« Memory that Drives! New Insights into Forecasting Performance of Stock Prices from SEMIFARMA-AEGAS Model »

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Memory that Drives!
New Insights into Forecasting Performance of Stock Prices from SEMIFARMA-AEGAS Model

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Abstract: *Stock price forecasting, a popular growth-enhancing exercise for investors, is inherently complex – thanks to the interplay of financial economic drivers which determine both the magnitude of memory and the extent of non-linearity within a system. In this paper, we accommodate both features within a single estimation framework to forecast stock prices and identify the nature of market efficiency commensurate with the proposed model. We combine a class of semiparametric autoregressive fractionally integrated moving average (SEMIFARMA) model with asymmetric exponential generalized autoregressive score (AEGAS) errors to design a SEMIRFARMA-AEGAS framework based on which predictive performance of this model is tested against competing methods. Our conditional variance includes leverage effects, jumps and fat tail-skewness distribution, each of which affects magnitude of memory in a stock price system. A true forecast function is built and new insights into stock price forecasting are presented. We estimate several models using the Skewed Student-t maximum likelihood and find that the informational shocks have permanent effects on returns and the SEMIFARMA-AEGAS is appropriate for capturing volatility clustering for both negative (long Value-at-Risk) and positive returns (short Value-at-Risk). We show that this model has better predictive performance over competing models for both long and/or some short time horizons. The predictions from SEMIRFARMA-AEGAS model beats comfortably the random walk model. Our results have implications for market-efficiency: the weak efficiency assumption of financial markets stands violated for all stock price returns studied over a long period.*

Keywords: Stock price forecasting; SEMIFARMA model; AEGAS model; Skewed Student-t maximum likelihood; Asymmetry; Jumps.

JEL Classification: C14, C58, C22, G17

"*If you torture the data long enough, it will confess.*"

- **Ronald H. Coase**, Essays on Economics and Economists

1. Motivation and contextualization

1.1 Motivation

Stock price movements invariably reflects the impacts of (co-) movements of myriads of factors viz., social, economic, political, and environmental (such as changes in weather). Given that investors are predominantly psychology-driven decision-making agents, their decisions are often derived from the realm of incomplete information and bounded rationality. This is one of the many reasons, why despite several seminal contributions to the determinants of stock prices, the tendency to include newer factors are increasing every day.[1] A time series econometrician faces then an upheaval task: to study the series over a stretch of time, identify a reality-approximating pattern by using state-of-the-art method and produce a nice predictive performance of the model. Just as Ronald Coase pointed out (in the above quote), 'if you torture the data long enough, it will confess'.

Indeed, over the past three and a half decades since Engle (1982) and Bollerslev (1986), financial economists have moved along the non-stationary econometric trajectories and have offered numerous powerful competing forecasting models to uncover the real nature of stock price movements. Unfortunately, stock price is one such financial metric which is not driven by a single event's momentum only (such as only political uncertainty or economic prosperity/recession, etc.). Rather, its ever-changing complex core that attracts anything 'psychological bound' of investors, means that there will no single econometric approach that can unravel the real dynamic nature of stock price movements. However, amidst all these dynamisms, the best way to understand its movements is to model its variance as conditional -allowing in part, to be determined by past variances and in part, by other factors (the class of Fama-French models, for instance). In other words, in general stock prices can reveal some strength in 'memory' – an ability of the system to remember past shocks. Finally, all the various known and unknown factors determining investors' sentiment also form a complex non-linear relationship. A preferable approach to produce realistic predictions would then be to combine both 'memory' and 'non-linearity' within a single modelling framework. This paper builds such a framework and aims to provide new insights into stock price movements.

At its core, our approach lays emphasis on modelling 'jumps' in stock prices along with a possible path dependence. By assuming a jump process for stock prices, we allow random movements of prices at all scales, no matter how small. Such a model often combines the usual geometric Brownian motion for the diffusion and a space-time Poisson process for the jumps such that jump amplitudes are uniformly distributed. Arguably, stock prices exhibit extreme sensitivity to news, in addition to of course, the structural changes in financial and economic dynamics. Such a sensitivity can be regarded as response to 'jumps' the source of which can be both endogenous and exogenous. Irrespective of the sources, a jump in stock prices often reflect the path dependence nature of the series: to what extent a strong/weak memory of the system predicts its future movements. Hence, in the current paper, we

[1] Some recent research investigates whether win or loss in a big match (such as football or rugby) leads to a rise/fall in stock prices the next day (Urquhart & Sakkas (2018)).

introduce a long-memory based conditional volatility model with asymmetry (and non-linearity). Our approach (to be discussed in Section 2 in details) exploits learning mechanism of the stock price system with 'memory' and embeds asymmetric nature of shocks on the conditional volatility of stock prices.

1.2 Identifying the missing link

The classical variants of Generalized Autoregressive Conditional Volatility (GARCH) model are extensively employed in the empirical architecture of stock price volatility modelling (viz., symmetric GARCH: Engle (1982) and Bollerslev (1986), asymmetric GARCH, such as exponential GARCH (EGARCH):Nelson (1990) and Threshold GARCH (TGARCH): Glosten, Jagannathan& Runkle (1993) and Zakoian (1994); the asymmetric power GARCH (APGARCH): Ding, Granger & Engle (1993); STGARCH model with regime switching: Hagerud (1997a), Gonzalez-Rivera (1998)). For details on the evolution of different GARCH-type models, see Bollerslev (2009) and Zhang & Wei (2010). These GARCH variants are based on properties of symmetry, asymmetry, nonlinearity, stationarity, persistence and structural breaks, but recent innovations have shown that *jumps* as another fundamental property in volatility (see for example, Harvey (2013), Yaya1, Bada & Atoi (2016), Charles & Darné (2017) and Babatunde, Yaya, & Akinlana (2019)).

The GARCH models are not robust enough to capture these large changes in financial time series, and therefore, they underestimate the magnitude effect of the returns. Andersen, Bollerslev & Dobrev (2007) originally propose non-parametric approaches based on Brownian Semi-Martingale for detecting jumps, but these methods cannot predict stock market volatility. Due to the presence of occasional jumps, Harvey & Chakravarty (2008) and Harvey (2013) propose Generalized Autoregressive Score (GAS) models - a class of observation driven time series models, where the time-varying parameters are functions of lagged dependent values and past observations. The parameters are stochastic and predictable given the past. These models capture these occasional jumps in financial time series with symmetric and asymmetric variants, using the score of the conditional density function to drive the dynamics of the time-varying parameter (see real, Koopman & Lucas (2013) and Creal, Schwaab, Koopman & Lucas (2014)).The distribution of innovations in GAS models are non-normal and the conditional variance is taken from the conditional score of the distribution with respect to the second moment.

Stock prices often exhibit complex dynamic properties and needs to be adequately flexible to describe its important characteristics. The GAS models have proved to be more robust in modelling and predicting fat tail and skewed data (Yaya, Bada &Atoi (2016), Opschoor, Janus, Lucas & Van Dick (2018) and Makatjane, Xaba & Moroke (2017)). To take into account conditional asymmetry, leverage effect and heavy tails, Laurent, Lecourt & Palm (2016)propose AEGAS model (also called Beta-Skew-t-AEGARCH), as an extension to GAS (Creal et al (2013)) by introducing time-varying parameters in the class of non-linear models with its exponential specification. This new class of volatility model is robust to outliers and occasional jumps by using the Skewed Student-t distribution to account for the occurrence of large changes in volatility. In the AEGAS model, the mechanism to update the parameters over time is provided by the scaled score of the likelihood function (Tafakori, Pourkhanali & Fard (2018)).

