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Traveler's dilemma : how the value of the luggage influences behavior

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Abstract

We go into classroom experiments on the Traveler' Dilemma in order to show the impact of the common knowledge of the value of the luggage. This value becomes a focal point that canalizes the behavior of the students. This leads us to commenting on the impact of such focal points both on the reasoning of the players and on the structure of the game. We construct a new game which models this impact and we study its Nash equilibrium.

Keywords: traveler's dilemma, Nash equilibrium, focal point, classroom experiment, fairness, honesty.

JEL Classification: C72

1. Introduction

The paper goes into classroom experiments on the traveler's dilemma game. In this game, two travelers ask an airline for a dollar amount, because the airline has lost their luggage. If m is the lowest claim, the company pays m+P to the lowest requesting claimant, and m-P to the other claimant. P can be seen as a reward for the traveler who asks for a lower reimbursement and as a penalty for the other claimant. In case of a tie, both travelers get the requested amount. Different games, with different levels of P, are proposed to the students: in some games, the students are unaware of the value of the lost luggage, in others, this value is common knowledge. The students justify in writing their behavior[1].

The Nash equilibrium is the same in all games studied, it neither depends on the value of P nor on the value of the lost luggage (and its common knowledge). Yet the students' reasoning and behavior are quite different in the different games. We highlight that the common knowledge of the value of the luggage, due to the focal points it introduces in the game, has a main impact on the reasoning and on the chosen claims. This leads us to constructing a new game with incomplete information on the lower bound of the traveler's strategy set. We study the new Nash equilibrium and show to what extent the common knowledge of the value of the luggage introduces more fairness in the traveler's dilemma.

In section 2 we recall the traveler's dilemma game and detail the classroom experiments. The obtained results are given in section 3. Section 4 focuses on the impact of focal points both on the reasoning and on the played actions. In section 5 we construct a new game with incomplete

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[1] Students are invited to justify their choices in writing but they are not compelled to do so. More than 2/3 of them explain their choices, at least partially.

information that shows how the common knowledge of the luggage can change the Nash equilibrium behavior of the players. Section 6 concludes.

2. Classroom experiments

The traveler's dilemma introduced by Basu (1994) is now a well-known game theory puzzle. Traveler 1 and traveler 2 ask simultaneously an airline for an integer dollar amount from 2 to an upper bound M, because the airline has lost their luggage. We call x_1 and x_2 the amounts requested respectively by traveler 1 and traveler 2. Both players' lost luggage are of same value. Both claimants get the lowest claim, plus P for the lowest requesting claimant, minus P for the other claimant. P is larger than or equal to 2. In case of a tie, both travelers get the requested amount. The rules of the game, namely the value of P, are common knowledge.

The airline justifies this strange reimbursement game as follows: not knowing the value of the luggage, it thinks that if both travelers ask for a same amount, then this amount is surely the true value of the luggage (and should be reimbursed to both travelers). In contrast, if the two amounts differ, then the traveler who asks for the highest amount surely lies about the value of the luggage (and should be penalized for lying). The other traveler surely says the truth (and will be rewarded for his honesty). Of course, as is well-known, this justification is fake. The rules of the game push each player to undercut the other player's amount x by one, in that $x-1+P > x$. So both travelers are led to the single Nash Equilibrium (2,2) where everybody asks for the lowest amount 2, regardless of the value of P, and regardless of the true value of the luggage.

This game has often been played in experimental settings (see for example Capra et al. (1999), Becker et al. (2005), Basu (2011), Rubinstein (2007)). Capra et al. (1999) and Becker et al. (2005) namely observe a strong difference in behavior when switching from low values of P to large values. When playing the traveler's dilemma several times (the played actions being observed after each round of play), the players converge to the Nash equilibrium (NE) for large values of P and to the highest possible amount M for low values of P.

In this paper, we again propose the standard traveler's dilemma, without repetition, to 218 students, with M=100 (in all games studied) and two different values of P, P=5 and P=35. In Games 1, 2, 3 and 4 each student is supposed to meet another student among the 217 other students. In Game 5 and in Game 6, the student's payoff only depends on his own action. In each game played, the students are invited to justify their choices in writing. Several examples of games are explained to the students before the beginning of the experiments, to be sure that the rules of the games are understood. The term common knowledge, used in Game 3 and in Game 4, is explained to the students.

In the two first games (in Game 1, P=5, in Game 2, P=35), the value of the luggage, V, is not mentioned to the players. To the very few students (3 or 4 among 218) who asked for it, we just said that it is not given in the game.

But Game 1 and Game 2 are just control experiments. We are mostly interested in two other games, Game 3 and Game 4, played by the same students: in these games the value of the luggage, V, is common knowledge and fixed at 70. In Game 3, P = 5 and in Game 4, P=35. Nothing changes as regards the NE. Regardless of the value of the luggage and regardless of the value of P, the players are pushed to undercut the amount of the other player by one, thus leading both travelers to asking for the lowest amount 2. Yet, as will be revealed by the results, the students' behavior in Game 3 and in Game 4 strongly differs from their behavior in Game 1 and in Game 2. The common knowledge of the value of the luggage has a main impact on the players' reasoning and behavior; it leads to amounts' distributions which are quite different from the distributions observed in the two control experiments.

Finally, in Game 5 and in Game 6, we simply test the students' honesty, in that we invite them to play two new control games. In both games, each student can ask for an integer amount from 2 to 100 for the lost luggage, and the airline simply reimburses this amount to the player. In Game 5, the students know that the value of their luggage is 70, in Game 6, they know that the value of their luggage is 20. Of course, in these two games, the optimal amount is 100 (in that it maximizes the player's payoff), but the honest amounts are 70 and 20, respectively in Game 5 and in Game 6. Game 5 and Game 6 are control games in that they help us analyzing the behavior and motivation of the students in Game 3 and in Game 4.

3. Results

The amounts' distributions of Game 1 and Game 2 are given in figure 1a and figure 1b.

