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Auteur

Verónica ACURIO VASCONEZ

Document de Travail n° 2020 – 08

Mars 2020

Bureau d'Économie Théorique et Appliquée
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www.beta-umr7522.fr

@beta_economics

Contact :
jaoulgrammare@beta-cnrs.unistra.fr

What if Oil was Less Substitutable?

Verónica ACURIO VASCONEZ*

March 2, 2020†

Abstract

The consequences of oil price shocks in the real economy have preoccupied economists since the 1970s and the absence of a reaction has stunned them in the 2000s. However, despite the huge literature devoted to the subject, no dynamic stochastic general equilibrium (DSGE) model has been able to capture, all at the same time, four of the well-known stylized effects observed after the oil price increase of the 2000s: the absence of recession, coupled with a low but persistent increase in the inflation rate, a decrease in real wages and low price elasticity of oil demand in the short run. One of the reasons is that theoretical papers assume a high degree of substitutability between oil and other factors, an assumption that is not backed up empirically. This paper enlarges the DSGE model developed in Acurio-Vásconez et al. (2015) by introducing imperfect substitutability between oil and other factors. The Bayesian estimation of the model over the period 1984:Q1-2007:Q3 suggests that the elasticities of substitution of oil are 0.086 in production and 0.014 in consumption. Furthermore, a sensitivity analysis of the estimated model points towards two main policy conclusions: (a) a stronger anti-inflationary Taylor rule can lead to a recession after an oil shock and; (b) wage flexibility could create a stronger increase in inflation and provoke a decrease in domestic consumption. This latter result contradicts the conclusions of Blanchard and Galí (2009) and Blanchard and Riggi (2013).

JEL Codes: D58, E32, E52, Q43

Keywords: New-Keynesian model, DSGE, oil, CES, stickiness, oil substitution.

1 Introduction

In the 1970s, it seemed that two of the most severe periods of recession in the U.S. after the Second World War had occurred just after sharp oil price increases. It is

*Email: veronica.acurio-vasconez@univ-lorraine.fr, Université de Lorraine, Université de Strasbourg, CNRS, BETA. 13 Place Carnot, CO No.026, 54035 Nancy Cedex, France.

†I am grateful to Cristiano Cantore, Gaël Giraud, Florent Mc Isaac and Pascale Duran-Vigneron for their useful comments. In addition I thank seminar participants at different conferences, specially in the IAEE conferences for their comments and reactions. All remaining errors are mine.

therefore no surprise that an enormous number of economists devoted efforts to looking for correlations between oil shocks and the business cycle, analyzing the impact of those shocks on developed economies, and studying the adjustments that they have induced. Hamilton (1983, 1988, 1996) found that seven of the eight postwar recessions in the U.S had been preceded by a dramatic increase in the price of oil, typically with a lag of around three-fourths of a year. Gisser and Goodwin (1986) found that crude oil prices had a significant impact on various macroeconomic indicators, often exceeding that of monetary policy and always that of fiscal policy. Bernanke et al. (1997) claimed, in contrast, that a substantial part of the recessionary impact of an oil shock resulted from the endogenous tightening of monetary policy, rather than from the increase in oil prices per se. Dotsey and Reid (1992) found that both oil price increases and movements in interest rates were significant in their statistical analysis of real GNP and employment. While not without its debates, the role of oil shocks in economic behavior was largely accepted by the mid-1990s.

However, oil prices rose from 34 dollars a barrel (real prices base 2013-2014) to 82 dollars between 2002 and 2007, an average increase of 19 percent per year, but inflation remained around 3 percent (1.3 percent increase between 2002 and 2007), while economic growth remained almost unchanged at a steady 2.7 percent annual growth rate. The unemployment rate stayed at around 5.3 percent, very close to its average over the previous 10 years, while real wages decreased by 0.4 percent between 2002 and 2005. A decrease in output and increase in the unemployment rate were both just visible by the end of 2008, in the aftermath of the subprime crisis. This apparently different reaction of the economy to oil price increases raised new questions about the interaction between oil and the real economy and, as in the 1970s, a large amount of research emerged in search of an explanation.

Some, such as Barsky and Kilian (2004), raised some ideas to explain the relationship between oil and the economy and studied the origin of oil shocks. They concluded that the alleged link between oil price changes and macroeconomic performance may have been overstated and that none of the major oil price increases since the 1980s had been associated with stagflation. In a survey, Hamilton (2011) noted that the key mechanism whereby energy price shocks affected the economy was through the disruption spending by consumers and firms on goods and services other than energy. Edelstein and Kilian (2009) tested the different transmission channels of energy shocks on consumer spending. They found that a part of the responses of real consumption aggregates could be attributed to shifts in precautionary savings and changes in the operating cost of energy-using durable goods. However, they also found that the energy price shocks of 1974, 1979-1981, 1990 and 2003-2007 were an important factor in explaining U.S. real consumption growth, but by no means the dominant factor. Hamilton (2009) studied the similarities and differences between the oil price increase in 2007-08 and earlier oil price shocks. He also pointed out that although the causes of oil shocks are different (physical supply disruptions in 1970s and strong demand combined with stagnating world production in 2007-2008), the consequences for the economy appeared to have been similar in both episodes. Blanchard and Galí (2009) approached the problem in two different manners: first by analyzing a Structural Vector Autoregressive Model (sVAR) and then by constructing a Dynamic Stochastic General Equilibrium Model (DGSE), which included oil in production and consumption. In a companion paper, Blanchard and Riggi (2013) estimated some of the parameters of a similar version of the DSGE model developed in Blanchard and Galí (2009), using minimum distance estimation techniques. Those two papers gave three possible explanations for the attenuation of oil shocks: (1) the reduction of the share of oil in production; (2) the flexibilization of real wages and; (3) improvements in monetary policy.

The existence of these debates makes us wonder whether we really understand how oil shocks spread to the economy and, most importantly, what kind of policy could be implemented to help lessen their effects. The motivation of this paper is therefore two-fold. First, to construct a model that represents the real economy more effectively and which is able to recover most of the stylized facts observed after the oil shock of 2000s and explain its transmission channels. Secondly, within this model, to analyze possible policy actions that can attenuate the impact of oil shocks.

Due to the increased popularity of DSGE models for policy analysis, I used this modeling technique to address these questions. However, to my knowledge, none of the few DSGE models that include oil and focus on the macroeconomic effects of oil shocks, such as Blanchard and Galí (2009), Kormilitsina (2011), Montoro (2012), Blanchard and Riggi (2013), Acurio-Vásconez et al. (2015), is able to recover, all at the same time, three of the well-known stylized facts observed after the oil shock in the 2000s: the absence of recession coupled with a low but persistent increase in the inflation rate, a decrease in real wages and a low price elasticity of oil demand in the short term. I believe that one of the reasons is that most of these models[1] assume a high degree of substitutability between oil and other factors, an assumption that is not backed up empirically.

The question of oil substitutability has seen increasing interest from economists over the last decade. One of the reasons for this can be found in Figure 1, which shows oil consumption per capita in the U.S. and the real oil price since 1970. Oil consumption per person remained near-constant from 1990 to 2012, even though its real price increased continuously. If we look at the U.S. industrial sector separately, the same phenomenon appears, as shown in Figure 2 showing oil consumption in the industrial sector in the U.S. Moreover, in this last case, there was an increasing trend in oil consumption from 1981 to 2008. Then, while it is true that firms and households can react to changes in oil prices (e.g. by shifting away from oil towards capital or labor in the case of firms and to final domestic goods consumption for households), this substitution has become less evident in the last few years in the U.S. One possible reason for this stagnation is the well known "rebound effect": even if there has been increasing oil productivity since the 1970s, resulting in an improvement in oil utilization, it has not generated any mitigation of global oil consumption, as illustrated by the automotive industry. A car with the same characteristics and power as a car 30 years ago now uses much less oil to travel the same distance. Nevertheless, today we can buy a better car at a relatively cheaper price than 30 years ago, but in the end it does not consume much less. Moreover, people who

[1] With the exception of Montoro (2012). However, the model of Montoro (2012) does not include capital accumulation, and the model is not estimated.

could not afford a car 30 years ago can now buy one, which again increases the quantity of oil use. Another reason is that the U.S. economy, like most industrialized economies, is still heavily oil dependent and that given current technologies, it is hard to substitute other energy sources for oil, at least in the short-term.

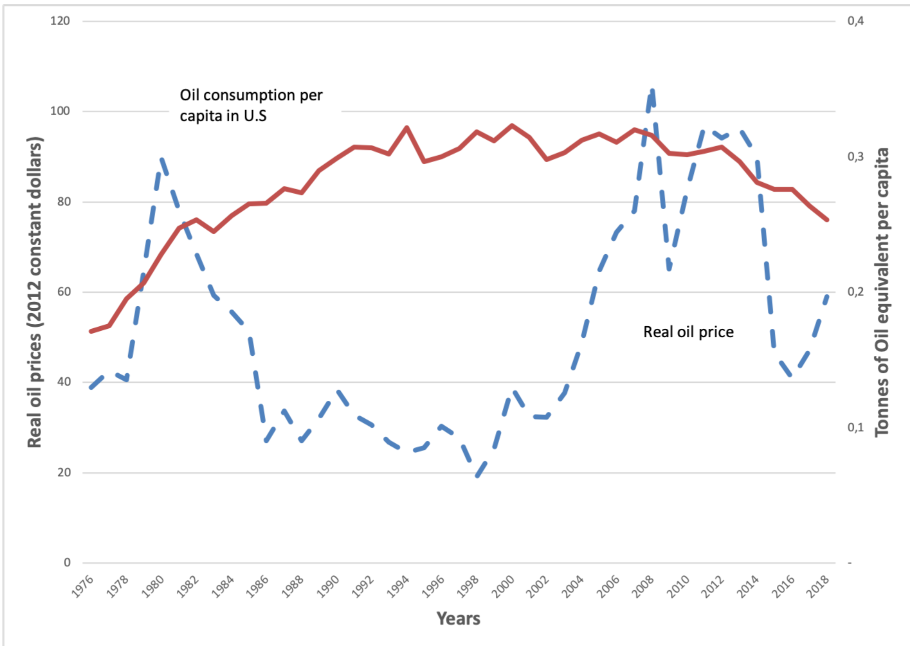

Figure 1: US Oil Consumption per Capita (toe) and real Spot Oil Prices base 2012

Source: BP Statistical Review of World Energy 2019, Federal Reserve of Saint Louis

The recent empirical literature has stated that energy is a critical input in industrialized economies and that it is not perfectly substitutable to other production factors. In Fouré et al. (2012), Hassler et al. (2012), van-der Werf (2008) and Kander and Stern (2012) among others, energy (or fossil energy) was introduced into the production function through a constant elasticity of substitution (CES) function with two factors: energy and a Cobb-Douglas combination of capital and labor. Each of these papers estimated the energy elasticity of substitution using different methods. Hassler et al. (2012) used a maximum-likelihood approach with data from the US; van-der Werf (2008) used linear regressions with data from several countries and industries; Kander and Stern (2012) constructed an extension of Solow's growth model and estimated its parameters using data from Sweden and linear regressions. Each of these papers gave different estimations for the energy elasticity of substitution with respect to labor and capital in different combinations, with values ranging from 0.004 to 0.64. However, they all rejected the assumption of a substitution elasticity equal to one. In Kumhof and Muir (2014), the authors used a CES function to model oil demand and interpreted the elasticity of substitution between oil and the composite factor as the long-run price elasticity of oil demand. This value was estimated in Helbling et al. (2011) and Benes et al. (2015) to be 0.08. Other examples can be found in Lindenberger and Kümmel (2010), who used a production function where output elasticities were not equal to cost shares and established that energy-dependent production functions reproduced past economic growth with a zero Solow residual, or in Hassler et al. (2012), who constructed a model of directed technical change, where the production function was Leontief and found that the economy directed its efforts toward input saving so as to economize on expensive or scarce inputs. Most recently, Henriet et al. (2014) introduced fossil fuel with CES functions in a Computational General Equilibrium Model (CGE). They estimated the elasticities of substitution with French data using cointegration methods and linear regressions and found that they were equal to 0.5 in both sectors, production and household consumption.

