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The limits to robust monetary policy in a small open economy with learning agents

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Abstract

We study in a small open economy New Keynesian model the consequences of adaptive learning for the design of optimal robust monetary policy. Compared to the rational expectations equilibrium, we find that the possiblity to conduct robust monetary policy is extremely limited in the open economy when private agents are learning. The misspecification that can be introduced into all equations of the model is very small and approaches zero at high speed as the learning gain rises.

Keywords: Robust control, model uncertainty, adaptive learning, optimal monetary policy, small open economy.

JEL Classification: C62, D83, D84, E52, E58.

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1 Introduction

The central bank (CB) conducts policy in a rapidly evolving macroeconomic environment affected by many national and international disturbances. The CB could have a limited understanding of many key features of this environment while private agents may not well understand how the complex economy functions as a whole.

The limited knowledge the CB and private agents have on the macroeconomic environment implies a great challenge for the conduct of monetary policy. The optimal monetary policy designed by assuming that all agents perfectly know the structure of the economy might not perform well in practice. Recent research suggests that the design of optimal monetary policy should carefully take into account the limited knowledge of the economy through a robust control approach and/or modeling the learning behavior of private agents. As highlighted by Schmidt-Hebbel and Walsh (2009), a key lesson learned from the research on monetary policy is that neither uncertainty nor learning can be ignored.

To deal with uncertain macroeconomic environment, policy makers should determine the model that is best describing the evolution of the economy and what the main drivers are as it changes rapidly over time. The robust control approach introduced by Hansen and Sargent (2001, 2003, 2007) gives the tools to design policy that would be robust to plausible deviations from the benchmark model such as the New Keynesian model (Clarida, Galí and Gertler 1999). Without a complete description of reality, central bankers are more inclined to base policy on principles that remain valid even if the model's assumptions are not well founded. In this scope, the optimal monetary policy is designed to perform well in worst-case scenarios by minimizing the consequences of misspecification in the policymaker's reference model. Generally, optimal interest rate policy is more aggressive under the robust control approach in both closed and open economy (Leitemo and Söderström 2008 a,b). This approach is related to (but is different from) the seminal contribution of Brainard (1967) who considers the impact of parameter uncertainty and advocates that the CB should be cautious by using less each policy instrument following an attenuation principle.[1]

Substituting the rational expectations (RE) hypothesis by the assumption that private agents use adaptive learning algorithms better reflects their limited knowledge of the economy. Molnár and Santoro (2014), and André and Dai (2017) have shown that adaptive learning gives rise to an intertemporal trade-off for the CB, leading monetary policy decisions and economic dynamics to radically differ from those under RE. One important conclusion found by André and Dai (2017) is that under learning, the CB should be liberal instead of being conservative. André and Dai (2018) have examined in a closed economy the design of optimal robust monetary policy when private agents are learning and find that learning significantly limits the possibility for the CB to conduct robust policy compared to RE.

This paper examines optimal robust discretionary monetary policy in a small open economy New Keynesian model with private agents who form expectations using adaptive learning algorithm. The aim of the paper is to study how adaptive learning affects the optimal robust monetary policy in the worst-case model. The robust control approach focuses on the worst-case scenario within a set of admissible models as economic agents are not able to attach probabilities to all plausible outcomes, which is translated into the presence of misspecification in the Phillips curve in the closed economy. In the open economy, the CB fears misspecification not only in the Phillips curve but also in the IS equation and the uncovered interest rate parity (UIP) because any shocks will affect the intratemporal trade-off through the presence of the exchange rate in the Phillips curve. Consequently, the equilibrium values of inflation, the output gap, the exchange rate and the interest rate depend on all sorts of shocks and model misspecification in all equations.

Our main results are: 1) Compared to the RE equilibrium, the possibility to conduct robust monetary policy is extremely limited in the open economy when private agents are

[1] The attenuation principle is also named the "conservatism principle" by Blinder (1998). According to this principle, the CB has to be cautious based on the fact that the choice of the instrument can have more severe consequences than in the absence of parameter uncertainty. Researchers who study robust monetary policy have reversed the meaning of "cautious" so that "being cautious or precautionary" signifies "to do more". In other words, the CB struggles to avoid worst outcomes by responding more aggressively to shocks (Söderström 2002, Gianonni 2007).

learning; 2) The misspecification that can be introduced into all equations of the model is very small and approaches zero at high speed as the learning gain rises.