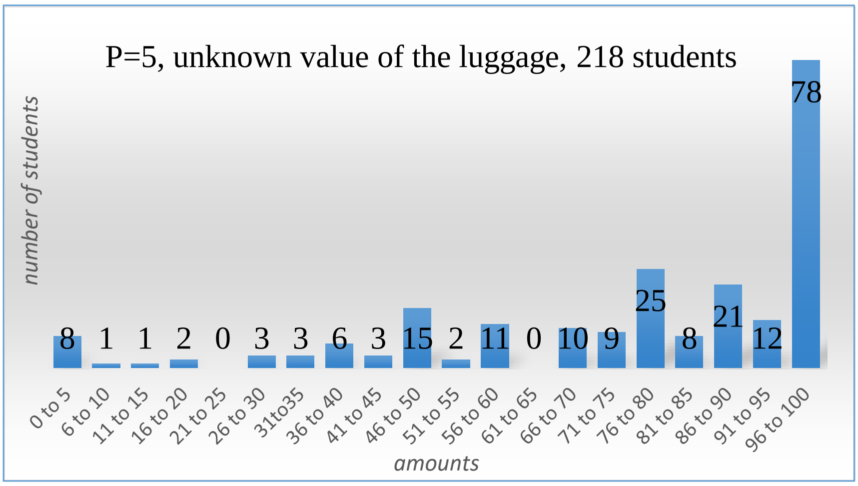

Figure 1a: Amounts played by the students in Game 1

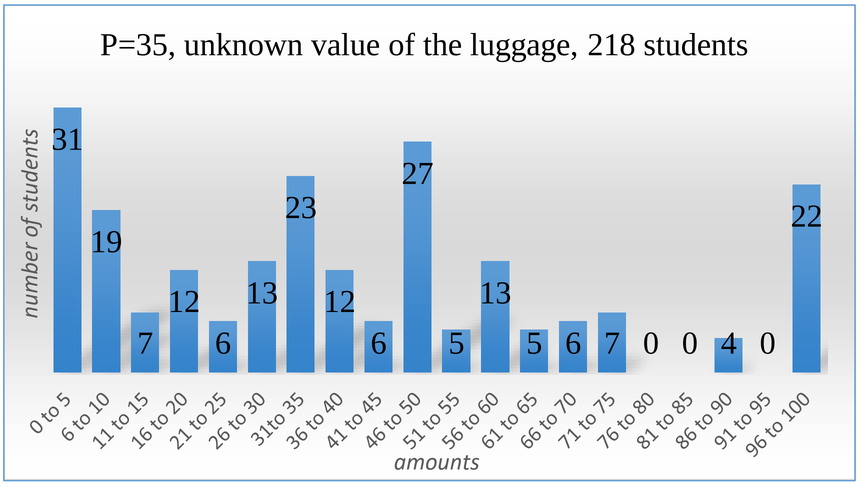

Figure 1b: Amounts played by the students in Game 2

As expected, we get results that are similar to that obtained in previous experiments (Capra et al. (1999), Becker et al. (2005)). When the students play without knowing the value of the luggage, they ask for large amounts when P is low (P=5): the mean amount in figure 1a is 78.09 and the median is 87. In contrast, when P is large (P=35), the students' claims strongly decrease: the mean amount in figure 1b is 40.85 and the median is 35. As expected, we observe that the students more often play the NE amount 2 when P is large : 10.09% of the students play 2 and 22.94% of them play 10 or less when P=35, whereas only 2.75% of the students play 2 and 4.13% of them play 10 or less when P=5. A salient observation is the dispersion of the played amounts in Game 2. For P=5, there is a significant dispersion (standard dispersion = 26) but 100 is clearly the mode of the distribution (28.9% of the students play 100 and 40.83% of them play an amount higher than or equal to 95). In contrast, for P=35, the students seem completely disoriented. The standard deviation is almost 30 (29.33), and the students clearly do not agree on the amount to ask for: no amount is really prominent, and no real mode emerges. The students hesitate among several amounts: 14.22% of the students play 5 or less, 9.63% of the students play 99 or 100, the statistical mode is 50 but it is only played by 11.47% of the students, 2 is played by 10.09% of the students and 35 is played by 9.63% of the students. The only unifying observation is that more than 7 students in 10 (71.56%) play 50 or less.

The amounts' distributions of Game 3 and Game 4 are given in figure 2a and figure 2b.

