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Mutual or stock insurance:
Solidarity in the face of insolvency

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Abstract

This paper analyzes the choice of the insurer, mutual or stock, for a heterogeneous population aware of insurers' probability of insolvency. The stock insurer sets individualized premiums and manages his probability of insolvency by means of a premium loading, in contrast to the mutual insurer who sets an average premium and allows a possibility to adjust the premium level ex-post. We show that under some conditions mutual insurer is optimally preferred by the entire population of high and low risk agents. The existence of pooling equilibrium depends on the relative weight of each group of risks in the population, and on the size of the risk loading. For a sufficiently small group of low risk agents, an increase in the risk loading provides an incentive to pool their risk with the high-risk agents through the mutual agreement.

Key words: mutualization, risk loading, insolvency, symmetrical information, high risks

JEL classification: D81, G22, G28

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1 Introduction

Insurers' probability of insolvency and current information accessibility are two issues that attract attention of regulators and general public alike.

In the economic literature, insurance contracts are mostly defined as an exchange of an uncertain wealth for a certain one by means of premium payment. This assumption excludes the possibility that the stock insurer will be unable to provide a full indemnity to each claimant. Moreover, the actuarially fair premiums are assumed to be equal to the individual expected loss, although the actuarial literature agrees on the necessity of a risk loading to manage the probability of insolvency. Laux and Muermann (2010) argue that in the perfect capital market it would indeed be optimal for the stock insurer to raise an infinite amount of external capital to avoid insolvency and sell full insurance contracts at the price equal to the policyholder's expected loss. Nevertheless, it is not the case in the actual market where the probability of insolvency should not be neglected.

At the same time, the use of information on the individual behavior and telematics data raise concerns about their potential impact on the insurance functioning, mutualization principle and solidarity. The literature suggests that low risk agents do not benefit from pooling their risk with high risk agents due to the adverse selection issue (Rothschild and Stiglitz, 1978). In the context of information asymmetry, mutuals serve as a screening mechanism allowing to separate high and low risks. Nevertheless, given the current data accessibility, the symmetrical information hypothesis becomes more relevant than before. Theory predicts that the low risk agents do not choose to pool their risk with the high risk agents because of the associated premium cross-subsidization. An insurer informed about his policyholders' risk type should be able to detect low risk agents and offer them full coverage for a lower premium. The individualization is therefore assumed to be advantageous for the insurers and the low risk agents.

The present paper examines the conditions making mutual insurance beneficial for a heterogeneous population of high and low risk agents. When insolvency is possible and the risk types are known, the choice between the mutual and the stock insurer is based on two criteria: price and expected indemnity. Other things being equal, agents are better covered in the mutual pool in case of insolvency, because the risk is spread across all the mutual participants. Alternatively, the stock insurance policyholders are unaffected by the insolvency except when they claim a loss, and the stock insurance premiums are individualized. We find that low risk agents are not necessarily better off staying with a stock insurer without high risk agents. The condition that makes a mixed mutual pool preferable for the low risks is related to the weight of their group in the population and the size of the risk loading. If the size of the portfolio decreases, it affects the premium size given a fixed level of the probability of insolvency. Consequently, an increase in the premium level provides incentives for the low risk agents to join the mutual pool with the high risk agents. Our results contribute to the current debate on information accessibility and its potential use in the evaluation of risk profiles.

Our paper adds to two strands of literature, the one on the coexistence between the mutual and the stock insurance, and the other one on the possibility of insolvency.

The coexistence between the stock and the mutual insurers has been studied in the adverse selection context. B. Smith and Stutzer (1990) show that high risk agents purchase a full coverage from stock insurer, while low risk agents choose a mutual contract to signal their type. Ligon and Thistle (2005) argue that low risk agents choose a mutual contract, specifically by forming small mutual pools, to prevent high risks from joining them. Bourlès and Henriet (2012) consider the case where the mutual risk-sharing is the only available form of insurance, and show that the mutual is more convenient for dealing with asymmetric information since there is less loss of efficiency comparing with the stock insurance form. Picard (2014) shows that participating policies associated with the mutual insurance prevent other insurers from cream-skimming low risks, since it will necessarily attract high risks as well. We are interested in the symmetric information case where both the mutual and the stock insurers can learn each agent's risk type before contracting, while the agents are aware of the probability of insolvency.

The insolvency as an exogenous factor is analyzed in Doherty and Schlesinger (1990) and Briys, Schlesinger, and Schulenburg (1991). Rees, Gravelle, and Wambach (1999) show that if consumers correctly perceive the endogenous probability of insurer's insolvency, it is always optimal for the latter to gather the capital to rule out the chance of insolvency. Mimra and Wambach (2019) model the endogenous insolvency given the information asymmetries and show that collecting more capital is not optimal as it might incentivize other insurers to cream-skim the low risk agents. Laux and Muermann (2010) show that in presence of the frictional cost of capital, it would be optimal for policyholders to collectively provide additional capital to improve risk sharing. However, stock insurance policyholders have an incentive to free-ride on the capital provided by others, which is not the case for the mutual form. In our model, the endogenous risk of insurer's insolvency comes from the limited financial reserves. The risk loading is used as a tool to manage the probability of insolvency. The role of the risk loading in the accumulation of reserves is discussed by Tapiero, Kahane, and Jacque (1986), Cummins (1991), M. Smith and Kane (1994) and Gatzert and Schmeiser (2012).

