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Document de Travail n° 2019 – 34

Septembre 2019

Bureau d'Économie Théorique et Appliquée BETA

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Environmental Quality and Monopoly Pricing*

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September 18, 2019

Abstract

This paper investigates various aspects of a monopolist's pricing and environmental quality choice, as two simultaneous decisions and with each as a separate decision, the other variable being exogenously fixed. Green quality is modeled as in Spence (1975), and the present analysis builds on his pioneering work. We contrast the private and the first-best socially optimal solutions. While the latter follows the intuitive property of assigning a higher price to higher quality, the former solution does so under a natural condition of log-supermodular demand. This condition is studied in some detail, and related to properties of an underlying utility function. We complete this characterization of optimal pricing by providing a counter-intuitive example where the two-dimensional interaction is such that the monopolist ends up charging a lower optimal price than the social planner, as well as producing a lower quality. Finally, we investigate respective sufficient conditions under which (i) the private and first-best solutions coincide, and (ii) either one is larger than the other.

JEL codes: Q50, L12, D42.

Key words and phrases: environmental quality, green goods, pricing quality, multi-distortion monopoly pricing, organic food.

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*The authors would like to thank two referees of this Journal for detailed feedback that helped improve significantly the presentation of the paper. This research was partially financed by the Cercle Gutenberg via the "Chaire Gutenberg: On environmental R&D and Imperfect Competition" at the University of Strasbourg, CNRS, BETA UMR 7522, F - 67000 Strasbourg, France.

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1 Introduction

Environmental quality has emerged in recent decades as a crucially important dimension of product differentiation for many consumer goods, ranging all the way from agricultural produce to high-tech products. There is a substantial body of evidence pointing quite unambiguously to the conclusion that a significant proportion of consumers in the industrialized world have a higher willingness to pay for green products, as opposed to traditional or brown products.[1] It follows that, at equal prices, (virtually) all consumers will prefer a green product to a brown product, or a greener product to a less green product. In other words, environmental quality naturally fits as part of vertical product differentiation in the usual terminology used in industrial organization.

In view of the increasing attention devoted to environmental issues, it comes as no surprise that a number of recent studies in environmental economics have investigated some aspect of environmental quality and its effects on market performance, along with various public policy initiatives. The issues that were considered include among others optimal standards, tax and other types of policy in vertically differentiated duopoly (e.g., Moraga-Gonzalez and Padron-Fumero, 2002, Lombardini-Riipinen, 2005, and Mantovani and Vergari, 2017 and Garcia-Gallego and Georgantzis, 2009), the scope of validity of the Porter hypothesis in strategic settings (e.g., André, González and Porteiro, 2009, and Lambertini and Tampieri, 2012a), and the effects of minimal environmental standards (e.g., Lambertini and Tampieri, 2012b).

All the aforementioned studies share a common modeling feature: They adopt the now-standard vertical differentiation model pioneered by Mussa and Rosen (1978), and further developed by Gabszewicz and Thisse (1979), Shaked and Sutton (1982). Thus these studies posit some aspects of environmental quality as the (aggregated) vertical attribute. In contrast, the present paper is an attempt to contribute to this strand of literature by adopting Spence's (1975) earlier and pioneering approach to operationalize product quality (see also Schmalensee, 1979).[2]

[1] According to a study by OECD (2002), 27% of consumers in OECD countries may be seen as "green consumers". There are many sectoral studies that confirm this general pattern (see e.g., Hidrue et. al, 2011). In addition, a large proportion of consumers refrain from purchasing products sold by firms that are known to be major polluters or that misrepresent the environmental content of their products. However, recent studies have also revealed that survey data on willingness to pay tends to display over-reporting, and that people are more willing to pay for products that include other, private benefits, such as improved health or taste (Chen et al., 2018).

[2] Although Spence's pioneering approach used to be quite familiar to economists working in industrial organization, as confirmed by the fact that it is covered in Varian's, Varian (1992), classical graduate textbook, it is fair to say that it has been eclipsed by the Mussa-Rosen model in recent decades.

There are some notable exceptions to the above qualification literature, including older work by Ebert and von dem Hagen (1998), which models the environmental quality of a polluting product by inserting emissions (as a negative measure of quality) as a separate argument in the inverse demand function (and the utility function). These authors then derive the socially optimal Pigouvian tax-subsidy policy for a monopoly market. In effect, unaware of Spence's work, these authors have chosen to model environmental quality in a similar manner (see also Petrakis, Sartzetakis and Xepapadeas, 2010). A macro-economic perspective is taken in Bosi and Desmarchelier (2017) and Bosi, Desmarchelier and Ragot (2019), while a trade-theoretic one is given by Lahiri and Symeonidis (2017). We shall discuss the detailed connection of our work to Ebert and von dem Hagen (1998) later on.

The two approaches to modeling product quality are not unambiguously ranked in terms of overall merit; rather, they each have their advantages and shortcomings. When it comes to environmental quality, Spence's model can be seen as advantageous in the following respects: (i) it allows for more varieties (or quality levels) than just two in the tractable version of the Mussa-Rosen model, (ii) it more readily allows for a general approach as opposed to relying on specific functional forms for the model primitives, and (iii) it dispenses with the simplifying (but crucial) assumptions of a covered market and of a unit purchase per consumer. Spence's model is particularly appropriate for produce (fruits and vegetables) that are differentiated in terms of environmental quality. A simple concrete example is organic/biological produce instead of regular produce. A bio tomato is of higher environmental quality when the hosting soil contains fewer chemicals, the planted seeds include fewer GM ingredients, the fertilizers and the protection agents used have less chemical content, etc.... Another class of products captured by Spence's formulation might be automobiles that are differentiated in terms of environmental performance, as assessed by some normalized measure of emissions. This differentiation could be due to the overall quality/performance of the catalytic converter, and/or to whether it is fuel, hybrid or electric powered, thus covering the range from regular cars to electric vehicles (EVs). However, as we do not include a damage term in the welfare functions, these examples form a perfect fit for monopoly pricing, but some (such as the car example) do not fit as well for the social planner problem, as discussed in more detail in Section 2.

The main purpose of this paper is to investigate various aspects of a monopolist's pricing and environmental quality choice, both when viewed as two simultaneous decisions and when chosen separately one at a time, the other variable being fixed. In so doing, we build on the results of Spence (1975) and Sheshinski (1976), after providing a summary of their main results. These authors reasoned in terms of output and quality and showed that the two monopoly choices are generally not socially optimal. In this setting, there are two distortions to monopoly pricing, as the firm prices with the marginal consumer in mind, while a social planner considers the average consumer. As a result, at fixed output, the monopolist may under or over provide quality relative to a social planner, depending on whether the marginal valuation of quality is below or above the average valuation of quality. More precisely, under (over)-provision prevails if the cross partial derivative of the inverse demand with respect to output and quality is (negative) positive.

For a preview of our results, we first note that the present treatment of Spence's problem is formulated in terms of price (instead of output) and quality choices. Although both Spence (1975) and Sheshinski (1976) were aware that, when both choices are made simultaneously, the optimal monopoly output may well end up higher than the social planner's, no actual example of such a counter-intuitive scenario was provided in their studies. If one thinks of quality choice by a monopolist as entailing a second market distortion, then it is of a different sort from the pricing distortion as the former can go either way, relative to the socially optimal level. We complement their analysis by providing an example where the two-dimensional interaction is such that the monopolist ends up charging a lower optimal price than the social planner, as well as producing a lower quality. Thus, even such a foregone conclusion in price theory as monopoly over-pricing (relative to a social planner) may be reversed in the presence of a concomitant quality choice.

As part of further investigation of the pricing behavior of a monopolist in the setting at hand, we find that, with price held fixed (e.g., as a consequence of a strong form of regulation), the monopolist always under-provides quality relative to the first-best solution. In addition, while a social planner always responds by increasing quality, the firm will do so under a sufficient condition on demand, that it be log-supermodular in price and quality. To establish that the firm may indeed respond with a lower quality without this sufficient condition, we provide an example with closed-form solutions where the latter, very counter-intuitive, property holds in a global sense. Likewise, if quality is held fixed at some exogenous level (e.g, through regulation in the form of a minimal quality standard) and this level is increased, the firm responds with a higher price under the same condition of log-supermodular demand. Without the latter property, the firm may price higher quality lower even in a global sense (as shown via an example). Since the socially optimal price always increases with quality (as seen above), the possible failure of this intuitive monotonicity property for the firm's price may be construed as yet another distortion due to monopoly pricing.

