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Individually-Consistent Sequential Equilibrium

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Abstract

We introduce a new equilibrium concept, called the *individually-consistent sequential equilibrium* (ICSE). This concept is more permissive than the sequential equilibrium (Kreps, Wilson (1982)) but more demanding than the self-confirming one (Fudenberg, Levine (1993)). We require that players share common beliefs on the actions planned to be played at all the information sets, but not on the potential deviations. Therefore, in contrast to Kreps, Wilson (1982), we allow different players to have in mind different perturbation systems, or alternative hypotheses. This is motivated by the fact that beliefs about unobserved events are by essence not verifiable.

Keywords: Sequential Equilibrium, Consistency, Perturbations, Beliefs

JEL: C72

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1 Introduction

Myerson wrote that "the goal of game-theoretic analysis is to try to predict the path of play" (Myerson (1991, p.38)), that is, to predict the strategies used by the different players, and therefore to predict the outcome of strategic interactions. Several solution concepts have been developed, among which the most famous is the Nash equilibrium. It is however well-known that the concept of Nash equilibrium sometimes relies on incredible (or implausible) threats from the part of the players. That is, there are some equilibrium strategy profiles that satisfy the definition of the Nash equilibrium, but that intuitively (or reasonably) should not be considered as such. The issue arises from the fact that there are no restrictions placed on what players might play at out-of-equilibrium information sets.

In order to discard non-intuitive equilibrium strategy profiles, a vast program of equilibrium refinements has been launched by Reinhard Selten (Selten (1965), Selten (1975)). Among these are the *subgame-perfect Nash equilibrium* (SPNE, Selten (1965)), the *(weak) perfect Bayesian equilibrium* (WPBE, discussed in Fudenberg, Tirole (1991), Bonanno (2016) or Watson (2016)), and the *sequential equilibrium* (SE, Kreps, Wilson (1982)), which is technically close to the *perfect equilibrium* (PE, Selten (1975)). The sequential equilibrium is the solution concept commonly used to solve extensive-form games. It requires consistency of beliefs at *all* information sets, even at those that find themselves out of the (equilibrium) strategy path. Most importantly, the players are required to share the same beliefs at out-of-equilibrium information sets, even about the numerical values of mathematical artifacts used to generate perturbations of strategy profiles, which are arbitrary by nature.

Based on the insights provided by the *self-confirming equilibrium* concept (Fudenberg, Levine (1993)), we argue for softer restrictions on out-of-equilibrium beliefs. Indeed, we argue that there is no *a priori* basis for requiring players to agree on the probabilities of other players' possible mistakes (or deviations). Our solution concept, called the *individually-consistent sequential equilibrium* (ICSE), accepts different perturbation systems for different players. Therefore, our concept is less demanding than the sequential equilibrium, but stronger than the self-confirming one, because we require *a minima* that the equilibrium strategy profile be a perfect Bayesian equilibrium of the game.

We proceed as follows. Section 2 introduces some notation and rapidly discusses the characteristics of the sequential equilibrium. Section 3 introduces our solution concept with a particular example. Section 4 discusses properties of our solution concept and possible extensions. Section 5 concludes.

2 Sequential Equilibrium

Let N represent a finite set of players (possibly comprising chance). Let T represent the set of all nodes of a given game (with X the set of non-terminal (decision) nodes, and Z the set of terminal nodes). Let

H denote the set of all possible information sets, with $h \in H$ a specific information set. Let $H_i \subseteq H$ denote the set of all possible information sets at which player i might be called upon to play.

For every $h \in H_i$, we note A_h the set of actions available to player i at information set h. A *pure* strategy for player i specifies one action at every of her information sets. Let $A_i = \times_{h \in H_i} A_h$. That is, A_i is the set of pure strategies available to player i, with typical element a_i. A *mixed strategy* for player i is a probability distribution over her set of pure strategies. We can write the set of all mixed-strategies for player i as $\Delta(A_i)$.[1] Therefore, the set of all possible mixed-strategy profiles is written $\times_{i \in N} \Delta(A_i)$, with typical element $\tau = (\tau_i)_{i \in N}$. A *behavioral strategy* for player i is not a probability distribution over her set of pure strategies, but a probability distribution over her possible actions, at each of her information sets. That is, a behavioral strategy for player i is a member of $\times_{h \in H_i} \Delta(A_h)$. The set of behavioral-strategy profiles is therefore $\times_{i \in N} \times_{h \in H_i} \Delta(A_h)$, with typical element $\pi = (\pi_i)_{i \in N}$. Finally, let $u_i : Z \to \mathbb{R}_+$ represent player i's payoff if terminal node $z \in Z$ is reached.