For the applications of the GAS models to economic and financial time series see, for example, Creal, Koopman & Lucas (2013) who present two examples to illustrate their GAS modeling framework based on square root information matrix scaling with Moody's credit rating data. Thus, Huang, Wang & Zhang (2014) compare the Realized ARCH and Realized GAS model under Gaussian and t distribution assumptions for the financial return and daily realized variance. Muela (2015) compare the performance of several Beta-skewed-t-EGARCH specifications in terms of Value at Risk on eight closing daily returns. On the other hand, Koopman, Lucas, & Scharth (2016) study the forecasting performance of nonlinear non-Gaussian state-space models, generalized autoregressive score models and autoregressive conditional moment models for predicting the volatility of twenty Dow Jones index stocks and five major stock indices over a period of several years. Olubusoye & Yaya (2016) investigate persistence and volatility pattern in the prices of crude oil and other distilled petroleum products for the US and the UK petroleum pricing markets. Whereas, Yaya, Bada&Atoi (2016) estimate volatility in the Nigerian Stock Market using the Beta-Skew-t-AEGARCH model and compare its forecasting performance over some other volatility models. Salisu (2016) employs the Beta-Skew-t-EGARCH framework proposed to model oil price volatility. Makatjane, Xaba, Moroke (2017) empirically investigate the behavior of the time-varying parameter by estimating the GAS model to the South Africa Sanlam stock price returns. Müller & Bayer (2017) propose a likelihood ratio test to select the Beta-Skew-t-EGARCH model with one or two volatility components and give an empirical illustration devoted to the DAX log-returns. Charles &Darné (2017) analyze volatility models in the presence of jumps in two crude-oil markets and evaluate the forecasting performance of the volatility models using the model confidence set approach, Finally, Tafakori, Pourkhanali & Fard (2018) evaluate the accuracy of several 100 one-day-ahead value at risk (VaR) forecasts for predicting Australian electricity returns using asymmetric exponential generalized autoregressive score (AEGAS) models.

The class of score-driven models have also recently become popular for analyzing financial time series, but these last works ignore the existence of dynamic behavior, especially long memory, in the conditional mean. Several studies find that the short-memory is contaminated by deterministic trend. Beran& Feng (2002a,b) incorporate a nonparametric deterministic trend on the ARFIMA model (SEMIFAR).Then,Feng, Yu & Beran (2007)combine the SEMIFAR model with GARCH errors to allow for time-varying conditional variance. Chikhi, Péguin & Terraza (2013) propose the SEMIFARMA model with HYGARCH errors to analyze the persistence of informational shocks in Dow Jones returns. Combining semiparametric long memory (SEMIFARMA) models with asymmetric exponential GAS (AEGAS) errors would provide a flexible class of model to capture the long memory structure in the conditional mean and the occasional jumps in the score-driven volatility, which includes leverage effect and fat tail-skewness distribution. Harvey (2013), among others, specify the GAS models with the heavily tailed and skewed conditional probability distribution. These models perform better than the classical GARCH models with larger values of log-likelihoods. Blasques, Koopman & Lucas (2014a) and Lambert & Laurent (2000, 2001) suggest using the maximum likelihood based on the skewed Student-*t* density proposed by Fernandez & Steel (1998) to estimate this GAS family of models.

As noted earlier, an imposing characteristic of a conditional volatility model is its memory characteristics. If and when the system reveals certain patterns (such as herding), this means that some time series observations within the price data depict certain degree of associations (in our case, it can be certain magnitude of dependence between past and present). Led by this, the main objective of this paper is to propose a mixture of long memory structure with nonparametric deterministic trend and the occasional jumps, leverage effect and fat tail-skewness distribution in the daily stock price returns. Our approach – the SEMIFARMA-AEGAS model – is employed to three stock markets data, such asArgentina, Saudi Arabia and France. Our strategy thus, is to combine and estimate the SEMIFARMA model with asymmetric exponential GAS (AEGAS or Beta-Skew-t-AEGARCH) errors using Skewed Student-t maximum likelihood.

The remainder of this article is organized as follows: Section 2 focuses on presentation of the SEMIFARMA-AEGAS model used throughout our study. Section3 outlines the daily price data of Argentinean, Saudi and French stock markets and discusses their statistical properties. Our estimation results are shown in section 4. In section 5, we evaluate the forecasting performance of best fitting GAS Models in stock markets, including the semiparametric long memory in the conditional mean equation. We thus try to compare the predictive quality of SEMIFARMA-GAS, SEMIFARMA-EGAS and SEMIFARMA-AEGAS models with that of a random walk. The last section concludes the paper.

2. The SEMIFARMA model with AEGAS errors

We specify a semiparametric fractionally autoregressive moving average (SEMIFARMA)model (Beran & Feng (2002a) and Chikhi, Péguin & Terraza (2013)) with asymmetric exponential generalized autoregressive score (AEGAS(1,1)) errors, also called Beta-Skew-t-AEGARCH (Creal, Koopman & Lucas (2013), Harvey (2013, 2014), Harvey & Sucarrat (2013)and Laurent, Lecourt & Palm (2016)) defined as follows

$$\phi(B)(1-B)^{d_2}\left\{(1-B)^{d_1}Y_t-g(x_t)\right\}=\theta(B)\varepsilon_t \qquad (1)$$

with $\varepsilon_t = z_t\sigma_t$, $\sigma_t > 0$ (2)

and $\log\sigma_t^2 = \omega+\alpha_1 u_{t-1}+\gamma_1 I_{t-1}+\varphi_1\log\sigma_{t-1}^2$ (3)

with $(1-B)^{d_2}=\sum_{k=0}^{\infty}(-1)^k\dfrac{\Gamma(d_2+1)}{\Gamma(k+1)\Gamma(d_2-k+1)}B^k = 1-\sum_{k=1}^{\infty}c_k(d_2)B^k$; $c_1(d_2)=d_2$, $c_2(d_2)=\frac{1}{2}d_2(1-d_2)$ and $\Gamma(.)$ is the gamma function. Furthermore, the roots of polynomials $\phi(B)=1-\phi_1B-\phi_2B^2-....-\phi_pB^p$ and $\theta(B)=1-\theta_1B-\theta_2B^2-....-\theta_qB^q$ in the lag operator with degrees p and q respectively, are outside the unit circle. B is the lag operator, d_1 is an integer: $d_1\in\{0,1\}$, $x_t=t$ is the trend and g : $[0,1]\to R$ is the smoothing function, which represents a nonlinear deterministic trend. The process is stationary and invertible, $-\frac{1}{2}<d_2<\frac{1}{2}$. The long memory is included in the mean equation (1) through the parameter d_2.

The estimation of nonparametric deterministic trend function $g(x_t)$ is based on the kernel method using the following model (see Hall &Hart (1990), Ray & Tsay (1997) and Beran (1999)):

$$(1-B)^{d_1}Y_t = g(x_t)+X_t \qquad (4)$$

where X_t is a long memory stationary errors if $d_2>0$ and short memory errors if $d_2=0$. We consider the polynomial kernel defined by:

$$K(x)=\sum_{l=0}^{\tau}\alpha_l x^{2l} \text{ with} |x|\le 1 \qquad (5)$$

and $K(x)=0$ if $|x|>1$. Here we have $\tau \in \{0, 1, 2, ...\}$ and the α_t coefficients are such that $\int_{-1}^{1} K(x)dx = 1$ (see Beran & Feng (2002a, 2002b) for details on the estimation method).

The interest of SEMIFARMA models for financial series lies mainly in their ability to capture the long memory dynamics and the nonlinear deterministic trend in the conditional mean. For the Skewed Student-t distribution in the conditional variance (see Hansen (1994), Lambert & Laurent (2000, 2001) and Theodossiou (2002))

$$I_{t-1} = \text{sgn}\left(-z_t^*\right)\left(u_t + 1\right) \qquad (6)$$

with $$E\left(I_t\right) = \frac{1-\xi^2}{1+\xi^2} \qquad (7)$$

where $$u_t = \frac{(\nu+1)z_t z_t^*}{(\nu-2)g_t \xi^{I_t}} - 1 \quad \text{if} \quad z_t \sim SKST\left(0,1,\xi,\nu\right) \qquad (8)$$

I_t is an indicator measure asymmetry defined as

$$I_t = \text{sgn}\left(z_t^*\right) = I\left(z_t^* \geq 0\right) - I\left(z_t^* < 0\right) \qquad (9)$$

where $$z_t^* = sz_t + m \qquad (10)$$

and $$g_t = 1 + \frac{z_t^{*2}}{(\nu-2)\xi^{2I_t}} \qquad (11)$$

$$m = \frac{\Gamma\left(\frac{\nu-1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \qquad (12)$$

$$s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2} \qquad (13)$$

where ξ is the asymmetry parameter and ν is the degree of freedom of the distribution.