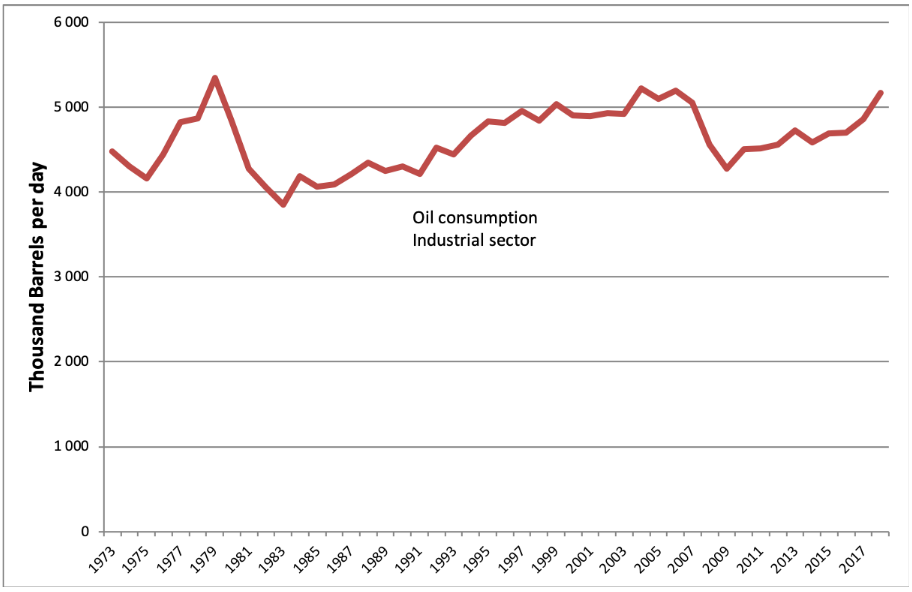

Figure 2: Total Petroleum Consumed by the Industrial Sector (Millions Barrels per Year)

Source: EIA. Table 3.7b. Petroleum Consumption: Industrial Sector

This paper takes this evidence into account and enlarges the model developed in Acurio-Vásconez et al. (2015), in which oil is incorporated into a DSGE model through a Cobb-Douglas function in the consumption flow and in the production function of intermediate firms. The production function I use here is an integrated CES function, constructed as in Hassler et al. (2012) and re-normalized as in Cantore and Levine (2012). This function includes oil, which is fully imported from a foreign economy, and a Cobb-Douglas combination of labor and capital. On the household side, I used a basic CES function that integrates final goods and oil to define the consumption flow.

Along with this framework, the model adds stickiness in nominal prices and wages. This last element allows me to analyze one of the conclusions given in Blanchard and Galí (2009) and Blanchard and Riggi (2013) regarding the softer impact on the economy after an oil shock, which is the reduction of wage rigidity. However, alternatively to the ad-hoc formulation of the real wage stickiness introduced in those papers, this paper adds nominal wage rigidity in a more conventional way, using a framework *à la* Calvo.

Once the model had been built and log-linearized around its steady state, it was estimated using Bayesian methods and quarterly U.S. data over the period 1984:Q1 - 2007:Q1. The estimated elasticities of substitution of oil were 0.086 in production and 0.014 in household consumption. These values exhibit the fact that oil is weakly substitutable to other quantities in both sectors. Another significant result of estimation was the posterior mean of oil share in consumption at steady state, which was estimated at 0.08, a value 5 times greater than that assumed in Blanchard and Galí (2009) and Blanchard and Riggi (2013).

The model was able to recover and explain the four well-known stylized facts after an oil price shock in the 2000s listed previously. Furthermore, this paper identified another channel to explain why we did not observe a decrease in GDP after the oil shock of the 2000s. If oil is not easily substitutable and is fully imported from a foreign economy, an increase in its price causes firms to produce more, partly in order to compensate for the increased oil bill. In this way, most of the domestic production and oil importation cancel each other out. The reaction of GDP to an oil shock could thus be close to nil. Furthermore, the use of a stronger anti-inflationist Taylor rule can lead to a recession after an oil shock. It is also important to note that due to the low substitutability of oil, in order to increase domestic production, firms also need to increase oil demand. This increase should not be problematic as long as the U.S. economy can import as much oil as needed. However, in a world where oil supply has entered a period of increased scarcity, the consequences could be a loss of output[2], as shown in Kumhof and Muir (2014), Bezdek et al. (2005), Reynolds (2002), among others.[3]

Moreover, a sensitivity analysis showed that a decrease in nominal wage rigidity in the estimated model, ceteris paribus, could lead to an increase in real wages, which then leads to higher prices, confining households to a worse trade-off between consumption and investment, in favor of investment. Then, contrary to the conclusions of Blanchard and Galí (2009) and Blanchard and Riggi (2013), wage flexibility can generate a greater increase in inflation and a decrease in consumption.

The rest of the paper is structured as follows. Section 2 presents the DSGE model. Section 3 describes the elements of the Bayesian estimation and examines its results. Section 4 analyzes the impulse response of the economy to a real oil price shock and discusses how the economy would respond under more flexible wages. Finally, Section 5 concludes.

2 Model

As in Acurio-Vásconez et al. (2015), this paper constructs a DSGE model that considers oil, labor and capital as inputs for intermediate firms, and where households can

[2] It should not be forgotten that the U.S economy is a major producer of oil nowadays and this result will be revised in a forthcoming companion paper where we allow for domestic energy production.

[3] See also references therein.

consume final domestic goods and oil. As was assumed in Blanchard and Galí (2009), Blanchard and Riggi (2013) and Acurio-Vásconez et al. (2015), oil is imported from a foreign country at an exogenous real price.[4] Price and wage stickiness were also introduced, and the model considers that the consumption flow and the intermediate production function are CES type. This section will first describe how households consume, work, hold capital and use oil. Then it will describe how firms use different inputs to produce intermediate goods that will be transformed by the final goods firm into a single aggregate final good. Finally, I will explain how the government intervenes in the economy.[5]

2.1 Households

We assumed a continuum of monopolistically competitive households indexed by $j \in [0, 1]$. Each of them consumes both oil and domestic goods, supplies a differentiated labor service to the production sector, invests in government bonds and capital, pays taxes, and receives profits from the firms in the economy.

At each period t, each household has an instantaneous utility function, which is assumed to be separable into consumption $C_t(j)$ and hours worked $L_t(j)$ and given by:

$$U(C_t(j), L_t(j)) = log(C_t(j) - hC_{t-1}) - \frac{L_t(j)^{1+\phi}}{1+\phi}$$

where $\phi \in \mathbb{R}^+$ is the inverse of the Frisch elasticity and $h \in [0, 1)$ represents the external habit formation parameter. Each household can consume two different types of goods: a domestic good at nominal price P_q that is produced inside the country and oil, which comes from a foreign country at nominal price P_e.[6] The consumption flow of household j is defined as:

$$C_t(j) := \left((1-x_c)^{1-\sigma} C_{q,t}^{\sigma}(j) + x_c^{1-\sigma} C_{e,t}^{\sigma}(j)\right)^{\frac{1}{\sigma}}$$

where $C_{e,t}(j)$ stands for the oil consumption of household j and

$$C_{q,t}(j) = \left(\int_0^1 C_{q,t}(i,j)^{\frac{\epsilon_p - 1}{\epsilon_p}}\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

represents the domestic consumption of household j, where $i \in [0, 1]$ indexes the type of good, $\epsilon_p > 0$ is the elasticity of substitution between domestic goods and x_c a distribution parameter. Defining $\sigma = \frac{\eta_c - 1}{\eta_c}$, η_c represents the elasticity of substitution

[4]Although variations in the price of oil can have a significant endogenous component, we based ourselves on this assumption because, as shown in the sVAR of Blanchard and Galí (2009), the correlation between quantities and oil prices is very weak when using quarterly data. Furthermore, the period of estimation stops in 2007, because after this date, the U.S. economy became a large exporter of shale gas, and this may have had an impact on oil prices.

[5]For more details on the model's construction, refer to the Appendix.

[6]Thus P_q could be interpreted as being core CPI, meaning CPI without gasoline and other energy goods.

between domestic goods and oil consumption. Note that when η_c is equal to one, the consumption flow collapses to being Cobb-Douglas in domestic and oil consumption; when η_c is equal to 0, there is a Leontief function between factors; and when η_c goes to $+\infty$, we get a linear function, meaning that the factors are perfect substitutes.

The j-th household allocates its expenditures among these different goods, i.e. it maximizes its consumption subject to its budget constraint $P_{c,t}C_t(j) = P_{q,t}C_{q,t}(j) + P_{e,t}C_{e,t}(j)$, where $P_{c,t}$ stands for the CPI price index.[7] Solving this problem gives the following consumption demand functions:

$$C_{q,t}(j) = (1 - x_c)\left(\frac{P_{q,t}}{P_{c,t}}\right)^{-\eta_c} C_t(j), \quad C_{e,t}(j) = x_c\left(\frac{P_{e,t}}{P_{c,t}}\right)^{-\eta_c} C_t(j) \tag{1}$$

and the equation for the CPI index:

$$P_{c,t} = \left((1 - x_c)P_{q,t}^{1-\eta_c} + x_c P_{e,t}^{1-\eta_c}\right)^{\frac{1}{1-\eta_c}} \tag{2}$$

On the other hand, a typical household j, seeks to maximize the following lifetime discounted utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t(j), L_t(j))\right]$$

where $\beta \in (0, 1)$ is the discount factor. Household j also holds an amount $B_t(j)$ of government bonds that pay a nominal short-run interest rate i_t, which is set by the Central Bank, lends capital $K_t(j)$ at price $P_{k,t}$[8] with real rental rate r_t^k and receives a nominal wage $W_t(j)$ for its work. Then, the j-th household's nominal budget constraint is:

$$P_{c,t}C_t(j) + P_{k,t}I_t(j) + B_t(j) \leq (1 + i_{t-1})B_{t-1}(j) + W_t(j)L_t(j) + D_t + r_t^k P_{k,t}K_t(j) + T_t \tag{3}$$

where D_t is the nominal profit of the firms in the economy,[9] T_t is the lump-sum transfers and $I_t(j)$ represents the investment of the j-th household. I will assume that the dynamics of capital accumulation follows:

$$I_t(j) := K_{t+1}(j) - (1 - \delta)K_t(j) \tag{4}$$

where $\delta \in (0, 1)$ is the depreciation rate.

Solving the maximization problem, we can derive Euler's and Fisher's equations,

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[7] Defined as the minimum expenditure required to buy one unit of C_t.

[8] As explained in Acurio-Vásconez et al. (2015), the assumption that the capital price is the same as the final good price prevents us from capturing decoupled bubble phenomena, such as the housing bubble that has affected most Western countries since the middle of the 1990s.

[9] I assume that each household owns an equal share of all firms and receives an aliquot share $D_t(j)$ of aggregate profits, i.e. the sum of dividends of all intermediate goods firms, so $D_t(j) = D_t := \int_0^1 D_t(i)di$ where i indexes the firms.

which take the following form:[10]

$$1 = \beta \mathbb{E}_t \left[(1+i_t) \frac{1}{\Pi_{c,t+1}} \frac{\lambda_{a,t+1}(j)}{\lambda_{a,t}(j)} \right] \tag{5}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{a,t+1}(j)}{\lambda_{a,t}(j)} \frac{1}{\Pi_{c,t+1}} \left(r^k_{t+1} + 1 - \delta \right) \Pi_{k,t+1} \right] \tag{6}$$

where $\Pi_{c,t} = \frac{P_{c,t}}{P_{c,t-1}}$ is CPI inflation, $\Pi_{k,t} = \frac{P_{k,t}}{P_{k,t-1}}$ is capital price inflation and $\lambda_{a,t}$ represents the marginal lifetime discounted utility function at t. Assuming external habit formation, we get:

$$\lambda_{a,t}(j) = \left[\frac{1}{C_t(j) - hC_{t-1}} \right]$$

In order to ensure that a solution for the household problem exists, the following transversality condition (no Ponzi game) will be imposed:

$$\lim_{k \to +\infty} E_t \left[\frac{B_{t+k}(j)}{\prod_{s=0}^{t+k-1} (1 + i_{s-1})} \right] \geq 0, \quad \forall t, \forall j$$

Let me now describe the first order condition for labor. Assume that each one of the households supplies a differentiated labor service to the production sector, meaning that the intermediate firms see each household's labor services, $L_t(j)$, as an imperfect substitute for the labor services of other households.

Following Erceg et al. (2000), I assume that there is a perfectly competitive labor "packer", which could be interpreted as an employment agency, which combines households' labor hours in the same proportion as firms would choose. The labor used by the intermediate goods producers is thus supplied by this labor "aggregator" that follows the following CES production function:

$$L^d_t := \left(\int_0^1 L_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

where $\epsilon_w > 0$ is the constant elasticity of substitution among different types of labor. The "packer" maximizes profits subject to the labor demand addressed to it, taking as given all differentiated labor wages $W_t(j)$ and the wage W_t, which is the price at which the "packer" sells one unit of labor index to the production sector. The first order condition of this problem yields the following equation:

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} L^d_t, \quad \forall j \tag{7}$$

which represents the aggregate demand for labor hours of household j. The zero profit condition implied by perfect competition states that:

$$W_t L^d_t = \int_0^1 W_t(j) L_t(j) dj$$

[10]Cf. Appendix for derivation.

Consequently, we have the following level price:

$$W_t = \left(\int_0^1 W_t^{1-\epsilon_w}(j)dj \right)^{\frac{1}{1-\epsilon_w}} \tag{8}$$

W_t can be interpreted as the aggregate wage index.

