$$\begin{aligned}
\frac{\partial H}{\partial l} &= \omega\eta l^{\eta(1-\varepsilon)-1}h^{(1-\eta)(1-\varepsilon)}-\lambda Bh=0 && (19)\\
\frac{\partial H}{\partial h} &= \omega(1-\eta)l^{\eta(1-\varepsilon)}h^{(1-\eta)(1-\varepsilon)-1}+\lambda B(1-l)=\theta\lambda-\dot{\lambda} && (20)\\
\frac{\partial H}{\partial \lambda} &= B(1-l)h=\dot{h} && (21)
\end{aligned}$$

Jointly with the transversality condition:

$$\lim_{t\to+\infty}e^{-\theta t}\lambda h=0$$

Consider (19),

$$\begin{aligned}
\lambda &= \frac{\omega\eta}{B}l^{\eta(1-\varepsilon)-1}h^{(1-\eta)(1-\varepsilon)-1} && (22)\\
\dot{\lambda} &= \lambda\left([\eta(1-\varepsilon)-1]\frac{\dot{l}}{l}+[(1-\eta)(1-\varepsilon)-1]\frac{\dot{h}}{h}\right) && (23)
\end{aligned}$$

Considering (22), and (21), it follows that (20) writes:

$$\dot{\lambda}=\left(\theta-B\left[1+\left(\frac{1-2\eta}{\eta}\right)l\right]\right)\lambda \tag{24}$$

Injecting (23) into (24), we obtain after rearranging:

$$\dot{l}=B\left(\frac{1-\eta}{\eta}\right)l^2+\frac{\theta-B(1-\eta)(1-\varepsilon)}{\eta(1-\varepsilon)-1}l$$

■

Proof of proposition 2

[9] The reader can also refer to Acemoglu (2009, Theorem 7.13, p.254).

Following Arrow and Kurz (1970), the set of necessary conditions in proposition 1 are sufficient if the maximized Hamiltonian $\tilde{H}$ with respect to the control variable l is concave with respect to the state variable h. Considering (19),

$$l = \left(\frac{\lambda B}{\omega \eta}\right)^{\frac{1}{\eta(1-\varepsilon)-1}} h^{\frac{1-(1-\eta)(1-\varepsilon)}{\eta(1-\varepsilon)-1}} \equiv \tilde{l}\left(\lambda, h\right)$$

implying:

$$\frac{h}{\tilde{l}} \frac{\partial \tilde{l}}{\partial h} = \frac{1-(1-\eta)(1-\varepsilon)}{\eta(1-\varepsilon)-1}$$

and then,

$$\tilde{H} = \frac{\omega}{1-\varepsilon} \tilde{l}^{\eta(1-\varepsilon)} h^{(1-\eta)(1-\varepsilon)} + \lambda B \left(1-\tilde{l}\right) h$$

That is,

$$\begin{aligned}
\frac{\partial \tilde{H}}{\partial h} &= \omega \frac{2\eta-1}{\eta(1-\varepsilon)-1} h^{(1-\eta)(1-\varepsilon)-1} \tilde{l}^{\eta(1-\varepsilon)} + \lambda B \left[1 - \frac{(1-\varepsilon)(2\eta-1)}{\eta(1-\varepsilon)-1}\tilde{l}\right] \\
\frac{\partial^2 \tilde{H}}{\partial h^2} &= \omega \frac{2\eta-1}{\eta(1-\varepsilon)-1} \left[\frac{\eta(\varepsilon-1)-\varepsilon}{\eta(\varepsilon-1)+1}\right] h^{(1-\eta)(1-\varepsilon)-2} \tilde{l}^{\eta(1-\varepsilon)} \\
&\quad -\lambda B \frac{\tilde{l}}{h} \frac{(1-\varepsilon)(2\eta-1)}{\eta(1-\varepsilon)-1} \frac{1-(1-\eta)(1-\varepsilon)}{\eta(1-\varepsilon)-1}
\end{aligned}$$

