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Testing Nonlinearity through a Logistic Smooth Transition AR Model with Logistic Smooth Transition GARCH Errors

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Abstract:

This paper analyzes the cyclical behavior of CAC 40 by testing the existence of nonlinearity through a logistic smooth transition AR model with logistic smooth transition GARCH errors. We study the daily returns of CAC 40 from 1990 to 2018. We estimate several models using nonparametric maximum likelihood, where the innovation distribution is replaced by a nonparametric estimate for the density function. We find that the rate of transition and the threshold value in both the conditional mean and conditional variance are highly significant. The forecasting results show that the informational shocks have transitory effects on returns and volatility and confirm nonlinearity.

Keywords:

LSTAR model, LSTGARCH model, nonparametric maximum likelihood, nonlinearity, informational shocks, time series analysis.

JEL Classification:

C14, C22, C58, G17

Introduction

Over the last twenty years, the interest in nonlinear time series models has been surely increasing. The presence of nonlinearity in financial series has important implications especially concerning the property of weak efficiency of markets. Indeed, if a series exhibits nonlinear structure, this implies significant nonlinear dependencies between the observations (Chikhi & Bendob (2018)). In applications to financial time series, models, which allow for regime-switching behavior have been most popular, especially the class of smooth transition autoregressive (LSTAR) models, popularized by Teräsvirta (1994), has enjoyed great success. A lot of work in this area has been devoted to estimation, specification, testing and applications such as forecasting (Potter (1999), Van Dijk et al. (2002b), Wahlström (2004), Chikhi & Diebolt (2009), Abedile & Shangodoyin (2006) and Umer, Sevi & Sevil (2018)). Smooth transition models may be appropriate to provide a privilege framework for the study of asymmetric stock market fluctuations. These models justify the sources of non-linearity of stock price adjustment by the presence of transaction costs.

For the history and applications of the STAR model to economic and financial time series see, for example, Granger & Teräsvirta (1993) and Teräsvirta (1994) who classify market into two phases of recession and expansion. Thus, Teräsvirta & Anderson (1992) forecast quarterly OECD industrial production series with STAR model. Sarantis (1999) tests nonlinearities in real effective exchange rates for 10 major industrialized countries and evaluates forecast accuracy of STAR model over the random walk model. Eitrheim & Teräsvirta (1996) evaluate the specification of STAR model by introducing a Lagrange multiplier (LM) test for the hypothesis of no error autocorrelation and LM-type tests for the hypothesis of no remaining nonlinearity and that of parameter constancy. Wahlström (2004) compares forecasts from the LSTAR model to those from a linear autoregressive model. In turn, Chikhi & Diebolt (2009) analyze the cyclical behavior of the German annual aggregate wage earnings using LSTAR model and show that the observed German annual aggregate wage movements appear as the result of transitory exogenous shocks. On the other hand, Zhou (2010) studies the STAR model in the presence of structural break in industrial production index of Sweden. Tayyab, Tarar & Riaz (2012) evaluates the suitability of the Smooth transition autoregressive (STAR) models specification for real exchange rate Modeling**.** Adebile & Shangodoyin. (2006) propose an alternative representation of the original version of the logistic STAR model. Whereas, Umer, Sevil & Sevil (2018) compare the performance of smooth transition autoregressive (STAR) and linear autoregressive (AR) models using monthly returns of Turkey and FTSE travel and leisure index. For review of threshold time series models in finance, see also Chen, So & Liu (2011).

The limitation of these works is that they don't capture the nonlinearity structure in the conditional variance. The assumption of white noise on the LSTAR model residuals ignores the presence of conditional heteroskedasticity; however, the financial series are generally characterized by a time-varying volatility that can be modeled by ARCH-type models (Engle (1982) and Bollerslev (1986)) that is often used to study the behavior of asset returns or innovations of the 'parent' model. Franses, Neele & Van Dijk (1998) and Lundbergh & Terräsvirta (1999, 2000) describe the nonlinear dynamics in both the conditional mean and the conditional variance by combining the Smooth Transition Autoregressive (STAR) models (Granger &Teräsvirta (1993) and Teräsvirta (1994)) with GARCH errors (Bollerslev (1986)) and with the Smooth Transition GARCH errors (Hagerud (1997) and

Gonzalez-Rivera (1998)), which have been widely used in forecasting. Thus, some authors have used the STGARCH or the STAR-STGARCH to study financial time series. Concerning the STGARCH model, several authors have introduced these specifications (Hagerud (1997), Gonzalez-Rivera (1998), Anderson et al. (1999) and Medeiros & Veiga (2009)) widely used in models of conditional mean, to model the asymmetric response of conditional variance to positive and negative news. Lubrano (2001) uses a Bayesian approach to estimate the STGARCH model. Yaya & Shittu (2016) model banks share prices using the STGARCH to capture nonlinear, asymmetric and symmetric properties of Nigerian banks stocks and to determine the volatility behavior of each bank. Regarding the STAR-STGARCH modeling, Chan, Marinova & McAleer (2002) analyses trends in the development of more ecological-friendly technologies using STAR-GARCH model. Chan & McAleer (2003) investigate several empirical issues regarding quasi-maximum likelihood estimation of STAR models with STGARCH errors. They show that different algorithms produce different estimates for the same model in the presence of extreme observations and outliers. Reitz & Westerhoff (2007) propose an empirical commodity market model with heterogeneous speculators using STAR-GARCH model. Pavlidis, Paya& Peel (2010) examine the impact of conditional heteroskedasticity and investigate the performance of several heteroskedasticity robust versions. The mean-variance equations are then compounded as STAR-GARCH model. Guo & Cao (2011) develop a smooth transition GARCH model with an asymmetric transition function, which allows for an asymmetric response of volatility to the size and sign of shocks, and an asymmetric transition dynamics for positive and negative shocks. Chan & Theoharakis (2011) estimate m-regimes STAR-GARCH model using quasi-maximum likelihood (QMLE) with parameter transformation. Ben Haj Hamida & Haddou (2014) propose to study exchange-rate dynamics for the Maghreb countries using the STAR-STGARCH model. Finally, Livingston & Nur (2018) use the Bayesian inference for the smooth transition autoregressive STAR(k)–GARCH (l, m) models.