For a GAS(1,1) model, the equation for time-varying parameters $\psi_t = \sigma_t^2$ is the autoregressive function $\psi_t = \omega + B_1\psi_{t-1} + A_1\kappa_{t-1}$. Harvey & Chakravarty (2008) and Creal, Koopman & Lucas (2013) propose to update the time-varying parameters with $\kappa_t = S_t \nabla_t$ where ∇_t is the score with respect to the parameter ψ_t with $\nabla_t = \partial \log f\left(Y_t \mid \psi_t, \psi_{t-1}, Y_{t-1}, X_t; \theta\right) / \partial \psi_t$ and S_t is a time-dependent scaling matrix. The Normal-GARCH model corresponds to a Normal-GAS(1,1) (i.e. $z_t \sim N(0,1)$ with $S_t = 2$, $A_1 = \alpha_1$, $B_1 = \alpha_1 + \beta_1$, $\psi_t = \sigma_t^2$ and $\nabla_t = 0.5\left(z_t^2 - 1\right)\sigma_t^2$). Note that $u_t = z_{t-1}^2 - 1$ is proportional to the score of the conditional distribution of ε_t with respect to σ_{t-1}^2 .For the choice of time dependent scaling, Creal, Koopman & Lucas (2013) recommend using $S_t = 1$ or $S_t = \left(E_{t-1}\nabla_t \nabla_t'\right)^{-1}$ while Harvey & Chakravarty (2008) set $S_t = 2$.

Some authors suggest using the Skewed Student-t innovation distribution. Lambert & Laurent (2000, 2001) apply and extend the skewed-Student density proposed by Fernandez & Steel (1998) to the GARCH framework. The procedure for Maximum Likelihood Estimation (MLE) of GAS family models was presented in Blasques, Koopman & Lucas (2014a). The strong consistency and asymptotic normality of maximum likelihood are also studied. Consequently, we propose the Skewed Student-t maximum likelihood to estimate a SEMIFARMA model jointly with AEGAS (Beta-t-EGARCH) error from the Skewed Student-t distribution using the BFGS algorithm (Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970)) implemented by Laurent (2013). The Skewed Student-t log-likelihood function is written as

$$L = T\left\{\log\Gamma\left(\frac{\nu+1}{2}\right)-\log\Gamma\left(\frac{\nu}{2}\right)-0.5\log\left[\pi(\nu-2)\right]+\log\left(\frac{2}{\xi+\frac{1}{\xi}}\right)+\log(s)\right\}$$
$$-0.5\sum_{t=1}^{T}\left\{\log\sigma_t^2+(1+\nu)\log\left[1+\frac{(sz_t+m)^2}{\nu-2}\xi^{-2I_t}\right]\right\} \qquad (14)$$

where

$$I_t=\begin{cases}1 \text{ if } z_t\geq-\frac{m}{s}\\ -1 \text{ if } z_t<-\frac{m}{s}\end{cases} \qquad (15)$$

And optimized with respect to the unknown dynamic parameters, contained in the vector $\psi^{'}=\left(d_2,\phi_1,...,\phi_p,\theta_1,...,\theta_q,\omega,\alpha_1,\gamma_1,\varphi_1\right)$ and ν in the first order model(For details on skewed student density see Lambert & Laurent (2001)and For the GAS estimation see Blasques, Koopman & Lucas (2014a, 2014b, 2014c)).

3. Data characteristics

For our empirical exercise, we consider stock market indices of three selected markets. The stock indices considered are TASI (Saudi Arabia), Merval (Argentina), and CAC SMALL (France). The data sample of Saudi Arabia is from January, 2000 to October, 2018 ($T=5000$); the data sample of the Argentina is from May, 2002 to January, 2019 ($T=4093$)and the data of France cover a historical period from January, 1999 to July, 2018 ($T=4999$). Daily stock prices of Merval are collected from Yahoo finance https://fr.finance.yahoo.com and the data series of TASI and CAC SMALL are drawn from https://www.investing.com.Unit root tests results (Philips & Perron (1988), Kwiatkowski, Phillips and Elliott, Rothenberg & Stock (1996)) show that all the logarithmic series are characterized by a unit root (see Table 1). The logarithmic series are finally differentiated to obtain the daily percentage returns at time t (see Figure 1)

$$r_t=100\times\left(\ln P_t-\ln P_{t-1}\right)$$

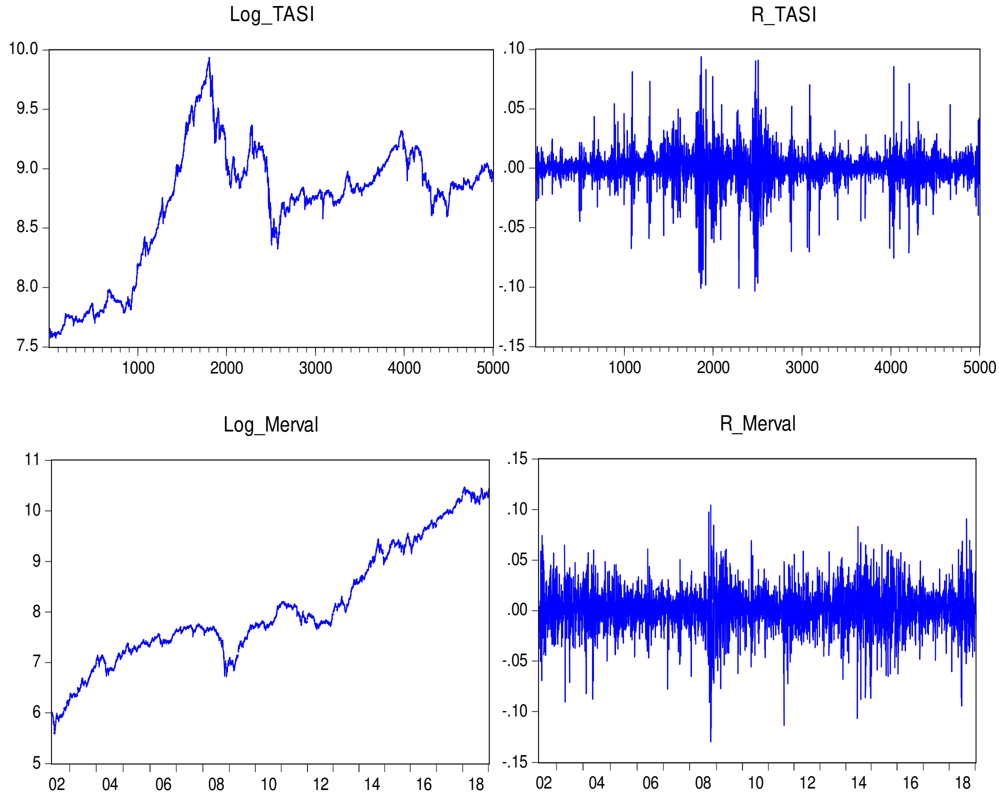
where P_t and P_{t-1} are daily stock price at two successive days t and $t-1$, respectively.

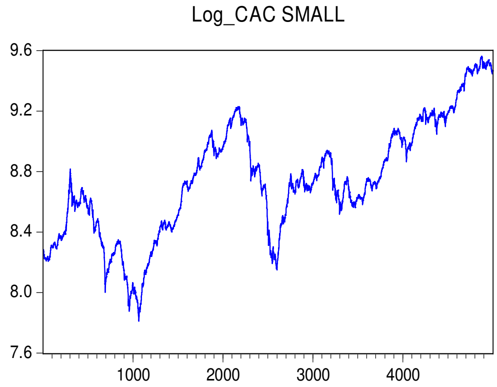
Table 1 – Identifying non-stationarity in the series