A *system of beliefs* is a function $\mu : X \to [0,1]$ such that, $\forall h \in H, \Sigma_{x \in h} \mu(x) = 1$. That is, a system of beliefs associates to each node x in an information set h a number between 0 and 1 that has to be interpreted as representing the relative probability of the player finding herself at that specific node x, given that she finds herself in that specific information set h. Beliefs are usually assumed to be common knowledge, and are therefore the same between the players (see Fudenberg, Tirole (1991, p.240)). We will later discuss this assumption, and call for softer restrictions.

Different solution concepts place different restrictions on what the players' beliefs can be. The *weak Perfect Bayesian Equilibrium* (WPBE) is the weakest refinement. It states that strategies should be sequentially rational,[2] and that players' beliefs should obey Bayes' rule "whenever possible". In the WPBE, Bayes' rule needs to be applied at every information set that happens with positive probability, given $\pi = (\pi_i)_{i \in N}$.[3] The SE requires *consistency* of beliefs at *all* information sets, even at those that find themselves out of the (equilibrium) strategy path. To generate beliefs that are *consistent* (or rational) at every information set, Kreps, Wilson (1982) require that the belief vector μ be the limit of a sequence of belief vectors derived from Bayes' rule applied to a sequence of *fully mixed* strategy profiles (strategy profiles that put positive probability to every action in every information set). Let's denote by $\times_{h \in H} \Delta^0(A_h)$ the set of all fully mixed behavioral strategies. Formally, a pair (μ, π) is *consistent* if and only if there exists some sequence $(\hat{\mu}^k, \hat{\pi}^k)_{k=1}^{\infty}$ such that:

[1] Formally, $\Delta(A_i) = \{p : A_i \to \mathbb{R} | \Sigma_{a \in A_i} p(a) = 1 \text{ and } p(a) \geq 0, \forall a \in A_i\}$.

[2] Sequential rationality requires that players, *given* their beliefs, choose the action(s) that maximize their expected payoffs at every information set they might be called upon to play. We denote by $i(h)$ the decision-maker at information set h, and by $-i(h)$ all decision-makers at every other information set (including i). Formally, we say that a strategy profile $\pi = (\pi_i)_{i \in N}$ is *sequentially rational at some information set* h, for a given system of beliefs μ, if the following condition holds:

$$\mathbb{E}[u_{i(h)} | h, \mu, \pi_{i(h)}, \pi_{-i(h)}] \geq \mathbb{E}[u_{i(h)} | h, \mu, \tilde{\pi}_{i(h)}, \pi_{-i(h)}], \forall \tilde{\pi}_{i(h)} \in \Delta(A_h).$$

We say that a strategy profile π is *sequentially rational* given μ if it is sequentially rational at all information sets, given the beliefs.

[3] The Perfect Bayesian Equilibrium (PBE) requires (at least) that the strategy profile be a WPBE in every strict subgame of the original game. Sometimes, stronger restrictions are imposed; see Fudenberg, Tirole (1991).

1. $\hat{\pi}^k \in \times_{h \in H} \Delta^0(A_h), \forall k \in \{1,2,3,...\}$,
2. $\hat{\mu}_h^k(x) = \frac{P(x|\hat{\pi}^k)}{\sum\limits_{y \in h} P(y|\hat{\pi}^k)}, \forall h \in H, \forall x \in h, \forall k \in \{1,2,3,...\}$,[4]
3. $\pi_{i(h)}(a_h) = \lim\limits_{k \to \infty} \hat{\pi}_{i(h)}^k(a_h), \forall i \in N, \forall h \in H_i, \forall a_h \in A_h$,[5]
4. $\mu_h(x) = \lim\limits_{k \to \infty} \hat{\mu}_h^k(x), \forall h \in H, \forall x \in h$.