Some authors assume that the innovations follow the Normal distribution, which cannot accommodate fat-tailed properties commonly existing in financial time series. Many existing studies point out that this problem can lead to inconsistent estimates. The Student’s t-distribution and General Error Distribution (GED) can be the two most popular alternatives with the intension of capturing the heavy-tailed returns. However, in most cases, the innovation distribution is unknown and often replaced by a nonparametric estimate and thus the estimation procedure becomes semiparametric (see Pagan & Ullah (1999) and Di & Gangopadhyay (2014)).

Our research, in contrast to studies that use parametric Gaussian distribution, Student’s t-distribution or General Error Distribution (GED), employs nonparametric maximum likelihood method to estimate semi-parametrically our model. We apply this technique to explain cyclical behavior, examine informational shocks and describe the nonlinear dynamics in CAC 40 returns using LSTAR-LSTGARCH models. The short-term predictability of CAC 40 index provides evidence for inefficiency of Paris stock market (in a weak level) with limited rationality, which emerges arbitrage opportunities.

The paper is structured as follows. Next section outlines the daily CAC 40 price data and discusses its statistical properties. Section 3 is devoted to semiparametric modelling of the daily return series of CAC 40; we compare the predictive quality of AR-GARCH, LSTAR-GARCH and LSTAR-LSTGARCH models with that of a random walk. The last section concludes.

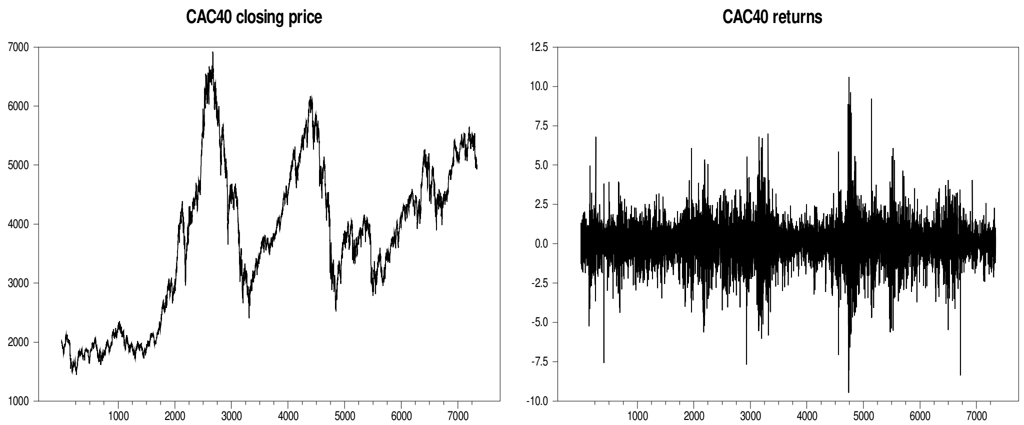
1. Dataset used and its statistical properties

The data used in this paper consists of the daily closing CAC40 price index downloaded from ABC Bourse (https://www.abcbourse.com/marches/) covering a historical period from January 2, 1990 to November 30, 2018 including 7340 observations. In order to better understand the characteristics of the CAC40 series, it is necessary to examine some descriptive statistics. As is usual in financial time series, the linear and nonparametric unit root tests are employed to test the stationarity behavior (Kwiatkowski, Phillips, Schmidt & Shin (1992), Breitung (2002), Elliott, Rothenberg & Stock (1996)).The corresponding results are presented in table 1. All the unit root tests accept at the 5% level the hypothesis stating that the logarithmic CAC40 series contain a unit root. It is therefore concluded that the series is finally differentiated to obtain the daily percentage returns of CAC40 at time t (see figure 1)

$$r_t = 100 \times \left(\ln P_t - \ln P_{t-1} \right)$$

Where P_t and P_{t-1} are daily CAC40 price at two successive days t and $t-1$ respectively.

Figure 1 –Daily CAC40 (Level and returns)



In order to test the presence of structural breaks and identify the dynamics of asymmetric adjustment, we use other unit root tests (Lee & Strazicich (2004) and Enders & Granger (1998)). For the returns of CAC40, the Lee-Strazicich LM unit root test strongly reject the null hypothesis in favor of trend stationarity with one break. Also, the stationarity is confirmed by the result of asymmetric unit root and linearity test proposed by Enders & Granger (1998)) based on the threshold autoregressive (TAR) models. According to the test results in table 1, we reject the null hypothesis of equality $H_0 : \rho_1 = \rho_2$ and the series of returns exhibits TAR type asymmetric adjustment. The joint F-statistic ϕ and the maximum t-statistic between $\rho_1 = 0$ and

$\rho_2 = 0$ (t-max) confirm these results and the process is concluded to be statistically nonlinear stationary with asymmetric dynamics. It is possible that this apparent nonlinearity could be due to structural breaks or outliers, as described in the Lee-Strazicich unit root test results.