Considering (22), it follows that:

$$\frac{\partial^2 \tilde{H}}{\partial h^2} = \omega \tilde{l}^{\eta(1-\varepsilon)} h^{(1-\eta)(1-\varepsilon)-2} \left[(2\eta-1) \frac{\varepsilon - \eta(\varepsilon-1)}{1+(\varepsilon-1)\eta}\right]$$

That is,

$$\frac{\partial^2 \tilde{H}}{\partial h^2} < 0 \Longleftrightarrow (2\eta-1) \frac{\varepsilon-\eta(\varepsilon-1)}{1+(\varepsilon-1)\eta} < 0$$

Case 1: *Distaste effect* $(\varepsilon < 1)$

Interestingly, in this case, $\varepsilon - \eta(\varepsilon-1) > 0$. Moreover, $1+(\varepsilon-1)\eta > (<) 0$ if and only if $\eta < (>) 1/(1-\varepsilon)$. In this case, concavity requires $2\eta - 1 < (>) 0$, that is $\eta < (>) 1/2$. Interestingly, since $\varepsilon < 1$, then $1/(1-\varepsilon) > 1$. That is, $\eta < 1/2$ implies $\eta < 1/(1-\varepsilon)$ while $\eta > 1/(1-\varepsilon)$ implies $\eta > 1/2$.

Case 2: *Compensation effect* $(\varepsilon > 1)$

Interestingly, in this case, $1+(\varepsilon-1)\eta > 0$. Moreover, $\varepsilon - \eta(\varepsilon-1) > (<) 0$ if and only if $\eta < (>) \varepsilon/(\varepsilon-1)$. In this case, concavity requires $2\eta-1 < (>) 0$, that is $\eta < (>) 1/2$. Interestingly, since $\varepsilon > 1$, then $\varepsilon/(\varepsilon-1) > 1$. That is, $\eta < 1/2$ implies that $\eta < \varepsilon/(\varepsilon-1)$ while $\eta > \varepsilon/(\varepsilon-1)$ implies $\eta > 1/2$. ■

Proof of proposition 3

From parameter spaces described in (9) and (10), let us consider 4 cases:

(a) $\varepsilon < 1$ jointly with $\eta < 1/2$.
(b) $\varepsilon < 1$ jointly with $\eta > 1/(1-\varepsilon)$.
(c) $\varepsilon > 1$ jointly with $\eta < 1/2$.
(d) $\varepsilon > 1$ jointly with $\eta > \varepsilon/(\varepsilon - 1)$.

In what follows, we discuss conditions for which $\bar{l} \in (0,1)$ in each of those 4 cases.

$$\bar{l} = \frac{\eta}{B(\eta-1)} \frac{\theta - B(1-\eta)(1-\varepsilon)}{(1-\varepsilon)\eta - 1}$$

Focus on case (a). It is obvious that

$$\frac{\eta}{B(\eta-1)} < 0 \text{ and } (1-\varepsilon)\eta - 1 < 0$$

and then, $\bar{l} > 0$ if and only if:

$$B < \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

Moreover, $\bar{l} < 1$ if and only if:

$$\frac{\theta\eta - B(1-\eta)}{B(\eta-1)((1-\varepsilon)\eta-1)} < 0 \Leftrightarrow \theta\left(\frac{\eta}{1-\eta}\right) < B$$

That is, in case (a), $\bar{l} \in (0,1)$ if and only if:

$$\theta\left(\frac{\eta}{1-\eta}\right) < B < \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

Interestingly,

$$\frac{\theta}{(1-\eta)(1-\varepsilon)} - \theta\left(\frac{\eta}{1-\eta}\right) = -\theta\frac{(1-\varepsilon)\eta - 1}{(1-\varepsilon)(1-\eta)} > 0$$

and then, the range $\left(\theta\left(\frac{\eta}{1-\eta}\right), \frac{\theta}{(1-\eta)(1-\varepsilon)}\right)$ is not empty.

Focus on case (b). It is obvious in this case that $\bar{l} < 1$ if and only if:

$$B < \theta\left(\frac{\eta}{1-\eta}\right) < 0$$

which is impossible. It follows that $\bar{l} > 1$ in case (b).

