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Does Predictive Ability of an Asset Price Rest in 'Memory'? Insights from a New Approach

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Abstract: *Despite an inherent share of unpredictability, asset prices such as in stock and Bitcoin markets are naturally driven by significant magnitudes of memory; depending on the strength of path dependence, prices in such markets can be (at least partially) predicted. Being able to predict asset prices is always a boon for investors, more so, if the forecasts are largely unconditional and can only be explained by the series' own historical trajectories. Although memory dynamics have been exploited in forecasting stock prices, Bitcoin market pose additional challenge, because the lack of proper financial theoretic model limits the development of adequate theory-driven empirical construct. In this paper, we propose a class of autoregressive fractionally integrated moving average (ARFIMA) model with asymmetric exponential generalized autoregressive score (AEGAS) errors to accommodate a complex interplay of 'memory' to drive predictive performance (an out-of-sample forecasting). Our conditional variance includes leverage effects, jumps and fat tail-skewness distribution, each of which affects magnitude of memory both the stock and Bitcoin price system would possess enabling us to build a true forecast function. We estimate several models using the Skewed Student-t maximum likelihood and find that the informational shocks in asset prices, in general, have permanent effects on returns. The ARFIMA-AEGAS is appropriate for capturing volatility clustering for both negative (long Value-at-Risk) and positive returns (short Value-at-Risk). We show that this model has better predictive performance over competing models for both long and/or some short time horizons. The predictions from this model beats comfortably the random walk model. Accordingly, we find that the weak efficiency assumption of financial markets stands violated for all price returns studied over longer time horizon.*

Keywords: Asset price; Forecasting; Memory; ARFIMA-AEGAS; Leverage effects and jumps; Market Efficiency.

JEL Classification : C14, C58, C22, G17.

1. Introduction

1.1 The broader context

Why asset prices (such as stock and the recent phenomenon, cryptocurrencies) fluctuate systematically, is a long-drawn story, having its primary root in behavioral economics and finance, where agents (investors) are driven by market sentiments and display a boundedly rational behavior leading to choices which are not necessarily optimal. Although cryptocurrencies are yet to own a definite financial theoretic model, its increasing importance among institutional investors[1] nudges its closer to any acceptable financial theory that explains stock price fluctuations. Decades long research and policy practices have led to a sole aim: understand asset price behavior with a robust theory to model and provide as accurate a prediction as possible. This paper does not intend to dig into the depth of this black box, rather undertakes a celebrated route held by time series econometricians: model and study the historical trajectory of the series itself in case there are no other available information to provide a conditional forecast. We reckon there are important information available (see for instance, Gillaizeau et al. (2019), among others) to condition the movement of stock and cryptocurrencies (such as Bitcoin), however, our sole aim in this paper to provide a better predictive power based mainly on the modelling of historical path of the series themselves. Our contribution, of course, is the recognition of an explicit role of 'memory' that drives a part of the volatility in asset prices.

Memory has an interesting virtue: it is often snared within the complexity of transient cycles – ones which dominate the real behavior of the series and sets a natural limit to true predictions. Yi et al. (2019) and Phan et al. (2015) in this journal discuss (out-of-sample) forecasting performance, where Yi et al. (2019) adopt cycle-decomposed predictors to forecast stock prices. Our purpose in this paper is to introduce a new approach to forecast stock and Bitcoin prices by offering a complex interplay of `memory' and volatility. The forecast performance of our approach is rigorously compared with competing models. Finally, implications of our model performance are drawn regarding investment decisions.

Indeed, stock and Bitcoin price movements in any country setting essentially reflects global movements and co-movements of myriads of factors viz., social, economic, political, and environmental (such as the effects of weather and sports events on stock prices). The fact

[1]https://www.coindesk.com/rising-institutional-investment-setting-pace-for-future-crypto-growth

is that investors are predominantly psychology-driven decision-making agents, where the decisions are invariably derived from the realm of incomplete information and bounded rationality. This is one of the many imposing reasons, why despite several seminal contributions to the determinants of stock and Bitcoin prices, the tendency to include newer factors are increasing every day.[2] A time series econometrician faces then an upheaval task: to study the series over a stretch of time, identify a reality-approximating pattern by using state-of-the-art method and produce a nice predictive performance of the model. Just as Ronald Coase pointed out (in the above quote), 'if you torture the data long enough, it will confess'. Indeed, over the past three and a half decades since Engle (1982) and Bollerslev (1986), financial economists have moved along the non-stationary econometric trajectories and have offered numerous powerful competing forecasting models to uncover the real nature of stock and Bitcoin price movements. Unfortunately, stock and Bitcoin price is one such financial metric which is not driven by a single event's momentum only (such as only political uncertainty or economic prosperity/recession, etc.). Rather, its ever-changing complex core that attracts anything 'psychological bound' of investors, means that there will no single econometric approach that can unravel the real dynamic nature of stock and Bitcoin price movements. However, amidst all these dynamisms, the best way to understand its movements is to model its variance as conditional -allowing in part, to be determined by past variances and in part, by other factors (the class of Fama-French models, for instance). In other words, in general stock and Bitcoin prices can reveal some strength in 'memory' – an ability of the system to remember past shocks. Finally, all the various known and unknown factors determining investors' sentiment also form a complex non-linear relationship. A preferable approach to produce realistic predictions would then be to combine both 'memory' and 'non-linearity' within a single modelling framework. This paper builds such a framework and aims to provide new insights into stock and Bitcoin price movements.

At its core, our approach lays emphasis on modelling 'jumps' in stock and Bitcoin prices along with a possible path dependence. Assuming a jump process for stock and Bitcoin price, we allow random movements of prices at all scales, no matter how small. Such a model often combines the usual geometric Brownian motion for the diffusion and a space-time Poisson process for the jumps such that jump amplitudes are uniformly distributed. Arguably, stock and Bitcoin prices exhibit extreme sensitivity to news, in addition to of course, the structural

[2]Some recent research investigates whether win or loss in a big match (such as football or rugby) leads to a rise/fall in stock prices the next day (Urquhart & Sakkas (2018)).

changes in financial and economic dynamics. Such a sensitivity can be regarded as response to ‘jumps’ the source of which can be both endogenous and exogenous. Irrespective of the sources, a jump in stock and Bitcoin prices often reflect the path dependence nature of the series: to what extent a strong/weak memory of the system predicts its future movements. Hence, in the current paper, we introduce a long-memory based conditional volatility model with asymmetry (and non-linearity). Our approach (to be discussed in Section 2 in details) exploits learning mechanism of the stock and Bitcoin price system with ‘memory’ and embeds asymmetric nature of shocks on the conditional volatility of stock and Bitcoin prices.

1.2 Identifying the missing link

The classical variants of Generalized Autoregressive Conditional Volatility (GARCH) model are extensively employed in the empirical architecture of price volatility (viz., symmetric GARCH: Engle (1982) and Bollerslev (1986), asymmetric GARCH, such as exponential GARCH (EGARCH): Nelson (1990) and Threshold GARCH (TGARCH): Glosten, Jagannathan & Runkle (1993) and Zakoian (1994); the asymmetric power GARCH (APGARCH): Ding, Granger & Engle (1993); STGARCH model with regime switching: Hagerud (1997a), Gonzalez-Rivera (1998)). For details on the evolution of different GARCH-type models, see Bollerslev (2009) and Zhang & Wei (2010). These GARCH variants are based on properties of symmetry, asymmetry, nonlinearity, stationarity, persistence and structural breaks, but recent innovations have shown that jumps another fundamental property in volatility (see for example, Harvey (2013), Yaya, Bada & Atoi (2016), Charles & Darné (2017) and Babatunde, Yaya, & Akinlana (2019)). The GARCH models are not robust enough to capture these large changes in financial time series, and therefore, they underestimate the magnitude effect of the returns. Andersen, Bollerslev & Dobrev (2007) originally propose non-parametric approaches based on Brownian Semi-Martingale for detecting jumps, but these methods cannot predict volatility. Due to the presence of occasional jumps, Harvey & Chakravarty (2008) and Harvey (2013) propose Generalized Autoregressive Score (GAS) models - a class of observation driven time series models, where the time-varying parameters are functions of lagged dependent values and past observations. The parameters are stochastic and predictable given the past. These models capture these occasional jumps in financial time series with symmetric and asymmetric variants, using the score of the conditional density function to drive the dynamics of the time-varying parameter (see Creal, Koopman & Lucas (2013) and Creal, Schwaab, Koopman &

Lucas (2014)). The distribution of innovations in GAS models are non-normal and the conditional variance is taken from the conditional score of the distribution with respect to the second moment.

Asset prices often exhibit complex dynamic properties and needs to be adequately flexible to describe its important characteristics. The GAS models have proved to be more robust in modelling and predicting fat tail and skewed data (Yaya, Bada & Atoi (2016), Opschoor, Janus, Lucas & Van Dick (2018) and Makatjane, Xaba & Moroke (2017)). To accommodate conditional asymmetry, leverage effect and heavy tails, Laurent, Lecourt & Palm (2016) propose AEGAS model (also called Beta-Skew-t-AEGARCH), as an extension to GAS (Creal et al (2013)) by introducing time-varying parameters in the class of non-linear models with its exponential specification. This new class of volatility model is robust to outliers and occasional jumps by using the Skewed Student-t distribution to account for the occurrence of large changes in volatility. In the AEGAS model, the mechanism to update the parameters over time is provided by the scaled score of the likelihood function (Tafakori, Pourkhanali & Fard (2018)).

For applications of GAS models to economic and financial time series see, for example, Creal, Koopman & Lucas (2013) who present two examples to illustrate their modeling framework; viz., square root information matrix scaling with Moody's credit rating data. Huang, Wang & Zhang (2014) compare the Realized ARCH and Realized GAS model under Gaussian and *t*-distribution assumptions for the financial return and daily realized variance. Muela (2015) compare the performance of several Beta-skewed-t-EGARCH specifications in terms of Value at Risk on eight closing daily returns. On the other hand, Koopman, Lucas, & Scharth (2016) study the forecasting performance of nonlinear non-Gaussian state-space models, generalized autoregressive score models and autoregressive conditional moment models for predicting the volatility of twenty Dow Jones index stocks and five major stock indices over a period of several years. Olubusoye & Yaya (2016) investigate persistence and volatility pattern in the prices of crude oil and other distilled petroleum products for the US and the UK petroleum pricing markets. Whereas, Yaya, Bada & Atoi (2016) estimate volatility in the Nigerian Stock Market using the Beta-Skew-t-AEGARCH model and compare its forecasting performance over some other volatility models. Salisu (2016) employs the Beta-Skew-t-EGARCH framework proposed to model oil price volatility. Makatjane, Xaba, Moroke (2017) empirically investigate the behaviour of the time-varying parameter by estimating the GAS model to the South Africa Sanlam stock price returns.

Müller & Bayer (2017) propose a likelihood ratio test to select the Beta-Skew-t-EGARCH model with one or two volatility components and give an empirical illustration devoted to the DAX log-returns. Charles & Darné (2017) analyze volatility models in the presence of jumps in two crude-oil markets and evaluate the forecasting performance of the volatility models using the model confidence set approach, Finally, Tafakori, Pourkhanali & Fard (2018) evaluate the accuracy of several 100 one-day-ahead value at risk (VaR) forecasts for predicting Australian electricity returns using asymmetric exponential generalized autoregressive score (AEGAS) models.

The class of score-driven models have recently become popular for analyzing financial time series, but these last works ignore the existence of dynamic behavior, especially long memory, in the conditional mean. Several studies find that the empirical return series exhibit long-range persistence (Granger & Joyeux (1980), Hosking (1981) and Chikhi, Péguin & Terraza (2013)). Combining long memory (ARFIMA) models with asymmetric exponential GAS (AEGAS) errors would provide a flexible class of model to capture the long memory structure in the conditional mean and the occasional jumps in the score-driven volatility, which includes leverage effect and fat tail-skewness distribution. Harvey (2013), among others, specify the GAS models with the heavily tailed and skewed conditional probability distribution. These models perform better than the classical GARCH models with larger values of log-likelihoods. Blasques, Koopman & Lucas (2014a) and Lambert & Laurent (2000, 2001) suggest using the maximum likelihood based on the skewed Student-*t* density proposed by Fernandez & Steel (1998) to estimate this GAS family of models.