We are interested in the symmetric information case with an endogenous probability of insolvency depending on the premium level. Fagart, Fombaron, and Jeleva (2002) and Charpentier and Le Maux (2014) are most closely related to the issue we treat. Fagart, Fombaron, and Jeleva (2002) examine the coexistence of both types of insurers in a competitive setting and show that multiple configurations are possible at the equilibrium. In particular, both mutual and stock insurers can coexist if the stock insurer is limited in size because of the fixed capital stock. However, the authors focus on the competition in the insurance market with multiple stock and mutual firms. We model the market with one mutual and one stock insurer to focus on their structural difference. Charpentier and Le Maux (2014) examine the attractiveness of the government provided insurance program compared to the limited-liability insurer in the context of the natural catastrophe insurance. Our issue is close to the one in Charpentier and Le Maux (2014), since their government provided insurance program is similar in its functioning to the mutual insurance. The difference between our works stems from the fact that we consider the heterogeneity in the probability of loss rather than correlation between risks.

Our paper is structured as follows. The model illustrating the difference in functioning between a mutual and a stock insurer is presented in Section 2. In Section 3, we derive the conditions such that the low risk agents prefer to pool their risk with the high risk agents by purchasing a mutual insurance contract. We pursue with a discussion in Section 4. The concluding remarks are provided in Section 5.

2 The model with two types of insurers and endogenous insolvency

Consider a population of n independent and risk averse agents with the same preferences represented by the von Neumann-Morgenstern utility function U defined over final wealth, $U' > 0\,, U'' < 0$. Each agent is endowed with a non-random initial wealth w and faces a risk of losing a monetary amount $L\,, L < w$. The individual loss probability of an agent k is noted $p_k\,, k = 1, ..., n$. Thus, without insurance, the final wealth of an agent k is a lottery $\widetilde{w}_k = (w, 1 - p_k\,; w - L, p_k)$.

Insurance is considered to be mandatory. It is provided by one mutual insurer and one stock insurer. An agent k chooses either to join the mutual pool managed by the mutual insurer or to purchase a contract from the stock insurer. We assume that all the capital is provided by policyholders through the premiums. An insurer collects premiums at the beginning of the period and pays claims at the end. The total amount of claims may exceed the amount of collected premiums, thus both insurers face the risk of ruin represented by a simple one-period model. Information on individual loss probability and insurer's insolvency probability is complete and symmetrical: both the agent and the insurer she chooses to contract with have information about the agent's individual loss probability and the insurer's probability of insolvency.

2.1 Endogenous probability of insolvency

The insolvency occurs when the insurer does not have enough funds to provide full coverage to all the claimants. The probability of insolvency, therefore, measures the chances that the total amount of loss will be larger than the total premiums amount. Consider $\tilde{N}$ the random variable denoting the number of claimants in the mutual pool, or equally in the stock insurance portfolio, with N the realization of $\tilde{N}$. We denote q the probability of insolvency. If there are n policyholders in the mutual pool, or equally in the stock insurer's portfolio, and π_k is the premium of an agent k, then the insurer's probability of insolvency is

$$q = \Pr\left(\tilde{N}L > \sum_{k=1}^{n} \pi_k\right). \tag{1}$$

We denote by $\tilde{X} = \frac{\tilde{N}}{n}$ the share of claimants. The random variable $\tilde{X}$ is defined over the interval $[0, 1]$ with density function $f(x)$ and the distribution function $F(x)$. Using it, we can rewrite equation (1) as

$$q = \Pr\left(\tilde{X} > \frac{\sum_{k=1}^{n} \pi_k}{nL}\right) = \Pr\left(\tilde{X} > \overline{x}\right) = 1 - F(\overline{x}), \tag{2}$$

where the term $\overline{x}$ in equation (2) denotes the largest share of claimants that is fully insurable, which is the maximum percentage of policyholders that can receive a total coverage. Each of the possible states of insolvency is associated with the share of claimants x such that $x > \overline{x}$. The probability of insolvency is therefore endogenous and depends in particular on the total number of policyholders, the premium structure and the policyholder's risk level. The latter has an impact both on the distribution of the number of claimants and the capital available to the insurer through the premiums, as it will be discussed further in this section.

A non-zero probability of insolvency implies a possibility of risk retention, which can be interpreted as negative externalities. Indeed, in case of insolvency, the indemnity is not full. Consequently, some policyholders might bear additional expenses on top of the premium. The impact of the insurer's insolvency on the given policyholder depends not only on the stochastic amount of claims, but also on the insurer's type. A policyholder's final wealth is a lottery conditional on the type of insurer she chooses to contract with, since the mutual and the stock insurers have different pricing strategies and different mechanisms to manage the possibility of insolvency. Those differences are presented in the rest of this section.

2.2 Stock insurer: individualized premiums and the risk loading

The stock insurer sets individualized premiums based on the policyholder's individual risk level. The premium is calculated according to the policyholder's expected loss. Moreover, the stock insurer can manage his probability of insolvency ex-ante by charging the premium with a risk loading[1]. The latter enables the accumulation of financial reserves, which constitute a buffer fund providing insurer's risk-bearing capacity (Cummins, 1991). The

[1]The risk loading is an actuarial measure and is different from the expense loading, which is used to cover insurance administrative costs.

stock insurance premium for an agent k , π_k^S , is set according to her expected loss $p_k L$ and charged with a risk loading c[2]:

$$\pi_k^S = p_k L + c\,.$$

Other things being equal, the probability of insolvency decreases with the risk loading, while the risk loading decreases with the size of the portfolio for a given probability of insolvency.