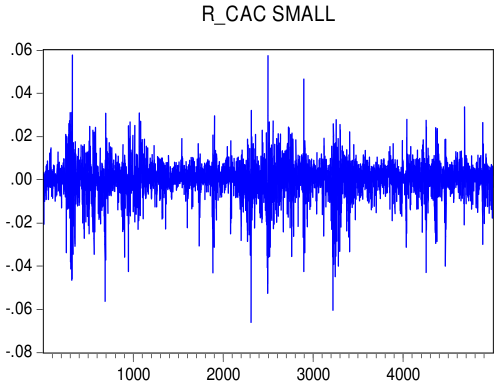
Series	Test	Logarithmic		Returns	
		Test stat.	Critical value	Test stat.	Critical value
TASI	PP	-1.959	-2.861	-65.532	-1.94
	KPSS	0.349	0.463	0.297	0.463
	ERS	0.010	3.26	70.105	3.26
Merval	PP	3.177	-1.94	-60.989	-2.862
	KPSS	1.762	0.463	0.102	0.463
	ERS	0.014	3.26	220.245	3.26
CAC SMALL	PP	1.371	-1.94	-55.891	-1.94
	KPSS	1.720	0.463	0.113	0.463
	ERS	0.011	3.26	40.361	3.26

***Note:** The asymptotic critical value at 5%are computed using Mackinnon's (1990) method. The table reports the results of Philips-Perron unit root test. We accept the unit root hypothesis H_0 for daily logarithmic series and reject it for daily returns. For Philips-Perron, Elliott-Rotenberg-Stock (ERS) and KPSS tests, the spectral estimation is based on the Bartlett kernel using the Andrews bandwidth. For KPSS test, H_0 is the null hypothesis of stationarity.*

Figure 1 –Evolutions of stock market indices and returns







Interesting observations emerge: from Figure 1, we note some sharp jumps and volatility clustering in the stock returns. As shown in Table 2, Argentina and Saudi Arabia show the highest risk, as measured by the standard deviation (2.023% and1.416% respectively) followed by France (0.867%). All series exhibit negative skewness. The observed asymmetry may indicate the presence of nonlinearities in the evolution process of all returns. In addition, all series also show excess kurtosis: the Jarque-Bera test (Jarque&Bera (1987) strongly rejects the null hypothesis of normality. On the other hand, there is an ARCH effect in the data since the ARCH-LM statistic is greater than the critical value of chi-square distribution with 1 degree of freedom at 1%. for all series.

Table 2 – Summary statistics for daily stock market returns

Countries	Std. Dev (%)	Skewness	Kurtosis	JB stat.	ARCH(1)
Saudi Arabia	1.416	-0.88	13.447	23382.58***	327.632***
Argentina	2.023	-0.376	6.302	1956.21***	147.009***
France	0.867	-0.948	9.170	8677.555***	415.59***

Note: **** indicates a rejection of null hypothesis of normality and homoscedasticity at the 1% level.*

In Table 3, we present the BDSBDS (Brock et al. (1996)) statistics to gauge whether stock returns are non-linear in nature. As evident, the BDS statistics are strictly greater than the critical value at 5% for all the embedding dimensions and thus all stock returns are non-linearly dependent. Moreover, the variance ratio statistic (Lo & Mac Kinlay (1988)) is significant for all stock returns as well: the critical probabilities are less than 0.05 for joint test Max |z| (at period 2) and the runs statistic is greater than the critical value of normal distribution at 5% (see Table 4). Consequently, we reject the random walk hypothesis, indicating that stock market price can be predicted in the short term. The weak efficiency assumption of financial markets seems violated for all series.

Table 3 – BDS test results on the stock market returns

m	Saudi Arabia		Argentina		France	
	BDS stat.	Prob.	BDS stat.	Prob.	BDS stat.	Prob.
2	24.1248	0.0000	11.8887	0.0000	23.4456	0.0000
3	30.6071	0.0000	16.1949	0.0000	29.8697	0.0000
4	34.9670	0.0000	18.7195	0.0000	33.5874	0.0000
5	39.0515	0.0000	20.8757	0.0000	37.0353	0.0000
6	43.4160	0.0000	22.7408	0.0000	40.4203	0.0000
7	48.3278	0.0000	24.4674	0.0000	43.9935	0.0000
8	53.8998	0.0000	26.3077	0.0000	48.0178	0.0000
9	60.6845	0.0000	28.1552	0.0000	52.6945	0.0000
10	68.4637	0.0000	30.3447	0.0000	57.9909	0.0000

Note: *The BDS statistics are calculated by the fraction of pairs method with* ε *equal to 0.7. m represents the embedding dimension.*

Table 4 – Variance Ratio Estimates and Test Statistics of Random Walk Hypothesis for stock market returns

Countries	Variance Ratio test		Runs test	
	Value	*p*-Value	Runs stat.	*p*-Value
Saudi Arabia	12.6863	0.0000	-6.0105	0.0000
Argentina	16.2340	0.0000	-1.9789	0.0478
France	14.700	0.000	-10.9891	0.0000

Note: *p-Value of variance ratio statistic represents a probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom*

These previous tests highlighted the presence of significant non-zero autocorrelations in the short term and lead us to reject the *i.i.d* hypothesis. However, it is impossible to exploit these autocorrelations to establish speculative rules leading to abnormal profits. Given this situation, we test the presence of autocorrelations by considering longer horizons. By plotting the periodogram of this series (see Figure 2) (with Parzen window), we note that this is a sign of long-memory since the spectral density is concentrated around low frequencies and tends to infinity when the frequency tends to zero.

Figure 2 –Periodograms of daily returns

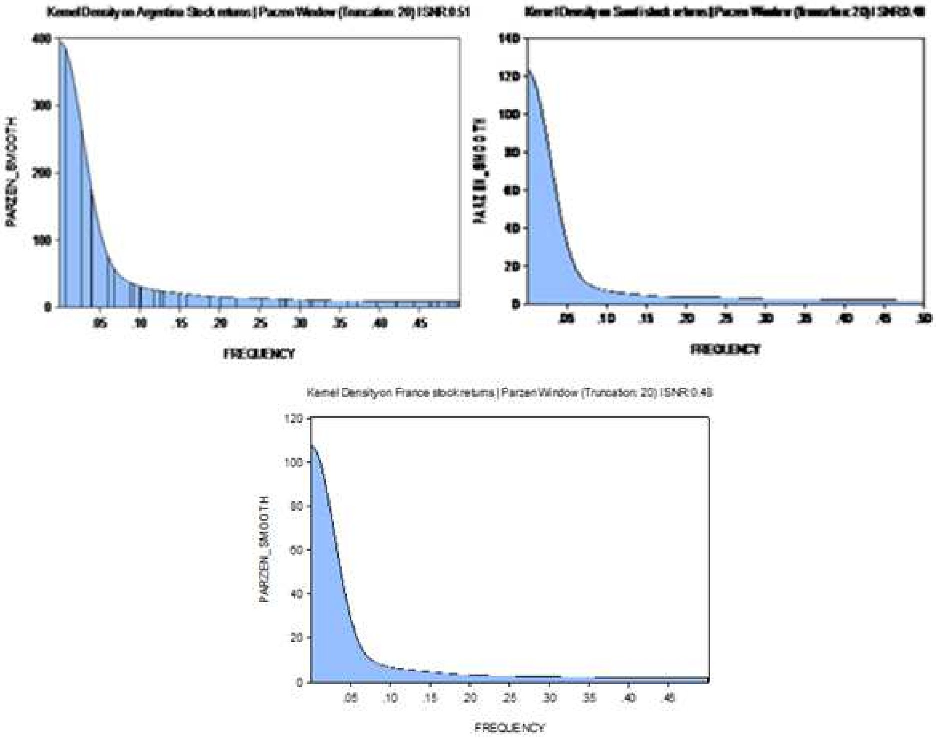


Table 5 – Results from the ARFIMA(0,d,0) estimation ondaily returns

Countries	GPH			RH		
	d	t-stat.	Prob.	*D*	t-stat.	Prob.
Saudi Arabia	0.0427	3.1925	0.0014	0.0291	2.9098	0.0036
Argentina	0.0494	3.1294	0.0018	0.0346	3.1335	0.0017
France	0.1634	12.2010	0.0000	0.1495	14.947	0.0000

Note: *RH: Robinson-Henry. GPH: Geweke-Porter-Hudak. d is the estimated Long memory parameter with a power of 0.8.*

From Table 5, it is clear that all the daily series of stock returns are generated by a long memory process. The values of Student statistic (with a power of 0.8) are strictly greater than the critical value of normal distribution at 5%. In addition, the memory parameter estimated by a Gaussian semiparametric method (Robinson & Henry (1999)) is positive and significant. The estimation results are confirmed by the GPH (Geweke & Porter-Hudak (1983)) method. The presence of a long memory indicates that agents can anticipate their returns to a sufficiently long-time horizon and the return will not revert to its fundamental value.

