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TESTING THE WEAK FORM EFFICIENCY OF THE FRENCH ETF MARKET WITH LSTAR-ANLSTGARCH APPROACH USING A SEMIPARAMETRIC ESTIMATION

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Abstract. In this paper, we consider the daily Xtrackers CAC 40 UCITS from 2009 to 2020 for the analysis as it is supposed to capture more information compared to other French stock markets. After application of different statistical tests including BDS test, Hinich bispectrum test, Tsay test for linearity, long memory test and automatic serial correlation tests, we try to test the weak form efficiency of French ETF market through a logistic smooth transition AR model with nonlinear asymmetric logistic smooth transition GARCH errors using semiparametric maximum likelihood where the innovation distribution is replaced by a nonparametric estimate based on the kernel density function. After analyzing the forecasting results, we show that the price fluctuations appear as the result of transitory shocks and the predictions provided by the LSTAR-ANSTGARCH model are better than those of other models for some time horizons. The predictions from this model are also better than those of the random walk model; accordingly, the XCAC 40 price is not weak form of efficient market for the entire period because its successive return are nonlinearly dependent and doesn't generate randomly.

Keywords: *LSTAR model, ANLSTGARCH model, semiparametric maximum likelihood, nonlinearity, informational shocks, kernel, bandwidth, market efficiency*

JEL Classification: C14, C12, C22, C58, G14

INTRODUCTION

Financial market efficiency is certainly one of the most discussed theories in the financial field. The financial market efficiency hypothesis states that the current prices reflect all available information about the actual value of the underlying assets. However, following the different past crises, there has been a disconnection between the stock price and its fundamental value (Lardic & Mignon, 2006;, Huber et al., 2008; Colmant et al., 2009). The asset prices don't reflect the best estimate of agents in the market The idea is based on the importance of predicting future prices and their ability to reflect immediately all available relevant information. In other words, the future stock returns have some predictive relationships with the available information of present and historical stock returns. In this case, the n*onlinear models are known to be efficient for financial time series forecasting (*Franses et al., 2000; Kyrtsou & Terraza, 2003; Antwi et al., 2019; Ouyang et al., 2020).

The interest in nonlinear time series models has been increasing. The presence of nonlinearity in stock price series has important implications for informational efficiency. Indeed, if a series exhibits nonlinear structure, this implies significant nonlinear dependencies between the observations (Chikhi & Bendob, 2018). In applications to financial time series, models, which allow for regime-switching behavior have been most popular, especially the class of smooth transition autoregressive (LSTAR) models, introduced by Teräsvirta (1994). A lot of work in this area has been devoted to estimation, specification, testing and applications such as forecasting (Potter, 1999; Van Dijk et al., 2002b; Wahlström, 2004; Chikhi & Diebolt, 2009b; Adebile & Shangodoyin, 2006; Umer et al., 2018). Smooth transition models, which justify the sources of non-linearity, may be appropriate to provide a privilege framework for the study of asymmetric stock market fluctuations.

For the history and applications of the STAR model to economic and financial time series see, for example, Granger & Teräsvirta (1993) & Teräsvirta (1994) who classify market into two phases of recession and expansion. Thus, Teräsvirta & Anderson (1992) use STAR model to predict quarterly OECD industrial production series. Skalin & Teräsvirta (2002) study nonlinearity in business cycle using the model and Baum et al. (1999) and Liew et al. (2004) in real exchange rates. Sarantis (1999) detects nonlinearities in real effective exchange rates for 10 major industrialized countries and evaluates forecast accuracy of STAR model over the random walk model. Eitrheim & Teräsvirta (1996) introduce LM tests for the hypothesis of no error autocorrelation, for the hypothesis of no remaining nonlinearity and that of parameter constancy to evaluate the specification of STAR model. Acemoglu & Scotts (1994) study the relation between business cycle-regimes and nonlinearity in the UK labour market. Öcal (2000) applies STAR model to test the nonlinearities in growth rates of UK macroeconomic time series. Escribano & Jordá (2001) investigate the selection of STAR model by varying some of the parameters and conditions in the models. Wahlström (2004) compares forecasts from the LSTAR model to those from a linear autoregressive model. In turn, Chikhi & Diebolt (2009b) analyze the cyclical behavior of the German annual aggregate wage earnings using LSTAR model. Zhou (2010) evaluates the STAR model in the presence of structural break in industrial production index of Sweden. Mourelle, Cuestas & Gil-Alana (2011), Shittu & Yaya (2011) and Yaya (2013) apply STAR model to Nigerian inflation series. Tayyab, Tarar & Riaz (2012) evaluate the suitability of the STAR model specification for real exchange rate Modeling**.** Adebile & Shangodoyin. (2006) propose an alternative representation of the original version of the logistic STAR model. Whereas, Umer, Sevil & Sevil (2018) compare the performance of STAR and linear AR models using monthly returns of Turkey and FTSE travel and leisure index. Aliyev (2019) examines the efficiency of Turkish stock market using the STAR model and evaluates its forecasting performance. For review of threshold time series models in finance, see also (Chen et al., 2011).

The limitation of these works is that they don't capture the nonlinearity structure in the conditional variance. The assumption of white noise on the LSTAR model residuals ignores the presence of conditional heteroskedasticity; however, the financial series are generally characterized by a time-varying volatility that can be modeled by ARCH-type models (Engle, 1982; Bollerslev, 1986) that is often used to study the behavior of asset returns or innovations of the 'parent' model. Franses et al. (1998) and Lundbergh & Terräsvirta (1999, 2000) combine the Smooth Transition

Autoregressive (STAR) models (Granger & Teräsvirta, 1993; Teräsvirta, 1994) with GARCH errors (Bollerslev, 1986) and with the Smooth Transition GARCH errors (Hagerud, 1997; Gonzalez-Rivera, 1998). Thus, some authors have used the STGARCH or the STAR-STGARCH to study empirically financial time series. Concerning the STGARCH model, several authors have introduced these specifications (Hagerud, 1997; Gonzalez-Rivera, 1998; Anderson et al., 1999; Medeiros & Veiga, 2009) to model the impact of news on volatility. Lubrano (2001) estimates the STGARCH model using a Bayesian approach. Yaya & Shittu (2016) test nonlinearity and asymmetry in the volatility of bank share price using the STGARCH. Regarding the STAR-STGARCH modeling, Chan et al (2002) analyze trends in the development of more ecological-friendly technologies using STAR-GARCH model. Chan & McAleer (2003) compare algorithms for quasi-maximum likelihood estimation of STAR-STGARCH model in the presence of extreme observations and outliers. Reitz & Westerhoff (2007) use the STAR-GARCH model to study the impact of heterogeneous speculators on commodity market. Pavlidis et al (2010) examine the impact of conditional heteroskedasticity and investigate the performance of different heteroskedasticity robust versions. Guo & Cao (2011) include asymmetry effects in the transition dynamics of STGARCH model. Chan & Theoharakis (2011) estimate m-regimes STAR-GARCH model using quasi-maximum likelihood with parameter transformation. Ben Haj Hamida & Haddou (2014) study exchange-rate dynamics for the Maghreb countries using the STAR-STGARCH model. Midilic (2016) applies the STAR-GARCH model using Iteratively Weighted Least Squares (IWLS) algorithm to forecast daily US Dollar/Australian Dollar and FTSE Small Cap index returns. Livingston & Nur (2018) use the Bayesian inference for the smooth transition autoregressive STAR–GARCH models. Finally Bildirici, Bayazit & Ucan (2020) suggest the Logistic Smooth Transition Generalised Autoregressive Conditional Heteroskedasticity long-short term memory (LSTARGARCHLSTM) method to analyze the volatilities of WTI, Brent and Dubai crude oil prices under the influence of the COVID-19 pandemic and the concurrent oil conflict between Russia and Saudi Arabia.

