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Simple Matching Protocols for Agent-based Models

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Abstract

The main purpose of this article is to show how simple matching protocols suitable for agent-based models can be developed from scratch. Keeping the feature of the underlying economy at minimum, I develop, detail, and present the code for three matching processes. Their small size and flexibility may act as a stimulus to non-expert students to undertake such stream of literature and address a variety of research topics..

JEL Classifications: A20, C63, E10, O10, O30.

Keywords: Agent-based Modelling, Matching Protocols, Computer Simulation, Linear Matrix Algebra, *R*.

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Contents

1 Introduction

The aim of the paper is pedagogical and comes out as a side project of Borsato (2020, 2021). In those contributions, the author develops an agent-based, stock-flow consistent model from scratch to analyse in which way the functional distribution of income between wages and profits, and the rate of innovative search at firm level interplay in determining the Secular Stagnation in productivity growth that characterizes the USA nowadays. The development of this, as well as any other, agent-based model is a time-consuming task that requires the devising of matching protocols between agents, whose interaction at the microeconomic level gives rise to the multifaceted global stylized facts for growth rates, employment, income distribution, and institutions (Tesfatsion, 2006; Tesfatsion and Judd, 2006).

The essay develops some basic matching process that is useful when designing agent-based models. With this contribution, the purpose is at least threefold. Firstly, I describe in the simplest way some quick and basic matching protocol that finds application in the agent-based literature. This description helps not (yet) expert students build their own models and adapt the following ideas to their own aims. Secondly, I show that such diverse protocols require a few weak assumptions and can be integrated into more complex environments. Through their implementation, scholars may contribute to the field of financial fragility and bank regulation (Delli Gatti et al., 2010), innovation and technological change (Dosi et al., 2010), or structural change and consumption preferences (Ciarli et al., 2010). Although the code is written for *R*-like environments and implements linear matrix algebra, modelers can adapt the related *philosophy* to the software they prefer and to their needs.[1] Finally, albeit there recently were a few other didactical contributions with the aim of encouraging students to approach and enrich non-neoclassical economics, at the best of my knowledge little is about agent-based models and papers most focus on aggregate stock-flow consistent modelling, for instance. My paper tackles one of the hardest issues in the agent-based literature, namely the development and application of the matching process with which economic agents interact with one another.

The paper is strictly connected in scope to Caiani et al. (2016b), Veronese Passarella (2019), and Carnevali (2021). Caiani et al. (2016b) is a textbook and introduces students to the basic toolkits of agent-based modelling. Once the philosophy surrounding this framework is presented, these toolkits are applied to the analysis of financial markets. Even though I draw upon this textbook in elaborating some of the following procedures, I apply them in different contexts and make them easier to grasp. In contrast, Carnevali (2021) and Veronese Passarella (2019) present small-to-medium scale stock-flow consistent (SFC hereafter) models which could facilitate the incorporation of additional and complex building blocks into the corresponding structures. Despite their obvious limitations for pedagogical purposes,

[1] I used *R*-studio, version 1.2.5033.

their models allow for comparative analyses and conditional forecasts, and might constitute the benchmark for early-careers researchers. Students interested in focusing on the interactions between heterogeneous agents can easily re-adapt the following protocols to match the framework in Carnevali (2021) and Veronese Passarella (2019). The essay is organised as follows: Section II deals with the literature; Section III describes, firstly, the features of the basic economy I work with and, secondly, three toy matching protocols that constitute the core of the article; last Section sums up and concludes.

2 Relation with the Literature

This work inserts into the Agent-based Computational Economics (ACE hereafter). A distinctive feature of this literature is that no dynamic optimisation technique is used to analyse the behaviour of any agent in the economy, because it is recognised that modern economies are complex systems of production.[2] In particular, this stream of research builds macroeconomic models from the bottom-up, in which microfoundations are strongly based upon the actual empirical microeconomic evidence. Following Dosi and Roventini (2019), we envisage four main features of any agent-based model. First, there is no *isomorphism* between micro and macro: the aggregate dynamics of the system cannot be reduced to the simple behaviour of some identifiable component. Higher levels of aggregation lead to the emergence of self-sustained growth paths with persistent fluctuations at business-cycle frequencies, statistical regularities as firms' size distribution, or completely new structures like markets and institutions. In this picture, *macro is not micro times n*. Second, bounded rationality and Knightian uncertainty deeply affects the environment where agents have to take their decisions. Being unable to form rational expectations on how the system works, the agents must rely on heuristics and rules-of-thumb. Third, imperfect information also means that individuals have a narrow set of other agents to interact with, and often establish durable relationships based on trust and reciprocity to solve problems of asymmetric information (Bowles, 2009, Ch. 2). And fourth, economic systems display self-organised criticality, in that the accumulation of imbalances might trigger degenerative dynamics even after seemingly innocuous shocks. From what said, macroeconomic events pour on local interactions, and agents' learning occurs through the changes in their behaviour based on experience. ACE models are therefore juxtaposed to the standard Walrasian methodology, since agents are designed with more autonomy and with the ability of self-organizing.[3] An increasing number of papers adopts this methodology to address fiscal and monetary

[2] LeBaron and Tesfatsion (2008) define *agents* as an encapsulated set of data and behaviours representing an entity residing in a computationally constructed world.

