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Document de Travail n° 2019 – 20

Mai 2019

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Théorique et Appliquée
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Which kind of ability bolsters legitimacy?

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May 22, 2019

Then Jesus added, "I can guarantee this truth: A prophet isn't accepted in his hometown." Luke 4:24 (GW) The Bible

Abstract

The aim of this paper is to study in a theoretical perspective how the choice of the ability on which an executive is evaluated to be promoted may be a crucial stake. We show that a procedure where an executive is selected on a managerial ability will allow to increase his own wage, compared to a procedure where he needs to demonstrate ability on the same task than his employee. The intuition is that it would neutralize the issue of rivalry with the employee by preserving the self confidence of the employee in spite he has failed at being promoted, making him easier to incentivize. The consequence is that selecting leaders on their ability to outperform their employee will tend to favor the emergence of a leadership culture of humility during the promotion process in a sense that opponents will strategically reduce their performance to preserve the self-confidence of their employee and then make him less costly to incentivize. On the contrary, selecting leaders on managerial ability will favor the emergence of a leadership culture of demonstration of strength.

Keywords: Legitimacy, leadership, tournament, contract theory, moral hazard, personnel economics

JEL-Classification: D00,D86, J50, M50, M51

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1 Introduction

This paper deals with the choice of the ability on the basis of which an executive should be selected to bolster his legitimacy optimally. The trade-off for the firm is to know if it should favor a "business" ability, that is required for the employee and that is very technical (like accounting skills in a audit firm, a researcher skills for a professor...), or a "managerial" ability which is specific to the executive function but not necessary hard to acquire (like general knowledge, or administrative abilities...). The stakes for the firm are plural. First, choosing the right procedure would be a way to make leaders more accepted by their subordinates and as a consequence it could be a way to legitimate high inequality of remuneration between them. Second, the firm could want to reveal a competence through the promotion process that will maximize production. Finally, the stake could be to create an enthusiastic atmosphere before the promotion in itself, where every employees are incentivized to work hard and reveal competence (the promotion seen as a final step and as a final reward may have a significant impact on the motivation before the promotion itself). Furthermore, in the eyes of a future executive, the ability assessed by the firm will also become a major stake for himself as it could influence the level of competence he needs to reveal during a promotion process. We argue it could become part of a leadership strategy.

Let us explain more extensively the two legitimacies treated in this paper and the procedures of selection of executives supposed to bolster them. First, selecting on business ability is what we call a *meritocratic procedure* and leads to choose as leader the best employee or the one who has the best employee 's ability. It will then bolsters what we call a *meritocratic legitimacy*. It could be a very intuitive way to promote people within a department in a organization using past achievement and CV as criterias of promotion. It would then suggest that in spite of hierarchy, the professional and education profile among workers are quite homogenous within the department as every employee have almost the same profession. But this selection process could also represent education and more specifically short and professionally oriented courses, designed to meet specific demands of companies focused on operational tasks. On the contrary, favoring managerial ability is what we call an *aristocratic procedure* and would bolster an *aristocratic legitimacy.* It could suggest one or several probation stages when an employee is given a chance to demonstrate a different ability beyond his routine task that requires business ability. It could be also some-thing more institutionalized such as internal contest for civil servant in public administration for instance. High education and selection through generalist trainings in prestigious universities could be seen also as an *aristocratic procedure* in a sense that it places people upstream in the hierarchy and contrary to professionalizing courses those trainings often teach general knowledge that is not specifically related to the operational task. Furthermore, we note that within a firm *meritocratic procedures* are potentially less costly than *aristocratic procedures* as it is feasible to assess business ability through a productive task whereas assessing managerial ability will mobilize staff on a non productive task which could generate an opportunity cost.

To modelize this, we assume a preliminary tournament before a production stage. Before the tournament the firm chooses which ability it wants to assess. Opponents and the firm don't know abilities of opponents with certainty but they only have an *a prior* belief about it which is common knowledge. Winning or loosing will reveal information and both opponents will revise by bayesian inference their initial beliefs about being competent or not on the ability that the firm has chosen to evaluate. However, the belief on the ability that is not assessed will remain unchanged. The winner will then become the principal in stage 2. This stage is captured by a principal agent framework with a function of production comprised of three tasks. Both of them are completed by the principal: the first task requires a very technical business ability and a second task requiring a managerial skill that is easier to master. Lastly, the third task is a business task completed by the employee. The two business tasks are substitutable to each other which means that the work of one team member can compensate the failure or repair the mistake of the other. In contrast, the managerial task is complementary with the other two tasks. It means that in spite of its ease, it is crucial for the production. We also assume that among the two substitutable tasks that require business ability, the one the principal is in charge of is harder (i.e requires more ability to be completed) than the one the employee completes. It justifies *a priori* that we should wonder about the necessity of selecting leaders on their business abilities. The efforts of the principal in both tasks is not modeled such that only his ability will count for the production. His strategic decision is to propose a wage to the agent to incentivize high effort from him. We argue this function of production captures a situation, in consulting and audit firm notably or also more generally in small teams, where the executive continues to be part of the operational process of production but is assigned harder tasks while he also has to supervise the work of others and master some management task. It can also capture the situation of a political leader that needs to manage a party but also needs to propose a political program and embody an ideological current (which would be a transposition of the business task we mention previously).

With this framework we get the following result. The first result is that the *aristocratic procedure* allows to make the agent accept a higher inequality of wage between him and his principal compared to *meritrocratic procedure.* The reason is that in the *meritocratic procedure*, after failure, the agent will be discouraged because he will have to execute the same task that the one which made him lose the tournament. In other words, becoming agent means implicitly being an "incompetent" employee. Thus, to maintain the income incentivizing it will be necessary to propose a higher wage to compensate the fact that in the eyes of the agent, his chances of making a high production are weak. Besides, this procedure will incentivize free riding behavior as the agent will tend to rely on the principal business ability. These issues will not appear in the *aristocratic procedure* because the selection task is not the same than the task completed by the agent thereafter. Thus, the production stage is seen as a fresh start where the motivation remains unaffected.

The second result is that during the tournament the *meritocratic procedure* incentivizes less to exert high effort than the *aristocratic procedure.* Indeed, we demonstrate that whatever the efforts of their opponents, the marginal return of effort of both opponents in the *meritocratic procedure* will be lower. The intuition is that opponents have interest to make a low performance during the tournament to contain to a minimum the discouragement of their opponents in order to offer them a wage as small as possible in the future. That is why they will reduce their efforts.

The third result is that for the firm it will be optimal to select the executive on managerial ability if it wants to maximize production in the second stage. First of all, the intuition is that revealing competence in business ability for the principal implies revealing also incompetence for the agent. Thus the net effect on the global production will be modest. Whereas selecting on managerial ability avoids this issue. As the managerial task is assumed complementary, increasing managerial ability would favor a fertile ground for the other tasks of the team to be productive simultaneously. Besides, as the *aristocratic procedure* is more incentivizing during tournament, it allows to reveal more competence at the equilibrium. Thus the production in stage 2 is guaranteed to be higher in second stage in the aristocratic procedure precisely because this procedure allows to reveal more competence.

This model suggests several applications. If we interpret the first result, it sheds new light on the mechanism of the rejection of authority and particularly the rejection by the peers. It suggests that for a subordinate, seeing some one else with "comparable" competence, or even social belonging, becoming promoted as his executive would be lived as a humiliation and it will entail an non-cooperative behavior. On the contrary some one with a different profile, with a non comparable competence, will be accepted because his promotion would preserve the pride of the subordinate and it would neutralize rivalry's issues. As a consequence, if we interpret competence as an identity or a social belonging, it would suggest that an organization with social class (in a sense that each social class would be defined by a common competence shared among members) would entail more submission to authority than an egalitarian organization. It also suggests that hierarchy *between* professions, notably established with aristocratic procedures such as high education, would be much more accepted that hierarchy *within* a profession, which would be established with meritocratic procedures.

The second result can be interpreted such that a *meritocratic procedure* will favor the emergence of a culture of humility whereas an *aristocratic procedure* will favor a culture of performance. Indeed in a *meritocratic procedure*, opponents will be incentivized to reduce their efforts to hide their strengths in order to preserve the self-confidence of their future team partners. On the contrary in *aristocratic procedures* the perspective of being propelled to the top of the hierarchy with very competitive remuneration without any monetary concessions will incentivize a demonstration of strength. This strategy of humility could be seen also as a leadership strategy that would aim to favor an *identification mechanism*. Indeed, observing the promotion of some one with an ability very close to his own ability would give to the agent good hope for his future achievement as an employee. On the contrary, in *aristocratic procedures*, the strategy of demonstration of strength could be seen as a totally opposite leadership strategy based on a *differentiation mechanism* that would aim to favor the image of the leader as an exceptional and rare being. Indeed, he will be able to make "extraordinary" achievement that a regular employee would not be able to achieve. It will be reassuring and so without making the employee feel humiliated.

The second result suggests also that among the procedure of selection, a high education system based on generalist training will be more efficient to reveal ability than a system based on professionally oriented courses, designed to train on business tasks. This is because the reward in case of success is much more incentivizing than in professionalizing courses. It suggest that in some industry or in some profession where meritocratic procedures are chosen through professionalizing courses, an egalitarian culture would emerge that would lead to a more egalitarian allocation of income between executives and employees. As a consequence, it suggests that professionalizing courses would favor more academic failure than generalist trainings (as victory would be much less incentivizing).

Finally, we believe the second result could explain a situation of blockage and stasis in a fratricidal struggle notably for instance ideological vacuum in a political party when it comes to appoint a leader for a future national election. Indeed, we can interpret the signal at the end of the tournament not only as information about a competence, but also as information on the subjective preference of the organization on a project or an idea. Thus, the model can capture a situation where two persons compete to become the leader of a party and have different political programs (which is a transposition of the business task that we have discussed previously) but they have the *a priori* belief that both programs have equally chances to be favored by the organization (which is a transposition of the uncertainty on business ability). Loosing the competition will send a signal that their political programs will not be taken into account and valued by the organization.[1] In the perspective of new national elections the loser might demand higher concession on ministerial position to support the new leader during the campaign. Anticipating that, rivals might have interest in forsaking the strategy of investing effort in their political programs. The organization might anticipate that and choose another criteria based on their personalities and their ways of governing (which is a transposition of managerial abilities). This could imply notably ideological vacuum during internal campaign for primary election in a political parti.

The originality and contribution of this work is to focus on the side effect of tournament by focusing on the point of view of the looser and the incentive issue that it implies. Thus, we focus on the consequence of failure as a mechanism to reveal incompetence whereas career concern literature focus on the consequence of success in demonstrating competence (Gibbons, 1996; Gibbons and Murphy, 1992; Jeon, 1996; MacLeod, 2007). As a consequence, the winner of the competition could then have to assume the failure of his opponent as a burden, as some thing very costly in term of incentive. With the same bayesian framework, we have an opposite result than classical career concern insight in a sense that opponents may have no interest in revealing high ability during the tournament. Besides our work shed a new light on issue of merit (effort) as a criteria that legitimate inequality of wage treated notably by the literature on unfairness aversion and procedural justice (Bolton et al., 2005; Ku and Salmon, 2013).[2] Indeed, because we focus on the idea of comparability and

[1]The assessment of political programs could also be delegated to electors like in a primary election for instance.

[2]Papers on unfairness aversion deal with how people might have a preference for equiteable allocation of revenues. Papers on procedural justice study how people might care about the procedure by which the allocation of incomes and power are decided

rivalry, effort will be seen as a reasonable source of legitimacy if it does not imply a humiliation in the view of the person not promoted. Thus effort will then allow to legitimate a higher inequality of remuneration.

The rest of the paper is organized as follows. Section 2 deals with the literature. Section 3 describes the theoretical framework and section 4 will explore the results. Finally, section 5 concludes.

2 Literature

As mentioned above, the issue of this work is related to the literature on career concern and reputation (Gibbons, 1996; Gibbons and Murphy, 1992; Jeon, 1996; MacLeod, 2007). Gibbons (1996) deals with how agents could accept to exert very high effort in the short term without reward, in order to reveal competence and expect a higher reward in the future. Jeon (1996) uses this same idea but shows that it would be efficient for an organization to mix young worker and old workers in the same team. Indeed, as competence of both team partners are complementary, the competence previously revealed by the old worker would exacerbate the performance of the young worker and help revealing his ability in a sense that it would make the signal of the assessing technology less noisy.

Bar-Isaac and Deb (2014) deals with how a firm could be faced to consumers with heterogenous preferences with respect to quality and how her optimal strategy will not be necessary to maximize quality. Each consumer may have different expectation in term of quality and this creates different market segments. If the firm cannot discriminate by price, increasing her quality will make her capture less of the surplus of some of the segment that requires less quality. Even though it does not deal about competence directly the unknown quality can be revealed by an action through a bayesian inference such that the theoretical framework are comparable to ours. Besides the issue is close to our work because it points out the idea that the reputation is a coarse notion that deserves to be studied in more details because not necessary objective. In our work, we focus on what kind of reputation is the more efficient between an ability already required from the subordinate or a new ability specific to the executive responsibility.

Our work is also related to Benabou and Tirole's work on "belief in a just world" (Benabou and Tirole, 2006). Their paper deals with how deny of "bad new" may make people agree with less redistributive taxes. It also explains how on the contrary lucid and pessimist people will choose a more redistributive tax rate. Deny in this paper is treated as a cognitive choice (i.e a costly decision about treatment of information) of hiding one's self negative information received about how effort will be rewarded in the future. It is assumed that people do not know with certainty to what extent economics success is due to personal effort and controllable actions or due to external factors inherited. Failure tends then to reinforce by bayesian inference the belief that they effort will not be "fairly" rewarded, namely that external factors are more important. Deny will consist in hiding oneself "bad" information to limit this bayesian inference. Our work does not deal specifically with deny because opponents do not choose to hide themselves information however the aristocratic procedure makes the learning on the ability assessed by the tournament completely pointless. Thus, the effect on neutralizing the bayesian inference will be identical. The behavior of submitting to authority (or accepting a very low wage) depends on how "bad news" (i.e. loosing the tournament), are not taken into account and for this it is very comparable to the behavior of choosing a low tax rate in the paper by Benabou and Tirole.

Besides, our work is related to the literature on fairness and inequitey aversion (Bolton et al., 2005; Englmaier and Wambach, 2010; Falk and Fischbacher, 2006; Falk et al., 2008; Fehr et al., 2013; Fehr and Schmidt, 1999, 2000; Schurter and Wilson, 2009). The "procedure of justice" has already been the object of many experimental papers (Bolton et al., 2005; Ku and Salmon, 2013). They show that the role of merit (measured in their experiment by a performance at a test) is ambivalent as a criteria of acceptation of inequality of incomes. Ruffle (1998) studies a dictator game is which there is a previous stage where participants passed a test. He shows that the proposer takes into account the performance of the one who received his proposal to choose the amount he wish to give. Cherry et al. (2002) study how dictators tend to give less equiteable share to recipients if they have themselves pass a previous test that put them in a good rank position. Ku and Salmon (2013) decide to separate participants into two categories: the advantaged ones (i.e participants who receive a high endowment) and the disadvantaged one (i.e participants who receive a low endowment). They study the willingness of the disadvantaged people to transfer money to the advantaged one with respect to the procedure of selection chosen in the experiment that split participants. They show notably that the merit of advantaged people did not make disadvantaged people accepting a higher monetary transfer (in one of their treatments they select participants to belong the advantaged group by measuring their performances at a test).