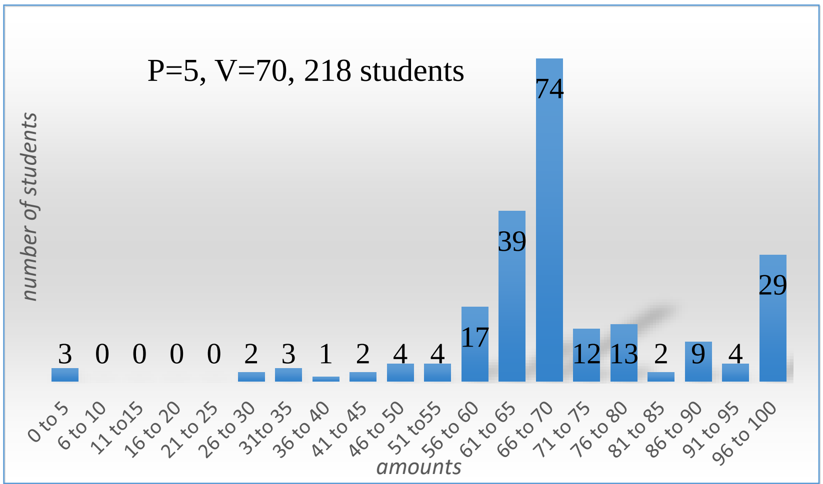

Figure 2a: Amounts played by the students in Game 3

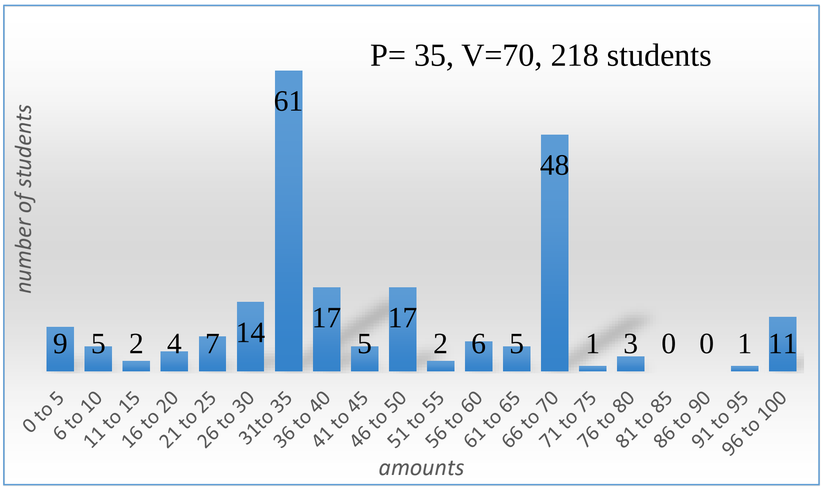

Figure 2b: Amounts played by the students in Game 4

These distributions contrast with that obtained in Game 1 and Game 2. It derives from figures 2a and 2b that the common knowledge of the value of the luggage, V, introduces focal points into the game, that canalize the behavior even though V is of no impact on the NE (which is (2,2)). For P=5, the most played amounts are around 65 and 70; almost 50% of the students (49.08%) play the four values 64, 65, 69 and 70 (16.97% play 64 or 65, 32.11% play 69 or 70) and 59.17% of the students play an amount from 60 to 70. For P= 35, the most played amounts are around 35 and 70; 46.33% of the students play the four values 34, 35, 69 and 70 (26.61% play 34 or 35 and 19.72% play 69 or 70). So almost half of the students only play four amounts in both games. We can also observe a kind of shrinking of the (used) strategy set. It is as if most students reduced their strategy set to actions that allow them to come close to the value of the luggage, with or without P: so 90.83% of the students play 60 or more when P=5 (so that they can reach at least the value 65 with P), and 86.70% of the students play 30 or more when P=35 (so that they can reach at least the value 65 with P). In the control experiments, Game 1 and Game 2, these percentages are lower, respectively 78.9% and 64.68%.

It logically follows from above that the dispersion of the amounts in Game 3 and in Game 4 is much less important than in Game 1 and in Game 2.The standard deviation switches from 26 to 16.74 when switching from Game 1 to Game 3, and from 29.33 to 22.66 when switching from Game 2 to Game 4. What is more, the distribution in Game 4 is clearly a bi-modal distribution, the two modes being 35 and 70. We can distinguish in the initial population two different populations with similar variance but different mean. So we split the students into two groups: the (141) students who play an amount from 2 to 52 (≈ 35 +(70-35)/2), and the (77) students who play an amount from 53 to 100. The mean amount is 32.93 (close to 35) and the standard deviation is 11.76 in the first group, the mean amount is 73.27 (close to 70) and the standard deviation is 12.03 in the second group. It follows that the pooled standard deviation is 11.91, so the amounts are concentrated around 35 and 70.
Clearly, both in Game 3 and in Game 4, students do not play the NE (2,2).

A remark on honesty has to be introduced at this level. As a matter of fact, it may be objected that payoffs should be replaced by utilities, the player’s happiness possibly increasing with honesty, in that this replacement may perhaps change the NE.

So we give the amounts’ distributions of Game 5 and Game 6 in figure 3a and figure 3b.