Table 1 – Unit root tests

Unit root tests				
Test			Logarithmic	Returns
ERS			0.0075	20.9229
H_0 : Unit root			(3.26)	(3.26)
Breitung			0.00001	0.0525
H_0 : Unit root			(0.0104)	(0.0104)
KPSS			0.1242	0.0618
H_0 : Stationarity			(0.146)	(0.463)
Asymmetric unit root and linearity test for dailyreturns of the CAC40				
				Returns
Enders-Granger H_0 : Unit root	Selected lags			1
	Attractor			-0.0145
	t-max			-48.6895
	ϕ			1965.3915
	$\rho_1 = \rho_2$			5.9869
Unit root test with structural break				
	Break type		Logarithmic	Returns
Lee-Strazicich H_0 : Unit root	Crash (A)	τ	-1.9994	-40.9447
			(-3.23)	(-3.23)
		Break point	10/28/1997	08/28/2000
		Selected lags	5	4
	Break (C)	τ	-2.6614	-41.6379
			(-3.9857)	(-4.0126)
		Break point	06/04/1998	12/24/1999
		Selected lags	5	4

Notes: *(.): The asymptotic critical value at 5%.The table reports the results of Breitung's nonparametric unit root test, which only gives the critical values for T = 100, T = 250 and T = 500. Therefore, the critical values used here are the ones for n = 500. We accept the unit root hypothesis H_0 for daily logarithmic series and reject it for daily returns.* τ is the minimum Lee-*Strazicich test statistic. For Elliott-Rotenberg-Stock (ERS) and KPSS tests, the spectral estimation is based on the Bartlett kernel using the Andrews bandwidth.* ρ_1 *and* ρ_2 *are coefficients of first lag values of each regimes.* ϕ *is the joint F-test for* $\rho_1 = \rho_2 = 0$. *t-max is the larger of the two t-statistics on* $\rho_1 = 0$ *and* $\rho_2 = 0$.

Table 2 reports the descriptive statistics and shows that daily CAC 40 return series exhibit significant negative skewness and leptokurtosis. The Jarque-Bera test (Jarque & Bera (1987)) confirms the non-normality of the distribution (see also Figure 2). Rejection of normality partially reflects the dependencies in the moments of returns series. The observed asymmetry may indicate the presence of nonlinearities in the evolution of daily returns. In addition, the ARCH-LM test result shows that CAC 40 returns are characterized by the presence of ARCH effect according to the results of normality test.

Table 2 – Summary statistics for daily CAC 40 returns

Skewness	Kurtosis	Jarque-Bera	ARCH(1)	ARCH(2)
-0.0643	7.7141	6812.2497	266.678	559.677
(0.0244)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Notes: *(.): The p-Value. We reject the assumption of normality H_0 because the Jarque-Bera statistic is greater than the critical value of* chi-square *distribution with 2 degrees of freedom at 5%. Moreover, we reject the homoscedasticity assumption H_0 (There is an ARCH effect in the data because the ARCH-LM statistic is greater than the critical value* of *chi-square* distribution with 1 and 2 degrees of freedom at 5%.).

Figure 2 – Kernel estimation of density

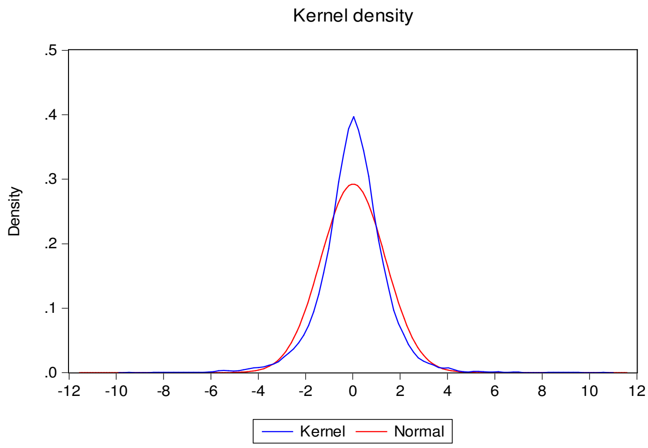


Table 3 – BDS test results on the series of returns

m	BDS stat.	Prob.
2	14.435	0.000
3	20.366	0.000
4	25.028	0.000
5	28.659	0.000
6	31.977	0.000
7	35.498	0.000
8	39.342	0.000
9	43.481	0.000
10	48.015	0.000
11	53.405	0.000
12	59.501	0.000
13	66.603	0.000
14	74.874	0.000
15	84.543	0.000

Notes: *The BDS statistics are calculated by the fraction of pairs method with ε equal to 0.7.m represents the embedding dimension. The BDS statistics are strictly greater than the critical value at 5%for all the embedding dimensions.*

As seen in table 3, the random walk hypothesis is clearly rejected and the returns of CAC 40 are non-linearly dependent. The BDS (Brock et al. (1996)) test generally brings out the presence of significant non-zero autocorrelations in the short term. This test leads us to reject the *i.i.d* hypothesis, but do not detect the presence of long memory structure. Given this situation, we test the presence of dependencies by considering longer horizons. As it is observed from Table 4, test results for fractional integration show the evidence that the process is anti-persistent and return series exhibits short-term memory, but it does not have the behavior of ARMA. The memory parameter estimated by the Andrews-Guggenberger (Andrews & Guggenberger (2003)), Robinson-Henry (Robinson & Henry (1998)) and the GPH (Geweke & Porter-Hudak (1983)) methods is negative and significant. The absence of a long memory indicates that agents can only anticipate their returns to a short time horizon. Indeed, the observed movements appear as the result of transitory exogenous shocks which affect the Paris market.