For a given probability of insolvency $\overline{q}$, the risk loading $c\left(\overline{q}, n\right)$ is such that

$$\Pr\left(\tilde{X} > \frac{\sum_{k=1}^{n}(p_k L + c)}{nL}\right) = \overline{q}\,,$$

or equivalently

$$F\left(\frac{\sum_{k=1}^{n} p_k}{n} + \frac{c}{L}\right) = 1 - \overline{q}\,.$$

Thus, the size of the risk loading depends on the desirable probability of insolvency and the total number of policyholders in the portfolio. In particular, it is decreasing with $\overline{q}$ and n: $\frac{\partial c}{\partial \overline{q}} < 0$ and $\frac{\partial c}{n} < 0$[3]. Note that the maximum value of the risk loading corresponding to $\overline{q} = 0$ is $c^{max} = (1-p)L$ with $\overline{p} = \frac{\sum_{k=1}^{n} p_k}{n}$.

We now write the expected utility of an agent insured by a stock insurer. For the stock insurer, the risk of insolvency corresponds to the risk of default. If the amount of collected premiums is lower than the amount of the aggregate claims, the stock insurer reimburses the claimants to the extent of the available funds and goes bankrupt. The stock insurance indemnity $I\left(.\right)$ depends on the realization of the random variable $\tilde{X}$ representing the share of claimants. If the maximum possible indemnity per head given the share of claimants is at least as high as the loss L, then the stock insurer is solvent and the indemnity is full: $I\left(x\right) = L$ if $x \leq \overline{x}$. If the maximum possible indemnity per head given the share of claimants is lower than the loss L, the insurer is insolvent, and each claimant receives a partial coverage derived from a pro rata calculation: $I\left(x\right) = \frac{\sum_{k=1}^{n} \pi_k^S}{nx} < L$, if $x > \overline{x}$.

[2]To better emphasize the difference between the stock and the mutual insurer, we assume here that the risk loading is not proportional to individual risk.

[3]Following the law of large numbers and the central limit theorem, the size of the risk loading becomes increasingly small when the size of the pool tends toward infinity.

Consequently, for a given policyholder, the impact of stock insurer's insolvency depends on the realization of her individual loss. The expected utility of a stock insurance policyholder can be written as

$$\mathbb{E}U(\tilde{w}_k^S) = \int_0^1 xU\left(w - \pi_k^S - L + I(x)\right) f(x)dx + \int_0^1 (1-x)U(w - \pi_k^S) f(x)dx\,,$$

with x being a share of claimants in the stock insurance portfolio, and therefore the policyholder k's chances of being a claimant. There are $1-x$ percent chances that she will not file a claim and her only expense will be the premium she paid, and there are x percent chances that she will become a claimant, in which case her wealth is conditional on the stock insurer's insolvency[4]. If the stock insurer is insolvent and she files a claim, her final wealth will be determined by the total share of claimants in the portfolio.

Another way of presenting the lottery on final wealth in the stock insurance case is given by Figure 1. If the agent k chooses the stock insurance contract, her final wealth $\tilde{w}_k^S$ will depend on her chances to become claimant, p_k , and on the stock insurer's probability of insolvency $\overline{q}$. The resulting multi-stage lottery can be represented as a one-stage lottery with probabilities $\overline{q}p_k$ and $(1-\overline{q}p_k)$.

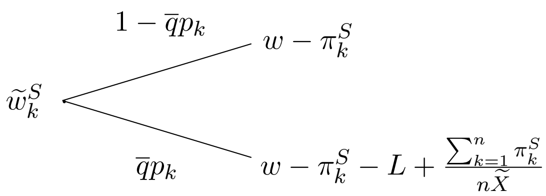

Figure 1: Policyholder k's final wealth in the stock insurance portfolio

2.3 Mutual insurer: average premiums and ex post adjustment

Depending on the mutual policy, the mutual premium can provide various degrees of differentiation. In comparison with the stock insurance premium, it is not necessarily individualized. The mutual belongs to its policyholders, hence there is less differentiation, since there is no pressure from stockholders for making a profit. The mutual premium

[4]For a stock insurance policyholder k, the probability to be affected by insolvency, qp_k, is therefore more relevant than the actual probability of insolvency q, since she is affected by her insurer's insolvency only when she files a claim herself.

can vary from a perfectly individualized premium ($\pi_k^M = p_k L$) to an average premium based on the aggregate risk estimation, $\pi^M = \overline{p}L$, with $\overline{p} = \frac{\sum_{k=1}^n p_k}{n}$ in the mutual pool of size n. For the sake of simplicity, we assume that the mutual insurer charges an average premium.

As opposed to the stock insurer, the mutual insurer does not fix his probability of insolvency by charging the premium with the risk loading. He has a possibility to adjust the premium level ex-post at the end of the period, if the collected premiums are not sufficient to cover the total amount of loss. This mechanism allows the mutual insurer to exclude the risk of default, in spite of a positive probability of insolvency that we denote q^M:

$$q^M = \Pr\left(\tilde{N}L > n\overline{p}L\right) = \Pr\left(\tilde{X} > \overline{p}\right) .$$

The probability of mutual insurer's insolvency is therefore the probability of an ex post premium adjustment.