[3] A key feature is Von Hayek (1937)'s notion of *spontaneous order*: chaotic processes at the microeconomic level may entail some form of regularity at the aggregate perspective. Markets are viewed as places for learning and discoveries, hence a place for innovation and imitation.

policies (Dosi et al., 2016), the debate on Secular Stagnation (Borsato, 2020, 2021), financial fragility (Delli Gatti et al., 2005, 2010), innovation-driven economic growth (Caiani et al., 2019; Dosi et al., 2010), structural changes (Ciarli et al., 2010, 2019; Lorentz et al., 2016); labour market policy (Deissenberg et al., 2008; Napoletano et al., 2012), and climate change (Lamperti et al., 2018a,b). This Section does not obviously aim at summarizing all the literature in which the matching protocols might be introduced.[4] Among this broadening stream, I single out Delli Gatti et al. (2005, 2010), Riccetti et al. (2015), Dosi et al. (2010), Caiani et al. (2016a), and the EURACE project.

Delli Gatti et al. (2005) studies the interaction between heterogeneous financially fragile firms and the banking system. The continuously changing configuration and the patterns of interactions are at the origin of business-cycle fluctuations. Since the framework matches a good spectrum of observed stylized facts, the authors explore the link between firms' size distribution and aggregate growth rates. They find that the power law distribution of firms' size lays at the root of the Laplace distribution in growth rates. Furthermore, this power law influences the age of existing firms, the amount of profits and debt during business-cycle phases of booms and busts. Delli Gatti et al. (2010) and Riccetti et al. (2015) work in the same vein and investigates the interplay between financial factors and business fluctuations. They propose a model with heterogeneous agents whose financial fragility amplify business-cycle fluctuations through the complex interaction with the banking system. Riccetti et al. (2015), in particular, focuses on the emergence of endogenous business cycles as consequence of the interactions between real and financial factors. Remarkable is the link between firms leverage and banks' lending activity: on the one hand, both may succeed in reducing unemployment rates, but on the other hand, excessive leverage and increasing bank exposure makes economy financially fragile.

Dosi et al. (2010) concerns to the $K+S$ family of models, in which the authors investigate the way microeconomic innovations turn out in and influence global outcomes. It bridges the Schumpeterian tradition of innovation-driven economic growth with the Keynesian theories of demand generation. This contribution is additionally an exercise in general disequilibrium analysis, since it goes beyond Walrasian frameworks, that did not mean to address and describe how production, pricing, and trade actually unfold in real-world economies.

Caiani et al. (2016a) provides a fully decentralized and stock-flow-consistent economy, in which consistency is applied since the microeconomic level to account for the structural interrelatedness of agents. Although the model does not concern to growth questions, it is promising in the field of bank regulation and macro-prudential issues. This contribution offers interesting guideposts to calibrate, validate, and adapt the basic framework to alternative research questions.

[4] Exhaustive reviews are Fagiolo and Roventini (2016) and Dawid and Delli Gatti (2018). The afore-mentioned Dosi and Roventini (2019) juxtaposes agent-based models to DSGE models.

Finally, I mention the EURACE project, a massive attempt to design and implement an agent-based macroeconomic platform for the whole European economy (Deissenberg et al., 2008). This attempt encompasses and subsequently unifies several environments, such as consumption and investment goods markets, labour markets, and markets for financial assets. Inspired by real-world empirical evidence, EURACE adopts realistic assumptions about agent's bounded rationality and limited information managing capacity. The framework contributes to several areas of research, like the interaction between technical change and income inequality as underlying causes of the recent increase in market concentration and market power (Terranova and Turco, 2021); the relationship between debt deleveraging, credit money, and financial instability (Cincotti et al., 2010; Raberto et al., 2011) and the effectiveness of different types of cohesion policies with respect to convergence of regions (Dawid et al., 2014).

3 Protocols for Households and Firms

Every macroeconomic agent-based model is characterized by a general structure, that defines the actors involved, and by a timeline of events reflecting their evolution through time. For what concerns to the general structure, there is a population of agents whose size might change across time. Each agent is denoted with a set of variables and parameters that identifies their changing behaviour and their *modus operandi*. For example, a variable may specify her consumption preferences, how they evolve or how they are influenced by changing in labour income or wealth, and so on. Moreover, some exogenous coefficient may reflect the macroeconomic state of the system, like tax rates, capital requirements, etc. Once the modeler set initial conditions, agents start interacting with each other and, as they keep accumulating knowledge and information about the surrounding environment, revise their behaviour accordingly. The final aggregation of individual decisions lead the analyst to grasp that the "*aggregate of interacting entities yield emergent properties*, which cannot be mapped down to the (conscious or unconscious) behaviours of some identifiable underlying components" (Dosi and Roventini, 2019, p. 2; italics in original). To my specific purpose, I focus on a one-good two-class closed economy with no government sector that is populated by a multitude of heterogeneous interacting agents. For the sake of simplicity, I have N households, in which $N - F$ are workers that offer labour inelastically at the going wage rate, while the remaining F households are capitalists such that each owns a single firm. Regardless of their status, everybody consumes and saves according to her propensity to save out of income. What is left from consumption in the form of savings, if any, is accumulated as cash holdings. For what concerns to the production side of the economy, entrepreneurs organize the production process through the hiring of workers. I abstract from any investment decisions and assume that the single good is produced by means of labour only. Furthermore, I set any financial side of the economy away. Output is expressed at constant prices.