3 Theoretical framework

We consider a two-stage game. The first stage is a tournament between two opponents such that the winner will become the principal and the looser will become the agent in the second stage. The second stage is a principal-agent framework with a working principal where three tasks are completed simultaneously. There are two similar tasks: one completed by the principal and the other by the agent, and a last task which is different and only exerted by the principal. Regarding the contribution of the principal in the global output, only his abilities in each of his task counts and we do not formalize his effort. As we only want to focus on the moral hazard in the side of the agent and not on the effort of the principal, the effort of the principal is ignored and not modeled to avoid to complicate the analysis unnecessarily. On the contrary, the contribution of the agent will depend on his ability and on his effort. But his effort is not contractible. Thus, the principal has to choose the wage he will give to the agent, while the agent has to choose his level of effort.

The firm can choose two different procedures of selection. The assessment technology is exogenous such that the choice will be about what kind of ability required for the principal (for each of his task) is more efficient to choose. The firm does not observe the effort of opponents during the tournament and the effort of the agent, thus the objective will be to maximize production in stage 2 and to incentivize maximum effort during tournament.

Here is the sequence of the game in stage 1:

- The firm and the opponents share a common *a priori* belief about the abilities of the competitors.
- The firm choose between the two procedures of selection (aristocratic or meritocratic)
- The tournament begins and opponents choose their efforts simultaneously
- The opponents observe the effort chosen and the performances achieved by each other. The firm only observes the performance and promotes as the principal. the opponent that makes the highest performance.
- The opponents make a bayesian inference to revise their beliefs on competitor abilities. Thus the posterior belief is common between competitors as well.

Here is the sequence of the game in stage 2:

- The principal proposes a variable wage with respect to the performance of the team (agent and principal) without being able to observe agent's effort but having a revised belief on their respective abilities.
- Agent choses his effort with respect to the wage proposed by the principal, having a revised belief on their respective abilities.
- The performance of the team is observed. The payments of both the principal and the agent occur.

3.1 Stage 1: a tournament that can assess two different abilities

The first stage is a tournament that selects opponents for the position of agent and principal in the second stage. There are two types de competences that can be evaluated: a managerial competence necessary to complete the managerial task $m_i \in \{0, 1\}$ such that γ_i is the probability to own the high level of competence; and a business competence necessary to complete the business task $b_i \in \{0, 1\}$ such that λ_i is the probability to own the high level of competence. To evaluate consists in determining m_i or b_i which can be interpreted as grades. We denote $(\pi^{\gamma}_{e_i}, \underline{\pi}^{\gamma}_{e_i})$ the probabilities of completing the managerial task whereas i has the high competence or the low competence given his level of effort e_i. Symmetrically, we denote $(\pi^{\lambda}_{e_i}, \underline{\pi}^{\lambda}_{e_i})$ the probabilities of completing the business task whereas i has the high competence or the low competence given his level of effort e_i. It means that if we evaluate managerial skills we have $P(m_i = 1 \mid e_i) = \gamma_i \pi^{\gamma}_{e_i} + (1 - \gamma_i)\underline{\pi}^{\gamma}_{e_i}$ and if we evaluate business skills we have $P(b_i = 1 \mid e_i) = \lambda_i \pi^{\lambda}_{e_i} + (1 - \lambda_i)\underline{\pi}^{\lambda}_{e_i}$. The organization cannot evaluate simultaneously the two competences such that it will have to chose on what task to assess the future principal. This assumption is made for tractability. Besides, it allows to avoid some eviction between the two tasks that could have occurred otherwise.

Definition 1. We define as an *aristocratic procedure* the fact of selecting principal on managerial ability and *aristocratic legitimacy*, the nature of legitimacy acquired by revealing such managerial ability.

Definition 2. We define as a *meritocratic procedure* the fact of selection principal on business ability and *meritocratic legitimacy*, the nature of legitimacy acquired by revealing such business ability.

First, we assume that the opponents have *ex ante* the same beliefs in both competences such that $\gamma_i = \lambda_i$ and they share the same belief with the firm about who is advantaged or not before the competition such that $\lambda_i = 1 - \lambda_j$ and $\gamma_i = 1 - \gamma_j$. Second, we assume that the assessing technology during first stage is identical whatever the competence evaluated such that $(\pi^{\gamma}_{e_i}, \underline{\pi}^{\gamma}_{e_i}) = (\pi^{\lambda}_{e_i}, \underline{\pi}^{\lambda}_{e_i})$. As a consequence we will have $P(m_i = 1 \mid e_i) = P(b_i = 1 \mid e_i)$. Thus, winning or loosing the competition will give the same information about owning the competence whatever the type of competence that is evaluated because the "quality" of the tournament's evaluation is assumed identical. This assumption is a technical hypothesis designed to make the comparison between the two procedures more tractable because relying only on second stage expected payoff.

For convenience, from now we will denote q_i the production level for any task such that $q_i \in \{m_i, b_i\}$ and x_i the probability of owning the competence required such that $x_i \in \{\gamma_i, \lambda_i\}$. Besides, we will denote $(\pi_{e_i}, \underline{\pi}_{e_i})$ the probabilities of succeed with high or low competence whatever the task such that $\pi_{e_i} \in \{\pi^{\gamma}_{e_i}, \pi^{\lambda}_{e_i}\}$ and $\underline{\pi}_{e_i} \in \{\underline{\pi}^{\gamma}_{e_i}, \underline{\pi}^{\lambda}_{e_i}\}$. For the purpose of future results, we assume complementarity between competence and effort such that:[3]

$$\begin{cases} \pi_{e_i} = e_i\theta_H + \epsilon \\ \underline{\pi}_{e_i} = e_i\theta_L + \epsilon \end{cases} \tag{1}$$

We assume of course that $\theta_H > \theta_L$ with θ_H the high ability such that $P(\theta = \theta_H) = x_i$, and θ_L the low ability such that $P(\theta = \theta_L) = 1 - x_i$. Besides, ϵ is a parameter that follow a Bernoulli distribution commonly known by both opponents and the firm. It guarantees $\pi_0 > 0$ and $\underline{\pi}_0 > 0$. Thus ϵ represents the probability of producing $q_i = 1$ given $e_i = 0$ and guarantees that even without putting high effort the competition remains random such winning or loosing the competition is still a noisy signal. We also assume $\theta_H > 0$ and $\theta_L > -\epsilon$ which means that θ_L can be negative but always superior to $-\epsilon$ to guarantee that $\underline{\pi}_{e_i} > 0$. Assume θ_L negative would mean it would be an incompetence that sabotage effort.

The winner of the tournament will be the one who completes the task whereas his opponent does not. Thus we have $P(q_i = 1 \mid e_i) = x_i\pi_{e_i} + (1 - x_i)\underline{\pi}_{e_i}$ and for i

[3] Assuming complementarity between competence and effort is a technical assumption necessary to justify in further results that completing the task by exerting high effort will reveal competence.

the probability of winning $P_W(e_i)$ and the probability of loosing $P_L(e_i)$ are:

$$P_W(e_i) = P\Bigg[q_i = 1 \cap q_j = 0 \mid (e_i, e_j)\Bigg] = x_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - x_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})$$
$$P_L(e_i) = P\Bigg[q_i = 0 \cap q_j = 1 \mid (e_i, e_j)\Bigg] = x_i(1 - \pi_{e_i})\underline{\pi}_{e_j} + (1 - x_i)(1 - \underline{\pi}_{e_i})\pi_{e_j}$$

Both opponents can observe the effort $(e_i, e_j) \in \{0, 1\}^2$ of each other only after the tournament and observe the result of the tournament as well. Thus, they can make a bayesian inference to learn about their relative ability with respect to their victories or defeats. By applying bayesian rule, we denote $\hat{x}_{i,v} = \hat{x}_{i,v}(e_i, e_j)$ the ex-post belief in case of victory with $\hat{x}_{i,v} \in \{\hat{\lambda}_{i,v}, \hat{\gamma}_{i,v}\}$, and $\hat{x}_{i,d} = \hat{x}_{i,d}(e_i, e_j)$ the ex-post belief in case of defeat with $\hat{x}_{i,d} \in \{\hat{\lambda}_{i,d}, \hat{\gamma}_{i,d}\}$:

$$\hat{x}_{i,v} = \frac{x_i \pi_{e_i}(1 - \underline{\pi}_{e_j})}{x_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - x_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})}$$
$$\hat{x}_{i,d} = \frac{x_i(1 - \pi_{e_i})\underline{\pi}_{e_j}}{x_i(1 - \pi_{e_i})\underline{\pi}_{e_j} + (1 - x_i)(1 - \underline{\pi}_{e_i})\pi_{e_j}}$$

Applying bayesian rule if i wins and if the firm chooses to evaluate on managerial ability we will have $\hat{\gamma}_{i,v} \neq \gamma_i$ and $\hat{\lambda}_{i,v} = \lambda_i$. On the contrary, if the firm chooses to evaluate on business ability we will have $\hat{\gamma}_{i,v} = \gamma_i$ and $\hat{\lambda}_{i,v} \neq \lambda_i$. These constitutes the formalization of the consequence of the firm's decision.

3.2 Stage 2: a hierarchical relationship with a working executive

In second stage, by convention we assume opponent i has won the tournament and becomes the principal and opponent j becomes the agent. We define the function of production which is totally different than the function of production in first stage except that the competence evaluated during tournament is still required during stage 2. This assumption is quite intuitive because otherwise the tournament would not constitute a relevant signal for the firm. Three tasks are exerted but only two competences are required and only one of them is evaluated during the tournament. Each of the three tasks can only take two values: 1 if it is completed and 0 otherwise:

- Task 1 (T_1) is a task exerted by the principal i which is related to a managerial competence. We denote the probability to complete the task: $P(T_1 = 1) =$

$a\hat{\gamma}_{i,v} + \underline{a}(1-\hat{\gamma}_{i,v})$ where $(\underline{a}, a)$ are the low and high managerial competence and $\hat{\gamma}_{i,v}$ is the ex-post probability of having this competence after the tournament. For convenience we assume $\underline{a} = 0$. As we previously specified, only the ability of the principal counts in the second stage and his effort does not count in the function of production.

- Task 2 (T_2) is also a task exerted by the principal i but it is this time related to a business competence. We denote the probability to complete the task: $P(T_2 = 1) = b\hat{\lambda}_{i,v} + \underline{b}(1 - \hat{\lambda}_{i,v})$ where $(\underline{b}, b)$ are the low and high business competence and $\hat{\lambda}_{i,v}$ is the ex-post probability of having this competence. For convenience we also assume $\underline{b} = 0$. As in task 1 the effort of the principal is not taken into account.

- Task 3 (T_3) is a task exerted by the agent j and is related to business competence as well. We denote the probability to complete the task: $P(T_3 = 1) = \pi_{d_j}\hat{\lambda}_{j,d} + \underline{\pi}_{d_j}(1 - \hat{\lambda}_{j,d})$ where $(\underline{\pi}_{d_j}, \pi_{d_j})$ are the high and low business competence given a level of effort $d_j \in \{0, 1\}$ and $\hat{\lambda}_{j,d}$ is the probability of having this competence. As $\hat{\lambda}_{i,v} = 1 - \hat{\lambda}_{j,d}$ we have $P(T_3 = 1) = \pi_{d_j}(1 - \hat{\lambda}_{i,v}) + \underline{\pi}_{d_j}(\hat{\lambda}_{i,v})$ As a contrary to precedent tasks we assume $\underline{\pi}_{d_j} > 0$.

We assume that $\underline{\pi}_{d_j} > 0$ because otherwise they would not be any moral hazard in the side of agent during stage 2. However, for simplicity we will assume there is no hazard moral in the side of the manager which allow us to assume $\underline{a} = 0$ and $\underline{b} = 0$. We also assume that $a < b$ which means that the business task exerted by the principal is more "technical" that the managerial task in a sense that it is more sensitive to competence. Eventually we assume also that $b > \pi_{d_j} - \underline{\pi}_{d_j}$ which means that the business task of the principal is more "technical" than the task of the agent. These two assumptions are important because they justify *a priori* that executive could be selected on business competence.

Then we define the function of production $S \in \{0, 1\}$ as follows:

$$\begin{cases} S = 1 \text{ if } T_1 = 1 \text{ and } T_2 = 1 \\ \qquad \text{or } T_1 = 1 \text{ and } T_3 = 1 \\ S = 0 \text{ otherwise} \end{cases} \tag{2}$$

This function of production describe in expression (2) means that T2 and T3 are similar and substitute to each other, whereas T1 is complementary to those two tasks. It means it is important that at least one of the two business tasks is completed with the managerial task. It captures the idea that any mistake in T2 or T3 can be compensated or corrected by the work of the other. Whereas, T1 is crucial anyway. Then we have:

$$P(S = 1) = P(T_1 = 1)P(T_2 = 1) + P(T_1 = 1)P(T_3 = 1)$$

Then it leads to:

$$P(S = 1 \mid d_j) = a\hat{\gamma}_{i,v}\left[b\hat{\lambda}_{i,v} + \pi_{d_j}(1 - \hat{\lambda}_{i,v}) + \underline{\pi}_{d_j}\hat{\lambda}_{i,v}\right]$$

4 Results

This section is divided into three subsections. The first subsection aims to determine what choice of procedure will be the more efficient to maximize the production in second stage. This issue is related to what procedure maximizes global welfare as the production is fully shared between the principal and the agent. The second subsection deals with what king of legitimacy allows to decrease monetary incentive by the principal by being a substitute to variable wage to implement high effort. The third subsection deals with the issue for the firm in first stage: what kind of procedures is more incentivizing during the tournament. The stake will be to make or not to make a demonstration of strength for competitors to bolster their legitimacies.

4.1 The nature of legitimacy and the efficiency of production

In this section we find conditions that guarantee that an *aristocratic procedure* is more efficient to maximize production in second stage. As the second stage is a principal agent-framework, the production is fully distributed between the principal and the agent such that maximizing the production means implicitly maximizing the global welfare as well. Thus, we focus on a first stage choice for the firm with respect to the equilibrium in second stage assuming that the principal has proposed an incentive wage sufficient to implement high effort from the agent. This backward induction approach allows us to assume that the agent will exert high effort.

As a consequence we must study on a preliminary approach, the influence of the choice of procedures on the beliefs. After the tournament, both opponents observe the level of efforts chosen. Thus, they will revise their beliefs on their abilities with respect to their efforts and performances.

Lemma 1. *Winning the tournament reinforces the belief in having high ability and this effect is stronger if high effort is exerted:* $\hat{x}_{i,v}(1, e_j) > \hat{x}_{i,v}(0, e_j) > x_i$. *However, loosing the tournament reinforces the belief in having low ability and this effect is stronger if high effort is chosen:* $\hat{x}_{i,d}(1, e_j) < \hat{x}_{i,d}(0, e_j) < x_i$.

Proof. See Appendix A □

Lemma 1 means that whatever the procedure, winning the competition reinforce the belief in being competent and so even more if high effort has been exerted. Besides it means that loosing the competition reinforces the belief in being incompetent and so even more when exerting high effort. This is due the complementarity between effort and competence. The more the performance is high the more it reveals that a high competence was necessary because competence exacerbates the effect of effort.

Lemma 2. *The firm has the choice between two procedures. In the aristocratic procedure, at the the end of the tournament both opponents share the following beliefs:*

- *in case of victory* $\hat{\gamma}_{i,v} > \gamma_i$ *and* $\hat{\lambda}_{i,v} = \lambda_i$ *but also* $\hat{\gamma}_{i,v} > \lambda_i$
- *in case of defeat* $\hat{\gamma}_{i,d} < \gamma_i$ *and* $\hat{\lambda}_{i,d} = \lambda_i$ *but also* $\hat{\gamma}_{i,d} < \lambda_i$.