In comparison with the stock insurer, the impact of the mutual insurer's insolvency is the same regardless the realization of the individual risk, because the ex-post adjustment, if necessary, applies to all the policyholders. If the mutual insurer is insolvent ($x > \overline{p}$), each policyholder will retain an equal share $\frac{1}{n}$ of the total financial shortage. The financial shortage being equal to $NL - n\overline{p}L$, each mutual participant will retain an amount of the shortage per head equal to $xL - \overline{p}L$ [5]. Thus, the expected utility of final wealth is the same for each policyholder:

$$\mathbb{E}U(\tilde{w}^M) = \int_0^1 U\left(w - \overline{p}L - A(x)\right) f(x) dx .$$

with $A(x)$ equal to zero if the mutual insurer is solvent and equal to $xL - \overline{p}L$ if he is not. The ex-post adjusted payment in case of insolvency, xL, is equal to the average loss per head in the pool, which illustrates the solidarity principle implicit in the mutual risk sharing.

Another way of presenting the lottery on final wealth in the mutual insurance case is given by Figure 2. If the agent chooses the mutual insurance contract, her final wealth

[5] In particular, each claimant receives a partial coverage $L - (xL - \overline{p}L)$, and each other policyholder contributes an additional amount $(xL - \overline{p}L)$.

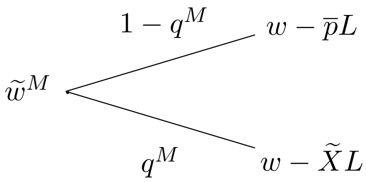

Figure 2: Policyholder k's final wealth in the mutual pool

$\widetilde{w}^M$ is unconditional on her loss and depends only on the mutual insurer's probability of insolvency q^M.

2.4 Additional assumptions

For the sake of simplicity, we assume that insurance is compulsory. It is not optimal for an agent to purchase an insurance contract if no one else does. Purchasing an insurance coverage alone implies paying a premium without reducing the risk, making it worse than without insurance at all. For this reason, we assume that each agent has to choose one insurer or another.

We also assume that the decision to purchase the stock or the mutual insurance contract is identical for all the agents of the same risk type. First, the same argument applies as for the mandatory insurance assumption. Next, the possibility of insolvency affects the expected indemnity, which is conditional on the global amount of claims. Hence, the expected indemnity is determined by the size and the composition of the mutual pool or the stock insurance portfolio. Consequently, each agent's individual decision to choose the stock or the mutual insurer depends not only on the differences between the two, but also on the other agents' choice. The choice problem is therefore different for each additional policyholder considered sequential decision problem. Thus, we consider that the choice of the insurer's type is identical for all the agents of the same risk type, and it is made simultaneously by each homogeneous group of agents.

In the next section, we examine possible market equilibria in terms of the choice of the insurer's type made by a heterogeneous population.

3 The choice of the insurer's type

We represent a heterogeneous population as n independent individual risks with two risk types: low risks (l) and high risks (h). The proportion of low risks in the population is denoted θ. There are $n_l = \theta n$ low risk agents and $n_h = (1-\theta)n$ high risk agents. We assume that the decision to purchase a stock or a mutual insurance contract is identical for all the agents of the same risk type. Thus, we describe the choice of the insurer's type as a two players game, where each player is the entire low or high risk group. The players are denoted Player L and Player H respectively. This representation implies that, in what follows, $k = h, l$. Hence, each agent's individual loss probability is defined by p_k , $1 > p_h > p_l > 0$.

The players decide simultaneously to purchase one of the two available contracts. The set of possible actions for each player is $\{M, S\}$, where M stands for the purchase of the mutual insurance contract, and S for the stock insurance contract. The payoff matrix is presented in Table 1. If the players choose different insurers, the composition of the mutual pool and the stock insurance portfolio stays homogeneous, and the expected utility of each player will depend on the risk distribution of the variable $X_k = \frac{N_k}{n_k}$, with $k = h, l$. If both choose the same insurer, the global risk is defined by the distribution of $X = \frac{N_h+N_l}{n_h+n_l} = \frac{N}{n}$.

		Player L	
		M	S
Player H	M	$\left(\mathbb{E}U(\tilde{w}_H^M)\,;\,\mathbb{E}U(\tilde{w}_L^M)\right)$	$\left(\mathbb{E}U(\tilde{w}_H^M)\,;\,\mathbb{E}U(\tilde{w}_L^S)\right)$
	S	$\left(\mathbb{E}U(\tilde{w}_H^S)\,;\,\mathbb{E}U(\tilde{w}_L^M)\right)$	$\left(\mathbb{E}U(\tilde{w}_H^S)\,;\,\mathbb{E}U(\tilde{w}_L^S)\right)$

Table 1: Payoff matrix

Now we explore the optimal choice of the insurer's type for both risk groups. First, we consider a homogeneous population. Based on Charpentier and Le Maux (2014), we prove the following result.

Proposition 1. *When the population is homogeneous and the stock insurer does not introduce a risk loading ($c = 0$), at equilibrium, the entire population is covered by the mutual insurer.*

Proof. See Appendix. □

First, when the population is homogeneous, the agents have the same loss probability ($p_k = p$). If the stock insurance risk loading is equal to zero ($c = 0$), the mutual and the stock insurance premiums are equal ($\pi^M = \pi^S = pL$), and so is the maximum insurable loss ($\overline{x} = p$). In this setting, the lottery on final wealth resulting from the purchase of the stock insurance contract is a mean-preserving spread of the lottery resulting from the purchase of the mutual insurance contract. Consequently, the mutual insurance contract is preferred by all risk-averse utility maximizers. Other things being equal, agents benefit from the mutual principle of sharing the total loss among all the participants.

