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Legitimacy and Incentives in a Labour Relationship

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"Ill-gotten gains never leads to prosperity". Popular proverb

Abstract

We design a two-stage model where the winner of a tournament becomes the executive of his former opponent. We call procedural legitimacy, the legitimacy an executive obtains if he is promoted through a competition with no unfair treatment. The aim of this paper is to study how effort may paradoxically bolster or undermine this type of legitimacy with respect to technological assumptions. Besides we show that, in bayesian terms, winning the competition reinforces the belief of having been advantaged but it also reinforces the belief that the looser will be disadvantaged in the future and thus be less productive. This will tend to make winning the competition by being advantaged much less profitable. To incentivize more effort during the competition, the firm has to design a procedure where opponents are not evaluated by their peers but by neutral and external people. Thus the competition will not bring information on a potential inequality of treatment in the future. We argue civil servant examination in public administration and human resources departments are designed partially for this reason.

Keywords: Legitimacy, leadership, tournament, contract theory, moral hazard, personnel economics

JEL-Classification: D00,D86, J50, M50, M51

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1 Introduction

The aim of this paper is to study a form of legitimacy that we denote *procedural legitimacy* not based on competence or performance but assuming that an executive will be seen as legitimate if he was promoted through a competition that guaranteed equality of opportunities (i.e a competition where no opponent benefit of preferential treatment). The stake for the organization is that this king of legitimacy could be an implicit incentive that could be potentially less costly than regular incentive such as variable wage. Indeed, a executive "legitimate" will send a good signal to the employee on the expected return of his effort as he would infer that only effort and not external criteria will count to get rewarded in the organization. This approach is notably justified by the work on "procedural justice" in social psychology (Tyler and Blader, 2003) and also in experimental economics (Ku and Salmon, 2013; Bolton et al., 2005). This concept states that people care about the fairness of procedures by which decisions are made and allocation of incomes is established. Thus the objective of this paper is to study how some particular procedures of promotion may bolster or undermine this procedural legitimacy, what will be the consequences in term of implicit incentive, and what would be the consequence for the future leader on the choice of his actual effort as a ex-ante mean to bolster his legitimacy.

To tackle these issues, we build a two-stage game where the first stage is a tournament that parts two candidates for an executive position, and the second stage is the hierarchical relationship strictly speaking where the looser is the employee of the one who has been promoted. The opponent during the tournament choose a effort to produce an outcome and the winner is the one who produces the most. We will assume that opponents have an a priori suspicion that their colleagues could be advantaged (discrimination, nepotism, racism, sexism...) during the tournament. This suspicion is not necessary based on tangible elements but is potentially presupposed. It is modeled by a probability of being advantaged and this probability will be considered as the indicator of the "degree" of procedural legitimacy. Furthermore, the advantage is treated as an asset that enhances the efficiency of effort such that opponents will revise their beliefs in being advantaged by bayesian learning observing effort and performance after the tournament. As a consequence their objectives will be not only to choose efforts in order to win the tournament but also to manage the signal about their potential advantages that their performances could send. Finally, the second stage is modeled as a regular principal-agent relationship: the principal (i.e the winner of the preceding tournament) does not exert effort and proposes a variable wage to incentivize high effort from the agent (i.e the loser). The output is observed at the end and the effort of the agent is of course unobservable.

Regarding the firm (an external player), its objective is to maximize efforts of opponents during tournament by choosing optimally between two procedures of promotion. We denote as "internal procedure" a process where people in charge promotion are the same than people that will assess later the work of the one who will remain employee. It captures a very intuitive promotion process corresponding to what occurs where people are assessed by their peers. The "external procedure" is a procedure where people that assess candidates at the executive position are not the same than people that will evaluate thereafter the employee. It correspond to a more institutionalized process where promotion is delegated to a human resource department for instance or to civil service entrance examination. As a consequence, the firm does not assess directly itself and is not responsible directly for any inequitable treatment. Besides, this inequality of treatment does not occur necessarily but it is just an exogenous suspicion from opponents that the firm has to deal with.

With this framework in place, we get the following main results. The first result does not focus for now on the issue of the optimal procedure but remains focused on the point of view of opponents strategies and only in the framework of an internal procedure. We show that the role of effort in the first stage is ambivalent to bolster legitimacy. Indeed, if we assume complementarity between effort and the presupposed advantage, namely a case where the advantage is comparable to a mean of production (such as more effective working tool, a strategic information, key contact etc.), then the more effort will be high, the more it will exacerbate the effect of advantage. Thus a high performance will be more suspicious and the one who is advantaged will betray himself. A bit like a sportsman doped that would outperform his opponents too obviously. However, if we assume substitutability between effort and advantage, namely a case where the performance expected for one opponent would be less demanding (such as starting with a head start), then exerting high effort for achieving a high performance indicates on the contrary that being advantaged is unlikely because the advantage would otherwise avoid exerting to much effort.

The second result shows that there is a possible substitution in the second stage between the legitimacy of an executive and the variability of wage which is classically used as an incentive to implement high effort from agent and reduce hazard moral. Which means that a legitimate executive would need to provide less costly monetary incentive relatively to a non legitimate one. However, this result holds only if we assume complementarity between effort and the advantage and not substitutability. Indeed it would imply that the advantage exacerbates the marginal return of effort exactly like would do a competence. Thus the looser of the tournament will demand a higher wage to compensate the fact that in his point of view his chances of success are little regarding he feels disadvantaged. In other words, the principal will have to "payback" the agent to maintain his motivation in some ways.

The third major result is about the issue of the optimal procedure assuming substitutability between effort and advantage. In internal procedures, we show that it is rational in bayesian term that whatever the level of effort achieved by the one who has been promoted, he will always be more confident about being advantaged than before competing. In other words, he will always "feel guilty". He will then anticipate that his agent will be disadvantaged by the firm in the future and that he will be less productive. This is what we call the *cost of illegitimacy* and it will tend to inhibit effort upstream during the competition stage. The solution for the firm would be to establish a procedure that would neutralize the "negative" effect of bayesian learning by delegating the promotion issue to an independent entity. That would be the purpose of an external procedure. Indeed, in that case, every inequalities of treatment assumed by opponents during the tournament would not bring information about a potential unfair treatment in the future as the assessors will be different people. This changing of assessors would serve as a fresh start. Thus being aware in advance that the cards will be reshuffled after the tournament will highly incentivizing. We believe human resources department and civil service entrance examination are partially designed to serve as a scape goat for this reason. If we interpret stage 1 as a career and promotion before stage 2 as a long term achievement, we can infer that external procedures will favor the general adhesion to the organization and the acceptation of a higher degree of social heterogeneity (in a sense that in our analyze social heterogeneity implies mechanically an *a priori* suspicion) and a higher degree of unequal treatment.

Our main contribution is to develop a theoretical approach on procedures of justice based on theory of incentive, a concept that has been treated - to the best of our knowledge, only in experimental paper. For that reason this work is an attempt to build microeconomics foundations of legitimacy using the theoretical framework of an "unfair" tournament. We deepen the idea of competition with bias used by Bolton et al. (2005), and include it in the theoretical framework of the theory of incentive. We also deepen the idea of Ku and Salmon (2013) that merit is ambivalent to make people accept inequality and we propose a theoretical framework to explain this using the bayesian revision of the belief in being disadvantaged. We also add the idea that the identity of people in charge of assessment will matter to explain behavior when faced to unfairness. That will even constitute the core of an efficient procedure. This approach makes us moving away from the literature that assumes social preference for ideal of justice (Englmaier and Wambach, 2010; Cox et al., 2007; Falk and Fischbacher, 2006) as our work is only focused on the motivational aspect of unfairness and the incentive issue.[1]

[1]The work on social preferences for ideal of justice assume that people have a preference in their utility functions for a certain allocation of revenues like they would have a preference for any kind of good or services.

The rest of the paper is organized as followed. Section 2 will deal with the literature related to our work in this paper . Section 3 will describe the theoretical framework. Section 4 will deal with results. Section 5 concludes.

2 Literature

This paper is related to the following literature. Several experimental papers in economics are directly related to the notion of procedure of justice. Other papers are indirectly related in the sense that they study the link between merit and reward. The results are not always consistent to each other. Ruffle (1998) shows in a dictator game, that the one who was chosen to allocate revenues took into account the merit of the person who receive it. The merit is measured by a score at a test before the experiment. Symmetrically, Cherry et al. (2002) and Hoffman et al. (1994) show that, still in the framework of a dictator game, the one who allocates revenues feels legitimate to divert incomes from the other player if he previously succeeded at a performance test. Cappelen et al. (2007) made the distinction between three fairness ideals: strict egalitarianism, liberal egalitarianism and libertarianism. They study a dictator game where there is a production stage prior to the allocation stage and they find that merit is a subjective concept. It depends on how people weight the importance of random variables and controllable variables in the production. They assumed that controllable variables are the amounts people invest during the production stage whereas random variables are the rates of return of those investments.

Charness and Rabin (2002) and Engelmann and Strobel (2004) show inequity aversion decreases if allocation of incomes increases the global efficiency of the economy.[1] That is to say, a growing inequality is accepted if everyone gets his revenue increased. Bolton et al. (2005) compare a situation with simultaneously unequal distribution, generated by a ultimatum game, and unfair procedures. They study how the receiver rejects the offer of the proposer in several cases: if the proposer can choose himself an unequal distribution

[1] Inequity aversion is concept whereby people have preferences equitable allocation such that they are willing to pay to reduce inequalities.

or if the income inequality depends on a bias established in advance (to create bias, they use loaded dice) or if it depends on a random criteria. They prove that the case with random criteria is the one which has the less rejection rate whereas the voluntary unfair allocation is highly rejected and the case with the bias is intermediate. Ku and Salmon (2013) show that several institutional elements could affect the acceptation of an unreasonable rise of richer people revenue. They part participants into two groups: one is constituted of people that are well endowed (or "favored") at the beginning of the experiment and the other one is less endowed (or "unfavored"). Unfavored participants will be asked to accept or not to give a proportion of their endowments to the favored participants. They are told that this transfer is actually an investment that will finally improve their own revenues but in a smaller proportion than the revenue of participant that were favored. Four situations have been taken into account: in the first case, the favored group selection is random; in the second one, the selection is based on merit; in the third case, the selection is based on belonging in a specific social group; and in the last case, selection rewards uncooperative behavior. The experimental papers of Ku and Salmon (2013) and Bolton et al. (2005) are indeed directly related the theoretical work in social psychology on the procedure of justice.

The theoretical literature is less extensive. Indeed, to the best of our knowledge no papers study, strictly speaking, procedures of justice. However, there are some papers on inequity aversion. Falk and Fischbacher (2006) build a model that explains reciprocity assuming that part of utility function is depending on a "kindness term" that depends on outcome transfer and intention. Cox et al. (2007) propose another model on reciprocity and justice. They define the concept of emotional state as key parameter of the marginal rate of substitution between the payoff of one individual and the payoff of other people. Englmaier and Wambach (2010) study specifically moral hazard in a principal-agent model with inequity aversion. Inequity aversion is adding to the traditional payoff as an extra cost, which is assumed to be increasing and convex with unequal distribution. Most of the theoretical work on iniquity aversion assume that individual has a social preference for justice. They analyze kindness, selfishness, reciprocity and unfairness aversion as a classical trade off between the direct utility of wealth and the negative utility related to unfairness.

3 Theoretical framework

We consider the following two-stage game. At date 1, an *unfair* tournament occurs between two agents i and j. A *fair* competition is assumed to be a competition with no favoritism of any kind (gender, religion, social network...). We denote λ_i as the ex-ante probability for i not to be disadvantaged with $i \in \{1,2\}$ and $\lambda_i \in [0,1]$. It is an *a priori* belief that will be revised. Furthermore λ_j is the ex-ante probability j not to be disadvantaged with $j \in \{1,2\}$ and is defined such as $\lambda_j = 1 - \lambda_i$. Both agents share the same beliefs *a priori* such that i observe λ_j and reciprocally. Besides the firm observes λ_i and λ_j. It means that the firm is not responsible for any favoritism in particular because its objective is to choose the procedure of promotion not to assess opponents. Its role will be to choose who should be in charge of evaluation rather than assessing strictly speaking as we will explain further. Furthermore, favoritism does not occur necessary in the model but we assume, first, that opponents are *a priori* suspicious about it even though it is not based of real proof or actual fact and, second, that the firm is aware of this suspicion. Theoretically, the favoritism is treated such as a unequal distribution of assets before the competition. It is of course an enrich concept that includes any kind of social capital or information, or any kind of effective working tool. It can be also interpreted as a head start. Being disadvantaged is assumed to reduce the efficiency of effort for the agent who is victim. So, if $\lambda_i = 1$, it means that i has the belief he is completely advantaged and j is completely disadvantaged. Symmetrically, if $\lambda_i = 0$, it means that i has the belief he is completely disadvantaged and j is completely advantaged.

At the end of the first stage, the agent who performed the highest output is promoted by the firm and becomes principal during the second stage. At stage 2 each agent revises his belief of being disadvantaged taking into account the performances and each effort chosen during stage 1. We denote λ_i^{expost} this updated belief for player i. The second stage is a principal-agent model.

Definition 1. The principal is perceived as *legitimate* during the second stage if after the tournament the agent believes the principal won thanks to his effort and not because he was advantaged. Formally, if agent i wins, he will be *legitimate* if after observing the results, it is less likely that he was advantaged than disadvantaged: $\lambda_i^{expost} \leq \frac{1}{2}$. By extension, the lowest λ_i^{expost} will be, the more the principal will be seen as *legitimate*.

Two procedures of promotion can be chosen by the firm: an internal and an external procedural. In internal procedures people in charge of assessment during the tournament are the same than people in charge of assessing production in stage 2. As a consequence during stage 2, principal and agent will use their revised beliefs λ_i^{expost} such that $\lambda_i^{expost} \neq \lambda_i$. However, in external procedures people in charge of the evaluation during the tournament are different such that an unfair treatment during stage 1 does not give information about a potential unfair treatment in stage 2. Thus principal and agent will use a unrevised belief λ_i^{expost} such that $\lambda_i^{expost} = \lambda_i$. The objective of the firm is to maximize effort during the competition by choosing the optimal procedure.

Here is the sequence of the game:

- Stage 1:
  - i and j share the same belief on λ_i and λ_j. The firm observe theses beliefs.
  - The firm choose between an internal or an external procedure without observing efforts in stages 1 and 2.
  - Opponents choose their efforts during the tournament simultaneously.
  - The opponent that makes the best performance is promoted. The firm does not observe efforts but only performances.
  - After the tournament, opponents observe efforts exerted and performance of each other.