The condition of log-supermodular demand in price and a relevant parameter (a demand shifter) has emerged in other economic contexts and is clearly an important property for a family of demand functions to possess (see e.g, Vives, 1999, Amir and Stepanova, 2006, or Cosandier et. al., 2018). In particular, it has the exact economic interpretation that the price elasticity of demand is increasing in the parameter, here product quality. In other words, demand becomes more inelastic as product quality is increased. This is arguably the most natural definition of what product quality ought to be thought of, when expressed via a property of the demand function. This is an appropriate definition whether quality is of an environmental sort or otherwise.[3] In light of the importance of this property, we provide a novel micro-economic foundation via the commonly used construct of a representative consumer and derive simple conditions on the utility function ensuring that the resulting demand function satisfies the natural properties of monotonicity in quality and log-supermodularity in price and quality.

A final issue analyzed in the present paper is the derivation of sufficient conditions under which the monopolist will provide the first-best socially efficient level of quality (without any regulation). In so doing, we build on Spence's analysis of a class of multiplicative demands where the dependence on price is hyperbolic and elasticity is independent of price and monotonic in quality. For this class of demands, we derive a full characterization of the comparison between the private and public solutions. The underlying key property is again the log-supermodularity of demand.

We conclude this Introduction with two final remarks. The first is that our decision to couch product quality in terms of environmental attributes is motivated partly by the desire to introduce the strand of literature on product quality pioneered by Spence (1975) to environmental economics. Our conclusions otherwise apply a priori equally well to traditional notions of product quality, and thus should be of interest to the literature on vertical product differentiation in industrial organization as well. The second remark is that we use in a very simple form the results from supermodular optimization, a natural technique here so as to avoid specifying a two-dimensional second-order condition for the optimization problems at hand; see e.g., Topkis (1998) or Vives (1999) for the relevant definitions of the notions and results we use here, in rather elementary ways.

This paper is organized as follows. Section 2 describes the model. Section 3 is devoted to the analysis of the two problems with one decision variable at a time, while Section 4 deals with the case of simultaneous choices. Section 5 contains a micro foundation for the demand function and the key property of log-supermodularity. Section 6 discusses the effects of suitable public policy.

[3] The condition of log-supermodular demand is slightly stronger than the rotation condition introduced by Myatt and Johnson (2006) in their study of the foundations of product design.

Section 7 offers a short conclusion. Finally, Section 8 contains all the proofs of this paper.

2 The model

While Spence conceived of product quality in rather general and abstract terms, in the present paper, we shall think of quality as being of an environmental nature. Thus higher quality will mean that a good was produced with a more environmentally friendly production process. This greener good may be achieved through a new technology or via the use of end-of-pipe filters as part of the old technology. To fix ideas, one may think simply about fruits or vegetables with a bio label of variable degree. The lowest quality would then be the regular (brown) version of the product, and quality would then increase as more natural ingredients (and thus fewer artificial inputs) are used in the agricultural process. We shall return to a discussion of the types of industries that fit the model under consideration later on in this section.

Following the classical Spence (1975) model, we consider a monopoly firm with constant unit cost of production operating in a market with a demand function $D(p,a)$, where $p \geq 0$ is the price charged by the firm, and a is a measure of product quality. To produce a good of quality a entails a constant marginal (or unit) cost of $c(a)$ depending on the chosen quality of its product, a. The possible range of qualities is postulated to be an interval $[a_{\min}, a_{\max}]$.

The monopolist's profit function in price-quality space is then

$$\pi(p,a) = [p - c(a)]D(p,a). \tag{1}$$

The following assumptions will be maintained throughout this paper.

(A1) *The demand function D is twice continuously differentiable and satisfies* $D_p(p,a) < 0$ and $D_a(p,a) > 0$ for all (p,a).

(A2) *The unit cost function $c : [a_{\min}, a_{\max}] \longrightarrow \mathbb{R}_+$ is twice continuously differentiable and satisfies* $c'(\cdot) \geq 0$.

These assumptions are standard in the related literature. The part $D_p(p,a) < 0$ captures the usual law of demand, and $D_a(p,a) > 0$ means that higher quality leads to higher demand.[4]

The cost structure reflects constant returns to scale in production for all possible quality levels, i.e., $c(a)$ is the constant unit cost. As higher quality is more costly to produce, it is natural to

[4]Throughout the paper, subscripts will denote partial differentiation with respect to the indicated variable.

require that $c'(\cdot) \geq 0$. Thus, the process of infusing products with greener content is subject to decreasing returns to scale, with the limit case of constant returns to scale still allowed.

Finally, $a_{\max}$ is an exogenously given maximal feasible green quality, given the present state of know-how. This assumption is mostly for technical convenience. Likewise, the quality $a_{\min}$ can be thought of as the brown product, corresponding to the status quo when only pure cost minimization, and no environmental considerations, guide the production process.

This formulation of quality-dependent demand is quite general, and could reflect multiple aspects of product differentiation including the environmental aspect. One particular way to conceptualize the model is to think about environmental quality as measuring the quality and/or quantity of end-of-pipe equipment used to clean up the emissions formed as a byproduct of manufacturing an otherwise fairly homogeneous product. This gives a specific meaning to the unit cost, in the form $c(a) = c_0 + c_1(a)$ where c_0 is the constant marginal cost of the brown product ($a_{\min}$), and $c_1(a)$ is the abatement cost corresponding to green quality a with $c_1(a_{\min}) = 0$. The fact that demand increases with quality then reflects pure environmental concerns, which still leads to consumers' higher willlingness to pay (as discussed before).[5]

Consumer surplus is defined as usual by

$$S(p,a) = \int_p^{+\infty} D(t,a)\, dt,$$

and total surplus (or social welfare) is the sum of producer and consumer surpluses

$$W(p,a) = \int_p^{+\infty} D(t,a)\, dt + [p - c(a)]\, D(p,a)\,. \tag{2}$$

The first-best social planner (or regulatory) solution aims to maximize social welfare with respect to price p and quality a. The absence of a damage term in the social welfare function may be justified as follows. First, it may simply be a reflection of the fact that the environmental damage due to the production of the good is already included in the utility function of the representative consumer and thus in the derived demand $D(p,a)$. In other words, demand would be lowered by the right amount corresponding to the good's environmental harm.[6]This is tacitly the justification of Ebert

[5]Examples of this special class of green differentiation include for instance electricity produced with solar power or wind versus coal-powered electricity. We thank a referee for this suggestion.

[6]An important tacit assumption of the present paper is that product quality is perceived correctly and similarly by the consumers, the firm and the social planner. This may be justified by invoking the role of certification via suitable eco-labels for green products. Shewmake et al. (2015) provide a micro foundation for the case where consumers might erroneously perceive the environmental quality of goods.

and von dem Hagen (1998), whose welfare function also did not include an explicit damage term.[7] This presupposes that consumers internalize the environmental damage externality the same way a utilitarian social planner would. This would not be a commonly satisfied assumption, but may nevertheless closely fit some Northern European (in particular the Nordic) countries.

A second justification applies to certain classes of products for which environmental damage is of such a limited magnitude that it is compensated by the Earth's natural regenerative properties. For instance, in the case of fruits and vegetables, quality is perceived by the majority of consumers as increasing as one goes from genetically modified (GM) crops to regularly grown (open field) crops, to greenhouse production, to a variety of green or sustainable production methods.[8] The latter may include any of the following features to variable extent: less pesticide, no GM ingredients, no synthetic fertilizers, and less irrigated water, among others (Chen et al., 2018).[9] Similar examples may be given for other farm/agricultural products, such as free range chickens versus caged chickens, their respective eggs, hormone-fed beef versus naturally-grazing beef, etc...