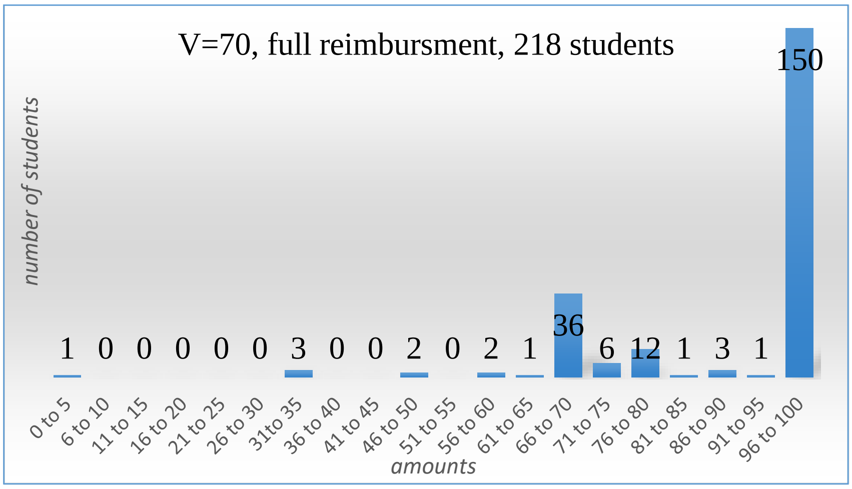

Figure 3a: Amounts played by the students in Game 5

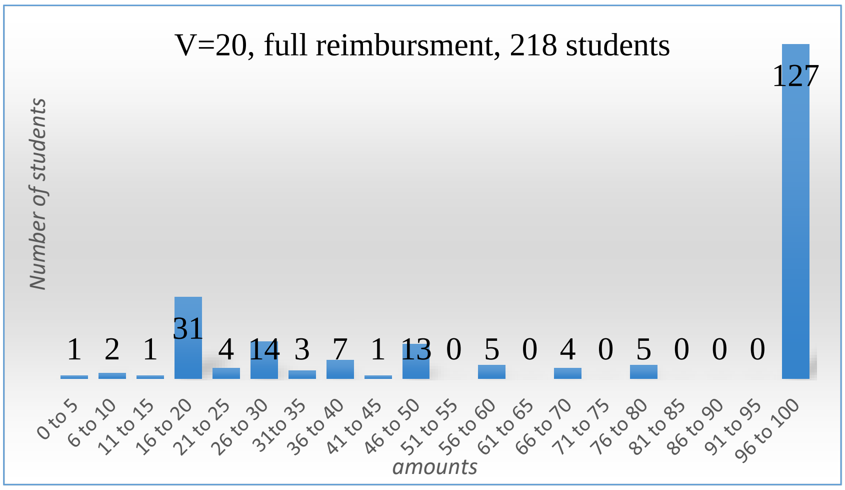

Figure 3b: Amounts played by the students in Game 6

A first answer to the objection is that introducing honesty in the utility function will not change the NE. Honesty drives the players to not ask for more than the value of their luggage (in that their utility decreases in amounts higher than V), but this does not change the fact that each traveler is induced to undercut the other traveler's amount by one: so the NE is again (2,2). The second answer comes from the control experiments Game 5 and Game 6: the students are not very honest with the airline! In these games, the students get the amount they ask for, knowing that their luggage is worth 70 (Game 5), 20 (Game 6). The mean amounts are 90.69 in Game 5 and 73.1 in Game 6, and both medians are equal to 100. Only 20.64% of the students are honest in Game 5, in that they ask for 70 or less, and only 16.06% of the students are honest in Game 6, in that they ask for 20 or less. So, clearly, honesty is not really a component of happiness of the students, and we should not be in a hurry to introduce it into a utility function. In fact, honesty plays an important role in the reasoning process, but not in the utilities.

4. Focal points

Focal points usually help players to coordinate on a NE in a game where there are several NE (see for example Schelling (1960), Sugden (1993), Blume and Gneezy (2010), Parravano and Poulsen (2015)…). In these papers, salient features of some equilibria and/or a kind of team or alter ego cognitive reasoning help (or help not) some NE to become focal (and so to be played with a high probability).

The aim in this paper is to highlight that focal points may even be more crucial in a game where there is only one NE, but nobody wants to play it. By focal points we mainly mean actions (here amounts) with salient features that help the players to converge to them. But we also mean salient characteristics of behavior and beliefs that help the players to choose among the amounts.

We first talk about focal points in Game 1 and in Game 2, where there is no common knowledge of the value of the luggage. When P=5, the maximal amount M (=100) is clearly the natural focal amount to which players converge (28.9% of the students play 100 in Game 1 and 50% of the students play 90 or more). The maximal amount 100 has many salient properties: it is the amount that maximizes the payoff of both players, when the two players consider each other as an alter ego, with the same aims and actions. Playing 100 is the cooperative way of playing, and if both players reason as a team, and so take a same single decision, they will of course play

100. Many of the students, in their comments, also mention that 100 is the only possible amount (except of 2) to converge on, when no communication is possible. They are right: why should two students spontaneously converge toward 78? More originally, 100 is also the best way of playing in a *3 player zero sum game* where the airline is seen as an enemy whose interest is opposed to that of the two travelers. x_1 and x_2 being traveler 1 and traveler 2's played amounts, the airline loses x_1+P+x_2-P, and the travelers win this total amount. So the two travelers' best payoff, when they are seen as a unique (team) player opposed to the airline, is achieved for x_1=x_2=100. It is interesting to observe that some students see the airline as the real opponent and therefore play 100 even when P=35.

In Game 2, there is no obvious focal point: no amount has the attraction force that has 100 in Game 1. 99 and 100 together are only played by around 10% of the students and three amounts, 2, 50 and 35 are each played by around 10% of the students. This reveals that each of these three amounts has some special properties, but that these properties are not sufficiently salient to become obvious for all the students.
The amount 2 is salient, not because it is the NE amount, but because it is the only amount such that, if you ask for it, then you cannot get less than it. By playing 2, you are sure to get at least 2 and possibly 37. As a consequence, it is also the only amount that never yields a negative payoff. Playing 2 is the cautious way of playing and the students who play 2 play it because of this fact (and not because 2 is stable to unilateral deviations). So 2 is spontaneously played in a context where cautiousness is a salient component of the players' behavior: as soon as you are cautious or as soon as you think that the other traveler is cautious, you spontaneously ask for 2.
It is more difficult to understand the attraction force of 50 and 35. Students playing 50 say that 50 is (close to) the middle of the possible amounts and that they are sure to catch P half of time. Among the students playing 35 and commenting their choice, there are at least three groups of students. The first make an error, they wrongly think that they cannot get less than 0 by asking for 35. The second are sure, for unknown reasons, that the other students will ask for more than P (and so they hope to catch P). The third, much less numerous, are very social players who justify their action – the fact that they do not play less than 35- by saying that they would be unhappy if their choice resulted in a negative payoff for the other player. So 35 has a true salient feature: it is the minimal amount to play if you wish that the *other* traveler does not get a negative payoff. But, as said above, this is not the students' main motivation to play 35. We are rather tempted to add that some students, disoriented by Game 2, play 35 just because it is one of the three game data (2, P and M).

We draw a parallel with some papers on focal NE (see for example Parravano and Poulsen (2015)); in these papers it is observed that convergence toward a Pareto dominant NE is possible when the payoffs are symmetric but becomes more difficult when payoffs are asymmetric (namely when there are several Pareto dominant NE that are not in favour of the same player). We get something similar in spirit in the travelers' dilemma. In this game (100,100) is a Pareto dominant symmetric outcome (even if it is not a NE) and the asymmetry is in the payoffs the players get when they unilaterally deviate from (100,100). As a matter of fact, a unilateral deviation to 99 leads both travelers to switching from the symmetric payoffs (100,100) to the asymmetric payoffs (99+P, 99-P) (or (99-P, 99+P)). The value of P is crucial: when P is small in comparison to 100 (P=5), many travelers feel that the difference (asymmetry) between both payoffs, equal to 2P, is small enough to neither lead them to deviating from 100 to 99, nor to be afraid of this deviation (by the other traveler). So they can play 100. But when P is large, then the asymmetry is too important to allow them to converge to (100,100). To summarize, in the travelers' dilemma, the asymmetry that prevents from converging to the Pareto dominant symmetric outcome (M,M) is not an asymmetry in the focal (non equilibrium) outcome (M,M), but the perceived asymmetry in the payoffs obtained after a unilateral deviation. This perceived asymmetry depends on the ratio P/M, a ratio that has no role to play in the NE concept.

We now switch to Game 3 and to Game 4, where the value of the luggage, V, here fixed at 70, is common knowledge[2]. This value clearly becomes a focal amount in the game, because it has both a real economic meaning and a strong emotional content.