- Stage 2:
  - The winner of the tournament offers a variable wage to the loser that will continue to exert a task. Effort of the loser is non contractible and the principal does not work.
  - Effort is exert and the principal, the agent and the firm observe the output.

Now that we have outline the general structure of the model, let us focus on explaining in more details the first stage and the second stage.

3.1 Stage 1: a tournament with suspicion

Let us first explain the interaction arising at the first stage of the game. We consider that i can exert a costly effort e_i that takes only two values (i.e $e_i \in \{0,1\}$). Exerting high effort during the first stage implies a high cost $\psi > 0$. It is assumed to be the same for each opponent i. Exerting low effort implies no cost. There are two levels of production q_i such that $q_i \in \{0,1\}$.

In the case where the high effort is chosen, we define $\pi_1 = P(q_i = 1 \mid e_i = 1 \mid \bar{i})$ as the probability to produce 1 for the i given he exerts the high level of effort and given he has been fully advantaged (formalized by $\bar{i}$). Thus $1 - \pi_1$ is the probability to produce 0. Besides, we define $\underline{\pi}_1 = P(q_i = 1 \mid e_i = 1 \mid \underline{i})$ as the probability to produce 1 for player i given he exerts the high level of effort and given he has been completely disadvantaged (formalized by $\underline{\text{i}}$). Thus, $1 - \underline{\pi}_1$ is the probability to produce 0 such as $\pi_1 > \underline{\pi}_1$.

In the case where the low effort is chosen, we define $\pi_0 = P(q_i = 1 \mid e_i = 0 \mid \bar{i})$ as the probability to produce 1 for i given he exerts the low level of effort and given he has been completely <u>not</u> disadvantaged and $1 - \pi_0$ the probability to produce 0. Besides, we define $\underline{\pi}_0 = P(q_i = 1 \mid e_i = 0 \mid \underline{i})$ as the probability to produce 1 for i given he exerts the low level of effort and given he has been completely disadvantaged. $1 - \underline{\pi}_0$ is the probability to produce 0 such as $\pi_0 > \underline{\pi}_0$.

Furthermore it is reasonable to assume that $\pi_1 > \pi_0$ and $\underline{\pi}_1 > \underline{\pi}_0$. Besides we will assume that $\pi_1 - \pi_0 \geq \underline{\pi}_1 - \underline{\pi}_0$ which is a technological assumption. If $\pi_1 - \pi_0 > \underline{\pi}_1 - \underline{\pi}_0$ it means that the advantage make the marginal return of effort more effective which presupposes that the advantage is complementary with effort. It corresponds to an advantage on asset like information, social capital or more effective working tool for instance. Assuming $\pi_1 - \pi_0 < \underline{\pi}_1 - \underline{\pi}_0$ would have mean that advantage would have sabotaged the productivity of effort which is of course not relevant. Finally, the particular case $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$ means that the marginal efficiency of effort is constant over being disadvantaged or not. It is interpretable as substitutability between effort and advantage. In that case, advantage is more "obvious" like a head start that make expectation for one opponent less demanding than for the other one for instance.

For simplicity, we note:

$$\pi_{e_i} = P(q_i = 1 \mid e_i \mid \bar{i})$$

And:

$$\underline{\pi}_{e_i} = P(q_i = 1 \mid e_i \mid \underline{i})$$

At the end of stage 1, the winner is the player who produces an output strictly greater than his opponent. In case of a tie, both agents loose and get zero. At the tournament, each opponent has to choose a level of effort anticipating the strategy of his competitor, and taking into account the probability of being disadvantaged and the expected level of production. As shown in Figure 1, the probability of winning and losing of i given he is advantaged are respectively :

$$P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j) \mid \bar{i}) = \pi_{e_i}(1 - \underline{\pi}_{e_j})$$

$$P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j) \mid \bar{i}) = (1 - \pi_{e_i})\underline{\pi}_{e_j}$$

In the same way we get:

$$P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j) \mid \underline{i}) = \underline{\pi}_{e_i}(1 - \pi_{e_j})$$

$$P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j) \mid \underline{i}) = (1 - \underline{\pi}_{e_i})\pi_{e_j}$$

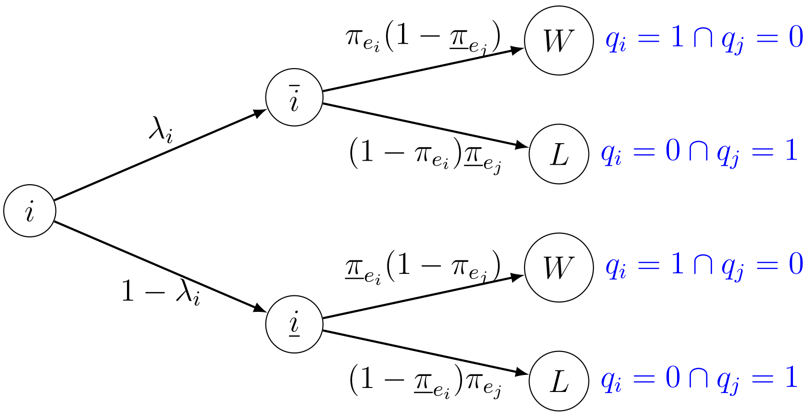

Figure 1: Conditional Probability Tree for i

Having characterized the tournament stage, we now turn to the second stage of the game.

3.2 Stage 2: the hierarchical relationship

Internal procedure

In the case of internal promotion procedures, people in charge of assessing opponents during competition are the same that people in charge of assessing the production in stage 2. So winning or loosing competition gives information about the future treatment in stage 2. Thus, both opponents will start stage 2 with a revised belief on been disadvantaged based on a bayesian learning process. Each player will take into account the performance of himself and his opponent and the effort they both exerted to update their beliefs as effort is observable between each other after the tournament . We mention however that the firm never obverses those efforts but knows that it will be rational from the opponent to update their beliefs.

During stage 2, we have a principal-agent relationship so that if j becomes agent then λ_j becomes λ_j^{expost}. In the interest of clarity, λ_j^{expost} will be denoted $\lambda_{j,P}^{e_i,e_j}$: the revised belief that refers to the probability for j of having been advantaged, given he won the tournament and given he exerted effort e_j and i exerted effort e_i. Symmetrically, λ_j^{expost} will be denoted $\lambda_{j,A}^{e_i,e_j}$ the revised probability of having been advantaged in case of defeat.

That is to say if j becomes agent in stage 2 we get:

$$\lambda_{j,A}^{e_i,e_j} = P(\overline{j} \mid (e_i, e_j) \mid q_i = 1 \cap q_j = 0)$$

And if j becomes principal in stage 2 we have:

$$\lambda_{j,P}^{e_i,e_j} = P(\overline{j} \mid (e_i, e_j) \mid q_i = 0 \cap q_j = 1)$$

Furthermore, as both opponents share the same prior belief and observe the same performance and efforts we know that:

$$\lambda_{j,A}^{e_i,e_j} = \lambda_{i,P}^{e_i,e_j} \text{ and } \lambda_{j,P}^{e_i,e_j} = \lambda_{i,A}^{e_i,e_j}$$

Besides we have in case of failure for j:

$$\begin{aligned} P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j)) = \lambda_j P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j) \mid \overline{j}) \\ + (1 - \lambda_j) P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j) \mid \underline{j}) \end{aligned}$$

and in case of success:

$$\begin{aligned} P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j)) = \lambda_j P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j) \mid \overline{j}) \\ + (1 - \lambda_j) P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j) \mid \underline{j}) \end{aligned}$$

Then we can write by bayesian inference:

$$\lambda_{j,A}^{e_i,e_j} = \frac{\overbrace{P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j) \mid \overline{j})}^{\text{Probability } j \text{ looses given he was advantaged}}}{\underbrace{P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j))}_{\text{Probability } i \text{ wins}}}$$

$$\lambda_{j,P}^{e_i,e_j} = \frac{\overbrace{P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j) \mid \overline{j})}^{\text{Probability } i \text{ looses given he was advantaged}}}{\underbrace{P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j))}_{\text{Probability } i \text{ looses}}}$$

As established in definition 1, the more $\lambda_{j,A}^{e_i,e_j}$ increases, the more the principal is perceived as *legitimate* by j. Indeed, if $\lambda_{j,A}^{e_i,e_j}$ increases it implies mechanically that $\lambda_{i,P}^{e_i,e_j}$ decreases which means it is less likely he was advantaged during the competition. Based on the strategies set during stage 1 we have then four different levels of *legitimacy*: $\lambda_{j,A}^{1,1}, \lambda_{j,A}^{0,1}, \lambda_{j,A}^{1,0}, \lambda_{j,A}^{0,0}$.

In case of failure, we have for each couple of strategies $(e_i, e_j) \in \{0,1\}$:

$$\lambda_{j,A}^{1,1} = \frac{\lambda_j(1-\pi_1)\underline{\pi}_1}{\lambda_j(1-\pi_1)\underline{\pi}_1 + (1-\lambda_j)(1-\underline{\pi}_1)\pi_1}$$
$$\lambda_{j,A}^{0,1} = \frac{\lambda_j(1-\pi_1)\underline{\pi}_0}{\lambda_j(1-\pi_1)\underline{\pi}_0 + (1-\lambda_j)(1-\underline{\pi}_1)\pi_0}$$
$$\lambda_{j,A}^{1,0} = \frac{\lambda_j(1-\pi_0)\underline{\pi}_1}{\lambda_j(1-\pi_0)\underline{\pi}_1 + (1-\lambda_j)(1-\underline{\pi}_0)\pi_1}$$
$$\lambda_{j,A}^{0,0} = \frac{\lambda_j(1-\pi_0)\underline{\pi}_0}{\lambda_j(1-\pi_0)\underline{\pi}_0 + (1-\lambda_j)(1-\underline{\pi}_0)\pi_0}$$

And in case of success we have:

$$\lambda_{j,P}^{1,1} = \frac{\lambda_j\pi_1(1-\underline{\pi}_1)}{\lambda_j\pi_1(1-\underline{\pi}_1) + (1-\lambda_j)\underline{\pi}_1(1-\pi_1)}$$
$$\lambda_{j,P}^{0,1} = \frac{\lambda_j\pi_0(1-\underline{\pi}_1)}{\lambda_j\pi_0(1-\underline{\pi}_1) + (1-\lambda_j)\underline{\pi}_1(1-\pi_1)}$$
$$\lambda_{j,P}^{1,0} = \frac{\lambda_j\pi_1(1-\underline{\pi}_0)}{\lambda_j\pi_1(1-\underline{\pi}_0) + (1-\lambda_j)\underline{\pi}_1(1-\pi_0)}$$
$$\lambda_{j,P}^{0,0} = \frac{\lambda_j\pi_0(1-\underline{\pi}_0)}{\lambda_j\pi_0(1-\underline{\pi}_0) + (1-\lambda_j)\underline{\pi}_0(1-\pi_0)}$$

Once we did that we can establish the function of production in stage 2. As in stage 1, the production can only take two value $\overline{S} = 1$ or $\underline{S} = 0$ and is stochastic. Besides the production is only produced by the agent and his effort is not contractible. The principal will offer a variable wage in order to incentivize high effort before the agent exerts effort. In case of high effort exerted in stage 2 we denote $\pi_1^P(\lambda_{j,P}^{e_i,e_j})$ the probability of producing 1 if j is agent and i is principal given that the production is produced only by the agent:

$$\pi_1^P(\lambda_{j,P}^{e_i,e_j}) = \lambda_{j,P}^{e_i,e_j}\pi_1 + (1-\lambda_{j,P}^{e_i,e_j})\underline{\pi}_1 \tag{1}$$

Besides we denote $\pi_0^P(\lambda_{j,P}^{e_i,e_j})$ the probability of producing in case of low effort we have:

$$\pi_0^P(\lambda_{j,P}^{e_i,e_j}) = \lambda_{j,P}^{e_i,e_j}\pi_0 + (1-\lambda_{j,P}^{e_i,e_j})\underline{\pi}_0. \tag{2}$$

Similarly if j wins and i becomes agent in second stage, we denote $\pi_1^A(\lambda_{i,A}^{e_i,e_j})$ the production produced by i with high effort such that:

$$\pi_1^A(\lambda_{i,A}^{e_i,e_j}) = \lambda_{i,A}^{e_i,e_j}\pi_1 + (1-\lambda_{i,A}^{e_i,e_j})\underline{\pi}_1 \tag{3}$$

and we denote $\pi_0^A(\lambda_{i,A}^{e_i,e_j})$ the production produced by i with low effort:

$$\pi_0^A(\lambda_{i,A}^{e_i,e_j}) = \lambda_{i,A}^{e_i,e_j}\pi_0 + (1-\lambda_{j,P}^{e_i,e_j})\underline{\pi}_0. \tag{4}$$

When we will establish the payoff in stage 1 using backward induction we will need to take into account the case when i wins and the case when he looses respectively weighed by the probability of winning and loosing. Indeed, the belief of being disadvantaged will not be the same after loosing or winning such that the expected payoff and the probability of producing the high level of production in stage 2 will not be the same either.

This mechanism of belief reinforcement through bayesian inference is central in this paper. Indeed if i becomes principal, he could have an interest in the reinforcement of the most positive belief for j that will motivates him to make a higher effort during stage 2. That why choosing his effort during stage 1 could be motivated not only by the interest of winning the tournament but also as an incentive strategy in stage 2 in case he becomes principal. This result will depend on a technological assumption seen in the next section.

Now that we have set the framework in the case of an internal procedure and notably the importance of bayesian inference we will describe an external procedure.

External procedure

In an external promotion procedure, the bayesian inference is no longer relevant. It means that people in charge of evaluation in the first stage are different than the one in charge of evaluation in second. As a consequence, winning or loosing the competition does not give information about discrimination in the future and the bayesian inference is not relevant anymore (i.e $\lambda_i^{expost} = \lambda_i$). Except this, the model in case of an external procedure is identical with the model of an internal procedure.

As previously specified the firm does not evaluate strictly speaking neither during the tournament nor the production in stage 2, but its role is to choose who will be in charge of evaluation. Assigning to a human resources department the responsibility of promotion process can be seen as an external procedure for instance. The human resources department will them define some general criterias and will evaluate employees independently of their regular colleagues or executives criterias or preferences. Civil service entrance examination can be also seen as an external procedure. For example it is case for the contest of "Capes" or "Aggrégation du secondaire" in french administration to become a teacher in high school. Of course, those kind of procedure are not necessary purely external in a sense that it could be completed by a last round like an interview with a future colleague for instance. But this is not necessary an issue for our work in a sense that our work has a normative purpose as well.