Some authors assume that the innovations follow the Normal distribution, which cannot accommodate fat-tailed properties frequently existing in financial time series. Many studies indicate that this problem can lead to inconsistent estimates. The Student's t-distribution and General Error Distribution can be the two most popular alternatives with the intension to capture the heavy-tailed returns. In this case, the density function is known and the maximum likelihood estimator of GARCH parameters can be obtained parametrically under regularity conditions (see Gonzalez-Rivera & Drost, 1999; Francq & Zakoian, 2004). However, in most cases, the innovation distribution is unknown and often replaced by a nonparametric estimate and thus the estimation procedure becomes semiparametric (see Pagan & Ullah, 1999; Newey & Steigerwald, 1997; Mukherjee, 2006; Di & Gangopadhyay, 2014). This approach assumes a nonparametric form of the density function (Engle & Gonzalez-Rivera, 1991; Drost & Klaassen, 1997) and avoids the inaccuracy of its incorrect specification and improves the estimation efficiency (Gonzalez-Rivera & Drost, 1999; Berkes & Horvath, 2004).

Our research, in contrast to studies that use parametric distributions, employs nonparametric maximum likelihood method to estimate semi-parametrically our model. We apply this technique to test weak form market efficiency, examine informational shocks and describe the nonlinear dynamics in the Xtrackers CAC 40 UCITS that covers the French ETF market using LSTAR-ANLSTGARCH model. We test the short-term predictability of the traded asset (XCAC) and the weak-form inefficiency of French ETF market with limited rationality, which emerges arbitrage opportunities. We apply different statistical tests including BDS, long memory, Hinich bispectrum and Tsay tests, After that, we examine the martingale difference hypothesis (MDH) using the automatic portmanteau (AQ) test of Escanciano and Lobato (2009).

The paper is structured as follows. Next section focuses on presentation of the LSTAR-ANLSTGARCH model and its semiparametric estimation. Section 3 outlines the daily XCAC price data and discusses its statistical properties. Section 4 is devoted to semiparametric modelling of the daily return series of XCAC; we compare the predictive quality of AR-GARCH, LSTAR-GARCH and LSTAR-ANLSTGARCH models with that of a random walk. The last section concludes the study outlining our findings.

1. ESTIMATION METHODOLOGY AND DATA

1.1. Methodology: The LSTAR-ANLSTGARCH specification and semiparametric estimation

We consider a logistic smooth transition autoregressive model (with two regimes) with asymmetric nonlinear logistic smooth transition GARCH errors, called LSTAR-ANLSTGARCH (Chan & Mcaleer, 2003) given as

$$\begin{aligned} Y_t = &\left(\phi_{10} + \sum_{i=1}^{p} \phi_{1i} Y_{t-i}\right) \times \left(1 - G\left(Y_{t-d}; \gamma_{mean}, c_{mean}\right)\right) \\ &+ \left(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} Y_{t-i}\right) \times G\left(Y_{t-d}; \gamma_{mean}, c_{mean}\right) + \varepsilon_t \end{aligned} \tag{1}$$

with $$\varepsilon_t = u_t \sigma_t, \ \sigma_t > 0, \ u_t \sim iid(0,1) \tag{2}$$

and

$$\begin{aligned} \sigma_t^2 = &\left(w_{10} + \sum_{k=1}^{q} \alpha_{1k} \varepsilon_{t-k}^2 + \sum_{j=1}^{r} \beta_{1j} \sigma_{t-j}^2\right) \times \left(1 - H\left(\varepsilon_{t-1}; \gamma_{vol}, c_{vol},\right)\right) \\ &+ \left(w_{20} + \sum_{k=1}^{q} \alpha_{2k} \varepsilon_{t-k}^2 + \sum_{j=1}^{r} \beta_{2j} \sigma_{t-j}^2\right) \times H\left(\varepsilon_{t-1}; \gamma_{vol}, c_{vol},\right) \end{aligned} \tag{3}$$

where ϕ_{10}, ϕ_{20} are the constants and ϕ_{1i}, ϕ_{2i}, $i = 1,...,p$ are the autoregressive coefficients of order p. The parameters and the conditions of existence of classical GARCH specification hold for the ANSTGARCH model, which realize smooth changing dynamics (Yaya & Shittu, 2016). The logistic form of the two transition functions $G\left(Y_{t-d}; \gamma_{mean}, c_{mean}\right)$ and $H\left(\varepsilon_{t-1}; \gamma_{vol}, c_{vol}\right)$ causes the nonlinear dynamics in both the conditional mean and the conditional variance equations, given as (Bildirici & Ersin, 2015)

$$G\left(Y_{t-d};\gamma_{mean},c_{mean}\right)=\left[1+\exp\left(-\gamma_{mean}\left(Y_{t-d}-c_{mean}\right)\right)\right]^{-1},\ \gamma_{mean}>0 \tag{4}$$

$$H\left(\varepsilon_{t-1};\gamma_{vol},c_{vol}\right)=\left[1+\exp\left(-\gamma_{vol}\left(\varepsilon_{t-1}-c_{vol}\right)\right)\right]^{-1},\ \gamma_{vol}>0 \tag{5}$$

To avoid identification problems in both the conditional mean and the conditional variance equations, the slope parameters γ_{mean} and γ_{vol}, which determine the speed of transition function, are strictly positive with $\gamma_{mean}=1,...,100$. c_{mean} and c_{vol} are the threshold parameters. The two logistic functions $G\left(Y_{t-d};\gamma_{mean},c_{mean}\right)$ and $H\left(\varepsilon_{t-1};\gamma_{vol},\mathrm{c}_{vol}\right)$ are twice differentiable continuous functions bounded between $[0,1]$ lower and upper bounds for different values of the transition variables Y_{t-d} and ε_{t-1} and their distance to the thresholds c_{mean} and c_{vol} with $d=1,2,...,p$. Bildirici & Ersin (2015) observe that the transition is relatively slow for low values of the slope parameters γ_{mean} and γ_{vol}, though the transition between regimes speeds up as γ_{mean} and γ_{vol} take larger values. It is noted that the Asymmetric nonlinear Logistic STGARCH process, developed by Anderson et al (1999) and Nam et al (2002), generalizes the LSTGARCH model introduced by Hagerud (1997) and Gonzalez-Rivera (1998). Then, for positive variance in the ANLSTGARCH model, it is required that $w_{10}>0$, $\alpha_{1k}\geq 0$, $\beta_{1j}\geq 0$, $w_{10}+w_{20}>0$, $\alpha_{1k}+\alpha_{2k}>0$ and $\beta_{1j}+\beta_{2j}>0$. If $\gamma_{vol}=0$ the transition function $H(.)$ is equal to 0.5 and hence the asymmetric nonlinear LSTGARCH model reduces to a single-regime GARCH model.

Financial time series are often characterized by non-Gaussian distributions. Diverse quasi maximum likelihood methods based on many assumptions on the error distribution have been studied in the literature but the true error distribution is unknown. However, the shape parameter of the density function is often incorrect. This leads to estimate non-parametrically the density function (Di & Gangopadhyay, 2014).