In the meritocratic procedure they will share the following beliefs:

- *in case of victory* $\hat{\lambda}_{i,v} > \lambda_i$ *and* $\hat{\gamma}_{i,v} = \gamma_i$ *but also* $\hat{\lambda}_{i,v} > \gamma_i$
- *in case of defeat* $\hat{\lambda}_{i,d} < \lambda_i$ *and* $\hat{\gamma}_{i,d} = \gamma_i$ *but also* $\hat{\lambda}_{i,d} < \gamma_i$.

Lemma 2 means that choosing an *aristocratic procedure* reinforces the beliefs that the winner owns the managerial competence and the looser does not. However, it preserves the prior belief in having the business ability unaffected. It means that the looser of the competition does not receive a negative signal on his ability to be a competent agent in the future. Whereas, in the *meritocratic procedure* the looser will demonstrate it is less likely he owns the business competence than he previously though. This can be interpreted as a loss of self-confidence if we define self-confidence as the belief in owning a high competence.

Now, let us denote $\Pi^{\gamma}_{j,1} = P(S = 1 \mid d_j = 1)$ the production in second stage in case of an *aristocratic procedure* when agent exerts high effort and $\Pi^{\lambda}_{j,1}$ in case of a *meritocratic procedure*. The stake for the firm is to choose a procedure that maximizes production in second stage (as already mentioned, we will see further in the paper that the firm also have the objective of incentivizing high effort during the tournament). In other word the purpose is to reveal what king of competence the selection procedure should reveal. For now we assume $b > \pi_{d_j} - \underline{\pi}_{d_j}$ which means that the business task of the principal is more "technical" and hard to complete than the task of the agent in a sense that it is more sensible to ability. It means it is *a priori* in the interest of the firm that the principal has more business ability than his agent because it increases the probability that at least one of the the two business tasks (T2 or T3) will be completed. We also assumed that $b > a$ which means that the managerial task is easier than the business task. Those two assumptions imply that it could be *a priori* efficient to select the principal on his business ability.

We get:

$$\Pi^{\gamma}_{j,1} > \Pi^{\lambda}_{j,1} <=> a\hat{\gamma}_{i,v}\Big[\lambda_i(b+\underline{\pi}_1-\pi_1)+\pi_1\Big] > a\gamma_i\Big[\hat{\lambda}_{i,v}(b+\underline{\pi}_1-\pi_1)+\pi_1\Big]$$
$$<=> \hat{\gamma}_{i,v}\lambda_i(b+\underline{\pi}_1-\pi_1)+\hat{\gamma}_{i,v}\pi_1 > \gamma_i\hat{\lambda}_{i,v}(b+\underline{\pi}_1-\pi_1)+\gamma_i\pi_1$$

As $\lambda_i = \gamma_i$ it is straightforward that:

$$\Pi^{\gamma}_{j,1} > \Pi^{\lambda}_{j,1} <=> (\hat{\gamma}_{i,v}-\hat{\lambda}_{i,v})\gamma_i(b+\underline{\pi}_1-\pi_1)+(\hat{\gamma}_{i,v}-\gamma_i)\pi_1 > 0$$

As $\hat{\gamma}_{i,v}-\gamma_i > 0$ according to lemma 2, $\Pi^{\gamma}_{j,1} > \Pi^{\lambda}_{j,1}$ will be verified if and only if $\hat{\gamma}_{i,v} \geq \hat{\lambda}_{i,v}$. Namely, the production will be higher with the *aristocratic procedure* if and only if this procedure allows to reveal a higher or an equal level of ability than the *meritocratic procedure*. However, if the *aristocratic procedure* reveals less ability in the first stage it will not be always verified that this procedure maximizes production in the second stage.

Proposition 1. *To maximize the production in second stage, the firm will choose the aristocratic procedure, that is* $\Pi^{\gamma}_{j,1} > \Pi^{\lambda}_{j,1}$*, even though the business task of the principal is more technical than the business task of the agent,* $b > \pi_{d_j} - \underline{\pi}_{d_j}$*, and more technical than the managerial task,* $b > a$*. This result holds if and only if at the equilibrium the aristocratic procedure reveals an equal or superior level of ability than the meritocratic procedure that is* $\hat{\gamma}_{i,v} \geq \hat{\lambda}_{i,v}$*.*

Nothing at the moment allows us to know which procedure will reveal more ability in equilibrium. This point will be treated in the last subsection. But it is interesting to understand that the objective of production in second stage is conditioned by the incentive issue of procedure in the first stage. Indeed, the choice of the procedure will change the effort of opponents during the first stage such that they will not reveal the same ability and nothing guarantees that in equilibrium $\hat{\gamma}_{i,v} = \hat{\lambda}_{i,v}$.

However, for the sake of the global interpretation of this result, let us assume for one moment that $\hat{\gamma}_{i,v} = \hat{\lambda}_{i,v}$ is verified. The intuition behind proposition 1 would be that bolstering $\hat{\lambda}_{i,v}$ increases the expected efficiency of the principal in task 2 but reduces the expected efficiency of agent in task 3. Even though the task 2 of the principal is more technical and sensible to ability by assumption, the global effect on production of the competition is mitigated by the information the opponent gets about the incompetence of the agent. Let us remind that there are two possibilities to produce a high level of production: completing T1 (managerial task) and T2 (business task of the principal) *or* completing T1 and T3 (business task of the agent). Thus, bolstering $\hat{\lambda}_{i,v}$ increases the chances that T1 and T2 are completed, however it decreases the likelihood that T1 and T3 to be completed. Whereas, this eviction effect will not occur by bolstering $\hat{\gamma}_{i,v}$, as T1 is complementary to the other tasks. As a consequence bolstering $\hat{\gamma}_{i,v}$ increases simultaneously the chances that T1 and T2 are completed and that T1 and T3 are completed.

Now, it is quite intuitive that this result will also holds if $\hat{\gamma}_{i,v} > \hat{\lambda}_{i,v}$. Indeed, it is quite intuitive that if the effect of an increase of $\hat{\gamma}_{i,v}$ is more efficient because it counts twice, that is all the more the case if this increase is superior to the increase of $\hat{\lambda}_{i,v}$. On the contrary, if the increase of $\hat{\lambda}_{i,v}$ is superior this could perfectly compensate the double effect of the increase of $\hat{\gamma}_{i,v}$ in the aristocratic procedure.

4.2 The nature of legitimacy and inequality of wage

The purpose of this section is to show how aristocratic legitimacy may be a substitute to variable wage as an incentive mechanism; whereas meritocratic legitimacy on the contrary increases moral hazard. Thus, we will focus on the equilibrium behavior in second stage.

Now we assume in stage 2 that the principal can only offer a wage w_j in case of high production and that in case of low production the wage will be null. This will be equivalent to a limited liability constraint in a sense that the principal will be limited to make the agent accountable for his failure. Thus, it will make moral hazard an issue. The payoff of the principal is then very simple $P(S = 1 \mid d_j = 1)(1 - w) + P(S = 0 \mid d_j = 1)(0 - 0) = P(S = 1 \mid d_j = 1)(1 - w_j)$. If we denote

$P(S=1 \mid d_j \in \{0,1\}) = \Pi_{j,d_j}$, the program of the principal is then:

$$\max_{w_i} \quad \Pi_{j,1}(1-w_j)$$

$$w.r.t. \qquad \Pi_{j,1}w_j - \psi_2 \geq \Pi_{j,0}w_j \tag{3}$$

$$\Pi_{j,1}w_i - \psi_2 \geq 0 \tag{4}$$

This program is straightforward to resolve. Indeed, only saturating the incentive constraint, that is to say equation (4), is sufficient and leads to $w_j = \frac{\psi_2}{\Pi_{j,1}-\Pi_{j,0}}$. If we denote $\overline{w}_j$ the equilibrium wage, this leads directly to lemma 3.

Lemma 3. *The equilibrium wage in second stage is:*

$$\overline{w}_j = \frac{\psi_2}{a\hat{\gamma}_{i,v}[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \Delta\underline{\pi})]}$$

As a consequence,

- $\frac{\partial \overline{w}_j}{\partial \hat{\gamma}_{i,v}} < 0$*: increasing the managerial competence, that is to say* $\hat{\gamma}_{i,v}$ *increases, allows to decrease the incentive wage*
- $\frac{\partial \overline{w}_j}{\partial \hat{\lambda}_{i,v}} > 0$*: increasing the business competence, that is to say* $\hat{\lambda}_{i,v}$ *increases, obliges to improve incentive wage.*

Proof. See appendix B □

Lemma 3 means that the incentive wage does not depend on the business ability b of the principal but only on his managerial ability a and on the business ability of the agent. So far, we compared only relative values by focusing on the variation of $\overline{w}_j$. Let us now focus on absolute value and denote $\overline{w}_j^{\lambda}$ the incentive wage in case of a *meritocratic procedure* and $\overline{w}_j^{\gamma}$ the incentive wage in case of an *aristocratic procedure*. This leads to proposition 2.

Proposition 2. *If* $\lambda_i = \gamma_i$ *then* $\overline{w}_j^{\lambda} > \overline{w}_j^{\gamma}$*: the aristocratic procedure always allows to decrease incentive wage relatively to the meritocratic procedure.*

Proof. See appendix C □

Proposition 2 means that the incentive wage will *always* be inferior in the *aristocratic procedure*. It is due to the complementarity between task 1 and task 3 whereas task 2 and task 3 are substitutable to each other. The interpretation is that the managerial task of the principal exacerbates the marginal return of the agent's effort in task 3 such that having a principal with high managerial abilities makes him more optimistic about contributing to make the production of the team completed. On the contrary, if the principal as a high business ability it will not enhance the agent's productivity. Then the agent will rely on him and it will tend to incentivize him to free riding behavior.

Besides, as agent compares himself with the principal with respect to business task, increasing certainty about principal's competence on this task decreases agent self confidence simultaneously. So, the more the principal will be good at the business task, the more the agent will think he is himself incompetent. This *intimidation mechanism* is the core of this paper and can be seen as a unexpected side effect of tournament. He will then be more pessimistic about his chances of completing task 3 and will demand a higher wage to exert high effort. The fact the gain will be less likely in the eyes of the employee will have to be compensated by the fact the gain will be higher. In some ways, the looser of the competition becomes a burden for the winner whereas in the *aristocratic procedure* this incentive issue is not relevant because the agent has his self-confidence about his business ability preserved in a sense that he belief in owning business ability remains unchanged (lemma 2).

This proposition undertones that a principal selected on a managerial task will be more "accepted" in a sense that he will less need to use strong incentives to motivate the agent. It is because the ability the principal owns does not send a bad signal to the agent on his own business ability. He cannot compare himself to the principal such that he does not feel disparaged by his promotion. In other words, after the competition, the self-confidence of the looser is preserved because the task he will complete once agent is different than the selection task. For him, stage 2 will become a fresh start. The consequence is that in a *meritocratic procedure*, the leader will be rejected by his peers because they will feel disregarded of not being promoted. It also suggest that an holistic organization organized with social class (in a sense that a social class would be defined by a shared competence among people that constitute it) will favor more acceptation of leader that a more egalitarian organization. Indeed some one "different", with a different ability or even a different social background or social belongings will be more accepted because it will neutralize rivalries issues.

It also suggests that the hierarchy *between* profession will be more accepted that a hierarchy *within* a profession. A solution would be to choose a criteria different than "experience" or an ability based on passed achievement but to choose a neutral skill specific to leaders tasks to select the leader. In an organization such as audit firms it would be relevant at the moment of co-opting to use commercial abilities or managerial abilities rather than technical skills in accounting for instance to justify promotion in order to maintain the looser motivated. This could be a way to institutionalize the transformation from a profession of employee to another profession: the profession of manager.

4.3 Culture of humility and culture of performance

The purpose of this section is to study the behavior of opponents during the tournament with respect to the procedure implemented by the firm. The firm wants to choose the procedure that incentivizes the most to exert high effort for both competitors. This issue is directly related to how the more incentivizing procedure will be a crucial stake to maximize global production in stage 2 (cf. proposition 1). As a preliminary work, we need to establish the payoff with respect to a constant effort in first stage in case of success at the tournament (principal side) but also in case of failure (agent side). Finally we will compare the marginal return of effort of the first stage in the two procedures to study the incentive issue.

We denote P_{P_i} the payoff of opponent i in second stage if he becomes principal and $W_j = \Pi_{j,1}\frac{\psi_2}{\Pi_{j,1}-\Pi_{j,0}}$ the expected wage given by i to j. It is straightforward to show that:

$$
\begin{aligned}
P_{P_i} =& \Pi_{j,1} - W(\hat{\gamma}_{i,v}, \hat{\lambda}_{i,v}) \\
P_{P_i} =& a\hat{\gamma}_{i,v}\left[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\right]\left[1 - \frac{\psi_2}{a\hat{\gamma}_{i,v}[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}\right]
\end{aligned}
$$

Furthermore, we denote P_{A_i} the payoff of i if he becomes agent in second stage.

$$
\begin{aligned}
P_{A_i} =& W(\hat{\gamma}_{j,v}, \hat{\lambda}_{j,v}) - \psi_2 \\
P_{A_i} =& a\hat{\gamma}_{j,v}\left[\hat{\lambda}_{j,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\right]\frac{\psi_2}{a\hat{\gamma}_{j,v}[\Delta\pi - \hat{\lambda}_{j,v}(\Delta\pi - \underline{\Delta\pi})]} - \psi_2
\end{aligned}
$$

Lemma 4. *For opponent i's payoff we have the following properties:*

- $\frac{\partial P_{P_i}}{\partial \hat{\gamma}_{i,v}} > \frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}}$
- $\frac{\partial P_{A_i}}{\partial \hat{\gamma}_{i,d}} > \frac{\partial P_{A_i}}{\partial \hat{\lambda}_{i,d}}$

Proof. See appendix D □

So far we focused on derivatives issues with respect to both natures of legitimacy, let us now study and compare the payoffs in the two procedures. For this, we denote $P^{\gamma}_{P_i} = P^{\gamma}_{P_i}(e_i)$ and $P^{\lambda}_{P_i} = P^{\lambda}_{P_i}(e_i)$ respectively the payoff of i if he is the principal in second stage in case of an *aristocratic procedure* and a *meritocratic procedure* with respect to the choice of effort $e_i \in \{0, 1\}$ during the tournament. Then we have $P^{\lambda}_{P_i}(e_i) = \Pi_{j,1}[\hat{\lambda}_{i,v}(e_i)] - W[\hat{\lambda}_{i,v}(e_i)]$ and $P^{\gamma}_{P_i}(e_i) = \Pi_{j,1}[\hat{\gamma}_{i,v}(e_i)] - W[\hat{\gamma}_{i,v}(e_i)]$. Similarly we denote $P^{\gamma}_{A_i} = P^{\gamma}_{A_i}(e_i)$ and $P^{\lambda}_{A_i} = P^{\lambda}_{A_i}(e_i)$ respectively the payoff of i if he is the agent in second stage in case of an *aristocratic procedure* and a *meritocratic procedure* with respect to the choices of efforts of both opponents. Thus we have $P^{\gamma}_{A_i}(e_i) = W[\hat{\gamma}_{i,v}(e_i)] - \psi_2$ and $P^{\lambda}_{A_i}(e_i) = W[\hat{\lambda}_{i,v}(e_i)] - \psi_2$. Then lemma 4 leads to proposition 3.