The result of Proposition 1 applies to the situations in which the proportion θ of the low risks in the population is equal to zero or one. This result can be extended to the heterogeneous population. Since the utility function $U(.)$ is continuous and so is the expected utility function $\mathbb{E}U(.)$, there exists a proportion of the low risk agents sufficiently large (or sufficiently small) so that the mutual insurer is still preferred by the low risk group (high risk group) when the risk loading is equal to zero.

Proposition 2. *When the risk loading is equal to zero, there exist the critical values $\hat{\theta}$ and $\hat{\hat{\theta}}$ such that:*

i) when $\theta < \hat{\theta}$, the high risk agents prefer the mutual insurer to the stock;

ii) when $\theta > \hat{\hat{\theta}}$, the low risk agents prefer the mutual insurer to the stock.

Proof. See Appendix. □

In terms of the two players game, Proposition 2 implies that the strategy S is strictly dominated by the strategy M for the group of type k agents when the group is large enough, i.e. when θ is close to one of the bounds, and the risk loading is equal to zero. For instance, if the proportion of the low risk agents is sufficiently high ($\theta > \hat{\hat{\theta}}$), it is optimal for them to choose the mutual insurer alone rather than the stock insurer with the high risk agents. In other words, the payoff from the strategy profile (S, M) is higher than the payoff of the strategy profile (S, S).

If it is optimal for the low risk agents to choose the mutual insurer, it is optimal for the high risk agents to choose the mutual insurer as well. Indeed, the strategy profile

(S, M) is never an equilibrium. If the low risk agents choose the mutual insurer, the high risk agents benefit from joining them. It is optimal for the high risk agents to pool their risk with the low risk agents, since the mutual pool provides a lower premium to the high risk agents and decreases the expected rest on charge in case of insolvency. Thus, if the proportion of the low risk agents is higher than the critical value $\hat{\hat{\theta}}$, and the risk loading is equal to zero, both the high and the low risk agents optimally choose the mutual insurer, and the strategy profile (M, M) is an equilibrium.

On the contrary, it is not necessarily optimal for the low risk agents to join the high risk agents in the mutual pool. For instance, if the proportion of the low risk agents is sufficiently small ($\theta < \hat{\theta}$), and the risk loading is equal to zero, the high risk agents prefer the mutual insurer to the stock. In this case, for the low risk agents, the choice of the mutual insurer implies that the global risk distribution in the mutual pool will be close to X_h. In other words, the average risk level in the population tends to the high risk level, $\overline{p} \to p_h$.

Now consider an increase in the size of the risk loading. It can further be shown that the risk loading has a positive impact on the policyholder's expected utility only if the probability of insolvency is lower than some critical value $\hat{x}$.

Proposition 3. *An increase in the stock insurance premium has a positive (negative) impact on the policyholder's expected utility if* $\overline{x} < \hat{x}$ *(*$\overline{x} > \hat{x}$*).*

Proof. See Appendix. □

Proposition 3 implies that an increase in the risk loading c has a positive or a negative impact on the policyholder's expected utility, depending on the stock insurer's initial probability of insolvency. An increase in the risk loading generates an accumulation of reserves forming the buffer fund, thus increasing the maximum insurable share of claimants $\overline{x}$ and consequently decreasing the probability of insolvency. At the same time, an increase in the risk loading has a negative impact on the expected utility because of the premium increase. If the probability of insolvency is high ($\overline{x} < \hat{x}$), the positive impact associated with the decrease in the probability of insolvency outweighs the negative impact of the premium increase. The overall impact is negative otherwise.

Following Proposition 3 and the properties of the risk loading function, which is negatively related to the portfolio size for a given value of the probability of insolvency, a sufficient increase in the stock insurance risk loading provides an incentive for the low risk agents to join the high risk agents in the mutual pool.

Proposition 4. *The mutual insurance is preferred by the entire heterogeneous population of high and low risk agents if $\theta < \hat{\theta}$ and $c > \hat{c}$.*

If the low risk agents choose the stock insurer, given that their group is small, the risk loading will compensate for the number of policyholders, given a predefined level of the probability of insolvency. For the values of the risk loading higher than the critical value $\hat{c}$, the relative benefit of the stock insurer's decreased probability of insolvency is outweighed by the premium level to the point that the mutual insurance provides a higher expected utility. Consequently, the stock insurer is not necessarily more attractive for the low risk agents than the mutual insurer.

4 Discussion

Insolvency represents a subject of interest especially since the reform of the European Union insurance regulation and the Solvency II Directive which came into effect in 2016. In particular, it introduces the Solvency Capital Requirement. The latter is defined as the amount of capital to be held by an insurer to meet his obligations to policyholders and beneficiaries over the following year with a probability of at least 99.5%. According to the report made by European Insurance and Occupational Pensions Authority, 180 insurers were affected by an insolvency from 1999 to 2016. The primary cause being cited is the technical provisions evaluation risk, while the most commonly reported early identification signal is the deteriorating capital strength and/or low solvency margin (EIOPA, 2018).