We now note some related important facts regarding the above examples of organic food or agriculture. The first is that the green version of the product also provides a health benefit that is known to lead to higher willingness to pay amongst consumers, beyond that generated by the green attributes. There is survey evidence that the health effect often has a stronger impact on consumers' willingness to pay than the environmental effect (Chen et al., 2018). Finally, the underlying complexity of this green product differentiation has led to the emergence of a much-needed concomitant eco-labeling certification industry (Mason, 2006).

In the sequel, we shall consider three separate but related problem pairs, each involving both the monopolist and the social planner. In the basic problem pair, as in Spence (1975), the firm chooses the quality of its product a and the price p so as to

$$\max \left\{ \pi \left(p,a\right) :p\geq 0,a\in \left[a_{\min },a_{\max }\right] \right\} , \tag{3}$$

where $a_{\min} \geq 0$, and the social planner chooses simultaneously the quality of the product a and the

[7] These authors suggested that including a usual damage term in an additively separable manner in the welfare function (as is often done in environmental economics) ignores interaction effects between consumption and pollution.

[8] Importantly, the listed order also corresponds to increasing marginal costs of production as quality increases (e.g., GM crops have the lowest cost and the most organic crops the highest cost).

[9] These different methods lead to different levels of erosion of the hosting soil, and these effects may be reflected in the firm's cost function $c(a)$. However, the resulting negative effects on air and water quality would not be taken into account by the monopolist, but ought to be by a social planner when they are of significant magnitude and likely to be beyond the earth's regenerative capabilities.

price p to

$$\max\{W(p,a) : p \geq 0, a \in [a_{\min}, a_{\max}]\}. \tag{4}$$

In each of the other two problem pairs, the same two decision-makers consider their same respective objective functions, but with one of the two decisions exogenously fixed in turn, and optimize only with respect to the other decision. We begin with the analysis of the latter two problems, as they constitute building blocks for the two-dimensional problem.

3 Setting price with exogenous quality and vice-versa

In this section, we consider the two subproblems where the monopolist chooses one of its two decision variables only, with the other held fixed at some exogenous level (for reasons to be specified below). The next subsection also provides a discussion of the key property of log-supermodularity of demand, to be invoked extensively below, in each of the three problems.

3.1 The case of fixed quality

We begin by treating the more important case where the quality level is exogenously fixed, and consider the standard problem of monopoly pricing, namely

$$\max\{[p - c(a)]D(p,a) : p \geq 0\}.$$

Denote the argmax by $p^m(a)$, i.e., the optimal price correspondence of the monopolist (as quality changes exogenously).[10]

The social planner chooses the price p^s so as to

$$\max\{W(p,a) : p \geq 0\}.$$

Denote the argmax by $p^s(a)$ i.e., the optimal price correspondence of the social planner.

We investigate the effects of changing exogenous quality on pricing. This is of interest on its own right for consumer and environmental economics. Indeed, since it is generally believed that high quality or green products sell for higher prices, it is worth uncovering exact conditions when this actually happens. At the same time, often, quality may indeed be considered fixed by the firm, due to regulatory requirements, such as a binding minimal quality standard.[11]

[10] For greater generality that will turn out to be needed for some of our results, we do not impose concavity assumptions on profits. Hence $p^m(a)$ may well be multi-valued (i.e., a correspondence).

[11] Alternatively, product quality may be considered fixed by the firm as a short-run constraint, while product innovation proceeds to determine future versions (or qualities) of the product for the medium or the long-run.

A direct consequence of the first order conditions of the social planner (see 11 below) follows.[12]

Proposition 1 *Under Assumptions (A1)-(A2), for a given quality level a,*
(i) the price correspondence of the social planner $p^s(a)$ *is increasing in quality* a*, since* $p^s(a) = c(a)$*.*
(ii) We always have $p^m(a) \geq p^s(a)$*.*

Thus the firm makes zero profit at the social optimum since the social planner always sets price at marginal cost. Two direct consequences are that (i) at a given quality level, the monopoly price is necessarily larger than the socially optimal price, and (ii) the latter is increasing in quality.

In contrast, from the perspective of the monopoly firm, the dependence of the optimal price on exogenous quality is a significantly more complex issue, as we now see. To this end, we shall need the notion of (strict) log-supermodularity of demand, which will play a major role in this paper. This property is defined by the log of demand, or $\log D(p,a)$, being a (strictly) supermodular function of (p,a), or using the cross partial test, $[\log D(p,a)]_{pa} > 0$, which boils down to

$$D(p,a)\, D_{pa}(p,a) - D_p(p,a)\, D_a(p,a) > 0 \text{ for all } (p,a). \tag{5}$$

This assumption is easily seen to be quite general, since we always have in (5) that $D_p(p,a)\, D_a(p,a) \leq 0$ under Assumption (A1).[13] Thus the condition (5) imposes only that the demand function not be too submodular in (p,a), or that the cross partial $D_{pa}(p,a)$ not be too negative. Though very broad, the scope is not universal as we shall demonstrate later on.

To garner some insight into the level of generality of this assumption, we provide several examples of well-known or plausible demand functions that satisfy it. The most commonly used demand function $D(p,a) = a - bp$ is log-supermodular.[14] More generally, any monotonic transformation of the latter demand, of the form $d(a-bp)$ with $d'(\cdot) > 0$ and $d''(\cdot) < 0$ is log-supermodular. A general additive class of demands with this property is given by $D(p,a) = s(a) - g(p)$ for any functions g and s with $g'(p) > 0$ and $s'(a) > 0$. One such well-known example is $D(p,a) = a - gp^\lambda$, $\lambda > 0$.

Yet another important broad class of demand functions that works is the multiplicatively separable family $D(p,a) = d(p)s(a)$ for any pair of one-variable functions $d(\cdot)$ and $s(\cdot)$ satisfying

[12] Throughout the paper, "increasing" will mean "weakly increasing". We will say that a correspondence is increasing if every one of its selections is (weakly) increasing.

[13] By Topkis's (1978) cross partial test, log-supermodularity of demand, or supermodularity of log-demand, is equivalent to the familiar complementarity property that $[\log D(p,a)]_{pa} \geq 0$, for all (p,a), which is easily seen to correspond to the non-strict version of (5). Note that the strict version is assumed here only for convenience.

[14] The fact that a may be taken as a measure of quality for the general class of linear demands has been explicitly recognized e.g., by Hackner (1996), or Amir et. al. (2018).

$d'(\cdot) < 0$ and $s'(\cdot) > 0$. It is easy to see that $[\log D(p,a)]_{pa} = 0$ (since $\log D(p,a) = \log d(p) + \log s(a)$), so that (5) holds as a limit case with equality.[15] Among the most commonly used demand functions in this class are $D(p,a) = a(b-p)$, $D(p,a) = ae^{-p}$, and the widely used hyperbolic class $D(p,a) = a/p^{\alpha}$ or $D(p,a) = a/(p+1)^{\alpha}$, $\alpha > 0$. This class also includes some of the demand functions used in the example of the next sections.

Importantly, this assumption has a very natural economic interpretation for the effect of quality on people's willingness to pay: Indeed, it is precisely equivalent to assuming that higher quality strictly increases the price elasticity of demand.[16] This constitutes a natural assumption, indeed even a precise definition, for the notion of product or environmental quality in the context at hand.

The next result provides two distinct but closely related sufficient conditions for the intuitive conclusion that the firm's price is increasing in its product quality. Here, we restrict attention to prices $p \geq c(a)$, as any $p < c(a)$ would never be chosen by the firm as it is dominated by $p = c(a)$.

Proposition 2 *Assume that the demand function satisfies either one of the two conditions*

$$c'(a)\, D_p^2(p,a) + D(p,a)\, D_{pa}(p,a) - D_p(p,a)\, D_a(p,a) > 0 \text{ for all } a \text{ and } p \geq c(a), \tag{6}$$

or

$$D_a(p,a) + [p - c(a)]\, D_{pa}(p,a) - c'(a)\, D_p(p,a) > 0, \text{for all } (p,a). \tag{7}$$

Then the firm's price correspondence $p^m(a)$ is globally increasing in quality a.