70 is the only fair economic amount in the game. In a fair world, the travelers and the airline should aim respectively to ask for and to reimburse 70. It follows from this fact that it is natural, hence focal, for a traveler to claim 70, at least if he considers the other traveler as an alter ego who also asks for the same amount. And indeed 70 has a strong attraction force: 29.82% of the students play 70 when P=5, 16.51% of the students play 70 when P=35. More generally, we observe that the mean claim moves towards 70 when switching from Game 1 to Game 3 (the mean amount decreases from 78.09 to 71.61), and when switching from Game 2 to Game 4 (the mean amount increases from 40.85 to 47.18). And for more than ¾ of the students (77.52%), the difference between the two claims for P=5 and P=35 is lower when V is common knowledge than when it is unknown.

70 is the most salient amount in Game 3 and in Game 4 but the students also take into account the risk to lose P, especially when P is large. It is therefore not astonishing that many students play the amount that allows to get 70, but by catching P (14.22% of the students play 65 when P=5, and 22.02% of them play 35 when P=35). So 65 in Game 3 and 35 in Game 4 become themselves focal values that generate a kind of *level-k* reasoning. Some of the students conjecture that the other traveler plays 65 in Game 3, 35 in Game 4, so they best respond by playing 64, respectively 34, and so on, down to 60, respectively 30 : 11.01%, respectively 11.47% of the students play amounts from 60 to 64 in Game 3, respectively from 30 to 34 in Game 4.

But 70 is also salient because it is a psychological, emotional, lower bound[3]. For some players, it is simply not possible to ask for an amount which doesn't allow to come close to 70, with or without P. So 70 is the fair amount to be paid back, but it is also a lower bound, you have to not go beneath: it is the minimal amount to recover from the airline. This explains why many students play 70 even when P=35: they understand the risk of being undercut, but they mention that 70 is the only amount played by themselves and their alter ego, that allows both to get 70. 70 becomes the new cooperative, team reasoning amount to win. Many students also clearly explain that they play 70 (or 69) because they are convinced that the *other* students will never ask for less than the value of the luggage. So they make a *level-k* reasoning. They are *level-1* players who suppose that the *level-0* players are "emotional" players, i.e. players that cannot ask for less than the amount they feel entitled to. These *level-1* students are sure to catch P as soon as they switch below 70, which explains that 29.36% of the students play below 70 but above 35 when P=35. Finally, among the students who play 35 (when P=35), but refuse to play below 35, some explain their behavior as follows: "If the other student plays more, I am sure to

[2] Morone and Morone (2016) also introduce a focal point in their traveler's dilemma experiments; but this focal point has no link with the value of the luggage and does not encourage the travelers to converge on it.
[3] In this sense 70 may be seen as an instinctive action (according to the categorization proposed by Rubinstein (2007)). But 70 is far from being only an instinctive action, in that it is also the result of a strategic reasoning, as explained later.

be paid back the value of the luggage. If he plays 35 like me, then, unfortunately, I only get half of the value of the luggage. But if he plays less, he is a very stupid fellow who accepts being cheated by the airline". In other words, standing up to the airline who aims to reimburse you less than the value of the luggage becomes itself a focal behavior. So the common knowledge of the value of the luggage naturally shrinks the strategy set: about 9 students in 10 play 60 or more in Game 3 and 30 or more in Game 4.

It is funny to go into the link between honesty and 70. 70 generates an honest behavior, which itself becomes focal, but not because travelers are honest, but because they are afraid of the honesty of the other traveler! We already know that only one fifth of the students are honest when the luggage is worth 70 (game 5) and less than one sixth of them when it is worth 20 (Game 6). Even if we add to the honest people those who do not ask for more than 80 in Game 5 and 30 in Game 6, the percentages of honest students only reach 28.9% in Game 5 and 24.31% in Game 6. Yet, despite this lack of honesty, 70 is a strong attractor. We already know that about 50% of the students play the four values allowing to reach 69 or 70, with or without P, in Game 3 and in Game 4. Moreover, for P=35, nearly 4/5 of the students (79.36%) ask for an amount from 30 to 70. And we can add that 61.93% of the students who play 70 or less in Game 4, are strongly dishonest in Game 5 and in Game 6, in that they ask for more than 80 in game 5 and for more than 30 in game 6 (this percentage grows to 70.18% if we count the students strongly dishonest in at least one of the two games). Clearly, especially for P=35, the students do not play 70 or less than 70 because they are honest, but because they are afraid of the possible honesty of the other traveler, which compels them to play as if they were honest. And the students say that they fear that the honesty of the other traveler is increasing in P!

The students' comment highlights a main difference between Game 3 and Game 4. In Game 3, the students are much less afraid of the honesty of the other students, so that about 1/5 of them (18.81%) keep on playing 90 or more, and more than ¼ of them (26.15%) play 80 or more. They play in this way because, on the one hand, they are not honest (they still want to get the maximal amount 100), on the other hand they are not afraid of the honesty of the other student. They write that if the other student is honest and asks for 70, then, by playing a large amount, they get 65, which is not the end of the world (at best, in this context, they could have won 74 by playing 69). And they also observe that if the other student is dishonest too and plays a large amount, then they are very happy to play a large amount (even if the other traveler's amount is lower). And the students are right: they win more than 74 as soon as the other traveler plays at least 80. In other words, when P is small, being dishonest has no strong negative consequence, whereas being honest can lead to missing great payoff opportunities. This explains that only 3/5 (59.17%) of the students play from 60 to 70 in game 3, whereas 4/5 of the students play from 30 to 70 in game 4. In fact, when P is small, 100 remains a focal amount, even though 70 becomes the most focal amount.

This is not true for P=35. For P=35, playing a large amount, say 90 or more, leads to a payoff of 35 when the other player is honest and plays 70; the best action, 69, yields the payoff 104, which is much larger than the payoff obtained by the student when he plays large amounts, and when the other player claims a large amount but lower than that of the student.