I shall express some equations that help me frame the protocols. The demand for labour of the i-th firm, $n_{d,i}$ is equal to:

$$n_{d,i} = \frac{y_i}{a} \tag{1}$$

in which y_i is the amount of production and a is labour productivity, set for simplicity equal to 1. The wage rate $w_{r,i}$ is set randomly by each firm but cannot be lower than an exogenously set subsistence amount. Labour costs are then:

$$wb_i = w_{r,i} \cdot n_{d,i} \tag{2}$$

in which wb_i stands for wage bill. I define profits at firm level, f_i as:

$$f_i = y_i - wb_i \tag{3}$$

For what concerns to price setting, firms fix prices, p_i, as mark-up over unit labour costs:

$$p_i = (1 + \mu_i)\,\frac{w_{r,i}}{a} \tag{4}$$

in which the mark-up μ_i varies randomly between firms.

The related R-code may start as below, whose first step consists of writing Eq. (1) through Eq. (4) so to briefly describe the economy I deal with. In the following R-script I define all the involved variables and parameters. This first block of code might be divided in three parts. Firstly, I set the main parameters of the model, like the number of workers and firms, or how many time periods the model is going to be simulated for. Secondly, I define the variables that, by definition, change through time. Consumption and saving functions, wage rates, and prices belong to this category, for instance. Finally, there is a for loop that projects Eq. (1) to Eq. (4) for as many periods as defined in the corresponding parameter setting.

```
rm(list=ls(all=TRUE)) #Clear all
library(Matrix)

#PARAMETER SETTING

#Number of periods
```

```
Time = 500
#Number of agents
N = 60
#Number of firms
F = 10

#Labour productivity
a = 1
#Propensity to consume out of wealth
alpha3 = 0.1
#Numbers of possible partners
chi = 3
#Coefficient in the exponential fuction
chi_1 = 4

#Pseudo-random number generator
set.seed(10)

#VARIABLES
#Worker's propensity to consume
alpha1 = matrix(data=runif(N-F, min=0.7,max=0.8), ncol=1, nrow=N-F)
#Capitalist's propensity to consume
alpha2 = matrix(data=runif(F, min=0.4, max=0.6), ncol=1, nrow=F)
#Diagonal matrix for alpha1
alpha1_diag = matrix(data=0, ncol=N-F, nrow=N-F)
#Diagonal matrix for alpha2
alpha2_diag = matrix(data=0, ncol=F, nrow=F)
#Total household consumption
c_agents = matrix(data=0, ncol=N, nrow=1)
#Consumption out of income
c_inc = matrix(data=0, ncol=N, nrow=F)
#Capitalist consumption out of income
c_e = matrix(data=0, ncol=F, nrow=F)
#Worker consumption out of income
c_w = matrix(data=0, ncol=F, nrow=N-F)
#C_w - transpose
cw_t = matrix(data=0, ncol=N-F, nrow=F)
```

```
#Consumption out of wealth
c_mh = matrix(data=0, ncol=N, nrow=F)
#Total household consumption
c = matrix(data=1, ncol=N, nrow=F)
#Savings per period
d_mh = matrix(data=0, ncol=N, nrow=F)
#Entrepreneurial income
e_inc = matrix(data=0, ncol=F, nrow=F)
#Entrepreneurial profits
f = matrix(data=0, ncol=1, nrow=F)
#Diagonal matrix for f
f_diag = matrix(data=0, ncol=F, nrow=F)
#Link for Firms-Consumers
link_fc = matrix(data=0, ncol=1, nrow=N)
#Mark-up
mu = matrix(data=0, ncol=1, nrow=F)
#Household wealth
mh = matrix(data=0, ncol=N, nrow=F)
#Demand for labour
nd = matrix(data=0, ncol=1, nrow=F)
#Unit price
p = matrix(data=0, ncol=1, nrow=F)
#Vector of 1s
vcap = matrix(data=1, ncol=1, nrow=F)
#Wage rate
w = matrix(data=runif(F, min=0.3, max=0.5), ncol=1, nrow=F)
#Wage bill
wb = matrix(data=0, ncol=1, nrow=F)
#Labour income
w_inc = matrix(data=0, ncol=N-F, nrow=F)
#Labour income - transpose
winc_t = matrix(data=0, ncol=F, nrow=N-F)
#Diagonal matrix for w
w_diag = matrix(data=0, ncol=F,nrow=F)
#Production = consumption
y = matrix(data=1, ncol=1, nrow=F)
#Disposable income
```

```
ydh = matrix(data=0, ncol=N, nrow=F)




#Time loop
for (t in 2:Time) {



#Model setup

#Demand for labour
nd = ceiling(y/a)
#Wage bill
wb = w*nd
#Entrepreneurial profits
f = y - wb
#Mark-up
mu = runif(F, min=0.1, max=0.2)
#Unit price
p = (1+mu)*w/a #} #End of time loop
```