Let $\theta=(\theta_{mean},\theta_{vol})$ be the parameter vector of models (1) and (3) where $\theta_{mean}=(\phi_{10},\phi_{20},\phi_{11},..,\phi_{1p},\phi_{21},..,\phi_{2p},\gamma_{mean},c_{mean})'$ is the parameter vector of conditional mean equation and $\theta_{vol}=(\alpha_{11},..,\alpha_{1q},\alpha_{21},..,\alpha_{2q},\beta_{11},..,\beta_{1r},\beta_{21},..,\beta_{2r},w_{10},w_{20},\gamma_{vol},c_{vol})'$ is that of conditional variance equation. The vector θ is a suitable compact set in $R^{2p+2q+2r+8}$. We define the semiparametric kernel density function based on θ (see Di & Gangopadhyay, 2011)

$$\hat{f}(z)=\frac{1}{Th}\sum_{t=1}^{T}K\left(z-u_t(\hat{\theta})\right)/h \tag{6}$$

where $u_t(\hat{\theta})=\varepsilon_t/\sigma_t(\theta)$. $K(.)$ and h represent the kernel and the bandwidth, respectively. The semiparametric likelihood function at θ can be defined as

$$L(\theta) = \frac{1}{T}\sum_{t=1}^{T}\log\left[\frac{1}{\sigma_t(\theta)} f\left(\frac{\varepsilon_t(\theta)}{\sigma_t(\theta)}\right)\right] \tag{7}$$

with any initial estimate of the parameter vector θ, we can apply the two-step estimation procedure and derive a semiparametric estimate of θ, which is given by

$$\hat{\theta}_{SMLE} = \arg\min_{\theta\in\Theta} \hat{L}(\theta)$$

If $h \to 0$ and $Th^4 \to \infty$, the initial estimator is $\sqrt{T}$ – consistent and $Th \to \infty$ to insure the consistency of the kernel density estimator (see Härdle, 1990; Di & Gangopadhyay; 2014 for details on the consistency of the kernel density estimate and its derivatives and the asymptotic properties of semiparametric maximum likelihood).

1.2. Data description and statistical properties

The data used in this paper consists of the daily closing Xtrackers CAC 40 UCITS price that covers the French ETF market downloaded from https://www.investing.com covering a historical period from February 12, 2009 to October 30, 2020 including 2849 observations. In order to better understand the characteristics of the XCAC40 series, it is necessary to examine some descriptive statistics.

Figure 1 presents time series plots for our studied daily closing Xtrackers CAC 40 UCITS index. The data are transformed into logarithm form. As is usual in financial time series, the logarithmic CAC40 series contain a unit root*. Our series is therefore differentiated to obtain the daily percentage Xtrackers CAC 40 returns at time *t* (see also Figure 1)

$$r_t = 100 \times \ln\left(XCAC_t / \mathrm{XCAC}_{t-1}\right)$$

2.

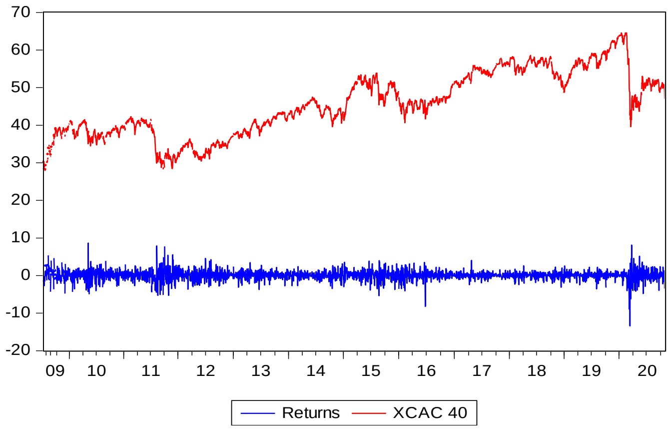

Fig. 1. Time series plots for daily French ETF market index and returns

The descriptive statistics of daily XCAC 40 return series in Table 1 reveal that the average return is positive and the French ETF market exhibits the high risk degree as measured by the standard deviation (134.289%). On the other hand, the series is

* The results of the unit root tests are not reported here but are available on request.

negatively skewed and the data are asymmetric in nature. The value of kurtosis is greater than 3, indicating leptokurtic and a more peaked distribution. as it is known in financial time series, the Jarque-Bera test (Jarque & Bera, 1987) confirms the non-normality of the distribution. Rejection of normality partially reflects the nonlinear dependencies in the moments of returns series. The ARCH-LM test result thus shows that XCAC 40 returns are characterized by the presence of ARCH effect.

Table 1. Summary statistics for daily Xtrackers CAC 40 returns

Mean	Std. Dev. (%)	Skewness	Kurtosis	JB	ARCH(1)
0.0158	134.289	-0.468	10.814	7350.285 (0.000)	41.804 (0.000)

Notes: Tests are performed by the authors using Eviews 9.0 software. (.): The p-Value. We reject the assumption of normality H_0 because the Jarque-Bera (JB) statistic is greater than the critical value of chi-square distribution with 2 degrees of freedom at 1%. Moreover, we reject the homoscedasticity assumption H_0 (There is an ARCH effect in the data because the ARCH-LM statistic is greater than the critical value of chi-square distribution with 1 degree of freedom at 1%.).

In order to test the existence of a nonlinear structure in Xtrackers CAC 40 stock returns and detect the *nonlinear behavior* of volatility, we use the Hinich bispectrum test (Hinich & Patterson, 1989) for linearity and Gaussianity and the Tsay test for neglected nonlinearities (Tsay, 2001; Tiao & Tsay, 1994; Luukkonen et al., 1988). In view of Table 2, the Gaussianity and the linearity statistics are strictly greater than the critical value of standard normal and that of chi-square distribution at 5%, with two degrees of freedom, respectively. The null hypothesis of linearity and Gaussianity is strongly rejected for returns and volatility. In addition, the Tsay test, which can be considered for LST variant against STAR or TAR, confirms nonlinearity because the F-statistics are greater than the critical value at 5%. We find the presence of logistic smooth transition in the returns and volatility processes. It is thus due essentially to the large variance change in the time period.

Table 2. Hinich bispectrum and Tsay tests for linearity

Series	Hinich bispectrum test					Tsay test
	Frame Size	Lattice Points	Test Quantile	Linearity	Gaussianity	F_{Tsay}^{4}
Returns	53	169	0.8	654.134 (0.000)	113381.916 (0.000)	5.638 (0.000)
Volatility	53	169	0.8	646.035 (0.000)	111357.455 (0.000)	37.883 (0.000)

Notes: Tests are performed by the authors using RATS 9.20 software. The numbers in the table are nonparametric Hinich bispectral test statistics with the null hypothesis H_0 of linearity and Gaussianity, obtaining the chi-squared statistic for testing the significance of individual bispectrum estimates by exploiting its asymptotic distribution. The numbers in the parenthesis are critical probabilities. F_{Tsay}^{4} is the Tsay Ori-F test for neglected non-linearities in an autoregression. We test more specifically against STAR using 4 lags.