I will assume that households set their wages following a Calvo setting framework. In each period t, only a fraction $(1-\theta_w)$ of households can re-optimize their nominal wage $(W_t(j) = W_t^o(j))$. The remaining ones leave their wage as before $(W_t(j) = W_{t-1}(j))$. Each household that can change its wage will choose $W_t^o(j)$ in order to maximize:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k U \left(C_{t+k|t}(j), L_{t+k|t}(j) \right) \right]$$

under the same budget constraint described in (3) and the labor demand defined in (7). Note that $C_{t+k|t}(j)$ and $L_{t+k|t}(j)$ respectively denote the consumption and labor supply in period $t+k$ of a household that last resets its wage in period t. The solution of this problem yields:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \lambda_{a,t+k}(j) L_{t+k|t}(j) \left[\frac{W_t^o}{P_{c,t+k}} - \mathcal{M}_w MRS_{t+k|t}(j) \right] \right] = 0$$

where $\mathcal{M}_w = \frac{\epsilon_w}{\epsilon_w - 1}$ is the wage markup and $MRS_{t+k|t}(j) := -\frac{U_L\left(C_{t+k}(j), L_{t+k|t}(j)\right)}{\lambda_{a,t+k}(j)}$ the marginal rate of substitution between consumption and hours worked in period $t+k$ for the household that can reset its wage in t.

Finally, this assumption of Calvo wage setting gives us the following "Aggregate wage relationship":

$$W_t = \left(\theta_w W_{t+1}^{1-\epsilon_w} + (1-\theta_w) W_t^{o1-\epsilon_w} \right)^{\frac{1}{1-\epsilon_w}}$$

2.2 Final Good Firm

There is a continuum of intermediate goods indexed by $i \in [0,1]$, that are used in the production of the single final aggregate good (which will be the domestic consumption commodity). This firm has a CES production function given by:

$$Q_t := \left(\int_0^1 Q_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

For simplicity, I assume that no energy is needed in the production of the final good.

Given all the intermediate good prices $(P_{q,t}(i))_{i \in [0,1]}$ and the final good price $P_{q,t}$, the final good firm chooses the quantities of intermediate goods $(Q_t(i))_{i \in [0,1]}$ in order to maximize its profit. The solution of this problem gives:

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t, \quad \forall i$$

which is the demand for good i.

Remark that the production function of the final good firm is constant return to scale and this firm is perfectly competitive, meaning that the zero profit condition holds at equilibrium. We therefore obtain the following equation for the price of the final aggregate good:

$$P_{q,t} = \Big(\int_0^1 P_{q,t}(i)^{1-\epsilon_p} di \Big)^{\frac{1}{1-\epsilon_p}}.$$

2.3 Intermediate Good Firms

There is a continuum of monopolistically competitive intermediate goods producers indexed by $i \in [0,1]$, that produce a differentiated good. Each of them is represented by a nested CES production function involving oil, capital and labor, as in Hassler et al. (2012) in the following scheme:[11]

$$Q_t(i) := \big(x_p (A_{E,t} E_t(i))^\rho + (1-x_p)(A_{LK,t} K_t(i)^\alpha L_t^d(i)^{1-\alpha})^\rho \big)^{1/\rho} \tag{9}$$

where $E_t(i)$ is the quantity of oil used, $K_t(i)$ is the capital rented and $L_t^d(i)$ is the amount of the "packed" labor input rented by the intermediate firm i. Variables $A_{E,t}$ and $A_{LK,t}$ represent respectively a measure of oil productivity and the total factor productivity (TFP). The latter measures the productivity of the combination of labor and capital. The "share" of capital in the composite factor is measured by $\alpha \in [0,1]$. Defined $\rho = \frac{\eta_p - 1}{\eta_p}$, then η_p is the elasticity of substitution between the utilization of oil and the composite factor (of capital and labor). Finally, x_p is a distribution parameter. Similarly to the consumption flow, when η_p is equal to zero, then the composite factor and oil are complements; when η_p is equal to 1, then we get a Cobb-Douglas function of these two factors; and when η_p tends to $+\infty$, both factors are perfect substitutes. Both technologies processes are assumed to be $AR(1)$ processes:

$$log(A_{E,t}) = \rho_{ae} log(A_{E,t-1}) + e_{ae} \quad \text{and} \quad log(A_{LK,t}) = \rho_{alk} log(A_{LK,t-1}) + e_{alk}$$

where $e_{ae} \sim \mathcal{N}(0, \sigma_{ae}^2)$ and $e_{alk} \sim \mathcal{N}(0, \sigma_{alk}^2)$.

Each firm maximizes its profit. I will study this problem in two stages: (1) each firm takes prices $P_{e,t}$, $P_{k,t}$, W_t, the real rental rate of capital r_t^k and demand $Q_t(i)$ as given, then it chooses quantities of oil $E_t(i)$, labor $L_t^d(i)$, and capital rent $K_t(i)$ in perfectly competitive factor markets in order to minimize cost. (2) Firm i chooses price $P_{q,t}(i)$ in order to maximize its profit. I will consider staggered prices *à la*

[11] van-der Werf (2008) showed that the nesting structure that fits the data best is when labor and capital are combined first and then the composite factor is combined with oil. He also showed that a nested combination of capital and labor is appropriate, but in the interests of simplicity, I followed Hassler et al. (2012) and Fouré et al. (2012) and assumed that capital and labor are combined in a Cobb-Douglas function.

Calvo. The first order conditions of the minimization problem give:

$$\begin{aligned}
E_t(i): &\quad P_{e,t} = \lambda_t(i) x_p A_{E,t}^{\rho} Q_t(i)^{1-\rho} E_t(i)^{\rho-1} \\
L_t^d(i): &\quad W_t = \lambda_t(i)(1-\alpha) Q_t(i)^{1-\rho}(1-x_p) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho} L_t^d(i)^{(1-\alpha)\rho-1} \\
K_t(i): &\quad r_t^k P_{k,t} = \lambda_t(i)\alpha Q_t(i)^{1-\rho}(1-x_p) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho-1} L_t^d(i)^{(1-\alpha)\rho}
\end{aligned}$$

and so:

$$\begin{aligned}
\text{Marginal Cost } (MC_t) = \quad \lambda_t &:= \frac{P_{e,t}}{x_p A_{E,t}^{\rho} Q_t(i)^{1-\rho} E_t(i)^{\rho-1}} \\
&:= \frac{W_t}{(1-\alpha) Q_t(i)^{1-\rho}(1-x_p) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho} L_t^d(i)^{(1-\alpha)\rho-1}} \\
&:= \frac{r_t^k P_{k,t}}{Q_t(i)^{1-\rho}\alpha(1-x_p) A_{LK,t}^{\rho} K_t(i)^{\alpha\rho-1} L_t^d(i)^{(1-\alpha)\rho}}
\end{aligned} \tag{10}$$

Because the intermediate firm technology is constant return to scale, it can be demonstrated that the marginal cost does not depend on i: all firms receive the same technology shock and all firms rent inputs at the same price.

In the second stage, intermediate firms choose the price that maximizes their profits. I consider that those prices are set under the same pricing scheme as households' wages. In each period, a fraction $(1-\theta_p)$ of firms can change their prices $(P_{q,t}(i) = P_{q,t}^o(i))$, while the remaining ones leave their prices unchanged $(P_{q,t}(i) = P_{q,t-1}(i))$. Each firm that can reset its price will choose the same new one, so the choice of $P_{q,t}^o(i)$ will not depend on i. The first order condition of this problem gives us:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta_p^k d_{t,t+k} Q_{t+k|t}^o \left(P_{q,t}^o - \mathcal{M}_p mc_{t+k|t}^o \right) \right] = 0 \tag{11}$$

where $Q_{t+k|t}^o := \left(\frac{P_{q,t}^o}{P_{q,t+k}} \right)^{-\epsilon_p} Q_{t+k}$ for every $k \geq 0$, $MC_{t+k|t}^o := MC_{t+k}$, $d_{t,t+k}$ is the stochastic discount factor from date t to $t+k$ defined as:

$$d_{t,t+k}(j) := \beta^k \frac{\lambda_{a,t+k}(j)}{\lambda_{a,t}(j)} \frac{P_{c,t}}{P_{c,t+k}}$$

and $\mathcal{M}_p = \frac{\epsilon_p}{\epsilon_p - 1}$ is the price gross markup.

We also get the following "Aggregate price relationship:"

$$P_{q,t}^{1-\epsilon_p} = \left(\theta_p P_{q,t-1}^{1-\epsilon_p} + (1-\theta_p) P_{q,t}^{o\;1-\epsilon_p} \right)$$

2.4 GDP, Monetary Policy and Government

As in Acurio-Vásconez et al. (2015), I defined real GDP (Y_t) as follows:

$$P_{c,t} Y_t = P_{q,t} Q_t - P_{e,t} E_t$$

Let $\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$ be domestic inflation. Let us suppose that the Central Bank sets the nominal short-term interest rate by the following monetary policy:

$$1 + i_t = (1 + i_{t-1})^{\phi_i} \left(\frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right)^{1-\phi_i} \varepsilon_{i,t},$$

where Y represents the steady state of Y_t, $log(\varepsilon_{i,t}) = \rho_i log(\varepsilon_{i,t-1}) + e_{i,t}$ and $e_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$.[12]

Finally, the Government budget constraint is given by:

$$(1 + i_{t-1}) \int_0^1 B_{t-1}(j)dj + G_t = \int_0^1 B_t(j)dj + T_t,$$

where G_t stands for nominal government spending. I assume that real government spending $G_{r,t} = \frac{G_t}{Pq,t}$ is an exogenous process given by:

$$log(G_{r,t}) = (1 - \rho_g)(log(\omega Q)) + \rho_g log(G_{r,t-1}) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}$$

where ω represents the share that the government takes from domestic output Q_t for its own spending, Q represents the steady state of Q_t, and $e_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$ is Gaussian white noise.

2.5 Real Prices and Stochastic Processes

All real variables are defined in relation to domestic prices P_q. The real price of oil, $S_{e,t}$, and the real price of capital, $S_{k,t}$, are thus given by

$$S_{e,t} := \frac{P_{e,t}}{P_{q,t}}, \quad S_{k,t} := \frac{P_{k,t}}{P_{q,t}}$$

Following Blanchard and Galí (2009) and Acurio-Vásconez et al. (2015), I suppose that the real price of oil is exogenous. Finally, following Acurio-Vásconez et al. (2015), I assume that the real price of capital is exogenous as well. Each of them follows an $AR(1)$ process in the form:

$$log(S_{e,t}) := \rho_{se} log(S_{e,t-1}) + e_{e,t}, \quad log(S_{k,t}) := \rho_{sk} log(S_{k,t-1}) + e_{k,t}$$

where $e_{e,t} \sim \mathcal{N}(0, \sigma_e^2)$ and $e_{k,t} \sim \mathcal{N}(0, \sigma_k^2)$ are Gaussian white noise.

3 Parameter Estimates

3.1 Setting

Aggregation and the steady state calculation are shown in the Appendix, along with the log-linear version of the model. The time period is a quarter. The model

[12] Remark that in this definition, the parameter ϕ_y measures the reaction of the Central Bank to the deviation of GDP from its steady state.

is estimated with Bayesian estimation techniques.[13] The period of estimation goes from 1984:Q1 to 2007:Q1. As explained in Acurio-Vásconez et al. (2015), the dataset starts in 1984 because the well-known structural break occurred at this date. As explained in the introduction, the dataset stops in 2007 because after this date, the U.S economy became an oil-exporter, but also because of the 2007-2008 crisis.

The estimation could be made with the same six quarterly macroeconomic U.S. time series used in Acurio-Vásconez et al. (2015) as observable variables: real GDP, real investment, hours worked, GDP deflator, oil expenditure in production and the Federal Funds Rate. However, using just these six series and the six shocks previously described, the η_c parameter which measures the elasticity of substitution of oil in consumption is not identifiable. This lack of identification in the estimation could lead to incorrect results. In order to be able to identify all the parameters besides the calibrated ones, I added two series to the six aforementioned: real domestic consumption and real wages;[14] and two ad-hoc shocks: one on the dynamic equation for wage inflation and one on the dynamic equation for price inflation.[15] These shocks could be interpreted as a wage markup and a price markup shock and are assumed to follow $ARMA(1,1)$ processes respectively of form:

$$\varepsilon_{w,t} = \rho_w \varepsilon_{w,t-1} + e_{w,t} - \nu_p e_{w,t-1}, \quad \varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1}$$

where $e_{w,t} \sim \mathcal{N}(0, \sigma_w^2)$ and $e_{p,t} \sim \mathcal{N}(0, \sigma_p^2)$ are Gaussian white noises.

3.2 Prior Distribution of Parameters

Before estimation, five parameters were calibrated according to the literature. The discount factor, β, was set to 0.99, so that the risk-less annual return is about 4 percent. The depreciation rate, δ, was calibrated at 0.025 which means 10 percent annual depreciation. The government spending output share, ω, was fixed at 18 percent. I calibrated ϵ_p and ϵ_w at 8, which gives us a price and wage markup approximately equal to 1.14.[16] Those values are summarized in Table 1.

Following Cantore et al. (2017), I assumed that elasticities of substitution, η_c and η_p, follow a gamma distribution with the mean being the estimated value in van-der Werf (2008) for the U.S., which is equal to 0.54 and standard deviation 1[17].