3.1 Matching 1: Firms-Workers Network in the Labour Market

The setting above paves the floor to two matching protocols, in which the first applies to the labour market whereas the second relates consumers to entrepreneurs. Entrepreneurs hire workers to produce the consumption good. For simplicity, hiring workers consists of single-period agreements between agents and takes place randomly. Workers are therefore randomly allocated to the *i*-th firm according to its labour demand $n_{d,i}$. The first step to build such a matching is to create a $F \times (N - F)$ matrix, called $\boldsymbol{net_w}$. Cells take value 1 if a link between a firm and a worker is established and 0 otherwise. I then sample random cells and set them equal to 1: precisely, every row will count a number of 1s corresponding to the labour demanded by the *i*-th firm.[5] The second step concatenates the network matrix with a vector of ones:

$$\boldsymbol{net}_{fw} = [vcap, \boldsymbol{net_w}] = \left[(1, \ldots, 1)', \boldsymbol{net_w}\right] \tag{5}$$

[5]Careful readers will notice that the script below set the number of 1s as equal to $(n_{d,i} - 1)$: it holds since the entrepreneur works in her own firm too, so the *net* labour demand is $(n_{d,i} - 1)$.

The vector of ones, $vcap$, refers to capitalists and assigns to each of them the same firm throughout the simulation. Moreover, it allows to have a full network matrix in which the overall number of 1s actually corresponds to aggregate employment. Each simulation step will change values inside $\boldsymbol{net_w}$ – and so inside $\boldsymbol{net_{fw}}$ – but keeping fixed the first column of the latter matrix that refers to capitalists.[6] Last step of the first matching is about demand schedules for the consumption good. I have supposed in the above that the two classes consume with two different propensities out of disposable income, say $\alpha_{1,j}$ for workers and $\alpha_{2,j}$ for entrepreneurs.[7] In contrast, I set for convenience the propensity to consume out of wealth, α_3, as equal and constant among agents. I can therefore express the demand schedules as follows:[8]

$$c_{w,j,t} = \alpha_{1,j} \cdot w_{r,j,t-1} \tag{6}$$

$$c_{e,j,t} = \alpha_{2,j} \cdot \left(w_{r,j,t-1} + f_{j,t-1}\right) \tag{7}$$

$$c_{mh,j,t} = \alpha_3 \cdot m_{h,j,t-1} \tag{8}$$

$c_{w,j,t}$, $c_{e,j,t}$, and $c_{mh,j,t}$ refer to the consumption out of worker's income, to the consumption out of entrepreneurial income, and to the consumption out of wealth for the j-th agent, respectively. Firms pay to their workers a wage rate, $w_{r,i}$, that varies randomly across firms. I can make a $F \times F$ matrix, say $\boldsymbol{w_{r,\text{diag}}}$, that contains wage values in its main diagonal. Such a matrix may then be multiplied by $\boldsymbol{net_w}$, so to have a matrix for labour incomes in which every worker receives the wage from the firm she belongs to. I label this further $F \times (N - F)$ matrix with $\boldsymbol{w_{inc}}$:

$$w_{r,i} \rightarrow \boldsymbol{w_{r,\text{diag}}} = \begin{bmatrix} w_{r,1} & \dots & 0 \\ \vdots & w_{r,i} & \vdots \\ 0 & \dots & w_{r,F} \end{bmatrix} \tag{9}$$

$$\boldsymbol{w_{inc}} = \boldsymbol{w_{r,\text{diag}}} \times \boldsymbol{net_w} \tag{10}$$

$\boldsymbol{w_{inc}}$ might be, for instance:

$$\boldsymbol{w_{inc}} = \begin{bmatrix} w_{r,1} & \dots & 0 \\ \vdots & w_{r,i} & \vdots \\ 0 & \dots & w_{r,F} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & \dots & 1 \\ \vdots & 1 & 0 & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} = \begin{bmatrix} w_{r,1} & 0 & \dots & w_{r,1} \\ \vdots & w_{r,i} & 0 & \vdots \\ 0 & 0 & \dots & w_{r,F} \end{bmatrix} \tag{11}$$

Now, if every worker is assigned a marginal propensity to consume, $\alpha_{1,j}$, I am able to

[6] It follows that $\boldsymbol{net_{fw}}$ is a $F \times N$ matrix.

[7] As in the standard Keynesian literature, $\alpha_{1,j} > \alpha_{2,j}$ strictly holds.