Each of the two alternative sufficient conditions is easily seen to hold with great generality. Indeed, for (6), as the first term is always positive, the conclusion may hold even when the log-supermodularity of demand (or condition (5)) fails.[17] Similarly, as to (7), the first and third terms are both always positive, so the only requirement is again that $D_{pa}(p,a)$ not be too negative. Thus, the two conditions in this Proposition are qualitatively very similar, but not identical. A natural

[15] Our results are also valid in case (5) is taken with a weak inequality. The strict inequality is assumed only for convenience, as it allows for shorter proofs and the avoidance of more technical arguments..

[16] Denoting by $\varepsilon(p,a) = \frac{pD_p(p,a)}{D(p,a)}$ the price elasticity of demand, one easily checks that

$$\frac{\partial \varepsilon(p,a)}{\partial a} = \frac{p\,[D(p,a)\, D_{pa}(p,a) - D_p(p,a)\, D_a(p,a)]}{[D(p,a)]^2}.$$

Hence, for (p,a) such that $p > 0$ and $D(p,a) > 0$, condition (5) is equivalent to $\frac{\partial \varepsilon(p,a)}{\partial a} > 0$.

[17] On the other hand, if one were to think of (6) as a condition on demand that guarantees that $p^m(a)$ is globally increasing *for all possible quality cost functions,* then the log-supermodularity of demand is exactly what is needed (this is easily seen to be the case with the limit case of a constant cost function, i.e., $c(a) = c$ for all a).

preference for (6) is due to its association with a very precise economic interpretation, in that the part $DD_{pa} - D_p D_a > 0$ if and only if the price elasticity is increasing in quality (though the latter inequality is thus only a sufficient condition for (6)).

Comparing the private and social problems, we see that while price and quality are always complements at the social level, they are so at the private level for a broad class of demand functions, but not quite a universal one. In other words, a social planner will always price higher for better quality, while a monopoly firm will have a tendency to do so, but this is guaranteed only under the condition (6) or (7) and, in particular, under the intuitive assumption of log-supermodularity of demand, i.e., when the price elasticity of demand is strictly increasing in quality.

The broad class of multiplicatively separable demands fits as a special case of Proposition 2.

Corollary 3 *For any demand function of the form $D(p,a) = d(p)s(a)$, with $d'(\cdot) < 0$ and $s'(\cdot) > 0$, the price correspondence $p^m(a)$ is increasing in quality a.*

In section 8.8, we explore whether there is any theoretical support for the highly counter-intuitive possibility of price globally decreasing in product quality. We provide an affirmative answer to this question, along with an explicit example of this counter-intuitive fact.

3.2 The case of fixed price

We now consider the reverse problem of monopoly choice of quality for a given fixed price

$$\max \{[p - c(a)] D(p,a) : a \in [a_{\min}, a_{\max}]\}.$$

Such a situation arises for instance if a monopolist supplier of a good or a service is subject to government pricing regulation in its traditional role of price setter. The most obvious example is a binding price ceiling. This would reduce the firm's problem to a choice of quality. In addition, in many industries, both goods and services, price remains fixed, at least for pre-specified time intervals, due e.g., to contractual commitments in vertical relationships or to other industry-specific characteristics. There are quite a few such examples of such goods and services, including some sports events, movies, plays and other performing arts; basic haircuts; tax filing; lawyer services (priced per hour), automobile repair and servicing, basic visit to a primary care physician, etc... Therefore, it appears that a theory of optimal quality choice by a price-constrained monopolist is of quite some independent interest.

Before looking at the case of fixed price, it is instructive to present the related case of fixed output when the firm chooses quality. This scenario is of interest as Spence's pioneering work reasoned in terms of output to shed light on the properties of this model, as we now briefly review.

Spence argued that, in addition to the usual mark-up above the marginal cost of production, the firm might also set quality too high or too low relative to the social optimum. This second inefficiency result is better understood for the firm's decision problem written in terms of quantity (instead of price) and quality. Denote by $P(q,a)$ the inverse demand where q is the quantity produced by the firm, and a the quality of the good, as in our formulation. Social welfare can be written as a function of the quantity-quality pair,

$$\widetilde{W}(q,a) = \int_0^q P(t,a)\,dt - qP(q,a) + \pi(q,a). \tag{8}$$

Therefore, the marginal impact of an increase in quality on social welfare is

$$\frac{\partial \widetilde{W}}{\partial a}(q,a) = \int_0^q P_a(t,a)\,dt - qP_a(q,a) + \frac{\partial \pi(q,a)}{\partial a}.$$

Setting $\frac{\partial \pi}{\partial a} = 0$, the sign of $\frac{\partial \widetilde{W}}{\partial a}$ depends on the impact of quality on consumer surplus which, in turn, depends on the relative magnitudes of the average valuation of quality increments, $\frac{1}{q}\int_0^q P_a(t,a)\,dt$, and the marginal valuation of quality increments by the marginal consumer, $P_a(q,a)$. When the former exceeds the latter, the firm sets quality too low. This comparison depends on the sign of the second order cross derivative P_{qa}, as shown by Spence (1975).

Proposition 4 *Spence (1975)*
(i) At given q, the firm under (over)-supplies quality relative to the social optimum if

$$\frac{1}{q}\int_0^q P_a(t,a)\,dt > (<) P_a(q,a) \text{ for all } a. \tag{9}$$

A sufficient condition for (9) is that $P_{qa}(q,a) < (>) 0$ for all (q,a).
(ii) At fixed price p, the firm always undersupplies quality relative to the social optimum.

As pointed out by Varian (1992), Proposition 4 is easily understood graphically. An increase in quality generates two effects on the demand curve. First, it shifts up. Second, the inverse demand curve becomes flatter (resp. steeper) if $P_{qa} < 0$ (resp., $P_{qa} > 0$). In this case, for fixed output, the monopolist undersupplies (resp. oversupplies) quality relative to the social optimum.[18]

[18] These rotations of the demand curve stem from changes in the dispersion of consumers' valuations. More precisely,

It is the tendency for low quality reflected in this result that has led regulatory authorities to specify minimum quality standards in the context of price regulation. Spence (1975) and Sheshinski (1976) discuss some aspects of this type of regulation of a monopoly firm in some detail.

As a final question in this subsection, we look at the problem of whether a price-constrained firm (through regulation) will choose a higher or lower quality as the price it is forced to charge increases. This is obviously the reverse question from the one addressed in subsection 8.8. It turns out that this problem satisfies analogous properties as the reverse one.

Proposition 5 *(i) If the demand function satisfies either condition (6) or (7), the quality correspondence* $a^m(p)$ *of the firm is globally increasing in price* p.
(ii) If the demand function satisfies (19) or (20), $a^m(p)$ *is globally decreasing in price* p *(while interior).*
(iii) The quality correspondence $a^s(p)$ *of the social planner is increasing in* p, *as* $a^s(p) = c^{-1}(p)$.

Indeed, since the condition of log supermodularity of demand treats the price and quality variables symmetrically, Propositions 2 and 10 also imply that, treating price as a parameter and quality as the decision variable, the monopolist will choose an optimal quality that is increasing in price if condition (6) or (7) holds, but instead decreasing in the price if condition (19) or (20) holds. Naturally, the more intuitive situation is the one where a higher quality is associated with a higher price, and this is the scenario that is predicated on a very general assumption on the demand function, just as in the reverse problem.[19] Due to this symmetry between the two single-decision problems, we also have the following analog of Corollary 3.

Corollary 6 *For any demand function of the form* $D(p,a) = d(p)s(a)$, *with* $d'(\cdot) < 0$ *and* $s'(\cdot) > 0$, *the quality correspondence* $a^m(p)$ *is increasing in price* p.

---

an increase in the dispersion of consumers' valuations rotates the demand curve clockwise ($P_{qa} > 0$). The impact of these clockwise rotations on the monopolist's behavior has been thoroughly studied by Johnson and Myatt (2016). They prove that in many settings, profits are U-shaped functions of the dispersion. High dispersion is complemented by niche production and low dispersion is complemented by mass-market supply.