This difference between Game 3 and Game 4 is perfectly understood by the students: 21.10% of them simultaneously play an amount higher or equal to 80 for P=5 but shift to an amount lower or equal to 70 for P=35.

In some way, when V is common knowledge, the students apply a kind of min max regret reasoning (see Halpern and Pass (2012) for more details), which leads them to comparing possible obtained payoffs with the best payoffs they could get by playing in a different way; yet, in addition to the notion of min max regret, the students now have a reference point in their reasoning, the payoff they get when the other traveler is honest. Observe that in this reasoning, the ratio P/M has again a role to play.

From a theoretical point of view, we can add that, when P is large, it is not even necessary to believe that the other traveler may be honest to be induced to play less than 70. For example, suppose that traveler 1(T1) is dishonest and thinks that traveler 2 (T2) is dishonest, that T2 is dishonest and thinks that T1 is dishonest, but that T1 thinks that T2 thinks that T1 is honest: it follows from these beliefs that T1 thinks that T2 will play 70 or less than 70, forcing himself to play less than 70, even if both travelers are completely dishonest and do not think that the other traveler is honest. In fact, as long as the dishonesty of both travelers is not common knowledge, both travelers are led to playing 70 or less than 70, which highlights how the common knowledge of V impacts the reasoning and behavior of the travelers even in a dishonest world.

To summarize, the common knowledge of V leads to focal amounts (V and V-P), but also to focal characteristics of behavior (the need of fairness, honesty and beliefs of honesty, standing up to the airline). All these focal points nicely canalize the behavior of the students and help them to converge on amounts that are fairer, much larger than the minimal amount 2.

5. Incomplete information on the lower bound

The value of the luggage, because of its emotional and economic content, may induce a change in the perceived strategy set. We observed that some students refuse to play less than 70 just because they want to recover the value of the luggage. These students act as if their strategy set were reduced to [70, 100]. Other students refuse to go below 35 (P=35) or 65 (P=5), even though they are aware of the fact that the other player may ask for a lower amount, so they reduce their strategy set to [70-P, 100]. But most of the students only believe that the *other* students will not play an amount lower than 70 or lower than 70-P, and they act accordingly: they do not hesitate to play 70-P-1. It derives from this fact that if the students played Game 4 repetitively, by observing the behavior after each round, they might, after few repetitions, often play an amount lower than 35. Even in our experiment without repetitions, there are some students who play 2, explaining that, given that the other students will surely play 35, they are sure to get more than half of the value of the luggage (and so they get more than two students who both play 35). So the question is: what is the real impact of the emotional and economic content of V in the long run? Can it stand up to the repetition of the game? In Game 4, it may not, namely because 2+P > V/2. But things may be different for M=150, P=35 and V=120. It may be that more travelers would refuse to converge to 2, even in a repeated game setting. Because the emotional content of the value of the luggage is linked to a real economic content, and so because it is not just emotion, it cannot be omitted, even in a repeated game.
In other words, students may perceive the game as a game with incomplete information on the lowest amount the other traveler accepts to ask for, because he wants to be paid back at least this amount. All happens as if we study the following game. Assume that traveler i, i=1,2, can ask for an integer dollar amount from t_i to 100; t_i is uniformly distributed on a set of integers, from 2 to S, where S may be the value of the luggage, V, or V-P. t_i is the lowest amount traveler i can claim and it will be called traveler i's type. Each traveler knows his type but not the other traveler's type; he assumes that the other traveler's lower bound is any integer from 2 to S with probability 1/(S-1). This lack of information reflects the fact that each traveler assumes that the

other traveler may refuse to claim less than a given part of the value of the luggage, but does not know which part. So, if S=V, a traveler of type S refuses to ask for less than V, because he cannot accept to be paid back less than the value of the luggage. Travelers with lower bounds lower than V accept to be paid back less than V and can therefore ask for lower amounts. It may be conjectured that, in this context, travelers play more than 2, even if they are of type 2, that is to say if they can play 2 if they want to do so (like in the standard traveler's dilemma).

Solving this game in the discrete setting reveals to be difficult (namely because of the deviations in full units). So we propose here to study the game in a continuous setting, where t_i is uniformly distributed on [0, S][4] (with S≤V). We assume M≥V.
Call $b_2(t_2)$ traveler 2's equilibrium strategy and b traveler 1's action when his type is t. We reasonably suppose that $b_2(t_2)$ increases in t_2. So traveler 1 of type t gets b+P when $b_2(t_2)>b$, i.e. $t_2>b_2^{-1}(b)$ and he gets $b_2(t_2)$-P when $t_2 <b_2^{-1}(b)$. Ties cannot happen in the continuous setting.

Traveler 1 of type t solves the following optimization program:

$$\max_b \int_0^{b_2^{-1}(b)} (b_2(t_2) - P)f(t_2)dt_2 + \int_{b_2^{-1}(b)}^{S} (b+P)f(t_2)dt_2$$

u.c. $b \geq t$
where f(x) is the uniform density function.
We call λ the Kuhn-Tucker multiplier, so we get:

$$\frac{1}{b_2'\left(b_2^{-1}(b)\right)}\left(b_2\left(b_2^{-1}(b)\right) - P\right)\frac{1}{S} - \frac{1}{b_2'\left(b_2^{-1}(b)\right)}(b+P)\frac{1}{S} + F(S) - F(b_2^{-1}(b)) + \lambda = 0$$

where F(x) is the cumulative distribution.
We look for a symmetric NE, so we get:

$$\frac{1}{b'(t)}(b-P) - \frac{1}{b'(t)}(b+P) + S - t + S\lambda = 0$$

It follows $b'(t) = \frac{2P}{S-t+S\lambda}$
When b(t) = t, i.e. when the constraint is binding, we have S-t-2P+Sλ = 0; it derives from the positivity of λ that b(t) = t when, either S-2P < 0, or S-2P > 0 and t ≥ S-2P.
And b(t) = -2Pln(S-t) +k when S-2P > 0 and t ≤ S-2P, where k is the constant that equalizes S-2P with -2Pln(S-(S-2P))+k, so k = S-2P+2Pln(2P).
It is easily checked that b(t) = -2Pln(S-t)+S-2P+2Pln(2P) is larger than or equal to t for any t in [0, S-2P] (because -2Pln(S-t)-t is decreasing in t for t in [0, S-2P]).