As noted earlier, an imposing characteristic of a conditional volatility model is its memory characteristics. When the system reveals certain patterns (such as herding), this means that some time series observations within the price data depict certain degree of associations (in our case, it can be certain magnitude of dependence between past and present). Led by this, the main objective of this paper is to propose a mixture of long memory structure and the occasional jumps, leverage effect and fat tail-skewness distribution in the daily price returns. Our approach – the ARFIMA-AEGAS model – is employed to three stock markets, such as Argentina, Saudi Arabia and France and five Bitcoin markets. Our strategy thus, is to combine and estimate the ARFIMA model with asymmetric exponential GAS (AEGAS or Beta-Skew-t-AEGARCH) errors using Skewed Student-t maximum likelihood.

The remainder of this article is organized as follows: Section 2 focuses on outlining the conceptual foundation of our proposed ARFIMA-AEGAS. Section 3 presents the daily stock and Bitcoin price data and discusses their statistical properties. Our estimation results are shown in section 4. In section 5, we evaluate the forecasting performance of best fitting GAS Models in stock and Bitcoin markets, including the long memory in the conditional mean equation. We thus try to compare the predictive quality of ARFIMA-GAS, ARFIMA-EGAS, ARFIMA-AEGAS and ARFIMA-EGARCH models with that of a random walk. The last section concludes the paper.

2. Model

We specify a fractionally autoregressive moving average (ARFIMA) model (Granger & Joyeux (1980) and Hosking (1981)) with Generalized Autoregressive Score [GAS(1,1)] errors, also called Beta-Skew-t-GARCH (Harvey & Chakravarty (2008), Creal, Koopman & Lucas (2013) and Harvey (2013)). This is defined as follows

$$\phi(B)(1-B)^{d_2}\{(1-B)^{d_1}Y_t - g(x_t)\} = \theta(B)\varepsilon_t \tag{1}$$

where
$$\varepsilon_t = z_t\sigma_t\ ,\quad \sigma_t > 0 \tag{2}$$

and
$$\sigma_t^2 = w + \alpha_1 u_{t-1}\sigma_{t-1}^2 + \varphi_1\sigma_{t-1}^2 \tag{3}$$

The interest of ARFIMA models for financial series lies mainly in their ability to capture the long memory behavior in the conditional mean

$$(1-B)^{d_2} = \sum_{k=0}^{\infty}(-1)^k \frac{\Gamma(d_2+1)}{\Gamma(k+1)\Gamma(d_2-k+1)} B^k = 1-\sum_{k=1}^{\infty} c_k(d_2)B^k$$

$$c_1(d_2) = d_2\ ,\ c_2(d_2) = \frac{1}{2}d_2(1-d_2)$$

where $\Gamma(.)$ is the gamma function. The roots of polynomials $\phi(B) = 1-\phi_1 B - \phi_2 B^2 - .... - \phi_p B^p$ and $\theta(B) = 1-\theta_1 B - \theta_2 B^2 - .... - \theta_q B^q$ in the lag operator with degrees p and q respectively, are outside the unit circle. B is the lag operator, d_1 is an integer: $d_1 \in \{0,1\}$. The process is stationary and invertible, $-\frac{1}{2} < d_2 < \frac{1}{2}$. The long memory is included in the mean equation (1) through the parameter d_2. $\varphi_1 = \alpha_1 + \beta_1$ and α_1 , β_1 , w are the classical GARCH parameters, with $\alpha_1 \geq 0$, $\beta_1 \geq 0$, $w > 0$. For the Skewed Student-t distribution in the conditional variance (see Hansen (1994), Lambert & Laurent (2000, 2001) and Theodossiou (2002)) :

$$u_t = \frac{(\nu+1)z_t z_t^*}{(\nu-2)g_t \xi^{I_t}} - 1 \quad \text{if} \quad z_t \sim SKST\,(0,1,\xi,\nu) \tag{4}$$

where
$$z_t^* = s z_t + m \tag{5}$$

and
$$g_t = 1 + \frac{z_t^{*2}}{(\nu-2)\xi^{2I_t}} \tag{6}$$

$$m = \frac{\Gamma\left(\frac{\nu-1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \tag{7}$$

$$s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2} \tag{8}$$

where ξ is the asymmetry parameter and ν is the degree of freedom of the distribution.

For a GAS(1,1) model, the equation for time-varying parameters $\psi_t = \sigma_t^2$ is the autoregressive function $\psi_t = \omega + B_1\psi_{t-1} + A_1\kappa_{t-1}$. Harvey & Chakravarty (2008) and Creal, Koopman& Lucas (2013) propose to update the time-varying parameters with $\kappa_t = S_t\nabla_t$ where ∇_t is the score with respect to the parameter ψ_t with $\nabla_t = \partial \log f\,(Y_t \,|\, \psi_t, \psi_{t-1}, Y_{t-1}, \mathrm{X}_t; \theta)/\partial\psi_t$ and S_t is a time dependent scaling matrix. The Normal-GARCH model corresponds to a Normal-GAS(1,1) (i.e. $z_t \sim N(0,1)$ with $S_t = 2,\ A_1 = \alpha_1,\ B_1 = \alpha_1 + \beta_1$, $\psi_t = \sigma_t^2$ and $\nabla_t = 0.5\left(z_t^2 - 1\right)\sigma_t^2$). Note that $u_t = z_{t-1}^2 - 1$ is proportional to the score of the conditional distribution of ε_t with respect to σ_{t-1}^2 . For the choice of time dependent scaling, Creal, Koopman& Lucas (2012) recommend using $S_t = 1$ or $S_t = \left(E_{t-1}\nabla_t\nabla_t'\right)^{-1}$ while Harvey & Chakravarty (2008) set $S_t = 2$.

We also consider the EGAS (Beta-Skew-t-EGARCH) model, which is given as (see Harvey (2013))

$$\log\sigma_t^2 = w + \alpha_1 u_{t-1} + \varphi_1 \log\sigma_{t-1}^2 \tag{9}$$

Introducing the leverage effect, we have the AEGAS (Beta-Skew-t-AEGARCH) model (see Laurent, Lecourt & Palm (2016))

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \gamma_1 I_{t-1} + \varphi_1 \log \sigma_{t-1}^2 \tag{10}$$

where I_t is an indicator measure asymmetry defined as

$$I_t = \operatorname{sgn}\left(z_t^*\right) = I\left(z_t^* \geq 0\right) - I\left(z_t^* < 0\right) \tag{11}$$

For the Skewed Student-t distribution

$$I_{t-1} = \operatorname{sgn}\left(-z_t^*\right)\left(u_t + 1\right)$$

(12)

with
$$E\left(I_t\right) = \frac{1-\xi^2}{1+\xi^2} \tag{13}$$

Some authors suggest using the Skewed Student-t innovation distribution. Lambert & Laurent (2000, 2001) apply and extend the skewed-Student density proposed by Fernandez & Steel (1998) to the GARCH framework. The procedure for Maximum Likelihood Estimation (MLE) of GAS family models was presented in Blasques, Koopman & Lucas (2014a). The strong consistency and asymptotic normality of maximum likelihood are also studied. Consequently, we propose the Skewed Student-t maximum likelihood to estimate an ARFIMA model jointly with AEGAS (Beta-t-EGARCH) error from the Skewed Student-t distribution using the BFGS algorithm (Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970)) implemented by Laurent (2013). The Skewed Student-t log-likelihood function is written as

$$L = T\left\{\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - 0.5\log\left[\pi\left(\nu-2\right)\right] + \log\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + \log(\mathrm{s})\right\}$$

$$-0.5\sum_{t=1}^{T}\left\{\log\sigma_t^2 + \left(1+\nu\right)\log\left[1+\frac{\left(sz_t+m\right)^2}{\nu-2}\xi^{-2I_t}\right]\right\} \tag{14}$$

where

$$I_t = \begin{cases} 1 \text{ if } z_t \geq -\dfrac{m}{s} \\ -1 \text{ if } z_t < -\dfrac{m}{s} \end{cases} \tag{15}$$

and optimized with respect to the unknown dynamic parameters, contained in the vector $\psi' = \left(d_2, \phi_1, ..., \phi_p, \theta_1, ..., \theta_q, \omega, \alpha_1, \gamma_1, \varphi_1\right)$ and ν in the first order model (For details on skewed student density see Lambert & Laurent (2001) and For the GAS estimation see Blasques, Koopman & Lucas (2014a, 2014b, 2014c)).

3. Data characteristics

Our database consists of daily stock market indices of three selected markets and five Bitcoin returns. The stock indices considered are TASI (Saudi Arabia), Merval (Argentina), and CAC SMALL (France). The daily percentage Bitcoin return data applied here are AUD (Australian Dollar), CAD (Canadian Dollar), EUR (Euro), GBP (British Pound Sterling) and USD (US Dollar). The data sample of Saudi Arabia is from January, 2000 to October, 2018 ($T = 5000$); the data sample of the Argentina is from May, 2002 to January, 2019 ($T = 4093$); the data of France cover a historical period from January, 1999 to July, 2018 ($T = 4999$) and the Bitcoin data sample cover a historical period from January 1, 2015 to March 13, 2019 corresponding to 1533 observations (see Figure 1). Daily stock prices of Merval are collected from Yahoo finance https://fr.finance.yahoo.com, the data series of TASI and CAC SMALL are drawn from https://www.investing.com and the Bitcoin data are gathered from Coincheck platform (see for instance, Gillaizeau et al. (2019) for details on data sources and their limitations). We use Bitcoin prices in five different markets such as Euros (EUR), Australian Dollars (AUD), US Dollars (USD), Canadian Dollars (CAD), and British Pounds (GBP). Our estimation involves Bitcoin returns series.

As a preliminary check before we estimate our model, we study if the series are non-stationary in nature or contrastingly, if the series are governed by some memory. Following convention, we perform unit root tests using Philips & Perron (1988), Kwiatkowski, Phillips, Schmidt & Shin (1992) and Elliott, Rothenberg & Stock (1996). In Table 1, we present these results: we find that the three logarithmic stock exchange index series are characterized by a unit root and the five Bitcoin return series are stationary. The logarithmic index series are finally differentiated to obtain the daily percentage returns at time t (see Figure 1)

$$r_t = 100 \times \left(\ln P_t - \ln P_{t-1}\right)$$

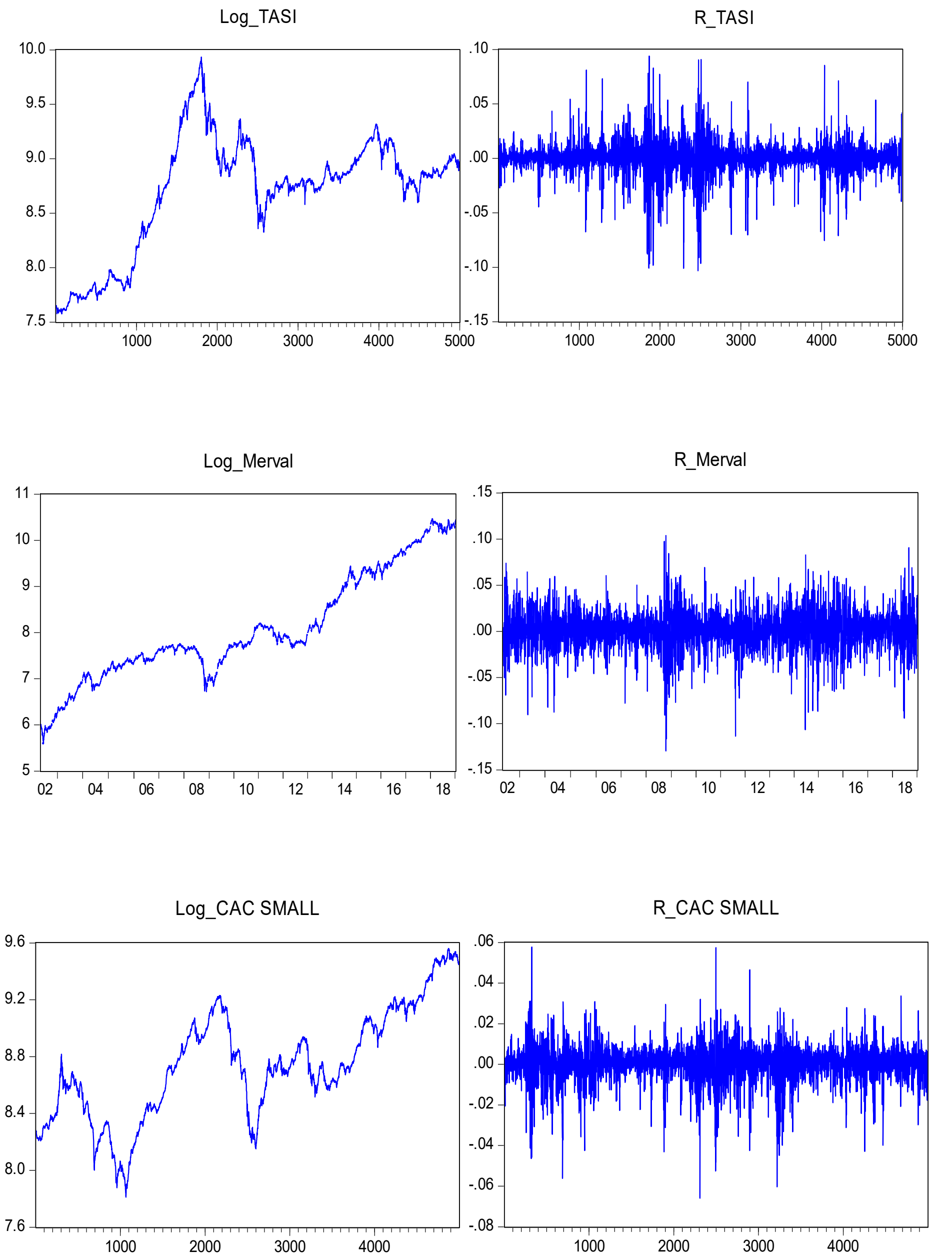
where P_t and P_{t-1} are daily stock price at two successive days t and $t-1$, respectively.