Table 4 – Results from the ARFIMA(0,d,0) estimation on daily CAC 40 returns

	GPH	Robinson-Henry	Andrews-Guggenberger
$\widehat{d}$	-0.0058	-0.0185	-0.0591
Student statistics	-3.6173	-2.2482	-3.35400

Notes: $\widehat{d}$ *is the estimated Long memory parameter with a power of 0.8.*

In order to verify the existence of an underlying nonlinear structure in CAC 40 stock returns and detect the nonlinear behavior of volatility, we use the Hinich bispectrum test (Hinich & Patterson (1989)) for linearity and Gaussianity and the Tsay test for neglected nonlinearities (Tsay (1986, 1991, 2001), Tiao & Tsay (1994) and Luukkonen, Saikkonen & Terasvirta (1988)). In view of Table 5, the Gaussianity and the linearity statistics are strictly greater than the critical value of standard normal and that of chi-square distribution at 5%, with two degrees of freedom, respectively. We reject the null hypothesis of linearity and Gaussianity. In addition, the Tsay test result confirm nonlinearity because the F-statistics are greater than the critical value at 5%. We find evidence of threshold behavior in the returns and volatility series. It is due to the large variance change in the time period. Consequently, these results indicate the presence of strong nonlinear structure in the evolution process of returns and volatility and confirm those of the Enders-Granger unit root test, which rejects the linear hypothesis in favor of the nonlinearity and asymmetry assumptions. The series very likely has TAR behavior.

Table 5 – Hinichbispectrumand Tsaytests for linearity

Series	Hinichbispectrum test					Tsay test
	Frame Size	Lattice Points	Test Quantile	Linearity	Gaussianity	F^4_{Tsay}
Returns	85	462	0.80000	8.7268 (0.0000)	6263.8270 (0.0000)	6.3586 (0.0000)
Volatility	85	462	0.800	1056.706 (0.0000)	169638.125 (0.0000)	31.1934 (0.0000)

__Notes:__ The numbers in the table are nonparametric Hinich bispectral test statistics with the null hypothesis H_0 of linearity and Gaussianity, obtaining the chi-squared statistic for testing the significance of individual bispectrum estimates by exploiting its asymptotic distribution. The numbers in the parenthesis are critical probabilities. F^4_{Tsay} is the Tsay Ori-F test for neglected non-linearities in an autoregression. We test more specifically against STAR using 4 lags.

3. Semiparametric modelling

In order to describe the nonlinear dynamics in both the conditional mean and the conditional variance, the modelling of CAC 40 series could be turned towards smooth transition autoregressive models (Lukkonen, Saikkonen & Terasvirta (1988), Teräsvirta & Anderson (1992), Tiao & Tsay(1994) and Teräsvirta (1994)) which could be combined with smooth transition GARCH errors (Gonzales-Riviera (1996), Hagerud (1997) and Chan & McAller (2003)) using nonparametric maximum likelihood, where the innovation distribution is unknown and replaced by a nonparametric estimate for the density function (Pagan & Ullah (1999) and Di & Gangopadhyay (2014)). In practical terms, we estimate AR, LSTAR jointly with GARCH and LSTGARCH models. The estimation procedure becomes semiparametric. These semiparametric approaches require two steps. In a first step, we incorporate an initial estimate of the model parameter to produce a residual. In a second step, we use the residuals to estimate the nonparametric likelihood, which is after maximized to obtain the final estimate of the model parameter.

We first use the nonlinear least square method based on the Gauss-Newton algorithm to estimate the LSTAR model (Van Dijk, Teräsvirta & Franses (2000)). We select the appropriate transition variable (Öcal & Osborn (2000)). The first stage in the modeling cycle is to test linearity against LSTAR nonlinearity by selecting a linear model with residuals using sum of squared residuals. The selected linear model obtained by the general-to-specific procedure and based on the mentioned criteria is assumed to form the null hypothesis for testing linearity (Chikhi & Diebolt (2009)). The LSTAR linearity test is carried out for different candidate transition variables. If the linearity is rejected against LSTAR, we select the appropriate transition variable and proceed to estimate the LSTAR parameters. The linearity tests and the grid search results are displayed in Table 6.

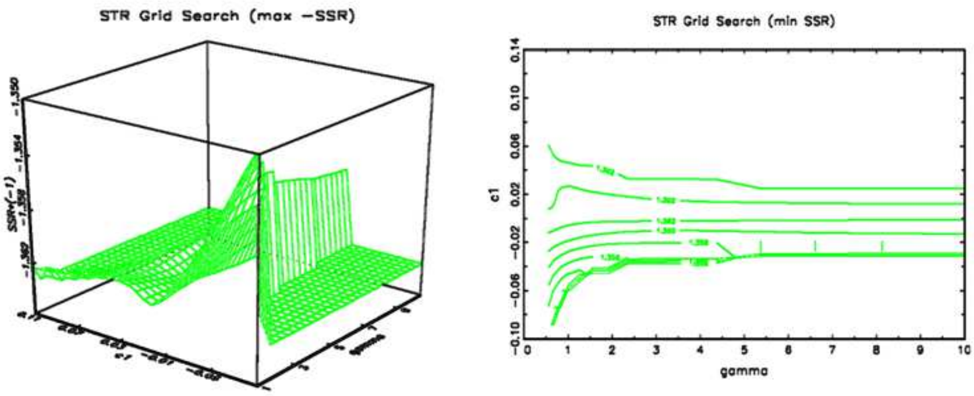
Table 6 – The linearity tests and Grid search results for the specification of the STAR model