[13] All estimations are done with Dynare version 4.5.3 (Dynare (2011)), in Matlab 2016, using the Monte-Carlo based optimization routine. Two tests are available to check the stability of sample generation using the MCMC algorithm, implemented in Dynare: The MCMC diagnostic (Univariate convergence diagnostic, Brooks and Gelman (1998)) and a comparison between and within moments of multiple chains.

[14] Domestic consumption is measured as being the real PCE minus the real PCE of Gasoline and other energy goods. Real wages are measured with the real hourly compensation series. An extended explanation of the series and its transformation can be found in the Appendix.

[15] In equations (33) and (42) of the log-linearized model.

[16] Those values are commonly used for the U.S economy. See, for exemple, Smets and Wouters (2007), Erceg et al. (2000) and references therein.

[17] No empirical work to my knowledge has try to estimate the value of oil substitution in consumption, here η_c, for U.S. Then I assumed that its prior was the same as for the elasticity of substitution in production.

Table 1: Calibrated Parameters

β	δ	ϵ_p	ϵ_w	ω
0.99	0.025	8	8	0.18

This prior concentrates the probability mass around the prior mean and allows the parameter to move from 0 to $+\infty$. Distribution parameters in the CES functions, x_c and x_p, have to be estimated as well. Following Cantore and Levine (2012), and as shown in the Appendix, at steady state, parameter x_c is equal to the share of oil consumption in total consumption expenditures. Then it is assumed to be Normal distributed with standard deviation 0.03, and mean 3 percent, which is the mean of the series generated by dividing the nominal Personal Consumption Expenditures: Gasoline and other energy goods by the Personal Consumption Expenditure (PCE) in the observation period. As pointed out by Cantore and Levine (2012), in order to be able to estimate the distribution parameter in the production function, x_p, a renormalization is necessary.[18] We can re-write equation (9) as:

$$\frac{Q_t(i)}{Q} := \left(\alpha_e \left(\frac{A_{E,t} E_t(i)}{A_E E} \right)^{\rho} + (1 - \alpha_e) \left(\frac{A_{LK,t} K_t(i)^{\alpha} L_t^d(i)^{1-\alpha}}{A_{LK} K^{\alpha} {L^d}^{1-\alpha}} \right)^{\rho} \right)^{1/\rho}$$

where variables without a time subscript represent the steady state of the equally named variable and α_e represents oil output elasticity at its steady state.[19] Additionally, it can be shown that:

$$x_p = \alpha_e^{\frac{1}{\eta_p}} \left(\frac{\mathcal{M}_p S_e}{A_E} \right)^{\frac{\eta_p - 1}{\eta_p}}$$

If we assume that the steady state of the real oil price, S_e, and the steady state of the productivity oil shock, A_E, are equal to one, then we get:[20]

$$\alpha_e = \frac{\mathcal{M}_p * \text{Oil Cost share}}{1 + \text{Oil Cost share}} \tag{12}$$

As pointed out by Kumhof and Muir (2014), the cost share of oil in the last years is around 3.5 percent which, with a markup of 1.14, gives us a steady state for oil output elasticity equal to 3.9 percent. Then, following Cantore et al. (2017), I assumed that oil output elasticity, α_e, follows a Normal distribution with mean 3.9 percent and standard deviation 0.03. The remaining parameter priors are taken as in Smets and Wouters (2007).

[18] Cf. Cantore and Levine (2012) for a more detailed explanation of this topic.

[19] Cf. Appendix for details of the calculations.

[20] Remark that equation (12) shows that in this model, oil output elasticity is greater than the cost share.

3.3 Estimation Results

Table 2 reports the prior and posterior distributions for each parameter along with the mean and the 10 and 90 percentiles of the posterior distribution. In the same way, Table 3 presents the estimates of the prior and posterior distributions of shock processes. Note that for estimation proposes, the observable series have been multiplied by 100, meaning that 1 represents a standard deviation of 1 percent.

There are some important issues to be highlighted in the estimation. Regarding the estimation results of the main behavioral parameters summarized in Table 2, it turns out that the mean value for the elasticity of substitution of oil is equal to 0.086 in the production sector and 0.014 in the consumption sector. These outcomes confirm the empirical results about the degree of oil substitution in U.S.: oil is poorly substitutable to other factors in both sectors. In addition, the estimated steady state for oil output elasticity, α_e, is equal to 0.013 and the share of oil consumption out of total production for households is 0.08. It is worth highlighting that although the posterior mean of oil output elasticity is close to the calibrated value of this parameter in Blanchard and Galí (2009) and Blanchard and Riggi (2013), the posterior mean of oil consumption in household consumption was almost 5 times bigger than that assumed in Blanchard and Galí (2009) and Blanchard and Riggi (2013). Other important results are the mean of the "share" parameter of capital, α, and the Calvo parameter in wages, θ_p, which are estimated at 0.38 and 0.71 respectively, fairly close to the literature. This degree of wage stickiness states that the average duration of a price contract is somewhat less than a year. The degree of wage stickiness is estimated to be 0.88, implying a price contract duration of roughly two years.

Table 2: Prior and Posterior Distribution of Structural Parameters

Parameter		Prior distribution	Posterior distribution		
			Mean	90% HPD interval	
"Share" parameter of capital	α	Normal(0.3,0.05)	0.3835	0.3666	0.4000
Elast. substitution in production	η_p	Gamma(0.54,1)	0.0855	0.0344	0.1419
Elast. substitution in consumption	η_c	Gamma(0.54,1)	0.0141	0.0010	0.0354
Oil share in consumption	x_c	Normal(0.03,0.03)	0.0799	0.0486	0.1092
Oil output elasticity	α_e	Normal(0.039,0.03)	0.0131	0.0100	0.0171
Inverse Frisch elasticity	ϕ	Normal(2,0.75)	3.1107	2.2912	4.0707
Taylor rule response to inflation	ϕ_π	Normal(2,0.25)	1.3023	1.000	1.5569
Taylor rule response to GDP	ϕ_y	Normal(0.12,0.05)	0.4253	0.4092	0.4381
Taylor rule inertia	ϕ_i	Beta(0.75,0.1)	0.5138	0.5000	0.5325
Calvo price parameter	θ_p	Beta(0.5,0.1)	0.7107	0.6314	0.7858
Calvo wage parameter	θ_w	Beta(0.5,0.1)	0.8845	0.8663	0.9000
Habit formation	h	Beta(0.7,0.1)	0.5265	0.5000	0.5547

Turning to the estimated processes for the exogenous shock variables reported in Table 3, a number of observations are worth making. The oil price shock is the third most persistent, with $AR(1)$ coefficient equal to 0.94. Finally, the shocks with the highest standard errors are, in descending order: the price of oil, government spending shocks and oil productivity.

Table 3: Prior and Posterior Distribution of Shock Parameters

Parameter		Prior distribution	Posterior distribution		
			Mean	90% HPD interval	
Autoregressive parameters					
Real oil price	ρ_{se}	Beta(0.5,0.2)	0.9434	0.9190	0.9683
Real capital price	ρ_{sk}	Beta(0.5,0.2)	0.7480	0.6907	0.7922
Government	ρ_g	Beta(0.5,0.2)	0.9148	0.8816	0.9506
Monetary	ρ_i	Beta(0.5,0.2)	0.9766	0.9624	0.9913
Oil productivity	ρ_{ae}	Beta(0.5,0.2)	0.5472	0.4103	0.7002
TFP	ρ_{alk}	Beta(0.5,0.2)	0.9063	0.8700	0.9457
Oil Prod. in Gov	$\rho_{ae,g}$	Beta(0.5,0.2)	0.1488	0.0296	0.2538
TFP in Gov.	$\rho_{alk,g}$	Beta(0.5,0.2)	0.6263	0.3642	0.9339
Price markup1	ρ_p	Beta(0.5,0.2)	0.9800	0.9673	0.9939
Wage markup1	ρ_w	Beta(0.5,0.2)	0.7992	0.6979	0.9022
Price markup2	ν_p	Beta(0.5,0.2)	0.6113	0.3754	0.8321
Wage markup2	ν_w	Beta(0.5,0.2)	0.8730	0.8203	0.9224
Standard deviations					
Real oil price	σ_{se}	Inv_Gamma(1,2)	3.0640	1.8321	4.3336
Real capital price	σ_{sk}	Inv_Gamma(1,2)	0.8031	0.6844	0.9108
Government	σ_g	Inv_Gamma(1,2)	2.1428	1.8845	2.3901
Monetary	σ_i	Inv_Gamma(1,2)	0.1877	0.1616	0.2138
Oil productivity	σ_{ae}	Inv_Gamma(1,2)	2.0761	1.7952	2.3552
TFP	σ_{alk}	Inv_Gamma(1,2)	0.4779	0.4167	0.5373
Price markup	σ_p	Inv_Gamma(1,2)	0.1674	0.1343	0.1974
Wage markup	σ_w	Inv_Gamma(1,2)	0.8317	0.7157	0.9409

4 Simulations and Results

There are eight sources of potential exogenous shocks in this economy: the real price of oil, real price of capital, government expenditure, monetary policy, both types of technologies and the wage and price markups. Once the model has been estimated, in this section I study the reaction of the economy to a real oil price shock and its sensitivity to changes in some parameters.

4.1 What if Oil was Less Substitutable?

Figure 3 shows the impulse response functions (hereafter IRFS) of the economy to a one standard deviation increase in the real price of oil equal to 3.06 percent. We observe that the model is able to recover four of the stylized facts observed after an oil shock in the 2000s. As expected, an increase in the real price of oil leads to a contemporaneous increment in the marginal cost of intermediate firms, which produces a raise in domestic prices and so domestic inflation. We therefore also see an increase in contemporaneous nominal interest rate due to the Taylor rule. Due to rational expectations, and with the parametrization used for the Taylor rule, agents should also anticipate an increment in the real rental rate of capital tomorrow: the interest rate increases more than expected inflation.

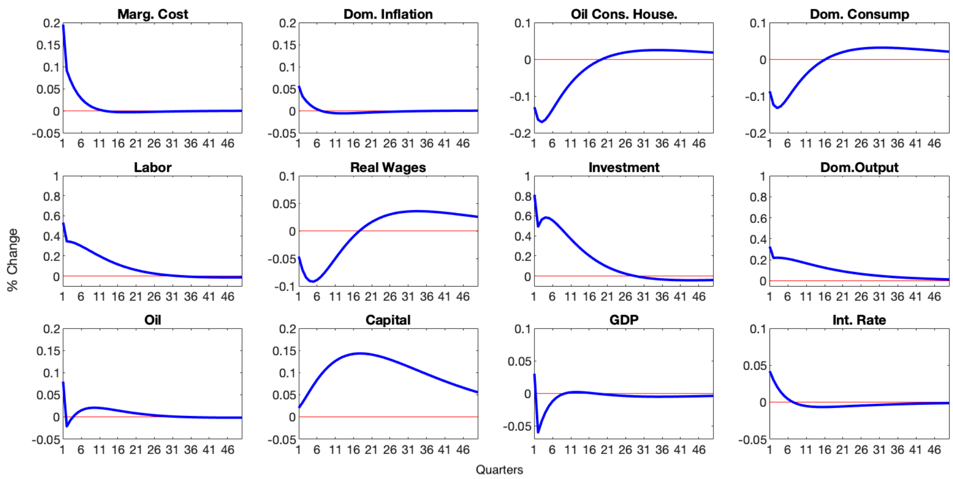

Figure 3: Response to one Standard Deviation Shock on Real Price of Oil

In a world where the oil factor is easily substitutable, we should observe a reduction in oil consumption and an increase in other factors, because of a substitution effect. Nonetheless, when oil is not easily substitutable, the only way to produce is by paying the higher oil bill. Furthermore, in this model the only way to buy oil is by exporting more domestic goods, meaning producing more. However, in order to export more, firms have to increase their domestic production and thus increase not only capital and labor demand, but also oil demand. This is exactly what we observe in the IRFS of Figure 3. The decrease in real wages can be explained as a consequence of high wage rigidity, combined with an increase in domestic inflation.

We may wonder why the firms do not decrease production in the first place. As shown in the IRF of investment and consumption, investment goes up while both consumptions goes down. The decrease in consumption is easy to explain by the fact that oil consumption and domestic consumption are not substitutable and so an increase in the price of one of them should trigger a decrease in the consumption of both. The increase in investment is more difficult to assess, however. This may be explained by the fact that consumers anticipate an increase in the real rate of capital that is large enough that they decide to increase investment today in order to convert it into capital tomorrow, which results in an increase in domestic demand. Finally, the increase in output is slightly stronger than the increase in oil importation, and so we do not observe a decrease in GDP. However, we should not forget that a big part of the increase in domestic production is only to pay for the higher oil bill. Note also that the investment, domestic output, GDP and oil consumption increases are very short-lived.

As argued in the introduction, the possibility of increasing domestic output should not be a problem as long as the domestic economy can import as much oil as needed. However, this oil increase might be problematic in our world where oil supply has entered a period of increased scarcity.

Note also that the magnitude of the initial increase in oil demand and its subsequent decline is weak, confirming the sluggish price elasticity of oil demand in the short run. For instance, a shock in the real price of oil equal to 3 percent provokes an increase of 0.08 percent in oil demand, so that the price elasticity of oil is equal to $\frac{0.08}{3.06} = 0.03 \approx 0$.