The BDS statistics presented in Table 3 strongly reject the *i.i.d* assumption, which gives a clear indication of the existence of nonlinear dependencies in XCAC 40 return series for all embedding dimensions. This test leads us to reject the *i.i.d* hypothesis but do not detect the presence of long-term dependencies. Given this situation, we test the presence of long memory. As it is observed from Table 4, test results for fractional integration show the evidence that the return series exhibits short memory, but it does not have the behaviour of ARMA. The memory parameter estimated by the Andrews-Guggenberger (Andrews & Guggenberger, 2003), Robinson-Henry (Robinson & Henry, 1998) and the GPH (Geweke & Porter-Hudak, 1983) methods is negative but not significant in all methods. The absence of a long memory indicates that agents can only anticipate their returns to a short time horizon. Indeed, the informational shocks have transitory effects on French ETF returns.

Table 3. BDS test results on the series of Xtrackers CAC40 returns

m	**BDS stat.**	**Prob.**
2	10.840	0.000
3	15.255	0.000
4	18.273	0.000
5	20.570	0.000

Notes: Tests are performed by the authors using Eviews 9.0 software. The BDS statistics are calculated by the fraction of pairs method with ε equal to 0.7. m represents the embedding dimension. The BDS statistics are strictly greater than the critical value at 5% for all the embedding dimensions.

Table 4. Results from the ARFIMA(0,d,0) estimation

	GPH	**Robinson-Henry**	**Andrews-Guggenberger**
$\hat{d}$	-0.008	-0.011	-0.126
t-stat.	-0.446	-0.833	-1.268

Notes: Tests are performed by the authors using Ox 7.20 and RATS 9.20 softwares. $\hat{d}$ is the estimated Long memory parameter with a power of 0.8.

We also present the results of the automatic variance ratio (AVR) test of Kim (2009), the automatic portmanteau (AQ) test of (Escanciano & Lobato, 2009) and the serial correlation test of Deo (2000), which is robust to conditional heteroskedasticity. These tests indicate whether or not rejects the weak-form efficiency for French ETF market. Table 5 summarizes the test statistics. The high degree of predictability and implicitly of inefficiency of French ETF market is observed. The automatic variance ratio test, the automatic portmanteau test and the robustified Box-Pierce test statistics suggest a surprising increase of the degree of inefficiency for all aggregation levels. Over the entire period, the martingale hypothesis is clearly rejected on the French ETF market at 0.05 significant level. Regarding the Deo's test statistic, we note that ETF returns are predictable in the short term and the French ETF market is inefficient. So, we reject the weak-form efficiency for French ETF market.

Table 5. Automatic serial correlation test results for daily Xtrackers CAC 40 returns

AVR stat.	AQ stat.	*Q1*	*Q5*	*Q10*	*Q15*	*Q20*	Deo's test stat
-9.609	4.115	4.115	14.526	18.585	27.920	32.681	34.268
(0.000)	(0.042)	(0.042)	(0.013)	(0.046)	(0.022)	(0.037)	(0.024)

Notes: Tests are performed by the authors using R software. (.): The p-Value. AVR stat. : Automatic variance ratio test of Kim (2009). AQ stat.: Automatic portmanteau test of Escanciano and Lobato (2009). Deo's test stat : Serial correlation test of Deo (2000).

2. EMPIRICAL RESULTS AND DISCUSSION

2.1. Estimation and diagnostics tests

In order to test the weak-form efficiency using the nonlinear framework, the modelling of XCAC 40 series could be turn towards smooth transition autoregressive models (Lukkonen et al., 1988; Teräsvirta & Anderson, 1992; Tiao & Tsay, 1994; Teräsvirta, 1994) which could be combined with asymmetric nonlinear smooth transition GARCH errors (Gonzalez-Rivera, 1996; Hagerud, 1997; Chan & McAller, 2003) using nonparametric maximum likelihood, where the innovation distribution is unknown and replaced by a nonparametric kernel density estimate (Pagan & Ullah, 1999; Di & Gangopadhyay, 2014). In practical terms, we estimate AR, LSTAR jointly with GARCH, GJR-GARCH, APGARCH and ANLSTGARCH models using semiparametric approach. Initially, we estimate the model to produce residuals. After that, we use these residuals to estimate the nonparametric kernel likelihood, which will be maximized to obtain the final estimate of the model parameters.

Firstly, we select the lags of linear AR in the conditional mean equation using the sum of squared residuals and the p-values. In this case, the maximum number of lags is restricted to be three and the largest lag to be considered is five. The second stage is to test linearity against LSTAR or ESTAR nonlinearity and select the optimal transition variable of the conditional mean equation by using minimum p-values of F test statistics of all candidate variables $\{r_{t-1}, r_{t-2}, ..., r_{t-5}\}$. In view of Table 6, we find that the sum of squared residuals is minimum for the variables $\{r_{t-1}, r_{t-2}, r_{t-3}\}$ and the F test rejects the linearity for the delays 1 and 2 in the significance degree of 1%. Either an LSTAR or ESTAR should cause rejection of linearity and rejection of H12. H12 is the appropriate statistic if ESTAR is the main hypothesis of interest. We show that H12 is accepted, but H03 is rejected, it means that the LSTAR model is appropriate.

Table 6. Lag selection of linear part and STAR type nonlinearity test results

Delay	STAR type nonlinearity					Lag selection of linear part	
	F-stat.	H01	H02	H03	H12	Selected lags	SSR
1	2.890 (0.034)	1.446 (0.229)	3.250 (0.071)	3.968 (0.046)	2.349 (0.095)	1,2,5	401.032
2	3.590 (0.013)	4.652 (0.031)	1.262 (0.261)	3.324 (0.036)	1.747 (0.186)	1,2,3	400.010
3	2.563 (0.053)	1.197 (0.273)	3.684 (0.055)	2.804 (0.094)	2.441 (0.087)	2,4,5	403.113
4	1.488 (0.215)	0.983 (0.321)	2.802 (0.094)	0.679 (0.409)	1.893 (0.150)	3,4,5	411.235
5	1.635 (0.179)	1.021 (0.312)	3.019 (0.082)	0.864 (0.352)	2.020 (0.132)	1,3,4	409.231

Notes: Tests are performed by the authors using RATS 9.20 and GAUSS 3.2 softwares. SSR : Sum of squared residuals. H01 is a test of the first order interaction terms only. H02 is a test of the second order interaction terms only. H03 is a test of the third order interaction terms only. H12 is a test of the first and second order interactions terms only. (.): The p-Value.

It is possible that the conditional variance is characterized by a nonlinear structure. The financial prices often exhibit nonlinear heteroscedastic behavior. For this reason, we first test the GARCH specification against the alternative of ANLSTGARCH. Table 7 shows that the volatility of the Xtrackers CAC 40 return series is adequately captured by the asymmetric nonlinear logistic smooth transition GARCH-type model. The values of the critical probabilities argue in favor of an ANLSTGARCH model. At this stage, we will study the conditional variance of CAC 40 returns by combining LSTAR model with ANLSTGARCH errors using nonparametric maximum likelihood.

Table 7. LM test for GARCH against the alternative of ANLSTGARCH

Model	LM
GARCH	3.628 (0.056)
ANLSTGARCH	154.087 (0.000)

Notes: (.): The critical probabilities.

It is well known in the literature that estimating the LSTAR-ANLSTGARCH parameters can be problematic due to computational difficulties using different optimization algorithms, which make Maximum Likelihood Estimator (MLE) difficult to obtain the best model in practice (see Dijk et al. 2002; Chan & McAleer, 2003).

Unusually, there has been very little research investigating the cause of the numerical difficulties in obtaining the parameter estimates and the number of regimes for LSTAR-ANLSTGARCH. This model has STAR type nonlinearity in both the conditional mean and variance and allows the smooth transitions between the regimes to be governed by a logistic function. Hence, we determine the differentiating characteristics of the volatility of the Xtrackers CAC 40.