Transition variable	*F*	*F4*	*F3*	*F2*	Suggested model	γ	*c*	*SSR*
r_{t-1}*	0.0000*	0.0003*	0.0000*	0.0682	LSTAR2	0.7558	-0.0740	0.0990
r_{t-2}	0.00001*	0.0000*	0.3504	0.6342	LSTAR1	1.3603	0.5544	1.3513

Notes: *γ is the rate of transition. c is the threshold variable. SSR: Sum of squared residuals. F is the fisher statistic.*

There is clear evidence that the linearity is clearly rejected in three out of four cases. The strongest evidence of LSTAR nonlinearity occurs when r_{t-1} is used as the transition variable. However, the grid search results show that SSR is minimized when r_{t-1} is considered as the switching variable (see also Figure 3).

Figure 3 – Graphical representation of grid search for start values



The results of the LSTAR model estimation by the Gauss-Newton are shown in Table 7. The results suggest that daily stock returns are generated by a nonlinear process. Some coefficients of linear and nonlinear part (Figure 5) are significantly different from zero. We also find that the rate of transition and the threshold variable in the transition function (Figure 4) are highly significant because the student statistics are greater than the critical value at significance level 5%. These last results confirm the nonlinearity in the conditional mean.

After estimating the model parameters, we evaluate it using misspecification tests. We focus here on testing serial dependence in the residuals and the ARCH effect. We note that the residuals (Figure 6) are not characterized by a Gaussian distribution and are leptokurtic (Figure 7). The asymmetry may indicate the presence of nonlinearities in the residuals or the squared residuals. However, these residuals can be modeled by GARCH models because the presence of an ARCH effect is confirmed by the result of the ARCH-LM test on LSTAR residuals ($nR^2 = 263.612 > \chi^2(1)$). Furthermore, as seen in Table 8, the series of the LSTAR residuals show strong nonlinear dependencies where the BDS statistics are strictly greater than the critical value1.96 for all the embedding dimensions.

Table 7– Estimation results of LSTAR model by nonlinear OLS

Parameters	LSTAR	
	Estimates	t-statistics
$\widehat{\phi}_{10}$	-0.04944	-0.8104
$\widehat{\phi}_{11}$	-0.32429	-1.4184
$\widehat{\phi}_{12}$	-0.8729	-2.474
$\widehat{\phi}_{20}$	0.0497	0.8104

$\widehat{\phi}_{21}$	1.0187	1.4578
$\widehat{\phi}_{22}$	0.8829	2.4794
γ	0.8097	5.3813
c	-0.0739	-7.6421
JB statistic	5437.8527 (0.000)	
Skewness	-0.1542	
Kurtosis	7.2063	
ARCH(8)	263.612 (0.000)	

Notes:*JB: Jarque-Bera statistic.* $\widehat{\phi}_{10}, \widehat{\phi}_{11}, \widehat{\phi}_{12}$ *are the parameters of linear part.* $\widehat{\phi}_{20}, \widehat{\phi}_{21}, \widehat{\phi}_{22}$ *are the parameters of nonlinear part.*

Figure 4 – Transition function of LSTAR model versus diff_ln_w_1 (upper panel) and over time (lower panel)