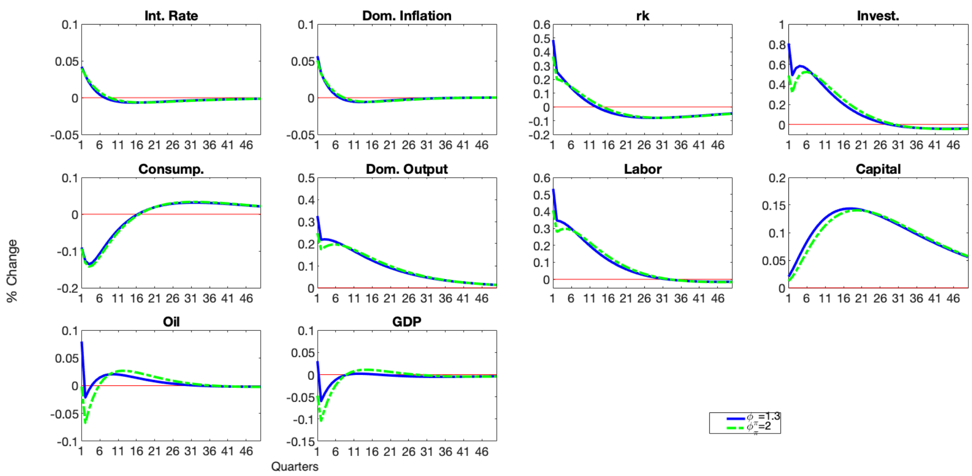

Figure 4: Response to one Standard Deviation Shock on Real Price of Oil—Persistence Comparison

Comparison of two models, namely a model with baseline 1.3 for the response of the Taylor rule to inflation (solid blue line) and its counterpart where the response to inflation is higher, equal to 2 (dashed green line)

As we argue that domestic production does not decrease because the increase in the nominal interest rate is higher than expected domestic inflation, the next step is to test how the IRFS change if the nominal interest rate reacts more strongly towards inflation, in such a way that the difference between interest rate and anticipated inflation is reduced. Assume a Taylor rule where the response to inflation is higher, $\phi_\pi = 2$ instead of 1.3. As shown in Figure 4, with a stronger anti-inflationist policy rule, the rise in the real rental rate of capital (dashed line) is somewhat smaller compared to the baseline (continuous line). Accordingly the increase in investment is much smaller than in the original case, so there is no need to increase production and we observe a decrease in oil importation rather than an increase. This is because even if firms still need to produce output to buy oil, there is no domestic demand for it, so the increase in domestic production is smaller. Then, we observe a smaller decrease in GDP, because of the smaller increase in domestic output. This result enters the debate raised by Bernanke et al. (1997) about the role of monetary policy in the attenuation of oil shocks. As in Bernanke et al. (1997), I find that the adverse effects of an oil price shock on output are amplified when the response of the funds rate, i here, is "stronger" ($\phi_\pi = 2$).

4.2 What if Nominal Wages were More Flexible?

Let us now study the sensitivity of the model to a change in the Calvo parameter of wages. The solid line in Figure 5 represents the IRFS of the model obtained with the estimated values, while the dashed line represents the IRFS of the model where nominal wages have been flexibilized, changing the estimated value of θ_w from 0.88 to 0.10, ceteris paribus. This experiment was carried out in order to analyze one of the conclusions given in Blanchard and Galí (2009) and Blanchard and Riggi (2013) regarding the smoothness of the economy in the face of an oil shock, being in their case, the flexibility of real wages.

As it should be expected, when nominal wages are more flexible, we observe an increase in real wages after an oil shock, because of the increase in labor demand. As labor is an input factor, this increases real marginal cost and so we observe a sharper rise in inflation. Accordingly, the Central Bank reacts more strongly and we observe a sharper increase in the real rental rate of capital. A worse tradeoff between investment and consumption thus takes place. Households prefer to invest rather than consume. On the other hand, the stronger increase in investment produces a stronger increase in domestic output and again, because of the low substitutability, oil demand increases further as well. Therefore, the increase in production results in a stronger but, once again, short-lived increase in GDP.

This experiment shows that in this model, and contrary to one of the conclusions given in Blanchard and Galí (2009) and Blanchard and Riggi (2013), the reduction of nominal wage rigidity causes an increase in real wages and, as a consequence, more inflation and lower consumption.

5 Conclusion

In recent years, the inclusion of energy or oil in theoretical models has seen a rapid development, but some questions and factors have still not been taken into account. One of these factors is oil substitutability. To the best of my knowledge, no DSGE

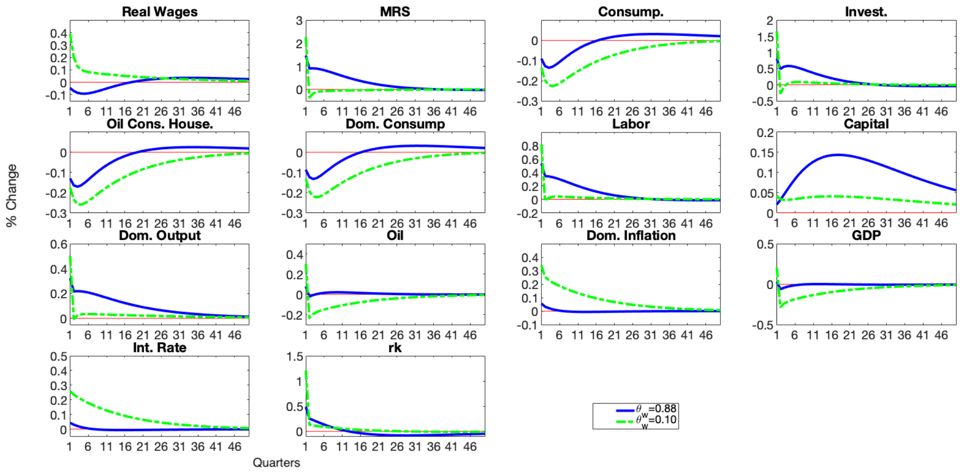

Figure 5: Response to one Standard Deviation Shock on Real Price of Oil—Wage Flexibility Comparison

Comparison of two models, namely, a model with “sticky” wages (solid blue line) and its counterpart with ”flexible wages” (dashed green line)

model that includes energy or oil has been able to recover, at the same time, four of the effects that the 2000’s oil shock generated in the U.S. economy. My assumption is that one possible reason for the lack of understanding is the assumption of perfectly substitutable oil.

Using a DSGE model, this factor is now taken into account through the introduction of oil into the production and consumption processes. Using Bayesian techniques and U.S data from 1984:Q1 to 2007:Q3, it can be proved that the elasticity of substitution in the U.S between oil and other factors is weak, results that are in line with empirical studies on the subject.

On the other hand, once this low substitutability has been introduced, the model is able to recover four well-known stylized facts after an oil price shock in the 2000s: the absence of a recession combined with a low level of inflation rate, a decrease in real wages and low price elasticity of oil demand. It also shows that with a stronger anti-inflationary monetary policy, GDP could suffer a contemporaneous slight decrease after an oil shock.

Furthermore, the model also includes nominal price and wage rigidities. As it turns out, a reduction in wage rigidity amplifies the response of the economy to an oil shock in terms of inflation and consumption, and shows that the increase obtained in GDP is possible, provided that there is the possibility to import as much as oil as needed.

Several extensions of this paper can be envisaged. First, one important factor has been neglected in this recent literature, namely unemployment. One natural extension could therefore be the inclusion of oil in match and search models. Second, one strong hypothesis should be eased, namely the assumption that oil is completely imported from a foreign country.

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A Appendix A: Model Derivation

A.1 Household's Maximization Problem

Each household j faces the following problem:

$$\max_{C_{q,t}(j),C_{e,t}(j)} P_{c,t}C_t(j) - P_{q,t}C_{q,t}(j) - P_{e,t}C_{e,t}(j),$$
$$\text{subject to :} C_t(j) = \left((1-x_c)^{1-\sigma}C_{q,t}^{\sigma} + x_c^{1-\sigma}C_{e,t}^{\sigma}\right)^{\frac{1}{\sigma}}$$

The first order condition with respect to $C_{q,t}(j)$ gives:

$$P_{c,t}\left((1-x_c)^{1-\sigma}C_{q,t}^{\sigma}(j) + x_c^{1-\sigma}C_{e,t}^{\sigma}(j)\right)^{\frac{1}{\sigma}-1} C_{q,t}^{\sigma-1}(j) - P_{q,t} = 0$$
$$P_{c,t}C_t^{1-\sigma}(j)C_{q,t}^{\sigma-1}(1-x_c)^{1-\sigma} = P_{q,t}$$
$$C_{q,t}(j) = (1-x_c)\left(\frac{P_{q,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} C_t(j)$$

In the same way, the first order condition with respect to $C_{e,t}(j)$ gives:

$$C_{e,t}(j) = x_c\left(\frac{P_{e,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} C_t(j)$$

And so one has:

$$P_{c,t}C_t(j) = P_{q,t}C_{q,t}(j) + P_{e,t}C_{e,t}(j)$$
$$P_{c,t} = (1-x_c)P_{q,t}\left(\frac{P_{q,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}} + x_cP_{e,t}\left(\frac{P_{e,t}}{P_{c,t}}\right)^{\frac{1}{\sigma-1}}$$
$$P_{c,t} = \left((1-x_c)P_{q,t}^{\frac{\sigma}{\sigma-1}} + x_cP_{e,t}^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}}$$

Given the description of household's problem, the Lagrangian function associated with it is:

$$\mathcal{L}_0 = \sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\Big[U(C_t(j),L_t(j)) - \widetilde{\lambda}(j)\left[P_{c,t}C_t(j) + P_{k,t}I_t(j)\right.$$
$$\left. + B_t(j) + (1+i_{t-1})B_{t-1}(j) + W_t(j)L_t(j) + r_t^kP_{k,t}K_t(j) + D_t + T_t\right]\Big]$$

where the household maximizes over $C_t(j)$, $B_t(j)$, $K_{t+1}(j)$, $W_t(j)$, $L_t(j)$, and where $\widetilde{\lambda}_t(j)$ is the Lagrangian multiplier associated. First order conditions gives:

$$C_t(j) : \widetilde{\lambda}_t(j) = \frac{1}{P_{c,t}}\mathbb{E}_t\left[\lambda_{a,t}\right] \tag{13}$$
$$B_t(j) : \widetilde{\lambda}_t(j) = \beta\mathbb{E}_t\left[(1+i_t)\widetilde{\lambda}_{t+1}(j)\right] \tag{14}$$
$$K_{t+1}(j) : \widetilde{\lambda}_t(j) = \frac{1}{P_{k,t}}\beta\mathbb{E}_t\left[\widetilde{\lambda}_{t+1}(j)\left(r_{t+1}^k + 1 - \delta\right)P_{k,t+1}\right] \tag{15}$$

With external habit formation:

$$\lambda_{a,t} = \frac{1}{C_t(j) - hC_{t-1}}$$

Substituting (13) in (14) and (15), and then simplifying, one obtains Euler's and Fisher's equations.

A.2 "Packer" Maximization Problem

The problem of the labor "packer" is:

$$\max_{L_t(j)} \quad W_t L_t^d - \int_0^1 W_t(j) L_t(j) dj,$$
$$\text{subject to :} L_t^d = \left(\int_0^1 L_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

The first order condition with respect to $L_t(j)$ yields:

$$W_t \left(\int_0^1 L_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1} - 1} L_t(j)^{\frac{\epsilon_w}{\epsilon_w - 1} - 1} - W_t(j) = 0$$
$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} L_t^d$$

By the zero profit condition one also has:

$$W_t L_t^d = \int_0^1 W_t(j) L_t(j) dj$$

Replacing the aggregated demand in this last equation one gets:

$$W_t L_t^d = \int_0^1 W_t(j) (\frac{W_t(j)}{W_t})^{-\epsilon_w} dj L_t^d$$
$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}$$

As for the optimal wage setting, let us assume that in each period t, only a fraction $(1 - \theta_w)$ of households can re-optimize their nominal wage ($W_t(j) = W_t^o(j)$). The remaining part lets its wage as before ($W_t(j) = W_{t-1}(j)$). Given a date t, suppose that the j-th household has to chose the wage $W_t^o(j)$. The household j does not care about future dates where it can re-optimize but only to the state where it cannot with probability θ_w^k, for all $k \geq 0$. Each household that can change its wage will chose $W_t^o(j)$ in order to maximize:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k U \left(C_{t+k|t}(j), L_{t+k|t}(j) \right) \right]$$

under the same budget constrain described in (3) and subject to:

$$L_{t+k|t}(j) = \left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w} L^d_{t+k} \tag{16}$$

Then the problem of household j is:

$$\begin{aligned}
&\max_{W_t(j)} \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k U\left(C_{t+k|t}(j), L_{t+k|t}(j)\right)\right], \\
&\text{subject to :} L_{t+k|t}(j) = \left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w} L^d_{t+k} \\
&\qquad P_{c,t}C_t(j) + P_{k,t}I_t(j) + B_t \\
&\qquad\qquad \leq (1+i_{t-1})B_{t-1}(j) + W_t(j)L_t(j) + D_t + r^k_t P_{k,t}K_t(j) + T_t
\end{aligned}$$