Now that we have established the theoretical framework in both stages, the difference between an internal and an external procedure, and notably the importance of bayesian inference, we are going to present our results in the next section.

4 Results

In this section we will present three major sets of results. In the first subsection, we will focus on the link between effort and procedural legitimacy as we define it in definition 1. We notably analyze how it could bolster or undermine the legitimacy of the winner. The second set of results deals with legitimacy as an incentive in stage 2, and how an agent may have a concern for legitimacy (or on the contrary be indifferent to it). The last set of results focuses on the incentivizing nature in first stage of both procedures (internal or external) that can choose the firm.

4.1 The ambivalent role of effort to bolster legitimacy

The purpose of this subsection is to study the link between effort and procedural legitimacy and notably how the role of effort can be ambivalent.

As we showed in the last section our theoretical framework suggests four level of legitimacies captured by four posterior beliefs of being disadvantaged after failure with respect to four couples strategies during stage 1: $\lambda_{j,A}^{1,1}, \lambda_{j,A}^{0,1}, \lambda_{j,A}^{1,0}, \lambda_{j,A}^{0,0}$. Now we are going to establish the mathematical conditions for $\lambda_{j,A}^{1,e_j} > \lambda_{j,A}^{0,e_j}$ which means that if i provides high effort instead of a low effort during the tournament and that he finally wins, his opponent j will have a stronger belief in not having been disadvantaged. This would be interpreted as effort bolstering procedural legitimacy as we define it in definition 1. Similarly, we can find the conditions that guarantee $\lambda_{j,A}^{e_i,0} > \lambda_{j,A}^{e_i,0}$ which means that for j loosing by exerting effort will bolster the legitimacy of the winner i.

Proposition 1. *(i) If i becomes the principal in stage 2, having exerted high effort during tournament bolsters his legitimacy that is to say, $\lambda_{j,A}^{1,e_j} > \lambda_{j,A}^{0,e_j}$, if and only if $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$. On the contrary, if $\frac{\pi_1}{\underline{\pi}_1} > \frac{\pi_0}{\underline{\pi}_0}$, then exerting high effort undermines legitimacy. However, winning the tournament, whatever his level of effort, undermines his legitimacy that is to say $\lambda_{j,A}^{e_i,e_j} < \lambda_j$. (ii) If j becomes agent, having exerted high effort during tournament bolsters legitimacy of the winner, that is to say $\lambda_{j,A}^{e_i,1} > \lambda_{j,A}^{e_i,0}$, if and only if $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$ and $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$.*

Proof. See Appendix A □

The intuition behind theses results are the following. Proposition 1 means that when i wins by exerting high effort, whatever the strategy of his opponent, it will reinforce the belief for j that he was advantaged. Intuitively, it means that effort bolsters legitimacy which is very intuitive but verified only under certain conditions. The conditions $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$ is interpreted as substitutability between effort and advantage. That is to say, the return of advantage is higher when the effort is low. Effort decreases the return of being advantaged. If we assume a perfect subtituability between the advantage and effort, that is to say if $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$, then $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$ is verified.

Further, Proposition 1 also leads to a very counterintuitive result if we assume the opposite condition $\frac{\pi_1}{\underline{\pi}_1} > \frac{\pi_0}{\underline{\pi}_0}$. This is an assumption of complementarity between effort and favoritism. Indeed, in that case exerting high effort in first stage decreases legitimacy in second stage. Because, effort *exacerbates* the effect of advantage. The effect of being disadvantaged is *worse* if effort of the opponent is high. The intuition is that making a very important performance will betray the one who has been advantaged. A little bit like a sportsman who has been doped and who outperforms too obviously his opponent.

Besides, proposition 1 asserts also that $\lambda_{j,A}^{e_i,e_j} < \lambda_j$, which means that loosing competition will always reinforce belief about being disadvantaged whatever the couple of strategies. It means that it is rational to be a "bad looser" whatever the effort exerted by his opponent even though he worked very "hard" and whatever his "merit". It will also be rational to be a bad looser even when loosing without exerting high effort. This result will be particularly useful in the last subsection to establish what procedure between an internal or an external procedure is the more incentivizing to exert high effort during the competition.

Finally, proposition 1 also sheds light on the case when j looses the competition by exerting himself a high effort instead of a low effort. This proposition states that the more he did work, the more it bolsters the legitimacy of the winner of the competition. However, this proposition holds only if we assume substitutability between effort and advantage. It means that for j loosing while he worked hard during the tournament will bolster the legitimacy of the winner. The intuition is that efforts of both opponents have the same effect: the more opponents work the less favoritism counts into the actual performance. So, outperforming the other becomes more related to effort. It is a quit counterintuitive result because one might believe at first that the legitimacy of the winner would have been higher if his effort was higher than the one of his opponent.

What is also interesting in that this last statement holds if we assume $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$. The assumption means that the nature of favoritism is linear and does not change the marginal return of effort. This is interpretable as if the opponent who is advantaged would benefit from a head start for instance. It would mean that assessors during the tournament would expect from him a less demanding outcome. This is straightforward that this assumption corresponds well to pure discrimination on gender or ethnical origin for instance. As a consequence, proposition 1 means that paradoxically, if discrimination is linear and one opponent benefit from a head start, then loosing the competition by exerting high effort tend to bolster legitimacy of the winner.

Now that we have exposed our first set of results on the ambivalent link between effort and legitimacy, we will in the next subsection focus the analysis on the link between legitimacy and the incentive wage in stage 2 by analyzing notably what conditions will make the agent concerned or indifferent about legitimacy.

4.2 Legitimacy and wage: concern or indifference of the agent for legitimacy

This subsection is a principal-agent framework. The principal cannot observe the effort of the agent such that effort is not contractible and he needs to offer a variable wage, that will be enough incentivizing and depending on the production observed. Indeed, as the production is stochastic, it is feasible to produce a high level of production while exerting low effort. Thus it could be rational for the agent to exert low effort and expecting receiving the high wage anyway.[1] Similarly to stage 1, the production can take two values $\bar{S} = 1$ or $\underline{S} = 0$ and effort can take only two values 1 and 0 as well.

4.2.1 Case without moral hazard

We will start by studying a very simple case where both the agent and the principal are risk neutral and where there is no limited liability constraint. This last assumption means that the principal could offer a negative wage $\underline{t}^*$ in case of low level of production and a positive wage $\bar{t}^*$ in case of high production as an incentive. In that case, the hazard moral is no longer an issue. Indeed, if we take into account the revised belief of effort return and if we denote ψ_2 the cost of high effort in stage 2, assuming that for low effort the cost is 0. The program of the principal is:

$$\max_{\bar{t},\underline{t}} \quad P_P^{e_i,e_j} = \pi_1^A(\lambda_{j,A}^{e_i,e_j})(\bar{S} - \bar{t}) + [1 - \pi_1^A(\lambda_{j,A}^{e_i,e_j})](\underline{S} - \underline{t})$$

$$s.c. \quad \pi_1^A(\lambda_{j,A}^{e_i,e_j})\bar{t} + [1 - \pi_1^A(\lambda_{j,A}^{e_i,e_j})]\underline{t} - \psi_2 \geq \pi_0^A(\lambda_{j,A}^{e_i,e_j})\bar{t} + [1 - \pi_0^A(\lambda_{j,A}^{e_i,e_j})]\underline{t} \tag{5}$$

$$\pi_1^A(\lambda_{j,A}^{e_i,e_j})\bar{t} + [1 - \pi_1^A(\lambda_{j,A}^{e_i,e_j})]\underline{t} - \psi_2 \geq 0 \tag{6}$$

[1]To read more work on moral hazard and contract theory see Laffont and Martimort (2009).

The incentive constraint (equation 5) is a constraint such that if it is verified, it will worthwhile for agent to exert high effort rather than low effort with respect to the wage $(\underline{t}, \bar{t})$ proposed by the principal. The participation constraint (equation 6) guarantees that exerting a low effort will be still worthwhile for the agent with respect with the wage. The results are very standard in the literature:

$$\underline{t}^* = -\frac{\pi_0^A(\lambda_{j,A}^{e_i,e_j})}{\Delta\pi(\lambda_{j,A}^{e_i,e_j})}\psi_2$$

$$\bar{t}^* = \frac{1-\pi_0^A(\lambda_{j,A}^{e_i,e_j})}{\Delta\pi(\lambda_{j,A}^{e_i,e_j})}\psi_2$$

Those results are found by saturating the incentive constraint and the participation constraint such that resolving the principal program is reduced to a 2 variables linear system of equations . Besides, the variable $\lambda_{i,A}^{e_i,e_j}$ regarding legitimacy and belief in being advantaged is treated as a fixed parameter in stage 2 as it depends on stage 1 behaviors. So, we find directly from the principal program:

$$P_P^{e_i,e_j} = \pi_1^A(\lambda_{j,A}^{e_i,e_j})(\bar{S} - \bar{t}^*) + [1 - \pi_1^A(\lambda_{j,A}^{e_i,e_j})](\underline{S} - \underline{t}^*)$$

Which gives a very standard result of contract theory, if we assume $\bar{S} = 1$ and $\underline{S} = 0$:

$$P_P^{e_i,e_j} = \pi_1^A(\lambda_{j,A}^{e_i,e_j}) - \psi_2$$

With $\pi_1^A(\lambda_{j,A}^{e_i,e_j}) = \lambda_{j,A}^{e_i,e_j}\pi_1 + (1-\lambda_{j,A}^{e_i,e_j})\underline{\pi}_1$ if i becomes the principal. We note that the principal's payoff will increase in $\lambda_{j,A}^{e_i,e_j}$ only because it increases the probability of getting the high level of production. But $\lambda_{j,A}^{e_i,e_j}$ does not change the cost of delegation ψ_2 which is a constant. This is because in this specific case with risk neutrality and no limited liability constraint, there is no informational rent.

Proposition 2. *In case of risk neutrality and no constraint of limited liability, the legitimacy of the principal decreases the spread between the high wage and the low wage: when $\lambda_{j,A}^{e_i,e_j}$ increases, $\Delta t^* = \bar{t}^* - \underline{t}^*$ decreases if and only if $\pi_1 - \pi_0 > \underline{\pi}_1 - \underline{\pi}_0$.*

Proof. See Appendix B □

Following Proposition 2, legitimacy is an incentive to high effort and an opportunity for the principal to substitute the wage gap, which is classically use as an incentive.

4.2.2 Case with moral hazard

Now we assume there is a limited liability constraint in stage 2 such that the principal cannot offer a negative wage in case of low production. In that case the agent has a positive informational rent and the issue of moral hazard is only partly solved in the standard approach. The program of the principal is then:

$$
\begin{aligned}
&\max_{\bar{t},\underline{t}} && \pi_1^A(\lambda_{j,A}^{e_i,e_j})(\bar{S}-\bar{t}) + [1-\pi_1^A(\lambda_{j,A}^{e_i,e_j})](\underline{S}-\underline{t}) \\
&s.c. && \pi_1^A(\lambda_{j,A}^{e_i,e_j})\bar{t} + [1-\pi_1^A(\lambda_{j,A}^{e_i,e_j})]\underline{t} - \psi_2 \geq \pi_0^A(\lambda_{j,A}^{e_i,e_j})\bar{t} + [1-\pi_0^A(\lambda_{j,A}^{e_i,e_j})]\underline{t} \\
& && \pi_1^A(\lambda_{j,A}^{e_i,e_j})\bar{t} + [1-\pi_1^A(\lambda_{j,A}^{e_i,e_j})]\underline{t} - \psi_2 \geq 0 \\
& && \bar{t} \geq 0 \\
& && \underline{t} \geq 0
\end{aligned}
$$

With $\bar{t} \geq 0$ and $\underline{t} \geq 0$ the limited liability constraints. For $\bar{S}=1$, $\underline{S}=0$ it leads to proposition 3:

Proposition 3. *In case of risk neutrality and limited liability, the equilibrium wage is:*

$$
\begin{aligned}
\underline{t}^* &= 0 \\
\bar{t}^* &= \frac{\psi_2}{\Delta\pi(\lambda_{j,A}^{e_i,e_j})}
\end{aligned}
$$

Thus, the agent has a concern for legitimacy, i.e $\frac{\partial \bar{t}^*}{\partial \lambda_{j,A}^{e_i,e_j}} > 0$, *if* $\pi_1 - \pi_0 > \underline{\pi}_1 - \underline{\pi}_0$. *On the contrary, the agent becomes indifferent to legitimacy, i.e* $\frac{\partial \bar{t}^*}{\partial \lambda_{j,A}^{e_i,e_j}} = 0$ *if* $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$. *The informational rent* $P_A = \psi_2 \frac{\pi_0^{e_i,e_j}}{\Delta\pi^{e_i,e_j}}$ *of the agent in stage 2 increases with legitimacy if and only if* $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$ *or if* $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$.

Proof. See Appendix C □

First, proposition 3 means that $\bar{t}$ is decreasing with legitimacy. Indeed, if principal choose high effort in stage 1, $\Delta\pi(\lambda_{i,A}^{e_i,e_j})$ will increase, so $\bar{t}$ will decrease. This is quit intuitive. If the principal increases his effort in stage 1, the belief for the agent in stage 2 will be more positive (see proposition 1) and that will increase mechanically the expected return of effort in stage 2. Legitimacy is then a substitute to monetary incentive implying that the principal may propose a lower wage.

Definition 2. We consider the agent as *concerned about legitimacy* if it is possible to increase incentive wage to make him exert high effort to compensate illegitimacy of the principal. By extension, we define agent as *indifferent to legitimacy* if his demanded wage is independent of legitimacy.

This illustrates perfectly the trade-off for the principal before he starts competition in stage 1. Does the cost of high effort in stage 1 worth it if it allows the principal to lower the wage he offers to the agent in stage 2? The technological condition in Proposition 3 means that the effort cancel partially the effect of being disadvantaged. Being legitimate (i.e $\lambda_{j,A}^{e_i,e_j}$ at a high level), the principal makes high effort much more worthwhile for the agent in comparison with low effort. In other words, he increases the marginal return of the agent's effort $\Delta\pi(\lambda_{j,A}^{e_i,e_j})$. So, it becomes much more interesting for the agent to exert high effort because this improves much more than low effort the probability of high level of production in a situation with legitimacy compared to a situation with no legitimacy. This is why $\bar{t}$ decreases with legitimacy. But, at the informational rent level, this is compensated by an increase of π_0. It becomes also, in absolute term this time, much more interesting to choose low effort because the probability of producing high level of production with low effort increases. As a consequence, the informational rent $P_A = \psi_2 \frac{\pi_0^{e_i,e_j}}{\Delta\pi^{e_i,e_j}}$ increases with legitimacy.