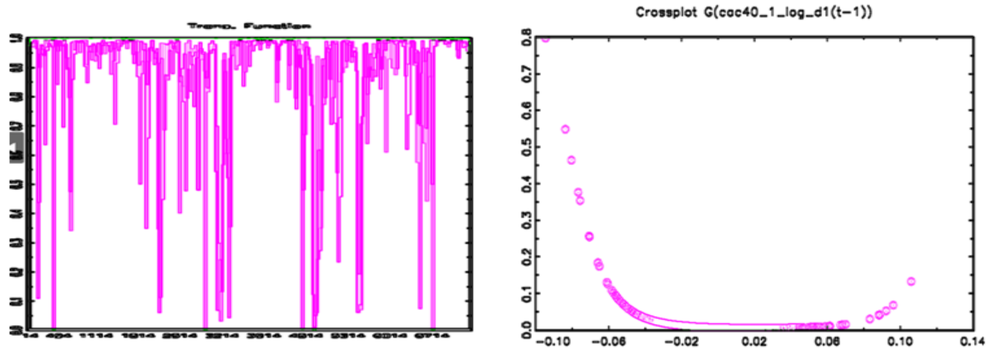


Figure 5 – Linear and nonlinear parts

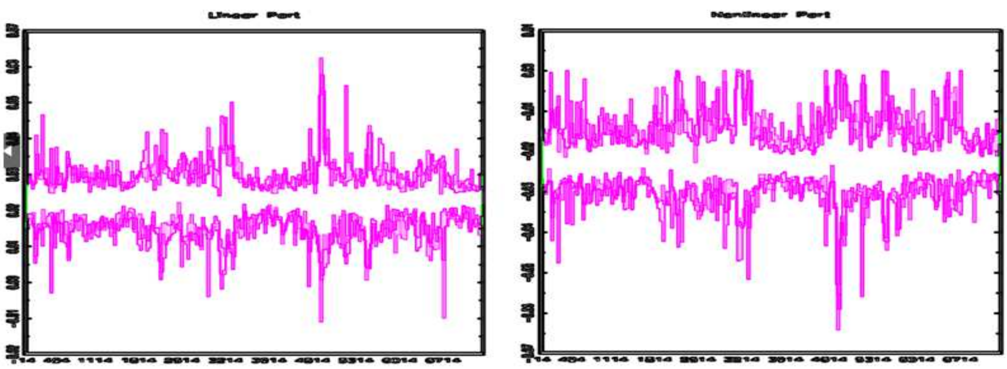


Figure 6 – LSTAR residuals

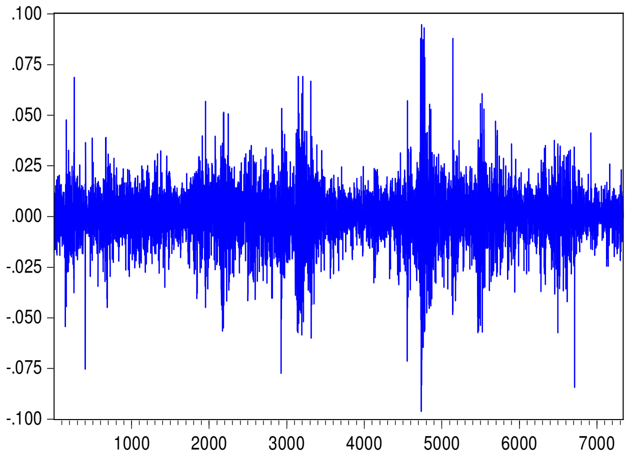


Figure 7 – Kernel estimation of density

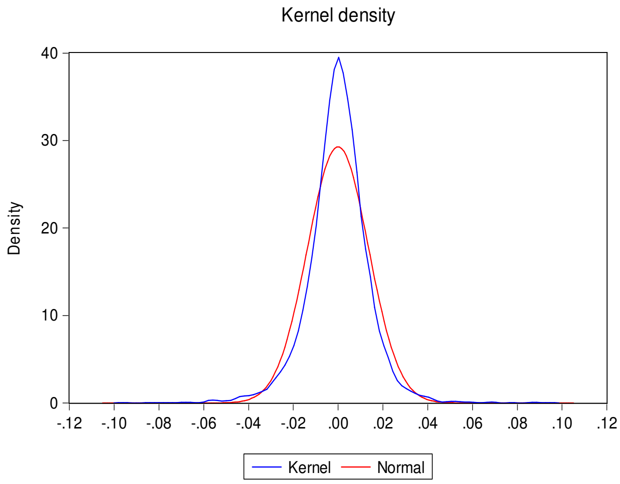


Table 8 – BDStest resultsonLSTAR residuals

Dimension	BDS Statistic	Prob.
2	14.1960	0.0000
3	20.1338	0.0000
4	24.9032	0.0000
5	28.5496	0.0000
6	31.8905	0.0000
7	35.4227	0.0000
8	39.2732	0.0000
9	43.4164	0.0000
10	47.9529	0.0000

Notes: *The BDS statistics are calculated by the fraction of pairs method with ε equal to 0.7. m represents theembedding dimension.*

It is likely that the conditional variance is characterized by a nonlinear structure. The financial asset prices often exhibit nonlinear heteroscedastic behavior. For this reason, we first test the GARCH specification against the alternative of LSTGARCH. Table 9 shows that the volatility of the CAC 40 returns series is adequately captured by the Logistic Smooth Transition GARCH-type model. The values of the critical probabilities reported in Table 9 argue in favor of an LSTGARCH model. At this stage, we will study the conditional variance of CAC 40 returns by combining LSTAR model with LSTGARCH errors using nonparametric maximum likelihood. The nonlinear models with LSTGARCH errors provide a flexible class of model to describe the nonlinear dynamics in both the conditional mean and the conditional variance.

Table 9 – LM test for GARCH against the alternative of LSTGARCH

Model	LM
GARCH	0.0988 (0.7802)
LSTGARCH	298.385 (0.000)

Notes: *(.): The critical probabilities.*

Table 10 – Semiparametric estimation using BHHH algorithm

Parameters	AR-GARCH	LSTAR-GARCH	LSTAR-LSTGARCH
$\widehat{\phi}_{11}$	0.0447 (3.5466)	0.0803 (4.1937)	0.0807 (1.5339)
$\widehat{\phi}_{12}$	-0.0254 (-2.1878)	0.0988 (1.9787)	-0.2464 (-2.1050)
$\widehat{\phi}_{21}$	-	2.4454 (1.3424)	-0.0696 (-1.1426)
$\widehat{\phi}_{22}$	-	5.5373 (1.9849)	0.2647 (2.3089)
$\widehat{\gamma}_{mean}$	-	0.0048 (2.5750)	2.6857 (2.7420)
$\widehat{c}_{mean}$	-	8.3744 (3.4476)	0.4608 (2.5078)
$\widehat{\omega}_1$	0.0279 (6.1009)	0.0276 (7.3094)	0.0130 (0.6663)
$\widehat{\alpha}_1$	0.0921 (10.5674)	0.0915 (14.6011)	0.0541 (7.7581)
$\widehat{\beta}_1$	0.8936 (91.3457)	0.8943 (127.3119	0.8809 (12.9342)
$\widehat{\omega}_2$	-	-	0.0105 (0.0825)
$\widehat{\alpha}_2$	-	-	0.0073 (6.3264)
$\widehat{\beta}_2$	-	-	0.9146 (8.3100)