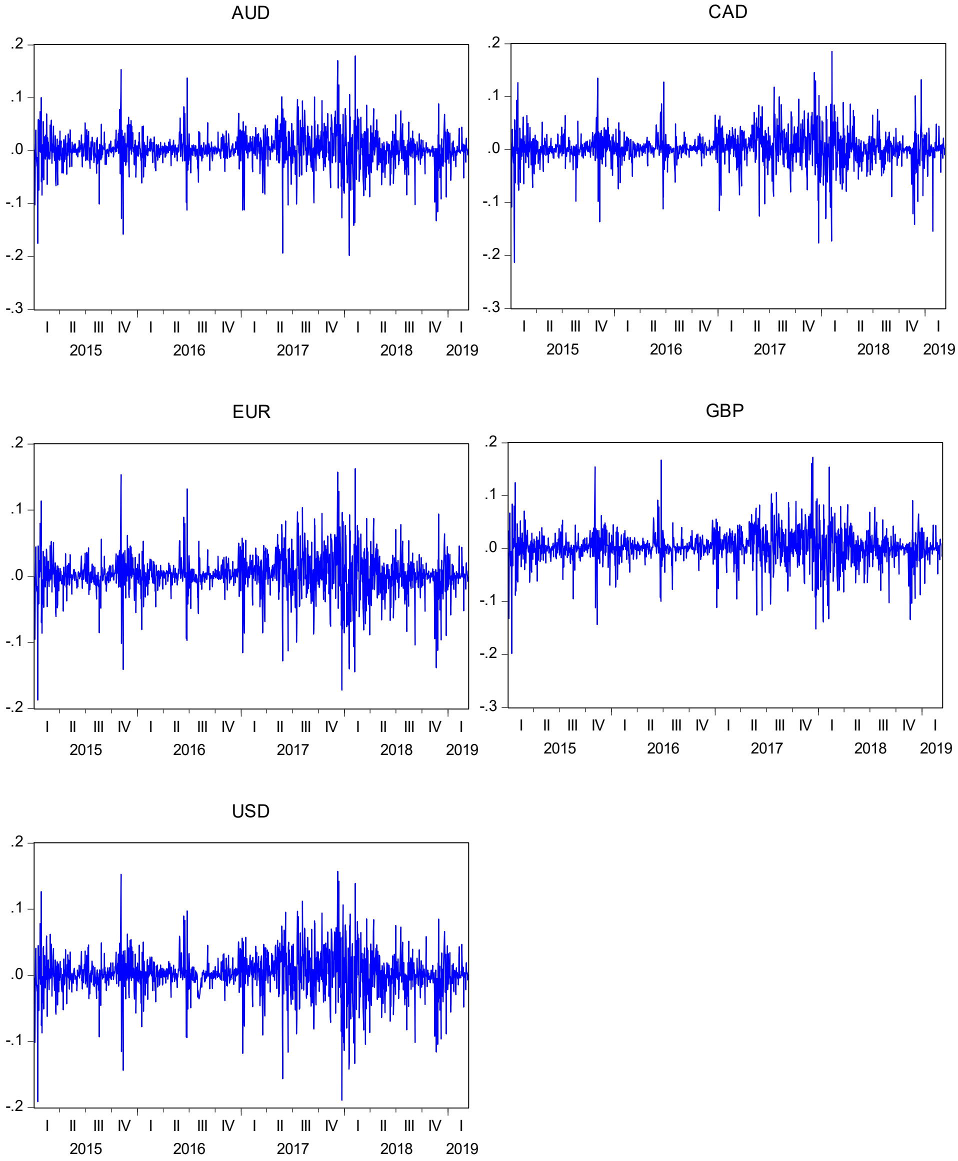
Table 1 – Identifying non-stationarity in the series

Series	Test	Logarithmic		Returns	
		Test stat.	Critical value	Test stat.	Critical value
TASI	PP	-1.959	-2.861	-65.532	-1.941
	KPSS	0.349	0.463	0.297	0.463
	ERS	0.010	3.26	70.105	3.26
Merval	PP	3.177	-1.941	-60.989	-2.862
	KPSS	1.762	0.463	0.102	0.463
	ERS	0.014	3.26	220.245	3.26
CAC SMALL	PP	1.371	-1.941	-55.891	-1.941
	KPSS	1.720	0.463	0.113	0.463
	ERS	0.011	3.26	40.361	3.26
Bitcoin AUD	PP	-	-	-31.149	-1.941
	KPSS	-	-	0.226	0.463
	ERS	-	-	50.452	3.26
Bitcoin CAD	PP	-	-	-31.965	-1.941
	KPSS	-	-	0.256	0.463
	ERS	-	-	63.821	3.26
Bitcoin EUR	PP	-	-	-30.856	-1.941
	KPSS	-	-	0.238	0.463
	ERS	-	-	81.521	3.26
Bitcoin GBP	PP	-	-	-31.561	-1.941
	KPSS	-	-	0.263	0.463
	ERS	-	-	65.847	3.26
Bitcoin USD	PP	-	-	-31.261	-1.941
	KPSS	-	-	0.228	0.463
	ERS	-	-	44.237	3.26

Notes: *The asymptotic critical value at 5% are computed using Mackinnon's (1990) method. The table reports the results of Philips-Perron unit root test. We accept the unit root hypothesis* H_0 *for daily logarithmic series and reject it for daily returns. For Philips-Perron, Elliott-Rotenberg-Stock (ERS) and KPSS tests, the spectral estimation is based on the Bartlett kernel using the Andrews bandwidth. For KPSS test,* H_0 *is the null hypothesis of stationarity.*

Figure 1 – Evolutions of stock market indices and returns





Interesting observations emerge: from Figure 1, we note some sharp jumps and volatility clustering in the returns. As shown in Table 2, the Bitcoin returns AUD, CAD, USD, GBP and EUR show the highest risk, as measured by the standard deviation (3.38%, 3.29%, 3.28%, 3.26% and 3.22% respectively) followed by the Argentina stock market - Merval- (2.023%). The French stock market show the lowest risk (0.867%). All series exhibit negative skewness. The observed asymmetry may indicate the presence of nonlinearities in the evolution process of all returns. In addition, all series also show excess kurtosis: the Jarque-Bera test (Jarque & Bera (1987) strongly rejects the null hypothesis of normality. On the other hand, there is an ARCH effect in the data since the ARCH-LM statistic is greater than the critical value of chi-square distribution with 1 degree of freedom at 1%. for all series.

Table 2 – Summary statistics for daily stock market and Bitcoin returns

Series	Std. Dev (%)	Skewness	Kurtosis	JB stat.	ARCH(1)
TASI	1.416	-0.880	13.447	23382.58***	327.632***
Merval	2.023	-0.376	6.302	1956.21***	147.009***
CAC SMALL	0.867	-0.948	9.170	8677.555***	415.59***
AUD	3.380	-1.002	9.756	6336.7***	101.34***
CAD	3.290	-0.899	9.034	5420.4***	125.25***
EUR	3.220	-1.276	13.707	12418.2***	61.302***
GBP	3.263	-0.377	5.038	1658.1***	182.70***
USD	3.287	-1.344	13.689	12432.1***	41.580***

Notes: **** indicates a rejection of null hypothesis of normality and homoscedasticity at the 1% level.*

In Table 3, we present the BDS (Brock et al. (1996))statistics to gauge whether stock returns are non-linear in nature. As evident, the BDS statistics are strictly greater than the critical value at 5% for all the embedding dimensions *m* and thus all returns are non-linearly dependent. Moreover, the variance ratio statistic (Lo & MacKinlay (1988)) is significant for all returns as well: the critical probabilities are less than 0.05 for all period(see Table 4). Consequently, we reject the random walk hypothesis, indicating that stock market price and Bitcoin returns can be predicted in the short term.

Table 3 – BDS test results on the stock market and Bitcoin returns

m	TASI	Merval	CAC SMALL	AUD	CAD	EUR	GBP	USD
2	24.124 (0.000)	11.888 (0.000)	23.445 (0.000)	16.236 (0.000)	15.219 (0.000)	16.553 (0.000)	16.963 (0.000)	15.280 (0.000)
3	30.607 (0.000)	16.194 (0.000)	29.869 (0.000)	17.876 (0.000)	17.199 (0.000)	18.380 (0.000)	18.304 (0.000)	17.158 (0.000)
4	34.967 (0.000)	18.719 (0.000)	33.587 (0.000)	19.416 (0.000)	18.601 (0.000)	20.126 (0.000)	19.993 (0.000)	18.905 (0.000)
5	39.051	20.875	37.035	21.016	20.054	21.833	21.585	20.664

	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
6	43.416	22.740	40.420	22.918	21.838	23.981	23.509	22.775
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
7	48.327	24.467	43.993	24.959	23.935	26.400	25.746	25.179
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
8	53.899	26.307	48.017	27.456	26.402	29.359	28.426	28.008
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
9	60.684	28.155	52.694	30.339	29.401	32.837	31.668	31.328
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
10	68.463	30.344	57.990	33.928	33.104	37.074	35.659	35.382
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: *The numbers in the parenthesis are the critical probabilities. The BDS statistics are calculated by the fraction of pairs method with* ε *equal to 0.7. m represents the embedding dimension.*

Table 4 – Variance Ratio test Statistics for stock market and Bitcoin returns

Series	Period	VR	z-stat.	Prob.
TASI	2	0.557	-12.686	0.000
	4	0.264	-11.874	0.000
	8	0.133	-9.443	0.000
Merval	2	0.521	-16.234	0.000
	4	0.259	-14.643	0.000
	8	0.130	-11.927	0.000
CAC SMALL	2	0.570	-14.700	0.000
	4	0.303	-13.291	0.000
	8	0.152	-10.923	0.000
AUD	2	0.663	-7.445	0.000
	4	0.307	-8.356	0.000
	8	0.166	-6.681	0.000
CAD	2	0.623	-7.716	0.000
	4	0.297	-7.998	0.000
	8	0.158	-6.429	0.000
EUR	2	0.653	-7.610	0.000
	4	0.317	-8.263	0.000

	8	0.166	-6.792	0.000
	2	0.638	-7.736	0.000
GBP	4	0.312	-8.202	0.000
	8	0.161	-6.836	0.000
	2	0.640	-8.261	0.000
USD	4	0.316	-8.455	0.000
	8	0.163	-6.829	0.000

Notes: *p-Value of variance ratio statistic represents a probability approximation using studentized maximum modulus with parameter value 3 and infinite degrees of freedom.*

These previous tests highlighted the presence of significant non-zero autocorrelations in the short term and lead us to reject the *i.i.d* hypothesis. However, it is impossible to exploit these autocorrelations to establish speculative rules leading to abnormal profits. Given this situation, we test the presence of autocorrelations by considering longer horizons. By estimating the fractional integration coefficient (see Table 5), we note that this is a sign of long memory since the values of Student statistic (with a power of 0.8) are strictly greater than the critical value of normal distribution at 5%. In other words, the memory parameter estimated by a Gaussian semiparametric method (Robinson & Henry (1999)) is positive and significant. The estimation results are confirmed by the GPH (Geweke & Porter-Hudak (1983)) method. The presence of a long memory indicates that we can anticipate the returns to a sufficiently long time-horizon and the return will not revert to its fundamental value.