A *SE* is defined to be any pair (μ, π) that is consistent and sequentially rational (Kreps, Wilson (1982, p.872)).[6]

What is crucial in the concept of SE is that the players are required to implicitly *agree* on the value of the ϵ used to generate perturbations of the strategy profiles. That is, while the ϵ are arbitrary in nature (they only represent mathematical artifacts), players still need to share the same beliefs about their numerical values. Kreps, Wilson (1982, p.873) justify this in the following way. Players begin the game with some (finite) sequence of hypotheses about how the game will be played. Their "primary hypothesis" is π. If ever they find themselves at an information set that was supposed to happen with zero probability, they will attempt to apply their "second most likely hypothesis" to try to understand how they might have arrived there. If this second hypothesis fails, then they attempt to apply their "third most likely hypothesis", and so on, until their situation can be rationalized. These alternative hypotheses are actually beliefs about the values of the ϵ's, that is, about the likely deviations of the other players. The strongest requirement is what they call *lexicographic consistency*, which is based on the following argument: "if there are rational secondary hypothesis, they should be unanimously held, just as in the primary hypothesis [π]" (Kreps, Wilson (1982, p.874)). Therefore, in their view, there must exist a consensus among the (rational) players about the different perturbation probabilities, or else the strategy profile can not be considered as a SE.

We argue that this requirement is too strong. Indeed, there seem to us to be no *a priori* basis for requiring players to agree on the probabilities of other players' possible mistakes (or deviations).[7] This is why we advocate for softer restrictions, by combining the insights provided by the sequential, as well as the *self-confirming* equilibrium (SCE, developed by Fudenberg, Levine (1993)). Fudenberg and Levine justify their concept by seeing it as the end result of a process of learning, in which players repeatedly observe the other players' actions (and thereby revise their beliefs about other players' strategies using *only* the observation of previous play), but never observe what happens (or would happen) at information sets that are never reached. Once this process of learning leads to an equilibrium (the SCE), players need to have correct expectations about what the others will play (the observed actions), but not necessarily

[4]With $P(x|\cdot)$ being computed using Bayes' rule.

[5]*i(h)* represents the player playing at information set h.

[6]For a definition of sequential rationality, see Kreps, Wilson (1982, p.871-2).

[7]If we consider deviations as signalling something, as in the case of forward induction, then this argument does not apply. However, in the case of the SE, ϵ's are explicitly considered as *perturbations*, and therefore potential mistakes made by the players.

about what players might play at contingencies that never arise. Moreover, because out-of-equilibrium play is never observed, different players need not have the same beliefs about what the others would do, would they reach a zero probability information set.

3 An Example of Individually-Consistent Sequential Equilibrium

This section introduces our solution concept with an example. Consider the following three-player game; particularly, consider the strategy profile $\{C_1, B_2, B_3\}$, which can be seen as the socially optimal situation. Probabilities at every node are represented in brackets.

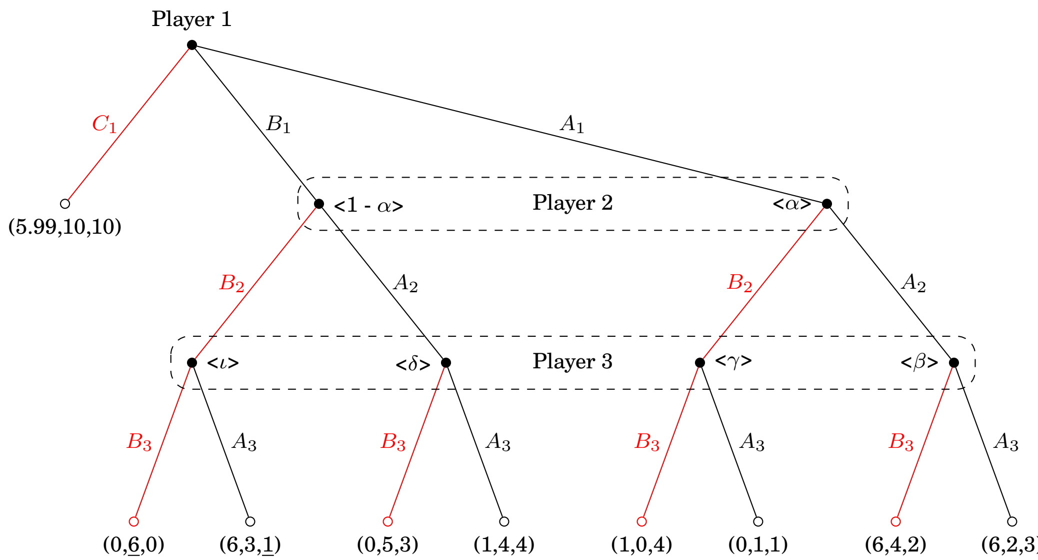

Figure 1: An example illustrating our solution concept.