Mutual and stock insurers have different approaches to the risk management, both in terms of probability of insolvency and the use of information on risk types. One most fundamental difference between mutual and stock insurance is reciprocity of mutual risk sharing. Mutual participants collectively retain the residual risk, spreading the financial shortage among all the participants. The stock insurer manages his probability of insolvency by adding a risk loading on top of the pure premium. In case of insolvency, the stock insurer distributes the available funds and eventually leaves the market.

The participating policies, which are essentially mutual insurance contracts, do not receive enough attention in the literature. Nevertheless, the risk of insolvency generates externalities among agents, which are better spread in the mutual insurance case. Doherty and Dionne (1993) argue that the mutualization principle provides more general efficiency, while the risk transfer is efficient when the law of large numbers applies. The application of the law of large numbers relies on the assumption of the large stock insurance portfolios, yet this condition is not always met. If the portfolio size is not sufficiently large, the risk loading is used to maintain the probability of insolvency on a predefined level.

In our framework, the capital is normalized to zero both for the stock and the mutual insurer. If we assume that both insurers have the equivalent positive amount of capital at the beginning of the period, it would imply the maximum insurable loss is higher than otherwise for both insurers. Consequently, if the amount of capital is high enough so that the probability of insolvency is sufficiently low even without the risk loading ($\overline{x} > \hat{x}$ when $c = 0$), it follows from Proposition 2 that any increase in the risk loading will generate a negative impact on the stock insurance policyholders' expected utility. As a result, the mutual insurance will be always preferred by a homogeneous population. If we assume only the stock insurer collects capital through the external stockholders prior to selling policies, it will decrease the stock insurer's probability of insolvency. In this case, the condition for the mutual insurer to be preferred will be harder to meet[6].

Furthermore, we assume that the mutual insurer exerts only negative premium adjustments. However, mutuals belong to their policyholders. Consequently, the latter can equally receive ex-post discounts in case of a financial surplus, if such a possibility is specified by the mutual policy. In this case, there is a possibility of both negative and positive ex-post premium adjustments. Our results can be extended to such contracts, which are in fact fully participating policies. The assumption of positive adjustment would relax the condition on the critical values by making the mutual insurer more attractive[7].

The existence of the pooling equilibrium with both high and low risk agents choosing the mutual insurer depends on the relative size of each group in the population. Naturally,

[6]It would not make the risk loading irrelevant, except if there is a possibility to gather an infinite amount of capital. Even then, a loading called the risk premium would be charged to compensate the investors, as argued by Doherty and Dionne (1993).

[7]Moreover, as demonstrated by Fagart, Fombaron, and Jeleva (2002), the contracts with both positive and negative premium adjustment are the efficient Pareto-optimal contracts.

it also depends on the distance between the two risk levels. The closer are the two groups in terms of the individual probability of loss, the higher is the range of values of θ and c allowing for the existence of the pooling equilibrium. The insurance companies start to use telematics and smart technologies to incentivize policyholders to exert preventive activities. As a result, current data availability can serve to promote self-protection and help policyholders to lower their exposure to risk. A successful decrease in the high risk agents' loss probability will reduce the distance between different risk groups and their relative weight in the population.

5 Concluding remarks

In the economic literature, insurance contracts are mostly defined as an exchange of an uncertain wealth for a certain one by means of premium payment. This definition implies the insurer is always solvent, which does not hold in reality. At the same time, the use of behavioral information for pricing and tailoring insurance contracts raise concerns about the mutualization principle and solidarity. Indeed, the individualization is assumed to be advantageous for the insurers and the low risk agents.

In this paper, we show that the low risk agents are not necessarily better off with a stock insurance contract. We provide conditions such that low risk agents choose the mutual pool with the high risk agents, even if the stock insurer is informed about their low risk level.

The stock insurer manages his probability of insolvency by charging a risk loading on top of the pure premium. That allows him to restrict his probability of insolvency to the desirable level, which can be suggested by the regulator. For a given level of the probability of insolvency, the size of the risk loading is determined by the number of policyholders. If the low risk group is sufficiently small, the associated size of the risk loading provides incentives to join the mutual pool with the high risk agents.

The choice between the mutual and the stock insurance depends on the social preferences as well. Moreover, the decision to choose the mutual insurance can be determined by the solidarity in the sense of the engagement rather than the risk level itself. The mutual insurers may therefore be more efficient in promoting preventive activities and addressing moral hazard issues. The ways of using behavioral information as a tool to enhance self-protection rather than discriminate by prices are the suggestion for a further research.

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Appendix

Proof of Proposition 1. The stock insurance premium without the risk loading and the mutual insurance premium are identical for a homogeneous population and equal to $\pi = pL$. The final wealth in the stock insurance case is a lottery such that a policyholder will have $w - \pi$ with $1 - x$ percent chance, and $w - \pi - L + I(x)$ with x percent chance, given that $I(x) = L$ if $x \leq p$ and $I(x) = \frac{\pi}{x}$ if $x > p$. The final wealth in the mutual insurance case is $w - \pi - A(x)$ with certainty, with $A(x) = 0$ if $x \leq p$ and $A(x) = xL - \pi$ if $x > p$.