Table 5 – Results from the ARFIMA(0,d,0) estimation

On daily returns

Series	GPH			Robinson-Henry		
	D	t-stat.	Prob.	*d*	t-stat.	Prob.
TASI	0.0427	3.1925	0.0014	0.0291	2.9098	0.0036
Merval	0.0494	3.1294	0.0018	0.0346	3.1335	0.0017
CAC SMALL	0.1634	12.2010	0.0000	0.1495	14.947	0.0000
AUD	0.1021	4.2547	0.0000	0.0950	5.1537	0.0000
CAD	0.1067	4.4495	0.0000	0.0927	5.1523	0.0000
EUR	0.1167	4.8657	0.0000	0.1022	5.6783	0.0000
GBP	0.1104	4.6014	0.0000	0.0986	5.4831	0.0000

USD	0.0887	3.6958	0.0000	0.0974	5.4111	0.0000

Notes: *GPH: Geweke-Porter-Hudak. d is the estimated Long memory parameter with a power of 0.8.*

4. Main results

In this section, we focus on modelling the dynamics of the return and the volatility of daily return series using the ARFIMA model jointly with GAS, exponential GAS and asymmetric exponential GAS errors. The estimation procedure is based on the Skewed Student-t maximum likelihood using the BFGS algorithm. As there are some sharp jumps in the volatility, it will be interesting to take into account this asymmetry in volatility estimation (see Salisu (2012), Yaya (2013) and Yaya & Gil-Alana (2014)). For the time dependent scaling S_t, we use the choice of Harvey & Chakravarty (2008) by setting $S_t = 2$. To facilitate inference about the null hypothesis of symmetry, we estimate $\log(\xi)$.

We estimate several models with different lags, such as an ARFIMA(p, d, q) jointly with a GAS(1,1), EGAS(1,1) and AEGAS(1,1) model. For each model, we calculate both Schwarz (1978) and Hannan & Quinn (1979) information criteria and the ARCH-LM statistic. The results of the model estimations by the Skewed Student-t maximum likelihood method are shown in Table6. For all studied return series, we find that the coefficients of the three models are highly significant, especially the fractional integration parameters, which are significant, positive and less than 1/2, indicating the presence of long-range dependence in the conditional mean equation illustrated in Figure 3, 4, 5, 6, 7, 8, 9 and 10. In particular, the fractional integration parameters of French stock market and Saudi stock market are higher than those of Argentina stock market, reflecting a more pronounced persistence effect. In addition, the estimated long memory coefficients of USD and EUR are higher than those of CAD, AUD and GBP. This result may be explained by the existence of strong persistence in these two Bitcoin return series. For all return series, the information criteria are minimum for the ARFIMA-AEGAS model. The asymmetric parameter γ is statistically significant and positive. Evidence regarding leverage effects implies that negative shocks imply a higher next period variance than positive shocks of the same magnitude. In other words, news has an asymmetric impact on volatility: bad news or negative shocks give more rise than good news or positive shocks.

This specification seems to be adequate to model the series since the ARFIMA-AEGAS residuals (see Figure 3, 4, 5, 6, 7, 8, 9 and 10) are characterized by the absence of conditional heteroskedasticity: there are no remaining ARCH effects in all the estimated models since the ARCH-LM statistics are strictly less than the critical value of χ_1^2 at 5%. It should be noted that the normality assumption of residuals is rejected because the Jarque-Bera statistics are strictly greater than the critical value of χ_2^2 at 5%. The conditional standard deviation is characterized by asymmetric dynamics with some sharp jumps for all series. Moreover, the series of standardized residuals show no dependence structure where the BDS statistics, reported in Table 7, are strictly less than the critical value of normal distribution at the 5% level for all embedding dimensions *m*.

Table 6 – Skewed Student-t maximum likelihood estimation – BFGS algorithm–

Parameters	TASI			Merval			CAC SMALL			AUD		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
d	0.068 (6.010)	0.068 (6.041)	0.089 (7.208)	0.041 (3.224)	0.044 (3.221)	0.054 (4.203)	0.133 (7.191)	0.163 (12.96)	0.160 (8.008)	0.058 (2.346)	0.057 (2.469)	0.056 (2.148)
ϕ_1	-	-	-	-	-0.237 (4.824)	-	0.048 (2.120)	-	0.044 (1.842)	-0.376 (-3.680)	-0.356 (-3.641)	-0.353 (-3.562)
θ_1	-	-	-	-	-	-	-	-	-	0.516 (6.203)	0.502 (6.278)	0.496 (6.181)
α_1	0.230 (10.13)	0.217 (13.28)	0.212 (11.93)	0.133 (5.979)	0.105 (6.185)	0.128 (11.02)	0.201 (10.92)	0.174 (14.31)	0.145 (13.80)	0.265 (8.150)	0.300 (8.137)	0.287 (6.645)
φ_1	0.999 (152.3)	0.973 (2115.2)	0.999 (2740.3)	0.970 (86.06)	0.999 (2080.3)	0.948 (1976.01)	0.971 (115.8)	0.956 (2342.2)	0.951 (2599.3)	0.980 (160.6)	0.958 (79.84)	0.998 (784.6)
γ	-	-	0.052 (5.823)	-	-	0.050 (6.355)	-	-	0.067 (10.02)	-	-	0.012 (3.098)
Asymmetry	0.115 (-8.180)	-0.120 (-7.747)	-0.088 (-5.881)	0.090 (-4.632)	--0.089 -4.711)	-0.079 (-4.148)	-0.188 (-10.16)	-0.183 (-10.31)	-0.159 (-8.573)	-0.053 (-2.185)	-0.057 (-2.408)	-0.047 (-1.844)
Tail	3.964 (14.81)	4.048 (14.45)	4.012 (14.90)	6.575 (8.896)	6.085 (9.079)	6.723 (9.031)	7.524 (9.526)	7.196 (10.56)	7.358 (10.79)	3.377 (9.589)	3.177 (8.949)	3.095 (9.276)
SC	-6.474	-6.474	-6.478^{+}	-5.176	-5.163	-5.186^{+}	-7.180	-7.182	-7.198^{+}	-4.551	-4.534	-4.556^{+}
HQ	-6.479	-6.480	-6.483^{+}	-5.182	-5.170	-5.193^{+}	-7.185	-7.187	-7.205^{+}	-4.566	-4.551	-4.571^{+}
JB stat.	11074.1^{*}	10107.4^{*}	11696.5^{*}	1093.1^{*}	3276.1^{*}	1309.9^{*}	765.83^{*}	821.01^{*}	749.86^{*}	1087900^{*}	1517400^{*}	2702400^{*}
ARCH(1)	0.447^{**}	0.418^{**}	2.641^{**}	1.300^{**}	1.679^{**}	0.213^{**}	0.236^{**}	0.096^{**}	0.932^{**}	0.197^{**}	0.034^{**}	0.128^{**}

Parameters	CAD			EUR			GBP			USD		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
d	0.070	0.063	0.060	0.072	0.066	0.062	0.067	0.063	0.058	0.068	0.065	0.060
	(2.607)	(2.676)	(2.566)	(2.553)	(2.586)	(2.476)	(2.265)	(2.426)	(2.309)	(2.232)	(2.329)	(2.166)
ϕ_1	-0.292	-0.264	-0.264	-0.376	-0.345	-0.360	-0.426	-0.401	-0.419	-0.378	-0.351	-0.368
	(-2.231)	(-2.355)	(-2.333)	(-3.373)	(-3.380)	(-3.610)	(-2.889)	(-2.960)	(-3.227)	(-3.261)	(-3.072)	(-3.271)
θ_1	0.405	0.395	0.391	0.535	0.516	0.526	0.587	0.572	0.585	0.534	0.514	0.527
	(3.509)	(4.067)	(4.048)	(5.865)	(6.226)	(6.501)	(4.848)	(5.142)	(5.528)	(5.542)	(5.402)	(5.621)
α_1	0.265	0.323	0.305	0.267	0.303	0.280	0.248	0.280	0.258	0.224	0.244	0.224
	(7.628)	(6.558)	(5.835)	(8.296)	(7.117)	(5.859)	(7.340)	(6.548)	(5.467)	(7.860)	(7.353)	(5.762)
φ_1	0.982	0.958	0.998	0.984	0.965	0.999	0.981	0.962	0.999	0.986	0.973	0.999
	(176.8)	(67.37)	(715.2)	(200.2)	(82.78)	(791.1)	(162.2)	(71.07)	(843.0)	(215.9)	(103.4)	(993.9)
γ	-	-	0.082	-	-	0.112	-	-	0.098	-	-	0.118
			(7.834)			(4.755)			(5.642)			(6.271)
Asymmetry	-0.034	-0.041	-0.035	-0.066	-0.069	-0.067	-0.073	-0.077	-0.074	-0.053	-0.053	-0.054
	(-1.358)	(-1.699)	(-1.381)	(-2.573)	(-2.768)	(-2.538)	(-2.942)	(-3.179)	(-3.022)	(-2.029)	(-2.100)	(-1.991)
Tail	3.200	3.051	2.960	3.284	3.092	2.998	3.383	3.143	2.996	3.539	3.374	3.306
	(9.572)	(8.607)	(8.966)	(9.200)	(8.472)	(9.133)	(9.686)	(8.940)	(10.59)	(7.936)	(7.643)	(8.013)
SC	-4.644	-4.633	-4.654^{+}	-4.632	-4.618	-4.636^{+}	-4.564	-4.549	-4.568^{+}	-4.531	-4.518	-4.533^{+}
HQ	-4.659	-4.650	-4.669^{+}	-4.647	-4.636	-4.651^{+}	-4.579	-4.566	-4.584^{+}	-4.546	-4.536	-4.549^{+}
JB stat.	941980*	1492100*	267450*	419670*	5743400*	8497900*	2624.4*	3035.1*	6102.6*	5201000*	7276700*	11241000*
ARCH(1)	0.231**	0.043**	0.119**	0.024**	0.0001**	0.024**	1.586**	2.717**	2.594**	0.0061**	0.0003**	0.012**

Notes: *Model 1: ARFIMA-GAS. Model 2: ARFIMA-EGAS. Model 3: ARFIMA-AEGAS. * indicates a rejection of null hypothesis of normality. ** indicates an acceptance of null hypothesis of homoscedasticity at the 1%. level. The values in parentheses are the Student statistics. + indicates the optimal Schwarz (SC) and the optimal Hannan-Quinn (HQ).*

Figure 3 – Residual analysis for ARFIMA-AEGAS (TASI returns)