The strategy profile $\{C_1, B_2, B_3\}$ is a Nash equilibrium of the game. It is however not a sequential equilibrium of the game. To see why, we need to perturb the strategy profile, and evaluate whether player 2's and player 3's actions are indeed best responses to C_1, given *consistent* beliefs at their respective information sets. The perturbed scenario is $\{(1-\epsilon_0^k-\epsilon_1^k)C_1+\epsilon_1^k B_1+\epsilon_0^k A_1, (1-\epsilon_2^k)B_2+\epsilon_2^k A_2, (1-\epsilon_3^k)B_3+\epsilon_3^k A_3\}$. Consistency requires that $\epsilon_0^k/(\epsilon_0^k+\epsilon_1^k)\to\alpha$, $\alpha\epsilon_2^k\to\beta$, $(1-\epsilon_2^k)\alpha\to\gamma$, $\epsilon_2^k(1-\alpha)\to\delta$ and $(1-\epsilon_2^k)(1-\alpha)\to\iota$ when the ϵ^k go to 0. It is therefore necessary that $\beta=\delta=0$, $\gamma=\alpha$, $\iota=1-\alpha$, and α can be any number in [0,1]. Given the actions of the other two players, player 2 would be willing to play B_2 only if $6(1-\alpha)\geq 5(1-\alpha)+4\alpha$, or $\alpha\leq\frac{1}{5}$. Player 3 would be willing to play B_3 only if $4\alpha\geq(1-\alpha)+\alpha$, or $\alpha\geq\frac{1}{4}$. Given that α cannot simultaneously be greater than $\frac{1}{4}$ and smaller than $\frac{1}{5}$, players' actions can not be reconciled with shared consistent beliefs, and the strategy profile therefore can not be considered as a sequential equilibrium. In fact, there does not exist any sequential equilibrium that supports the strategy C_1 for player 1 (and therefore supports the socially optimal arrangement).[8]

The key particularity of our solution concept is that we accept different perturbation systems for different players. It follows that the strategy profile $\{C_1, B_2, B_3\}$ is a solution of the game with respect to our notion of equilibrium, given that the only feature that prevents it from being a sequential equilibrium is that player 2 and player 3 (need to) have different beliefs about the numerical values of the perturbations ϵ_0^k and ϵ_1^k. Consider the following situation: let's assume that for player 2, $\alpha = \frac{1}{5}$ (and so $1-\alpha = \frac{4}{5}$), while for player 3, $\alpha = \frac{1}{4}$ (and so $1-\alpha = \frac{3}{4}$). This might the case if player 2 and player 3 have in mind the following perturbed scenarios: $\{(1-2\epsilon^k - 8\epsilon^k)C_1 + 8\epsilon^k B_1 + 2\epsilon^k A_1, (1-\epsilon^k)B_2 + \epsilon^k A_2, (1-\epsilon^k)B_3 + \epsilon^k A_3\}$ for player 2, and $\{(1-3\epsilon^k - 9\epsilon^k)C_1 + 9\epsilon^k B_1 + 3\epsilon^k A_1, (1-\epsilon^k)B_2 + \epsilon^k A_2, (1-\epsilon^k)B_3 + \epsilon^k A_3\}$ for player 3. While the beliefs about the values of the perturbations are different between the two players, having these particular beliefs makes both B_2 and B_3 best responses to C_1, and to the action played by the other. Note here that player 2's and player 3's beliefs are still going in the same direction: both believe that it is more likely that player 1 might deviate to B_1 rather than to A_1; the difference is only in relative terms. We discuss notions of *distance* and *direction* of beliefs in the following section.

4 Characteristics of the Individually-Consistent Sequential Equilibrium

Our solution concept is motivated by the fact that beliefs about unobserved events are by essence not verifiable. We accept the idea that there exists a consensus on the actions players play and plan to play at their information sets (the shared *primary hypothesis* according to Kreps, Wilson (1982)), but we do not require that the players share common beliefs on the other actions.

Formally, what distinguishes our concept from the SE is that we do not require the existence of only *one* sequence of perturbed strategy profiles on which all players need to agree, but allow for different perturbation systems for different players. Therefore, a pair (μ, π) is *individually-consistent* if and only if there exist some sequences $(\hat{\mu}_i{}^k, \hat{\pi}_i{}^k)_{k=1}^{\infty}$ such that, $\forall i \in N$:

1. $\hat{\pi}_i{}^k \in \times_{h\in H} \Delta^0(A_h), \forall k \in \{1,2,3,...\}$,
2. $\hat{\mu}_{hi}^k(x) = \frac{P(x|\hat{\pi}_i{}^k)}{\sum\limits_{y\in h} P(y|\hat{\pi}_i{}^k)}, \forall h \in H, \forall x \in h, \forall k \in \{1,2,3,...\}$,[9]
3. $\pi_{i(h)}(a_h) = \lim\limits_{k\to\infty} \hat{\pi}_{j,i(h)}^k(a_h), \forall j \in N, \forall h \in H, \forall a_h \in A_h$,
4. $\mu_{hi}(x) = \lim\limits_{k\to\infty} \hat{\mu}_{hi}^k(x), \forall h \in H, \forall x \in h$.