[8] Temporal index bases current consumption on past income.

compute the amount of consumption out of disposable income for all agents, $\boldsymbol{c_w}$:[9]

$$\boldsymbol{c_w} = \boldsymbol{\alpha_{1,\text{diag}}} \times \boldsymbol{w_{inc}^T} = \begin{bmatrix} \alpha_{11} \cdot w_{r,1} & \dots & 0 \\ 0 & \alpha_{1i} \cdot w_{r,i} & 0 \\ \vdots & 0 & \vdots \\ \alpha_{1,N-F} \cdot w_{r,1} & \dots & \alpha_{1,N-F} \cdot w_{r,F} \end{bmatrix} \tag{12}$$

In particular, the transpose matrix $\boldsymbol{w_{inc}^T}$ makes clear that the propensity to consume does vary across workers but does not with respect to the single firm: if the j-th worker has got a propensity to consume equal to 0.6, then this value is mantained regardless of the firm the agent decides to purchase the good.

The same procedure applies to capitalist's income, $\boldsymbol{e_{inc}}$, in that:

$$\boldsymbol{e_{inc}} = \boldsymbol{w_{r,\text{diag}}} + \boldsymbol{f_{\text{diag}}} \tag{13}$$

$$\boldsymbol{c_e} = \boldsymbol{\alpha_{2,\text{diag}}} \times \boldsymbol{e_{inc}} = \begin{bmatrix} \alpha_{21} \cdot (w_{r,1} + f_1) & \dots & 0 \\ 0 & \alpha_{2i} \cdot (w_{r,i} + f_i) & 0 \\ \vdots & 0 & \vdots \\ \alpha_{2F} \cdot (w_{r,1} + f_1) & \dots & \alpha_{2F} \cdot (w_{r,F} + f_F) \end{bmatrix} \tag{14}$$

in which $\boldsymbol{f_{\text{diag}}}$, $\boldsymbol{\alpha_{2,\text{diag}}}$, $\boldsymbol{e_{inc}}$ and $\boldsymbol{c_e}$ are all $F \times F$ matrices.

So doing, I combine $\boldsymbol{e_{inc}}$ and $\boldsymbol{w_{inc}}$ and get two full $F \times N$ matrices of disposable income and consumption, $\boldsymbol{y_{dh}}$ and $\boldsymbol{c_{inc}}$ respectively, that correspond to:

$$\boldsymbol{y_{dh}} = [\boldsymbol{e_{inc}}, \boldsymbol{w_{inc}}] \tag{15}$$

$$\boldsymbol{c_{inc}} = \left[\boldsymbol{c_e}, \boldsymbol{c_w^T}\right] \tag{16}$$

Since agents do not spend all their income in consumption, what is left is saved:

$$\boldsymbol{\Delta m_h} = \boldsymbol{y_{dh}} - \boldsymbol{c} \tag{17}$$

$\boldsymbol{\Delta m_h}$ is a $F \times N$ matrix and tracks savings per period, while $\boldsymbol{c}$ represents the total household consumption, made up of two components: consumption of out income $\boldsymbol{c_{inc}}$ and con-

[9] $\boldsymbol{\alpha_{1,\text{diag}}}$ is a $(N-F) \times (N-F)$ matrix obtained through the same procedure adopted for $\boldsymbol{w_{r,\text{diag}}}$.

sumption out of wealth $\boldsymbol{c_{mh}}$.[10] The latter is:

$$\boldsymbol{c_{mh}} = \alpha_3 \cdot \boldsymbol{m_h} \tag{18}$$

in which $\boldsymbol{m_h}$ is a $F \times N$ matrix representing the cumulative sum of savings $\boldsymbol{\Delta m_h}$.[11]

I organize the *R*-code in two steps:

1. Creation of random network: I create for every run a new matching schedule that enables me to allocate workers to random firms. Defining $\boldsymbol{net_w}$ and $\boldsymbol{net_{fw}}$ inside the time loop avoids the matrix to fill up of ones after few periods because of the random sampling. The matrix is empty at the beginning of each run.
2. Household's equations: I first take diagonal matrices for the propensities to consume, the wage rate and for entrepreneurial profits. I then compute and aggregate the consumption functions as developed in Eq. (6) through Eq. (18).

```
# Matching process 1: the labour market

# a) Creation of random network

#Workers allocation network
net_w = matrix(data=0, ncol=N-F, nrow=F)
#Full workers' network matrix
net_fw = matrix(data=0, ncol=N, nrow=F)

#Update workers' network
for (i in 1:F) {
  net_w[i, sample(N-F, max(nd[i]-1,0))] = 1
}
#Update full matrix
net_fw = cbind(vcap,net_w)



# b) Households equations

#Diagonal matrix for alpha1
alpha1_diag = diag(as.list(alpha1))
```

[10] Hence, $\boldsymbol{c} = \boldsymbol{c_{inc}} + \boldsymbol{c_{mh}}$.