Proposition 3. *Relatively to the aristocratic procedure, the meritocratic procedure provides insurance for the looser of the competition, that is to say for opponent i's payoff we have the following properties:*

- $P^{\gamma}_{P_i}(e_i) > P^{\lambda}_{P_i}(e_i)$
- $P^{\gamma}_{A_i}(e_i) < P^{\lambda}_{A_i}(e_i)$

Proof. See appendix E □

Proposition 3 means that the *meritocratic procedure* as a function of insurance for the looser because it contains the amplitude of the payoffs in case of victory or defeat and that for two reasons. Firstly it is a pure monetary insurance because loosing the competition reveals he is not competent and thus, it will require a higher wage to incentivize him (cf Proposition 2). Secondly it is also a cognitive insurance in a sense that it maintains an optimistic informational environment for the agent. Indeed, loosing the competition gives him information about the high business ability of his leader in task 2. As a consequence he will expect a more likely success in completing the task and receiving indeed his wage, thanks to the ability of the principal. For this reason the *meritocratic procedure* allows him to have a free riding behavior because he will receive his wage whatever task 2 (exerted by the principal) or task 3 (exerted by the agent) is completed. And so even without exerting high effort because he will always be able rely on the executive's business ability. This is due to the substitutability of the two business tasks but it is also due to the assumption that task 2 is more "technical" than task 3 which was necessary to justify *a priori* that the principal should be selected on his business ability instead of his managerial ability.

On the contrary, the *aristocratic procedure* does not provide insurance to the agent. Indeed, the managerial ability exacerbates the marginal return of the agent's effort such that it will not be necessary to provide a high wage to the agent to incentivize him. And so, in the same proportion that it exacerbates the probability of producing a high level of production for the whole team. It finally has no impact on the rent of the agent. In other words, the cognitive insurance is exactly compensated by the decrease of the incentive wage. Besides, in case of victory, the principal will not have to deal with an incentive issue due to the discouragement of the agent so that his payoff will be higher.

As a consequence, paradoxically, with respect to a constant effort the *meritocratic procedure* rewards more the losers (that is to say the "undeserving" opponent) and penalizes the winner (that is to say the "deserving" opponent) relatively to the *aristocratic procedure*.

From now we are going to compare not only the payoff with respect to a constant effort like we have done until now, but the marginal return of effort during tournament with the *aristocratic procedure* ΔP^{γ} and with the *meritocratic procedure* ΔP^{λ}. The purpose is to study the incentive issue of both procedures on efforts in the first stage. Indeed, it is assumed that the firm wants to choose the procedure of selection that will reveal as much as possible the ability of the winner and thus expect the competition to incentivize the highest effort. Indeed, it is quite intuitive that the firm designs a tournament to reveal as much competence as possible in order to maximize production in stage 2. We denote ΔP^{γ} the marginal return of effort in stage 1 in the *aristocratic procedure* and ΔP^{λ} the marginal return of the *meritocratic procedure*. The terms $P_W(1)$ and $P_L(1)$ are the probability of winning and loosing the tournament exerting high effort and the term $P_W(0)$ and $P_L(0)$ are the probability of winning and loosing the tournament exerting low effort. Thus we have:

$$\Delta P^{\gamma} = P_W(1)P_{P_i}^{\gamma}(1) - P_W(0)P_{P_i}^{\gamma}(0) + P_L(1)P_{A_i}^{\gamma} - P_L(0)P_{A_i}^{\gamma}$$
$$\Delta P^{\lambda} = P_W(1)P_{P_i}^{\lambda}(1) - P_W(0)P_{P_i}^{\lambda}(0) + P_L(1)P_{A_i}^{\lambda}(1) - P_L(0)P_{A_i}^{\lambda}(0)$$

The issue is not trivial because comparing absolute payoff in the second stage is not enough to infer marginal payoff in the first stage. Indeed, the second stage payoffs are endogenous and depend actually on the effort of the first stage as well, and so through the bayesian learning on competence. It is easy to decompose the spread between marginal payoff into a "first-stage effect" which is the variation due exclusively to the probability of winning and loosing the tournament and into a "second-stage effect" due exclusively to the variation of second stage's payoff with respect to effort of the first stage. That leads to:

$$\Delta P^{\gamma} - \Delta P^{\lambda} = \overbrace{\Big[P_W(1) - P_W(0)\Big]\Big[P^{\gamma}_{P_i}(0) - P^{\lambda}_{P_i}(0)\Big] + \Big[P_L(1) - P_L(0)\Big]\Big[P^{\gamma}_{A_i} - P^{\lambda}_{A_i}(0)\Big]}^{\text{first stage effect}}$$
$$\underbrace{+ P_W(1)\Big[P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0) - [P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)]\Big] - P_L(1)\Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big]}_{\text{second stage effect}}$$

Let us focus on the "first-stage effect" for now. We know that $P_W(1)-P_W(0) > 0$ and $P^{\gamma}_{P_i}(0) - P^{\lambda}_{P_i}(0) > 0$ (cf. proposition 3). Exerting high effort increases the likelihood of winning the competition and the spread of payoff between the aristocratic and the meritocratic procedures in case of victory is positive such that the global effect is positive:

$$\Big[P_W(1) - P_W(0)\Big]\Big[P^{\gamma}_{P_i}(0) - P^{\lambda}_{P_i}(0)\Big] > 0$$

Besides $P_L(1) - P_L(0) < 0$ and $P^{\gamma}_{A_i} - P^{\lambda}_{A_i}(0) < 0$ (cf proposition 3) which leads to:

$$\Big[P_L(1) - P_L(0)\Big]\Big[P^{\gamma}_{A_i} - P^{\lambda}_{A_i}(0)\Big] > 0$$

Increasing effort decreases the probability of loosing but the spread of payoff between the aristocratic and the meritocratic procedures in case of defeat is negative. Thus the opponent loses more but has less chances to lose such that the global effect remains positive.

Thus, $\Delta P^{\gamma} > \Delta P^{\lambda}$ will be verified if the last term, representing exclusively the "second stage effect", is positive. As $P_W(1) > P_L(1)$ this will be verified if:

$$\Big[P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0)\Big] - \Big[P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)\Big] - \Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big] + \Big[P^{\gamma}_{A_i}(1) - P^{\gamma}_{A_i}(0)\Big] > 0$$

We add the term $\Big[P^{\gamma}_{A_i}(1) - P^{\gamma}_{A_i}(0)\Big]$ because it is null and it improves the understanding of the expression. Indeed, this means that the marginal payoff of the second stage in the *aristocratic procedure* must be superior to the marginal payoff of the second stage in the *meritocratic procedure*. Thus the first stage will influence the result only through the bayesian inference and not through the probability of wining or losing. This is actually quite intuitive because for both procedures, the assessment technology remains the same such that the first stage will not be a key determinant to compare behaviors in the two procedures.

Lemma 5. $\Delta P^{\gamma} > \Delta P^{\lambda}$ *if in the meritocratic procedure, by exerting effort the marginal informational rent paid in case of victory is superior to the marginal informational rent received in case of defeat, that is:* $W_j^{\lambda}(1) - W_j^{\lambda}(0) \geq W_i^{\lambda}(1) - W_i^{\lambda}(0)$*.*

Proof. See appendix F □

Lemma 5 means that to compare the marginal payoff in the *aristocratic procedure* and the *meritocratic procedure* it will be enough to compare the increase of the rent paid in case of victory to the increase of the rent received in case of defeat with respect to the effort exerted during the tournament (efforts of the first stage interfering through the learning process on abilities). It is because the marginal production will be always higher in *aristocratic procedure* (cf. proposition 1), such that only the cost of delegation (related to moral hazard issue) in the global payoff becomes a stake in case he will be the principal. In other words, he knows that the production will be higher but he does not know if the incentive issue will be aggravated or not. It also means that it is sufficient to focus only on the *meritocratic procedure* mostly because the marginal payoff of agent in case of the *aristocratic procedure* is null.

Lemma 6. *In the meritocratic procedure, the marginal rent paid in case of victory will be higher than the marginal rent received in case of defeat that is:* $W_j^{\lambda}(1) - W_j^{\lambda}(0) \geq W_i^{\lambda}(1) - W_i^{\lambda}(0)$ *is verified if:*

- $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ *and* $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$
- *and if* $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ *and* $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$

Proof. See appendix G □

Lemma 6 means that in the meritocratic procedure, the spread between the rent paid in case of victory and the rent received in case of defeat depends on the assessment technology, or in other words, on the posterior beliefs. This is due to the fact that the function of production is the same in second stage. Only the weight of one competence relatively to the other will change. This implies that lemma 6 is verified if we have an "asymmetric" assessment technology, that is to say $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ and $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$ but also $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ and $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$. To understand the intuition of this result we can present expression $W_j^\lambda(1) - W_j^\lambda(0) \geq W_i^\lambda(1) - W_i^\lambda(0)$ in the following way:

$$\underbrace{\left[W_j^\lambda(1) - W_i^\lambda(1)\right]}_{\text{Loss of exerting high effort}} + \underbrace{\left[W_i^\lambda(0) - W_j^\lambda(0)\right]}_{\text{Gain of exerting low effort}} > 0 \tag{5}$$

Let us focus now on the point of view of agent i. The intuition is that, in case his opponent j exert low effort, $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ will imply that $W_j^\lambda(1) > W_i^\lambda(1)$. And then, $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$ will imply that $W_j^\lambda(0) = W_i^\lambda(0)$ such that expression (5) will be verified. Furthermore, in case this time opponent j exerts low effort, $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$ will imply $W_i^\lambda(0) > W_j^\lambda(0)$. Then, $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ will imply $W_i^\lambda(1) = W_j^\lambda(0)$ such that expression (5) will be also verified.

Let us interpret more extensively the first case when agent j exerts low effort. Expression $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ means that in the case agent j exert low effort, and if agent i exerts more effort than him, the competence agent i will reveal if he wins will be superior to the competence his opponent will reveal if his opponent wins. Yet, we know that the marginal rent received by the agent in stage 2 is increasing with the competence of the principal as the agent will rely on principal's ability and learn about his own lack of ability (cf. lemma 4) such that the principal will have to increase the wage of the agent to avoid free riding behavior. As a consequence for agent i, increasing his effort will imply that the marginal rent he would paid in case of victory will be superior to the marginal rent he would received in case of failure. Such that by increasing his effort, he will decrease his payoff if he becomes principal and increase his payoff if he becomes agent. Thus, he has an incentive to exert low effort to reveal as less as possible his ability. If he chooses to exert low effort, then $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$ that will imply $W_i^\lambda(0) = W_j^\lambda(0)$ such that the rent he would paid in case of victory would be exactly identical to the rent he would received in case of defeat.

Now, let us explain the case when his opponent agent j exert high effort. Indeed, in that case, $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$ will imply $W_i^\lambda(0) > W_j^\lambda(0)$. Expression $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$ means that if agent i exerts low effort, then his victory will be a weak signal about his ability in comparison with the signal his defeat will reveal about the ability of his opponent. It suggests that there is an opportunity cost of exerting high effort. Indeed, in case of defeat it becomes more interesting to be seen as absolutely incompetent to be able to completely rely on the winner's ability and receive a higher rent. For this reason, there is an incentive to exert low effort to reveal less ability. On the contrary, if he decides to exert high effort he would receive exactly the same rent in case of defeat than the rent paid in case of victory as $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ than implies $W_i^\lambda(1) = W_j^\lambda(0)$.

Proposition 4. *Relatively to the aristocratic procedure, the meritocratic procedure favors a culture of humility that is to say $\Delta P^\gamma > \Delta P^\lambda$ is verified if $\lambda_i = \frac{1}{2}$, $\pi_1 < 1-\underline{\pi}_1$ and $\pi_0 < 1-\underline{\pi}_0$:*

Proof. See appendix H □

Proposition 4 means that, if the condition $\lambda_i = \frac{1}{2}$, $\pi_1 < 1-\underline{\pi}_1$ and $\pi_0 < 1-\underline{\pi}_0$ are verified, the *meritocratic procedure* will reduce the marginal return of effort relatively to the *aristocratic procedure* whatever the effort of the opponent. It means the *meritocratic procedure* would be less incentivizing and would favor less the emergence of high effort as a dominant strategy for both competitors. It is because those conditions will allow the conditions of lemma 6 to hold that is: $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ and $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$ but also $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ and $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$. By ripple effect, it will imply that lemma 5 will be verified as well and $\Delta P^\gamma > \Delta P^\lambda$ will hold.

The condition $\lambda_i = \frac{1}{2}$ is straightforward to understand: it means that the competition is *fair* in a sense that none of the opponents starts being *a priori* advantaged. As we assumed $\lambda_i = \gamma_i$, it means it must be verified for both competences. It becomes then quite intuitive that $\hat{\lambda}_{i,v}(1,1) = \hat{\lambda}_{j,v}(1,1)$ and $\hat{\lambda}_{i,v}(0,0) = \hat{\lambda}_{j,v}(0,0)$ will be verified. Indeed, it is quite intuitive that if $\lambda_i = \frac{1}{2}$ (that is to say $\lambda_i = \lambda_j$) and if both opponents choose the same strategy, they will learn exactly the same information on there ability in case of victory or defeat such that $\hat{\lambda}_{i,v} = \hat{\lambda}_{j,v}$ will hold.

The additional conditions $\pi_1 < 1 - \underline{\pi}_1$ and $\pi_0 < 1 - \underline{\pi}_0$ are the one which guarantee that the assessment technology is not symmetric that is to say $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ and $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$. Expression $\pi_1 < 1 - \underline{\pi}_1$ means that with high effort the probability of achieving a high level of production with the high competence will be lower than the probability of achieving a low level of production with the low competence. Similarly, expression $\pi_0 < 1 - \underline{\pi}_0$ means that with a low effort this time, the probability of producing a high level of production with high competence is inferior to the probability of producing a low level of production with low competence. We interpret it means the incompetence is more disadvantageous than the competence is advantageous. These conditions may be interpreted as the conditions that guarantee the *a priori* belief in loosing the competition is higher than the *a priori* belief in winning. And so because even thought incompetence or competence are equally likely (because $\lambda_i = \frac{1}{2}$), being incompetent is more disadvantageous. As a consequence it will make success more informative on competence than failure is informative on incompetence. Indeed, the intuition is that if i wins, he will infer that a high probability of owning the competence must have compensated the fact that competence does not provide a high advantage on the production. Whereas in case of failure, the very disadvantageous effect of incompetence might be sufficient to explain defeat. Much more certainty on owning the competence is required to be optimistic about winning than certainty on owning incompetence is required to be sure about loosing because incompetence is more disadvantageous. In some ways, winning is more "surprising" and implies a high revision of belief relatively to defeat. Those assumptions guarantee that by increasing his effort, agent i will reveal more ability if he wins that his opponent will reveal ability if he becomes principal. As a consequence the marginal informational rent paid in case of victory will be superior to the marginal informational rent received in case of defeat. This will imply that lemma 6 will be verified because the increase of i's effort will make i less hard to incentivize in case he fails than his opponent j. This in turn will imply lemma 5 to be verified as well namely $\Delta P^{\gamma} > \Delta P^{\lambda}$.