Therefore, the relevant part of the Lagrangian for the j-th household is:

$$\mathcal{L}^w_0 = \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \left[-\frac{L^{1+\phi}_{t+k|t}(j)}{1+\phi} - \widetilde{\lambda}_{t+k}(j) W_t(j) L_{t+k|t}(j)\right]\right]$$

substituting (16) in this last equation one has:

$$\begin{aligned}
\mathcal{L}^w_0 = \mathbb{E}_t \Big[\sum_{k=0}^{\infty} (\beta\theta_w)^k \big[-\frac{1}{1+\phi} \left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w(1+\phi)} (L^d_{t+k})^{1+\phi} \\
- \widetilde{\lambda}_{t+k}(j) W_t(j) \left(\frac{W_t(j)}{W_{t+k}}\right)^{-\epsilon_w} L^d_{t+k}\big]\Big]
\end{aligned}$$

So the first order condition with respect to $W_t(j)$ yields:

$$\begin{aligned}
\mathbb{E}_t \Big[\sum_{k=0}^{\infty} (\beta\theta_w)^k \big[\epsilon_w \frac{W^o_t(j)^{-\epsilon_w(1-\phi)-1}}{W^{-\epsilon_w(1+\phi)}_{t+k}} (L^d_t)^{1+\phi} + (1-\epsilon_w)\widetilde{\lambda}_{t+k}(j) \left(\frac{W^o_t(j)}{W_{t+k}}\right)^{-\epsilon_w} L^d_{t+k}\big]\Big] = 0 \\
\mathbb{E}_t \Big[\sum_{k=0}^{\infty} (\beta\theta_w)^k \big[\epsilon_w W^o_t(j)^{-1} L^{1+\phi}_{t+k|t}(j) + (1-\epsilon_w)\widetilde{\lambda}_{t+k}(j) L_{t+k|t}(j)\big]\Big] = 0
\end{aligned}$$

Using equation (13) one then has:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \left[\frac{\lambda_{a,t+k}(j)}{P_{c,t+k}} W^o_t(j) L_{t+k|t}(j) - \mathcal{M}_w L^{1+\phi}_{t+k|t}(j)\right]\right]$$

where $L_{t+k|t}(j)$ denote labor supply in period $t+k$ of a household that last resets its wage in period t.

Note that $\lambda_{a,t+k}(j)$ represent the marginal lifetime discounted utility function for agent j. However, because of complet markets and assuming a symmetric equilibrium, this should be the same for each agent. Then one can drop the j. Furthermore, one can write $MRS_{t+k|t} := -\frac{U_L(C_{t+k}, L_{t+k|t})}{\lambda_{a,t+k}}$ as the marginal rate of substitution between consumption and leisure in period $t+k$ for the household that can reset its wage in t, this last condition can be rewritten as:

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \lambda_{a,t+k} L_{t+k|t} \left[\frac{W_t^o}{P_{c,t+k}} - \mathcal{M}_w MRS_{t+k|t} \right] \right] = 0$$

and so in the limiting case of full wage flexibility ($\theta_w = 0$), one has

$$\frac{W_t^o}{P_{c,t}} = \frac{W_t}{P_{c,t}} = \mathcal{M}_w MRS_{t|t}$$

That is why one can interpret $\mathcal{M}_w$ as being the desired gross wage markup. Then using equation (8) one has

$$\begin{aligned} W_t^{1-\epsilon_w} &= \int_0^1 W_t(j)^{1-\epsilon_w} dj \\ &= \int_{\text{can not reset wages}} W_t(j)^{1-\epsilon_w} dj + \int_{\text{set wages optimally}} W_t(j)^{1-\epsilon_w} dj \\ &= \theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^o)^{1-\epsilon_w}. \end{aligned}$$

A.3 Final Good Producer Problem's maximization

The problem of the Final Good Producer is:

$$\begin{aligned} &\max_{Q_t(\cdot)} P_{q,t} Q_t - \int_0^1 P_{q,t}(i) Q_t(i) di \\ \text{subject to} \;:\; \; & Q_t = \left(\int_0^1 Q_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \end{aligned}$$

Solving this problem one obtains (Cf. Acurio-Vásconez et al. (2015) for derivation):

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t, \quad \forall i \tag{17}$$

which is the demand of good i.

A.4 Intermediate Firms Relations

The cost minimization problem of firm i is:

$$\begin{aligned} \text{minimize cost: } & P_{e,t} E_t(i) + W_t L_t^d(i) + r_t^k P_{k,t} K_t(i) \\ \text{subject to } & E_t(i), L_t^d(i), K_t(i) \geq 0, \\ & \left(x_p A_{E,t}^\rho E_t(i)^\rho + (1-x_p) A_{LK,t}^\rho (K_t(i)^\alpha L_t^d(i)^{1-\alpha})^\rho \right)^{1/\rho} \geq Q_t(i) \end{aligned}$$

One has the following Lagrangian associated to this problem:

$$\mathcal{L}_0 := P_{e,t}E_t(i) + W_t L_t^d(i) + r_t^k P_{k,t} K_t(i)$$
$$-\lambda_t(i)\left(\left(x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p)A_{LK,t}^{\rho}(K_t(i)^{\alpha} L_t^d(i)^{1-\alpha})^{\rho}\right)^{1/\rho} - Q_t(i)\right)$$

which yields the first order conditions expressed on the paper.

Defining:

$$\text{marginal cost } (MC_t{:})\quad \lambda_t(i) := \frac{\dfrac{d(cost)}{d(worker)}}{\dfrac{d(output)}{d(worker)}} = \frac{\dfrac{d(cost)}{d(capital)}}{\dfrac{d(output)}{d(capital)}} = \frac{\dfrac{d(cost)}{d(energy)}}{\dfrac{d(output)}{d(energy)}}.$$

the relation (10) is determined. One also has:

$$\begin{aligned}
cost\,(Q_t(i)) :=& P_{e,t}E_t(i) + W_t L_t^d(i) + r_t^k P_{k,t}K_t(i)\\
=& \lambda_t(i) x_p A_{E,t}^{\rho} Q_t(i)^{1-\rho} E_t(i)^{\rho} + \lambda_t(i)(1-\alpha)Q_t(i)^{1-\rho}(1-x_p)A_{LK,t}^{\rho} K_t(i)^{\alpha\rho} L_t^d(i)^{(1-\alpha)\rho}\\
&+ \lambda_t(i)\alpha Q_t(i)^{1-\rho}(1-x_p)A_{LK,t}^{\rho} K_t(i)^{\alpha\rho} L_t^d(i)^{(1-\alpha)\rho}\\
=& \lambda_t(i)Q_t(i)^{1-\rho}\left(x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p)A_{LK,t}^{\rho}\left(K_t(i)^{\alpha} L_t^d(i)^{1-\alpha}\right)^{\rho}\right)\\
=& \lambda_t(i) Q_t(i)
\end{aligned}$$

In the other hand:

$$\begin{aligned}
Q_t(i)^{\rho} =& x_p A_{E,t}^{\rho} E_t(i)^{\rho} + (1-x_p)A_{LK,t}^{\rho}(K_t(i)^{\alpha} L_t^d(i)^{1-\alpha})^{\rho}\\
=& x_p A_{E,t}^{\rho}\left(\frac{P_{e,t}}{\lambda_t(i)Q_t(i)^{1-\rho}x_p A_{E,t}^{\rho}}\right)^{\frac{\rho}{\rho-1}} + (1-x_p)A_{LK,t}^{\rho}\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho} L_t^d(i)^{\rho}\\
=& x_p A_{E,t}^{\rho}\left(\frac{P_{e,t}}{\lambda_t(i)Q_t(i)^{1-\rho}x_p A_{E,t}^{\rho}}\right)^{\frac{\rho}{\rho-1}}\\
&+ (1-x_p)A_{LK,t}^{\rho}\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}\left(\frac{W_t L_t^d(i)^{\alpha\rho}}{\lambda_t(i)Q_t(i)^{1-\rho}(1-\alpha)A_{LK,t}^{\rho}K_t(i)^{\alpha\rho}}\right)^{\frac{\rho}{\rho-1}}
\end{aligned}$$

Combining the first order conditions for $L_t^d(i)$ and $K_t(i)$ one has:

$$\frac{W_t}{(1-\alpha)L_t^d(i)^{-1}} = \frac{r_t^k P_{k,t}}{\alpha K_t(i)^{-1}}$$

which yields to:

$$\frac{K_t(i)}{L_t^d(i)} = \frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}$$

Then:

$$\begin{aligned}
Q_t(i)^\rho =& x_p A_{E,t}^\rho \left(\frac{P_{e,t}}{\lambda_t(i)Q_t(i)^{1-\rho}x_p A_{E,t}^\rho}\right)^{\frac{\rho}{\rho-1}} + \\
&+ (1-x_p)A_{LK,t}^\rho \left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}\left(\frac{L_t^d(i)}{K_t(i)}\right)^{\alpha\rho\frac{\rho}{\rho-1}}\left(\frac{W_t}{\lambda_t(i)Q_t(i)^{1-\rho}(1-\alpha)(1-x_p)A_{LK,t}^\rho}\right)^{\frac{\rho}{\rho-1}} \\
=& x_p A_{E,t}^\rho \left(\frac{P_{e,t}}{\lambda_t(i)Q_t(i)^{1-\rho}x_p A_{E,t}^\rho}\right)^{\frac{\rho}{\rho-1}} \\
&+ (1-x_p)A_{LK,t}^\rho \left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\frac{-\alpha\rho}{\rho-1}}\left(\frac{W_t}{\lambda_t(i)Q_t(i)^{1-\rho}(1-\alpha)(1-x_p)A_{LK,t}^\rho}\right)^{\frac{\rho}{\rho-1}} \\
=& \left(\frac{1}{\lambda_t(i)Q_t(i)^{1-\rho}}\right)^{\frac{\rho}{\rho-1}}\Bigg(x_p A_{E,t}^\rho\left(\frac{P_{e,t}}{x_p A_{E,t}^\rho}\right)^{\frac{\rho}{\rho-1}} + (1-x_p)A_{LK,t}^\rho\left(\frac{W_t}{1-\alpha}\right)^{\frac{\rho}{\rho-1}-\frac{\alpha\rho}{\rho-1}} \\
&\qquad ((1-x_p)A_{LK,t}^\rho)^{\frac{-\rho}{\rho-1}}\left(\frac{\alpha}{r_t^k P_{k,t}}\right)^{\frac{-\alpha\rho}{\rho-1}}\Bigg) \\
\lambda_t(i)^{\frac{\rho}{\rho-1}} =& \left(\frac{1}{x_p A_{E,t}^\rho}\right)^{\frac{1}{\rho-1}} P_{e,t}^{\frac{\rho}{\rho-1}} + \left(\frac{1}{(1-x_p)A_{LK,t}^\rho}\right)^{\frac{1}{\rho-1}}\left(\frac{W_t}{1-\alpha}\right)^{\frac{(1-\alpha)\rho}{\rho-1}}\left(\frac{\alpha}{r_t^k P_{k,t}}\right)^{\frac{-\alpha\rho}{\rho-1}} \\
\lambda_t(i) =& \left(\left(\frac{P_{e,t}^\rho}{x_p A_{E,t}^\rho}\right)^{\frac{1}{\rho-1}} + \left(\frac{1}{(1-x_p)A_{LK,t}^\rho}\right)^{\frac{1}{\rho-1}}\left(\frac{W_t}{1-\alpha}\right)^{\frac{(1-\alpha)\rho}{\rho-1}}\left(\frac{\alpha}{r_t^k P_{k,t}}\right)^{\frac{-\alpha\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}
\end{aligned}$$

Then $\lambda(i)$ does not depend on i.

As for the price maximization, at date t, denote $Q_{t+k|t}(i)$ the output at date $t+k$ for a firm i that last resets its price in period t. As in the case of wages each firm that can reset its price will chose the same one, so the choice of $P_{q,t}^o(i)$ will not depend on i. The firm only cares about the future states in which it cannot re-optimize. Therefore the problem of the i-th firm is:

$$\max_{P_{q,t}(i)} \mathbb{E}_t\left[\sum_{k=0}^{\infty}\theta^k d_{t,t+k}\left[P_{q,t}(i)Q_{t+k|t}(i) - cost(Q_{t+k|t}(i))\right]\right]$$
$$\text{subject to}\quad Q_{t+k|t}(i) = \left(\frac{P_{q,t}(i)}{P_{q,t+k}}\right)^{-\epsilon_p} Q_{t+k}, \quad \forall k \geq 0.$$

where $d_{t,t+k} := \frac{\beta^t\widetilde{\lambda}_t}{\widetilde{\lambda}_{t+1}}$ is the stochastic discount factor.
The first order condition of this problem yields:

$$\mathbb{E}_t\left[\sum_{k=0}^{\infty}\theta_p^k d_{t,t+k}Q_{t+k|t}^o\left(P_{q,t}^o - \mathcal{M}_p mc_{t+k|t}^o\right)\right] = 0$$

In the limiting case of full flexibility ($\theta_p = 0$) equation (11) gives:

$$P_{q,t} = P^o_{q,t} = \mathcal{M}_p MC_t$$

that is why one can interpret $\mathcal{M}_p$ as the desired price markup.