[19] A dual economic interpretation of log supermodularity of demand holds here: It says that the quality elasticity of demand, defined by $\widehat{\varepsilon}(p,a) = \frac{aD_a(p,a)}{D(p,a)}$, increases in the (regulated) price. Indeed, one easily checks that

$$\widehat{\varepsilon}_p(p,a) = \frac{a\left[D(p,a)\,D_{pa}(p,a) - D_p(p,a)\,D_a(p,a)\right]}{\left[D(p,a)\right]^2}.$$

Hence, for (p,a) such that $p > 0$ and $D(p,a) > 0$, condition (5) is equivalent to $\widehat{\varepsilon}_p(p,a) > 0$. In other words, demand increases more (in percentage terms) in response to a quality increase when the price imposed on the firm is higher.

While the situation of a fixed price is less economically relevant than the case where quality is the fixed variable, it is worth noting that the above analysis applies equally to both cases. In addition, this result sheds some new light on the second market failure (in quality choice) of the monopoly regime with endogenous quality.

4 Setting both price and quality

In this section, we summarize the other results from Spence (1975) and Sheshinsky (1976) comparing the optimal choices of the monopolist to the social optimum under several scenarios, and provide some extensions of interest. In particular, we provide a sufficient condition on a class of demand functions for the monopoly price to be lower than the planner's price, as well as an explicit example of this counter-intuitive possibility.

The monopolist's first order conditions with respect to p and a in (3) are respectively

$$\begin{cases} [p^m - c(a^m)] D_p(p^m, a^m) + D(p^m, a^m) = 0, \\ [p^m - c(a^m)] D_a(p^m, a^m) - c'(a^m) D(p^m, a^m) = 0. \end{cases} \tag{10}$$

The planner's first order conditions with respect to p and a in (4) can be rewritten respectively as

$$\begin{cases} p^s = c(a^s) \\ \int_{p^s}^{+\infty} D_a(t, a^s)\, dt - c'(a^s) D(c(a^s), a^s) = 0 \end{cases}. \tag{11}$$

Even in this setting, the social planner always sets price equal to marginal cost, leading to zero profit. However, as seen by Spence (1975), the behavior of the monopolist deviates significantly from the usual case. Sheshinsky (1976) has shown that, if $P_{qa} < 0$ (so for fixed price, the monopolist undersupplies quality), one can exclude only the case

$$p^m < p^s \text{ and } a^m > a^s. \tag{12}$$

All the other three possible rankings of the two price-quantity pairs (p, a) chosen by the firm and the planner remain possible, i.e., are consistent with our assumptions (and Spence's, Spence (1975), and Sheshinski's, Sheshinski (1976)). Even adding our assumption of log-supermodular demand (so that price and quality are complements for the monopolist) does not rule out any other case beyond (12). One implication of (12) is that a necessary condition for the monopolist to choose a price lower than the social planner's is that the firm undersupply quality.[20] We provide an example

[20] One might be tempted to conclude from Proposition 4 that if $P_{qa} = 0$ (i.e., if P is additively separable in q and a), then the firm will choose the socially optimal price. However, this is correct at fixed output q, but not necessarily if output is also a choice variable.

below (new to the literature) where the latter case happens.

In addition, Spence (1975) has shown that for a class of price-iso-elastic demand functions with elasticity increasing in quality, quality is undersupplied by the monopolist. We build on this result and show that the monopolist then chooses a price lower than the social planner's. Furthermore, we derive a necessary and sufficient condition for the monopolist to choose a price lower than the social optimum, and an explicit example where this happens. Let

$$\overline{\pi}(a) = \max_p \pi(p,a) \text{ and } \overline{W}(a) = \max_p W(p,a).$$

When the monopolist's quality choice a^m is interior, i.e. $0 < a^m < +\infty$, as pointed out by Spence (1975) the question of whether the firm under or oversupplies quality relative to the social optimum translates formally into the question of whether the derivative $\overline{W}'(a^m)$ is strictly positive or strictly negative. This question can be raised for any demand function, but the case in which the elasticity is independent of price is illuminating.

4.1 Iso-elastic demand and quality choice

In this section, we consider a special class of demand functions whose elasticity depends on quality but not on price. In other words, such demands are iso-elastic in price, but not in quality. With $\varepsilon(a) = pD_p(p,a)/D(p,a)$ denoting the (usual) price elasticity of demand, this class is such that $D(p,a) = g(a)p^{\varepsilon(a)}$ where $g'(a) > 0$ and $\varepsilon(a) < -1$ for all a.[21] This class of demand functions is thus characterized by the fact that the price elasticity depends only on the quality choice. These two properties of the demand function, multiplicative separability and price iso-elasticity, allow for a full ranking of the price and quality choices by the firm and the planner according to the key property of log-supermodularity of demand, in contrast to the general case.

Not being multiplicatively separable, this demand is strictly log-supermodular (resp., log-submodular) in (p,a) if $\varepsilon'(a) > 0$ (resp., < 0) since $\log D(p,a) = \log g(a) + \varepsilon(a)\log p$, and therefore

$$[\log D(p,a)]_{pa} = \varepsilon'(a)/p \text{ for all } (p,a).$$

For this class of demand functions, the strict log-supermodularity/submodularity of demand is the key property that allows a full comparison of the private and socially optimal quality levels.

Proposition 7 *Assume that the monopolist chooses an interior quality level, i.e.,* $\overline{\pi}'(a^m) = 0$ *with* $0 < a^m < 1$. *If the price elasticity* $\varepsilon(a)$ *is such that*

[21] This reflects the well known property that the monopolist operates on the elastic part of the demand curve.

(i) D is strictly log-supermodular (or $\varepsilon'(\cdot) > 0$), then $a^m < a^s$.

(ii) D is strictly log-submodular (or $\varepsilon'(\cdot) < 0$), then $a^m > a^s$.

(iii) $\varepsilon'(a^m) = 0$, then $a^m = a^s$.

This corresponds to Proposition 3 in Spence (1975), with the new observation that the elasticity property behind the comparison of the two optimal qualities corresponds to the strict log-supermodularity or log-submodularity of demand. In addition, Proposition 7 includes the correction that the conclusions are only valid under the additional assumption that the quality level chosen by the monopolist is interior, namely neither minimal nor maximal. The necessity of this additional assumption is illustrated in section 8.8 by an example where, despite the fact that price elasticity strictly increases with quality, quality is set at the socially optimal level by the monopolist (in contradiction to Proposition 3 in Spence, 1975). This is due to the fact that the monopolist chooses the maximal quality level, which is also socially optimal.

For this class of demands, we now establish a necessary and sufficient condition for the counter-intuitive result that the monopolist charges a price lower than the social planner's. Though raised in the literature, this possibility was not confirmed in any formal sense, not even through examples.

Corollary 8 *The monopoly firm charges a price lower than the social planner's if and only if*

$$\frac{\varepsilon(a^m)}{\varepsilon(a^m)+1} < \frac{c(a^s)}{c(a^m)}. \tag{13}$$

To shed some light on this result, note that the function $\varepsilon \mapsto \frac{\varepsilon}{\varepsilon+1}$ is increasing over $(-\infty, -1)$ and takes values in $(1, +\infty)$ with $\lim_{\varepsilon \mapsto -\infty} \frac{\varepsilon}{\varepsilon+1} = 1$. Hence, the necessary and sufficient condition (13) is more likely to hold if the price elasticity is low. For the case of a strictly log-supermodular demand, or price elasticity strictly increasing with quality, we know from Proposition 7 that the monopolist undersupplies quality, i.e. $a^m < a^s$ and, $1 < \frac{c(a^s)}{c(a^m)}$. To illustrate that condition (13) may well hold in a familiar setting, we provide an explicit example with closed form solutions.

4.2 An example where the monopoly price is lower than the planner's

Here, we give familiar functional forms to the elasticity of demand and to the marginal cost to prove that the class of models fulfilling (13) includes plausible cases. Consider the specification

$$D(p,a) = p^{-\left(1+\frac{1}{a}\right)} \text{ and } c(a) = a^2,$$

i.e. $g(a) = 1$ and $\varepsilon(a) = -1 - 1/a$. In addition, we assume that the quality level $a \in [a_{\min}, a_{\max}]$ with $1 \leq a_{\min}$ and $2 \leq a_{\max} \leq 3$. The profit function is then

$$\pi(p, a) = (p - a^2)p^{-(1+\frac{1}{a})}.$$

As all prices $p < 1$ are dominated choices (since $\pi(p, a) < 0$), we can focus on prices $p \geq 1$. For all $(p, a) \in [1, +\infty) \times [a_{\min}, a_{\max}]$, Assumptions (A1) and (A2) hold.[22] In addition, demand is strictly log-supermodular, so (6) holds and price and quality are complements for the monopoly firm.