However if we assume $\frac{\pi_1}{\underline{\pi}_1} > \frac{\pi_0}{\underline{\pi}_0}$, the absolute effect of the probability of receiving the high wage while exerting low effort will be lower than the effect of legitimacy on the decreasing of $\bar{t}$. The intuition is that this assumption means a complementarity between effort and favoritism such that advantage will be fully leveraged only if high effort is exerted. So, at the informational rent level, the increasing effect of legitimacy on $\pi_0^{e_i,e_j}$ will be little, whereas it will remain a strong decreasing effect on $\bar{t}$.

What is interesting and very counterintuitive in Proposition 3 is that the conditions for reducing the informational rent with legitimacy are exactly the opposite of the condition for increasing legitimacy by effort in Proposition 1, respectively $\frac{\underline{\pi}_1}{\pi_1} < \frac{\underline{\pi}_0}{\pi_0}$ and

$\frac{\underline{\pi}_1}{\pi_1} > \frac{\underline{\pi}_0}{\pi_0}$. It means that if $\frac{\underline{\pi}_1}{\pi_1} < \frac{\underline{\pi}_0}{\pi_0}$ is verified, effort and advantage are complementary and the effort of principal in stage 1 decreases his legitimacy. Indeed, effort will betray the one who is advantaged by *exacerbating* the effect of the advantage on return (Proposition 1). So, if he wants to become legitimate to decrease the informational rent he would have to make low effort and keep low profile. However, if effort cancels partially the effect of being disadvantaged on return (i.e $\frac{\underline{\pi}_1}{\pi_1} > \frac{\underline{\pi}_0}{\pi_0}$), then the principal will increase his legitimacy if he makes high effort in stage 1 but, this legitimacy will increase the informational rent. Thus, exerting low effort during stage 1 becomes the only strategy to decrease informational rent and so whatever the technological assumption.

Assuming $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$ so that $\Delta\pi(\lambda_{j,A}^{e_i,e_j}) = \Delta\pi$, means that the surplus of production resulting from exerting high effort during stage 2 is the same whatever the legitimacy of the principal and the strategies of the two agents during stage 1. As we previously explained, the assumption can means favoritism can be seen as a head start, namely an advantage that makes the expectation of the firm for one opponent less demanding.

The payoff of the principal will be:

$$P_P = \pi_1(\lambda_{j,A}^{e_i,e_j}) - \pi_1(\lambda_{j,A}^{e_i,e_j})\frac{\psi_2}{\Delta\pi}$$

and the payoff of the agent will be:

$$P_A = \pi_0(\lambda_{j,A}^{e_i,e_j})\frac{\psi_2}{\Delta\pi}$$

In that case of course, the informational rent increases with legitimacy and so the cost of delegation for the principal. Indeed, $\Delta\pi$ is constant over $\lambda_{i,P}^{e_i,e_j}$ if we assume $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$, so the cost of delegation will only depend on $\lambda_{j,A}^{e_i,e_j}$ through $\pi_1(\lambda_{j,A}^{e_i,e_j})$. It means that $\bar{t}$ will not increase but his probability of occurrence will increase. However, this does not mean that the payoff of the principal will decrease with legitimacy. Indeed, as we assumed $P_P > 0$, it necessarily means that $\pi_1^{e_i,e_j}(1 - \frac{\psi_2}{\Delta\pi}) > 0$ and $1 - \frac{\psi_2}{\Delta\pi} > 0$, so the global effect of legitimacy on the cost of delegation will always be compensated by the increase of the expected level of production. Bolstering his legitimacy allows him to increase his payoff more than the payoff of the agent. So, it remains interesting for him to exert low effort. The proposition holds only if we assume the principal is not associated with discrimination and has not more information than looser of the competition about who has been advantaged.

4.3 Internal *versus* external promotion procedure

In this section we will compare the impact of efforts during the tournament of two different procedures of promotion: an internal and an external procedures. In an internal procedure, people in charge of the evaluation of opponents during the tournament remain the same than people that will evaluate them in the second stage. In external procedure, the organization outsources partially the promotion process such that people that will select the principal are not the same than people in charge of evaluating production in stage 2. Comparing the marginal return of both procedures allows us to elicit which one will be the more incentivizing for exerting high effort. In both cases we will establish a relation between the marginal return of effort and the *a priori* belief in being advantaged.

4.3.1 Internal procedure and aversion to "unfair" procedure

The purpose of this subsection is to study the behavior and strategy of i during the tournament with respect to λ_i. Besides, we particularly focus on the internal procedure which means that we need to take into account the legitimacy consideration in stage 2 through the bayesian inference.

From now we will assume $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$ so that $\Delta\pi^{e_i,e_j} = \Delta\pi$ to simplify the following calculations. Agents play a simultaneous game in which they chose the level of effort that maximizes their payoffs. Each couple of strategies $(e_i, e_j) \in \{0,1\}^2$ will lead to specific $\lambda_{j,P}^{e_i,e_j}$ (in case i wins) and $\lambda_{i,A}^{e_i,e_j}$(in case i looses). It means that for each couple of strategies, the winner of the tournament will have a different *legitimacy*. Besides, assuming $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$ implies that we have necessarily $\frac{\underline{\pi}_1}{\pi_1} \geq \frac{\underline{\pi}_0}{\pi_0}$. Thus according to proposition 1 not only winning by exerting high effort bolsters legitimacy but also loosing by exerting high effort bolsters the legitimacy of the winner. Finally, under assumption

$\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$, bolstering legitimacy is only motivated by a cognitive strategy (i.e a strategy that aims only to reveal or hide information) and not by a monetary incentive strategy. Indeed, this assumption of linearity implies the incentive wage will not vary with legitimacy (proposition 1). As a consequence, only the belief of having a high probability of receiving this wage in case of low effort will change. We have:

$$P_i^{e_i,e_j} = P_P^{e_i,e_j} \overbrace{P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j))}^{\text{Probability of winning}} + P_A^{e_i,e_j} \overbrace{P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j))}^{\text{Probability of loosing}} - \psi_{e_i}$$

We denote $P_i^{e_i,e_j}$ the average between the expected payoff i would get if he becomes the principal $P_P^{e_i,e_j}$, weighted by the probability of winning, and the expected payoff he would get if he becomes the agent $P_A^{e_i,e_j}$, weighted by the probability of loosing less the cost of effort $\psi_{e_i} \in \{\psi, 0\}$ during stage 1. The model is solved using *backward induction.*

We will first study in which conditions on the parameters $\lambda_i, \pi_1, \underline{\pi}_1, \pi_0, \underline{\pi}_0$ and ψ, expression $P_i^{1,e_j} - P_i^{0,e_j} > 0$ will hold, that is to say that the marginal return of effort is positive. It will mean that i will have an incentive to exert high effort whatever the effort of his opponent. We will secondly study in which conditions the couple of strategies $(e_i, e_j) = (1, 1)$ may become a Nash Equilibrium of the game that is to say $P_i^{1,1} > P_i^{0,1}$ and $P_j^{1,1} > P_j^{1,0}$ may be verified simultaneously.

Using equation (1) and equation (4), we can now write the global payoff $P_i^{e_i,e_j}$ for i at the beginning of stage 1 for any couple of strategies $(e_i, e_j) \in \{0, 1\}^2$ and ψ_1:

$$\begin{aligned} P_i^{e_i,e_j} = & \underbrace{\pi_1^P(\lambda_{j,P}^{e_i,e_j})(1 - \frac{\psi_2}{\Delta\pi})}_{\text{Principal's Payoff}} \overbrace{P(q_i = 1 \cap q_j = 0 \mid (e_i, e_j))}^{\text{Probability of winning}} \\ & + \underbrace{\pi_0^A(\lambda_{i,A}^{e_i,e_j})\frac{\psi_2}{\Delta\pi}}_{\text{Agent's Payoff}} \overbrace{P(q_i = 0 \cap q_j = 1 \mid (e_i, e_j))}^{\text{Probability of loosing}} - \psi_{e_i} \end{aligned}$$

Establishing a relation between $P_i^{1,e_j} - P_i^{0,e_j}$ and λ_i is actually a non trivial problem because several effects are bounded and depend on the ex-ante exogenous parameters of favoritism. Indeed, agents could have interest to actually be discriminated if they anticipate that in case of victory, the performance of the agent in second stage could be very high. Indeed, an agent who has low chances to be discriminated will be more productive.

The stake is to determine the conditions that guarantee that the opportunity of taking advantage of favoritism in the first stage would outweigh the legitimacy consideration in second stage.

We denote $r_i^{e_j}$ as the marginal return of high effort for i given he was fully advantaged ($\lambda_i = 1$) and given j exerts effort e_j such that:

$$r_i^{e_j} = \overbrace{(\pi_1 - \pi_0)(1 - \underline{\pi}_{e_j}) \quad \underbrace{\underline{\pi}_1(1 - \frac{\psi_2}{\Delta\pi})}_{\text{Payoff if } i \text{ is principal}}}^{\text{Marginal return if } i \text{ is principal and advantaged}} \quad + \quad \overbrace{(\pi_0 - \pi_1)\underline{\pi}_{e_j} \quad \underbrace{\pi_0 \frac{\psi_2}{\Delta\pi}}_{\text{Payoff if } i \text{ is agent}}}^{\text{Marginal Return if } i \text{ is agent and advantaged}}$$

Besides, we denote $\underline{r}_i^{e_j}$ as the marginal return of high effort for i given he was fully disadvantaged ($\lambda_i = 0$) and given agent j makes effort e_j such that:

$$\underline{r}_i^{e_j} = \overbrace{(\underline{\pi}_1 - \underline{\pi}_0)(1 - \pi_{e_j}) \quad \underbrace{\pi_1(1 - \frac{\psi_2}{\Delta\pi})}_{\text{Payoff if } i \text{ is principal}}}^{\text{Marginal return if } i \text{ is principal and disadvantaged}} \quad + \quad \overbrace{(\underline{\pi}_0 - \underline{\pi}_1)\pi_{e_j} \quad \underbrace{\underline{\pi}_0 \frac{\psi_2}{\Delta\pi}}_{\text{Payoff if } i \text{ is agent}}}^{\text{Marginal Return if } i \text{ is agent and disadvantaged}}$$

Then we look for the conditions that guarantee $P_i^{1,e_j} - P_i^{0,e_j}$ is positive.

Lemma 1. $P_i^{1,e_j} - P_i^{0,e_j} > 0$ *if and only if* $(r_i^{e_j} - \underline{r}_i^{e_j})\lambda_i + \underline{r}_i^{e_j} - \psi > 0$

Proof. See Appendix D ☐

Lemma 1 means there is a linear relation between the marginal return of effort and λ_i. If $r_i^{e_j} - \underline{r}_i^{e_j}$ is positive it means the more i will believe he is advantaged the more he will increase his effort. His interest is to be granted with preferential treatment and that his opponent is disadvantaged. It means that the negative effect of being advantaged on the payoff in second stage due to legitimacy consideration will never outweigh the positive effect during the first stage. However, if $r_i^{e_j} - \underline{r}_i^{e_j}$ is negative it means that the more i will believe he is advantaged the more he will decrease his effort. This case is the opposite case: the interest of i is then to be disadvantaged. It means that the legitimacy consideration in second stage will always outweigh the effect on the first stage.

Lemma 2. $P_i^{1,e_j} - P_i^{0,e_j} > 0$ *if and only if:*

- $\lambda_i > \hat{\lambda}_i$ *with* $\hat{\lambda}_i = \frac{\psi - \underline{r}_i^{e_j}}{r_i^{e_j} - \underline{r}_i^{e_j}}$
- $r_i^{e_j} > \psi > \underline{r}_i^{e_j}$
- $\pi_1 + \underline{\pi_0} > 1$

Proof. See Appendix E □

Lemma 2 gives the conditions that guarantee i will exert high effort whatever the effort of his opponent e_j. The major parameter to interpret in lemma 2 is the parameter $\hat{\lambda}_i$ which is the threshold to exert high effort for i. It is the minimum belief in being advantaged to accept to compete. It remains a function of the effort of the opponent j such that $\hat{\lambda}_i = \hat{\lambda}_i(e_j)$. It can be interpreted as the degree of aversion to "unfair" procedure of promotion for i. We can also interpret $(1 - \hat{\lambda}_i)$ as a degree of adhesion to the promotion procedure. Besides, if we conceptualize the second stage as a long term career achievement and first stage as the career itself, we can interpret also $(1 - \hat{\lambda}_i)$ as an adhesion to the organization in its entirety. Thus, following this idea, the more $(1 - \hat{\lambda}_i)$ will increase the more people will accept to be a part of the organization because in the long run they think it is more likely that effort will lead to a promotion. And so even though they are suspicious on being perhaps discriminated.

The other conditions are more technical. Let us focus on the first condition on ψ: $\underline{r}_i^{e_j} < \psi$. If we refer to the equation in lemma 1 it means that the surplus of payoff exerting high effort when being fully disadvantaged is negative because the marginal return in that case does not compensate the cost of effort. Thus, it becomes necessary to be a minimum optimistic about being advantage to exert high effort. The second condition $r_i^{e_j} > \psi$ means that being advantaged brings a return that more than compensates the cost of effort.

Finally, the condition $\pi_1 + \underline{\pi_0} > 1$ is the condition for $r_i^1 - \underline{r}_i^1 > 0$ to hold. Indeed, we have:

$$\begin{aligned} r_i^{e_j} - \underline{r}_i^{e_j} =& \Delta\pi \Big[(1 - \underline{\pi}_{e_j}) \underline{\pi}_1 - (1 - \pi_{e_j}) \pi_1 \Big] (1 - \frac{\psi_2}{\Delta\pi}) \\ &+ \Delta\pi \Big[\pi_{e_j} \underline{\pi}_0 - \underline{\pi}_{e_j} \pi_0 \Big] \frac{\psi_2}{\Delta\pi} \end{aligned} \tag{7}$$

Now, if we look at equation (7), $r_i^{e_j} - \underline{r}_i^{e_j}$ will be positive if $(1 - \underline{\pi}_{e_j})\underline{\pi}_1 - (1 - \pi_{e_j})\pi_1$ remains always superior to $\underline{\pi}_{e_j}\pi_0 - \pi_{e_j}\underline{\pi}_0$. That is to say the marginal probability of getting the principal payoff $1 - \frac{\psi_2}{\Delta\pi}$ is high enough to compensate the fact that the marginal probability of getting the agent wage $\frac{\psi_2}{\Delta\pi}$ is negative. We argue this is verified if $\pi_1 > 1 - \underline{\pi_0}$ holds. This will be verified notably if $\underline{\pi_0} > \frac{1}{2}$. We interpret this as the production to be "easy" in a sense that even with low effort and discrimination it is still more likely to complete the task than to fail.