Let us explain more extensively what could be the interpretation of proposition 4. The first interpretation would be in term of leadership strategy and managerial culture. Proposition 4 suggests how a culture of humility can emerge in an organization if a *meritocratic procedure* is chosen whereas a culture of performance would emerge if an *aristocratic procedure* is chosen. Indeed, in a *meritocratic procedure* opponents will anticipate that the more they outperform their competitors, the more they will discourage him in the future and the more they will have to offer him a high wage to incentivize him. As a consequence it becomes interesting to decrease effort in order to hide one's strength in some ways and to keep low profile. We interpret this as an *identification mechanism*, in a sense that the looser will still believe, in spite of defeat, that him and the winner have a close level of competence, and observing the success of his opponent, he will keep good faith in his future achievements as an employee. On the contrary, the *aristocratic procedure* will tend to favor a totally opposite leadership strategy based on a *differentiation mechanism* where the leader needs to be seen as an exceptional being capable of making extraordinary achievement that average people are not able to. In the *aristocratic procedure*, the more the winner outperformed his opponent, the more it will increase the productivity of the agent in the future without making him having a pessimistic belief about his ability in business task because of his failure at the tournament. This will imply that the more the principal has demonstrated a managerial ability, the more the incentive wage will be lower and the share of the output the principal will keep for himself will be higher.

Proposition 4 might shed a new light on academic failure in professionalizing courses. Indeed, we can interpret the *aristocratic procedure* as prestigious high education system based on generalist training (those training teaching knowledge not related to a specific operational task). The model suggests then that in such procedure of selection opponents are more incentivized to exert high effort and they are more incentivized to reveal ability. The reason is the perspective of being propelled at the top of the hierarchy without making monetary concession. Symmetrically, we can interpret the *meritocratic procedure* as professionalizing courses designed to train to specific demand focused on operational task. Proposition 4 suggests that the less attractive wage in case of success will not be very incentivizing such that opponents might choose not to exert effort and not to reveal competence. We might interpret this lack of motivation as a reason for academic failure in those kind of trainings.

We argue that proposition 4 can also explain a situation of blockage and stasis in a fratricidal struggle. Indeed we could interpret the noisy signal after tournament as a general preference of the organization on an idea or a project and not specifically on a competence. Competing would then consist in elaborating an idea or a project that would seduce the most the organization and that would be designed to be applied in case of victory. As a consequence, the model could capture a situation where two persons compete to become the leader of a team and have different political projects or operational visions for the team (which is a transposition of the business task) but they have the *a priori* belief that both projects have equally chances to be favored by the organization (which is a transposition of the uncertainty on business ability). Loosing the competition will send a signal that his vision for the team will not be taken into account and valued by the firm and will tend to discourage the looser. Anticipating that, rivals might have interest in not investing to much effort in differentiating one from another about their operational visions for the team. This operational vision can represents a political program for instance during primary election of political parti (in this example the political parti would delegate the evaluation to electors). Two opponents might have different political programs within the same political family that represents different political currents. Proposition 4 suggests that it will be in their interests to forsake the idea of investing to much effort in elaborating a high quality political program (i.e a political program that would be widely valued by the electors) in fear of discouraging to much their opponents in case of victory. Indeed, the latter would then interpret is defeat as a signal that his political ideas will not be taken into account in the future. Thus, if second stage represents for instance parliamentary election, the winner of the primary election might then have to make concession on ministerial positions or on member of the parliament positions if he wants his former rival to support him during the national election campaign (in that example, the proportion of ministerial position representing the political current of the looser is a transposition of the incentive wage in the general case). The organization might anticipate that and choose another criteria based on managerial ability or human qualities or more generally on criteria related to the personalities of the opponents and not their programs. The political parti might even define in advance (before the primary election) the major elements of the political program of the parti. It also can be a tacit agreement between the opponents that anticipate the future issue of gathering the different political currents and choose to focus the primary campaign on a different criteria. The consequence could be notably the emergence of ideological vacuum in political campaign.

5 Conclusion

The aim of this paper is to study the role of the nature of competence to bolster the legitimacy of an executive. We focus on the one hand on a managerial ability as a criteria of selection and on a second hand on a business ability as another criteria. The first one is easy to master but it is crucial for the production of the team (executive and employee), whereas the second one is very technical and requires a high ability but can be compensated by the ability of the employee. The implicit issue in this work is that the demonstration of competence of the executive may be either humiliating or reassuring for the employee. If it is humiliating it will imply a rejection, whereas if it is reassuring it will imply acceptation.

Our first result consists in showing that the demonstration of strength will be humiliating if the achievement is on the same task than the one the employee must master himself. However, it will be reassuring if the achievement is on a complementary task such as managerial task. For this reason, our work shed a new light on a substitute to regular monetary incentives. Indeed, the strategy of the firm will be focused on changing the informational environment on the self-confidence of the employee and his confidence in the ability of his executive. In general an model of incentive assume exogenous beliefs and monetary incentive must then take this into account. In this work we made theses beliefs endogenous. Thus revealing the business competence of the executive will reveal the incompetence of the agent and diminish the marginal return of his effort which would necessitate a higher wage in term of incentive. It will be necessary to propose a higher wage to compensate the fact that in the eyes of the agent, his chances of making a high production are little. On the contrary the managerial ability will have a opposite effect: it will increase the marginal return of the employee's effort and it will enable low remuneration.

Our second result is about the incentivize impact of the choice of the evaluation task on effort during the first stage. We showed that under certain assumption an *aristocratic procedure* is more incentivizing than a *meritocratic procedure*. We interpret that such that in a *meritocratic procedure*, contrary to an *aristocratic procedure*, it is more in the interest of opponents to reduce their performances by decreasing their effort in order to hide their abilities to each other. Then they will preserve the self confidence of their future employees in case of victory and it will be less costly to incentivize him. It suggests how a culture of humility could emerge in an organization whereas an *aristocratic procedure* would favor the emergence of a culture of performance.

Our third result is about the strategy of the firm if it wants to maximize the production once the executive is selected. Actually, it will be also efficient to choose managerial ability even though this ability is much easier to master than the business ability and even though the business task of the manager is harder than the one of employee. Its complementary nature will make it crucial to enhance to efficiency of the whole team whereas the business ability will only enhance this production of the executive. In other words, it is not efficient to reveal information about business ability of the manager because it reveals information about the incompetence of the employee. Besides, as an *aristocratic procedure* is more incentivizing it guarantees that at the equilibrium more ability will be revealed. This guarantees that at the equilibrium an *aristocratic procedure* will maximize production in second stage.

This model suggests several applications. The first result sheds new light on the issue of rejection of authority and more specifically the rejection by the peers. Indeed, it suggests that an employee will accept more a leader selected on a different ability than his own ability. Observing a peer being promoted instead of him would make him feel humiliated by the organization and will entail a lack of motivation to work. Now if we interpret business ability and managerial ability as characteristics of social belonging, it suggests that holistic societies organized by social class will imply more acceptation of authority than egalitarian society. It also suggests that the hierarchy *between* professions, (established through an *aristocratic procedure* such as high education) will be more accepted than the hierarchy *within* a profession (established through a *meritocratic procedure* such as demonstration of business ability in a career).

The second result might be interpreted such that education based on professionalizing courses designed to meet the demands of firms on business task will favor academic failure. Whereas high education based on more generalist trainings will allow to reveal more ability. Indeed, in professionalizing courses opponents anticipate that their rewards in case of victory could not be enough relatively to their rewards in case of failure such that it is not even worth working. What is quite counter intuitive in this prediction about academic failure is that this is not due to the actual competence of opponents (as both opponents are assumed *a priori* equally competent in business ability and managerial ability in both procedures) but it is due to egalitarian culture that emerges through the procedure of selection. On the contrary, in generalist training the reward in case of success is higher because inequality of wage will be much more accepted and as a consequence, opponents will be much more incentivized to reveal abilities.

We argue our second result could also explain a situation of blockage in fratricidal struggle and particularly in political parti when it comes to choose a leader and candidate for the next national election. Indeed, it is possible to interpret the signal at the end of the tournament not as a competence like we did previously, but as information on the preference of the organization on an idea, a project or a way of governing. Thus, exerting effort would consist in either elaborating a political program based on political ideology (which would be a transposition of the business ability we have previously discussed) or to develop a reputation on one's way of governing (which would be a transposition of managerial ability). Our result suggests that each opponent anticipates that if the organization widely prefer his program relatively to the other, the looser of the competition will infer that his political program will never be valued by the parti. In the perceptive of national election he might demand high concession (on ministerial position for instance) to bring his support to the new leader during the forthcoming campaign. This will be an incentive for a future leader to forsake the idea of exerting too much effort in elaborating a political program during the tournament to limit the extent of his victory. The parti might anticipate that and deliberately choose to select a leader on a different criteria such that his way of governing or more generally his personality. This could explain notably ideological vacuum in primary election for instance.

One of the limit of our work is how the first stage is modeled. Indeed first we did not formalize the behavior of the organization as a maximization program and we only focused on the spread between the marginal payoff of both procedures for opponents. This allowed us to capture the incentive issue for the organization during the tournament but it concealed some complexity. Indeed, the organization might support some cost implementing a procedure that would be different between an *aristocratic procedure* and a *meritocratic procedure*. We could notably imagine that an *aristocratic procedure* would generate a higher cost for a firm. Indeed, it could be easier for a firm to assess on business ability because it is precisely the ability on which workers are specialized. It could be also less costly as it could be sufficient to observe achievements of an employee on a productive task whereas the assessment of managerial ability requires a probation period which generates then an opportunity cost. This could explain notably why a *meritocratic procedure* could remain implemented in organization whereas an *aristocratic procedure* seems finally more incentivizing and productive.

Another limit of our work is assumptions on the production function in stage 2. We could imagine a case where there are no uncertainty on the managerial ability required for the principal. It would capture a case where the managerial task is so easy to master that it would be reasonable to assume that anybody would be able to master it. As a consequence, revealing ability of the principal on this task would be useless. This assumption would naturally modify the results of the model and we could imagine to find conditions that would make a *meritocratic procedure* more efficient for the global output and more incentivizing during the tournament. It could also be interesting to find the conditions that guarantee that in spite of the managerial task being mastered by anyone, it would be still more efficient to select leader on this criteria because of incentives issues in second stage due to rivalry's issues.

Finally, in further research we think it would be interesting to assume that principal and agent never compete against each other but enter the firm at a different time. This assumption would may be allow to neutralize the rivalry's issue like the choice of managerial ability as criteria of selection did, guaranteeing simultaneously that the organization could select on business competence. This would avoid the cost of establishing probation period to evaluate managerial ability. Besides, we could imagine that being aware his executive was selected on business ability would be highly motivating to work hard for an employee. A high employee turnover would then become a major stake for the firm to avoid rivalry to emerge.

Appendix

A/ Proof lemma 1

- We will have $x_{i,v}(1, e_j) > x_{i,v}(0, e_j)$ verified if $\frac{\partial x_{i,v}(e_i,e_j)}{\partial e_i} > 0$. We know that:

$$x_{i,v}(e_i, e_j) = \frac{x_i \pi_{e_i}(1 - \underline{\pi}_{e_j})}{x_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - x_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})}$$

We denote $D = x_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - x_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})$. And we know that $\pi_{e_i} = e_i\theta_H + \epsilon$ and $\underline{\pi}_{e_i} = e_i\theta_L + \epsilon$. Thus we have:

$$\frac{\partial x_{i,v}(e_i,e_j)}{\partial e_i} = \Big[x_i(1-\underline{\pi}_{e_j})\theta_H[x_i\pi_{e_i}(1-\underline{\pi}_{e_j}) + (1-x_i)\underline{\pi}_{e_i}(1-\pi_{e_j})] - x_i\pi_{e_i}(1-\underline{\pi}_{e_j})[x_i(1-\underline{\pi}_{e_j})\theta_H + (1-x_i)(1-\pi_{e_j})\theta_L] \Big] \Big/ D^2$$

Let us focus on the sign of the numerator N.

$$N > 0 = x_i(1-\underline{\pi}_{e_j})\theta_H \Big[x_i\pi_{e_i}(1-\underline{\pi}_{e_j}) + (1-x_i)\underline{\pi}_{e_i}(1-\pi_{e_j}) \Big] - x_i\pi_{e_i}(1-\underline{\pi}_{e_j}) \Big[x_i(1-\underline{\pi}_{e_j})\theta_H + (1-x_i)(1-\pi_{e_j})\theta_L \Big] > 0$$

$$N > 0 = x_i(1-\underline{\pi}_{e_j})\theta_H x_i\pi_{e_i}(1-\underline{\pi}_{e_j}) + x_i(1-\underline{\pi}_{e_j})\theta_H(1-x_i)\underline{\pi}_{e_i}(1-\pi_{e_j}) - x_i\pi_{e_i}(1-\underline{\pi}_{e_j})x_i(1-\underline{\pi}_{e_j})\theta_H - x_i\pi_{e_i}(1-\underline{\pi}_{e_j})(1-x_i)(1-\pi_{e_j})\theta_L > 0$$

It leads to:

$$N > 0 = x_i(1-\underline{\pi}_{e_j})\theta_H(1-x_i)\underline{\pi}_{e_i}(1-\pi_{e_j}) - x_i\pi_{e_i}(1-\underline{\pi}_{e_j})(1-x_i)(1-\pi_{e_j})\theta_L > 0$$

$$N > 0 = x_i(1-\underline{\pi}_{e_j})(1-x_i)(1-\pi_{e_j}) \Big[\theta_H\underline{\pi}_{e_i} - \pi_{e_i}\theta_L \Big] > 0$$

The sign of N only depends on the sign of $\theta_H\underline{\pi}_{e_i} - \pi_{e_i}\theta_L$

$$\theta_H\underline{\pi}_{e_i} - \pi_{e_i}\theta_L = \theta_H(e_i\theta_L + \epsilon) - \theta_L(e_i\theta_H + \epsilon)$$

$$\theta_H\underline{\pi}_{e_i} - \pi_{e_i}\theta_L = \theta_H\theta_L e_i + \theta_H\epsilon - \theta_L\theta_H e_i - \theta_L\epsilon$$

$$\theta_H\underline{\pi}_{e_i} - \pi_{e_i}\theta_L = \theta_H\epsilon - \theta_L\epsilon$$

$\theta_H\epsilon - \theta_L\epsilon > 0$ is always verified. Thus $\frac{\partial x_{i,v}(e_i,e_j)}{\partial e_i} > 0$ and $x_{i,v}(1,e_j) > x_{i,v}(0,e_j)$.