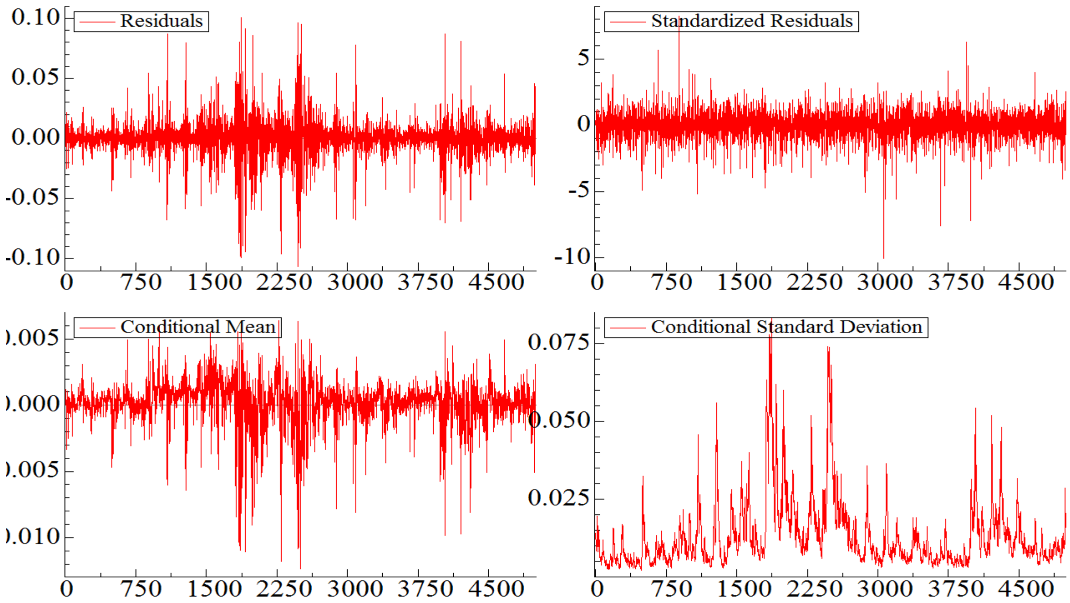


Figure 4 – Residual analysis for ARFIMA-AEGAS (Merval returns)

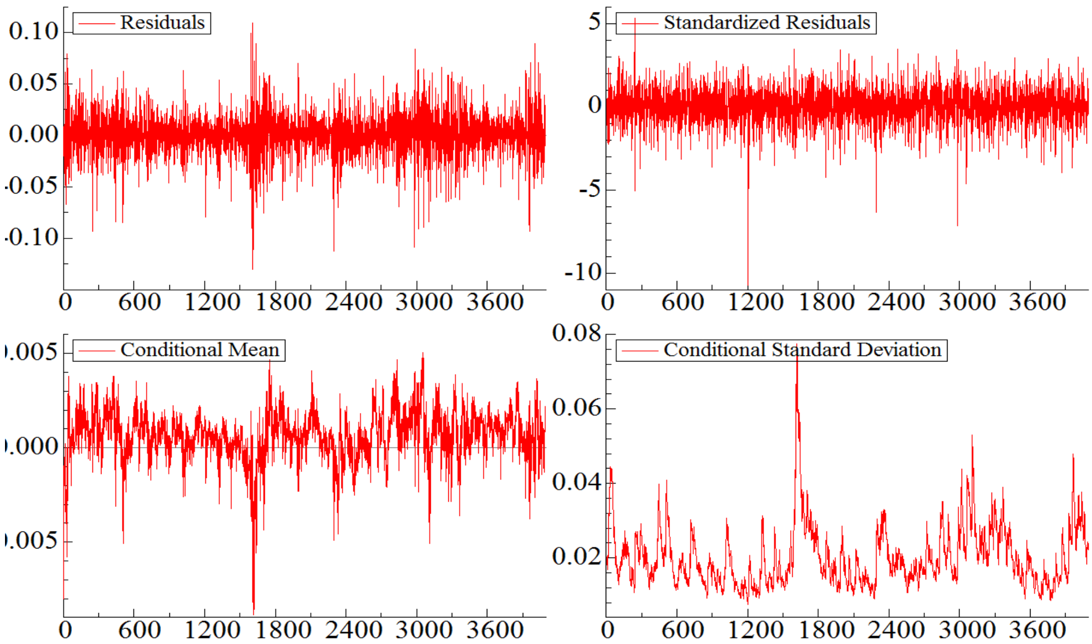


Figure 5 – Residual analysis for ARFIMA-AEGAS (CAC SMALL returns)

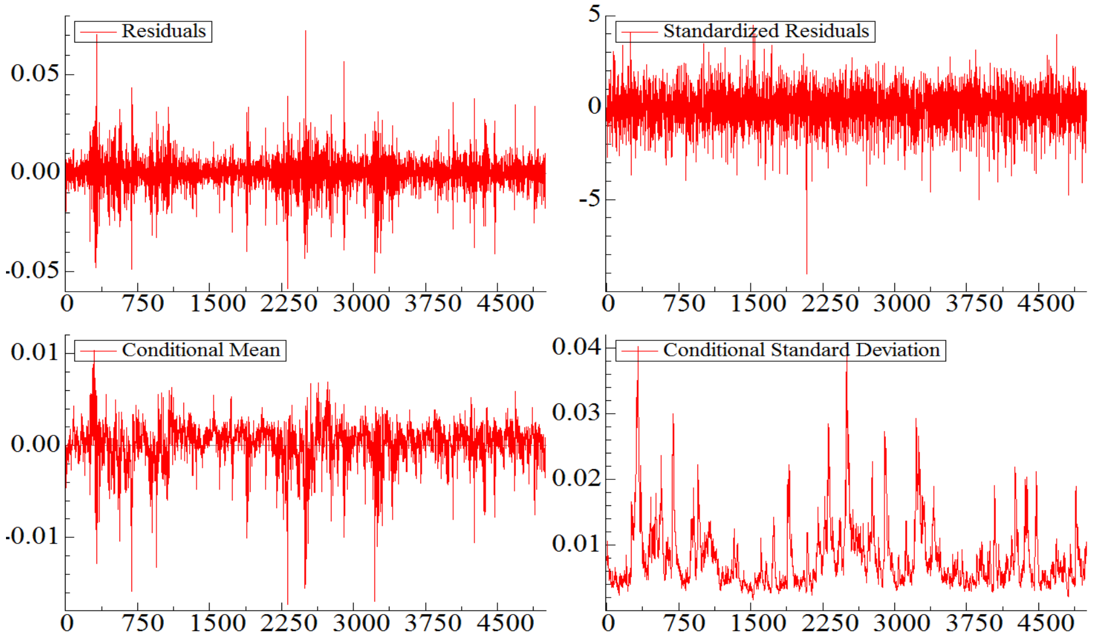


Figure 6 – Residual analysis for ARFIMA-AEGAS (AUD returns)

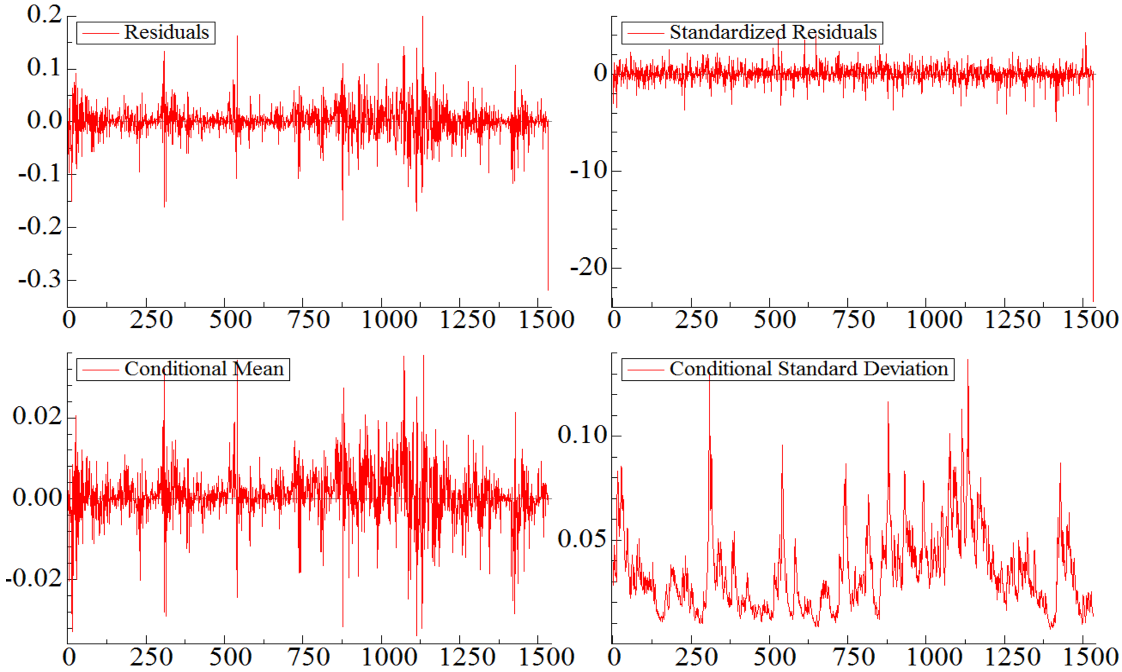


Figure 7 – Residual analysis for ARFIMA-AEGAS (CAD returns)

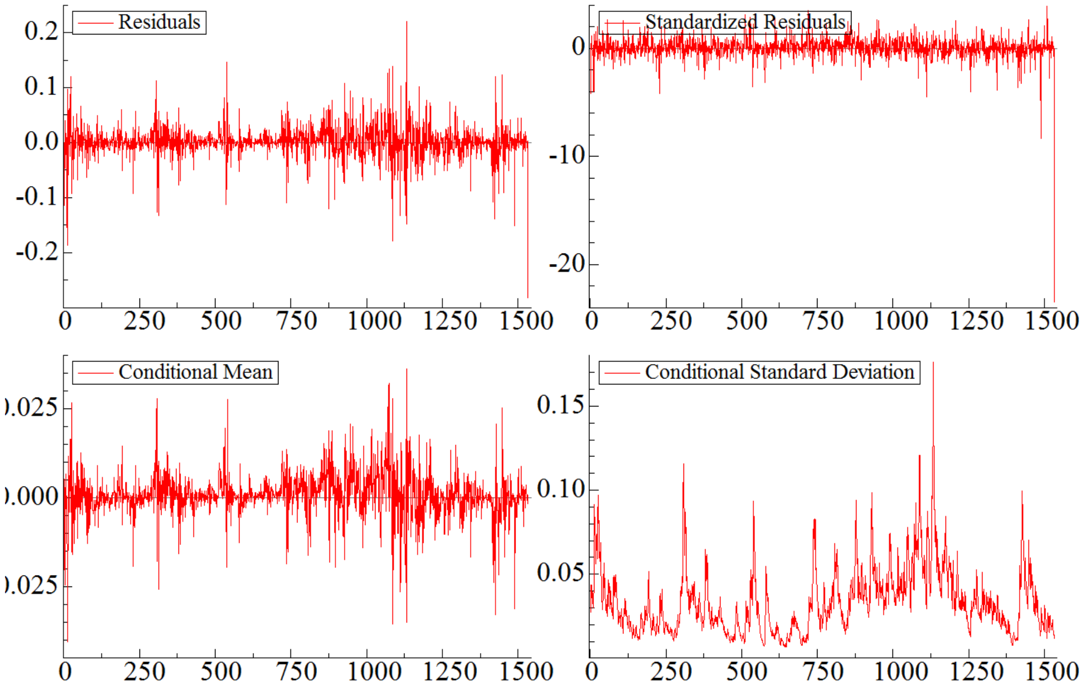


Figure 8 – Residual analysis for ARFIMA-AEGAS (EUR returns)

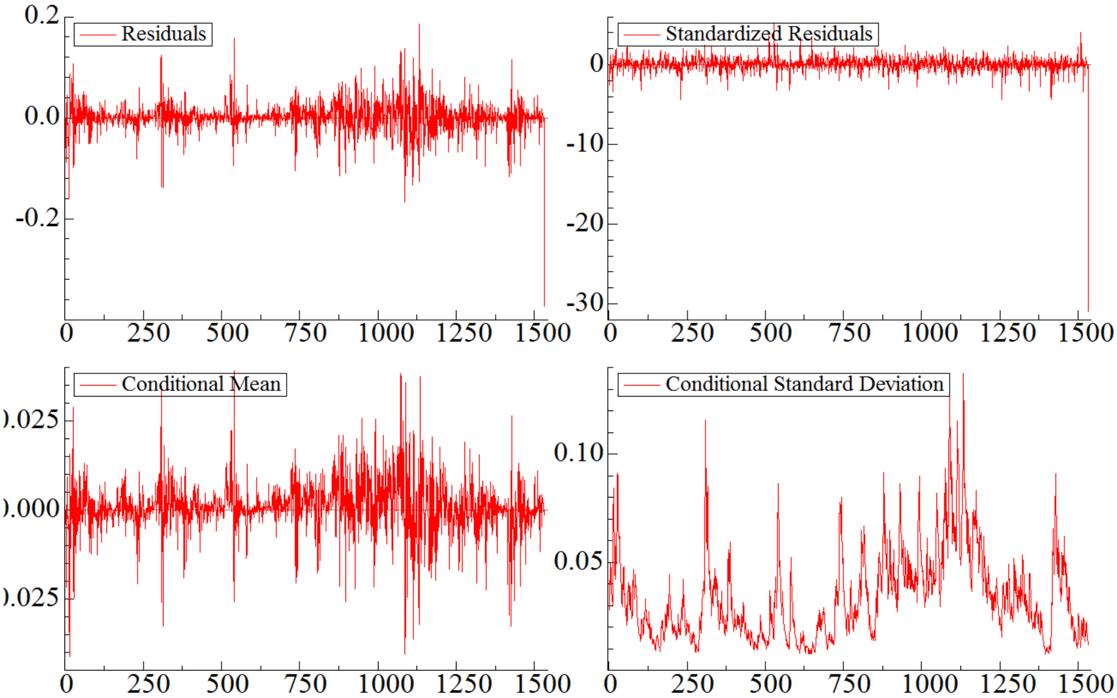


Figure 9 – Residual analysis for ARFIMA-AEGAS (GBP returns)