Now we are going to study the condition that guarantee the couple of strategies $(1, 1)$ could be a Nash Equilibrium.

Proposition 4. *The couple of strategies when both agents make high effort during stage 1 is a Nash Equilibrium that is to say $P_i^{1,1} - P_i^{0,1} > 0$ and $P_j^{1,1} - P_j^{1,0} > 0$ if and only if:*

- $1 - \lambda_i^* > \lambda_i > \lambda_i^*$ *with* $\lambda_i^* = \frac{\psi - \underline{r}_i^1}{r_i^1 - \underline{r}_i^1}$
- $r_i^1 > \psi > \underline{r}_i^1$
- $\pi_1 + \underline{\pi_0} > 1$

Proposition 4 means that high effort could be chosen by both agents and is a Nash Equilibrium under certain conditions. It is quit intuitive that we cannot find a dominant strategy equilibrium directly with lemma 2 because we would need opponents to be perfectly symmetric (except on their beliefs λ_i and λ_j where $\lambda_i \neq \lambda_j$). And this is not true if opponent chooses a different strategy because $r_i^1 \neq r_j^0$ and $\underline{r}_i^1 \neq \underline{r}_j^0$. However the couple of strategies $(e_i, e_j) = (1, 1)$ guarantees this symmetry. As both opponent are totally identical and choose the same strategy, it leads to $r_i^1 = r_j^1$ and $\underline{r}_i^1 = \underline{r}_j^1$. Besides, they share the same belief such that $\lambda_j = 1 - \lambda_i$. Thus it is straightforward using lemma 1 that $P_j^{1,1} - P_j^{1,0} > 0$ leads to $(r_i^{e_j} - \underline{r}_i^1)(1 - \lambda_i) + \underline{r}_i^1 - \psi > 0$ which leads to proposition 4.

Similarly to lemma 2, the major parameter to interpret is the threshold λ_i^*. As $\hat{\lambda}_i$ in lemma 2 is a function of the effort of the opponent j such that $\hat{\lambda}_i = \hat{\lambda}_i(e_j)$, we denote $\lambda_i^* = \hat{\lambda}_i(1)$ the threshold to exert high effort for i given the effort of the opponent j

is high. If we assume that opponents are naturally suspicious such that belonging to specific social group based on ethnic origin or religion, or differences in gender for instance, lead necessary to discrimination, then the spread $[1-\lambda_i^*;\lambda_i^*]$ can be an indicator of accepted social heterogeneity. The more the spread will be high, the more the two opponents will tend to "accept" competition in spite of discrimination, in a sense that they will exert high effort anyway.

Now that we have established that the marginal return of effort in an internal procedure is a linear relation with respect to the *a priori* belief in being discriminated we will focus on the marginal return in an external procedure.

4.3.2 External procedures and the cost of illegitimacy

Now we are going to study a case where people who evaluate the performance of opponents and who design the procedure of selection are *not* the same people who will evaluate the performance of the agent in the second stage once the principal is chosen. The purpose of this subsection will be to compare the marginal return between the two procedures to establish which one is the more incentivizing to exert high effort.

The consequence in external procedures is that the bayesian inference becomes useless for the agent. Learning information about having been discriminated or not during competition does not highlight what will occur in the future once becoming an employee. As a consequence we will assume at the beginning of the second stage that $\lambda_{i,A}^{e_i,e_j}=\lambda_i$ (i.e the belief of being advantaged in second stage remains the same than in first stage because only due to exogenous factors). Thus, except the absence of bayesian inference, the payoff of i will be similar than in the internal procedure because the function of production and the assessing technology are assumed to be identical. This allows us to denote $E_i^{e_i,e_j}(\lambda_i)$ the payoff of i in the first stage such that:

$$\begin{aligned}E_i^{e_i,e_j}(\lambda_i) =&\pi_1^P(\lambda_j)(1-\frac{\psi_2}{\Delta\pi})P(q_i=1\cap q_j=0\mid(1,e_j))\\&+\pi_0^A(\lambda_i)\frac{\psi_2}{\Delta\pi}P(q_i=0\cap q_j=1\mid(1,e_j))-\psi\end{aligned}$$

To facilitate understanding we will also denote from now $E_i^{1,e_j}(\lambda_i) - E_i^{0,e_j}(\lambda_i) = \Delta E_i(\lambda_i)$ and $P_i^{1,e_j} - P_i^{0,e_j} = \Delta P_i(\lambda_i)$. In this subsection we will establish the sign of $\Delta P_i(\lambda_i) - \Delta E_i(\lambda_i)$ with respect to λ_i which will allow to understand what procedure is the more incentivizing to exert effort. The method is divided in 3 steps. The first step is to establish the value of λ_i such that $\Delta P_i(\lambda_i)$ and $\Delta E_i(\lambda_i)$ are equal. The second step is to focus on $\Delta E_i(\lambda_i)$ and to find that it is concave with respect to λ_i. The last step is to use the result of lemma 1 and lemma 2 of the last subsection to conclude on the sign of $\Delta P_i(\lambda_i) - \Delta E_i(\lambda_i)$.

We will now focus on establishing the value of λ_i such that $\Delta P_i(\lambda_i) = \Delta E_i(\lambda_i)$. It is straightforward that $E_i^{e_i,e_j}(0) = P_i^{e_i,e_j}(0)$ and $E_i^{e_i,e_j}(1) = P_i^{e_i,e_j}(1)$ because if $\lambda_i = 0$ then $\lambda_i^{e_i,e_j} = 0$ and if $\lambda_i = 1$ then $\lambda_i^{e_i,e_j} = 1$. It is intuitive because if $\lambda_i = 1$ for instance, it means that there is no more uncertainty on being discriminated *a priori* and the competition does not bring new information. Thus the learning mechanism is not relevant. And if $\lambda_i = 0$, it is absolutely sure in the eyes of opponents that i will be discriminated such that the outcome of the tournament will not bring new information either. As a consequence we already know that in both case where there is no initial uncertainty (in $\lambda_i = 0$ and $\lambda_i = 1$) on being discriminated, the payoff in both procedures (and of the marginal payoff) will be identical.

Lemma 3. *$\Delta E_i(\lambda_i)$ is concave for $\lambda_i \in (0, 1)$ if $(1 - \frac{\psi_2}{\Delta\pi}) > \frac{\psi_2}{\Delta\pi}$.*

Proof. See Appendix F □

We can now establish the shape of $\Delta E_i(\lambda_i)$. Lemma 3 means that the relation between λ_i and the marginal return of effort is not linear but concave contrary to the case of an internal procedure. Actually $\Delta E_i(\lambda_i)$ is a second degree polynomial with a negative coefficient (figure 2). This lead directly to lemma 4.

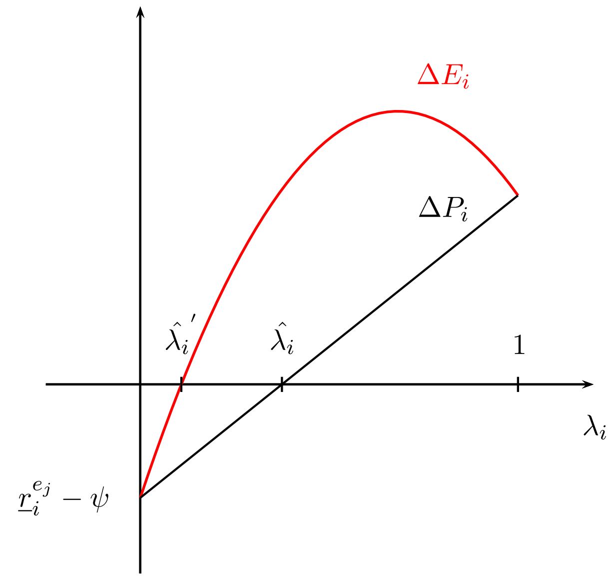

Figure 2: Marginal return of effort for both procedures with respect to λ_i

Lemma 4. *There is a value of λ_i denoted $\hat{\lambda}_i{}'$ such that $\Delta E_i(\hat{\lambda}_i{}') = 0$ if $r_i^{e_j} > \psi > \underline{r}_i^{e_j}$ is satisfied. It means that there is a threshold below which the belief about being advantaged will be not incentivizing.*

Lemma 4 means that there is minimum value for λ_i denoted $\hat{\lambda}_i{}'$ enough to incentivize high effort in an external procedure. As we can see in figure 2, this is guaranteed by the fact we know that $E_i^{e_i,e_j}(0) < 0$ and that $E_i^{e_i,e_j}(1) > 0$ since $r_i^{e_j} > \psi > \underline{r}_i^{e_j}$ Indeed this last condition guarantees in lemma 2 that $P_i^{e_i,e_j}(0) < 0$ and $P_i^{e_i,e_j}(1) > 0$. Thus as we know that $E_i^{e_i,e_j}(0) = P_i^{e_i,e_j}(0)$ and $E_i^{e_i,e_j}(1) = P_i^{e_i,e_j}(1)$, it implies $\overset{1}{E}{}_i^{e_i,e_j}(0) < 0$ and $E_i^{e_i,e_j}(1) > 0$.

Hence, as we can see in figure 2, knowing that $E_i^{e_i,e_j}(0) = P_i^{e_i,e_j}(0)$ and $E_i^{e_i,e_j}(1) = P_i^{e_i,e_j}(1)$ and that $\Delta E_i(\lambda_i)$ is concave (lemma 3) guarantee that it will always be superior to $\Delta P_i(\lambda_i)$ for $\lambda_i \in (0, 1)$ because $\Delta P_i(\lambda_i)$ is linear. That leads directly to proposition 5.

Proposition 5. *Relatively to external procedures, internal procedures:*

- *incentivizes opponents to decrease their efforts that is to say:* $\Delta E_i(\lambda_i) > \Delta P_i(\lambda_i)$ *is always verified for any* λ_i.
- *makes agent more averse to discrimination that is to say:* $\hat{\lambda}_i{}' < \hat{\lambda}_i$.

Proposition 5 means that opponents will always have a higher marginal payoff in the case of an external procedure. It means the external procedure will be more incentivizing to exert high effort. Comparing the two procedures allows us to highlight what is related exclusively to a learning behavior. Indeed, the biased probability of winning due to discrimination is identical in the two procedures and the function of production in second stage is identical as well. Thus the only difference is the existence or the absence of the learning process. As a consequence, understanding proposition 1 allows us to highlight the underlying mechanism in proposition 5. And, according to proposition 1 we get $\lambda_{i,A}^{e_i,e_j} < \lambda_i$. It means that even though effort is a way in internal procedures to defuse doubt on his legitimacy for the principal, it will be never be enough to entirely neutralize the effect of failure on the bayesian learning. In other words, the looser will always remain a "bad looser" whatever the intensity of the effort of his opponent (and his "merit") and the winner will always remain a "good winner" in a sense that he will always have more doubt about his own legitimacy after winning the competition even thought he put a high effort to win. In some ways, defeat will always make the looser more suspicious and victory will always make the principal feel more "guilty" whatever their efforts if they have bayesian rationality. It is interesting because we could expect that internal procedural would be more incentivizing as a high effort could dispel doubt about unfair treatment. But actually, this effect will not be sufficient if we assume bayesian rationality as suspicion and "guilt" will always be more significative after the tournament.

Hence in the *internal procedure*, as the proverb goes: "ill-gotten gains never leads to prosperity". Namely, there is a endogenous mechanism of "redressing injustice". Winning the tournament makes the principal believe he was more advantaged in the short term during the tournament than he previously though. But this advantage will be no longer useful and will even become a burden for him as he symmetrically receives information about how the new agent will be discriminated in the future and then will be less productive. That is the illegitimacy entails no extra incentive cost but has only a straightforward effect on the efficiency of the effort of the agent because his effort will not be valued by the firm. As a consequence, the consideration for his own legitimacy is captured by this expected cost that we can consider as the cost of his illegitimacy. In other words, he supports the liability of preferential treatment he received. In external procedures, this liability disappears as the changing of assessors becomes a "fresh start" for the agent. Thus the principal will perfectly benefit from his potential advantage during the tournament without assuming the responsibility in second stage, and it will be liberating as he will actually have no information about a potential unfair treatment for the agent in the future. The person in charge of assessment in an external procedure will be a scape goat in some ways. This explain why an external procedure will be more incentivizing. In some ways, the neutrality of an external procedure is appreciated and incentivizing for a future principal because it allows to "clear his conscience" in case of victory. In other words, it makes him believe that this unfair treatment will not be supported anymore by the looser of the tournament. The future principal knows in advance that even though he could have been advantaged in the first place, card will be reshuffled in the future and his agent may be treated fairly.

So, the benefit of neutrality is not where we could expect it to be. Neutrality of external procedures is not incentivizing because people would have a higher *a priori* belief about being advantaged. Indeed, in internal procedures it is assumed that they do not know either if they could be disadvantaged *a priori* such that they will have the same *a priori* belief in both procedures. However, the neutrality of external procedures will offer to the winner the opportunity to have the belief that his opponent will have a fresh start and could be totally fairly rewarded by the organization in the future. So, he will not support the cost of his illegitimacy as the unfair treatment will possibly occur only once. This expected cost of illegitimacy is of course a perceived cost as it is based on a subjective prior belief that is not necessarily based on tangible elements.

The direct consequence is that agent will be more averse to discrimination in internal procedure as he will be less incentivized by being potentially advantaged. Indeed, $\hat{\lambda}_i{}' < \hat{\lambda}_i$ means the threshold for exerting high effort will be higher. To some extent, discrimination "counts twice" in the internal procedure and this is why it is less acceptable in the view of opponents. Whereas, in the external procedure, stage 2 is a fresh start for the looser of the competition and it ensures the future principal from not supporting the cost of his illegitimacy. Like in the last subsection, we can also interpret $1 - \hat{\lambda}_i{}'$ as a degree of adhesion to the external procedure. If we interpret the promotion at the beginning of the second stage as a long run career achievement, and first stage as career itself we can interpret $1 - \hat{\lambda}_i{}'$ as a degree of adhesion to the whole organization. As a consequence proposition 5 implies that an external procedure will favor adhesion to the organization in its entirety.