A.5 Aggregation

By market clearing conditions one has:

$$K_t = \int_0^1 K_t(i)di, \quad L_t^d = \int_0^1 L_t^d(i)di, \quad E_t = \int_0^1 E_t(i)di,$$

Equation (17) yields:

$$\left(\left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} Q_t\right)^\rho = Q_t^\rho(i) = x_p A^\rho_{E,t} E_t(i)^\rho + (1-x_p)A^\rho_{LK}(K_t(i)^\alpha L_t^d(i)^{1-\alpha})^\rho$$

In the other hand using the equivalence from the first order conditions for the firms one has:

$$E_t(i)^{\rho-1} = \left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^\rho_{LK,t}}{W_t x_p A^\rho_{E,t}}\right)\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho} L_t^d(i)^{\rho-1} \tag{18}$$

Then

$$\begin{aligned}
\left(\left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} Q_t\right)^\rho =& x_p A^\rho_{E,t}\left(\left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^\rho_{LK,t}}{W_t x_p A^\rho_{E,t}}\right)\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}(L_t^d(i))^{\rho-1}\right)^{\frac{\rho}{\rho-1}} \\
&+ (1-x_p)A^\rho_{LK,t}\left(\frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho} L_t^d(i)^\rho \\
=& \Bigg[x_p A^\rho_{E,t}\left(\left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^\rho_{LK,t}}{W_t x_p A^\rho_{E,t}}\right)\left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\alpha\rho}\right)^{\frac{\rho}{\rho-1}} + \\
&(1-x_p)A^\rho_{LK,t}\left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\alpha\rho}\Bigg] L_t^d(i)^\rho
\end{aligned}$$

Let us note:

$$\begin{aligned}
\tilde{F}_t = \Bigg[& x_p A^\rho_{E,t}\left(\left(\frac{P_{e,t}(1-\alpha)(1-x_p)A^\rho_{LK,t}}{W_t x_p A^\rho_{E,t}}\right)\left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\alpha\rho}\right)^{\frac{\rho}{\rho-1}} + \\
&(1-x_p)A^\rho_{LK,t}\left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\alpha\rho}\Bigg]
\end{aligned}$$

One has:

$$\left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} Q_t = \tilde{F}_t^{\frac{1}{\rho}} L_t^d(i)$$

Taking the integral at both sides and then taking power ρ one has:

$$\left(\int_0^1 \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} diQ_t\right)^{\rho} = \tilde{F}_t(L_t^d)^{\rho} \tag{19}$$

In the other hand, taking the integral in both sides of (18) and then taking power ρ one has:

$$E_t^{\rho} = \left[\frac{P_{e,t}(1-\alpha)(1-x_p)A_{LK,t}^{\rho}}{W_t x_p A_{E,t}^{\rho}}\left(\int_0^1 \frac{K_t(i)}{L_t^d(i)}\right)^{\alpha\rho}\right]^{\frac{1}{\rho-1}}(L_t^d)^{\rho}$$

$$E_t^{\rho} = \left[\frac{P_{e,t}(1-\alpha)(1-x_p)A_{LK,t}^{\rho}}{W_t x_p A_{E,t}^{\rho}}\left(\frac{\alpha W_t}{r_t^k P_{k,t}(1-\alpha)}\right)^{\alpha\rho}\right]^{\frac{1}{\rho-1}}(L_t^d)^{\rho}$$

One also has:

$$\frac{W_t L_t^d}{1-\alpha} = \frac{r_t^k P_{k,t} K_t}{\alpha}$$

replacing these two last equations in (19) one finally gets:

$$\int_0^1 \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} diQ_t = \left(x_p A_{E,t}^{\rho} E_t^{\rho} + (1-x_p)A_{LK,t}^{\rho}(K_t^{\alpha}(L_t^d)^{1-\alpha})^{\rho}\right)^{1/\rho} \tag{20}$$

Define now

$$v_{p,t} := \int_0^1 \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon_p} di$$

By by Calvo price setting one has:

$$\begin{aligned} v_{p,t} =& P_{q,t}^{\epsilon_p}\int_0^1 P_{q,t}(i)^{-\epsilon_p} di \\ =& P_{q,t}^{\epsilon_p}\left[\int_{\text{no set}} P_{q,t-1}(i)^{-\epsilon_p} di + \int_{\text{set}} P_{q,t}^{o}(i)^{-\epsilon_p} di\right] \\ =& \theta_p \Pi_{q,t}^{\epsilon_p} v_{p,t-1} + (1-\theta_p)\left(\frac{P_{q,t}^{o}}{P_{q,t}}\right)^{-\epsilon_p} \end{aligned} \tag{21}$$

Taking integral on both sides of equation (7) one gets:

$$\int_0^1 L_t(j)dj = L_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} L_t^d dj \tag{22}$$

Define:

$$v_{w,t} := \int_0^1 \left(\frac{W_{q,t}(j)}{W_t}\right)^{-\epsilon_w} di$$

Hence:

$$L_t = v_{w,t} L_t^d$$

Then by Calvo setting, one gets:

$$v_{w,t} = \theta_w \left(\frac{W_t}{W_{t-1}}\right)^{\epsilon_w} v_{t-1}^{w} + (1-\theta_w)\left(\frac{W_t^{o}}{W_t}\right)^{-\epsilon_w} \tag{23}$$

A.6 Equilibrium

At equilibrium:

1. Households maximize their utility. I assume complete markets, separable utility in labor, and I consider a symmetric equilibrium where $C_t(j) = C_t$, $C_{q,t}(j) = C_{q,t}$, $C_{e,t}(j) = C_{e,t}$, $K_{t+1}(j) = K_{t+1}$, $\lambda_{a,t}(j) = \lambda_{a,t}$. Therefore the first order conditions associated to household's problem become:

$$\begin{aligned}\lambda_{a,t} =&\widetilde{\lambda}_t P_{c,t}\\ \widetilde{\lambda}_t =&\beta\mathbb{E}_t\left[(1+i_t)\widetilde{\lambda}_{t+1}\right]\\ \widetilde{\lambda}_t =&\beta\mathbb{E}_t\left[\widetilde{\lambda}_{t+1}(r^k_{t+k}+(1-\delta)\frac{P_{k,t+1}}{P_{k,t}})\right]\end{aligned}$$

The profit at equilibrium is:

$$D_t = P_{q,t}Q_t - W_tL^d_t - r^k_tP_{k,t}K_t - P_{e,t}E_t$$

so the budget constraint becomes:

$$P_{c,t}C_t + P_{k,t}I_t + G_t = P_{q,t}Q_t - P_{e,t}E_t$$

2. All markets clears.

3. Firms maximize their profits:

$$\begin{aligned}\frac{P_{e,t}}{x_pA^{\rho}_{E,t}Q^{1-\rho}_tE^{\rho-1}_t} =&\frac{W_t}{(1-\alpha)Q^{1-\rho}_t(1-x_p)A^{\rho}_{LK,t}K^{\alpha\rho}_t(L^d_t)^{(1-\alpha)\rho-1}}\\ =&\frac{r^k_tP_{k,t}}{Q^{1-\rho}_t\alpha(1-x_p)A^{\rho}_{LK,t}K^{\alpha\rho-1}_t(L^d_t)^{(1-\alpha)\rho}}\end{aligned}$$

4. Government budget constrain is fulfilled:

$$(1+i_{t-1})\int_0^1 B_{t-1}(i)di + G_t = \int_0^1 B_t(i)di + T_t$$

5. Equations (1), (2), (4), (20), (21), (22) and (23).

A.7 Steady State

Let Z denote the steady state of variable Z_t. The subscript r represents a nominal variable that has been deflated by the domestic price $P_{q,t}$ in order to represent a real variable.

Households, Inflations, Government Constraint and Investment

$$
\begin{aligned}
i =& \frac{1}{\beta} - 1, \quad r^k = \frac{1}{\beta} - 1 + \delta, \quad d = \beta \\
\frac{G_r}{Q} =& \omega, \quad I = \delta K, \quad Y = \frac{1}{S_c}(Q - S_e E) \\
C_q =& (1 - x_c) S_c^{\eta_c} C, \quad C_e = x_c \left(\frac{S_c}{S_e}\right)^{\eta_c} C \\
S_c =& \left((1 + x_c) + x_c S_e^{1-\eta_c}\right)^{\frac{1}{1-\eta_c}} \\
\Pi_c =& \Pi_q = \Pi_k = \Pi_{wr} = 1 \\
S_e =& S_k = A_E = A_{LK} = 1
\end{aligned}
$$

Firms

$$
\begin{aligned}
v_p =& 1, \quad Q^o = Q, \quad P_q^o = P_q, \quad MC_r = \frac{1}{\mathcal{M}_p} \\
Q =& \left[x_p A_{E,t}^{\rho} + (1 - x_p) A_{LK}^{\rho} (K^{\alpha} L^{1-\alpha})^{\rho}\right]^{\frac{1}{\rho}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{S_e}{x_p A_E^{\rho} Q^{1-\rho} E^{\rho-1}} =& \frac{W_r}{(1-\alpha) Q^{1-\rho} (1 - x_p) A_{LK}^{\rho} K^{\alpha\rho} (L^d)^{(1-\alpha)\rho - 1}} \\
=& \frac{r^k S_k}{Q^{1-\rho} \alpha (1 - x_p) A_{LK}^{\rho} K^{\alpha\rho - 1} (L^d)^{(1-\alpha)\rho}}
\end{aligned}
$$

We have also:

$$
\begin{aligned}
MC_r Q^{1-\rho} =& \frac{S_e}{x_p A_E^{\rho} E^{\rho-1}} \\
\frac{E}{Q} =& \left(\frac{\mathcal{M}_p S_E}{x_p A_E^{\rho}}\right)^{\frac{1}{\rho-1}}
\end{aligned}
$$

Using the value of x_p from normalization one has:

$$
\frac{E}{Q} = \frac{\alpha_e}{M_p S_e} \tag{24}
$$

Using the CPO from the firms' optimization problem one has:

$$
\frac{K}{Q} = \frac{\alpha(1 - \alpha_e)}{r^k \mathcal{M}_p S_k} \tag{25}
$$

$$
\frac{K}{L} = \frac{W_r \alpha}{(1-\alpha) r^k S_k} \tag{26}
$$

Using the production function and equations (24) and (26) one can show that:

$$
Q = \left(\frac{A_{LK}^{\rho} - \alpha_e^{1-\rho} (\mathcal{M}_p S_E)^{\rho}}{1 - \alpha_e} \frac{\alpha}{(1-\alpha) r_k S_k} \left(\frac{W_r}{Q}\right)^{\alpha\rho}\right)^{\frac{1}{\rho(1-\alpha)}}
$$

Budget Constraint

Denote $S_{c,t} = \frac{P_{c,t}}{P_{q,t}}$. From the budget constraint equation and using equations (24) and (25) one has:

$$\begin{aligned} S_c C =& Q - S_e E - \delta S_k K - \omega Q \\ =& Q - \frac{\alpha_e}{\mathcal{M}_p} Q - \frac{\delta\alpha(1-\alpha_e)}{r^k \mathcal{M}_p} Q - \omega Q \\ \frac{C}{Q} =& S_c^{-1} \underbrace{\left(1 - \frac{\alpha_e}{\mathcal{M}_p} - \frac{\delta\alpha(1-\alpha_e)}{r^k \mathcal{M}_p} - \omega\right)}_{:=a} \end{aligned}$$

Labor

$$v_w = 1, \quad L = L^d, \quad W^o = W, \quad \frac{\lambda_a W_r}{S_c} = \mathcal{M}_w L^\phi, \quad \lambda_a = \frac{1}{C(1-h)}$$

Then one has:

$$\frac{W_r}{Q} = \frac{W_r}{C}\frac{C}{Q} = (1-h)\mathcal{M}_w L^\phi a$$

Then using firms' optimization CPO and Q one has:

$$L = \frac{(1-\alpha)(1-\alpha_e)}{\mathcal{M}_p}\frac{Q}{W_r}$$

From where

$$L = \left(\frac{(1-\alpha)(1-\alpha_e)}{(1-h)\mathcal{M}_p\mathcal{M}_w a}\right)^{\frac{1}{1+\phi}}$$

and the exact value of the remaining variables follows.