Table 8. Estimation results

Parameters	**AR-GARCH**	**LSTAR-GARCH**	**LSTAR-ANLSTGARCH**
		Conditional mean	
$\hat{\phi}_{10}$	-	0.0004 (2.572)	-
$\hat{\phi}_{11}$	-0.037 (-1.980)	0.386 (10.170)	-0.031 (-1.12)
$\hat{\phi}_{12}$	-0.024 (-1.262)	0.257 (4.527)	0.936 (20.402)
$\hat{\phi}_{13}$	-	-	0.154 (3.421)
$\hat{\phi}_{20}$	-	-	-
$\hat{\phi}_{21}$	-	-	-
$\hat{\phi}_{22}$	-	0.003 (11.007)	-
$\hat{\phi}_{23}$	-	0.002 (1.947)	0.040 (1.117)
$\hat{\gamma}_{mean}$	-	0.078 (3.421)	0.047 (2.467)
$\hat{c}_{mean}$	-	10.540 (5.895)	0.068 (1.994)
		Conditional variance	
$\hat{w}_{10}$	0.023 (2.792)	0.000004 (15.703)	0.00001 (2.383)
$\hat{\alpha}_{11}$	0.105 (5.338)	0.126 (38.813)	0.077 (7.582)
$\hat{\beta}_{11}$	0.889 (46.46)	0.855 (339.886)	0.944 (62.531)
$\hat{\gamma}_{1}$	-	-	-
$\hat{\delta}$	-	-	-
$\hat{w}_{20}$	-	-	0.0008 (15.074)

$\hat{\alpha}_{21}$	-	-	0.093 (2.065)
$\hat{\beta}_{21}$	-	-	0.898 (3.283)
$\hat{\gamma}_{vol.}$	-	-	3.877 (9.520)
$\hat{c}_{vol.}$	-	-	-0.006 (-2.023)
$\hat{h}_{opt}$	0.118	0.118	0.118
L	-4451.486	-4346.271	-4255.148*
Schwarz	3.134	3.074	3.022+
HQ	3.130	3.063	3.011+
ARCH(1)	0.076 [0.781]	0.066 [0.796]	0.265 [0.606]

Notes: Model parameters are estimated by the authors using RATS 9.20 and Ox 7.20 softwares. [.]: The critical probability. The values in parentheses are the Student statistics. $\hat{\phi}_{10}$, $\hat{\phi}_{11}$, $\hat{\phi}_{12}$, $\hat{\phi}_{13}$, $\hat{\phi}_{14}$, $\hat{\phi}_{15}$, $\hat{\phi}_{20}$, $\hat{\phi}_{21}$, $\hat{\phi}_{22}$, $\hat{\phi}_{23}$ are the estimated LSTAR parameters. $\hat{\gamma}_{mean}$ is the estimated rate of transition (LSTAR). $\hat{c}_{mean}$ represents the estimated location variable of LSTAR model. $\hat{w}_{10}, \hat{\alpha}_{11}, \hat{\beta}_{11}, \hat{w}_{20}, \hat{\alpha}_{21}, \hat{\beta}_{21}$ are the estimated ANLSTGARCH parameters. $\hat{\gamma}_{vol.}$ is the estimated rate of transition (ANLSTGARCH). $\hat{c}_{vol.}$ is the estimated location variable of ANLSTGARCH model. L represents the estimated log-likelihood function. + indicates the optimal Schwarz (SC) and the optimal Hannan-Quinn (HQ). * indicates the maximum log-likelihood function. $\hat{h}_{opt}$ represents the optimal bandwidth for a kernel density estimator.

We estimate the AR-GARCH, LSTAR-GARCH and LSTAR-ANLSTGARCH models using a semiparametric version of maximum likelihood based on the Gaussian kernel with an optimal bandwidth. In view of Table 8, we find that the semiparametric log-likelihood function is maximum for the LSTAR-ANLSTGARCH model and most of its coefficients are generally significant. According to Schwarz and HQ information criteria (Schwarz, 1978; Hannan & Quinn, 1979), LSTAR-ANLSTGARCH model generally outperforms other models. In addition, the smoothness parameter and the threshold value in both the logistic smooth transition autoregressive and asymmetric nonlinear logistic smooth transition GARCH are significantly different from zero. Regarding the estimates of LSTAR-ANLSTGARCH model, the slope and the threshold parameters in both the conditional mean and the conditional variance equations are significant. The results show that the estimated value of transition speed of regimes generally indicates a rapid change, it means that the switching from recession into expansion is rapid. These results confirm that the conditional variance, which captures the heterogeneous and the volatility clustering is characterized by a nonlinear dynamics with regime switching behavior. It is also shown that the GARCH parameter of nonlinear part is positive and statistically significant, this implies that positive shocks produce high volatility than negative shocks of the same magnitude. The parameters of linear part are positive and statistically significant, it means that the model manages to capture the temporal dependence of the conditional variance. Furthermore, the sum of GARCH parameters in both the linear and the nonlinear part is less than 1. There is still volatility clustering indicating support for asymmetry. Thus, we find that a negative shock increases the conditional volatility more than a positive shock of the same magnitude. In other words, the unexpected shocks have an asymmetric effect on conditional volatility and the speed of adjustment with respect to the equilibrium is faster. On the other hand, the stock price will tend to move to the average price over time and the LSTAR-ANSTGARCH model is stable overall. It should be noted that the residuals of our selected model illustrated in Figure 2, are characterized by the absence of conditional heteroskedasticity: the ARCH-LM statistic is strictly less than the critical value of $\chi^2(1)$ at 1% for all candidate models.

Figure 3 shows higher volatility persistence of ANLSTGARCH. When the level of the true conditional standard deviation changes, the ANLSTGARCH switches from the low-volatility (high-volatility) state to the high-volatility (low-volatility) state, hence the ANLSTGARCH model is more flexible than the GARCH model in accommodating different sizes of shocks.