It is important to understand that discrimination does not occur necessarily in the model. But we only focus on the point of view of opponents that have an *a priori* suspicion that can be reinforced or not after the competition even though this resentment could be totally false. As a consequence, the issue for choosing the procedure of promotion is to anticipate that a climate of suspicion could emerge even though it could not necessary be founded on real proof, and to try to defuse it. Mostly, the issue for the firm is to anticipate that opponents have interest in seeing themselves legitimate in case of victory and do not want to assume the cost of their illegitimacies in the future.

Yet, a promotion process designed by a human resources department could be seen as an external procedure in as sense that employee would not be evaluated by their actual colleagues. Thus, we argue that for this reason such departments could be designed to become a scape goat and channel all the resentment after the failures of opponents. The same reasoning could be applied for internal contest for civil servant in public administration. This way the climate of suspicion would be defused. Furthermore the opponent will know that in case of victory they will not have to assume the liability of a potential unfair treatment and thus the cost of their illegitimacies.

Furthermore, we also can assume that opponents choose themselves the procedure of promotion. This way we could talk about an endogenous choice of procedure. Then they will choose an external procedure instead of an internal procedure which is a choice that can be interpreted as deny. Indeed, it would mean choosing to hide oneself information about a potential unfair treatment that could occur within the organization after the promotion. It reveals that it would be more efficient for opponents not to discover information about having been discriminated by their colleagues.

5 Conclusion

This paper investigates how to build microeconomics foundation of the concept of procedural legitimacy. A concept that to best of our knowledge has not been treated in a theoretical approach. The major contribution is to use bayesian approach instead of unfairness aversion to explain submission, choice of effort and reaction to unfairness.

In our work, we define the legitimacy of a leader through the fairness of the procedure of selection that promoted him. As a consequence, the indicator of legitimacy is the probability of having been advantaged during this promotion process. This probability will be revised such that the performance observed during the promotion process can send a signal on a potential advantage received. We argue that the role of effort to bolster this kind of legitimacy will depend on technological assumption. In case of substitutability between effort and advantage (i.e a case where the advantage is interpreted as a difference in the performance expected for one opponent relatively to the other), effort bolsters legitimacy. But paradoxically, in case of complementarity (i.e a case when the advantage is interpreted as an asymmetric allocation of a mean of production that one opponent could benefit to the detriment of other such as a more effective working tool or a key contact), effort reveals advantage and then undermines legitimacy. Indeed, in that case, a high performance will betray the advantaged received a little bit like a sportsman who has been doped and makes an extraordinary but suspicious achievement.

However, this legitimacy does not necessary have an impact on behavior in response to “unfair” allocation of incomes after the promotion during the hierarchical relationship. Indeed we show that after the promotion, once an agent sees his former opponent becoming his executive, then he could be indifferent to legitimacy issue in a sense that he will not demand a higher wage if his executive is perceived as non legitimate. This result holds in case of substitutability between effort and advantage but it will not hold if we assume complementarity. It is quit intuitive because in that case, advantage will interact with effort like a competence would do. Thus in the eyes of a suspicious agent, as the remuneration is conditional on high performance, his chances of being rewarded will be very low. He will then demand a higher wage to compensate that.

The last result of this paper is to show how the role of procedures of selection will be a crucial stake for the firm if its objective is to implement high effort during the promotion process. We compare the marginal return of two types of procedures: an internal procedure where people in charge of evaluate opponents during the tournament are the same that people that will assess the production of the one who will remain employee thereafter, and an external procedure where the assessors change between the tournament and the hierarchical relationship. Thus, in our approach the firm is not responsible for any favoritism and the suspicion of opponents may be based on non tangible fact or totally fictive. Intuitively, we could expect that an internal procedure would be more incentivizing because being aware that their future employees will still be evaluated in the future by the same people could be an incentive to dispel doubts on their potential advantages. Indeed, one might think that after the competition, the winner will have to deal with the consequences of the signal his performance could have sent about his potential advantage. Thus in this internal procedure framework, exerting high effort could be a strategy to demonstrate he was not advantaged making the internal procedure a very incentivizing procedure.

Actually our finding is the opposed: we show that it is the external procedure that will be more incentivizing. This result depends on the inability of effort to bolster legitimacy (or dispel doubts) if we assume bayesian rationality. Indeed, we show that loosing the competition will always make the looser more suspicious about having been disadvantaged that he was before the competition and so whatever the effort exerted by both opponents. Symmetrically, the winner will always be more confident about being advantaged, that is in some ways, he will always feel more “guilty”, and so whatever his level of effort. Thus, the winner will have mechanically more information about how his agent will be more likely to be disadvantaged in the future and then be less productive. This will tend to inhibit effort upstream during the tournament as it makes the fact of winning much less profitable. In an external procedure this behavior does not occur because presupposed unequal treatment during the tournament will not give information on a potential unfair treatment in the future because the assessors will change. In other words, being aware in advance that the cards will be reshuffled after the competition will be highly motivating.

This theoretical framework gives an indicator of sensibility to “unfair” treatment. We define it as the minimum belief in being advantaged that will be sufficient to incentivize effort. We show that in an external procedure this threshold will be lower than in an internal procedure. It means that in external procedures relatively to internal procedures, more suspicious opponents will accept to compete in spite of their suspicion. We can also interpret it as an indicator of the acceptable degree of social heterogeneity in an organization. Indeed, if we assume that opponents are very pessimistic and that social heterogeneity lead mechanically to an *a priori* suspicion of unequal treatment, then the external procedure guarantees a higher adhesion the organization (in a sense that it is more likely opponent will compete anyway).

As a consequence our results suggest that it would be in the interest of the organization to delegate promotion to an independent entity such as human resources department or to promote people through independent contests with independent assessors. Those institutions will serve as a scape goat. We argue that contests for civil servant in public administration for instance, like Capes or Aggregation du secondaire which are contest to become teacher in high school in french administration are design partially for this reason.

A possible extension to our model would be to complicate the objective of the firm by adding uncertainty on competence. Then there would be two sources of uncertainty simultaneously: unfair treatment and competence. In that case, it would be perhaps interesting to assume that internal procedures assess better the competence that external procedures such that there would be a trade off for the firm. Should the firm prefer to defuse suspicion by choosing an external procedure or to favor pure economic choice by choosing an internal procedure? The issue would not be trivial as a worse assessment of competence in the external procedure could be compensated by a higher motivation in an external procedure that potentially could reveal more competence as well.

Appendix

A/ Proof Proposition 1

- First let us focus on the case i wins. Indeed, $\lambda_{j,P}^{1,e_j} > \lambda_{j,P}^{0,e_j}$ leads by successive equivalence to:

$$\frac{\lambda_j(1-\pi_{e_j})\underline{\pi}_1}{\lambda_j(1-\pi_{e_j})\underline{\pi}_1+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_1} > \frac{\lambda_j(1-\pi_{e_j})\underline{\pi}_0}{\lambda_j(1-\pi_{e_j})\underline{\pi}_0+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_0}$$

$$\frac{\underline{\pi}_1}{\lambda_j(1-\pi_{e_j})\underline{\pi}_1+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_1} > \frac{\underline{\pi}_0}{\lambda_j(1-\pi_{e_j})\underline{\pi}_0+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_0}$$

$$\frac{1}{\lambda_j(1-\pi_{e_j})+(1-\lambda_j)(1-\underline{\pi}_{e_j})\frac{\pi_1}{\underline{\pi}_1}} > \frac{1}{\lambda_j(1-\pi_{e_j})+(1-\lambda_j)(1-\underline{\pi}_{e_j})\frac{\pi_0}{\underline{\pi}_0}}$$

  This leads $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$ because each term of this inequation is decreasing function. Finally we have $\frac{\underline{\pi}_1}{\pi_1} > \frac{\underline{\pi}_0}{\pi_0}$.

- Now let us focus on the case when i looses. Indeed, $\lambda_{i,A}^{1,e_j} > \lambda_{i,A}^{0,e_j}$ leads by successive equivalence to:

$$\frac{\lambda_i(1-\pi_1)\underline{\pi}_{e_j}}{\lambda_i(1-\pi_1)\underline{\pi}_{e_j}+(1-\lambda_i)(1-\underline{\pi}_1)\pi_{e_j}} > \frac{\lambda_i(1-\pi_0)\underline{\pi}_{e_j}}{\lambda_i(1-\pi_0)\underline{\pi}_{e_j}+(1-\lambda_i)(1-\underline{\pi}_0)\pi_{e_j}}$$

$$\frac{1-\pi_1}{(1-\pi_1)\left[\lambda_i\pi_{e_j}+(1-\lambda_i)\frac{1-\underline{\pi}_1}{1-\pi_1}\pi_{e_j}\right]} > \frac{1-\pi_0}{(1-\pi_0)\left[\lambda_i\pi_{e_j}+(1-\lambda_i)\frac{1-\underline{\pi}_0}{1-\pi_0}\pi_{e_j}\right]}$$

$$\frac{1}{\lambda_i\pi_{e_j}+(1-\lambda_i)\frac{1-\underline{\pi}_1}{1-\pi_1}\pi_{e_j}} > \frac{1}{\lambda_i\pi_{e_j}+(1-\lambda_i)\frac{1-\underline{\pi}_0}{1-\pi_0}\pi_{e_j}}$$

This is verified if $\frac{1-\underline{\pi}_1}{1-\pi_1} > \frac{1-\underline{\pi}_0}{1-\pi_0}$ because term of the inequation are a increasing function. By successive equivalence we have:

$$\frac{1-\underline{\pi}_1}{1-\pi_1} > \frac{1-\underline{\pi}_0}{1-\pi_0}$$

$$(1-\underline{\pi}_1)(1-\pi_0) > (1-\underline{\pi}_0)(1-\pi_1)$$

$$1-\underline{\pi}_1-\pi_0+\underline{\pi}_1\pi_0 > 1-\underline{\pi}_0-\pi_1+\underline{\pi}_0\pi_1$$

$$\underline{\pi}_0-\underline{\pi}_1+\underline{\pi}_1\pi_0 > \pi_0-\pi_1+\underline{\pi}_0\pi_1$$

It is verified if $\underline{\pi}_0-\underline{\pi}_1=\underline{\pi}_0-\underline{\pi}_1$, that is to say $\underline{\pi}_1-\underline{\pi}_0=\underline{\pi}_1-\underline{\pi}_0$, and if $\underline{\pi}_0\pi_1 > \underline{\pi}_1\pi_0$ that is to say if $\frac{\pi_1}{\underline{\pi}_1} < \frac{\pi_0}{\underline{\pi}_0}$.

- $\lambda_{j,P}^{e_i,e_j} < \lambda_j$

  By successive equivalence we have:

$$\frac{\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}}{\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_{e_i}} < \lambda_j$$

$$\frac{\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}-\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}\lambda_j-\lambda_j(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_{e_i}}{\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_{e_i}} < 0$$

$$\frac{(1-\pi_{e_j})\underline{\pi}_{e_i}-\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}-(1-\underline{\pi}_{e_j})\pi_{e_i}+\lambda_j(1-\underline{\pi}_{e_j})\pi_{e_i}}{\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}+(1-\lambda_j)(1-\underline{\pi}_{e_j})\pi_{e_i}} < 0$$

The denominator is always positive. Let us find conditions that guarantee the numerator is negative. By successive equivalence it leads to:

$$(1-\pi_{e_j})\underline{\pi}_{e_i}-\lambda_j(1-\pi_{e_j})\underline{\pi}_{e_i}-(1-\underline{\pi}_{e_j})\pi_{e_i}+\lambda_j(1-\underline{\pi}_{e_j})\pi_{e_i} < 0$$

$$(1-\pi_{e_j})\underline{\pi}_{e_i}(1-\lambda_j)-(1-\underline{\pi}_{e_j})\pi_{e_i}(1-\lambda_j) < 0$$

$$(1-\pi_{e_j})\underline{\pi}_{e_i}-(1-\underline{\pi}_{e_j})\pi_{e_i} < 0$$

$(1-\pi_{e_j})\underline{\pi}_{e_i} < (1-\underline{\pi}_{e_j})\pi_{e_i}$ is always verified because $\pi_{e_i} > \underline{\pi}_{e_i}$ and $1-\pi_{e_j} < 1-\underline{\pi}_{e_j}$. Thus $\lambda_{j,P}^{e_i,e_j} < \lambda_j$ is always verified.

B/ Proof Proposition 2

$$\Delta t^* = \bar{t}^* - \underline{t}^*$$

$$\Delta t^* = \frac{\psi_2}{\Delta\pi^{e_i,e_j}}(-\pi_0^{e_i,e_j} + 1 + \pi_0^{e_i,e_j})$$

$$\Delta t^* = \frac{\psi_2}{\Delta\pi^{e_i,e_j}}$$

Besides we know that:

$$\pi_1^{e_i,e_j} = \lambda_j^{e_i,e_j}\pi_1 + (1-\lambda_j^{e_i,e_j})\underline{\pi}_1$$

$$\pi_1^{e_i,e_j} = \lambda_j^{e_i,e_j}(\pi_1 - \underline{\pi}_1) + \underline{\pi}_1$$

And

$$\pi_0^{e_i,e_j} = \lambda_j^{e_i,e_j}(\pi_0 - \underline{\pi}_0) + \underline{\pi}_0$$

And

$$\Delta\pi^{e_i,e_j} = \lambda_j^{e_i,e_j}[(\pi_1 - \underline{\pi}_1) - (\pi_0 - \underline{\pi}_0)] + \underline{\pi}_1 - \underline{\pi}_0$$

Thus, $\Delta\pi^{e_i,e_j}$ increases and Δt^* decreases when $\pi_1 - \pi_0 > \underline{\pi}_1 - \underline{\pi}_0$.

C/ Proof Proposition 3

- As $\underline{t} \geq 0$ the limited liability constraint we infer that the first best wage in case of low production,i.e $-\frac{\pi_0}{\Delta\pi(\lambda_{j,A}^{e_i,e_j})}\psi_2$, will not be implementable. Thus $\underline{t}^* = 0$. Hence, it is straightforward that saturating the incentive constraint and integrating $\underline{t} = 0$ we have: $\Delta\pi(\lambda_{j,A}^{e_i,e_j})\bar{t} - \psi_2 = 0$ wich leads to $\bar{t}^* = \frac{\psi_2}{\Delta\pi(\lambda_{j,A}^{e_i,e_j})}$.