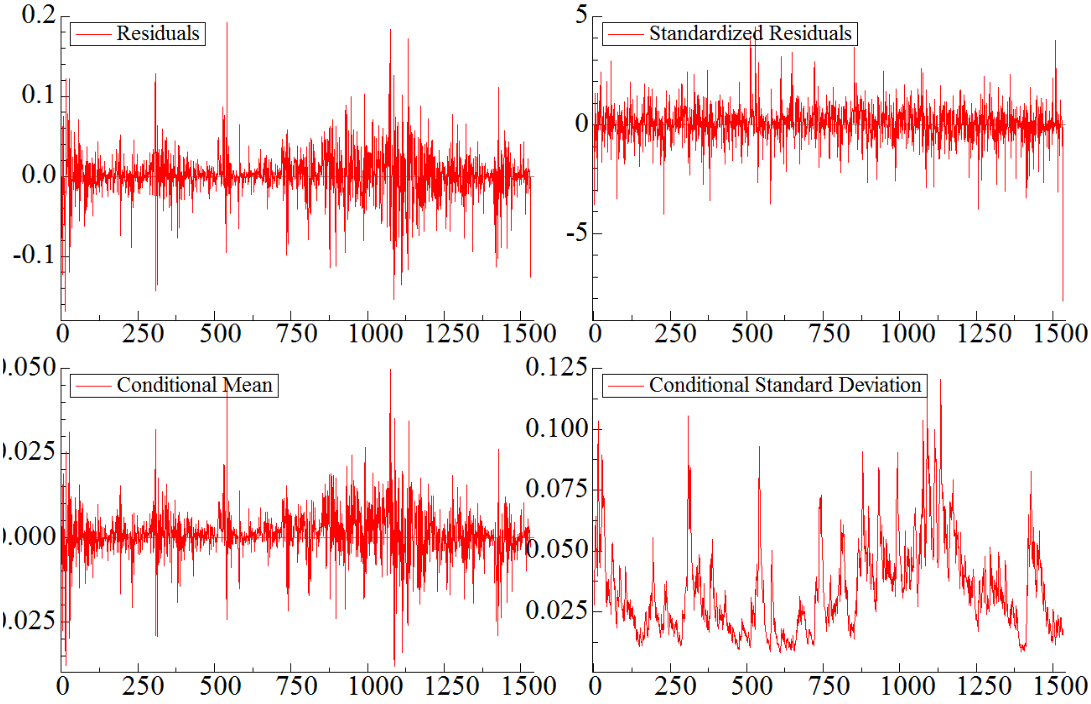


Figure 10 – Residual analysis for ARFIMA-AEGAS (USD returns)

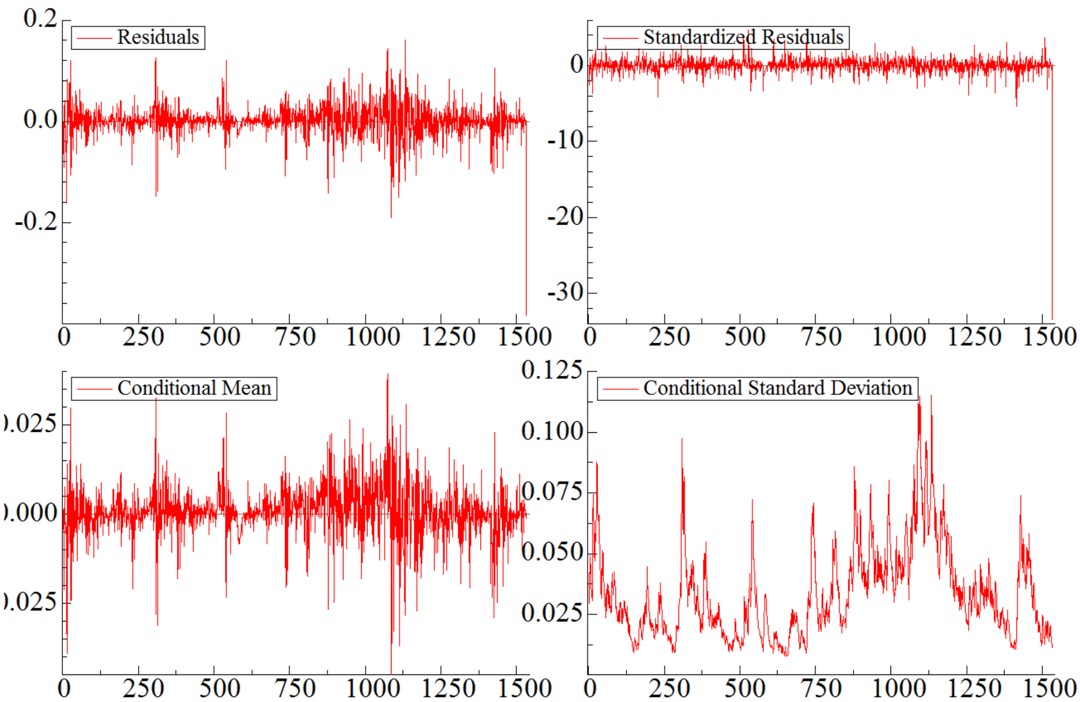


Table 7 – BDS test on standardized residuals

m	TASI	Merval	CAC SMALL	AUD	CAD	EUR	GBP	USD
2	0.700	-1.283	-0.262	0.432	0.979	1.213	-1.398	1.214
	(0.483)	(0.199)	(0.793)	(0.665)	(0.327)	(0.224)	(0.161)	(0.224)
3	0.994	-0.104	1.021	-0.167	0.623	0.548	1.327	0.570
	(0.320)	(0.916)	(0.307)	(0.866)	(0.532)	(0.583)	(0.184)	(0.568)
4	1.281	0.221	0.876	-0.401	0.289	0.040	0.762	-0.007
	(0.200)	(0.824)	(0.380)	(0.688)	(0.771)	(0.967)	(0.445)	(0.994)
5	0.993	0.595	0.986	-0.449	0.195	-0.319	0.321	-0.293
	(0.320)	(0.551)	(0.324)	(0.653)	(0.845)	(0.749)	(0.747)	(0.768)
6	0.799	0.692	0.938	-0.307	0.029	-0.662	0.037	-0.400
	(0.423)	(0.488)	(0.348)	(0.758)	(0.976)	(0.507)	(0.970)	(0.688)
7	0.594	0.754	0.901	-0.129	-0.123	-0.743	-0.060	-0.399
	(0.552)	(0.450)	(0.367)	(0.896)	(0.901)	(0.457)	(0.952)	(0.689)

Notes: *The numbers in the parenthesis are the critical probabilities. The BDS statistics are calculated by the fraction of pairs method with* ε *equal to 0.7. m represents the embedding dimension.*

5. Forecasting performance

To determine which model provides a reasonable explanation of cyclical behavior of stock returns, some diagnostic tests are performed at the outset. We first use the estimation results to compute Out-of-Sample value-at-Risk for the long and short trading position for confidence levels 95% and 99%, respectively. The results presented in Table 8 report the success/failure ratio, the Kupiec likelihood ratio (Kupiec (1995)) and the statistics for the dynamic quantile test (Engle &Manganelli (2004)). The LR statistics has the distribution χ^2 with one degree of freedom. The critical value of the Kupiec test for the most frequently adopted significance level 0.05 equals to 3.8415. The null hypothesis is rejected if the likelihood ratio exceeds the critical value (Piontek & Papla (2004)). For the ARFIMA model with skewed-Student AEGAS errors, the null hypothesis of the test is not rejected both in case of underestimating of potential loss and in case of overestimating VaR for the short and long positions, it means that the null hypothesis of Kupiec likelihood ratio test can be accepted for 99% and 95% confidence levels. However, the dynamic quantile Engle-

Manganelli test results indicate that the out-of-sample VaR forecast for all the daily returns obtained by the ARFIMA-GAS and ARFIMA-EGAS models gives unsatisfactory results and consequently fails this test for short and long trade positions. It seems that the ARFIMA-AEGAS is appropriate for capturing volatility clustering for both negative (long Value-at-Risk) and positive returns (short Value-at-Risk) for all series. The ARFIMA-AEGAS specification seems to be adequate to model the return series for both long Value-at-Risk and short Value-at-Risk. It should be noted that the null hypothesis of correct unconditional coverage is rejected for the classical ARFIMA-EGARCH and the dynamic quantile Engle-Manganelli test, in this model, gives unsatisfactory results for short and long trade positions.

Figure 11 illustrates the relation of the Value-at-risk with the return of prices. The upper line is the maximal amount that can be lost with a confidence level 97.5% over the period of time taken into consideration, when the business events are not favorable for the business activity (see Cera, Cera & Lito (2013)). The calculation of VaR through the use of the skewed-Student AEGAS model for the returns, has also advantages of the nature of forecasting the values of the VaR in the future. If the other factors remain constant, then the AEGAS model gives a very high level of approximation with the real values of the VaR.

Table 8 – Out-of-Sample Value-at-Risk Backtesting

Series	Model	Position	Kupiec LR test				Test of E.M	
TASI	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.940	8.323	0.003	19.394	0.003
			0.99	0.989	0.034	0.852	6.685	0.350
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.053	1.424	0.232	22.315	0.001
			0.01	0.011	0.777	0.377	5.666	0.461
	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.950	0.050	0.821	20.942	0.001
			0.99	0.992	3.591	0.058	5.654	0.462
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.046	1.171	0.279	7.651	0.264
			0.01	0.008	2.045	0.152	7.116	0.310
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.944	3.109	0.077	12.099	0.059
			0.99	0.992	2.508	0.113	5.548	0.475

		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.046	1.319	0.250	9.783	0.134
			0.01	0.009	0.149	0.698	10.176	0.117
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.956	4.369	0.036	26.405	0.000
			0.99	0.996	27.028	0.000	20.536	0.002
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.049	0.0008	0.976	5.743	0.452
			0.01	0.010	0.034	0.852	2.898	0.821
Merval	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.942	4.432	0.035	7.819	0.251
			0.99	0.988	0.963	0.326	8.024	0.236
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.051	0.123	0.725	11.569	0.072
			0.01	0.010	0.003	0.952	3.776	0.706
	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.94	0.845	0.357	2.632	0.853
			0.99	0.989	0.003	0.952	9.126	0.166
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.047	0.649	0.420	8.362	0.212
			0.01	0.008	0.551	0.457	5.103	0.530
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.942	1.992	0.158	8.207	0.223
			0.99	0.992	1.992	0.158	6.019	0.420
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.050	0.018	0.891	1.196	0.977
			0.01	0.008	0.823	0.364	7.895	0.245
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.951	0.194	0.659	42.611	0.000
			0.99	0.994	8.050	0.004	13.402	0.037
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.048	0.194	0.659	13.804	0.031
			0.01	0.009	0.174	0.676	21.842	0.001
CAC SMALL	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.944	3.120	0.077	15.434	0.017
			0.99	0.989	0.369	0.543	6.423	0.377
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.054	2.292	0.129	49.431	0.000

			0.01	0.011	0.780	0.377	2.973	0.812
	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.950	0.024	0.875	11.092	0.085
			0.99	0.992	2.504	0.113	3.584	0.732
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.047	0.895	0.344	35.281	0.000
			0.01	0.009	0.459	0.497	2.401	0.879
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.948	0.385	0.534	7.001	0.320
			0.99	0.992	2.504	0.113	9.054	0.170
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.051	0.310	0.577	10.862	0.092
			0.01	0.008	2.041	0.153	5.045	0.537
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.957	6.936	0.008	19.044	0.004
			0.99	0.995	16.572	0.000	13.699	0.033
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.054	2.104	0.146	11.117	0.084
			0.01	0.009	0.459	0.497	2.401	0.879
AUD	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.946	0.322	0.569	21.712	0.001
			0.99	0.995	5.405	0.020	4.410	0.621
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.024	0.009	0.924	8.946	0.176
			0.01	0.001	3.533	0.060	6.913	0.328
	ARFIMA-EGAS	Short position	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.943	1.358	0.243	26.682	0.000
			0.99	0.994	3.451	0.063	14.854	0.021
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.043	1.273	0.259	21.169	0.001
			0.01	0.011	0.166	0.683	22.664	0.000
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.941	2.284	0.130	12.509	0.051
			0.99	0.993	1.539	0.214	9.982	0.189
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.045	0.567	0.451	1.958	0.962
			0.01	0.012	0.827	0.363	4.910	0.670
	ARFIMA-	Short	Quantile	Success	Kupiec	Prob.	Stat.	Prob.