[11] The propensity to consume α_3 could have been set as $\alpha_{1,j}$ or $\alpha_{2,j}$. Previous reasoning would have applied in this case too. Moreover, $\boldsymbol{m_{h,t}} = \boldsymbol{m_{h,t-1}} + \boldsymbol{\Delta m_{h,t}}$.

```
#Diagonal matrix for alpha2
alpha2_diag = diag(as.list(alpha2))
#Diagonal matrix for alpha3
alpha3_diag = diag(alpha3, nrow=N, ncol=N)
#Diagonal matrix for w
w_diag = diag(as.list(w))
#Diagonal matrix for f
f_diag = diag(as.list(f))

#Workers consumption out of income
c_w = alpha1_diag%*%winc_t
#C_w transpose
cw_t = t(c_w)
#Labour income
w_inc= w_diag%*%net_w
#W_inc transpose
winc_t = t(w_inc)

#Capitalist consumption out of income
c_e = alpha2_diag%*%e_inc
#Capitalist income
e_inc = (w_diag+f_diag)
#No consumption with losses
e_inc[e_inc<0]=0

#Join consumption components
c_inc = cbind(c_e,cw_t)
#Consumption out of wealth
c_mh = alpha3*mh
#Consumption function
c = c_inc + c_mh

#Disposable income
ydh = cbind(e_inc,w_inc)
#Savings
d_mh = ydh - c
#Wealth
```

```
mh = mh + d_mh

#Household total consumption
c_agents = apply(c, 2, sum)
```

3.2 Matching 2: Firms-Consumers Network in the Goods Market

The second matching process finds its first application in Delli Gatti et al. (2010) and Riccetti et al. (2015), among the others, but I refer to Caiani et al. (2016b, pp. 64 − 67) for an exhaustive application on the analysis of financial markets, precisely the study of financial fragility. In this case, firms choose their bank according to the interest rate charged on loans. I here restrict the exposition and then conform the process to the current setting.

The main idea is that agents meet on the goods market and act following the same protocol: potential consumers observe a subset of prices from a restricted and random set of suppliers which reflects their imperfect information. They choose the best seller according to the lowest selling price. Households have each period the *chance* to switch to another supplier with a positive probability that depends on price differentials:

$$Prob = \begin{cases} 1 - e^{\chi_1 \cdot \frac{p_{new} - p_{old}}{p_{new}}} & \text{if} \quad p_{new} < p_{old} \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Eq. (19) says the larger the price differential between the old and the new price, the higher the probability to switch toward the new producer; χ_1 is a parameter that influences the speed with which the exponential function e decreases to zero. The assumption considers the empirical fact that consumers establish a durable relationship based on trust and reciprocity to solve problems of asymmetric information (Bowles, 2009).

The *R*-script may be divided in three parts:

1. Creation of the link between customers and suppliers: it consists of assigning to each firm a random partner in the goods market. I use the function that draws a random number from a standard uniform distribution and transforms it to an integer between 0 and F. They represent the indexes of randomly assigned firms. The matching is fully random in the first period.
2. True matching protocol: I first select χ potential partners and record their selling price. Then I remark the best seller and the past one. Once they are recorded, they are compared: if the old partner sells at a lower price, nothing changes and I go to the next step; if the new partner charges a better price, I then apply the Bernoulli experiment

based on Eq. (19). When the outcome of this experiment is positive, the customer switches to the new seller.

3. Aggregation: last step simply aggregates the consumption expenditure of each household and distributes it to the selected partner.

```
# Matching process 2: the goods market

# a) Creation of the link for buyers and sellers
link_fc[1:N] = ceiling(runif(N)*F)



# b) Matching protocol

for (j in 1:N) {

  #Select potential partners
  newfc = ceiling(runif(chi)*F)
  #Select best among potential partners
  inew = min(p[newfc])
  #Pick up the old partner
  iold = p[link_fc[j]]

  #Compare old and new
  if (runif(1) <  (1-exp(chi_1*(inew-iold)/inew)) ) {
    #Switch to a new partner
    research = which(p[newfc]==min(p[newfc]))
    #Check if multiple partners
    if(length(research)>1){
      research = research[ceiling(runif(1)*length(research))]}
    #Update the link
    link_fc[j] = newfc[research[1]]
  }
  #Stick to the old partner
  else{link_fc[j]=link_fc[j]}}



# c) Aggregation of consumer expenditure
```

```
#Consumption faced by each firm
for (i in 1:F) {
  y[i] = sum(c_agents[link_fc==i])
}

} #End of time loop
```

3.3 Matching 3: Innovation and Transmission Mechanism

The last matching protocol I want to devise is about innovation and is somewhat different from the above. Although it can be integrated in the precedent framework without loss of generality, I sketch a simple process of innovation, and the related transmission mechanism, that considers interactions among firms only, with no reference to workers or consumers.[12] The economic literature always emphasized that innovation is very related with uncertainty since potential innovators do not know whether their effort and expenditures to promote technological development will succeed or not. Moreover, innovation turns out to be the most important force driving productivity and economic growth.[13] Although innovations occur at the micro-level of the economy through the introduction of novelties and many entrepreneurial decisions, its potentiality gets fully fledged at the industry or meso-level of economic activity (Dopfer et al., 2004).