4. Main results

In this section, we present and discuss the results from SEMIFARMA model jointly with GAS, exponential GAS and asymmetric exponential GAS errors. The estimation procedure is semiparametric in nature. In the *first step*, we incorporate an initial estimate of the nonlinear deterministic trend function, the optimal window and the cross-validation criteria (in equation (4)) by the nonparametric kernel methodology to produce a long memory stationary residuals. In the *second step,* we use the residuals to estimate the conditional mean and the conditional variance parameters. This estimation procedure is based on the Skewed Student-t maximum likelihood using the BFGS algorithm. As there are some sharp jumps in the volatility, it will be interesting to take into account this asymmetry in volatility estimation (see Salisu (2012), Yaya (2013) and Yaya & Gil-Alana (2014)).For the time dependent scaling S_t, we use the choice of Harvey & Chakravarty (2008) by setting $S_t = 2$ .To facilitate inference about the null hypothesis of symmetry, we estimate $\log(\xi)$.

We estimate several models with different lags, such as a SEMIFARMA (p, d, q) jointly with a GAS(1,1), EGAS(1,1) and AEGAS(1,1) model. For each model, we calculate both Schwarz (1978) and Hannan & Quinn (1979) information criteria and the ARCH-LM statistic. The results of the model estimations by the Skewed Student-t maximum likelihood method are shown in Table 6. We find that the coefficients of the three models are highly significant. The information criteria are minimum for the SEMIFARMA-AEGAS model and the asymmetric parameter γ is statistically significant at 5% level, indicating negative shocks imply a higher next period variance than positive shocks of the same magnitude. In addition, long memory coefficient for the equation of the mean illustrated in Figure 3, 4, 5 is also significant for all studied stock return series. The series of the SEMIFARMA-AEGAS residuals (see Figure 3, 4, 5) are characterized by the absence of conditional heteroskedasticity: there are no remaining ARCH effects in all the estimated models since the ARCH-LM statistics are strictly less than the critical value of χ_1^2 at 5%. It should be noted that the normality assumption of residuals is rejected because the Jarque-Bera statistics are strictly greater than the critical value of χ_2^2 at5% (see also Figure 3, 4, 5). The conditional standard deviation is characterized by an asymmetric dynamics with some sharp jumps for all series. Moreover, the series of standardized residuals show no dependence structure where the BDS statistics, reported in Table 7, are strictly less than the critical value of normal distribution at the 5% level for all embedding dimensions.

Table 6 – Skewed Student-t maximum likelihood estimation – BFGS algorithm–

	Saudi Arabia			Argentina			France		
Parameters	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
d	0.061 (5.089)	0.062 (5.301)	0.085 (6.866)	0.132 (3.254)	0.041 (3.347)	0.054 (4.203)	0.133 (7.191)	0.163 (12.96)	0.160 (8.008)
ϕ_1	-0.828 (-7.066)	-0.833 (-8.142)	-0.853 (-10.18)	0.652 (6.370)	-	-	0.048 (2.120)	-	0.044 (1.842)
θ_1	0.845 (7.710)	0.850 (8.937)	0.867 (11.06)	-0.744 (-8.649)	-	-	-	-	-
$\widehat{h}_{opt}$	0.257	0.257	0.257	0.257	0.257	0.257	0.257	0.257	0.257
$IMSE$	1.087	1.087	1.087	1.087	1.087	1.087	1.087	1.087	1.087
ω	0.021 (4.368)	2448.375 (3621.)	3122.605 (-6255.)	0.131 (3.071)	3163.216 (-4113.2)	4088.578 (3655.4)	2.080 (5.662)	429516.9 (8572.4)	486854.93 (29930.1)

α_1	0.231 (10.01)	0.219 (13.26)	0.213 (14.75)	0.136 (6.084)	0.127 (10.11)	0.128 (11.02)	0.201 (10.92)	0.174 (14.31)	0.145 (13.80)
φ_1	0.999 (151.2)	0.973 (2086.1)	0.966 (2078.2)	0.969 (86.12)	0.960 (2122.3)	0.948 (1976.01)	0.971 (115.8)	0.956 (2342.2)	0.951 (2599.3)
γ	-	-	0.0748 (8.779)	-	-	0.050 (6.355)	-	-	0.067 (10.02)
Asymmetry	0.117 (7.551)	0.122 (7.916)	0.099 (6.487)	0.090 (4.625)	0.090 (4.649)	0.079 (4.148)	0.188 (10.16)	0.183 (10.31)	0.159 (8.573)
Tail	3.969 (14.80)	0.062 (14.44)	4.274 (14.10)	6.637 (8.705)	6.723 (8.746)	6.723 (9.031)	7.524 (9.526)	7.196 (10.56)	7.358 (10.79)
SC	-6.472	-6.476	-6.490**	-5.174	-5.178	-5.186**	-7.180	-7.182	-7.198**
HQ	-6.479	-6.482	-6.498^{+}	-5.181	-5.184	-5.193^{+}	-7.185	-7.187	-7.205^{+}
JB stat.	11278.2*	10320.6*	7765.5*	1165.1*	1148.3*	1309.9*	765.83*	821.01*	749.86*
ARCH(1)	0.447*	0.418*	2.641*	1.140*	0.041*	0.213*	0.236*	0.096*	0.932*

Note: *Model 1: SEMIFARMA-GAS. Model 2: SEMIFARMA-EGAS. Model 3: SEMIFARMA-AEGAS. * indicates a rejection of null hypothesis of normality and homoscedasticity at the 1%. level. The values in parentheses are the Student statistics. + indicates the optimal Schwarz (SC) and the optimal Hannan-Quinn (HQ). IMSE: Minimum Integrated Mean Squared Error.* $\widehat{h}_{opt}$ *is the estimated optimal bandwidth.*

Figure 3 – Residual analysis for SEMIFARMA-AEGAS (TASI returns)

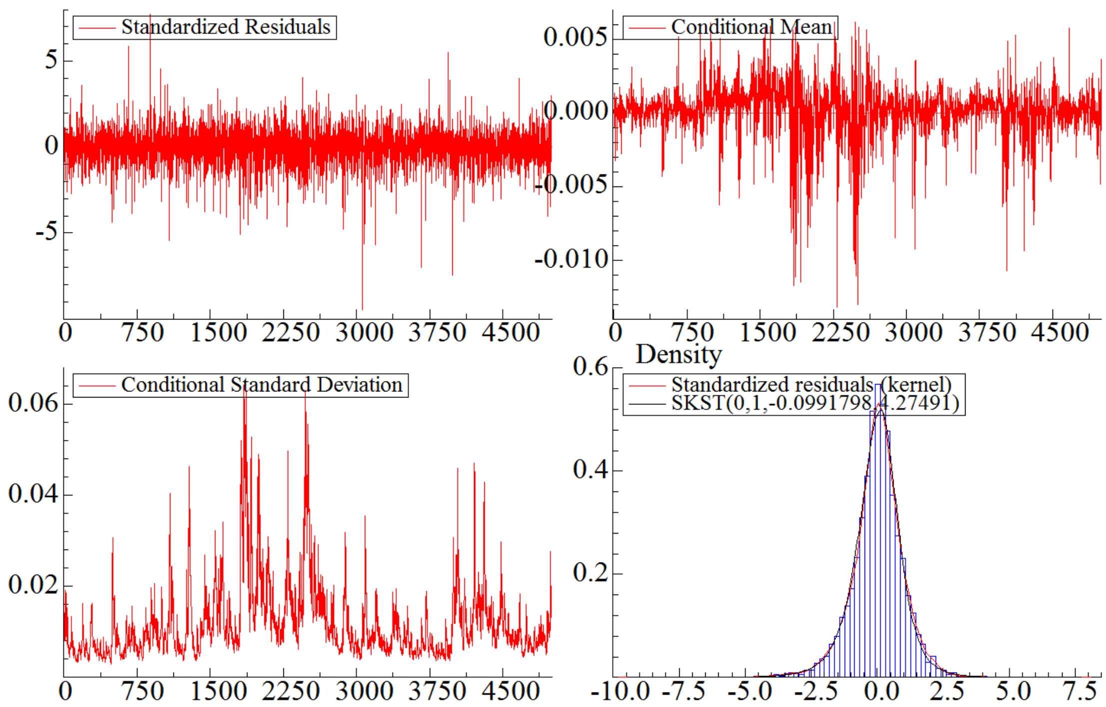


Figure 4 – Residual analysis for SEMIFARMA-AEGAS (Merval returns)