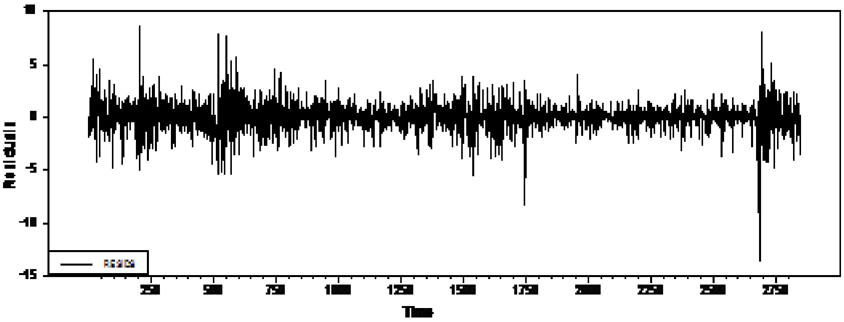

Fig. 2. LSTAR-ANLSTGARCH residuals

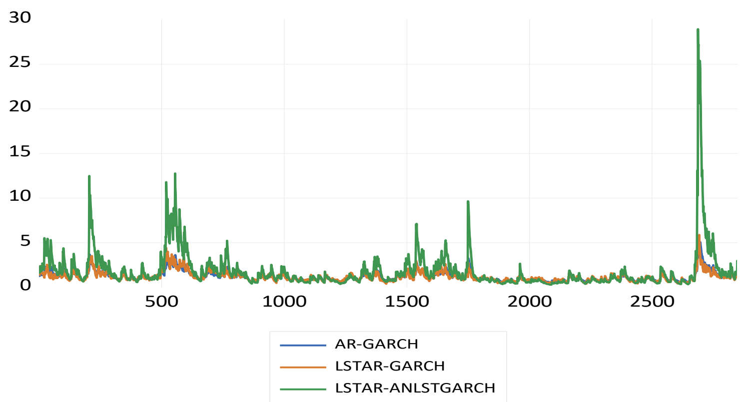

Fig. 3. Comparison of estimates of conditional standard deviations

2.2. Forecasts

In order to compare out-of-sample forecast power of LSTAR-ANLSTGARCH model in French ETF market, we use the mean square error (MSE) and the mean absolute error (MAE) with one step ahead method. Table 9 summarizes statistical comparisons of out-of-sample forecasts provided by the AR-GARCH, LSTAR-GARCH, LSTAR-ANLSTGARCH and the random walk models. We find that the LSTAR-ANLSTGARCH model tend to have better predictive results comparing to other models in most of forecasting time horizons. Moreover, the three models outperform the random walk model in all forecasting time horizons. However, all the models take into account the short-term memory in the conditional mean equation and the conditional volatility, considering that the predictive power for daily XCAC 40 returns reflects the impossibility to forecast up to the longest horizon. The forecast results don't show any sign of efficiency. Thus, the French ETF market doesn't follow random walk. By plotting the evolution of MSE with forecast time horizons (see Figure 5), we note that the LSTAR-ANLSTGARCH model tends to be better than other models. This is a sign of nonlinearity that we verified with statistical tests.

In order to evaluate the out-of-sample forecast accuracy of LSTAR-ANLSTGARCH over other candidate models on one hand and the random-walk on the other hand, we can also use the model confidence set (Hansen et al., 2011) to trim the group of models to a subset of equally superior models. The selection procedure begins with the allocation of the initial set of models M_0 to the set $M^*_{95\%}$. In other words, the model confidence set (MCS) function is initiated on all of the candidate models with a confidence level of 95%. If the null hypothesis of equal predictive ability (EPA) is rejected, we remove an inferior model from the group. The results are reported in Table 10.

Table 10 reports a list of models contained in $M^*_{95\%}$. It appears clear that the LSTAR-ANLSTGARCH models are the most consistently chosen by the MCS as the superior models. The results of the MCS selection procedure shows that LSTAR-ANLSTGARCH model is included in the MCS. The p-values clearly indicate that the null hypothesis of equal accuracy of the LSTAR-ANLSTGARCH model is strongly accepted and this model is included in $M^*_{95\%}$. It is also observed that the LSTAR-ANLSTGARCH model, which incorporates nonlinearity and possible asymmetric shocks, would be the most likely model to be selected and is favored for modelling XCAC 40 volatility since the p-value is maximum for LSTAR-ANLSTGARCH model, which creates asymmetrical responses of volatility for both negative and positive shocks. The asymmetry and nonlinearity effects detected on volatility seem to improve the volatility forecasts.

Table 9. Out-of-sample forecast statistics

Function	Horizon	Criteria (10^{-2})	AR-GARCH	LSTAR-GARCH	LSTAR-ANLSTGARCH	Random Walk

Conditional mean (Returns)	1 day	MSE	0.325*	0.424	0.359	0.517
		MAE	0.108*	0.129	0.179	0.253
	2 days	MSE	0.160*	0.272	0.208	0.329
		MAE	0.049*	0.065	0.096	0.118
	5 days	MSE	0.829	0.790	0.807*	1.024
		MAE	0.556	0.545	0.542*	0.892
	10 days	MSE	2.001	1.811	1.724*	2.291
		MAE	0.960	0.920	0.910*	1.147
	15 days	MSE	2.302	2.227	2.154*	3.145
		MAE	1.126	1.125*	1.127	1.312
Conditional variance (Volatility)	1 day	MSE	1.978	1.774	0.673*	-
		MAE	0.444	0.421	0.259*	-
	2 days	MSE	2.670	2.757	0.892*	-
		MAE	0.516	0.525	0.298*	-
	5 days	MSE	0.382*	2.481	2.460	-
		MAE	0.155*	0.355	0.354	-
	10 days	MSE	1.230	1.236	1.229*	-
		MAE	0.241	0.235*	0.236	-
	15 days	MSE	2.495	1.109	1.074*	-
		MAE	0.354	0.222	0.217*	-

Notes: Predictions are calculated by the authors using RATS 9.20 and Ox 7.20 softwares. * indicates the minimum criterion.

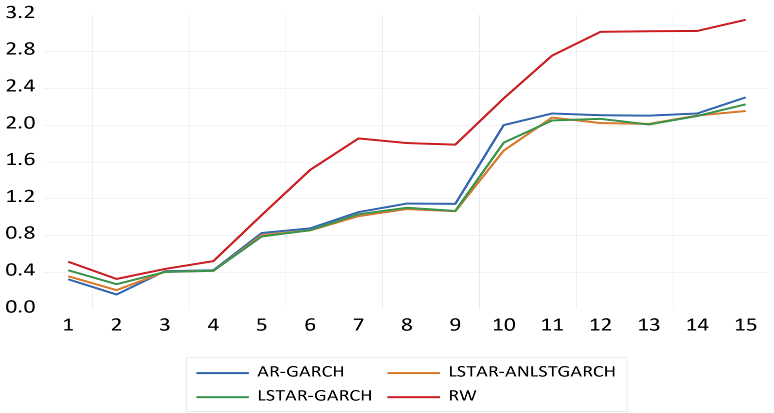

Fig. 4. Evolution of MSE criterion with forecast horizons

Table 10. MCS and p-values

Model	**p-value**	**Model contained in $M^*_{95\%}$**

Random Walk	0.000	LSTAR-ANLSTGARCH
AR-GARCH	0.000	
LSTAR-GARCH	0.048	
LSTAR-ANLSTGARCH	0.845^{*}	

Notes : The p-values that are marked with one $*$ indicate models that are included in $M^{*}_{95\%}$. The MCS is simulated with the R software. Models are calculated by the authors using R software. $M^{*}_{95\%}$ indicates the MCS function, which is initiated on all of the candidate models with a confidence level of 95%.

Given that the daily Xtrackers CAC 40 returns are characterized by the presence of nonlinear dynamics in the equations of the mean and by the asymmetric effects in the conditional volatility, the LSTAR-ANLSTGARCH modelling allows computation of better forecasts than the other models and the random walk. The returns are short-term predictable. The agents cannot anticipate their returns to a long time horizon. Indeed, the observed movements appear as the result of transitory shocks, which affect the French ETF market. The XCAC 40 returns will come back to their previous fundamental value and the shock will be persistent in the short term. In addition, the series is characterized by the existence of nonlinearities in the volatility. Consequently, there is an asymmetric impact of positive and negative information on the level of future variance and the weak efficiency assumption of financial markets seems violated for XCAC 40 returns. The investor is able to earn excess return on the basis of some secretly held private, public or historical information.

CONCLUSION

This study has examined the weak form of efficiency on the French ETF market using a LSTAR-ANLSTGARCH approach. Firstly, several statistical tests including Hinich bispectrum test, Tsay test for linearity, BDS test, long memory test, automatic variance ratio test, automatic portmanteau test and Deo's test were applied for analysis and results. After application of these tests, it has found that Xtrackers CAC 40 is not weak form of efficient market because its successive return are nonlinearly dependent and doesn't generate randomly. We find also evidence of threshold behavior and short memory structure in the returns and volatility series.