Proposition 1
In the continuous setting, if S-2P ≤ 0, then the NE is similar to the NE of the standard traveler's dilemma, in that each traveler plays the lowest possible amount he can play.
If S-2P > 0, then, at the NE, the traveler of type t, with t∈[0, S-2P], plays b(t)= -2Pln(S-t)+S-2P+2Pln(2P), so he plays more than the lowest possible amount he can play. The traveler of type t, with t∈[S-2P, S], plays b(t)=t.

[4] We set 0 (instead of 2) for the lowest possible amount. In this continuous setting, we can also suppose P>1 (instead of P≥2 in the discrete setting). So the NE in the standard traveler's dilemma becomes (0,0) and each traveler gets a NE payoff 0.

For t in [0, S-2P], b(t) is increasing in S and b(t)-t is decreasing in t, hence is maximal for t=0. So, when the travelers are standard travelers, i.e. travelers of type 0 who can choose any amount in [0, M], they can play much more than 0, given that they play -2Pln(S)+S-2P+2Pln(2P).
There are two natural values for S, which are V and V-P. For both values, the distance between the NE amount and 0 increases with the value of the luggage.
Let us illustrate the results with an example, by setting S=V=50.
For P>25, travelers of type 0 will play 0, as in the standard traveler's dilemma. In fact P is too large and we come back to some students' behavior who observe that by playing the minimal amount (here 0), they are almost sure[5] to be paid back half of the value of the luggage, which dissuades them from asking for more.
For P<25 a traveler of type 0 can play more than 0: he plays -2Pln(V)+V-2P+2Pln(2P), which is decreasing in P. For example, for P=15, he plays 4.68, for P=5 he play 23.91, for P=2 he plays 35.9. So, the smaller is P, the more a standard traveler (a traveler of type 0) affords himself to play a large amount. In other words, the more it becomes difficult to get back a reasonable part of the value of the luggage when playing 0, because 0+P is very far from the value of the luggage, the more a standard traveler plays a large amount. This expresses an expected result: when catching P is not enough to be paid back a reasonable part of the value of the luggage by asking for 0 (P<<V), then, by assuming that the other traveler cannot accept to play less than the amount t (uniformly distributed on [0, V]), a traveler of type 0 can afford to ask for much more than 0 at the NE.
What is more, when S>2P, each type of traveler gets a NE payoff which is higher than the NE payoff in the standard traveler's dilemma, which is 0.

Proposition 2
For S≥ 2P:
- Each traveler of type t, with t∈[0, S-2P], gets the positive NE payoff S-P+2Pln(2P/S).
-The NE payoff for a traveler of type t, with t∈[S-2P, S] decreases in t and is equal to:

$$\frac{S}{2}-\frac{2P^2}{S}+P+2Pln\left(\frac{2P}{S}\right)-\frac{t^2}{2S}+\frac{(S-2P)t}{S}$$

For S>2P this payoff is positive for any t in [S-2P, S], for S=2P it becomes equal to 0 for t=S. The mean NE payoff is $S-P+2Pln\left(\frac{2P}{S}\right)-\frac{4P^3}{3S^2}$*. It is lower than S, growing in S and equal to S/3 for S=2P.*
For S<2P:
Each traveler of type t gets $\frac{-t^2}{2S}+\frac{(S-2P)t}{S}+P$.
This payoff is decreasing in t, from P for t=0 to (S-2P)/2 for t=S. It is negative for t higher than $S-2P+\sqrt{(S-2P)^2+2PS}$ *. But the mean NE payoff is always equal to S/3.*

Proof : see Appendix A

It follows from proposition 2 that this model introduces more fairness into the traveler's dilemma. Let us set S=V, in that V is surely the most natural value for S.
When P is large so that V<2P, some types of traveler, at the NE, can get a negative payoff (this does not happen in the standard traveler's dilemma) but the mean payoff is V/3, which means

[5] In this game, a traveler of type 0 is sure to catch P by playing 0.

that, even for large values of P, a traveler is paid back V/3 (mean payoff), i.e. the third of the value of the luggage.
When V>2P then each type of traveler gets a positive NE payoff. Each traveler of type t, with t $\in$ [0, V-2P], gets the same payoff $V - P + 2Pln\left(\frac{2P}{V}\right)$. The travelers of type t, with t$\in$]V-2P,V], get a lower payoff but their payoff it is always larger than the positive payoff $V - P + 2Pln\left(\frac{2P}{V}\right) - \frac{2P^2}{V}$. It immediately follows that the travelers can be paid back a large part of their luggage when V is large in comparison to P. For example, in the example above, for V=50 and P=15, the mean NE payoff is 17.88. For V= 50 and P=2, the mean NE payoff grows to 37.89. For V=500 and P= 10, the mean NE payoff is 425.62, so becomes close to V.
Clearly, when there is incomplete information on the lowest amount a traveler accepts to ask for, then the reimbursed amount becomes fairer at equilibrium.