The small open economy New Keynesian model used in this paper is based on Galí and Monacelli (2005) and Leitemo and Söderström (2008b). Regarding the assumptions used and the main results obtained, our paper is closely related to two strands of literature. The first is the literature on optimal robust monetary policy under RE and the second studies the implications of adaptive learning for the design of monetary policy.

The robust control approach adopted in this paper considers additive model misspecification. Taking worst-case scenarios into consideration, the CB tends to amplify rather than attenuate the response of optimal policy to shocks in a closed economy (e.g., Giannoni and Woodford 2002, Onatski and Stock 2002, Giordani and Söderlind 2004, Leitemo and Söderström 2008a, and Gonzalez and Rodriguez 2013). The fact that the central banker is more aggressive to avoid particularly costly outcomes triggers inflation persistence (Qin, Sidiropoulos and Spyromitros 2013). Such persistence justifies appointing a dovish central banker if he has a greater concern about misspecification of the Phillips curve (Dai and Spyromitros 2010). However, if the CB fears misspecification in the true degree of shock persistence orthe potential output (Tillmann 2009, 2014), it should be more hawkish.

An alternative approach to robustness is to consider multiplicative Knightian uncertainty implying that the uncertainty is located in one or more specific parameters of the model, and the true values of these parameters are bounded between minimum and maximum plausible values (Giannoni 2002, 2007, Onatski and Stock 2002, and Tetlow and von zur Muehlen 2004). Numerical simulations show that under parameter uncertainty, the robust interest rate rule generally reacts more strongly to changes in inflation and the output gap, with greater inertia than in the absence of such uncertainty. The CB is less cautious than in Brainard's model, as it fears worst-case scenarios.

A growing number of studies in the literature on monetary policy shares with our paper the assumption of adaptive learning (Evans and Honkapohja 2009). The main motivation for incorporating such an assumption is that the RE hypothesis is unrealistic because it requires private agents to be highly skilled in collecting and processing data, and understanding the structure of the economy, particularly when the economic environment is uncertain. As Bernanke (2007) has stressed, the traditional RE model of inflation and inflation expectations is less helpful for thinking about an economy whose structure is constantly evolving in ways that are imperfectly understood by both the public and policymakers. Moreover, model uncertainty makes it even more difficult for private agents to properly forecast how economic variables evolve, providing thus a stronger rationale for learning behavior. However, adopting learning assumption does not mean that we put into question the rationality of private agents but suggests the latter is limited. It is acknowledged that DSGE models with learning assumption outperform these with the RE hypothesis (Slobodyan and Wouters 2012, Ormeño and Molnár 2015).[2]

The emergence of learning as a working hypothesis raises a tremendous challenge for monetary authorities because when private agents are learning, the optimal monetary policy designed with the RE hypothesis could perform poorly (Milani 2008, and Orphanides and Williams 2008). When monetary policy is conducted through exogenous Taylor rules, it is shown that learning helps selecting among all possible equilibria obtained under RE, and in this sense it can be viewed as a process that converges towards RE equilibrium (Bullard and Mitra 2002, Evans and Honkapohja 2003, 2006, Machado 2013, Airaudo, Nisticò and Zanna 2015). An advantage of learning is that it solves the disinflationary-booms anomaly in the New Keynesian model under RE (Moore 2016). By considering optimal monetary policy decisions, our paper is closely linked to Molnár and Santoro (2014) who investigate optimal discretionary monetary policy when agents are learning in the benchmark New Keynesian model, and Evans and Honkapohja (2006) and Mele, Molnár and Santoro (2014) who study the implications of learning for monetary policy under commitment.

[2] The assumption of adaptive learning is supported by empirical studies. According to Trehan (2011), Trehan and Lynch (2013), and Ormeño and Molnár (2015), consumers and firms react sluggishly to persistent shifts in the inflation rate, suggesting that they slowly adapt their inflation forecast.