An *individually-consistent sequential equilibrium* (ICSE) is any pair (μ, π) that is both individually-consistent and sequentially rational.

[8]The proof can be found in the Appendix.

[9]$\hat{\mu}_{hi}(x)$ denotes player i's beliefs at information set h.

The beliefs of a same player are consistent, in that her beliefs, at every information set, derive from a same system of perturbations. Nothing precludes different players from sharing the same beliefs about the numerical values of the perturbation parameters, but we do not require that they have to do so. Players agree on the planned actions; they may only disagree on the beliefs assigned to actions that are never played or planned to be played. The idea is that, given that these actions are never observed, no player has any real information about them. So, in the above example, player 2 may think that player 1, if deviating from C_1, plays A_1 and B_1 with the probabilities $\frac{2}{10}$ and $\frac{8}{10}$, whereas player 3 may think that player 1 plays A_1 and B_1 with the probabilities $\frac{3}{12}$ and $\frac{9}{12}$. Even if player 2 knows player 3's beliefs, she is not compelled to change her beliefs, given that player 3 has no more information than herself, so she might be right and he might be wrong. In the same way, player 3, even if he knows player 2's beliefs, is not compelled to change his beliefs, given that player 2 has no more information than himself, so he might be right and she might be wrong. It derives from this fact that there is no reason to be more restrictive on beliefs.

Yet, if players belong to the same community and feel ill at ease when having opposite beliefs, we may possibly introduce some restrictions, based on notions of *direction* and *distance*.

As a matter of example, if we require beliefs to go in the same direction (as was the case in the studied example), then players might be less likely to try to converge on the same beliefs, and this even if they have the opportunity to communicate; this might justify the acceptance of different beliefs. If player 2 believes that player 1 is more likely to deviate towards B_1 than towards A_1 (in the above example, $\alpha = \frac{1}{5}$ and $1 - \alpha = \frac{4}{5}$) and then learns that player 3 believes the same thing (even if the weights placed on the perturbation parameters are different; in the above example, $\alpha = \frac{1}{4}$ and $1 - \alpha = \frac{3}{4}$), she might be less likely to revise her beliefs compared to a situation in which she learns that player 3 believes a deviation towards A_1 is more likely (assuming accuracy motives from the part of the players).

Requiring a same direction of beliefs amounts to ordering the beliefs at each information set in the same way, i.e., to add a fifth property:

5. $\forall h \in H, \forall x$ and $x' \in h, \forall i$ and $j \in N$, if $\mu_{hi}(x) \geq \mu_{hi}(x')$, then $\mu_{hj}(x) \geq \mu_{hj}(x')$.

Restrictions based on distance follow a similar logic. In the above studied example, player 3 believes that player 1, if deviating, plays A_1 with probability $\frac{1}{4}$, whereas player 2 believes that player 1 plays A_1 with probability $\frac{1}{5}$. Yet $\frac{1}{4}$ and $\frac{1}{5}$ are not very distant: they are both close to what could be a virtual common probability $\frac{4.5}{20}$. Another way to express this small distance would be to look at the payoffs. As a matter of fact, by replacing, in Figure 1, the underlined numbers 1 and 6 by 0.87 and 6.17 respectively, we get a game where players 2 and 3 can share the same beliefs about player 1's deviation (with $\alpha = \gamma = \frac{4.5}{20}$), that is to say a game where (C_1, B_2, B_3) becomes a SE. Therefore, small differences in outcomes may translate small differences in beliefs, in a way that has not been defined up to now.

5 Conclusion

We introduce a new solution concept, called the *individually-consistent sequential equilibrium* (ICSE), which combines the insights provided by the concepts of *sequential* (Kreps, Wilson (1982)) and *self-confirming* (Fudenberg, Levine (1993)) equilibrium. While the sequential equilibrium requires *lexicographic consistency*, our solution concept accommodates situations in which different players have in mind different perturbations systems. This is motivated by the fact that beliefs about unobserved events are by essence not verifiable.