6. Concluding remarks

The model in section 5 has to be interpreted carefully in that there are at least two differences between the discrete and the continuous setting: the first is that, in the continuous game, a traveler of type t has a probability 0 of meeting another traveler of the same type, the second is that the amounts are not integers, so can continuously increase in t. In a discrete model, the amounts can only jump in increments of 1, which leads several types to playing the same amount, a fact which cannot happen in the continuous model. Yet the spirit of the continuous game is the same than that of the discrete game.
We insist on the fact that introducing incomplete information on the lowest amount a traveler can ask for, is not an artificial ad hoc change in the structure of the game. It is a natural change linked to the economic meaning of the value of the luggage V: why should a player accept to be paid back much less than V? Doubts about the ability of another traveler to accept such unfairness are well-founded, and so is the incomplete information on the lowest possible amount the other traveler can accept to claim. The game results in more fairness, at least if P is lower than V/2, in that all types of travelers will be paid back more than in the standard traveler's dilemma. Even in the unfavorable context where P is larger than V/2, the mean Nash equilibrium reimbursement is V/3, i.e. the third of the value of the luggage. To this extent, the common knowledge of the value of the luggage reveals to be a useful tool to introduce more fairness.
Our new way to write the traveler's dilemma, linked to the introduction of common knowledge of the value of the luggage, is a way to reconcile the Nash equilibrium with the students' behavior. Other authors have also proposed changes in the structure of the game in order to get new more acceptable Nash equilibria.
For example, in Ispano (2015), there is no maximal amount M, so the travelers can propose any amount up to infinity, which leads to a mixed Nash Equilibrium whose support is the whole strategy set. Introducing no upper bound changes however the logic of the traveler's dilemma, in that it disrupts iterative dominance and it prevents the travelers from focusing on a same maximal amount.
Basu (1994) argues that if one could switch from a precise set of integers to a more fuzzy set where some amounts are called "large" and others "low", so that the travelers could only choose among "large" and "small" amounts, then (large, large) would spontaneously be the Nash equilibrium. To some extent, Basu's idea, even if very imprecise, fits with the students'

comments and behavior. On the one hand, the students say, for example, that playing “large” amounts when P=5 seems not very risky, whereas it seems less risky to play “low” amounts when P=35. On the other hand, many students do not really work with an integer set, in that most of them mainly work with the multiples of 10! So 69.27%, respectively 47.25%, 59.17% and 48.17% of the students play multiples of 10 in Game 1, respectively in Game 2, Game 3 and Game 4, even though the multiples of 10 are only 10% of the possible amounts. And respectively 86.24%, 76.15%, 85.32%, and 81.19%, i.e. 4/5 of the students only play multiples of 5, even though these numbers only represent 1/5 of all possible amounts. And of course, if the strategy set is reduced to the amounts 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 and even to the multiples of 5, then, when P=5, (100,100) is a Nash equilibrium (as are the couples (x, x) where x is a multiple of 10 or 5), because 90+5 < 100 and 95+5 ≤ 100 (and more generally x+5 < next multiple of 10 and x+5 ≤ next multiple of 5). So the set of Nash equilibria becomes larger in Game 1 and in Game 3. With this smaller strategy set, the coordination on (100,100) in Game 1 simply illustrates the (easy) coordination on the most natural Nash equilibrium (because it is the Pareto dominant Nash equilibrium in this new game) and the coordination on (70,70) in Game 3 simply illustrates the coordination on the most focal Nash equilibrium in Game 3. A strategy set reduced to multiples of 5 or 10 however doesn’t change the standard traveler’s dilemma Nash equilibrium when P=35: this also explains why it is much less easy for the students, even if they think in multiples of 10 or 5, to coordinate in Game 2.
Let us add a last remark. In our paper we introduce incomplete information on the lower bound of the traveler’s strategy set. This allows us to reconcile the Nash equilibrium with a behavior that consists in playing amounts which are larger than the lowest possible amounts. Yet, as long as S (and so V) has no link with the upper bound M (except of being lower than it), the Nash equilibrium does not depend on M. This is not satisfying from a behavioral point of view, given that, as claimed in section 4, it can be conjectured that the travelers’ behavior also depends on the ratio P/M.
So the traveler’s dilemma remains a challenging game to study.

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Appendix A

We first study the context with $S \geq 2P$.
Each traveler of type l, with $l \in [0, S-2P]$, gets the NE payoff :

$$\int_0^l (-2Pln(S-t) + S - 3P + 2Pln2P)\, f(t)dt + (S-l)(-2Pln(S-l) + S - P + 2Pln2P)/S$$

Where f(.) is the uniform density function. This formula becomes:

$$\frac{[2P(S-t)\,ln(S-t) + 2Pt]_0^l}{S} + \frac{l(S-3P+2Pln2P)}{S} + \frac{(S-l)(-2Pln(S-l) + S - P + 2Pln2P)}{S}$$

Developing this formula leads to the NE payoff (S-P)+2Pln(2P/S).

Each traveler of type l, with l ∈[S-2P, S], gets the NE payoff :
$\int_0^{S-2P}(-2Pln(S-t)+S-3P+2Pln2P)\,f(t)dt+\int_{S-2P}^{l}(t-P)f(t)dt \;\; +\frac{(l+P)(S-l)}{S}$.
By developing this formula, we get the NE payoff: $\frac{S}{2}-\frac{2P^2}{S}+P+2Pln\left(\frac{2P}{S}\right)-\frac{l^2}{2S}+\frac{(S-2P)l}{S}$.
This formula is decreasing in l given that l ∈[S-2P, S]. By replacing l by S we get the payoff :
$S-P+2Pln\left(\frac{2P}{S}\right)-\frac{2P^2}{S}$
To get the mean NE payoff we call A=(S-P)+2Pln(2P/S) and B= $\frac{S}{2}-\frac{2P^2}{S}+P+2Pln\left(\frac{2P}{S}\right)$.
The mean NE payoff is equal to:
A(S-2P)/S+B2P/S + $\int_{S-2P}^{S}(-\frac{t^2}{2S}+\frac{(S-2P)t}{S})f(t)dt$
By developing this formula, we get: $S-P+2Pln\left(\frac{2P}{S}\right)-\frac{4P^3}{3S^2}$
This payoff is lower than S, growing in S and equal to S/3 for S=2P.
We now study the context where S<2P.
Given that b(l) = l for each type l, the NE payoff of a traveler of type l becomes:
$\int_0^{l}(t-P)f(t)dt+(l+P)(S-l)/S$ *, equal to* $\frac{-l^2}{2S}+\frac{(S-2P)l}{S}+P$.
This payoff is decreasing in l, given that S-2P <0, it decreases from P for l=0 to (S-2P)/2 for l=S, and it is negative for l higher than $S-2P+\sqrt{(S-2P)^2+2PS}$.
The mean NE payoff is:
$\int_0^{S}\left(\frac{-t^2}{2S}+\frac{(S-2P)t}{S}+P\right)f(t)dt$.
This payoff is equal to S/3, regardless of the values of S and P (>S/2)).

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