The remainder of the paper is structured as follows. Section 2 outlines the model. Section 3 derives equilibrium solutions under RE. Section 4 explores the effects of constant-gain learning on robust monetary policy and the equilibrium. Section 5 concludes.

2 The model

We use a New Keynesian model of a small open economy similar to the one derived by Galí and Monacelli (2005) and Leitemo and Söderström (2008b) for the baseline. We consider following Leitemo and Söderström the robust control problem of the CB that conducts policy under discretion, implying that the CB fears misspecification in all structural model equations. As Leitemo and Söderström, we also add a time-varying premium on foreign bond holdings. This time-varying premium is an important source of uncertainty in open economies and represents misspecification (specification errors) in the terminology of Hansen and Sargent (2001) in the UIP condition. The optimal monetary results from a sequential Nash game between the CB conducting robust policy to minimize the social loss and the nature (or malevolent agent) determining the level of model misspecification to maximize the social loss.[3]

2.1 The structural equations

The small domestic country freely trades with the rest of the world (foreign country), constituted of a continuum of foreign economies. We assume that foreign and domestic countries share preferences and technology. Domestic and foreign firms produce traded consumption goods, using labor as the sole input. Households derive their utility from consuming both domestic and foreign goods, and have a marginal decreasing disutility in labor supply to firms.

[3]Alternatively, the CB and the malevolent agent can play a Stackelberg game with the first acting as a Stackelberg leader. Notice that if the malevolent agent is the Stackelberg leader, the CB could adjust its policy according to the scenario designed by the malevolent agent (Hansen and Sargent, 2003). It results that the approach in terms of model misspecification would lose its interest.

Denote by e_t the log-linearized real exchange rate, defined in terms of domestic-price level. We have by definition

$$e_t = s_t + p_t^f - p_t, \tag{1}$$

with s_t being the nominal exchange rate, p_t^f the price level of the goods produced in the foreign country and p_t the price level of domestically produced goods.

The real exchange rate is directly related to the inflation rate in the domestic goods sector, π_t,[4] via the New Keynesian Phillips curve:[5]

$$\pi_t = \beta E_t^* \pi_{t+1} + \kappa x_t - \phi e_t + h_t^\pi + \varepsilon_t^\pi, \tag{2}$$

where x_t denotes the output gap representing the log deviation of the flexible-price equilibrium level of domestic output from the steady-state output, $0 < \beta < 1$ the discount factor, and the expectation operator E_t^* stands for private agents' expectations conditional on information set available at time t with the asterisk signaling that these agents may form rational expectations or not. The composite parameter κ is the output-gap elasticity of inflation and encompasses the effects of the output gap on real marginal costs and thus on inflation. We have $\kappa = \hat{\kappa}(\eta + \sigma)$, with η representing the elasticity of the representative household's labor supply, and $\hat{\kappa} \equiv \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}(1+\varphi)$. Here, φ reflects the inverse of steady-state Frisch elasticity of labor supply and ϑ the share of firms that do not optimally adjust but simply update in period t their previous price by the steady-state inflation rate. The real exchange rate affects the Phillips curve via real marginal costs with a composite coefficient $\phi = -\frac{\hat{\kappa}\omega(1-\omega)(1+\varphi)[1-(2-\omega)\zeta\sigma]}{\vartheta}$.[6] When households choose labor supply, they care

[4]Note that π_t is different from the inflation rate of the consumer price index that also takes into account the inflation of foreign goods consumed by residents. In the closed economy, π_t represents both producer and consumer price inflation rates.

[5]For the microfoundations of the model, see Leitemo and Söderström (2008b).

[6]The composite parameter ϕ is positive as long as $(1-\omega) + (2-\omega)\omega\zeta\sigma > 1$. The latter is generally true according to Leitemo and Söderström (2008b). Notice that estimating a variant of the Phillips curve of Galí and Monacelli (2005), Mihailov, Rumler and Scharler (2011) found that inflation can be either positively or negatively correlated with the expected change in the real exchange rate with the coefficients ranging from -0.26 to 0.47 for different European countries.

about the purchasing power of their wage deflated by the consumer price index that also includes prices of imported goods, implying that the equilibrium wage depends on the real exchange rate. The noise $\varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$ is an *iid* cost-push shock. The term h_t^π represents the misspecification in the Phillips curve.