- We will have $x_{i,d}(1,e_j) < x_{i,d}(0,e_j)$ verified if $\frac{\partial x_{i,d}(e_i,e_j)}{\partial e_i} < 0$. We know that:

$$x_{i,d}(e_i,e_j) = \frac{x_i(1-\pi_{e_i})\underline{\pi}_{e_j}}{x_i(1-\pi_{e_i})\underline{\pi}_{e_j} + (1-x_i)(1-\underline{\pi}_{e_i})\pi_{e_j}}$$

We denote $E = x_i(1-\pi_{e_i})\underline{\pi}_{e_j} + (1-x_i)(1-\underline{\pi}_{e_i})\pi_{e_j}$. Thus we have:

$$\frac{\partial x_{i,d}(e_i,e_j)}{\partial e_i} = \Big[-x_i\underline{\pi}_{e_j}\theta_H[x_i(1-\pi_{e_i})\underline{\pi}_{e_j} + (1-x_i)(1-\underline{\pi}_{e_i})\pi_{e_j}] - x_i(1-\pi_{e_i})\underline{\pi}_{e_j}[-x_i\underline{\pi}_{e_j}\theta_H - (1-x_i)\pi_{e_j}\theta_L] \Big] \Big/ E^2$$

Let us focus on the numerator that we denote F.

$$
\begin{aligned}
F = & - x_i \underline{\pi}_{e_j} \theta_H \Big[x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} + (1-x_i)(1-\underline{\pi}_{e_i}) \pi_{e_j} \Big] \\
& - x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} \Big[- x_i \underline{\pi}_{e_j} \theta_H - (1-x_i) \pi_{e_j} \theta_L \Big] \\
F = & - x_i \underline{\pi}_{e_j} \theta_H x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} - x_i \underline{\pi}_{e_j} \theta_H (1-x_i)(1-\underline{\pi}_{e_i}) \pi_{e_j} \\
& + x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} x_i \underline{\pi}_{e_j} \theta_H + x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} (1-x_i) \pi_{e_j} \theta_L
\end{aligned}
$$

Then it leads to:

$$
\begin{aligned}
F = & - x_i \underline{\pi}_{e_j} \theta_H (1-x_i)(1-\underline{\pi}_{e_i}) \pi_{e_j} + x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} (1-x_i) \pi_{e_j} \theta_L \\
F = & x_i \underline{\pi}_{e_j} (1-x_i) \pi_{e_j} \Big[- \theta_H (1-\underline{\pi}_{e_i}) + \theta_L (1-\pi_{e_i}) \Big]
\end{aligned}
$$

As $\theta_H > \theta_L$ and $1-\underline{\pi}_{e_j} > 1-\pi_{e_j}$ then $-\theta_H(1-\underline{\pi}_{e_j}) + \theta_L(1-\pi_{e_j}) < 0$ and $F < 0$. As a conclusion we have $\frac{\partial x_{i,d}(e_i,e_j)}{\partial e_i} < 0$ so $x_{i,d}(1,e_j) < x_{i,d}(0,e_j)$ is verified.

- Let us prove that $x_{i,d}(e_i,e_j) < x_i$. We have:

$$
\frac{x_i (1-\pi_{e_i}) \underline{\pi}_{e_j}}{x_i (1-\pi_{e_i}) \underline{\pi}_{e_j} + (1-x_i)(1-\underline{\pi}_{e_i}) \pi_{e_j}} < x_i
$$

After simplifying by x_i, this expression is equivalent to:

$$
\begin{aligned}
(1-\pi_{e_i}) \underline{\pi}_{e_j} (1-x_i) - (1-x_i)(1-\underline{\pi}_{e_i}) \pi_{e_j} & < 0 \\
(1-\pi_{e_i}) \underline{\pi}_{e_j} - (1-\underline{\pi}_{e_i}) \pi_{e_j} & < 0
\end{aligned}
$$

Which is always verified because $(1-\pi_{e_i}) < (1-\underline{\pi}_{e_i})$ and $\underline{\pi}_{e_j} < \pi_{e_j}$.

- Let us prove that $x_{i,v}(e_i,e_j) > x_i$. We have:

$$
\frac{x_i \pi_{e_i} (1-\underline{\pi}_{e_j})}{x_i \pi_{e_i} (1-\underline{\pi}_{e_j}) + (1-x_i) \underline{\pi}_{e_i} (1-\pi_{e_j})} > x_i
$$

After simplifying by x_i, this expression is equivalent to:

$$
\begin{aligned}
\pi_{e_i} (1-\underline{\pi}_{e_j})(1-x_i) - (1-x_i) \underline{\pi}_{e_i} (1-\pi_{e_j}) & > 0 \\
\pi_{e_i} (1-\underline{\pi}_{e_j}) - \underline{\pi}_{e_i} (1-\pi_{e_j}) & > 0
\end{aligned}
$$

Which is always verified because $(1-\pi_{e_i}) < (1-\underline{\pi}_{e_i})$ and $\underline{\pi}_{e_j} < \pi_{e_j}$.

B/ Proof lemma 3

Let us prove that $w^* = \frac{\psi_2}{a\hat{\gamma}_{i,v}[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \Delta\underline{\pi})]}$

We have:

$$\begin{aligned}\Pi_1 - \Pi_0 =& a\hat{\gamma}_{i,v}\Big[b\hat{\lambda}_{i,v} + \pi_1(1-\hat{\lambda}_{i,v}) + \underline{\pi}_1\hat{\lambda}_{i,v}\Big] \\ & - a\hat{\gamma}_{i,v}\Big[b\hat{\lambda}_{i,v} + \pi_0(1-\hat{\lambda}_{i,v}) + \underline{\pi}_0\hat{\lambda}_{i,v}\Big] \\ \Pi_1 - \Pi_0 =& a\hat{\gamma}_{i,v}\Big[b\hat{\lambda}_{i,v} + \pi_1(1-\hat{\lambda}_{i,v}) - b\hat{\lambda}_{i,v} \\ & - \pi_0(1-\hat{\lambda}_{i,v}) + \hat{\lambda}_{i,v}(\underline{\pi}_1 - \underline{\pi}_0)\Big] \\ \Pi_1 - \Pi_0 =& a\hat{\gamma}_{i,v}\Big[(1-\hat{\lambda}_{i,v})\Delta\pi + \hat{\lambda}_{i,v}\underline{\Delta\pi}\Big] \\ \Pi_1 - \Pi_0 =& a\hat{\gamma}_{i,v}\Big[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \Delta\underline{\pi})\Big]\end{aligned}$$

As $w^* = \frac{\psi_2}{\Pi_1 - \Pi_0}$, thus: $w^* = \frac{\psi_2}{a\hat{\gamma}_{i,v}[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \Delta\underline{\pi})]}$

C/ Proof proposition 2

Let us demonstrate that $\overline{w}_i^\lambda > \overline{w}_i^\gamma$ if $\lambda_i = \gamma_i$. We denote:

- $\overline{w}_i^\lambda(x') = \overline{w}_i^\lambda(\hat{\gamma}_{i,v} = x_0, \hat{\lambda}_{i,v} = x') = \frac{\psi_2}{ax_0[\Delta\pi - x'(\Delta\pi - \Delta\underline{\pi})]}$
- $\overline{w}_i^\gamma(x) = \overline{w}_i^\lambda(\hat{\gamma}_{i,v} = x, \hat{\lambda}_{i,v} = x_0) = \frac{\psi_2}{ax[\Delta\pi - x_0(\Delta\pi - \Delta\underline{\pi})]}$

With $(x_0; x) \in (0,1)^2$ such that $x \geq x_0$ and $(x_0; x') \in (0,1)^2$ such that $x' \geq x_0$. If $x = x_0$ then $\overline{w}_i^\lambda(x_0) = \overline{w}_i^\gamma(x_0) = \frac{\psi_2}{ax_0[\Delta\pi - x_0(\Delta\pi - \Delta\underline{\pi})]}$. As $\lambda_i = \gamma_i$ we denote x_0 as the exante self confidence in managerial ability (in case of an aristocratic procedure) and in business ability (in case of a meritocratic procedure). We also denote x' as the expost self confidence in business ability in the *meritocratic procedure* and x the expost self confidence in managerial ability in the *aristocratic procedure*. $x \neq x'$ because in spite of the assessing technology being identical in both procedures, both procedures do not reveal the same level of ability (as equilibrium efforts in stage 1 might be different). Then we have:

- $x' > x_0 <=> \overline{w}_i^\lambda(x') > \overline{w}_i^\lambda(x_0)$ as $\frac{\partial \overline{w}_i^\lambda(x)}{\partial x} > 0$ (cf lemma 3)

- $x > x_0 <=> \overline{w}_i^{\gamma}(x) < \overline{w}_i^{\gamma}(x_0)$ as $\frac{\partial \overline{w}_i^{\gamma}(x)}{\partial x} < 0$ (cf lemma 3)

Yet $\overline{w}_i^{\lambda}(x_0) = \overline{w}_i^{\gamma}(x_0)$ so $\overline{w}_i^{\lambda}(x') > \overline{w}_i^{\gamma}(x)$ thus $\overline{w}_i^{\lambda} > \overline{w}_i^{\gamma}$.

D/ Proof Lemma 4

1) Let us prove that $\frac{\partial P_{P_i}}{\partial \hat{\gamma}_{i,v}} > \frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}}$.

$$P_{P_i} = a\hat{\gamma}_{i,v}\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big]\Big[1 - \frac{\psi_2}{a\hat{\gamma}_{i,v}[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}\Big]$$

$$P_{P_i} = a\hat{\gamma}_{i,v}\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big] - \frac{\psi_2 a\hat{\gamma}_{i,v}\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big]}{a\hat{\gamma}_{i,v}[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}$$

$$P_{P_i} = a\hat{\gamma}_{i,v}\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big] - \frac{\psi_2\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big]}{[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}$$

Thus $\frac{\partial P_{P_i}}{\partial \hat{\gamma}_{i,v}} = a\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big]$ and is positive.

$$\frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}} = a\hat{\gamma}_{i,v}(b + \underline{\pi}_1 - \pi_1) - \Big[-\psi_2\frac{[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 + \pi_1][-(\Delta\pi - \underline{\Delta\pi})] - [b + \underline{\pi}_1 + \pi_1][\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}{[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]^2}\Big]$$

$$\frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}} = a\hat{\gamma}_{i,v}(b + \underline{\pi}_1 - \pi_1) - \Big[\psi_2\frac{[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 + \pi_1](\Delta\pi - \underline{\Delta\pi}) + [b + \underline{\pi}_1 + \pi_1][\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}{[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]^2}\Big]$$

As a consequence we have:

$$\frac{\partial P_{P_i}}{\partial \hat{\gamma}_{i,v}} - \frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}} > 0 <=> a\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big] - a\hat{\gamma}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \Big[\psi_2\frac{[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 + \pi_1](\Delta\pi - \underline{\Delta\pi}) + [b + \underline{\pi}_1 + \pi_1][\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]}{[\Delta\pi - \hat{\lambda}_{i,v}(\Delta\pi - \underline{\Delta\pi})]^2}\Big]$$

Which is verified if $a\Big[\hat{\lambda}_{i,v}(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big] - a\hat{\gamma}_{i,v}(b + \underline{\pi}_1 - \pi_1) > 0$. Yet, this is always verified because it is equivalent to $\frac{\partial \Pi_1}{\partial \hat{\gamma}_{i,v}} - \frac{\partial \Pi_1}{\partial \hat{\lambda}_{i,v}} > 0$ which is always verified (cf proposition 1).

2) Let us prove that $\frac{\partial P_{A_i}}{\partial \hat{\gamma}_{j,v}} < \frac{\partial P_{A_i}}{\partial \hat{\lambda}_{j,v}}$ and $\frac{\partial P_{A_i}}{\partial \hat{\gamma}_{i,v}} > \frac{\partial P_{A_i}}{\partial \hat{\lambda}_{i,v}}$

We have:

$$\begin{aligned}P_{A_i} =& a\hat{\gamma}_{j,v}\left[\hat{\lambda}_{j,v}(b+\underline{\pi}_1-\pi_1)+\pi_1\right]\frac{\psi_2}{a\hat{\gamma}_{j,v}[\Delta\pi-\hat{\lambda}_{j,v}(\Delta\pi-\underline{\Delta\pi})]}-\psi_2\\ P_{A_i} =& \frac{\psi_2\left[\hat{\lambda}_{j,v}(b+\underline{\pi}_1-\pi_1)+\pi_1\right]}{[\Delta\pi-\hat{\lambda}_{j,v}(\Delta\pi-\underline{\Delta\pi})]}-\psi_2\end{aligned}$$

It is straight forward that $\frac{\partial P_{A_i}}{\partial \hat{\gamma}_{j,v}} = 0$

$$\begin{aligned}\frac{\partial P_{A_i}}{\partial \hat{\lambda}_{j,v}} =& -\psi_2\frac{[\hat{\lambda}_{j,v}(b+\underline{\pi}_1+\pi_1][-(\Delta\pi-\underline{\Delta\pi})]-[b+\underline{\pi}_1+\pi_1][\Delta\pi-\hat{\lambda}_{j,v}(\Delta\pi-\underline{\Delta\pi})]}{[\Delta\pi-\hat{\lambda}_{i,v}(\Delta\pi-\underline{\Delta\pi})]^2}\\ \frac{\partial P_{A_i}}{\partial \hat{\lambda}_{j,v}} =& \psi_2\frac{[\hat{\lambda}_{i,v}(b+\underline{\pi}_1+\pi_1](\Delta\pi-\underline{\Delta\pi})+[b+\underline{\pi}_1+\pi_1][\Delta\pi-\hat{\lambda}_{i,v}(\Delta\pi-\underline{\Delta\pi})]}{[\Delta\pi-\hat{\lambda}_{i,v}(\Delta\pi-\underline{\Delta\pi})]^2}\end{aligned}$$

Thus $\frac{\partial P_{A_i}}{\partial \hat{\lambda}_{j,v}} > 0 = \frac{\partial P_{A_i}}{\partial \hat{\gamma}_{j,v}}$. We also have:

$$P_{A_i} = \frac{\psi_2\left[(1-\hat{\lambda}_{i,d})(b+\underline{\pi}_1-\pi_1)+\pi_1\right]}{[\Delta\pi-(1-\hat{\lambda}_{i,d})(\Delta\pi-\underline{\Delta\pi})]}-\psi_2$$

Which leads to:

$$\frac{\partial P_{A_i}}{\partial \hat{\lambda}_{i,d}} = -\psi_2\frac{[(1-\hat{\lambda}_{i,d})(b+\underline{\pi}_1-\pi_1)+\pi_1](\Delta\pi-\underline{\Delta\pi})+[b+\underline{\pi}_1-\pi_1][\Delta\pi-(1-\hat{\lambda}_{i,d})(\Delta\pi-\underline{\Delta\pi})]}{[\Delta\pi-(1-\hat{\lambda}_{i,d})(\Delta\pi-\underline{\Delta\pi})]^2}$$

Which is always negative. Thus we have $\frac{\partial P_{A_i}}{\partial \hat{\lambda}_{i,d}} < 0 = \frac{\partial P_{A_i}}{\partial \hat{\gamma}_{i,d}}$

E/ Proof of Proposition 3

Let us prove that $P^{\gamma}_{P_i} > P^{\lambda}_{P_i}$ and $P^{\gamma}_{A_i} < P^{\lambda}_{A_i}$. We denote:

- $P^{\gamma}_{P_i}$ so that:

$$P^{\gamma}_{P_i}(y) = P_{P_i}(\hat{\gamma}_{i,v}=y;\hat{\lambda}_{i,v}=y_0) = ay\left[y_0(b+\underline{\pi}_1-\pi_1)+\pi_1\right]-\frac{\psi_2\left[y_0(b+\underline{\pi}_1-\pi_1)+\pi_1\right]}{[\Delta\pi-y_0(\Delta\pi-\underline{\Delta\pi})]}$$

- $P^{\lambda}_{P_i}$ so that:

$$P^{\lambda}_{P_i}(y) = P_{P_i}(\hat{\gamma}_{i,v} = y_0; \hat{\lambda}_{i,v} = y) = ay_0\Big[y(b+\underline{\pi}_1-\pi_1)+\pi_1\Big]-\frac{\psi_2\Big[y(b+\underline{\pi}_1-\pi_1)+\pi_1\Big]}{[\Delta\pi - y(\Delta\pi - \underline{\Delta\pi})]}$$

Thus $P^{\gamma}_{P_i}(y_0) = P^{\lambda}_{P_i}(y_0)$. Now we have two cases:
- In the case where $\frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}} > 0$

$$\begin{aligned} y >& y_0 \\ \text{implies } P^{\gamma}_{P_i}(y) >& P^{\gamma}_{P_i}(y_0) \\ \text{and } P^{\lambda}_{P_i}(y) >& P^{\lambda}_{P_i}(y_0) \end{aligned}$$

As $P^{\gamma}_{P_i}(y_0) = P^{\lambda}_{P_i}(y_0)$ we can infer that $P^{\gamma}_{P_i}(y) > P^{\lambda}_{P_i}(y)$ because $\frac{\partial P_{P_i}}{\partial \hat{\gamma}_{i,v}} > \frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}}$, that is to say managerial ability "increases faster" the payoff.