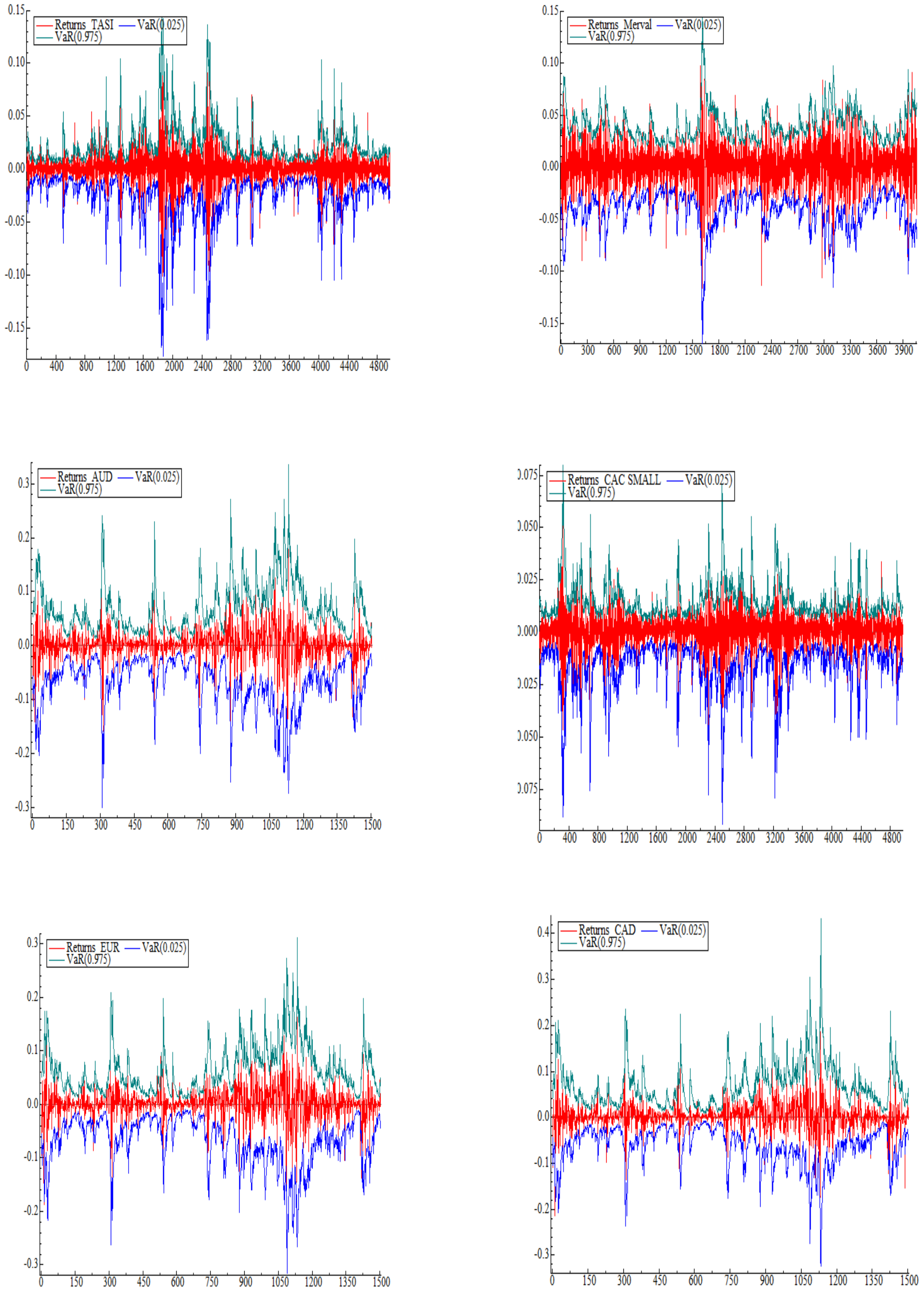
	EGARCH	positions	0.95	0.935	5.835	0.015	38.497	0.000
			0.99	0.988	0.437	0.508	51.943	0.000
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.053	0.336	0.562	26.482	0.000
			0.01	0.017	6.416	0.011	48.476	0.000
	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.931	9.471	0.002	37.063	0.000
			0.99	0.986	1.329	0.248	8.906	0.178
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.054	0.664	0.414	58.803	0.000
			0.01	0.011	0.437	0.508	41.287	0.000
CAD	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.938	4.163	0.041	12.139	0.058
			0.99	0.990	0.072	0.787	6.008	0.422
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.045	0.544	0.460	7.909	0.244
			0.01	0.009	0.072	0.787	6.008	0.422
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.941	1.950	0.162	6.910	0.438
			0.99	0.992	0.897	0.343	7.339	0.394
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.045	0.567	0.451	1.625	0.977
			0.01	0.011	0.437	0.508	8.254	0.310
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.944	0.869	0.351	17.687	0.007
			0.99	0.992	0.897	0.343	1.161	0.978
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.044	1.005	0.315	31.486	0.000
			0.01	0.010	0.022	0.880	29.148	0.000
EUR	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.945	0.664	0.414	50.389	0.000
			0.99	0.993	1.539	0.214	20.183	0.002
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.044	1.005	0.315	28.003	0.000
			0.01	0.011	0.437	0.508	27.631	0.000
	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.940	2.913	0.087	12.536	0.051
			0.99	0.995	5.405	0.020	17.940	0.006

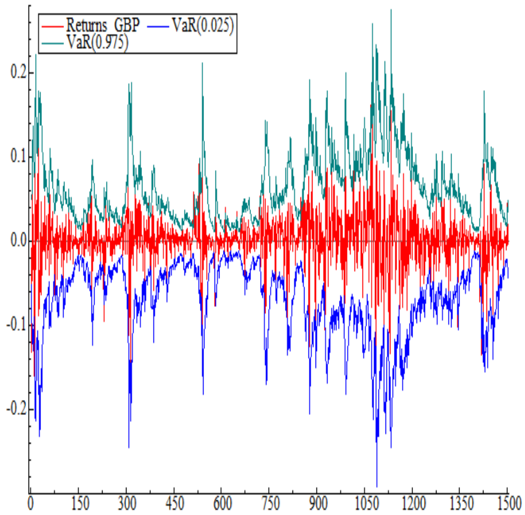
Currency	Model	Position	Quantile	Success/Failure	Kupiec	Prob.	Stat.	Prob.
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.043	1.509	0.219	7.925	0.243
			0.01	0.011	0.557	0.455	1.984	0.921
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.947	0.212	0.644	3.018	0.883
			0.99	0.993	2.383	0.122	5.293	0.624
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.042	1.575	0.209	11.420	0.076
			0.01	0.011	0.166	0.683	10.682	0.153
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.933	7.552	0.005	46.656	0.000
			0.99	0.988	0.437	0.508	28.331	0.000
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.055	0.869	0.351	54.501	0.000
			0.01	0.015	3.456	0.063	71.428	0.000
GBP	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.956	1.509	0.219	16.860	0.009
			0.99	0.994	4.003	0.045	15.021	0.020
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.048	0.065	0.798	16.827	0.009
			0.01	0.005	4.003	0.045	3.439	0.751
	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.948	0.049	0.823	28.309	0.000
			0.99	0.990	0.148	0.700	36.951	0.000
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.051	0.117	0.732	36.828	0.000
			0.01	0.009	0.148	0.700	41.838	0.000
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.948	0.117	0.732	13.398	0.062
			0.99	0.988	0.437	0.508	4.491	0.721
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.053	0.336	0.562	0.313	0.999
			0.01	0.009	0.013	0.908	5.987	0.424
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.937	4.324	0.037	23.42	0.000
			0.99	0.985	2.649	0.103	57.256	0.000
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.056	1.101	0.294	32.181	0.000

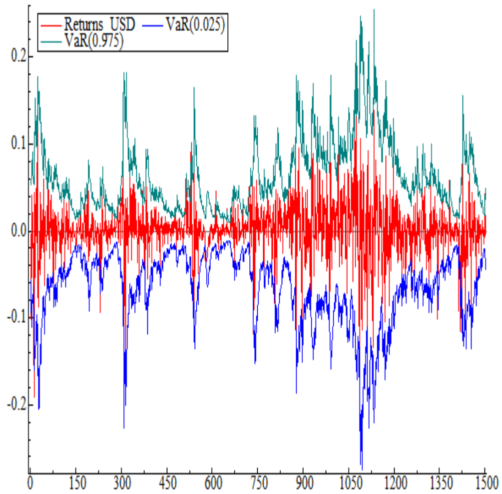
			0.01	0.016	5.344	0.020	68.21	0.000
USD	ARFIMA-GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.952	0.245	0.620	7.672	0.263
			0.99	0.994	2.853	0.091	2.609	0.856
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.043	1.220	0.269	9.955	0.126
			0.01	0.007	0.662	0.415	7.954	0.241
	ARFIMA-EGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.949	0.010	0.918	27.312	0.000
			0.99	0.989	0.022	0.880	30.484	0.000
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.044	1.005	0.315	22.822	0.000
			0.01	0.011	0.166	0.683	100.37	0.000
	ARFIMA-AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.950	0.0003	0.985	2.447	0.931
			0.99	0.989	0.022	0.880	1.816	0.969
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.044	1.005	0.315	1.624	0.977
			0.01	0.011	0.437	0.508	10.684	0.152
	ARFIMA-EGARCH	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			0.95	0.941	2.284	0.130	33.257	0.000
			0.99	0.988	0.437	0.508	56.183	0.000
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			0.05	0.049	0.0003	0.985	22.004	0.001
			0.01	0.015	4.356	0.036	68.377	0.000

Notes: *In the Dynamic Quantile Regression, p=5.E.M: Dynamic Quantile Test of Engle and Manganelli (2004).*

Figure 11 – Out-of-Sample Value-at-Risk forecasts







We further probe these results by performing the Out-of-sample tests of forecasting accuracy using the minimum loss functions on ARFIMA-GAS, ARFIMA-EGAS, ARFIMA-AEGAS and ARFIMA-EGARCH for skewed Student-t distribution and the random walk model. The forecast evaluation measures used include root mean square error (RMSE) and mean absolute error (MAE). The forecast performance and the corresponding ranking for all the models are summarized in Tables 9 and 10. The results indicate that, whatever the forecast horizon, the random walk model is beaten by all the other models. It is also observed that the ARFIMA-AEGAS model tend to have better predictive results comparing to ARFIMA-EGAS and ARFIMA-GAS with some time horizons. We see that asymmetry effects with jumps detected on volatility seem to improve the volatility forecasts. For all series, the ARFIMA-AEGAS model outperforms the classical ARFIMA-EGARCH. This can be explained by the fact that the ARFIMA-EGARCH model specifically captures the long-memory in the conditional mean and asymmetries in the volatility, which are taken into account exponentially and completely neglects the outliers and occasional jumps. Furthermore, the model rankings presented in Table 10, indicate that the skewed-Student AEGAS is the preferred model for all returns. The GAS models capture the asymmetric behavior and the volatility clustering phenomenon in the presence of long-run dynamic dependencies in the conditional mean equation.