The New Keynesian IS equation is given by

$$x_t = E_t^* x_{t+1} - \sigma^{-1}(r_t - E_t^* \pi_{t+1}) - \delta \left(E_t^* e_{t+1} - e_t\right) + h_t^x + \varepsilon_t^x, \tag{3}$$

where r_t the nominal short-term interest rate. Notice that $\sigma \equiv \frac{\hat{\sigma}}{1-\omega}$ with $\hat{\sigma}$ representing the risk aversion of households, $0 \leq \omega \leq 1$ the share of foreign goods in domestic consumption, and δ a composite parameter defined by $\delta \equiv \frac{1}{\sigma}\left[\frac{\Omega}{(1-\omega)} - 1\right]$ with $\Omega \equiv (1-\omega)\left[(1-\omega) + (2-\omega)\omega\zeta\sigma\right]$, where ζ stands for the elasticity of substitution across domestic and foreign goods. The composite parameter δ is the elasticity of the output gap with respect to the expected change in the real exchange rate, reflecting the substitution effect induced by such a change on the demand of domestically produced goods.[7] We introduce an *iid* demand shock $\varepsilon_t^x \sim N(0, \sigma_x^2)$ and a term h_t^x denoting the misspecification in the IS equation.

Finally, we define the real UIP condition, where the expected rate of real depreciation is related to the real interest rate differential:

$$r_t - E_t^* \pi_{t+1} = E_t^* e_{t+1} - e_t + h_t^e + \varepsilon_t^e, \tag{4}$$

where foreign variables are set to zero; h_t^e denotes the misspecification in the UIP equation, and $\varepsilon_t^e \sim N(0, \sigma_e^2)$ an *iid* real exchange rate disturbance. When $\varepsilon_t^e > 0$, it means that investors require a positive risk premium on domestic bonds compared to foreign bonds.

Here, we consider the worst-case model where the CB sets the interest rate to minimize

[7] Note that Ω and δ are positive if $(1-\omega) + (2-\omega)\omega\zeta\sigma > 1$, which is shown by Leitemo and Söderström (2008b) to be typically satisfied.

its loss function while a fictitious malevolent agent in the sense of Hansen and Sargent (2007) selects the specification errors to maximize loss.[8] Such an agent represents the policy maker's worst fears about model misspecification.

The worst-case scenario is the outcome that the CB is most afraid of and against which it conducts robust policy. The model misspecification cannot arise independently of random noises that affect model equations and are positively dependent on the variance of such noises (Giordani and Söderlind 2004). This is because if the variance of the disturbance in one equation was null, then the misspecification would be detected at once. Therefore, the larger the variance of the disturbance, the larger the specification error that cannot be detected.

The key factor that differentiates the behavior of the open economy from that of the closed economy and hence their transmission mechanism of monetary policy is the presence or not of the real exchange rate in the Phillips curve.

In the present model, movements in the exchange rate negatively affect inflation, for a given output gap. An increase in the exchange rate (depreciation of domestic currency) raises domestic consumer prices and reduces the real wage for a given nominal wage. This incites households to supply less labor and enjoy more leisure. Meanwhile, the depreciation increases the relative price of foreign goods in terms of domestic goods and hence foreign and domestic demands for domestic products due to substitution effects. This pushes firms to hire more workforces to increase production.

The resulting disequilibrium in the labor market is corrected by an increase in nominal wages, leading to higher production costs and hence inflation of product prices. Given expected future exchange rate, a depreciation of domestic currency increases the output gap and has a negative effect on inflation since such depreciation will increase the aggregate demand for domestically produced goods that must be equal to national production. This

[8] An alternative approach is to consider the 'approximating model' (Hansen and Sargent 2007) postulating that while the policy rule and agents' expectations reflect the CB's preference for robustness, there is no model misspecification in the reference model that turns out to be correct.

constraint imposed in the Small Open Economy model of Leitemo and Söderström (2008b) implies that the price of domestic goods must be lower. A decrease in the inflation rate of domestic producer prices is needed given the parameter values of the model to ensure a higher real wage in the current period, allowing thus domestic firms to hire more workers and produce more goods.[9]

The fact that the transmission mechanism of monetary policy is qualitatively different from those of a closed economy crucially depends on the formulation of the open-economy Phillips curve. In Galí and Monacelli (2005), the open-economy model with perfect international risk sharing and a Phillips curve that does not incorporate the real exchange rate is isomorphic to the closed economy, so that all closed-economy results are qualitatively similar to those in the open economy.