At given quality a, it can be verified that the prices set by the monopolist and the planner are

$$p^m(a) = (1 + a)a^2 \text{ and } p^s(a) = a^2. \tag{14}$$

Hence, we have

$$\overline{\pi}(a) = a^{\frac{a-2}{a}}(1 + a)^{-\frac{(1+a)}{a}} \text{ and } \overline{W}(a) = a^{\frac{a-2}{a}}.$$

Both $\overline{W}$ and $\overline{\pi}$ are convex on $[1, 3]$ and thus have corner maxima. Comparing the corner values, one can find $a_{\min}$ and $a_{\max}$ such that $1 \leq a_{\min} < 1.5$, $2 \leq a_{\max} \leq 3$ and the monopolist chooses the lowest quality, i.e. $\overline{\pi}(a_{\min}) > \overline{\pi}(a_{\max})$. Yet the largest quality level is socially optimal, i.e. $\overline{W}(a_{\max}) > \overline{W}(a_{\min})$, and (13) holds. As illustration, let $a_{\min} = 1$ and $a_{\max} = 3$, we get

$$\begin{cases} a^m = 1 < a^s = 3 \\ p^m = 2 < p^s = 9 \end{cases}.$$

Thus, the firm sets a lower price and quality than the planner, in line with Corollary 8.

5 A microeconomic foundation

In this section, we offer a simple micro-economic foundation for the demand function at hand, when the utility function depends directly on product quality. In so doing, we build on Ebert and von dem Hagen (1998) and the main goal is to derive sufficient conditions on the utility function that yields a direct demand function satisfying the key properties of monotocity in quality and log-supermodularity in price and quality (i.e., condition (5) here).

Consider a representative consumer with income w maximizing a quasi-linear utility function $U(q, a) + y$, where q stands for the quantity of the good under consideration (with price p), and y

[22] One can check that for all $(p, a) \in [1, +\infty) \times [a_{\min}, a_{\max}] \geq 1$, $D_a(p, a) = (\ln p)\, p^{-(1+\frac{1}{a})}/a^2 > 0$, and $D_p(p, a) = -(1 + a)p^{-(\frac{1+2a}{a})}/a < 0$.

is the numeraire good (with price normalized to 1).[23]

The consumer problem is then

$$\max_{q,y\geq 0} U(q,a)+y$$

subject to

$$pq+y=w.$$

We make the usual assumption that

(A3) *U is three times continuously differentiable and satisfies* $U_q>0$, $U_a>0$, and $U_{qq}<0$.

Solving the constraint to get $y=w-pq$, and substituting into the utility, the consumer problem is equivalent to (the unconstrained problem)

$$\max_{q\geq 0} U(q,a)+w-pq.$$

Proposition 9 *Under Assumption (A3) and with sufficiently large income w,*

(i) $D(p,a)$ is decreasing in p.

(ii) $D(p,a)$ is strictly increasing in a if $U_{qa}>0$ for all (q,a).

(iii) $D(p,a)$ is log-supermodular in (p,a) if U satisfies

$$U_{qq}U_{qa}+q(U_{qqq}U_{qa}-U_{qqa}U_{qq})<0 \text{ for all } (q,a)\,. \tag{15}$$

With part (i) of this Proposition being a standard result, we now comment on the other two parts. Part (ii) says that the natual property that higher quality leads to higher demand amounts to having the marginal utility of the good increasing in its quality, i.e, that consumption and quality are (Edgeworth) complements in the utility function.[24]

The meaning of condition (15) directly on the utility function is as follows. It holds if and only if marginal utility U_q satisfies the gross complements condition from standard consumer theory (see e.g., Amir and Bloch, 2009 for an overview) with respect to quantity and quality.[25] We stress

[23] The Law of Demand follows here from the fact that, for a quasi-linear utility function, the income effect vanishes. Therefore, the price effect reduces to the substitution effect, which is negative from standard consumer theory.

[24] This is equivalent to the utility function being supermodular in q and a, or $U_{qa}>0$ (e.g., Amir et al., 2018).

[25] For extra clarity, we recall some standard facts from classical consumer theory in the two-good case.

(i) Two goods x and y are gross complements if the demand for good x decreases with the price of good y.

(ii) A sufficient condition for x and y to be gross complements is that the utility function $V(x,y)$ satisfies the gross complements condition, which is $V_xV_y+x(V_{xx}V_y-V_{xy}V_x)<0$ for all (x,y).

that this condition is more restrictive than in the usual consumer theory context since it is the marginal utility function that must satisfy it, and not the utility function itself. At the same time, the condition actually holds quite broadly. Indeed, among the commonly used utility functions, it is satisfied, among many others, by the quadratic utility, the Cobb-Douglas utility, the entire class of multiplicatively separable utility of the form $U(q,a) = g(a)u(q)$, with $g' > 0$, $u' < 0, u'' < 0$, if u' is log-concave (i.e., if $u'u''' - (u'')^2 \leq 0$). The latter is quite a general class of utility functions. As examples of this class, we have the subclass of CARA utility of the form $U(q,a) = g(a)e^{-q}$, with $g' > 0$, among others. Finally, condition (15) is the translation back onto the utility function of the economically intuitive property that demand elasticity should be increasing in quality.

In conclusion, Parts (i) and (ii) together capture precisely what the meaning of higher quality should be, when expressed in terms of general properties of an underlying utility function. Therefore, these two conditions provide a full justification for the two general assumptions in this paper that are meant to capture product quality.

6 Regulation via taxation or subsidization

In this section, we review some results on optimal regulation via taxation or subsidization, which were obtained by Ebert and Von dem Hagen (1998), and modify them appropriately for our setting. Since the modifications are minor, we shall simply describe the results and leave out the analytical arguments, due to their close proximity to those of Ebert and Von dem Hagen (1998).

In section 4, we pointed out that, when the monopolist sets both price and environmental quality, the only scenario that can be ruled out is where the firm sets a lower price than the social planner for a higher environmental quality. All other scenarios might emerge (lower price for lower quality, larger price for larger quality and larger price for lower quality). Nevertheless, by introducing two policy instruments, one can prove, following Ebert and Von dem Hagen (1998), that the first-best allocation is obtained by always subsidizing consumption while environmental quality is either subsidized or taxed. In our framework, the purpose of the social planner is to maximize social welfare defined by (2). In other words, no additional social benefit of environmental quality is introduced. Here, we follow Ebert and Von dem Hagen (1998) where a damage function of pollution is introduced only when the utility of the representative consumer is separable in consumption and pollution, so that demand for the consumption commodity is independent of pollution. In our framework, the underlying utility function of the representative consumer is nonseparable in consumption and environmental quality, therefore market demand depends on environmental quality and therefore already takes into account the social benefit of environmental quality, as argued earlier. The commodity tax corrects the usual market failure induced by the mark-up above the marginal cost of production: One has to subsidize the monopolist in order to reach the socially optimal level of production. On the other hand, the Pigouvian tax on environmental quality has to take into account two opposite effects of environmental quality. As pointed out by Spence (1975) and recalled above, at a given price level, the monopolist always undersupplies environmental quality with respect to the social optimum, but there is also an indirect effect of environmental quality due to the revision of the price strategy. The conclusion of our set-up is that, to solve the market failure generated by the monopolist's choice of environmental quality, if the marginal impact of quality increments on the marginal cost of production is larger (lower) than the marginal consumer's valuation of quality increments, one has to subsidize (tax) the monopolist. In other words, under this condition (the reverse condition) the second inefficiency induced by the choice of the environmental quality increases (decreases) the usual welfare loss of the monopoly.