- In the case where $\frac{\partial P_{P_i}}{\partial \hat{\lambda}_{i,v}} < 0$

$$\begin{aligned} y >& y_0 \\ \text{implies } P^{\gamma}_{P_i}(y) >& P^{\gamma}_{P_i}(y_0) \\ \text{and } P^{\lambda}_{P_i}(y) <& P^{\lambda}_{P_i}(y_0) \end{aligned}$$

As $P^{\gamma}_{P_i}(y_0) = P^{\lambda}_{P_i}(y_0)$ it is straightforward that $P^{\gamma}_{P_i}(y) > P^{\lambda}_{P_i}(y)$.

Besides, we denote:

- $P^{\gamma}_{A_i}$ so that:

$$P^{\gamma}_{A_i}(y) = P_{A_i}(\hat{\gamma}_{i,d} = y; \hat{\lambda}_{i,d} = y_0) = \frac{\psi_2\Big[(1-y_0)(b+\underline{\pi}_1-\pi_1)+\pi_1\Big]}{[\Delta\pi - (1-y_0)(\Delta\pi - \underline{\Delta\pi})]} - \psi_2$$

- $P^{\lambda}_{A_i}$ so that:

$$P^{\lambda}_{A_i}(y) = P_{A_i}(\hat{\gamma}_{i,d} = y_0; \hat{\lambda}_{i,d} = y) = \frac{\psi_2\Big[(1-y)(b+\underline{\pi}_1-\pi_1)+\pi_1\Big]}{[\Delta\pi - (1-y)(\Delta\pi - \underline{\Delta\pi})]} - \psi_2$$

Thus $P^{\gamma}_{A_i}(y_0) = P^{\lambda}_{A_i}(y_0)$.
As we know that $\frac{\partial P_{A_i}}{\partial \hat{\lambda}_{i,d}} < 0$ and $\frac{\partial P_{A_i}}{\partial \hat{\gamma}_{i,d}} = 0$

$$
\begin{aligned}
y <& y_0 \\
\text{implies } P^{\gamma}_{A_i}(y) =& P^{\gamma}_{A_i}(y_0) \\
\text{and } P^{\lambda}_{A_i}(y) >& P^{\lambda}_{A_i}(y_0)
\end{aligned}
$$

As $P^{\gamma}_{A_i}(y_0) = P^{\lambda}_{A_i}(y_0)$ it is straight forward that $P^{\gamma}_{A_i}(y) < P^{\lambda}_{A_i}(y)$.

G/ Proof Lemma 5

Let us prove that $\Delta P^{\gamma} > \Delta P^{\lambda}$ if $W^{\lambda}_j(1) - W^{\lambda}_j(0) \geq W^{\lambda}_i(1) - W^{\lambda}_i(0)$ and $\theta_L \geq 0$.

1) Let us prove that $\Delta P^{\gamma} > \Delta P^{\lambda}$ if $\Big[P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0)\Big] - \Big[P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)\Big] - \Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big] > 0$.
We have:

$$
\begin{aligned}
\Delta P^{\gamma} =& P_W(1)P^{\gamma}_{P_i}(1) - P_W(0)P^{\gamma}_{P_i}(0) + P_L(1)P^{\gamma}_{A_i}(1) - P_L(0)P^{\gamma}_{A_i}(0) \\
\Delta P^{\gamma} =& P_W(1)P^{\gamma}_{P_i}(1) - P_W(1)P^{\gamma}_{P_i}(0) + P_W(1)P^{\gamma}_{P_i}(0) - P_W(0)P^{\gamma}_{P_i}(0) + P_L(1)P^{\gamma}_{A_i}(1) - P_L(0)P^{\gamma}_{A_i}(0) \\
\Delta P^{\gamma} =& P_W(1)\Big[P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0)\Big] + P^{\gamma}_{P_i}(0)\Big[P_W(1) - P_W(0)\Big] + P^{\gamma}_{A_i}\Big[P_L(1) - P_L(0)\Big]
\end{aligned}
$$

As $P^{\gamma}_{A_i}(1) = P^{\gamma}_{A_i}(0)$. We also have:

$$
\begin{aligned}
\Delta P^{\lambda} =& P_W(1)P^{\lambda}_{P_i}(1) - P_W(1)P^{\lambda}_{P_i}(0) + P_W(1)P^{\lambda}_{P_i}(0) - P_W(0)P^{\lambda}_{P_i}(0) \\
&+ P_L(1)P^{\lambda}_{A_i}(1) - P_L(1)P^{\lambda}_{A_i}(0) + P_L(1)P^{\lambda}_{A_i}(0) - P_L(0)P^{\lambda}_{A_i}(0) \\
\Delta P^{\lambda} =& P_W(1)\Big[P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)\Big] + P^{\lambda}_{P_i}(0)\Big[P_W(1) - P_W(0)\Big] \\
&+ P_L(1)\Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big] + P^{\lambda}_{A_i}(0)\Big[P_L(1) - P_L(0)\Big]
\end{aligned}
$$

That leads to:

$$
\begin{aligned}
\Delta P^{\gamma} - \Delta P^{\lambda} =& \Big[P_W(1) - P_W(0)\Big]\Big[P^{\gamma}_{P_i}(0) - P^{\lambda}_{P_i}(0)\Big] + \Big[P_L(1) - P_L(0)\Big]\Big[P^{\gamma}_{A_i} - P^{\lambda}_{A_i}(0)\Big] \\
&+ P_W(1)\Big[P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0) - [P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)]\Big] - P_L(1)\Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big]
\end{aligned}
$$

i) Now let us find the conditions that guarantee that $P_W(1) - P_W(0) > 0$.

$$P_W(1) \geq P_W(0)$$
$$<=> \lambda_i \pi_1 (1-\underline{\pi}_{e_j}) + (1-\lambda_i)\underline{\pi}_1(1-\pi_{e_j}) \geq \lambda_i \pi_0 (1-\underline{\pi}_{e_j}) + (1-\lambda_i)\underline{\pi}_0(1-\pi_{e_j})$$

As $\pi_1 > \pi_0$ is always verified, the proposition holds if $\underline{\pi}_1 \geq \underline{\pi}_0$ that is $\theta_L + \epsilon \geq \epsilon$, which leads to $\theta_L \geq 0$.

ii) Now let us find condition that guarantee that $P_L(1) - P_L(0) \leq 0$

$$P_L(1) \leq P_L(0)$$
$$<=> (1-\lambda_i)(1-\underline{\pi}_1)\pi_{e_j} + \lambda_i(1-\pi_1)\underline{\pi}_{e_j} \leq (1-\lambda_i)(1-\underline{\pi}_0)\pi_{e_j} + \lambda_i(1-\pi_0)\underline{\pi}_{e_j}$$

This will be verified if $(1-\lambda_i)(\underline{\pi}_1 - \underline{\pi}_0)\pi_{e_j} + \lambda_i(\pi_1 - \pi_0)\underline{\pi}_{e_j} \geq 0$. Like previously, as we know that $\pi_1 > \pi_0$ holds, the proposition is guaranteed if if $\underline{\pi}_1 \geq \underline{\pi}_0$ that is $\theta_L \geq 0$.

As a consequence, we know that $P_W(1) - P_W(0) > 0$ and $P^{\gamma}_{P_i}(0) - P^{\lambda}_{P_i}(0) > 0$ (cf proposition 3) and besides $P_L(1) - P_L(0) < 0$ and $P^{\gamma}_{A_i} - P^{\lambda}_{A_i}(0) < 0$ (cf proposition 3) which leads to $\Big[P_L(1) - P_L(0)\Big]\Big[P^{\gamma}_{A_i} - P^{\lambda}_{A_i}(0)\Big] > 0$. Thus, $\Delta P^{\gamma} > \Delta P^{\lambda}$ if $P_W(1)\Big[P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0) - [P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)]\Big] - P_L(1)\Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big] > 0$.

Now, let us find conditions that guarantee that $P_W(1) \geq P_L(1)$:

$$P_W(1) \geq P_L(1)$$
$$<=> \lambda_i \pi_1 (1-\underline{\pi}_{e_j}) + (1-\lambda_i)\underline{\pi}_1(1-\pi_{e_j}) \geq \lambda_i(1-\pi_1)\underline{\pi}_{e_j} + (1-\lambda_i)(1-\underline{\pi}_1)\pi_{e_j}$$
$$<=> \lambda_i\pi_1 - \lambda_i\pi_1\underline{\pi}_{e_j} + (1-\lambda_i)\underline{\pi}_1 - (1-\lambda_i)\underline{\pi}_1\pi_{e_j} \geq \lambda_i\pi_{e_j} - \lambda_i\pi_1\underline{\pi}_{e_j} + (1-\lambda_i)\underline{\pi}_{e_j} - (1-\lambda_i)\underline{\pi}_1\pi_{e_j}$$
$$<=> \lambda_i\pi_1 + (1-\lambda_i)\underline{\pi}_1 \geq \lambda_i\pi_{e_j} + (1-\lambda_i)\underline{\pi}_{e_j}$$

It will be verified if and only if $\lambda_i(\pi_1 - \pi_0) + (1-\lambda_i)(\underline{\pi}_1 - \underline{\pi}_0) \geq 0$. If $e_j = 1$ it will always be verified. If $e_j = 0$ it will be verified if $\underline{\pi}_1 - \underline{\pi}_0 \geq 0$ that is $\theta_L + \epsilon \geq \epsilon$, which leads to $\theta_L \geq 0$. Thus $P_W(1) \geq P_L(1)$ will be verified if $\theta_L \geq 0$.

As a consequence, as $P_W(1) > P_L(1)$ for $\theta_L \geq 0$, $\Delta P^\gamma - \Delta P^\lambda$ will be verified if $P^\gamma_{P_i}(1) - P^\gamma_{P_i}(0) - \left[P^\lambda_{P_i}(1) - P^\lambda_{P_i}(0)\right] - \left[P^\lambda_{A_i}(1) - P^\lambda_{A_i}(0)\right] > 0$.

2) let us prove now that $P^\gamma_{P_i}(1) - P^\gamma_{P_i}(0) - \left[P^\lambda_{P_i}(1) - P^\lambda_{P_i}(0)\right] - \left[P^\lambda_{A_i}(1) - P^\lambda_{A_i}(0)\right] > 0$ if $W^\lambda_j(1) - W^\lambda_j(0) \geq W^\lambda_i(1) - W^\lambda_i(0)$.

We have by spliting the principal "side" of the marginal payoff:

$$\begin{aligned} &P^\gamma_{P_i}(1) - P^\gamma_{P_i}(0) - \left[P^\lambda_{P_i}(1) - P^\lambda_{P_i}(0)\right] - \left[P^\lambda_{A_i}(1) - P^\lambda_{A_i}(0)\right] > 0 \\ <=> \; &\Pi^\gamma_j(1) - \Pi^\gamma_j(0) - \left[W^\gamma_j(1) - W^\gamma_j(0)\right] - \left[\Pi^\lambda_j(1) - \Pi^\lambda_j(0)\right] \\ &+ \left[W^\lambda_j(1) - W^\lambda_j(0)\right] - \left[W^\lambda_i(1) - W^\lambda_i(0)\right] > 0 \end{aligned}$$

As $W^\gamma_j(1) - W^\gamma_j(0) = 0$ it will be verified if :

- $\Pi^\gamma_j(1) - \Pi^\gamma_j(0) \geq \Pi^\lambda_j(1) - \Pi^\lambda_j(0)$
- and $W^\lambda_j(1) - W^\lambda_j(0) \geq W^\lambda_i(1) - W^\lambda_i(0)$

Let us prove that $\Pi^\gamma_j(1) - \Pi^\gamma_j(0) \geq \Pi^\lambda_j(1) - \Pi^\lambda_j(0)$. It will be verified if $\frac{\partial \Pi^\gamma_{j,1}[\hat{\gamma}_{i,v}(e_i)]}{\partial e_i} > \frac{\partial \Pi^\lambda_{j,1}[\hat{\lambda}_{i,v}(e_i)]}{\partial e_i}$

Yet $\frac{\partial \Pi^\gamma_{j,1}[\hat{\gamma}_{i,v}(e_i)]}{\partial e_i} > \frac{\partial \Pi^\lambda_{j,1}[\hat{\lambda}_{i,v}(e_i)]}{\partial e_i}$ is equivalent to:

$$\frac{\hat{\gamma}_{i,v}}{\partial e_i} \frac{\partial \Pi^\gamma_{j,1}[\hat{\gamma}_{i,v}(e_i)]}{\partial \hat{\gamma}_{i,v}(e_i)} > \frac{\hat{\lambda}_{i,v}}{\partial e_i} \frac{\partial \Pi^\lambda_{j,1}[\hat{\lambda}_{i,v}(e_i)]}{\partial \hat{\lambda}_{i,v}(e_i)}$$

As the assessing technology is by assumption identical whatever the procedure we have $\frac{\hat{\gamma}_{i,v}}{\partial e_i} = \frac{\hat{\lambda}_{i,v}}{\partial e_i}$. Thus the expression is equivalent to:

$$\begin{aligned} \frac{\partial \Pi^\gamma_{j,1}[\hat{\gamma}_{i,v}(e_i)]}{\partial \hat{\gamma}_{i,v}(e_i)} &> \frac{\partial \Pi^\lambda_{j,1}[\hat{\lambda}_{i,v}(e_i)]}{\partial \hat{\lambda}_{i,v}(e_i)} \\ \lambda_i(b + \underline{\pi}_1 - \pi_1) + \pi_1 &> \gamma_i(b + \underline{\pi}_1 - \pi_1) \\ (\lambda_i - \gamma_i)(b + \underline{\pi}_1 - \pi_1) + \pi_1 &> 0 \end{aligned}$$

As $\lambda_i = \gamma_i$ because the *a priori* belief in both competences are assumed identical before the tournament, the proposition is always verified.

Thus, the expression $P^{\gamma}_{P_i}(1) - P^{\gamma}_{P_i}(0) - \Big[P^{\lambda}_{P_i}(1) - P^{\lambda}_{P_i}(0)\Big] - \Big[P^{\lambda}_{A_i}(1) - P^{\lambda}_{A_i}(0)\Big] > 0$ will be verified only if $W^{\lambda}_j(1) - W^{\lambda}_j(0) \geq W^{\lambda}_i(1) - W^{\lambda}_i(0)$

3) As a conclusion, if $W^{\lambda}_j(1)-W^{\lambda}_j(0) \geq W^{\lambda}_i(1)-W^{\lambda}_i(0)$ and if $\theta_L \geq 0$ (conditions that guarantee $P_W(1) \geq P_L(1)$, and $P_W(1) \geq P_W(0)$, and $P_L(1) \leq P_L(0)$) it implies $\Delta P^{\gamma} > \Delta P^{\lambda}$.