The expected wealth in the stock insurance case is

$$\mathbb{E}(\tilde{w}^S) = \int_0^1 x(w - \pi - L + I(x))f(x)dx + \int_0^1 (1 - x)(w - \pi)f(x)dx\,,$$

which can be rewritten as

$$\mathbb{E}(\tilde{w}^S) = \int_0^1 x(w-\pi)f(x)dx - \int_0^1 xLf(x)dx \\ + \int_0^1 xI(x)f(x)dx + (w-\pi) - \int_0^1 x(w-\pi)f(x)dx\,.$$

Since $I(x) = L$ if $x \leq p$ and $I(x) = \frac{\pi}{x}$ if $x > p$, the equation yields

$$\mathbb{E}(\tilde{w}^S) = w - \pi - \int_0^1 xLf(x)dx + \int_0^{\overline{x}} xLf(x)dx + \int_{\overline{x}}^1 x\frac{\pi}{x}f(x)dx\,.$$

Finally, the expected wealth under the stock insurance contract is

$$\mathbb{E}(\tilde{w}^S) = w - L\int_{\overline{x}}^1 xf(x)dx - \pi F(\overline{x})\,.$$

The expected wealth resulting from the purchase of the mutual insurance contract is denoted

$$\mathbb{E}(\tilde{w}^M) = \int_0^1 (w - \pi - A(x)f(x)dx\,,$$

which can be rewritten as

$$\mathbb{E}(\tilde{w}^M) = w - \pi - \int_0^1 A(x)f(x)dx\,.$$

Since $A(x) = 0$ if $x \leq p$ and $A(x) = xL - \pi$ if $x > p$, the previous equation yields

$$\mathbb{E}(\tilde{w}^M) = w - \pi - \int_{\overline{x}}^1 (xL - \pi)f(x)dx\,.$$

Finally, the expected wealth under the mutual insurance contract is

$$\mathbb{E}(\tilde{w}^M) = w - L\int_{\overline{x}}^1 xf(x)dx - \pi F(\overline{x}) = \mathbb{E}(\tilde{w}^S)\,.$$

Both lotteries have the same mean. Thus, the lottery on final wealth in the stock insurance case is a mean-preserving spread of the lottery in the mutual insurance case. The latter second-degree stochastically dominates the former. Following Rothschild and Stiglitz (1970), we conclude that the latter will be preferred by all risk averse agents:

$\mathbb{E}U(\tilde{w}^M) \geq \mathbb{E}U(\tilde{w}^S)$ for all concave functions U. $\square$

Proof of Proposition 2. From Proposition 1, we know that $\mathbb{E}U(\tilde{w}^M) > \mathbb{E}U(\tilde{w}^S)$, other things being equal. Thus, for $k = h, l$, we have:

$$U(w - p_k L) \Pr(\tilde{X}_k < p_k) + \int_{p_k}^{1} U(w - xL) f(x) dx >$$
$$U(w - p_k L) - \int_{p_k}^{1} x \Big[U(w - p_k L) - U(w - p_k L - L + \frac{p_k L}{x}) \Big] f(x) dx \,.$$

If the entire heterogeneous population of high and low risk agents is in the stock insurance portfolio, we have $\overline{x} = \theta p_l + (1 - \theta) p_h$, and $I(x) = \frac{(\theta p_l + (1-\theta) p_h) L}{x}$, $\forall x > \overline{x}$. Then, a homogeneous group of k-type agents prefers the mutual insurer if:

$$U(w - p_k L) \Pr(\tilde{X}_k < p_k) + \int_{p_k}^{1} U(w - xL) f(x) dx > U(w - p_k L) -$$
$$\int_{\theta p_l + (1-\theta) p_h}^{1} x \Big[U(w - p_k L) - U(w - p_k L - L + \frac{(\theta p_l + (1 - \theta) p_h) L}{x}) \Big] f(x) dx \,.$$

When $\theta \to 0\,, \theta p_l + (1 - \theta) p_h \to p_h$, and $\mathbb{E}U(\tilde{w}^M | (M, S)) > \mathbb{E}U(\tilde{w}_h^S | (S, S))$: the high risk agents prefer the mutual insurer insuring only them rather then the stock insurer insuring the low risk agents as well. In the same way, when $\theta \to 1\,, \theta p_l + (1 - \theta) p_h \to p_l$, and $\mathbb{E}U(\tilde{w}^M | (S, M)) > \mathbb{E}U(\tilde{w}_l^S | (S, S))$.

By continuity, $\exists \theta < \hat{\theta}$ such that $\mathbb{E}U(\tilde{w}^M | (M, S)) > \mathbb{E}U(\tilde{w}_h^S | (S, S))$, and $\exists \theta > \hat{\hat{\theta}}$ such that $\mathbb{E}U(\tilde{w}^M | (S, M)) > \mathbb{E}U(\tilde{w}_l^S | (S, S))$. $\square$

Proof of Proposition 3. From an application of Charpentier and Le Maux (2014). The expected utility under the stock insurance contract is:

$$\mathbb{E}U(\tilde{w}^S) = \int_0^1 x U\left(w - \pi^S - L + I(x)\right) f(x) dx + \int_0^1 (1 - x) U\left(w - \pi^S\right) f(x) dx \,,$$

which can be rewritten as

$$\mathbb{E}U(\tilde{w}^S) = U\left(w - \pi^S\right) - \int_0^1 x \Big[U\left(w - \pi^S\right) - U\left(w - \pi^S - L + I(x)\right) \Big] f(x) dx \,,$$

with $I(x) = L$ if $x \leq \overline{x}$ and $I(x) = \frac{\pi^S}{x}$ if $x > \overline{x}$. Then, we can rewrite the expected utility expression as

$$\mathbb{E}U(\tilde{w}^S) = U\left(w - \pi^S\right) - \int_{\overline{x}}^1 x\Big[U\left(w - \pi^S\right) - U\left(w - \pi^S - L + \frac{\pi^S}{x}\right)\Big]f(x)dx\,,$$

where $\pi^S = p_k L + c\,$.