- Let us find the change of direction of $\bar{t}^*$ with respect to $\lambda_{j,A}^{e_i,e_j}$ that we denote for the sake of clarity λ. We have:

$$\begin{aligned}\bar{t}^* =& \frac{\psi_2}{\lambda(\pi_1 - \pi_0) + (1-\lambda)(\underline{\pi}_1 - \underline{\pi}_0)}\\ \bar{t}^* =& \frac{\psi_2}{\lambda\Big[(\pi_1 - \pi_0) - (\underline{\pi}_1 - \underline{\pi}_0)\Big] + (\underline{\pi}_1 - \underline{\pi}_0)}\end{aligned}$$

Thus if $\pi_1 - \pi_0 > \underline{\pi}_1 - \underline{\pi}_0$ it is straightforward that $\bar{t}^*$ decreases with λ. Besides, if if $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$ then $\bar{t}^*$ is independent of λ

- Let us find the change of direction of the informational rent that we denote P_A:

We got an expected payoff for the agent ex-ante:

$$P_A = \pi_1^{e_i,e_j}\bar{t} + (1 - \pi_1^{e_i,e_j})\underline{t} - \psi_2$$

That leads to a simple expression of the informational rent:

$$P_A = \psi_2 \frac{\pi_0^{e_i,e_j}}{\Delta\pi(\lambda_{j,A}^{e_i,e_j})}$$

Let us find the change of direction of P_A with λ. Indeed, since $\pi_0^{e_i,e_j} = \lambda(\pi_0 - \underline{\pi}_0) + \underline{\pi}_0$ and $\Delta\pi(\lambda_{j,A}^{e_i,e_j}) = (\Delta\pi - \underline{\Delta\pi})\lambda + \underline{\Delta\pi}$ with $\underline{\Delta\pi} = \underline{\pi}_1 - \underline{\pi}_0$, we have:

$$\frac{\partial P_A}{\partial\lambda} = \frac{\psi_2(\pi_0 - \underline{\pi}_0)\Big[(\Delta\pi - \underline{\Delta\pi})\lambda + \underline{\Delta\pi}\Big] - \psi_2\Big[\lambda(\pi_0 - \underline{\pi}_0) + \underline{\pi}_0\Big](\Delta\pi - \underline{\Delta\pi})}{\Big[(\Delta\pi - \underline{\Delta\pi})\lambda + \underline{\Delta\pi}\Big]^2}$$

The denominator is positive, thus let us focus on the sign of the numerator that we denote N:

$$\begin{aligned}N > 0 <=> &\psi_2\Big[(\pi_0 - \underline{\pi}_0)(\Delta\pi - \underline{\Delta\pi})\lambda + \underline{\Delta\pi}(\pi_0 - \underline{\pi}_0)\\ &- \lambda(\pi_0 - \underline{\pi}_0)(\Delta\pi - \underline{\Delta\pi}) - \underline{\pi}_0(\Delta\pi - \underline{\Delta\pi})\Big] > 0\end{aligned}$$

We simplify by $(\pi_0 - \underline{\pi}_0)(\Delta\pi - \underline{\Delta\pi})\lambda$:

$$N > 0 <=> \psi_2 \Bigg[\underline{\Delta\pi}(\pi_0 - \underline{\pi}_0) - \underline{\pi}_0(\Delta\pi - \underline{\Delta\pi}) \Bigg] > 0$$

We simplify by ψ_2 and develop:

$$N > 0 <=> \underline{\Delta\pi}\pi_0 - \underline{\Delta\pi}\underline{\pi}_0 - \underline{\pi}_0\Delta\pi + \underline{\pi}_0\underline{\Delta\pi} > 0$$

We simplify by $\underline{\pi}_0\underline{\Delta\pi}$:

$$N > 0 <=> \underline{\Delta\pi}\pi_0 - \underline{\pi}_0\Delta\pi > 0$$

We develop $\Delta\pi$ and $\underline{\Delta\pi}$ and by successive equivalence we have :

$$N > 0 <=> \underline{\pi}_1\pi_0 - \underline{\pi}_0\pi_0 - \underline{\pi}_0\pi_1 + \underline{\pi}_0\pi_0 > 0$$

$$N > 0 <=> \underline{\pi}_1\pi_0 > \underline{\pi}_0\pi_1$$

$$N > 0 <=> \frac{\pi_0}{\underline{\pi}_0} > \frac{\pi_1}{\underline{\pi}_1}$$

Thus, to conclude, $\frac{\partial P_A}{\partial \lambda} > 0$ if and only if $\frac{\pi_0}{\underline{\pi}_0} > \frac{\pi_1}{\underline{\pi}_1}$ is verified.

- Let us find the change of direction of P_A with λ if we assume $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0$. Then $P_A = \pi_0^{e_i,e_j}\frac{\psi_2}{\Delta\pi}$ where $\frac{\psi_2}{\Delta\pi}$ is constant. Then we have:

$$\frac{\partial P_A}{\partial \lambda} > 0 <=> (\pi_0 - \underline{\pi}_0)\frac{\psi_2}{\Delta\pi} > 0$$

  Which is always verified.

D/ Proof Lemma 1

Let us prove that: $P_i^{1,e_j} > P_i^{0,e_j}$ if and only if $(r_{i,ND}^{.,e_j} - r_{i,D}^{.,e_j})\lambda_i + r_{i,D}^{.,e_j} - \psi > 0$

We have:

$$\begin{aligned} P_i^{1,e_j} =& \pi_1^P(\lambda_{j,P}^{1,e_j})(1 - \frac{\psi_2}{\Delta\pi})P(q_i = 1 \cap q_j = 0 \mid (1, e_j)) \\ &+ \pi_0^A(\lambda_{i,A}^{1,e_j})\frac{\psi_2}{\Delta\pi}P(q_i = 0 \cap q_j = 1 \mid (1, e_j)) - \psi \end{aligned}$$

and

$$\begin{aligned} P_i^{0,e_j} =& \pi_1^P(\lambda_{j,P}^{0,e_j})(1 - \frac{\psi_2}{\Delta\pi})P(q_i = 1 \cap q_j = 0 \mid (0, e_j)) \\ &+ \pi_0^A(\lambda_{i,A}^{0,e_j})\frac{\psi_2}{\Delta\pi}P(q_i = 0 \cap q_j = 1 \mid (0, e_j)) \end{aligned}$$

with $\pi_1^P(\lambda_{j,P}^{e_i,e_j}) = \lambda_{j,P}^{e_i,e_j}\pi_1 + (1-\lambda_{j,P}^{e_i,e_j})\underline{\pi}_1$ and $\pi_0^A(\lambda_{i,A}^{e_i,e_j}) = \lambda_{i,A}^{e_i,e_j}\pi_1 + (1-\lambda_{i,A}^{e_i,e_j})\underline{\pi}_1$ for all $e_i \in \{0,1\}$.

Besides with have:

$$\lambda_{j,P}^{e_i,e_j} = \frac{P[q_i = 1 \cap q_j = 0 \mid (e_i,e_j) \mid \bar{j}]}{P(q_i = 1 \cap q_j = 0 \mid (x,e_j))}$$

and

$$1-\lambda_{j,P}^{e_i,e_j} = \frac{P[q_i = 1 \cap q_j = 0 \mid (e_i,e_j) \mid \underline{j}]}{P(q_i = 1 \cap q_j = 0 \mid (e_i,e_j))}$$

and

$$\lambda_{i,A}^{e_i,e_j} = \frac{P[q_i = 0 \cap q_j = 1 \mid (e_i,e_j) \mid \bar{i}]}{P(q_i = 0 \cap q_j = 1 \mid (e_i,e_j))}$$

and

$$1-\lambda_{i,A}^{e_i,e_j} = \frac{P[q_i = 0 \cap q_j = 1 \mid (e_i,e_j) \mid \underline{i}]}{P(q_i = 0 \cap q_j = 1 \mid (e_i,e_j))}$$

Thus this leads to:

$$\begin{aligned}
P_i^{1,e_j} =& \Big[\pi_1 P[q_i = 1 \cap q_j = 0 \mid (1,e_j) \mid \bar{j}] \\
&+ \underline{\pi}_1 P[q_i = 1 \cap q_j = 0 \mid (1,e_j) \mid \underline{j}]\Big] \frac{1-\frac{\psi_2}{\Delta\pi}}{P(q_i = 1 \cap q_j = 0 \mid (1,e_j))} P(q_i = 1 \cap q_j = 0 \mid (1,e_j)) \\
&+ \Big[\pi_0 P[q_i = 0 \cap q_j = 1 \mid (1,e_j) \mid \bar{i}] \\
&+ \underline{\pi}_0 P[q_i = 0 \cap q_j = 1 \mid (1,e_j) \mid \underline{i}]\Big] \frac{\frac{\psi_2}{\Delta\pi}}{P(q_i = 0 \cap q_j = 1 \mid (1,e_j))} P(q_i = 0 \cap q_j = 1 \mid (1,e_j)) \\
&- \psi
\end{aligned}$$

that is to say:

$$\begin{aligned}
P_i^{1,e_j} =& \Big[\pi_1\lambda_j\underline{\pi}_1(1-\pi_{e_j}) + \underline{\pi}_1(1-\lambda_j)\pi_1(1-\underline{\pi}_{e_j})\Big](1-\frac{\psi_2}{\Delta\pi}) \\
&+ \Big[\pi_0\lambda_i(1-\pi_1)\underline{\pi}_{e_j} + \underline{\pi}_0(1-\lambda_i)(1-\underline{\pi_1})\pi_{e_j}\Big]\frac{\psi_2}{\Delta\pi} \\
&- \psi
\end{aligned}$$

Symmetrically we have for $e_i = 0$

$$
\begin{aligned}
P_i^{0,e_j} =& \Big[\pi_1 P[q_i = 1 \cap q_j = 0 \mid (0, e_j) \mid \bar{j}] \\
&+ \underline{\pi}_1 P[q_i = 1 \cap q_j = 0 \mid (0, e_j) \mid \underline{j}] \Big] \frac{1 - \frac{\psi_2}{\Delta\pi}}{P(q_i = 1 \cap q_j = 0 \mid (0, e_j))} P(q_i = 1 \cap q_j = 0 \mid (0, e_j)) \\
&+ \Big[\pi_0 P[q_i = 0 \cap q_j = 1 \mid (0, e_j) \mid \bar{i}] \\
&+ \underline{\pi}_0 P[q_i = 0 \cap q_j = 1 \mid (1, e_j) \mid \underline{i}] \Big] \frac{\frac{\psi_2}{\Delta\pi}}{P(q_i = 0 \cap q_j = 1 \mid (0, e_j))} P(q_i = 0 \cap q_j = 1 \mid (0, e_j))
\end{aligned}
$$

That is to say:

$$
\begin{aligned}
P_i^{1,e_j} =& \Big[\pi_1 \lambda_j \underline{\pi}_0 (1 - \pi_{e_j}) + \underline{\pi}_1 (1 - \lambda_j) \pi_0 (1 - \underline{\pi}_{e_j}) \Big] (1 - \frac{\psi_2}{\Delta\pi}) \\
&+ \Big[\pi_0 \lambda_i (1 - \pi_0) \underline{\pi}_{e_j} + \underline{\pi}_0 (1 - \lambda_i)(1 - \underline{\pi}_0) \pi_{e_j} \Big] \frac{\psi_2}{\Delta\pi}
\end{aligned}
$$

Let us focus now on $P_i^{1,e_j} > P_i^{0,e_j}$. We know $\lambda_j = 1 - \lambda_i$ and $\pi_1 - \pi_0 = \underline{\pi}_1 - \underline{\pi}_0 = \Delta\pi$.

Then we have:

$$
\begin{aligned}
P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> & \Big[\pi_1 [\lambda_j \underline{\pi}_1 (1 - \pi_{e_j}) - \lambda_j \underline{\pi}_0 (1 - \pi_{e_j})] \\
&+ \underline{\pi}_1 [(1 - \lambda_j) \pi_1 (1 - \underline{\pi}_{e_j}) - (1 - \lambda_j) \pi_0 (1 - \underline{\pi}_{e_j})] \Big] (1 - \frac{\psi_2}{\Delta\pi}) \\
&+ \Big[\pi_0 [\lambda_i (1 - \pi_1) \underline{\pi}_{e_j} - \lambda_i (1 - \pi_0) \underline{\pi}_{e_j}] \\
&+ \underline{\pi}_0 [(1 - \lambda_i)(1 - \underline{\pi_1}) \pi_{e_j} - (1 - \lambda_i)(1 - \underline{\pi}_0) \pi_{e_j}] \Big] \frac{\psi_2}{\Delta\pi} \\
&- \psi > 0 \\
P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> & \Big[\pi_1 \lambda_j (1 - \pi_{e_j}) \Delta\pi + \underline{\pi}_1 (1 - \lambda_j)(1 - \underline{\pi}_{e_j}) \Delta\pi \Big] (1 - \frac{\psi_2}{\Delta\pi}) \\
&+ \Big[\pi_0 \lambda_i (1 - \pi_1 - 1 + \pi_0) \underline{\pi}_{e_j} + \underline{\pi}_0 [(1 - \lambda_i)(1 - \underline{\pi_1} - 1 + \underline{\pi}_0) \pi_{e_j} \Big] \frac{\psi_2}{\Delta\pi} \\
&- \psi > 0
\end{aligned}
$$

Thus that leads to:

$$
\begin{aligned}
P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> & \Big[\pi_1 (1 - \lambda_i)(1 - \pi_{e_j}) \Delta\pi + \underline{\pi}_1 \lambda_i (1 - \underline{\pi}_{e_j}) \Delta\pi \Big] (1 - \frac{\psi_2}{\Delta\pi}) \\
&+ \Big[- \pi_0 \lambda_i \underline{\pi}_{e_j} \Delta\pi - \underline{\pi}_0 (1 - \lambda_i) \pi_{e_j} \Delta\pi \Big] \frac{\psi_2}{\Delta\pi} \\
&- \psi > 0
\end{aligned}
$$

We can now separate the part that is relative to the case when i is discriminated from the part relative to the case when i is advantaged:

$$
\begin{aligned}
P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> & \left[\Delta\pi(1-\lambda_i)(1-\pi_{e_j})\pi_1(1-\frac{\psi_2}{\Delta\pi}) - \Delta\pi(1-\lambda_i)\pi_{e_j}\underline{\pi}_0\frac{\psi_2}{\Delta\pi}\right] \\
& + \left[\Delta\pi\lambda_i(1-\underline{\pi}_{e_j})\underline{\pi}_1(1-\frac{\psi_2}{\Delta\pi}) - \Delta\pi\lambda_i\underline{\pi}_{e_j}\pi_0\frac{\psi_2}{\Delta\pi}\right] \\
& - \psi > 0
\end{aligned}
$$