The protocol portrays an economy in which firms invest on R&D to improve productivity. Determinants of R&D investments are beyond the scope of the present essay, and I hypothesize that firms invest in each period a random amount of funds drawn from a standard normal distribution. I then record the total expenditure the i-th firm did along its history, from t_0 to t, with $t = 1, \ldots, T$. Since negative values are meaningless, I round them up to zero. Such a simple though very abstract mechanism allows me to replicate some empirical fact of the literature. There is a lot of evidence, indeed, that points out the co-existence of R&D-performing firms with R&D non-performing firms and, among the former, the co-existence of firms that invest a lot of funds with others that invest much less (Dosi and Nelson, 2010; Pavitt, 1984). Difference in innovating behaviours will then result in persistent productivity differentials (Bartelsman and Doms, 2000).
I provide the reader with some basic definitions: I denote with a_{ii} the labour productivity in the i^{th} firm as result of its R&D effort, with $\boldsymbol{a}_{ij}$ the labour productivity in the i^{th} firm as

[12] Borsato (2020, 2021) further develops the present reasoning and adapts it to a general SFC framework including both Matching 1 and Matching 2.
[13] The literature on innovation is endless; I suggest Aghion and Howitt (2008); Dosi (1982, 1988); Dosi and Nelson (2010); Kline and Rosenberg (2010); Pavitt (1984, 1987); Rosenberg (1972, 1982); Schumpeter (1934, 1943) and Nelson and Winter (1982).

result of the imitation process, and a_i the effective labour productivity in the i^{th} firm at some point in time, that is equal to the maximum between a_{ii} and $\boldsymbol{a_{ij}}$. For simplicity, I assume their equality at the very beginning of the analysis, precisely $a_{i,t_0} = a_{ii,t_0} = \boldsymbol{a_{ij,t_0}} = 1$. Firms incur new loans to improve their technology levels. The literature often emphasizes the R&D expenditure as share of output as the determinant for the growth in productivity or for the innovation rate in the economy (Tavani and Zamparelli, 2017). In this contribution, however, I want to stress the role of the total amount of funds invested on innovative research. In fact, two firms may devolve the exact share but if the absolute amount differs, the larger firm will have higher probability to innovate than the smaller one. As said, the more a firm invests on innovative activities, the more probable it innovates. To represent this process, I can define a logistic probability distribution, λ_i, as an increasing function of the amount invested in R&D:

$$\lambda_i = \frac{1}{1 + e^{-\varepsilon \cdot \Sigma_{k=0}^{t} rd_{i,k}}} \tag{20}$$

Eq. (20) is the probability to innovate and it is a sinusoidal function approaching to 1 as cumulative R&D investments grow over time, $rd_{cum} = \Sigma_{k=0}^{t} rd_{i,k} \rightarrow \infty$. This condition means that the probability each firm has to innovate strictly depends on how much the same firm spends. The logistic function has been used quite often in the literature to illustrate the progress of creation and diffusion of an innovation through its life cycle (De Tarde, 1903). Precisely, the introduction of new products or processes in the economies spurs an intense amount of research and development leading to strong improvements in cost reduction and quality. The *mid-term* outcome consists of a rapid growth of that industry. Clear examples from the past are railroads, urban electrification, cars, light bulbs, and so on. However, once those improvements exhausted, new products or processes are so widespread that markets saturate. Back to Eq. (20), it is important to underline that λ changes from firm to firm, pointing that the ability and probability to introduce innovations are a direct function of own R&D effort. This way is tantamount to introduce path dependency and irreversibility in the model.

To know whether innovation occurs, every firm is assigned a random number drawn from a standard uniform distribution, $p_{inn,i} = \zeta_{1,i}$. If this number is smaller than the threshold λ_i, the firm innovates. Innovation takes place in the economy as an improvement in labour productivity. Labour productivity, $a_{ii,t}$, is a direct function of the outlay in innovation activities:

$$a_{ii,t} = a_{ii,t-1} + a_1 \cdot rd_{i,t-1} \tag{21}$$

in this way I take into account firm's ability to learn from past achievements.