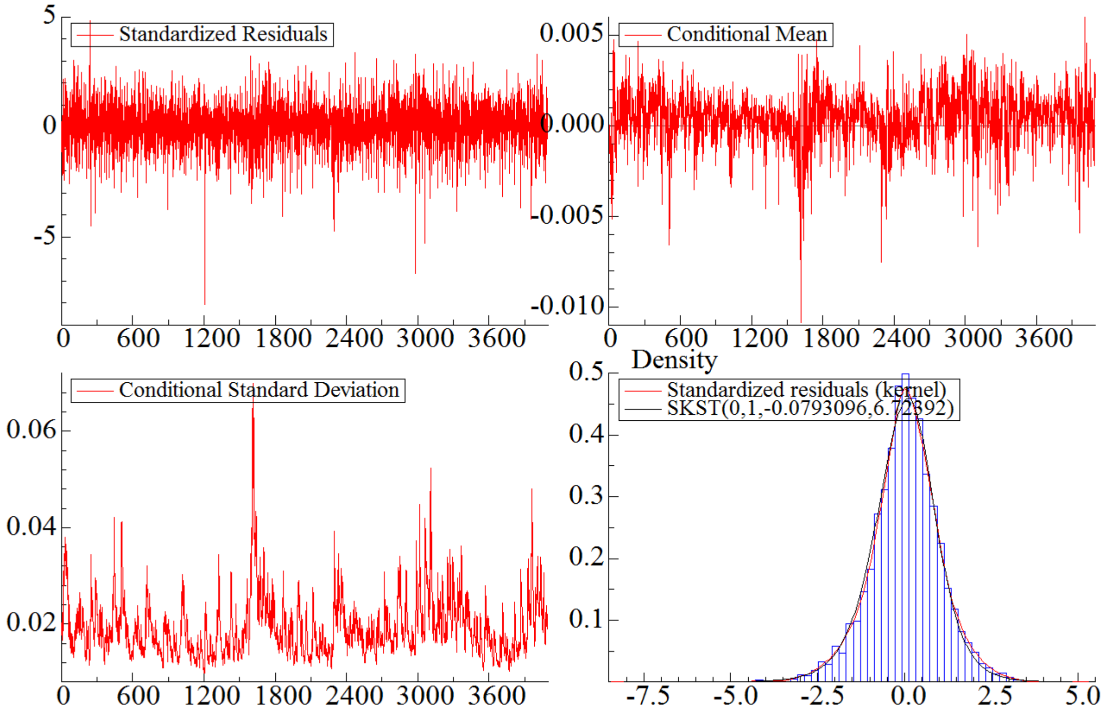


Figure 5–Residual analysis for SEMIFARMA-AEGAS (CAC SMALL returns)

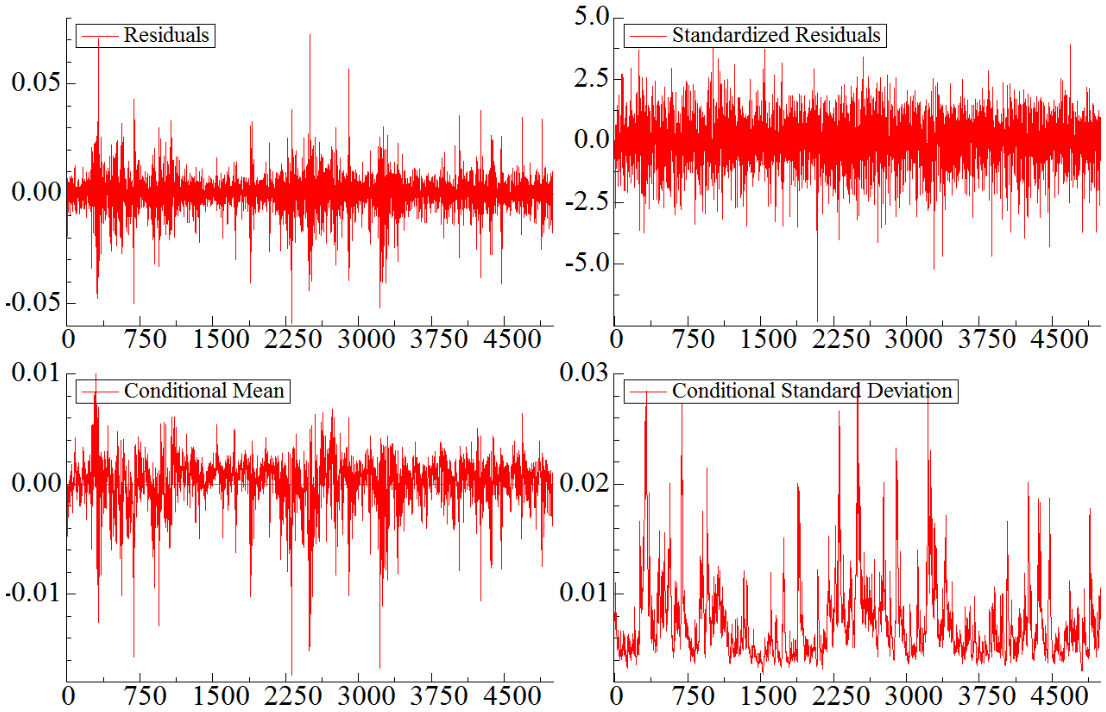


Table 7 – BDS test on standardized residuals of SEMIFARMA-AEGAS

m	Saudi Arabia		Argentina		France	
	BDS stat.	Prob.	BDS stat.	Prob.	BDS stat.	Prob.
2	0.7001	0.4838	-1.2838	0.1992	-0.2621	0.7932
3	0.9941	0.3202	-0.1049	0.9165	1.0214	0.3070
4	1.2814	0.2000	0.2214	0.8248	0.8766	0.3807
5	0.9938	0.3203	0.5953	0.5516	0.9864	0.3239
6	0.7998	0.4238	0.6928	0.4884	0.9386	0.3479
7	0.5940	0.5525	0.7548	0.4504	0.9014	0.3674
8	0.2839	0.7765	0.8061	0.4202	0.7143	0.4750
9	0.1329	0.8942	0.7069	0.4796	0.6689	0.5035
10	-0.0555	0.9557	0.7315	0.4644	0.5744	0.5656

Note: *The BDS statistics are calculated by the fraction of pairs method with* ε *equal to 0.7.* m *represents the embedding dimension.*

5. Forecasting performance

To determine which model provides a reasonable explanation of cyclical behavior of stock returns, some diagnostic tests are performed at the outset. First, we use the estimation results to compute *in-sample value-at-Risk* for the long and short trading position for confidence levels 95% and 99%, respectively. The results presented in Table 8 report the success/failure ratio, the Kupiec likelihood ratio (Kupiec (1995)) and the statistics for the dynamic quantile test (Engle & Manganelli (2004)). The LR statistics has the distribution χ^2 with one degree of freedom. The critical value of the Kupiec test for the most frequently adopted significance level 0.05 equals to 3.8415. The null hypothesis is rejected if the likelihood ratio exceeds the critical value. For the SEMIFARMA model with skewed-Student GAS and skewed-Student AEGAS errors, the null hypothesis of the test is not rejected both in case of underestimating of potential loss and in case of overestimating VaR for the short and long positions, it means that the null hypothesis of correct unconditional coverage can be accepted for the 95% and 99% levels of confidence. However, the dynamic quantile Engle-Manganelli test results indicate that the in-sample VaR forecast for the daily TASI and CAC SMALL returns obtained by the SEMIFARMA-GAS model gives unsatisfactory results and consequently fails this test at95% confidence level for short and long trade positions. It seems that the SEMIFARMA-AEGAS is appropriate for capturing volatility clustering for both negative (long Value-at-Risk) and positive returns (short Value-at-Risk) for all series.