Our concept is therefore more permissive than the sequential equilibrium, but more demanding than the self-confirming one. Because we require sequential rationality, the ICSE always *a minima* picks perfect Bayesian equilibria (PBE), while self-confirming equilibria need not even be Nash.

We discuss potential restrictions on beliefs, based on the notions of *direction* and *distance*. These restrictions, while constraining the range of strategy profiles that can be considered as solutions, have an intuitive flavor. Indeed, if we assume accuracy motives from the part of the players, then these restrictions might actually justify the acceptance of different beliefs for different players. Further work will generalize their properties.

Appendix

We show that there are no sequential equilibria supporting the action C_1 for player 1 (and therefore the socially optimal situation) in the game shown in Figure 1.

Case 1: Player 2 plays A_2. Therefore player 1 plays A_1.

Case 2: Player 2 plays B_2.

(i): If player 3 plays A_3, player 1 plays B_1.

(ii): If player 3 plays B_3, then player 1 might want to play C_1, but we have already shown that the beliefs that would support this equilibrium are not mutually consistent.

(iii): If player 3 plays A_3 and B_3, then $\gamma = \frac{1}{4}$, and so necessarily $\alpha = \frac{1}{4}$. But then, player 2 would prefer to play A_2. To show this, first let r be the probability that player 3 plays A_3. By playing A_2, player 2's expected payoff is $\frac{1}{4}(2r+4(1-r)+\frac{3}{4}(4r+5(1-r)) = \frac{1}{4}(14r+19(1-r))$. By playing B_2, player 2's expected payoff is $\frac{1}{4}r+\frac{3}{4}(3r+6(1-r)) = \frac{1}{4}(10r+18(1-r))$, which is strictly inferior to the expected gain of playing A_2.

Case 3: Player 2 plays A_2 and B_2.

(i): Player 3 plays A_3. In this case, it would not be profitable for player 2 to randomize, given that playing only A_2 would allow her to always gain strictly more.

(ii): Player 3 plays B_3. Given player 2's indifference between A_2 and B_2, it is necessary that $\alpha = \frac{1}{5}$.

To show that with these beliefs, player 3 would want to deviate, first note q the probability that player 2 would play A_2. Then by playing A_3, player 3's expected gain would be $\frac{1}{5}(1+2q)+\frac{4}{5}(1+3q)=\frac{1}{5}(5+14q)$. By playing B_3, player 3's expected gain would be $\frac{1}{5}(4-2q)+\frac{4}{5}(3q)=\frac{1}{5}(4+10q)$, which is strictly inferior to the expected gain player 3 would receive by playing A_3.

(iii): Player 3 plays A_3 and B_3. Let r be the probability that player 3 plays A_3, and q the probability that player 2 plays A_2. Let ϵ_0 and ϵ_1 be the perturbations associated to A_1 and B_1 respectively. The expected gain of playing A_2 for player 2 is $\epsilon_0(2r+4(1-r))+\epsilon_1(4r+5(1-r))=\epsilon_0(4-2r)+\epsilon_1(5-r)$. The expected gain of playing B_2 for player 2 is $\epsilon_0 r+\epsilon_1(3r+6(1-r))=\epsilon_0 r+\epsilon_1(6-3r)$. Equalizing these expected gains yields $\epsilon_0(4-2r)+\epsilon_1(5-r)=\epsilon_0 r+\epsilon_1(6-3r)$, or $\epsilon_0(4-3r)=\epsilon_1(1-2r)$.

The expected gain of playing A_3 for player 3 is $\epsilon_0(3q+(1-q))+\epsilon_1(4q+(1-q))=\epsilon_0(1+2q)+\epsilon_1(1+3q)$. The expected gain of playing B_3 for player 3 is $\epsilon_0(2q+4(1-q))+\epsilon_1 3q=\epsilon_0(4-2q)+\epsilon_1 3q$. Equalizing these expected gains yields $\epsilon_0(1+2q)+\epsilon_1(1+3q)=\epsilon_0(4-2q)+\epsilon_1 3q$, or $\epsilon_0(3-4q)=\epsilon_1$. It follows that $\epsilon_1=\epsilon_0(3-4q)=\epsilon_0\frac{4-3r}{1-2r}$. Therefore, $3-4q=\frac{4-3r}{1-2r}$, so $4q=3-\frac{4-3r}{1-2r}=\frac{3-6r-4+3r}{1-r}=\frac{-1-3r}{1-r}$, which is strictly inferior to 0; an impossible event.

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