In a second time, we investigated the presence of nonlinearities in the French ETF returns. In this context, we proposed a semiparametric estimation for LSTAR with ANLSTGARCH errors. We implemented the nonparametric maximum likelihood method to estimate exactly this class of models by taking into account the phenomenon of persistence and nonlinearity for the conditional variance. From the results, informational shocks have transitory effects on volatility and the LSTAR-

ANLSTGARCH model shows a superiority over the AR-GARCH, LSTAR-GARCH and the random walk models. Using the model confidence set, the forecasts show a clear improvement compared to the random walk model at all horizons; consequently, weak-form efficiency of financial markets seems violated for the XCAC 40 returns. Thus, recent works on semiparametric modeling through ANLSTGARCH process may provide new evidence to better understand the nonlinear dynamics and the asymmetric character of financial series. This semiparametric maximum likelihood estimator is a special case of the general quasi-maximum likelihood in the sense that the parametric form of the density in quasi maximum likelihood is replaced by a consistent kernel density estimate.

The agents have heterogeneous behaviors that vary according to their initial endowments, their individual constraints and their usual activities. In addition, transaction costs are not only variable from one agent to another and based on transaction orders, but they can also define specific thresholds for each investor. The LSTAR-ANLSTGARCH model can reproduce the *regime*-switching behavior in the presence of heterogeneous transaction costs and distinct expectations of agents. The smooth transition between regimes can be attributed to the transaction volumes and heterogeneity of investor expectations.

REFERENCES

Acemoglu, D., & Scott, A. (1994). Asymmetries in the Cyclical Behaviour of UK Labour Markets, *Economic Journal*, 104(427), 1303-23.

Adebile, O.A., & Shangodoyin, D.K (2006). Forecasting performance of logistic STAR model: an alternative version to the original LSTAR models. *Model Assisted Statistics and Applications*, 1(3), 139-146.

Aliyev, F. (2019). Testing Market Efficiency with Nonlinear Methods: Evidence from Borsa Istanbul. *International Journal of Financial Studies*, 7(2), 1-11.

Anderson, H.M., Nam, K., & Vahid, F. (1999). Asymmetric nonlinear smooth transition GARCH models. *Rothman, P., editor, Nonlinear time series analysis of economic and financial data, Kluwer, Boston*, 191-207.

Andrews, D.W.K., & Guggenberger, P. (2003). A Bias-Reduced Log-Periodogram Regression Estimator for the Long-memory Parameter. *Econometrica*, 71, 675-712.

Antwi, E., Gyamfi, E.N,, & Kyei, K.A. (2019). Modeling and forecasting Ghana’s inflation rate under threshold models. *Journal of Developing Areas,* 53(3), 93-105.

Baum, C.F., Barkoulas, J.T., & Caglayan, M. (1999). Long Memory or Structural Breaks: Can Either Explain Nonstationary Real Exchange Rates under the Current Float?. *Journal of International Markets, Institutions and Money*, 9, 359–376.

Ben Haj Hamida, H., & Haddou, H. (2014). Dynamique de la Volatilité du Taux de Change dans les Pays du Maghreb: Une Approche non Linéaire. *International Conference on Business, Economics, Marketing & Management Research*, 2, 81-86.

Berkes, I., & Horváth, L. (2004). The efficiency of the estimators of the parameters in GARCH processes. *Ann. Statist*, 32(2), 633-655.
Bildirici, M., Bayazit, N.G., & Ucan Y. (2020). Analyzing Crude Oil Prices under the Impact of COVID-19 by using LSTARGARCHLSTM. *Energies*, 13(11), 2980.
Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31, 307-327.
Brock, W.A., Dechert, W.D., Scheinkman, J.A., & LeBaron, B. (1996). A Test for independence Based on the Correlation Dimension. *Econometric Review*, 15, 197-235.
Chan, F., & McAleer, M (2003). Estimating smooth transition autoregressive models with GARCH errors in the presence of extreme observations and outliers. *Applied Financial Economics*, 13(8), 581-592.
Chan, F., & Theoharakis, B (2011). Estimating m-regimes STAR-GARCH model using QMLE with parameter transformation. *Mathematics and Computers in Simulation*. 81 (7), 1385-1396.
Chan, F., Marinova, D., & McAleer, M. (2002). STAR-GARCH Models of Ecological Patents in the USA. *International Congress on Environmental Modelling and Software*.
Chen, C.W.S., So, M.K.P., & Liu, F.C. (2011). A review of threshold time series models in finance. *Statistics and Its Interface*, 4, 167–181.
Chikhi, M., & Bendob, A. (2018). Nonparametric NAR-ARCH Modelling of Stock Prices by the Kernel Methodology. *Journal of Economics and Financial Analysis*, 2, 105-120.
Chikhi, M., & Diebolt, A. (2009). Transitory Exogenous Shocks in a Non-Linear Framework: Application to the Cyclical Behaviour of the German Aggregate Wage Earnings. *Historical Social Research*, 34(1), 354-366.
Colmant, B., Roland, G., & Szafarz, A. (2009). Efficience des marchés: concepts, bulles spéculatives et image comptable. *ULB Institutional Repository* - Université Libre de Bruxelles -.
Di, J., & Gangopadhyay, A. (2014). One-step Semiparametric Estimation of the GARCH Model. *Journal of Financial Econometrics*, 12(2), 382–407.
Diebold, F.X., & Mariano, R.S. (1995). Comparing Predictive accuracy. *Journal of Business and Economic Statistics*, 13(3), 253-263.
Drost, F.C., & Klaassen, C.A.J. (1997). Efficient estimation in semiparametric GARCH models. *Journal of Econometrics*, 81(1), 193-221.
Eitrheim, Ø., & Teräsvirta, T. (1996). Testing the adequacy of smooth transition autoregressive models. *Journal of Econometrics*, 74(1), 59-75.
Engle, F.R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4), 987–1007.
Engle, F.R., & Gonzalez-Rivera, G. (1991). Semiparametric ARCH Models. *Journal of Business & Economic Statistics*, 9(4), 345-59.
Escanciano, J.C., & Lobato, I..N. (2009). An automatic portmanteau test for serial correlation. *Journal of Econometrics*, 151(2), 140-149.

Escribano, A., & Jordá, O. (2001). Testing nonlinearity: Decision rules for selecting between logistic and exponential STAR models. *Spanish Economic Review*, 3, 193–209.

Francq, C., & Zakoïan, J.M. (2004). Maximum likelihood estimation of pure GARCH and ARMA-GARCH processes. *Bernoulli*, 10(4), 605-637.

Franses, P.H., Neele, J., & Van Dijk, D. (1998). Forecasting Volatility with Switching Persistence GARCH Models. *Econometric Institute Report*, EI-9819.

Franses, P.H., Franses, R.F.P.H., & Van Dijk, D. (2000). Non-linear time series models in empirical finance. *Cambridge University Press*, Cambridge.

Geweke, J., & Porter-Hudak, S. (1983). The estimation and application of long-memory time series models. *Journal of Time Series Analysis*, 4, 221-238.

Gonzales-Riviera, G. (1996). Smooth Transition GARCH Models, *Working Paper*. Department of Economics, University of California at Riverside.