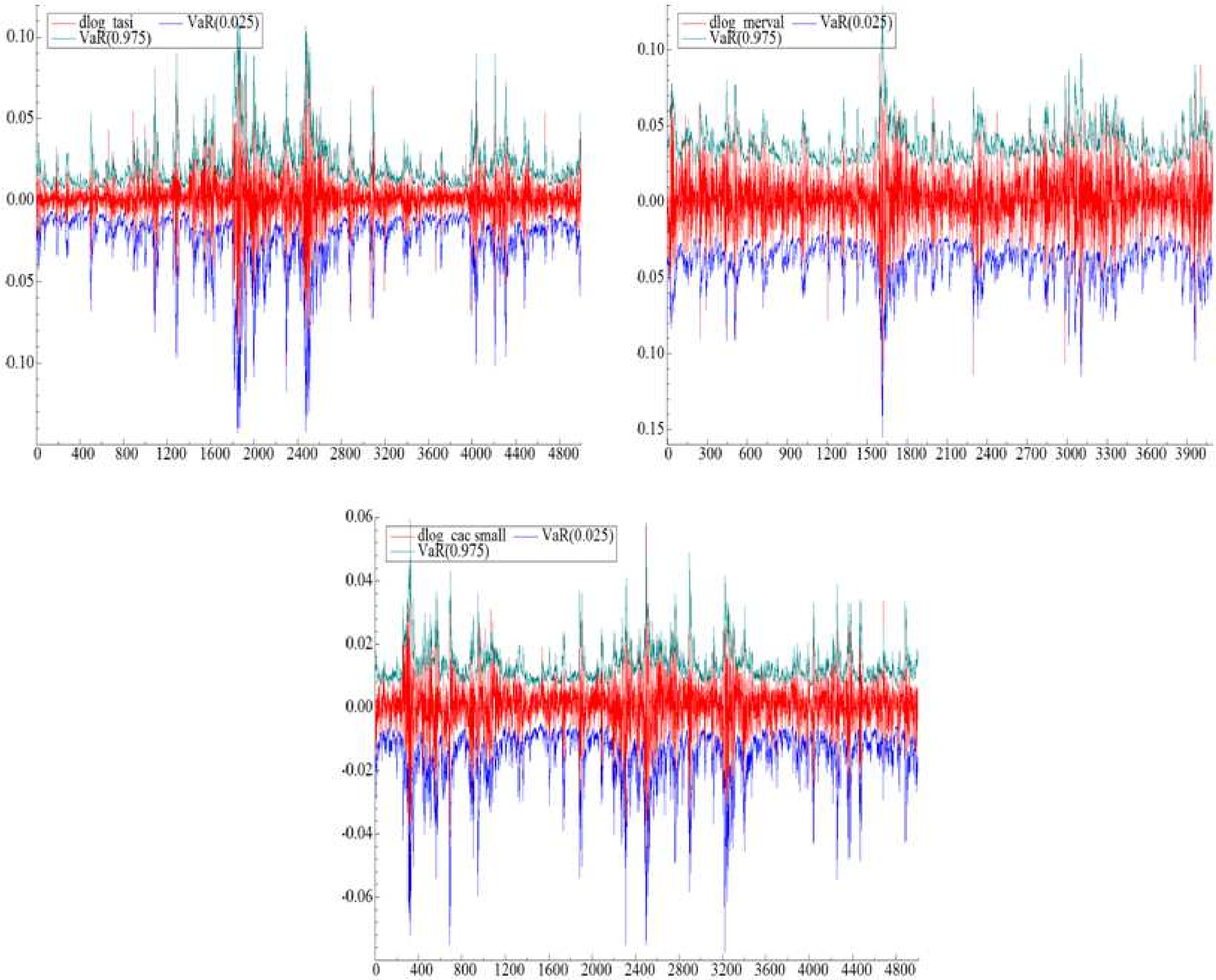
Figure 6 illustrates the relation of the Value-at-risk with the return of stock prices. The upper line is the maximal amount that can be lost with a confidence level 97.5% over the period of time taken into consideration, when the business events are not favorable for the business activity (see Cera, Cera & Lito (2013)). The calculation of VaR using skewed-Student AEGAS model has also advantages of the nature of forecasting the values of the VaR in the future. If the other factors remain constant, then the AEGAS model gives a very high level of approximation with the real values of the VaR.

Table 8 – In-sample Value-at-Risk Backtesting

Series	Model	Position	Kupiec LR test				Test of E.M	
TASI	SEMIFARMA -GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			95%	0.9443	3.2023	0.0735	13.641	0.0339
			99%	0.9906	0.1843	0.6676	7.6106	0.2680
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			5%	0.0500	0.00001	0.9974	15.732	0.0152
			1%	0.0104	0.0805	0.7765	6.2444	0.3963
	SEMIFARMA -AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			95%	0.9455	1.9929	0.1580	6.165	0.4087
			99%	0.9922	2.6400	0.1042	11.975	0.0625
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			5%	0.0502	0.0046	0.9457	1.1967	0.9770
			1%	0.0108	0.3166	0.5736	6.3905	0.3808
Merval	SEMIFARMA -GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			95%	0.9437	3.1967	0.0737	6.1385	0.4078
			99%	0.9890	0.3980	0.5281	8.0615	0.2336
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			5%	0.0503	0.0100	0.9201	9.7045	0.1376
			1%	0.0100	0.00015	0.9899	3.8000	0.7037
	SEMIFARMA -AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			95%	0.9425	2.5468	0.1072	7.8775	0.2472
			99%	0.9885	0.8708	0.3507	3.1235	0.7932
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			5%	0.0481	0.3007	0.5834	7.7516	0.2568
			1%	0.0085	0.9096	0.3402	3.3311	0.7662
CAC SMALL	SEMIFARMA -GAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			95%	0.9443	3.2141	0.0730	15.674	0.0156
			99%	0.9892	0.3182	0.5726	6.3717	0.3828
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			5%	0.0516	0.2735	0.6009	39.501	0.0000
			1%	0.0108	0.3182	0.5726	2.5601	0.8616
	SEMIFARMA -AEGAS	Short positions	Quantile	Success	Kupiec	Prob.	Stat.	Prob.
			95%	0.9447	2.7794	0.0954	10.027	0.1204
			99%	0.9894	0.1807	0.6707	12.896	0.0857
		Long positions	Quantile	Failure	Kupiec	Prob.	Stat.	Prob.
			5%	0.0526	0.7112	0.3990	10.212	0.1160
			1%	0.0092	0.3289	0.5663	2.3858	0.8810

***Note:** In the Dynamic Quantile Regression, p=5.E.M: Dynamic Quantile Test of Engle and Manganelli (2002).*

Figure 6 – In-sample Value-at-Risk forecasts



We further probe these results by performing the Out-of-sample tests of forecasting accuracy using the minimum loss functions on SEMIFARMA-GAS, SEMIFARMA-EGAS, SEMIFARMA-AEGAS for skewed Student-t distribution and the random walk model. The forecast evaluation measures used include mean square error (MSE) and mean absolute error (MAE). The MSE criterion is a quadratic scoring rule which measures the average magnitude of the error and the MAE criterion is more robust to outliers since it does not make use of square.