Table 9 – Comparison of predictive qualities

Series	Equation	Horizon	Criteria (10^{-3})	ARFIMA-EGARCH	ARFIMA-GAS	ARFIMA-EGAS	ARFIMA-AEGAS	Random Walk
TASI	Conditional mean	15 days	RMSE	2.1130	0.6696	0.6696	**0..6575**	3.9077
			MAE	14.320	14.280	14.310	**13.690**	18.542
		30 days	RMSE	1.6140	0.5105	0.5105	**0.5105**	4.9842
			MAE	10.670	10.591	10.581	**10.570**	21.637
		90 days	RMSE	1.2119	0.3657	**0.3657**	0.3661	15.2770
			MAE	7.4460	7.4310	**7.4310**	7.4770	23.6330
	Conditional Volatility	15 days	RMSE	0.0744	**0.0239**	0.0240	0.0312	-
			MAE	0.4593	**0.4506**	0.4578	0.5890	-
		30 days	RMSE	0.0519	0.0168	0.0167	**0.0167**	-
			MAE	0.3624	0.3033	0.2901	**0.2899**	-
		90 days	RMSE	0.0334	0.0105	0.0107	**0.0001**	-
			MAE	0.1775	0.2046	0.1618	**0.1578**	-
Merval	Conditional mean	15 days	RMSE	2.1450	0.6782	0.6822	**0.6781**	2.7683
			MAE	17.620	17.320	17.500	**17.300**	21.799
		30 days	RMSE	1.8290	**0.5792**	0.5793	0.6868	3.7974
			MAE	15.990	**14.030**	14.030	17.810	24.014
		90 days	RMSE	0.8192	0.8185	0.8184	**0.7859**	3.8292
			MAE	19.680	19.670	19.670	**19.605**	25.198
	Conditional Volatility	15 days	RMSE	0.0455	0.0415	0.0719	**0.0144**	-
			MAE	0.3885	0.3853	0.6960	**0.3808**	-
		30 days	RMSE	0.0711	**0.0167**	0.0220	0.1986	-
			MAE	0.6641	**0.4724**	0.6487	0.6266	-
		90 days	RMSE	0.0413	0.03521	0.0357	**0.0350**	-
			MAE	0.8165	0.6254	0.7382	**0.6124**	-
CAC SMALL	Conditional mean	15 days	RMSE	0.6906	0.2185	0.2185	**0.2180**	1.6950
			MAE	5.7240	5.6550	5.6620	**5.6260**	9.3980
		30 days	RMSE	0.6602	0.2087	0.2088	**0.2087**	1.7310
			MAE	5.2210	5.2200	5.2210	**5.2190**	9.4243
		90 days	RMSE	0.6120	**0.1928**	0.1931	0.1931	1.8044
			MAE	4.7870	**4.7740**	4.7780	4.7780	9.9630
	Conditional Volatility	15 days	RMSE	0.0083	0.0081	**0.0073**	0.0080	-
			MAE	0.0640	0.0607	**0.0606**	0.0632	-

		30 days	RMSE	0.0071	0.0068	0.0068	**0.0067**	-
			MAE	0.0452	0.0446	0.0387	**0.0316**	-
		90 days	RMSE	0.0063	0.0059	0.0057	**0.0056**	-
			MAE	0.0608	0.0548	0.0450	**0.0437**	-
AUD	Conditional mean	15 days	RMSE	0.0823	0.0821	0.0823	**0.0812**	0.1829
			MAE	0.0283	0.0282	0.0292	**0.0267**	0.1263
		30 days	RMSE	0.0605	**0.0452**	0.0713	0.0600	0.1857
			MAE	0.0198	**0.0183**	0.0194	0.0191	0.1298
		90 days	RMSE	0.0404	0.0401	0.0415	**0.0382**	0.1975
			MAE	0.0184	0.0170	0.0174	**0.0161**	0.1332
	Conditional Volatility	15 days	RMSE	0.0263	0.0260	0.0262	**0.0255**	-
			MAE	0.0077	0.0071	0.0069	**0.0064**	-
		30 days	RMSE	0.0184	0.0182	**0.0165**	0.0166	-
			MAE	0.0038	0.0031	**0.0014**	0.0023	-
		90 days	RMSE	0.0115	0.0106	0.0109	**0.0095**	-
			MAE	0.0053	0.0024	0.0015	**0.0014**	-
CAD	Conditional mean	15 days	RMSE	0.0734	0.0722	0.0732	**0.0716**	0.1684
			MAE	0.0253	0.0245	0.0250	**0.0236**	0.1192
		30 days	RMSE	0.0546	0.0537	0.0537	**0.0522**	0.1721
			MAE	0.0182	0.0171	0.0180	**0.0161**	0.1248
		90 days	RMSE	0.0426	0.0423	0.0437	**0.0407**	0.1855
			MAE	0.0208	0.0202	0.0202	**0.0187**	0.1363
	Conditional Volatility	15 days	RMSE	0.0207	0.0207	0.0201	**0.0163**	-
			MAE	0.0061	0.0059	0.0058	**0.0033**	-
		30 days	RMSE	0.0146	**0.0121**	0.0141	0.0133	-
			MAE	0.0069	**0.0032**	0.0038	0.0042	-
		90 days	RMSE	0.0112	0.0097	0.0090	**0.0071**	-
			MAE	0.0041	0.0021	0.0019	**0.0012**	-
EUR	Conditional mean	15 days	RMSE	0.0968	0.0962	0.0968	**0.0945**	0.1889
			MAE	0.0317	0.0313	0.0314	**0.0295**	0.1397
		30 days	RMSE	0.0699	0.0697	**0.0678**	0.0699	0.2018
			MAE	0.0212	0.0211	**0.0197**	0.0212	0.1443
		90 days	RMSE	0.0452	0.0450	0.0452	**0.0441**	0.2225
			MAE	0.0184	0.0182	0.0181	**0.0172**	0.1634
	Conditional Volatility	15 days	RMSE	0.0364	0.0360	0.0356	**0.0340**	-
			MAE	0.0100	0.0097	0.0088	**0.0080**	-
		30 days	RMSE	0.0269	0.0257	0.0256	**0.0248**	-

			MAE	0.0075	0.0063	0.0052	**0.0051**	-
		90 days	RMSE	0.0154	0.0143	0.0148	**0.0124**	-
			MAE	0.0031	0.0023	0.0021	**0.0015**	-
GBP	Conditional mean	15 days	RMSE	0.0342	0.0342	0.0351	**0.0311**	0.1373
			MAE	0.0169	0.0165	0.0169	**0.0128**	0.0879
		30 days	RMSE	0.0277	0.0276	0.0266	**0.0244**	0.1397
			MAE	0.0139	0.0134	0.0126	**0.0113**	0.1018
		90 days	RMSE	0.0260	0.0260	**0.0231**	0.0242	0.1482
			MAE	0.0159	0.0153	**0.0125**	0.0131	0.1521
	Conditional volatility	15 days	RMSE	0.0040	0.0038	0.0035	**0.0025**	-
			MAE	0.0018	0.0018	0.0019	**0.0014**	-
		30 days	RMSE	0.0029	0.0027	0.0027	**0.0017**	-
			MAE	0.0015	0.0009	0.0009	**0.0007**	-
		90 days	RMSE	0.0048	**0.0021**	0.0042	0.0037	-
			MAE	0.0034	**0.0012**	0.0032	0.0028	-
USD	Conditional mean	15 days	RMSE	0.0993	0.0991	0.0992	**0.0955**	0.2032
			MAE	0.0328	0.0315	0.0314	**0.0284**	0.1453
		30 days	RMSE	0.0718	0.0714	**0.0681**	0.0707	0.2105
			MAE	0.0212	0.0102	**0.0062**	0.0094	0.1566
		90 days	RMSE	0.0477	0.0469	0.0462	**0.0448**	0.2312
			MAE	0.0182	0.0118	0.0137	**0.0092**	0.1721
	Conditional volatility	15 days	RMSE	0.0394	**0.0378**	0.0383	0.0384	-
			MAE	0.0125	**0.0102**	0.0104	0.0116	-
		30 days	RMSE	0.0271	0.0265	0.0269	**0.0244**	-
			MAE	0.0055	0.0053	0.0061	**0.0051**	-
		90 days	RMSE	0.0157	0.0155	0.0153	**0.0145**	-
			MAE	0.0036	0.0023	0.0022	**0.0021**	-

Table 10 – Model rankings

Series	Criteria	ARFIMA-EGARCH	ARFIMA-GAS	ARFIMA-EGAS	ARFIMA-AEGAS
TASI	RMSE	4	2	3	1
	MAE	4	3	2	1
Merval	RMSE	4	2	3	1
	MAE	4	2	3	1
CAC SMALL	RMSE	3	2	2	1
	MAE	3	2	2	1
AUD	RMSE	4	2	3	1
	MAE	3	2	2	1
CAD	RMSE	4	2	3	1
	MAE	4	2	3	1
EUR	RMSE	4	2	2	1
	MAE	4	3	2	1
GBP	RMSE	4	2	3	1
	MAE	4	2	2	1
USD	RMSE	3	2	2	1
	MAE	3	2	2	1

Consequently, the price movements appear as the result of lasting shocks which affect the stock and Bitcoin markets. In other words, the consequences of a shock will be sustainable, the TASI, the Merval, the CAC SMALL and the Bitcoin returns will not come back to their previous fundamental value. The shock of returns will be persistent in the long term and the volatility exhibits nonlinearity and asymmetry effects with jumps.

6. Conclusions

In this paper, we have combined path-dependence nature of stock price with asymmetric volatility estimated and characterized by jumps. An ARFIMA model with skewed-Student AEGAS errors, we argued, has the potential to capture long-range persistence in the conditional mean and asymmetric jumps and volatility clustering in the conditional variance. This model offers better features of the dynamic volatilities and exploits nonlinear and asymmetric structures to model the existence of time-varying parameters. In this regard, we use the scaled score of the likelihood function. In addition, the asymmetric exponential GAS model serves as an extension of the GARCH family models which assume that the conditional distribution does not vary over time. It exploits the full likelihood of information. Taking a local density score step as a driving mechanism, the time-varying parameters increase and produced a clear indication of a leptokurtic behavior and a heavy tail in the financial series.

Our empirical exercise focused on the behavior of the time-varying parameter by estimating the ARFIMA-AEGAS model with the Skewed Student-t maximum likelihood. From the dynamic quantile Engle-Manganelli test results, the Out-of-sample Value-at-Risk forecast obtained by the ARFIMA-AEGAS model gives satisfactory results at 99% and 95% confidence level for short and long trade positions. Using the minimum loss functions, the ARFIMA-AEGAS model shows a clear superiority over all the other models. Particularly, the forecasts of the ARFIMA-AEGAS model show a clear improvement compared to the random walk model at all horizons. The observed movements appear as the result of lasting and asymmetric shocks, which affect these financial markets. Consequently, recent works on volatility modeling through asymmetric exponential GAS process, which captures volatility clustering for both negative and positive returns, seem particularly promising and may provide new evidence to better understand the nonlinear and asymmetric dynamics of financial series.

The proposed ARFIMA-AEGAS model, as seen in our exercise, possesses better predictive power over conventional methods for both stock and Bitcoin prices. Its implications for Bitcoin market worthy of note: ‘memory’ forms an essential component in cryptocurrency market in order for us to model ‘sentiment’ of investors. Both volatility and memory – combined within a single framework, offers rich insights into the way asset prices evolve and can be predicted in the absence of a strong theoretical asset pricing framework, such as Bitcoin market. Finally, as there is a strong evidence of memory in these markets, we conclude that both stock and Bitcoin markets are relatively inefficient (weakly efficient), so that an investor can exploit partly some magnitudes of memory to predict future returns. But then, it is this memory, in interaction with market forces, which otherwise have different degrees of convergence rate to a steady state mean, make the ‘weak efficient’ character weakening further. An immediate consequence is that interactive memory can make the system appear as highly non-linear and unpredictable. A challenge remains to disentangle components of the asset price series with strong memory and the rest to be modelled non-parametrically. This latter proposition can be a subject of further research.

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