Gonzalez-Rivera, G. (1998). Smooth-Transition GARCH Models. *Studies in Nonlinear Dynamics & Econometrics*, 3(2), 1-20.

Gonzalez-Rivera, G., & Drost, F.C. (1999). Efficiency comparisons of maximum-likelihood-based estimators in GARCH models. *Journal of Econometrics*, 93(1), 93-111.

Granger, C., & Teräsvirta, T. (1993). Modelling Non-Linear Economic Relationships. *Oxford University Press*.

Guo, Z.F., & Cao, L. (2011). An Asymmetric Smooth Transition GARCH Model. *International Journal of Applied Mathematics*, 41(4), 1-3.

Hagerud, G.E. (1997). Specification Tests for Asymmetric ARCH. *Working paper series in Economics and Finance*. Department of Finance, Stockholm School of Economics.

Hansen, P.R., Lunde, A., & Nason, J.M. (2011). The model confidence set. *Econometrica*, 79(2), 453-497.

Hinich, M.J., & Patterson, D.M. (1989). Evidence of nonlinearity in the trade-by-trade stock market return generating process. In W. A. Barnett, J. Geweke and K. Shell (eds.), *Economic Complexity: Chaos, Sunspots, Bubbles and Nonlinearity International Symposium in Economic Theory and Econometrics*, 383-409. Cambridge: Cambridge University Press.

Huber, J., Kirchler, M., & Sutter, M. (2008). Is more information always better: Experimental financial markets with cumulative information. *Journal of Economic Behavior & Organization*, 65(1), 86-104.

Jarque, C.M., & Bera, A.K. (1987). A test for normality of observations and regression residuals. *International Statistical Review* 55, 163–172.

Kyrtsou, C., & Terraza, M. (2003). Is it Possible to Study Chaotic and ARCH Behaviour Jointly? Application of a Noisy Mackey–Glass Equation with Heteroskedastic Errors to the Paris Stock Exchange Returns Series, *Computational Economics*, 21(3), 257-276.

Liew, V., Baharumshah, A., Zubaidi, L.E. (2004). Nonlinear Adjustment towards Purchasing Power Parity in ASEAN Exchange Rates. *IUP Journal of Applied Economics*, III, 7-18.

Livingston, J., & Nur, D. (2018). Bayesian inference for smooth transition autoregressive (STAR) model: A prior sensitivity analysis. *Communications in Statistics - Simulation and Computation*, 46(7), 5440-5461.

Lo, A.W., & MacKinlay, A.C. (1987a). Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test. *Working Paper*, 5-87*, Rodney L. White Center*, Wharton School, University of Pennsylvania.

Lubrano, M. (2001). Smooth Transition GARCH Models : a Bayesian Perspective. *Recherches Economiques de Louvain - Louvain Economie Review*, 67(3), 257-287.

Lundbergh, S., & Teräsvirta, T. (1999): Modelling economic high-frequency time series with STAR-STGARCH models. *Working Paper Series in Economics and Finance*, Stockholm School of Economics, 291, SSE/EFI.

Lundbergh, S., & Teräsvirta, T. (2000). Forecasting with smooth transition autoregressive models. *Working Paper Series in Economics and Finance*, Stockholm School of Economics, 390, SSE/EFI.

Luukkonen, R., Saikkonen, P., & Terasvirta, T. (1988). Testing Linearity against Smooth Transition Autoregressive Models. *Biometrika*, 70, 491-499.

Medeiros, M., & Veiga, A. (2009). Modeling Multiple Regimes in financial Volatility with a Flexible coefficient GARCH(1,1) Model. *Econometric Theory*, 25(1), 117-161.

Midilic, M. (2016). Estimation of STAR-GARCH models with iteratively weighted least squares. *Working Papers of Faculty of Economics and Business Administration*, Ghent University, Belgium, Ghent University, Faculty of Economics and Business Administration.

Mukherjee, K. (2006). Pseudo-likelihood estimation in ARCH models. *Canadian Journal of Statistics*. 34, 341-356.

Ouyang, T., Huang, H., He, Y., & Tang, Z. (2020). Chaotic wind power time series prediction via switching data-driven modes. *Renew Energy*, 145, 270–281.

Newey, W., & Steigerwald, D. (1997). Asymptotic Bias for Quasi-Maximum-Likelihood Estimators in Conditional Heteroskedasticity Models. *Econometrica*, 65(3), 587-600.

Öcal, N., & Osborn, D.R. (2000). Business cycle non-linearities in UK consumption and production. *Journal of Applied Econometrics*, 15(1), 27-43.

Öcal, N. (2000). Nonlinear Models for U.K. Macroeconomic Time Series. *Studies in Nonlinear Dynamics & Econometrics*, 4(3), 1-15.

Pagan, A., & Ullah, A. (1999). Nonparametric Econometrics. *Cambridge University Press*.

Pavlidis, E.G., Paya, I., & Peel, D.A. (2010). Specifying Smooth Transition Regression Models in the Presence of Conditional Heteroskedasticity of Unknown Form. *Studies in Nonlinear Dynamics & Econometrics*, 14(3), 1-40.

Reitz, S., & Westerhoff, F. (2007). Commodity price cycles and heterogeneous speculators: a STAR–GARCH model. *Empirical Economics* (Springer), 33(2), 231-244.

Robinson P M, Henry M (1998): Long and Short Memory Conditional Heteroscedasticity in Estimating the Memory Parameter of Levels. *Discussion* *paper STIDERC. Econometrics* EM/98/357, London School of Economics and Political Science.

Sarantis, N. (1999). Modeling non-linearities in real effective exchange rates. *Journal of International Money and Finance*, 18(1), 27-45.

Skalin, J., & Teräsvirta, T. (2002). Modelling asymmetries and moving equilibria in unemployment rates, *Macroeconomic Dynamics*, 6(2), 202-241.

Tayyab, M., Tarar, A., & Riaz, M. (2012). Application of Smooth Transition autoregressive (STAR) models for Exchange Rate. *Mathematical Theory and Modeling*, 2(9), 30-39.

Teräsvirta, T., & Anderson, H. (1992). Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models. *Journal of Applied Econometrics*, 7(S), S119-36.

Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of the American Statistical Association*, 89(425), 208–218.

Tiao, G.C., & Tsay, R.S. (1994). Some advances in non-linear and adaptive modelling in time-series. *Journal of Forecasting* 13, 109-131.

Tsay, R.S. (2001). Analysis of Financial Time Series: Financial Econometrics. *John Wiley & Sons*.

Umer, M., Sevil, T., & Sevil, G. (2018). Forecasting performance of smooth transition autoregressive (STAR) model on travel and leisure stock index. *Journal of Finance and Data Science*, 4(2), 90-100.

Van Dijk, D., Teräsvirta, T., & Franses, P.H. (2002). Smooth Transition Autoregressive Models: A survey of recent developments. *Econometric Reviews*, 21(1), 1-47.

Wahlström, S. (2004). Comparing forecasts from LSTAR and linear Autoregressive models. *Mathematical Statistics Stockholm University*.

Yaya, O.S., & Shittu, O.I. (2016). Specifying Asymmetric STAR models with Linear and Nonlinear GARCH Innovations: Monte Carlo Approach. *Journal of Modern Applied Statistical Methods*, 13(1), 410-430.

Zhou, J. (2010). Smooth Transition autoregressive Models: A study of the industrial production index of Sweden. *Working Paper*. Department of Statistics. UPPSALA Universitet.