Focus on case (c). In this case, it is obvious that $\bar{l} > 0$. Moreover, $\bar{l} < 1$ if and only if:

$$\frac{\theta\eta - B(1-\eta)}{B(\eta-1)((1-\varepsilon)\eta-1)} < 0 \Leftrightarrow \theta\left(\frac{\eta}{1-\eta}\right) < B$$

That is, since $\varepsilon > 1$ jointly with $\eta < 1/2$, then, $\bar{l} \in (0,1)$ if and only if:

$$B > \theta\left(\frac{\eta}{1-\eta}\right)$$

Focus on case (d). In this case, it obvious that $\bar{l} > 0$ if and only if:

$$B > \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

Moreover, $\bar{l} < 1$ if and only if:

$$\frac{\theta\eta - B(1-\eta)}{B(\eta-1)((1-\varepsilon)\eta-1)} < 0$$

Which is always true.

It follows that, since $\varepsilon > 1$ jointly with $\eta > \varepsilon/(\varepsilon-1)$, then, $\bar{l} \in (0,1)$ if and only if:

$$B > \frac{\theta}{(1-\eta)(1-\varepsilon)}$$

■

Proof of proposition 4

Simply remark that:

$$\varphi'\left(\bar{l}\right) = B\left(\frac{1-\eta}{\eta}\right)\bar{l} = -\varphi'(l^*) \tag{25}$$

Following proposition 3, cases (1.1), (1.2) and (2) ensure that $\bar{l} \in (0,1)$. That is, considering (25), the sign of $\varphi'\left(\bar{l}\right)$ is fully determined by the sign of $1-\eta$. In cases (1.1) and (1.2), $1-\eta > 0$ while, $1-\eta < 0$ in case (2). The proposition follows. ■

Proof of proposition 6

From (4):

$$\frac{\dot{P}}{P} = \frac{\dot{h}}{h} - \frac{\dot{l}}{l}$$

and then,

$$\frac{\dot{P}}{P} > 0 \Leftrightarrow \frac{\dot{h}}{h} > \frac{\dot{l}}{l}$$

Focus on case (2.2) of proposition (5), that is, $\eta < 1/2$ jointly with $l_0 > \bar{l}$. From (7) and (8),

$$\frac{\dot{P}}{P} > 0\,(<0) \Leftrightarrow \frac{\dot{h}}{h} > (<)\,\frac{\dot{l}}{l} \Leftrightarrow l < (>)\,\tilde{l}$$

Moreover, within case (2.2) of proposition (5), we know that:

$$B > \theta\left(\frac{\eta}{1-\eta}\right)$$

This ensures that $\bar{l} < \tilde{l} < 1$.

Focusing on (17), since $l_0 > \bar{l}$, $l(t) = \tilde{l}$ at $t = \tilde{t}$ such that:

$$\tilde{t} = \frac{\eta}{B(\eta - 1)\bar{l}} \ln \left(\frac{\tilde{l}}{l_0} \frac{l_0 - \bar{l}}{\tilde{l} - \bar{l}} \right)$$

Interestingly, $\eta < 1$ jointly with $l_0 < \tilde{l}$ ensures that $\tilde{t} > 0$.

As observed through proposition 4, since $\eta < 1/2$ and $l_0 > \bar{l}$, l increases from l_0 to 1. That is, choose l_0 such that $\bar{l} < l_0 < \tilde{l}$ implies that P increases, up to the date where $l = \tilde{l}$ (i.e. $t = \tilde{t}$) and then, decreases up to the date where $l = 1$ (i.e. $t = t^*$). Moreover, from (8), $\forall l < 1$, $\dot{h} > 0$. That is, an EKC occurs: first, both P and h increases and then, P decreases while h continues to increase. At the end, $l = 1$ at $t = t^*$ and the economy reaches a steady state such that $h = h(t^*)$ and $P = \alpha A h(t^*)$. It is interesting to remark that this EKC trajectory meet the transversality condition since the economy converges toward a steady state. ■

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