Following Leitemo and Söderström (2008b), the present framework keeps the real exchange rate in the Phillips curve and assumes imperfect access to international capital markets. These features imply that both demand shocks in the IS equation and the risk premium shocks on foreign exchange become new sources of macroeconomic volatility that are not present in the closed economy.[10] They break the isomorphism result and justify the study of robustness against misspecification in both the IS equation and the UIP condition besides the misspecification in the Phillips curve.

[9] The negative relationship between inflation and the real exchange rate obtained by Leitemo and Söderström (2008b) is somewhat counter-intuitive. We notice that Walsh (1999), and Razin and Yuen (2002) among others obtain a positive relationship between these two variables. However, both types of relationships could find empirical justification since Mihailov, Rumler and Scharler (2011) found that inflation can be either positively or negatively correlated with the expected change in the real exchange rate. Here, to facilitate the comparaison with the resultts of Leitemo and Söderström, we posit that the sign associated with e_t in the Phillips curve is negative.

[10] In the close economy, demand shocks do not contribute to macroeconomic volatility when the CB conducts optimal monetary policy since their effect on the aggregate demand is fully offset by an adequate change in the policy interest rate. This is impossible in the open economy since such shocks affect the trade-off between inflation and the output gap through the exchange rate due to the latter's presence in the Phillips curve.

2.2 Monetary policy objectives

The CB is assumed to have the same preference for inflation and output-gap stabilization as the society, whose expected loss function is given by:

$$L_t^s = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left(\pi_{t+i}^2 + \alpha x_{t+i}^2 \right), \tag{5}$$

where $\alpha > 0$ denotes the relative weight assigned to the objective of stabilizing the output gap. For simplicity, we assume that inflation target is equal to zero. The overly ambitious output target, which is common in the Barro-Gordon framework, is also set to zero in (5). Thus, the discretionary monetary policy set to minimize social loss (5) would avoid an average inflation bias when private agents form RE.

Given the model specifications set by the malevolent agent, the CB designs the robust discretionary policy for the worst possible model within a given set of plausible models. The CB allocates a budget χ_j^2, $j = \pi,\ x,\ e$, to the malevolent agent, for the misspecification to be created in the Phillips curve, the IS equation and the UIP condition, respectively.

The specification errors, h_t^j, with $j = \pi,\ x,\ e$, monitored by the malevolent agent are subject to following budget constraints:

$$E_t \sum_{j=0}^{+\infty} \beta^t \left(h_{t+i}^j \right)^2 \leq \chi_j^2,\ \ j = \pi,\ x,\ e. \tag{6}$$

In the absence of robust control, $\chi_j = 0$ for all j.

Under discretion, the CB designs a robust monetary policy that takes account not only of different shocks affecting the economy but also of model misspecification. The optimal robust monetary policy is obtained by solving the min-max problem:

$$\min_{\pi_t, x_t, e_t, r_t} \max_{h_t^j} L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left(\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi {h_{t+i}^\pi}^2 - \theta^x {h_{t+i}^x}^2 - \theta^e {h_{t+i}^e}^2 \right), \tag{7}$$

subject to the misspecified Phillips curve (2), IS equation (3), and UIP condition (4), and the malevolent agent's budget constraints (6). The penalty parameter $\theta^j > 0$, with $j = \pi$, x, e, controls the preference for robustness. The higher are θ^j, the lower the preference for model robustness. The specification errors h_t^j, with $j = \pi$, x, e, are inversely proportional to θ^j. The absence of concern for robustness corresponds to the case where $\theta^j \to \infty$, implying that $h_t^j \to 0$. In the following, we assume for simplicity that the malevolent agent's budget constraints (6) are not binding.