In addition, in contrast to the well known result in the literature on environmental regulation under imperfect competition, that the Pigouvian tax should be lower than marginal damage (in the case of a negative externality) or equivalently that the Pigouvian subsidy should be larger than the marginal benefit, here the Pigouvian subsidy on environmental quality is always lower than marginal benefit. This reflects a complementarity between the environmental quality and consumption. The subsidy gives the firm the incentive to increase the environmental quality of its product. Due to the complementarity between quality and consumption, consumers increase their demand. Therefore, the Pigouvian subsidy required to reach the first-best optimum is lower than the social marginal benefit of environmental quality. In Ebert and Von dem Hagen (1998) the utility of the representative consumer depends on the consumption of a commodity and pollution emissions in a nonseparable way. In this case, consumption and emissions could either be complements or substitutes (hence, the Pigouvian tax could either be larger or lower than the marginal damage).[26]

Finally, even if the marginal social benefit of quality increments in the optimal allocation equals zero, two instruments are needed to attain a first-best optimum and environmental quality is taxed.

[26] For similar exercises dealing with the green services industry under imperfect competition, see David and Sinclair-Desgagné (2010), and with a dynamic polluting oligopoly, see Benchekroun and Van Long (1998).

7 Conclusion

This paper has taken a fresh look at Spence's, Spence (1975), pioneering study comparing monopoly pricing and quality choice to a social planner's solution, but in the context of environmental or green quality. Typical examples of markets that fit the present analysis are organic foods and produce, grown or made using various sustainable production techniques and green inputs.

We have considered three separate problems: price choice under fixed quality, quality choice under fixed price, and simultaneous choice of price and quality. For the first two problems, we show that a sufficient condition for price and quality to be positively related is the log-supermodularity of demand, which has the precise economic interpretation that the price elasticity of demand is increasing in product quality, or the dual interpretation interchanging price and quality. In contrast, this conclusion holds for the social planner's analogous pair of problems in full generality. We argue that, although quite general, this condition does not hold universally, and provide an explicit example where the optimal monopoly price is globally decreasing in quality.

For the simultaneous choice of price and quality, we build on Spence's analysis by considering a special class of demand functions whose elasticity depends on quality but not on price. For this class, we show that the log-supermodularity of demand leads to under-provision of quality by the firm, and that the opposite property of log-submodularity of demand leads to the opposite result. We also provide a full characterization of the cases where the monopolist would choose a lower price than the social planner, as well as an explicit example with the latter outcome.

We provide a micro-economic foundation for the underlying quality-dependent demand function via a standard representative consumer's utility maximization. The properties of demand increasing in quality and log-supermodular in price and quality are then reflected in the utility function.

All together then, we conclude that the log-supermodularity of demand is a key general property which provides substantial insight towards an understanding of the various economic forces at work for the monopoly quality-price problem. This property has a natural economic interpretation in terms of quality.

8 Proofs

Here, we provide the proofs of the results that were not provided in the text, as well as an example where the monopoly choices coincide with the social optimum.

8.1 Proof of Proposition 1

From (11), we know that $p^s(a) = c(a)$. At given quality a, the firm chooses $p^m(a)$ such that

$$\pi(p^m(a), a) \geq 0 \Leftrightarrow [p^m(a) - c(a)] D(p^m(a), a) \geq 0.$$

Hence,

$$p^m(a) \geq c(a) = p^s(a).$$

8.2 Proof of Proposition 2

The proof uses Topkis's Theorem for monotone comparative statics (see Topkis, 1998, or Amir, 2005 for an elementary exposition). Taking the log of the profit function for any a and $p \geq c(a)$, we have $\log \pi(p, a) = \log(p - c(a)) + \log D(p, a)$. Hence, using the FOC, we have

$$[\log \pi(p, a)]_{pa} = \frac{c'(a)}{[p - c(a)]^2} + \frac{D(p, a) D_{pa}(p, a) - D_p(p, a) D_a(p, a)}{D^2(p, a)} > 0 \text{ iff (6) holds.}$$

The conclusion that (every selection of) the argmax correspondence is increasing in a then follows from a strengthening of Topkis's Theorem (see Amir, 1996 or Topkis, 1998 p. 79) using a strict form of complementarity, applied to $\max\{\log \pi(p, a) : p \in [c(a), +\infty)\}$, where the feasible set $[c(a), +\infty)$ is ascending in a since $c(a)$ is increasing in a.

If instead (7) holds, then

$$[\pi(p, a)]_{pa} = D_a(p, a) + [p - c(a)] D_{pa}(p, a) - c'(a) D_p(p, a) > 0 \text{ for all } (p, a). \tag{16}$$

The conclusion then follows from Topkis's Theorem (Topkis, 1998) applied to $\max\{\pi(p, a) : p \geq 0\}$.

8.3 Proof of Corollary 3

For $D(p, a) = d(p)s(a)$, we have $\log D(p, a) = \log d(p) + \log s(a)$. Hence $[\log D(p, a)]_{pa} = 0$. It follows that the sign of the LHS of (6) is the same as the sign of $\frac{c'(a)}{[p-c(a)]^2}$.

8.4 Proof of Proposition 7

At given quality a, the optimal price is the (interior) solution of the first-order condition (10)

$$p^m(a) = \frac{\varepsilon(a)}{[\varepsilon(a) + 1]} c(a) \tag{17}$$

since the second order condition $\pi_{pp}\left(\frac{\varepsilon(a)}{\varepsilon(a)+1} c(a), a\right) = [\varepsilon(a) + 1] \left[\frac{\varepsilon(a) c(a)}{\varepsilon(a)+1}\right]^{\varepsilon(a)-1} g(a) < 0$ holds.

In an analogous way, for a given quality level a, the interior price defined by (11) is a solution for the social planner since the second order condition $W_{pp}(c(a), a) = \varepsilon(a) g(a) [c(a)]^{\varepsilon(a)-1} < 0$ holds globally. Hence, the regulator charges as always its marginal cost, i.e.

$$p^s(a) = c(a). \tag{18}$$

Then, one easily checks that

$$\overline{\pi}(a) = \frac{[-\varepsilon(a) - 1]^{-\varepsilon(a)-1} g(a)}{[-\varepsilon(a)]^{\varepsilon(a)} c(a)^{-\varepsilon(a)-1}} \text{ and } \overline{W}(a) = \frac{g(a)}{[-\varepsilon(a) - 1] c(a)^{-\varepsilon(a)-1}}.$$

Combining the two yields

$$\overline{W}(a) = \left(\frac{\varepsilon(a) + 1}{\varepsilon(a)}\right)^{\varepsilon(a)} \overline{\pi}(a).$$

At any $a^m \in (a_{\min}, a_{\max})$ such that $\overline{\pi}'(a^m) = 0$, we have

$$\overline{W}'(a^m) = \varepsilon'(a^m) \overline{\pi}(a^m) \left(\frac{\varepsilon(a^m) + 1}{\varepsilon(a^m)}\right)^{\varepsilon(a^m)} \left(-\log \frac{\varepsilon(a^m)}{\varepsilon(a^m) + 1} - \frac{1}{\varepsilon(a^m) + 1}\right).$$

Since $-\log\left(\frac{\varepsilon}{\varepsilon+1}\right) - \frac{1}{\varepsilon+1} > 0$ for all $\varepsilon < -1$, the sign of $\overline{W}'(a^m)$ is the sign of $\varepsilon'(a^m)$. Since $\overline{W}(a)$ is strictly concave in a (as seen from the second order condition), this argument proves (i)-(iii).

8.5 Proof of Corollary 8

The condition follows directly from (17) and (18).

8.6 Proof of Proposition 9

The consumer's demand $D(p, a)$ fulfills the familiar first order condition

$$U_q(D(p, a), a) = p.$$

Using the implicit function theorem, and differentiating the FOC with respect to p, then a, and then p again, we get, after collecting terms,

$$\begin{cases} U_{qq} D_p = 1 \\ U_{qq} D_a + U_{qa} = 0 \\ (U_{qqq} D_a + U_{qqa}) D_p + U_{qq} D_{pa} = 0 \end{cases}$$

Using these conditions to solve for D_p, D_a and D_{pa}, one easily gets parts (i) and (ii) since

$$D_p = \frac{1}{U_{qq}(q^*, a)} < 0 \text{ and } D_a = -\frac{U_{qa}(q^*, a)}{U_{qq}(q^*, a)} > 0$$

and

$$DD_{pa} - D_p D_a > 0 \Leftrightarrow \frac{1}{U_{qq}^3}\left[qU_{qqq}U_{qa} - qU_{qqa}U_{qq} + U_{qa}U_{qq}\right] > 0.$$

Since $U_{qq} < 0$, we get the desired inequality in part (iii).