$$\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} = -U'(w - \pi^S) - \frac{d}{dc}\int_{\overline{x}}^1 x\Big[U(w - \pi^S) - U(w - \pi^s - L + \frac{\pi^S}{x})\Big]f(x)dx\,.$$

From Leibniz rule, we have:

$$\begin{aligned}\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} = -U'(w - \pi^S) - \int_{\overline{x}}^1 x\Big[&-U'(w - \pi^S) \\ &- \left(\frac{1}{x} - 1\right)U'\left(w - \pi^s - L + \frac{\pi^S}{x}\right)\Big]f(x)dx \\ &+ \frac{1}{L}\overline{x}\left[U(w - \pi^S) - U\left(w - \pi^S - L + \frac{\pi^S}{\overline{x}}\right)\right]f(\overline{x})\,,\end{aligned}$$

where $\frac{1}{L} = \frac{d\overline{x}}{dc}$. Since $\frac{\pi^S}{\overline{x}} = L$, we have:

$$\begin{aligned}\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} &= -U'(w - \pi^S) \\ &\qquad - \int_{\overline{x}}^1 x\Big[-U'(w - \pi^S) + \left(1 - \frac{1}{x}\right)U'\left(w - \pi^s - L + \frac{\pi^S}{x}\right)\Big]f(x)dx \\ &= -U'(w - \pi^S) + \int_{\overline{x}}^1 \Big[xU'(w - \pi^S) + (1 - x)U'\left(w - \pi^s - L + \frac{\pi^S}{x}\right)\Big]f(x)dx\end{aligned}$$

Using Taylor development, we have $f(x) \approx f(a) + f'(a)(x - a)$. For $a = w - \pi^S$, we have:

$$U'(w - \pi^S - L + \frac{\pi^S}{x}) \approx U'(w - \pi^S) + (-L + \frac{\pi^S}{x})U''(w - \pi^S)\,,$$

or else

$$(1-x)U'(w-\pi^S-L+\frac{\pi^S}{x})+xU'(w-\pi^S) \approx U'(w-\pi^S)+(1-x)(-L+\frac{\pi^S}{x})U''(w-\pi^S)\,.$$

Consequently,

$$\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} = -U'(w-\pi^S) + \int_{\overline{x}}^1 \left[xU'(w-\pi^S) + (1-x)U'\left(w - \pi^s - L + \frac{\pi^S}{x}\right)\right] f(x)dx$$
$$= -U'(w-\pi^S) + \int_{\overline{x}}^1 \Big[U'(w-\pi^S) + (1-x)(-L+\frac{\pi^S}{x})U''(w-\pi^S)\Big] f(x)dx\,,$$

which we can rewrite as:

$$\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} = -U'(w-\pi^S) + U'(w-\pi^S)\int_{\overline{x}}^1 f(x)dx +$$
$$\int_{\overline{x}}^1 (1-x)(-L+\frac{\pi^S}{x})U''(w-\pi^S)f(x)dx\,.$$

Given that $\int_{\overline{x}}^1 f(x)dx = 1 - \Pr(\tilde{X} \le \overline{x})$, we have:

$$\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} = -U'(w-\pi^S)\Pr(\tilde{X} \le \overline{x}) + U''(w-\pi^S)\int_{\overline{x}}^1 (1-x)(-L+\frac{\pi^S}{x})f(x)dx\,.$$

Denoting $-L = I(\overline{x})$ and $\frac{\pi^S}{x} = I(x)$ for $x > \overline{x}$, we have:

$$\int_{\overline{x}}^1 (1-x)(-L+\frac{\pi^S}{x})f(x)dx = -\int_{\overline{x}}^1 (1-x)(I(\overline{x}) - I(x))f(x)dx = -H(\overline{x})\,,$$

where $H(\overline{x})$ is a positive function such that $\frac{\partial H(\overline{x})}{\partial \overline{x}} < 0$. Then we have:

$$\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} = -U'(w-\pi^S)\Pr(\tilde{X} \le \overline{x}) - U''(w-\pi^S)H(\overline{x})\,.$$

The impact of the risk loading on the expected utility of wealth is positive if $\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} < 0$, which is the case when $-U'(w-\pi^S)\Pr(\tilde{X} \le \overline{x}) - U''(w-\pi^S)H(\overline{x}) < 0$, leading to the following condition:

$$\frac{\Pr(\tilde{X} \le \overline{x})}{H(\overline{x})} > -\frac{U''(w-\pi^S)}{U'(w-\pi^S)}\,,$$

where the right term is the Arrow-Pratt measure of risk aversion. If agents are risk-averse (U concave), then this term is positive. The left term is always positive, but it is not necessarily greater than the Arrow-Pratt measure of risk aversion. Given that $\frac{\Pr(\tilde{X} \le \overline{x})}{H(\overline{x})}$ is increasing with $\overline{x}$, $\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} < 0$ when $\overline{x}$ is large, and $\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} > 0$ when $\overline{x}$ is small. Consequently, there exists an inflection point $\hat{x}$ such that when $\overline{x} < \hat{x}$, $\frac{\partial \mathbb{E}U(\tilde{w}^S)}{\partial c} > 0$. $\square$