H/ Proof Lemma 6

As demonstrated in lemma 5, to prove that $\Delta P^{\gamma} > \Delta P^{\lambda}$, we need to prove that $W^{\lambda}_j(1) - W^{\lambda}_j(0) > W^{\lambda}_i(1) - W^{\lambda}_i(0)$. Let us prove it is verified if:

- $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ and $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$
- and if $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ and $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$

$W^{\lambda}_j(1) - W^{\lambda}_j(0) \geq W^{\lambda}_i(1) - W^{\lambda}_i(0)$ is equivalent to:

$$\Big[W^{\lambda}_j(1) - W^{\lambda}_i(1)\Big] + \Big[W^{\lambda}_i(0) - W^{\lambda}_j(0)\Big] > 0 \tag{6}$$

To find conditions that guarantee expression (7), we study two cases with respect to the strategy of the opponent agent j. We will find conditions such that:

1. in case $e_j = 1$, $W^{\lambda}_j(1) = W^{\lambda}_i(1)$ and $W^{\lambda}_i(0) > W^{\lambda}_j(0)$ that will imply that expression (7) will hold.
2. in case $e_j = 0$, $W^{\lambda}_j(1) > W^{\lambda}_i(1)$ and $W^{\lambda}_i(0) = W^{\lambda}_j(0)$ that will imply that expression (7) will hold as well.

We define $W(x)$ a function such that:

$$W(x) = \Big[x(b + \underline{\pi}_1 - \pi_1] + \pi_1\Big] \frac{\psi_2}{\Delta\pi - x(\Delta\pi - \underline{\Delta\pi})}$$

As a consequence:

- For $x = \hat{\lambda}_i(1, e_j)$:

  we have $W[\hat{\lambda}_i(1,e_j)] = W^{\lambda}_j(1) = \Big[\hat{\lambda}_i(1,e_j)(b + \underline{\pi}_1 - \pi_1) + \pi_1\Big] \frac{\psi_2}{\Delta\pi - \hat{\lambda}_i(1,e_j)(\Delta\pi - \underline{\Delta\pi})}$

- For $x = \hat{\lambda}_i(0, e_j)$:

  we have $W[\hat{\lambda}_i(0, e_j)] = W_j^\lambda(0) = \left[\hat{\lambda}_i(0, e_j)(b + \underline{\pi}_1 - \pi_1) + \pi_1\right] \frac{\psi_2}{\Delta\pi - \hat{\lambda}_i(0,e_j)(\Delta\pi - \underline{\Delta\pi})}$

- For $x = \hat{\lambda}_j(1, e_j)$:

  we have $W[\hat{\lambda}_j(1, e_j)] = W_i^\lambda(1) = \left[\hat{\lambda}_j(1, e_j)(b + \underline{\pi}_1 - \pi_1) + \pi_1\right] \frac{\psi_2}{\Delta\pi - \hat{\lambda}_j(1,e_j)(\Delta\pi - \underline{\Delta\pi})}$

- For $x = \hat{\lambda}_j(0, e_j)$:

  we have $W[\hat{\lambda}_j(0, e_j)] = W_i^\lambda(0) = \left[\hat{\lambda}_j(0, e_j)(b + \underline{\pi}_1 - \pi_1) + \pi_1\right] \frac{\psi_2}{\Delta\pi - \hat{\lambda}_j(0,e_j)(\Delta\pi - \underline{\Delta\pi})}$

Let us demonstrate that $W(x)$ is an increasing function.

$$W(x) = \left[x(b + \underline{\pi}_1 - \pi_1] + \pi_1\right] \frac{\psi_2}{\Delta\pi - x(\Delta\pi - \underline{\Delta\pi})}$$

$$W'(x) = \psi_2 \frac{(b + \underline{\pi}_1 - \pi_1)[\Delta\pi - x(\Delta\pi - \underline{\Delta\pi})] - [-(\Delta\pi - \underline{\Delta\pi})][x(b + \underline{\pi}_1 - \pi_1) + \pi_1]}{\left[\Delta\pi - x(\Delta\pi - \underline{\Delta\pi})\right]^2}$$

$$W'(x) = \psi_2 \frac{(b + \underline{\pi}_1 - \pi_1)[\Delta\pi - x(\Delta\pi - \underline{\Delta\pi})] + [\Delta\pi - \underline{\Delta\pi}][x(b + \underline{\pi}_1 - \pi_1) + \pi_1]}{\left[\Delta\pi - x(\Delta\pi - \underline{\Delta\pi})\right]^2}$$

W' is always positive.

As a consequence, as $W(x)$ is an increasing function:

1. to demonstrate $W_j^\lambda(1) \geq W_i^\lambda(1)$, that is $W[\hat{\lambda}_i(1, e_j)] \geq W[\hat{\lambda}_j(1, e_j)]$, we need to demonstrate $\hat{\lambda}_i(1, e_j) \geq \hat{\lambda}_j(1, e_j)$. Furthermore, do demonstrate $W_j^\lambda(1) = W_i^\lambda(1)$ we need to prove $\hat{\lambda}_i(1, e_j) = \hat{\lambda}_j(1, e_j)$.

2. to demonstrate $W_i^\lambda(0) \geq W_j^\lambda(0)$, that is $W[\hat{\lambda}_j(0, e_j)] \geq W[\hat{\lambda}_i(0, e_j)]$, we need to demonstrate $\hat{\lambda}_j(0, e_j) \geq \hat{\lambda}_i(0, e_j)$. Furthermore, to demonstrate $W_i^\lambda(0) = W_j^\lambda(0)$ we need to prove $\hat{\lambda}_j(0, e_j) = \hat{\lambda}_i(0, e_j)$

As a conclusion, if we study the cases with respect to the strategy of j, expression (7) will be verified:

1. in case $e_j = 1$, if $W_j^\lambda(1) = W_j^\lambda(1)$ and $W_j^\lambda(0) > W_j^\lambda(0)$ that is $\hat{\lambda}_i(1, 1) = \hat{\lambda}_j(1, 1)$ and $\hat{\lambda}_i(0, 1) < \hat{\lambda}_j(0, 1)$

2. in case $e_j = 0$ if $W_j^\lambda(1) > W_j^\lambda(1)$ and $W_j^\lambda(0) = W_j^\lambda(0)$ that is $\hat{\lambda}_i(1, 0) > \hat{\lambda}_j(1, 0)$ and $\hat{\lambda}_i(0, 0) = \hat{\lambda}_j(0, 0)$

I/ Proof Proposition 4

As demonstrated in lemma 6, to prove that $\Delta P^\gamma > \Delta P^\lambda$, we need to prove that:

$$\Big[W_j^\lambda(1) - W_i^\lambda(1)\Big] + \Big[W_i^\lambda(0) - W_j^\lambda(0)\Big] > 0 \tag{7}$$

For this we proved that:

- in case $e_j = 1$, $\hat{\lambda}_i(1,1) = \hat{\lambda}_j(1,1)$ and $\hat{\lambda}_i(0,1) < \hat{\lambda}_j(0,1)$ that will imply $W_j^\lambda(1) = W_j^\lambda(1)$ and $W_j^\lambda(0) > W_j^\lambda(0)$ such that will guarantee expression (7) will hold.

- and in case $e_j = 0$, $\hat{\lambda}_i(1,0) > \hat{\lambda}_j(1,0)$ and $\hat{\lambda}_i(0,0) = \hat{\lambda}_j(0,0)$ that will imply $W_j^\lambda(1) > W_j^\lambda(1)$ and $W_j^\lambda(0) = W_j^\lambda(0)$ that will also guarantee expression (7) will hold.

We will now prove it those conditions are guaranteed if $\lambda_i = \frac{1}{2}$, $\epsilon < \frac{1}{2}$ and $\theta_H + \theta_L < (1 - 2\epsilon)$ which is equivalent to $\lambda = \frac{1}{2}$, $\pi_1 < 1 - \underline{\pi}_1$ and $\pi_0 < 1 - \underline{\pi}_0$:

Let us find condition that guarantees $\hat{\lambda}_{i,v}(e_i, e_j) \geq \hat{\lambda}_{j,v}(e_i, e_j)$. By successive equivalence we have:

$$\hat{\lambda}_{i,v}(e_i, e_j) \geq \hat{\lambda}_{j,v}(e_i, e_j)$$

$$\frac{\lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j})}{\lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - \lambda_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})} \geq \frac{(1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i})}{(1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i}) + \lambda_i \underline{\pi}_{e_j}(1 - \pi_{e_i})}$$

Which is equivalent to:

$$\lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j})\Big[(1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i}) + \lambda_i \underline{\pi}_{e_j}(1 - \pi_{e_i})\Big] \geq$$

$$(1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i})\Big[\lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - \lambda_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})\Big]$$

$$\lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j})(1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i}) + \lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j})\lambda_i \underline{\pi}_{e_j}(1 - \pi_{e_i}) \geq$$

$$(1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i})\lambda_i \pi_{e_i}(1 - \underline{\pi}_{e_j}) + (1 - \lambda_i)\pi_{e_j}(1 - \underline{\pi}_{e_i})(1 - \lambda_i)\underline{\pi}_{e_i}(1 - \pi_{e_j})$$

If we assume $\lambda_i = \frac{1}{2}$ the expression is equivalent to:

$$\pi_{e_i}(1 - \underline{\pi}_{e_j})\underline{\pi}_{e_j}(1 - \pi_{e_i}) - \pi_{e_j}(1 - \underline{\pi}_{e_i})\underline{\pi}_{e_i}(1 - \pi_{e_j}) \geq 0$$

such that we know that :

$$\hat{\lambda}_{i,v}(e_i, e_j) \geq \hat{\lambda}_{j,v}(e_i, e_j) <=> \pi_{e_i}(1 - \underline{\pi}_{e_j})\underline{\pi}_{e_j}(1 - \pi_{e_i}) \geq \pi_{e_j}(1 - \underline{\pi}_{e_i})\underline{\pi}_{e_i}(1 - \pi_{e_j}) \quad (8)$$

And that:

$$\hat{\lambda}_{i,v}(e_i, e_j) \leq \hat{\lambda}_{j,v}(e_i, e_j) <=> \pi_{e_i}(1 - \underline{\pi}_{e_j})\underline{\pi}_{e_j}(1 - \pi_{e_i}) \leq \pi_{e_j}(1 - \underline{\pi}_{e_i})\underline{\pi}_{e_i}(1 - \pi_{e_j}) \quad (9)$$

As a consequence, we can now study several cases with respect to the strategy of agent j:

1. In case $e_j = 1$

   - if $e_i = 1$ let us demonstrate that $\hat{\lambda}_i(1, 1) = \hat{\lambda}_j(1, 1)$ which implies $W_j^\lambda(1) = W_i^\lambda(1)$:

     Indeed, then equation (8) becomes $\pi_1(1-\underline{\pi}_1)\underline{\pi}_1(1-\pi_1) \geq \pi_1(1-\underline{\pi}_1)\underline{\pi}_1(1-\pi_1)$ such that we have $\hat{\lambda}_{i,v}(1, 1) = \hat{\lambda}_{j,v}(1, 1)$ and then $W_j^\lambda(1) = W_i^\lambda(1)$.

   - if $e_i = 0$ let us demonstrate that $\hat{\lambda}_j(0, e_j) > \hat{\lambda}_i(0, e_j)$ which implies $W_i^\lambda(0) > W_j^\lambda(0)$:

     Indeed, then equation (9) becomes by successive equivalence:

     $$\pi_0(1 - \underline{\pi}_1)\underline{\pi}_1(1 - \pi_0) \leq \pi_1(1 - \underline{\pi}_0)\underline{\pi}_0(1 - \pi_1)$$
     $$(1 - \underline{\pi}_1)\underline{\pi}_1 \leq \pi_1(1 - \pi_1) \quad (10)$$

     This is equivalent to:

     $$(\pi_1 - \underline{\pi}_1) + \underline{\pi}_1^2 - \pi_1^2 \geq 0$$
     $$(\pi_1 - \underline{\pi}_1) + (\underline{\pi}_1 - \pi_1)(\underline{\pi}_1 + \pi_1) \geq 0$$
     $$(\pi_1 - \underline{\pi}_1)\Big[1 - (\underline{\pi}_1 + \pi_1)\Big] \geq 0$$

     As $(\pi_1 - \underline{\pi}_1) > 0$, the expression will be strictly verified if $1 - (\underline{\pi}_1 + \pi_1) > 0$ that is $\pi_1 < 1 - \underline{\pi}_1$. Thus it leads to $\theta_H + \theta_L < 1 - 2\epsilon$. As $\theta_L \geq 0$ (cf. lemma 5), it implies that $\theta_H + \theta_L > 0$. Thus $1 - 2\epsilon > 0$ must be verified that is $\epsilon < \frac{1}{2}$. The expression $1 - 2\epsilon > 0$ is also equivalent to $1 - \pi_0 - \underline{\pi}_0 > 0$ that is $\pi_0 < 1 - \underline{\pi}_0$.

     As a consequence, if $e_j = 1$ and $e_i = 0$, equation (9) will be verified and we have $\hat{\lambda}_j(0, e_j) > \hat{\lambda}_i(0, e_j)$ and then $W_i^\lambda(0) > W_j^\lambda(0)$

- As a consequence in case $e_j = 1$, if $\pi_1 < 1 - \underline{\pi}_1$ and $\pi_0 < 1 - \underline{\pi}_0$ then $W_j^\lambda(1) = W_i^\lambda(1)$ and $W_i^\lambda(0) > W_j^\lambda(0)$ such that $W_j^\lambda(1) - W_j^\lambda(0) > W_i^\lambda(1) - W_i^\lambda(0)$. It implies Proposition 4 is verified.

2. In case $e_j = 0$:

   - if $e_i = 1$, let us demonstrate that $W_j^\lambda(1) > W_i^\lambda(1)$, that is $\hat{\lambda}_i(1, e_j) \geq \hat{\lambda}_j(1, e_j)$:

     Indeed, then equation (8) becomes by successive equivalences:

$$\pi_1(1 - \underline{\pi}_0)\underline{\pi}_0(1 - \pi_1) \geq \pi_0(1 - \underline{\pi}_1)\underline{\pi}_1(1 - \pi_0)$$
$$\pi_1(1 - \pi_1) \geq (1 - \underline{\pi}_1)\underline{\pi}_1 \qquad (11)$$

     Expression 11 is identical to expression 10 in the previous item. As a consequence, it implies that if $e_i = 0$ and $e_j = 1$ then equation (8) will be verified and $W_j^\lambda(1) > W_i^\lambda(1)$ holds if $\pi_1 \leq 1 - \underline{\pi}_1$ and $\pi_0 \leq 1 - \underline{\pi}_0$.

   - if $e_i = 0$ let us demonstrate that $W_i^\lambda(0) = W_j^\lambda(0)$ that is $\hat{\lambda}_j(0, e_j) = \hat{\lambda}_i(0, e_j)$:

     Indeed, then the expression (9) becomes $\pi_0(1 - \underline{\pi}_0)\underline{\pi}_0(1 - \pi_0) \leq \pi_0(1 - \underline{\pi}_0)\underline{\pi}_0(1 - \pi_0)$ such that we have $\hat{\lambda}_j(0, e_j) = \hat{\lambda}_i(0, e_j)$ and then $W_i^\lambda(0) = W_j^\lambda(0)$.

   - As a consequence, in case $e_j = 0$, if expression $\pi_1 < 1 - \underline{\pi}_1$ and $\pi_0 < 1 - \underline{\pi}_0$ are verified then $W_j^\lambda(1) > W_i^\lambda(1)$ and $W_i^\lambda(0) = W_j^\lambda(0)$ such that $W_j^\lambda(1) - W_j^\lambda(0) > W_i^\lambda(1) - W_i^\lambda(0)$. It implies Proposition 4 is verified.

3. As a consequence, in both cases whatever the strategy of the agent j ($e_j = 1$ or $e_j = 0$), if $\pi_1 < 1 - \underline{\pi}_1$ and $\pi_0 < 1 - \underline{\pi}_0$ are verified, expression (7) will be verified and proposition 4 will hold.

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