We now have a linear relation with respect to λ_i:

$$
\begin{aligned}
P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> & (1-\lambda_i)\overbrace{\left[(\underline{\pi}_1-\underline{\pi}_0)(1-\pi_{e_j})\pi_1(1-\frac{\psi_2}{\Delta\pi}) - (\underline{\pi}_1-\underline{\pi}_0)\pi_{e_j}\underline{\pi}_0\frac{\psi_2}{\Delta\pi}\right]}^{\underline{r}_i^{e_j}} \\
& + \lambda_i\underbrace{\left[(\pi_1-\pi_0)(1-\underline{\pi}_{e_j})\underline{\pi}_1(1-\frac{\psi_2}{\Delta\pi}) - (\pi_1-\pi_0)\underline{\pi}_{e_j}\pi_0\frac{\psi_2}{\Delta\pi}\right]}_{r_i^{e_j}} \\
& - \psi > 0
\end{aligned}
$$

To conclude we have:

$$P_i^{1,e_j} > P_i^{0,e_j} <=> (r_i^{e_j} - \underline{r}_i^{e_j})\lambda_i + \underline{r}_i^{e_j} - \psi > 0$$

E/ Proof Lemma 2

Let us find conditions that guarantee:

$$P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> \lambda_i > \frac{\psi - \underline{r}_i^{e_j}}{(r_i^{e_j} - \underline{r}_i^{e_j})}$$

Start from lemma 1 it is straightforward if $r_i^{e_j} - \underline{r}_i^{e_j} > 0$ is verified and if $r_i^{e_j} > \psi > \underline{r}_i^{e_j}$ holds. Let find conditions that guarantee $r_i^{e_j} - \underline{r}_i^{e_j} > 0$.

$$
\begin{aligned}
r_i^{e_j} - \underline{r}_i^{e_j} > 0 <=> & \left[(\pi_1-\pi_0)(1-\underline{\pi}_{e_j})\underline{\pi}_1(1-\frac{\psi_2}{\Delta\pi}) - (\pi_1-\pi_0)\underline{\pi}_{e_j}\pi_0\frac{\psi_2}{\Delta\pi}\right] \\
& - \left[(\underline{\pi}_1-\underline{\pi}_0)(1-\pi_{e_j})\pi_1(1-\frac{\psi_2}{\Delta\pi}) - (\underline{\pi}_1-\underline{\pi}_0)\pi_{e_j}\underline{\pi}_0\frac{\psi_2}{\Delta\pi}\right] > 0 \\
r_i^{e_j} - \underline{r}_i^{e_j} > 0 <=> & (1-\underline{\pi}_{e_j})\underline{\pi}_1(1-\frac{\psi_2}{\Delta\pi}) - \underline{\pi}_{e_j}\pi_0\frac{\psi_2}{\Delta\pi} \\
& - (1-\pi_{e_j})\pi_1(1-\frac{\psi_2}{\Delta\pi}) + \pi_{e_j}\underline{\pi}_0\frac{\psi_2}{\Delta\pi} > 0
\end{aligned}
$$

That leads to the following equation:

$$r_i^{e_j} - \underline{r}_i^{e_j} > 0 <=> \Big[(1 - \underline{\pi}_{e_j})\underline{\pi}_1 - (1 - \pi_{e_j})\pi_1\Big](1 - \frac{\psi_2}{\Delta\pi})$$
$$+ \Big[\pi_{e_j}\underline{\pi}_0 - \underline{\pi}_{e_j}\pi_0\Big]\frac{\psi_2}{\Delta\pi} > 0$$

We find $\pi_{e_j}\underline{\pi}_0 - \underline{\pi}_{e_j}\pi_0 \leq 0$. Indeed it is equivalent to $\frac{\pi_{e_j}}{\underline{\pi}_{e_j}} \geq \frac{\pi_0}{\underline{\pi}_0}$ which is always true for $e_j = 0$ and true for $e_j = 1$ because of the assumption of substitutability between effort and favoritism.

As a consequence, and because $(1 - \frac{\psi_2}{\Delta\pi}) > \frac{\psi_2}{\Delta\pi}$, we need to find conditions that guarantee $(1-\underline{\pi}_{e_j})\underline{\pi}_1-(1-\pi_{e_j})\pi_1 \geq -\pi_{e_j}\underline{\pi}_0+\underline{\pi}_{e_j}\pi_0$. This leads by successive equivalence:

$$(1 - \underline{\pi}_{e_j})\underline{\pi}_1 - (1 - \pi_{e_j})\pi_1 \geq -\pi_{e_j}\underline{\pi}_0 + \underline{\pi}_{e_j}\pi_0 <=> \underline{\pi}_1 - \underline{\pi}_{e_j}\underline{\pi}_1 - \pi_1 + \pi_{e_j}\pi_1 + \pi_{e_j}\underline{\pi}_0 - \underline{\pi}_{e_j}\pi_0 \geq 0$$
$$<=> \underline{\pi}_1 - \pi_1 - \underline{\pi}_{e_j}(\underline{\pi}_1 + \pi_0) + \pi_{e_j}(\underline{\pi}_0 + \pi_1) \geq 0$$

Yet, $\underline{\pi}_1 + \pi_0 = \underline{\pi}_0 + \pi_1$ because $\underline{\pi}_1 - \underline{\pi}_0 = \pi_1 - \pi_0$. Thus we can write:

$$\underline{\pi}_1 - \pi_1 + (\pi_{e_j} - \underline{\pi}_{e_j})(\underline{\pi}_0 + \pi_1) \geq 0$$

Yet,$\pi_1 - \underline{\pi}_1 = \pi_{e_j} - \underline{\pi}_{e_j}$. Indeed, if $e_j = 1$ it is straightforward and if $e_j = 0$ the expression is $\underline{\pi}_1 - \pi_1 = \pi_0 - \underline{\pi}_0$ which is equivalent to $\underline{\pi}_1 - \underline{\pi}_0 = \pi_1 - \pi_0$. Thus we have:

$$\underline{\pi}_1 - \pi_1 + (\pi_{e_j} - \underline{\pi}_{e_j})(\underline{\pi}_0 + \pi_1) \geq 0 <=> -1 + \underline{\pi}_0 + \pi_1 \geq 0$$

As a consequence $r_i^{e_j} - \underline{r}_i^{e_j} > 0$ holds if $\underline{\pi}_0 + \pi_1 \geq 1$ is verified. To conclude:

$$P_i^{1,e_j} - P_i^{0,e_j} > 0 <=> \lambda_i > \frac{\psi - \underline{r}_i^{e_j}}{(r_i^{e_j} - \underline{r}_i^{e_j})}$$

is verified and if $r_i^{e_j} > \psi > \underline{r}_i^{e_j}$ and $\underline{\pi}_0 + \pi_1 \geq 1$ hold.

F/ Proof lemma 3

Let us prove that $\Delta E_i(\lambda_i)$ is concave while $\lambda_i \in (0,1)$. We have:

$$\begin{aligned}
E_i^{1,e_j}(\lambda_i) =& \pi_1^P(\lambda_j)(1-\frac{\psi_2}{\Delta\pi})P(q_i=1\cap q_j=0 \mid (1,e_j)) \\
&+ \pi_0^A(\lambda_i)\frac{\psi_2}{\Delta\pi}P(q_i=0\cap q_j=1 \mid (1,e_j)) - \psi \\
E_i^{1,e_j}(\lambda_i) =& \Big[\pi_1(1-\lambda_i)+\underline{\pi}_1\lambda_i\Big](1-\frac{\psi_2}{\Delta\pi})\Big[\lambda_i\pi_1(1-\underline{\pi}_{ej})+(1-\lambda_i)\underline{\pi}_1(1-\pi_{e_j})\Big] \\
&+\Big[\pi_0\lambda_i+\underline{\pi}_0(1-\lambda_i)\Big]\frac{\psi_2}{\Delta\pi}\Big[\lambda_i(1-\pi_1)\underline{\pi}_{e_j}+(1-\lambda_i)(1-\underline{\pi}_1)\pi_{e_j}\Big]-\psi
\end{aligned}$$

Then, for a low effort we have:

$$\begin{aligned}
E_i^{0,e_j}(\lambda_i) =& \Big[\pi_1(1-\lambda_i)+\underline{\pi}_1\lambda_i\Big](1-\frac{\psi_2}{\Delta\pi})\Big[\lambda_i\pi_0(1-\underline{\pi}_{ej})+(1-\lambda_i)\underline{\pi}_0(1-\pi_{e_j})\Big] \\
&+\Big[\pi_0\lambda_i+\underline{\pi}_0(1-\lambda_i)\Big]\frac{\psi_2}{\Delta\pi}\Big[\lambda_i(1-\pi_0)\underline{\pi}_{e_j}+(1-\lambda_i)(1-\underline{\pi}_0)\pi_{e_j}\Big]
\end{aligned}$$

Then we can write the marginal return:

$$\begin{aligned}
\Delta E_i(\lambda_i) =& \Big[\pi_1(1-\lambda_i)+\underline{\pi}_1\lambda_i\Big](1-\frac{\psi_2}{\Delta\pi})\Big[\lambda_i\Delta\pi(1-\underline{\pi}_{ej})+(1-\lambda_i)\Delta\pi(1-\pi_{e_j})\Big] \\
&+\Big[\pi_0\lambda_i+\underline{\pi}_0(1-\lambda_i)\Big]\frac{\psi_2}{\Delta\pi}\Big[\lambda_i(1-\pi_1-1+\pi_0)\underline{\pi}_{e_j} \\
&+(1-\lambda_i)(1-\underline{\pi}_1-1+\underline{\pi}_0)\pi_{e_j}\Big]-\psi \\
\Delta E_i(\lambda_i) =& \Big[(\underline{\pi}_1-\pi_1)\lambda_i+\pi_1\Big](1-\frac{\psi_2}{\Delta\pi})\Big[\lambda_i\Delta\pi(1-\underline{\pi}_{ej})+(1-\lambda_i)\Delta\pi(1-\pi_{e_j})\Big] \\
&+\Big[\lambda_i(\pi_0-\underline{\pi}_0)+\underline{\pi}_0\Big]\frac{\psi_2}{\Delta\pi}\Big[-\lambda_i\Delta\pi\underline{\pi}_{e_j}-(1-\lambda_i)\Delta\pi\pi_{e_j}\Big]-\psi \\
\Delta E_i(\lambda_i) =& \Big[(\underline{\pi}_1-\pi_1)\lambda_i+\pi_1\Big](1-\frac{\psi_2}{\Delta\pi})\Big[\lambda_i\Delta\pi(1-\underline{\pi}_{ej}-1+\pi_{e_j})+\Delta\pi(1-\pi_{e_j})\Big] \\
&+\Big[\lambda_i(\pi_0-\underline{\pi}_0)+\underline{\pi}_0\Big]\frac{\psi_2}{\Delta\pi}\Big[\lambda_i\Delta\pi(-\underline{\pi}_{e_j}+\pi_{e_j})-\Delta\pi\pi_{e_j}\Big]-\psi
\end{aligned}$$

Thus we can now develop the expression:

$$\begin{aligned}
\Delta E_i(\lambda_i) =& \Big[(\underline{\pi}_1-\pi_1)(\pi_{e_j}-\underline{\pi}_{e_j})\Delta\pi\lambda_i^2+(\underline{\pi}_1-\pi_1)\Delta\pi(1-\pi_{e_i})\lambda_i \\
&+\Delta\pi(\pi_{e_j}-\underline{\pi}_{e_j})\pi_1\lambda_i+\Delta\pi\pi_1(1-\pi_{e_j})\Big](1-\frac{\psi_2}{\Delta\pi}) \\
&+\Big[(\pi_0-\underline{\pi}_0)(\pi_{e_j}-\underline{\pi}_{e_j})\Delta\pi\lambda_i^2+(\pi_0-\underline{\pi}_0)\Delta\pi\pi_{e_i}\lambda_i \\
&+\Delta\pi(\pi_{e_j}-\underline{\pi}_{e_j})\underline{\pi}_0\lambda_i-\Delta\pi\underline{\pi}_0\pi_{e_j}\Big]\frac{\psi_2}{\Delta\pi}-\psi
\end{aligned}$$

We can now focus on the sign of the coefficient in front of the squared term $\gamma = (\underline{\pi}_1 - \pi_1)(\pi_{e_j}-\underline{\pi}_{e_j})\Delta\pi(1-\frac{\psi_2}{\Delta\pi})+(\pi_0-\underline{\pi}_0)(\pi_{e_j}-\underline{\pi}_{e_j})\Delta\pi\frac{\psi_2}{\Delta\pi}$:

$$
\begin{aligned}
\gamma =& \Delta\pi(\pi_{e_j} - \underline{\pi}_{e_j})\Bigg[(\underline{\pi}_1 - \pi_1)(1 - \frac{\psi_2}{\Delta\pi}) + (\pi_0 - \underline{\pi}_0)\frac{\psi_2}{\Delta\pi}\Bigg] \\
<=> \gamma =& \Delta\pi(\pi_{e_j} - \underline{\pi}_{e_j})\Bigg[- (\pi_1 - \underline{\pi}_1)(1 - \frac{\psi_2}{\Delta\pi}) + (\pi_0 - \underline{\pi}_0)\frac{\psi_2}{\Delta\pi}\Bigg]
\end{aligned}
$$

We know that $(\pi_1 - \underline{\pi}_1) = (\pi_0 - \underline{\pi}_0)$ so we have:

$$
\gamma = \Delta\pi(\pi_{e_j} - \underline{\pi}_{e_j})(\pi_1 - \underline{\pi}_1)\Bigg[- (1 - \frac{\psi_2}{\Delta\pi}) + \frac{\psi_2}{\Delta\pi}\Bigg]
$$

Thus the sign of γ depends exclusively on variable from second stage. By assumption $(1 - \frac{\psi_2}{\Delta\pi}) > \frac{\psi_2}{\Delta\pi}$ which means that the wage of the principal will be superior to the wage of the agent in second stage. So γ is negative and $\Delta E_i(\lambda_i)$ is concave as $\Delta E_i(\lambda_i)$ will be represented by an inverted parabola.

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