8.7 Proof of Proposition 10

The proof is just the mirror image of the proof of Proposition 2.

If (19) holds, we have

$$[\log \pi (p,a)]_{pa} = \frac{c'(a)}{[p - c(a)]^2} + \frac{D(p,a) D_{pa}(p,a) - D_p(p,a) D_a(p,a)}{D^2(p,a)} < 0.$$

From (20), it is easy to see that we now have instead

$$[\pi (p,a)]_{pa} = D_a(p,a) + [p - c(a)] D_{pa}(p,a) - c'(a) D_p(p,a) < 0 \text{ for all } (p,a).$$

The conclusion then follows (for the interior part of the solution, since the feasible set is ascending in a) from the dual result to the strengthening of Topkis's Theorem (Amir, 1996) applied to $\max\{\log \pi (p,a) : p \geq 0\}$ and. $\max\{\pi (p,a) : p \geq 0\}$ respectively.

8.8 Can price be globally decreasing in product quality?

Here we now explore whether there is any theoretical support for the highly counter-intuitive possibility of price globally decreasing in product quality. It turns out that sufficient conditions can be given for this to hold globally, but these are strongly restrictive, unlike those that validate the opposite conclusion of price increasing in product quality. We also provide an explicit example of this counter-intuitive fact.

Proposition 10 *Assume that the demand function satisfies either one of the two conditions*

$$c'(a) D_p^2(p,a) + D(p,a) D_{pa}(p,a) - D_p(p,a) D_a(p,a) < 0 \textit{ for all } a \textit{ and } p \geq c(a), \tag{19}$$

or

$$D_a(p,a) + [p - c(a)] D_{pa}(p,a) - c'(a) D_p(p,a) < 0 \textit{ for all } a \textit{ and } p \geq c(a). \tag{20}$$

Then the price response correspondence, $p^m(a)$, of the firm is globally decreasing in quality a.

The fact that a monopoly firm might set a high price for a low-quality product (or a brown good) and a lower price for a higher quality product (or a green good) is certainly provocative and very counter-intuitive from the standpoint of consumers. Indeed one often takes for granted that higher-quality products will be more expensive. This issue formed a key motivation behind an important theory of marketing for new experience goods, wherein high price and advertizing are perceived as credible signals of (otherwise unknown) product quality for consumers (Milgrom and Roberts, 1986). Interestingly, the strand of literature associated with the latter study does not take it for granted a priori that higher quality would always lead to higher price.

It might be surprising for economists as well for two different reasons. The first is that higher quality is posited to be more costly for the firm to produce, which is clearly a very natural assumption in most settings, including the environmental case. The second reason is that most consumers exhibit a higher willingness to pay for greener products (as argued in the Introduction, see also OECD, 2002), and a monopolist should be expected to therefore price green goods higher. Yet, the underlying clarifying intuition is that (19) implies that the price elasticity is (strongly) decreasing in quality (since the first term of (19) is positive), despite the associated upward shift in demand.

In addition, as should be expected, the above results are theoretically consistent with this possibility being relatively difficult to observe in practice, in the sense that (20) is an extremely restrictive condition on demand. It requires $D_{pa}(p, a)$ to be strongly negative, since the first and third terms in both (19) and (20) are always positive. In terms of economic interpretation, (19) requires that the price elasticity be strongly decreasing in quality, which is highly counter-intuitive.

It follows that finding actual explicit examples where the optimal price is globally decreasing in quality cannot be expected to be easy, as most of the well known demand functions have the property that the price elasticity is strictly increasing in quality (see our earlier examples and Vives, 1999), i.e., satisfy (5). It is easier to find examples where price decreases in quality locally rather than globally. Nevertheless, using a familiar class of demand functions, we provide a simple and plausible example where this property holds in a global sense.

Example 3.1 :

Consider a monopolist having demand and unit cost functions given by

$$D(p, a) = \frac{1}{(p - a + 1)^2}, p \geq 0, a \in [0, 1], \text{ and } c(a) = \frac{a}{4}.$$

Clearly, this hyperbolic demand satisfies (A1)-(A2), but not (6), (7), (19) and (20). The profit function is

$$\pi(p, a) = (p - \frac{a}{4})\frac{1}{(p - a + 1)^2}$$

The firm's first order condition in price, $\pi_p(p,a)=0$, reduces to $(1-p-a/2)/(p-a+1)^3=0$ or

$$p^m(a)=1-\frac{a}{2}. \tag{21}$$

In addition the second order condition holds since $\pi_{pp}\left(1-\frac{a}{2},a\right)=-(2-\frac{3}{2}a)^3<0$ for all $a\in[0,1]$.

The monopoly price is *globally* decreasing in quality.[27] The highest optimal price will be charged for the lowest possible quality that the firm can produce!

As a preview of the situation where price and quality are both chosen, we show that, in this example, the monopolist chooses the highest quality, which is also socially optimal, and charges a price larger than the social planner's. Indeed, denote by $\overline{\pi}(a)=\pi\left(1-\frac{a}{2},a\right)$. One can check that $\overline{\pi}(a)=\frac{1-3a/4}{(2-3a/2)^2}$ and $\overline{\pi}'(a)=\frac{3(1-3a/4)}{2(2-3a/2)^3}>0$ for all $a\in[0,1]$. Hence, $a^m=1$ and $p^m=1/2$.

The social welfare function is

$$W(p,a)=\frac{1}{p-a+1}+(p-\frac{a}{4})\frac{1}{(p-a+1)^2}.$$

Solving the first order conditions yields $a^s=1$ and $p^s=1/4$. Thus the planner's optimal price is lower than the monopolist's, as expected since the corresponding optimal qualities are the same.

In conclusion, although price and quantity are always positively related when chosen by a social planner, they need not be for a monopolist. There are robust and familiar circumstances under which the latter would price in a seemingly counter-intuitive way. Nevertheless, we stress that the restrictiveness of the underlying sufficient conditions suggests that this is not likely to be an empirically relevant (or widespread) phenomenon. On the other hand, even though this unusual property tends to fail globally, it may hold locally and possibly at relevant quality levels.

8.9 An example where the monopolist's quality is socially optimal

In this last part, we identify one circumstance for market demand not considered by Spence (1975) under which quality is set at the socially optimal level by the monopolist. This is a specific case of the model where the price elasticity of demand is independent of price. Assume that the minimal quality level is non-zero, i.e. $a\in[a_{\min},a_{\max}]$ with $a_{\min}>0$. More precisely, let demand and cost be

$$D(p,a)=p^{-\left(1+\frac{1}{a}\right)} \text{ and } c(a)=e^a.$$

The demand elasticity is independent of price and strictly increasing in quality. Hence, by Proposition 3 in Spence (1975), the monopolist should choose a strictly smaller quality than the social

[27] Note that this result is obtained despite this example does not fulfill the condition (20).

planner. However, this result relies on the assumption (not mentioned) that the quality level chosen by the monopolist is interior. Otherwise, if he chooses the largest feasible quality level, so will the social planner, as we now see. It can be verified that the monopoly and planner's prices are

$$p^m\left(a\right) = (1+a)e^a \text{ and } p^s\left(a\right) = e^a. \tag{22}$$

This yields

$$\overline{\pi}\left(a\right) = \frac{a}{e}e^{-\frac{1+a}{a}\ln(1+a)} \quad \text{and} \quad \overline{W}\left(a\right) = \frac{a}{e}.$$

Both these two functions are strictly increasing over $[a_{\min}, a_{\max}]$[28]. Hence,

$$\left\{\begin{array}{c} a^m = a^s = a_{\max} \\ p^s = e^{a_{\max}} < (1+a_{\max})e^{a_{\max}} = p^m \end{array}\right. .$$

[28] One can check that $\overline{\pi}'\left(a\right) = \frac{1}{a}e^{-1}\frac{\ln(a+1)}{(a+1)^{\frac{1}{a}(a+1)}} > 0$ and $\overline{W}'\left(a\right) = \frac{1}{e} > 0$ for all $a \in [a_{\min}, a_{\max}]$.

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