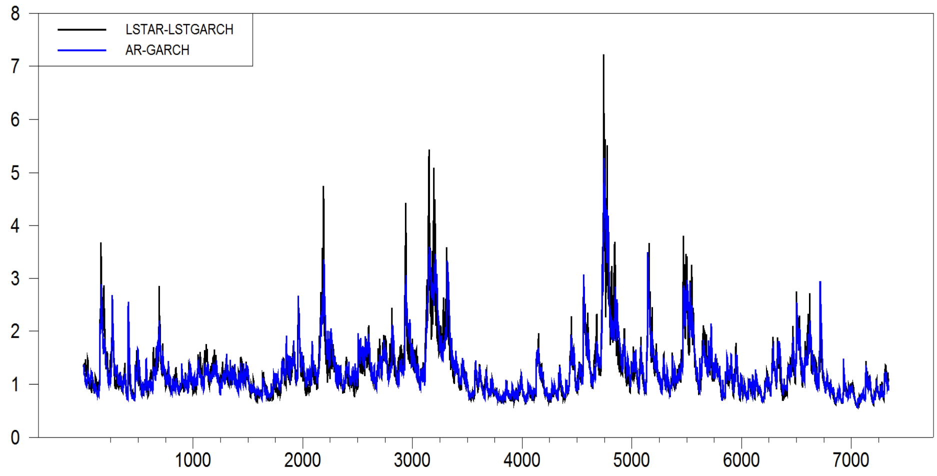
$\widehat{\gamma}_{\mathrm{vol.}}$	-	-	0.1817 (2.3796)
$\widehat{c}_{vol.}$	-	-	1.5726 (3.4263)
L	-11689.4394	-11686.2592	-11542.1713

Notes: *(.): The t-statistics.* $\widehat{\phi}_{11}, \widehat{\phi}_{12}, \widehat{\phi}_{21}, \widehat{\phi}_{22}$ *are the estimated STAR parameters.* $\widehat{\gamma}_{mean}$ *is the estimated rate of transition (STAR).* $\widehat{c}_{mean}$ *represents the estimated threshold value of STAR model.* $\widehat{\omega}_1, \widehat{\alpha}_1, \widehat{\beta}_1, \widehat{\omega}_2, \widehat{\alpha}_2$ *,* $\widehat{\beta}_2$ *are the estimated LSTGARCH parameters.* $\widehat{\gamma}_{\mathrm{vol.}}$ *is the estimated rate of transition (LSTGARCH).* $\widehat{c}_{vol.}$ *is the estimated threshold value of LSTGARCH model. L represents the estimated likelihood function.*

In view of Table 10, we find that the likelihood function is at maximum for the LSTAR-LSTGARCH model and the coefficients of this model are generally significant. In addition, the rate of transition and the threshold value in both the logistic smooth transition autoregressive and logistic smooth transition GARCH are significantly different from zero. The estimation results confirm that the conditional variance, which captures the heterogeneous and the volatility clustering is characterized by a nonlinear dynamics with regime switching behavior. It is shown that the GARCH parameter of nonlinear part is positive and statistically significant. this implies that positive shocks (appreciation) produce high volatility than negative shocks (depreciation) of the same magnitude. The parameter of linear part is positive and statistically significant. It means that the model manages to capture the temporal dependence of the conditional variance. In addition, the sum of GARCH parameters in both the linear and the nonlinear part is less than 1.There is still volatility clustering indicating support for asymmetry. Thus, we find that a negative shock increases the conditional variance more than a positive shock of the same magnitude. This highlights the asymmetric effect of unexpected shocks on conditional volatility. The transition parameter is generally quite high. This implies, in principle, that the speed of adjustment with respect to the equilibrium is faster. On the other hand, the coefficient of the threshold delay in the linear part of LSTAR is positive and that of nonlinear part is negative. These two parameters determine whether the weak and wide deviations have returned to the mean and the inclusion of transaction costs suggests that the large deviations of the long-run equilibrium, have a strong tendency to deviate from the equilibrium. The stock price will tend to move to the average price over time (mean-reversion). The LSTAR-STGARCH model is stable overall.

Figure 8 shows higher volatility persistence of LSTGARCH. When the level of the true conditional standard deviation changes, the LSTGARCH switches from the low-volatility (high-volatility) state to the high-volatility (low-volatility) state. Hence, the LSTGARCH model is more flexible than the GARCH model in accommodating different sizes of shocks.

Figure 8 – Comparison of estimates of conditional standard deviations



In order to evaluate the forecasting performance of fitting LSTAR-LSTGARCH Model in French stock market, we use the mean square error (MSE) and the mean absolute error (MAE). Table 11 contains statistical comparisons of out-of-sample forecasts provided by the AR-GARCH, LSTAR-GARCH, LSTAR-LSTGARCH and the random walk models. It is observed that MSE and MAE criteria generally give the same results. We find that the three models outperform the random walk model in all forecasting time horizons. The LSTAR-LSTGARCH model tend to have better predictive results comparing to LSTAR-GARCH and AR-GARCH in 2, 10, 20, 30 days. The LSTAR-LSTGARCH is beaten by the LSTAR-GARCH only in one day. Moreover, values of MSE and MAE criteria increase with horizons of 20 and 30 because all the models take into account the short-term memory in the conditional mean equation and the conditional volatility, considering that the criteria increase with the long prediction horizons. In other words, the predictive power for daily CAC 40 returns reflects the impossibility to forecast up to the longest horizon.