The imitation process is similar to the innovative one. Let us look at entrepreneurs as if they were walking on the street. The single person has got a certain probability to meet somebody else. For simplicity, one person cannot meet more than three people in the same period. Moreover, meetings are fully random. We can image each meeting as the single possibility to copy the technology of the competitor. The imitation process occurs with the same law followed by the innovation process. Individuals make use of only local knowledge and make transaction with positive probability as long as it is beneficial to them. I define a $F \times F$ network matrix, called $\boldsymbol{imi_{net}}$. Its cells take value 1 if a connection between two firms is established, and 0 otherwise. Once I got all the linkages, I record in $\boldsymbol{a_{ij}}$ all the potential productivity levels that a firm can reach by imitating the technology of its competitors. Then, the firm compares the productivity levels from imitation and in-house innovation, choosing the best-performing technique and updating its productivity accordingly. As before, every firm is assigned a number drawn from a standard uniform distribution, $p_{imi,i} = \zeta_{2,i}$, which is compared to the λ_i threshold above. This procedure represents an important feature of the model: the probability the firm has to imitate strictly depends on its amount of innovative investments. I do exclude *free riders* or opportunistic behaviours in this way. Therefore, if $p_{imi,i} < \lambda_i$ a firm may imitate when $rd_i > 0$. Then:

$$a_i = max\left[a_{ii}; \boldsymbol{a_{ij}}\right] \tag{22}$$

Once all variables and parameters are defined, the *R*-script can be split into three sections again:

1. General setting: I first set the amount each firm spends for innovative search; secondly, I compute the threshold function λ_i, the random numbers $p_{inn,i}$ and $p_{imi,i}$, and lastly the imitation network.
2. The innovation process: a simple comparison between $p_{inn,i}$ and λ_i declares whether innovation occurs at time t. In-house productivity gains are accordingly updated.
3. The imitation or transmission mechanism: I first compare $p_{imi,i}$ with λ_i. If the i-th firm has access to imitation, it assesses whether imitating competitors' technologies results convenient or not. Best-performing techniques start spreading throughout the economy.

```
rm(list=ls(all=TRUE)) #Clear all
library(Matrix)


#Set the time span, the number of firms and the number of workers
#Number of periods
Time = 200
```

```
#Number of firms
F = 50

#Productivity at t=0
a0 = 1
#Coefficient in the productivity equation
a1 = 0.5
#Coefficient in lambda function
epsilon = 0.015
#Meetings per unit time
meet = 3
#Pseudo-random number generator
set.seed(10)



#Variables

#R&D investments
rd = matrix(data=0, ncol=1, nrow=F)
#R&D memory matrix
memord = matrix(data=0, ncol=Time, nrow=F)
#Cumulative R&D investment
rdcum = matrix(data=0, ncol=Time, nrow=F)
#Logistic Threshold function
lambda = matrix(data=0, ncol=1, nrow=F)
#Random number from U(0,1)
p_inn = matrix(data=0, ncol=1, nrow=F)
#Random number from U(0,1)
p_inn = matrix(data=0, ncol=1, nrow=F)
#Actual Productivity
a = matrix(data=a0, ncol=1, nrow=F)
#Productivity from home innovation
aii = matrix(data=a0, ncol=1, nrow=F)
#Productivity from imitation
aij = matrix(data=a0, ncol=meet, nrow=F)
#Productivity from best competitor
aij_max <- matrix(data=0, ncol=1, nrow=F)
```

```
#Time loop
for (t in 2:Time) {

#Matching 3: innovation and trasmission mechanism

# a) General Setting

#R&D investment at firm level
rd = rnorm(F,0,1)
rd[rd<0]=0
#R&D history
memord[,t] = rd
#Historical cumulative R&D
rdcum = rowSums(memord)

#Threshold logistic function
lambda = 1/(1+exp(-epsilon*rdcum))
#Random number
p_inn = runif(F)
#Random number
p_imi = runif(F)
#Network among firms
iminet = matrix(data=0, ncol=F, nrow=F)

  for (i in 1:F) {
    #Random matching among competitors
    iminet[i, sample(F, meet)] = 1

# b) Innovation

  #With positive R&D
  if (rd[i]>0) {

   #Innovation occurrence
    if (p_inn[i] < lambda[i]) {
```

```
   #Home-Productivity evolution
    aii[i] = aii[i] + a1*rd[i]}



# c) Imitation

   #Imitation occurrence
    if (p_imi[i] < lambda[i]) {
     #Record competitor productivity
      aij[i,] = aii[iminet[i,]==1]
     #Choose best competitor
      aij_max[i] = max(aij[i,])
     #Set effective productivity
      a[i] = max(aii[i], aij[i])}
    else{a[i] = aii[i]}
     }else{a[i] = a[i]}
  }
} #End Time loop
```

4 Conclusive Remarks

The writing of this article has been suggested by the very recent contributions of Veronese Passarella (2019) and Carnevali (2021). They presented small-to-medium scale SFC models that could act as benchmark framework for PhD students, early-career researchers, or whatever practitioners that try to depart from the standard neoclassical modelling. With the main framework and the software also adopted by Caiani et al. (2016b), I have developed and detailed three matching protocols that find, or have already found, direct application in the ACE literature. The implementation of simple linear matrix algebra could result helpful to not (yet) expert students whose aim consists of developing agent-based models from scratch, or it might be a nudge to address very complex issues through the inception of these protocols in their analyses. Although such protocols are inherently simple, they might be re-arranged to more complex frameworks and integrated into Monte Carlo runs and sensitivity analyses. Finally, albeit the code is written for *R*-like environments and implements linear matrix algebra, modelers can adapt the related *philosophy* to the software they prefer and to their own needs.

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