B Appendix B: Log-linearized Model

Small case letters represent the log-deviation of each variable with respect its steady state, $z_t := log(Z_t) - log(Z)$. For the rental rate of capital (r_t^k) and the investment (I) the log-deviation will be noted $\widehat{r_t}$ and $\widehat{I_t}$ respectively. The model is simplified in order to have just real prices and quantities. The list of log-linear equations that characterize the equilibrium is:

$$s_{c,t} = \left(\left(\frac{S_e}{S_c} \right)^{1-\eta_c} \right) x_c s_{e,t} \tag{27}$$

$$c_{q,t} = c_t + \eta_c s_{c,t} \tag{28}$$

$$c_{e,t} = c_t + \eta_c (s_{c,t} - s_{e,t}) \tag{29}$$

$$\lambda_{a,t} = -\frac{1}{1-h}(c - hc_{t-1}) \tag{30}$$

$$i_t = \lambda_{a,t} - \mathbb{E}_t[\lambda_{a,t+1}] + \mathbb{E}_t[\pi_{c,t+1}] \tag{31}$$

$$i_t = (1 - \beta(1-\delta))\mathbb{E}_t[\widehat{r}_{t+1}] + \mathbb{E}_t[\pi_{k,t+1}] \tag{32}$$

$$\pi_{q,t} + \pi_{w_{r,t}} = \beta \mathbb{E}\left[\pi_{q,t+1} + \pi_{w_{r,t+1}}\right] + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\phi\epsilon_w)}(mrs_t + s_{c,t} - w_{r,t}) + \varepsilon_{w,t} \tag{33}$$

$$mrs_t = \phi l_t + \frac{1}{1-h}(c - hc_{t-1}) \tag{34}$$

$$mc_{r,t} = (\rho - 1)q_t - (\rho - 1)e_t - \rho a_{e,t} + s_{e,t} \tag{35}$$

$$s_{e,t} - \rho a_{e,t} - (\rho - 1)e_t = w_{r,t} - \rho a_{lk,t} - \alpha\rho k_t - ((1-\alpha)\rho - 1)l_t \tag{36}$$

$$l_t + w_{r,t} = k_t + \widehat{r}_t + s_{k,t} \tag{37}$$

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\left(\phi_\pi \pi_{q,t} + \phi_y y_t\right) + \varepsilon_i \tag{38}$$

$$q_t = \alpha_e(a_{e,t} + e_t) + (1-\alpha_e)(\alpha k_t + (1-\alpha)l_t + a_{lk,t}) \tag{39}$$

$$\delta \widehat{I}_t = k_{t+1} - (1-\delta)k_t \tag{40}$$

$$Qq_t - S_e E(e_t + s_{e,t}) = S_c C(s_{c,t} + c_t) + S_k I(\widehat{I} + s_{k,t}) + G_r g_{r,t} \tag{41}$$

$$\pi_q = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} mc_{r,t} + \beta\mathbb{E}[\pi_{q,t+1}] + \varepsilon_{p,t} \tag{42}$$

$$S_c Y(y_t + s_{c,t}) = Qq_t - S_e E(e_t + s_{e,t}) \tag{43}$$

$$\pi_{k,t} = \pi_{q,t} + s_{k,t} - s_{k,t-1} \tag{44}$$

$$\pi_{c,t} = \pi_{q,t} + s_{c,t} - s_{c,t-1} \tag{45}$$

$$\pi_{w_{r,t}} = w_{r,t} - w_{r,t-1} \tag{46}$$

$$s_{e,t} = \rho_{se} s_{e,t-1} + e_{se,t} \tag{47}$$

$$s_{k,t} = \rho_{sk} s_{k,t-1} + e_{sk,t} \tag{48}$$

$$g_{r,t} = \rho_g g_{r,t-1} + \rho_{gae} e_{ae,t} + \rho_{galk} e_{alk,t} + e_{g,t} \tag{49}$$

$$\varepsilon_{i,t} = \rho_i e_{i,t-1} + e_{ei,t} \tag{50}$$

$$a_{e,t} = \rho_{ae} a_{e,t-1} + e_{ae,t} \tag{51}$$

$$a_{lk,t} = \rho_{alk} a_{lk,t-1} + e_{alk,t} \tag{52}$$

$$\varepsilon_{w,t} = \rho_w \varepsilon_{w,t-1} - \nu_w e_{w,t-1} + e_{w,t} \tag{53}$$

$$\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} - \nu_p e_{p,t-1} + e_{p,t} \tag{54}$$

C Appendix C: Bayesian Estimation

C.1 Data Treatment

A total of eight series, corresponding to the eight structural shocks of the model, are taken as key macro-variables for the estimation. All the series are quarterly. A description of the original series's sources is presented in Table 4 and data is available upon request. The sample goes from 1984:Q1 to 2007:Q1.

Table 4: Original Sources

Serie	Description	Source
GDPC09	Real Gross Domestic Product, Chained Dollars (2009), Seasonally Adjusted, Annual Rate	Table 1.1.6 Bureau of Economic Analysis
GDPDEF	Implicit Price Deflators for Gross Domestic Product (2009), Seasonally Adjusted	Table 1.1.9. Bureau of Economic Analysis
PCE	Personal Consumption Expenditures by Major Type of Product, Seasonally Adjusted,Annual Rate	Table 2.3.5. Bureau of Economic Analysis
PCE_{oil}	Personal Consumption Expenditures by Major Type of Product: Gasoline and other energy goods, Seasonally Adjusted,Annual Rate	Table 2.3.5. Bureau of Economic Analysis
PFI	Private Fixed Investment by Type, Seasonally Adjusted,Annual Rate	Table 5.3.5. Bureau of Economic Analysis
CE16OV	Civilian Employment, 16 and over, Seasonally Adjusted, Thousands	LNS12000000 Bureau of Labor Statistics
CE16OV Index	CE160V (2009)=1	
LNS10	Population level, civilian noninstitutional population, 16 and over, Seasonally Adjusted, Thousands	LNS10000000 Bureau of Labor Statistics
LNS10 Index	LNS10 (2009)=1	
PRS85006023	Nonfarm Business, All Persons, Average weekly hours worked Duration (2009), Seasonally Adjusted	PRS85006023 Bureau of Labor Statistics
PRS85006103	Nonfarm Business, All Persons, Hourly Compensation Duration (2009), Seasonally Adjusted	PRS85006103 Bureau of Labor Statistics
FEDFUND	Federal funds effective rate, percent: Per Year, Average of Daily figures	Board of Governors of the Federal Reserve System
$TotalSAOil$	Constructed as in Acurio-Vásconez et al. (2015)	Acurio-Vásconez et al. (2015)

The observable variables include: (i) real GDP, (ii) real non-oil Consumption, (iii) real Private Fixed Investment, (iv) Hours Worked, (v) real Wages, (vi) Inflation, (vii) the Federal Funds Rate and (viii) Total Oil Use in Production. The model is stationary, so series that are not originally stationary, which are the fist five, have to be detrended. For that, I use linear trend techniques. The rest of the series are stationary, so I do not detrend them, but I takeout their respective mean for the estimation period. A detailed explanation of the manipulation of the data is presented on Table 5. For estimation and simulation, the model has been log-linearized, then the corresponding observable variables are given in natural logarithms and multiplied by 100.

Table 5: Observable Variables

Observed Variable	Transformation
invobs	$detrend\left(log\left(\frac{\frac{PFI}{GDPDEF}}{LNSIndex}\right) * 100\right)$
yobs	$detrend\left(log\left(\frac{GDPC09}{LNSIndex}\right) * 100\right)$
cqobs	$log\left(\frac{\frac{PCE-PCE_{oil}}{GDPDEF}}{LNSIndex}\right) * 100$
labobs	$log\left(\frac{PRS85006023*CE16OVIndex}{LNSIndex}\right) * 100 - mean\left(ln\left(\frac{PRS85006023*CE16OVIndex}{LNSIndex}\right) * 100\right)$
wobs	$log\left(\frac{\frac{PRS85006103}{GDPDEF}}{LNSIndex}\right) * 100 - mean\left(ln\left(\frac{\frac{PRS85006103}{GDPDEF}}{LNSIndex}\right) * 100\right)$
infobs	$log\left(\frac{GDPDEF}{GDPDEF(-1)}\right) * 100 - mean\left(ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right) * 100\right)$
iobs	$\left(log\left(1 + \frac{FEDFUND}{400}\right) - mean\left(ln\left(1 + \frac{FEDFUND}{400}\right)\right)\right) * 100$
eobs	$log\left(\frac{TotalSAOil}{LNSIndex}\right) * 100 - mean\left(log\left(\frac{TotalSAOil}{LNSIndex}\right) * 100\right)$

Finally, I have to identify the observable series to my model's variables. Note that the model have three different type of prices: a domestic price P_q, a CPI P_c, which is equal to the GDP deflator by definition, and a capital price P_k. Because all the observable series are deflated by the GDP deflator and the real variables in the model are deflated by the domestic price P_q, there are some concordances that have to be done. The final observation equations for the model are:

$$
\begin{aligned}
invobs_t =& \widehat{I}_t + s_{k,t} - s_{c,t} \\
ybos_t =& y_t \\
cqobs_t =& c_{q,t} - s_{c,t} \\
labobs_t =& l_t \\
eobs_t =& e_t \\
infobs_t =& \pi_{c,t} \\
iobs_t =& i_t \\
wobs_t =& w_{r,t} - s_{c,t}
\end{aligned}
$$

C.2 Distribution Parameters

Before estimation, one needs to identify what the distribution parameters in the CES function represent. Define $\omega_c = \frac{P_e C_e}{P_c C}$. Remark that ω_c could be calibrated from data[21]. From the steady state equations, one needs the following relationship to be satisfied:

$$\begin{aligned}
\frac{C_e}{C} =& x_c \left(\frac{P_e}{P_c}\right)^{\frac{1}{\sigma-1}} \\
\frac{P_e C_e}{P_c C} =& x_c \left(\frac{P_e}{P_c}\right)^{\frac{\sigma}{\sigma-1}} \\
\omega_c =& x_c S_e^{\frac{\sigma}{\sigma-1}} \left(\frac{P_c}{P_q}\right)^{\frac{-\sigma}{\sigma-1}} \\
\omega_c =& x_c S_e^{\frac{\sigma}{\sigma-1}} \left((1-x_c) + x_c S_e^{\frac{\sigma}{\sigma-1}}\right)^{-1}
\end{aligned}$$

Assuming a steady state equals to 1 for the real price of oil, S_e, one has $\omega_c = x_c$. In this way, the distribution parameter, x_c, represents the share of oil consumption out of household total consumption.

The identification of the parameter x_p is less straightforward. As pointed out in Cantore and Levine (2012), distribution parameters in CES production functions needs a renormalization in order to be estimated. In fact, Cantore and Levine (2012) showed that under the formulation of the CES function as in equation (9), the parameter x_p is a dimensional parameter and depends on the units chosen for factor inputs. In order to avoid this problem in estimation, I normalize this function as those authors do.

Remark that at steady state, the following equations hold:

$$\frac{Q}{E}\left(\frac{x_p A_E^{\rho} E^{\rho}}{Q^{\rho}}\right) = \frac{Se}{MC_r} \tag{55}$$

Define

$$\pi = \frac{x_e A_E^{p} E^{\rho}}{Q^{\rho}} \Rightarrow x_p = \pi\left(\frac{Q}{A_E E}\right)^{\rho} \tag{56}$$

As pointed out in Cantore and Levine (2012), π is the re-normalized distribution parameter. We just need to interpreted what does it mean in the model.

For that, remark that using equation (55) one also has

$$\begin{aligned}
\pi =& \frac{S_e}{MC_r}\frac{E}{Q} \\
\pi =& \frac{1}{MC_r}\frac{P_e E}{P_q Q} \Rightarrow \frac{E}{Q} = \pi \frac{1}{\mathcal{M}_p S_e}
\end{aligned} \tag{57}$$

[21] For $P_e C_e$ one can use the series for nominal personal consumption expenditures: Gasoline and other energy goods, and for $P_c C$, one can use the nominal Personal Consumption Expenditure. I will take ω_c as the mean of the generated series by $\frac{P_e C_e}{P_c C}$ in the estimation period

From where one has

$$x_p = \pi^{1-\rho}\left(\frac{\mathcal{M}_p S_e}{A_E}\right)^{\rho}$$

In the other hand, the steady state of the output elasticity of oil denoted by $\alpha_{e,t}$, is defined as

$$\begin{aligned}\alpha_e =& \frac{\partial Q}{\partial E}\frac{E}{Q} \\ =& x_p\left(\frac{A_E E}{Q}\right)^{\frac{\eta_p - 1}{\eta_p}} \\ =& x_p\left(\mathcal{M}_p \frac{S_e}{x_p A_E^{\rho}}\right)^{1-\eta_p}\end{aligned}$$

Then $\pi = \alpha_e$, i.e, the normalized parameter represent oil output elasticity.

As for the oil cost share and the output elasticity, at steady state, in this model it is defined as:

$$\begin{aligned}\text{oil cost share} :=& \frac{P_e E}{PcY} \\ =& \frac{P_e E}{P_q Q - P_e E} \\ =& \frac{\frac{P_e E}{P_q Q}}{1 - \frac{P_e E}{P_q Q}} \\ =& \frac{\alpha_e MC_r}{1 - MC_r} \\ =& \frac{\alpha_e}{\mathcal{M}_p - \alpha_e}\end{aligned}$$

Finally, following Kumhof and Muir (2014), I assume that the oil cost share is 3.5 percent. Then following this last relationship, I assume that the prior value for the output elasticity α_e is 3.9 percent.