Table 9 – Comparison of predictive qualities

Series	Equation	Horizon	Criteria (10^{-3})	SEMIFARMA-GAS	SEMIFARMA-EGAS	SEMIFARMA-AEGAS	Random Walk
TASI	Conditional mean	1 day	MSE	0.0956	0.0960	**0.0929**	0.1202
			MAE	9.7780	9.8050	**9.6390**	12.597
		15 days	MSE	0.4484	0.4484	**0.4324**	0.8241
			MAE	14.320	14.331	**13.690**	18.542
		30 days	MSE	0.2607	0.2607	**0.2607**	0.9687
			MAE	10.591	10.590	**10.581**	21.637
		90 days	MSE	0.1338	**0.1338**	0.1341	1.2364
			MAE	7.4310	**7.4310**	7.477	23.633
	Conditional Volatility	1 day	MSE	0.00089	0.00015	**0.000086**	-
			MAE	0.09467	0.1226	**0.09307**	-
		15 days	MSE	**0.000574**	0.000576	0.000979	-
			MAE	**0.4506**	0.4578	0.589	-
		30 days	MSE	0.000283	0.000282	**0.000281**	-
			MAE	0.3033	0.2901	**0.2899**	-
		90 days	MSE	0.000111	0.000115	**0.0001105**	-
			MAE	0.2046	0.1618	**0.1578**	-
Merval	Conditional mean	1 day	MSE	0.1016	0.09825	**0.00007446**	0.0372
			MAE	10.08	9.912	**0.1127**	12.615
		15 days	MSE	0.460	0.4655	**0.4599**	0.5903
			MAE	17.32	17.50	**17.30**	21.799
		30 days	MSE	**0.3355**	0.3357	0.4718	0.6360
			MAE	**14.03**	14.03	17.81	24.014
		90 days	MSE	0.6701	**0.6698**	0.6711	0.6877
			MAE	19.67	**19.67**	19.68	25.198
	Conditional Volatility	1 day	MSE	**0.000155**	0.000222	0.0016	-
			MAE	0.3941	0.4713	**0.2729**	-
		15 days	MSE	**0.000210**	0.00517	0.00173	-
			MAE	0.3853	0.696	**0.3808**	-
		30 days	MSE	**0.000281**	0.000485	0.03948	-
			MAE	**0.4724**	0.6487	0.6266	-
		90 days	MSE	0.00124	0.00128	**0.00123**	-
			MAE	0.6254	0.7382	**0.6124**	-
CAC SMALL	Conditional mean	1 day	MSE	0.000765	0.00108	**0.000717**	0.0231
			MAE	0.8749	1.042	**0.847**	2.5210
		15 days	MSE	0.04776	**0.04755**	0.04777	0.4831
			MAE	**5.626**	5.662	5.670	9.3980
		30 days	MSE	0.04358	0.0436	**0.04357**	0.5344
			MAE	5.220	5.221	**5.219**	9.4243
		90 days	MSE	**0.03721**	0.03731	0.03731	0.6471
			MAE	**4.774**	4.778	4.778	9.963
	Conditional Volatility	1 day	MSE	0.00002704	0.00002944	**0.00004703**	-
			MAE	0.06858	1.042	**0.052**	-
		15 days	MSE	0.00006873	**0.00006864**	0.00006953	-
			MAE	0.06079	**0.06066**	0.06321	-
		30 days	MSE	**0.00004585**	0.00004657	0.00004616	-
			MAE	0.0446	**0.0387**	0.04169	-
		90 days	MSE	0.00004061	0.00003346	**0.00003291**	-
			MAE	0.05485	0.04506	**0.043766**	-

Table 10 – Model rankings

Series	Criteria	SEMIFARMA-GAS	SEMIFARMA-EGAS	SEMIFARMA-AEGAS	Random walk
TASI	MSE	2	2	1	3
	MAE	2	2	1	3
Merval	MSE	1	3	2	4
	MAE	2	3	1	4
CAC SMALL	MSE	2	2	1	3
	MAE	2	2	1	3

Tables 9 and 10 report the forecast performance and the corresponding ranking for all the models. The results indicate that, whatever the forecast horizon, the random walk model is beaten by all the other models. We note that all the errors aren't of the same magnitude since the values of MSE are less than those of the MAE. It is also observed that MSE and MAE criteria generally give the same results. The SEMIFARMA-AEGAS model tend to have better predictive results comparing to SEMIFARMA-EGAS and SEMIFARMA-GAS with some time horizons. Furthermore, the model rankings presented in Table 10, indicate that the skewed-Student AEGAS is the preferred model for all stock returns. This model captures the asymmetric behavior and the volatility clustering phenomenon in the presence of long-run dynamic dependencies and nonparametric deterministic trend in the conditional mean equation.

In order to test the statistical significance of the forecasting improvements of SEMIFARMA-AEGAS predictions over the SEMIFARMA-GAS and the random-walk, we can also use the asymptotic test, the sign tests, Wilcoxon's test and the Morgan-Granger-Newbold test (Diebold & Mariano (1995)). As seen in Table 11, the p-values clearly indicate that the null hypothesis of equal accuracy of the three models is strongly rejected. It is observed that different predictive accuracy are accepted because the p-values are less than 0.05, it means that, in this case, the SEMIFARMA-AEGAS model beat the SEMIFARMA-GAS and the random walk process. The Diebold-Mariano statistics are significant, meaning that there is a difference in the forecasts computed from the SEMIFARMA-GAS and SEMIFARMA-AEGAS models. A negative sign of the statistics implies that SEMIFARMA-GAS model is dominated by SEMIFARMA-AEGAS model. The results indicate that asymmetry effects detected on volatility seem to improve the volatility forecasts. Indeed, the sign of the statistics is negative, implying that the asymmetry effects with jumps observed on volatility provide a better volatility forecast. Consequently, the price movements appear as the result of lasting shocks which affect the Saudi, Argentinean and French stock markets; in other words, the consequences of a shock will be sustainable, the TASI, the Merval and the CAC SMALL returns will not come back to their previous fundamental value. The shock of stock returns will be persistent in the long term and the volatility exhibits nonlinearity and asymmetry effects with jumps.

Table 11 –Comparing predictive accuracy: Diebold-Mariano test

Series	Test of equal accuracy	S_1	S_2	S_3	*MGN*
TASI	SEMIFARMA-AEGAS versus SEMIFARMA-GAS	-1.13 (0.87)	-2.97 (0.00)	-4.11 (0.00)	-7.03 (0.00)
	SEMIFARMA-AEGAS versus Random walk	-3.48 (0.00)	-10.22 (0.00)	-8.38 (0.00)	-9.25 (0.00)
Merval	SEMIFARMA-AEGAS versus SEMIFARMA-GAS	-0.72 (0.76)	-2.76 (0.00)	-1.95 (0.05)	-4.02 (0.00)
	SEMIFARMA-AEGAS versus Random walk	-5.71 (0.00)	-5.62 (0.00)	-3.40 (0.00)	-6.11 (0.00)
CAC SMALL	SEMIFARMA-AEGAS versus SEMIFARMA-GAS	-1.01 (0.84)	- 4.52 (0.00)	- 2.71 (0.01)	- 3.62 (0.00)
	SEMIFARMA-AEGAS versus Random walk	-3.76 (0.00)	-5.22 (0.00)	- 3.36 (0.00)	- 6.07 (0.00)

Note: The p-values are given in parentheses. S_1: Asymptotic test statistic, S_2: Sign test statistic, S_3: Wilkoxon test statistic, MGN: Morgan-Granger-Newbold test statistic. A positive (negative) sign of the statistics implies that model B dominates (is dominated by) model A. The prediction horizon used is 90. These tests are based on absolute forecast mean errors.

6. Conclusions

In this paper, we have combined path-dependence nature of stock price with asymmetric volatility estimated characterized by jumps. A SEMIFARMA model with skewed-Student AEGAS errors, we argued, has the potential to capture long-range persistence with nonparametric deterministic trend in the conditional mean and asymmetric jumps and volatility clustering in the conditional variance. This model offers better features of the dynamic volatilities and exploits nonlinear and asymmetric structures to model the existence of time-varying parameters. In this regard, we use the scaled score of the likelihood function. In addition, the asymmetric exponential GAS model serves as an extension of the GARCH family models which assume that the conditional distribution does not vary over time. It exploits the full likelihood of information. Taking a local density score step as a driving mechanism, the time-varying parameters increase and produced a clear indication of a leptokurtic behavior and a heavy tails in the financial series.

Our empirical exercise focused on the behaviour of the time-varying parameter by estimating the SEMIFARMA-AEGAS model with the Skewed Student-t maximum likelihood. From the dynamic quantile Engle-Manganelli test results, the in-sample Value-at-Risk forecast obtained by the SEMIFARMA-AEGAS model gives satisfactory results at 90% and 95% confidence level for short and long trade positions. Using the minimum loss functions, the SEMIFARMA-AEGAS model shows a clear superiority over all the other models. Particularly, the forecasts of the SEMIFARMA-AEGAS model show a clear improvement compared to the random walk model at all horizons. The observed movements appear as the result of lasting and asymmetric shocks, which affect the French, Argentina and Saudi markets. Consequently, recent works on volatility modeling through asymmetric exponential GAS process, which captures volatility clustering for both negative and positive returns, seem particularly promising and may provide new evidence to better understand the nonlinear and asymmetric dynamics of financial series.

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