Table 11 – Out-of-sample forecast statistics

	Horizon	Criteria	AR-GARCH	LSTAR-GARCH	LSTAR-LSTGARCH	Random Walk
Conditional mean (Returns)	1 day	MSE	0.00067	0.00056*	0.00062	0.0512
		MAE	0.08124	0.07883*	0.07892	0.1295
	2 days	MSE	0.00091	0.00087	0.00075*	0.0725
		MAE	0.02536	0.02212	0.02200*	0.1485
	10 days	MSE	0.00050	0.00046	0.00035*	0.1618
		MAE	0.05765	0.05689	0.05676*	0.2481
	20 days	MSE	0.00085	0.00081	0.00079*	0.2284
		MAE	0.05835	0.05743	0.05731*	0.3078
	30 days	MSE	0.00174	0.00163	0.00119*	0.2814
		MAE	0.06341	0.06332	0.06225*	0.3954
Conditional variance (Volatility)	1 day	MSE	0.00052	0.00048	0.00036*	-
		MAE	0.07206	0.07177	0.07165*	-
	2 days	MSE	0.00045	0.00029*	0.00041	-
		MAE	0.06635	0.06559*	0.06571	-
	10 days	MSE	0.00043	0.00038	0.00027*	-
		MAE	0.05642	0.05601	0.05589*	-

20 days	MSE	0.00090	0.00086	0.00077*	-
	MAE	0.08633	0.08512	0.08503*	-
30 days	MSE	0.00145	0.00125	0.00108*	-
	MAE	0.10334	0.09841	0.09132*	-

In order to test the statistical significance of the forecasting improvements of LSTAR-LSTGARCH predictions over the LSTAR-GARCH on one hand and the random-walk on the other hand, we can use also the tests based on the asymptotic test, the sign tests, the Wilcoxon's test and the Morgan-Granger-Newbold test (Diebold & Mariano (1995)). The null hypothesis is the equal predictive accuracy of the two models. The results are reported on Tables 12.

Table 12 - Comparing predictive accuracy: Diebold-Mariano test

Test of equal accuracy	S_1	S_2	S_3	MGN
LSTAR-LSTGARCH versus LSTAR-GARCH	- 1.62 (0.10)	- 13.05 (0.00)	- 5.77 (0.00)	- 20.62 (0.00)
LSTAR-LSTGARCH versus Random walk	- 0.63 (0.10)	-7.11 (0.00)	- 5.69 (0.00)	- 10.18 (0.00)

The p-values are given in parentheses. S_1: Asymptotic test statistic, S_2: Sign test statistic, S_3: Wilkoxon test statistic, MGN: Morgan-Granger-Newbold test statistic,. A positive (negative) sign of the statistics implies that model B dominates (is dominated by) model A.. The prediction horizon used is 30. These tests are based on absolute forecast errors.

As seen in Table 12, the p-values clearly indicate that the null hypothesis of equal accuracy of the three models is strongly rejected. It is observed that different predictive accuracy are accepted because the p-values are less than 0.05, it means that, in this case, the LSTAR-LSTGARCH model beat the LSTAR-GARCH and the random walk process. The Diebold-Mariano statistics are, in most cases, significant, meaning that there is a difference in the forecasts computed from the LSTAR-GARCH and LSTAR-LSTGARCH models. A negative sign of the statistics implies that LSTAR-GARCH model is dominated by LSTAR-LSTGARCH model. The nonlinearity effects detected on volatility seem to improve the volatility forecasts. Indeed, the sign of the statistics is negative, implying that the nonlinearities observed on volatility provide a better volatility forecast.

Given that the daily CAC 40 returns are characterized by the presence of nonlinear dynamics in the equations of the mean and by the asymmetric effects in the conditional volatility, the LSTAR-LSTGARCH modelling allows computation of better forecasts than the other models and the random walk. The returns are short-term predictable. The agents cannot anticipate their returns to a long time horizon. Indeed, the observed movements appear as the result of transitory shocks, which affect the Paris stock market. The CAC 40 returns will come back to their previous fundamental value and the shock will be persistent in the short term. This suggests that it will be possible *a priori* to establish remunerative strategies on the Paris stock market. In addition, the series is characterized by the existence of nonlinearities in the volatility. Consequently, there is an asymmetric impact of positive and negative information on the level of future variance and the weak efficiency assumption of financial markets seems violated for daily CAC 40 returns.

4. Concluding Remarks

We investigated the presence of nonlinearities in the CAC 40 returns. We proposed a semiparametric estimation for LSTAR with LSTGARCH errors. We implemented the nonparametric maximum likelihood method to estimate exactly this class of models by taking into account the phenomenon of persistence and nonlinearity for the conditional variance. From the results, informational shocks have transitory effects on volatility and the LSTAR-LSTGARCH model shows a clear superiority over the AR-GARCH and LSTAR-GARCH models for short horizons. Specifically, the forecasts of the logistic smooth transition model show a clear improvement compared to the random walk model at all horizons; consequently, low efficiency of financial markets seems violated for the CAC 40 returns studied over a short period. Thus, recent works on semiparametric modeling through LSTGARCH process may provide new evidence to better understand the nonlinear dynamics and the asymmetric character of financial series.

The agents have heterogeneous behaviors that vary according to their initial endowments, their individual constraints and their usual activities. In addition, transaction costs are not only variable from one agent to another and based on transaction orders, but they can also define specific thresholds for each investor. The LSTAR-LSTGARCH model can reproduce the regime-switching behavior in the presence of heterogeneous transaction costs and distinct expectations of agents. The smooth transition between regimes can be attributed to the transaction volumes and heterogeneity of investor expectations.

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