2.3 Learning rules of private agents

Given the complexity and the uncertainty that characterize the economy, it is hard for private agents to know the actual law of motion (ALM) for inflation, the output gap, and the exchange rate such that they learn the latter's evolution using an algorithm.[11] Consequently, they recursively estimate a Perceived Law of Motion (PLM), i.e., a noisy steady state in the terminology of Evans and Honkapohja (2001), which is consistent with the law of motion followed by the CB under RE. Believing that the steady-state levels of endogenous variables only depend on *iid* exogenous shocks, private agents perceive their expected levels as constant and know that the conditional and unconditional expectations of these variables are identical. This rationalizes that private agents estimate these variables via sample means.

Private agents form their expectations using the following learning algorithms (Molnár and Santoro 2014):

$$E_t^* \pi_{t+1} \equiv a_t = a_{t-1} + \gamma_t (\pi_{t-1} - a_{t-1}), \tag{8}$$

$$E_t^* x_{t+1} \equiv b_t = b_{t-1} + \gamma_t (x_{t-1} - b_{t-1}), \tag{9}$$

$$E_t^* e_{t+1} \equiv z_t = z_{t-1} + \gamma_t (e_{t-1} - z_{t-1}), \tag{10}$$

where $\gamma_t \in (0, 1)$ is the learning gain that is assumed to be constant henceforth, i.e.

[11] The modern literature on learning algorithms was pioneered by Marcet and Sargent (1989) who studied the convergence to RE equilibrium when agents form expectations using least-squares learning. For a survey of the literature, see Evans and Honkapohja (2001).

$\gamma_{t+1} = \gamma_t = \gamma$.[12] It corresponds to the speed of integration of new data into current expectations. The learning algorithms (8)-(10) establish a positive relationship between a variable's expectation and its last period value. Given that the last period value of a variable depends on past cost-push, demand and exchange-rate shocks, these algorithms make the expectations of endogenous variables dependent on all past shocks.

3 Benchmark equilibrium with rational expectations

We shortly present the benchmark equilibrium that recapitulates, with some small modifications in notations, the solution of Leitemo and Söderström (2008b).

3.1 Optimal monetary policy decisions

The Lagrangian of the min-max problem for the CB is:

$$\begin{aligned}
\min_{\pi_t, x_t, e_t, r_t} \max_{h_t^j} \mathscr{L}_t^{CB} \quad = \quad & E_t \sum_{i=0}^{+\infty} \beta^i \left\{ \tfrac{1}{2} \left[\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi {h_{t+i}^\pi}^2 - \theta^x {h_{t+i}^x}^2 - \theta^e {h_{t+i}^e}^2 \right] \right. \\
& -\lambda_{1,t+i} \left[\pi_{t+i} - \beta E_t \pi_{t+1+i} - \kappa x_{t+i} + \phi e_{t+i} - h_{t+i}^\pi - \varepsilon_{t+i}^\pi \right] \\
& -\lambda_{2,t+i} \left[x_{t+i} - E_t x_{t+1+i} + \sigma^{-1} (r_{t+i} - E_t \pi_{t+1+i}) \right. \\
& \qquad \left. + \delta \left(E_t e_{t+1+i} - e_{t+i} \right) - h_{t+i}^x - \varepsilon_{t+i}^x \right] \\
& \left. -\lambda_{3,t+i} \left[e_{t+i} - E_t e_{t+1+i} + (r_{t+i} - E_t \pi_{t+1+i}) - h_{t+i}^e - \varepsilon_{t+i}^e \right] \right\}. \qquad (11)
\end{aligned}$$

Deriving (11) with respect to r_t, π_t, x_t, e_t, h_t^j with $j = \pi,\ x,\ e$ yields the first-order conditions (FOC) that can be arranged to obtain the intratemporal trade-off condition (the optimal

[12] An alternative approach is to assume that γ_t is decreasing over time. Compared to decreasing-gain learning, constant-gain learning generally yields a better analytical tractability of the model.

targeting rule) and relate all specification errors with inflation:

$$x_t = -\left[\frac{\kappa}{\alpha} - \frac{\sigma\phi}{\alpha(1+\sigma\delta)}\right]\pi_t, \tag{12}$$

$$h_t^{\pi} = \frac{1}{\theta^{\pi}}\pi_t = -\frac{\alpha(1+\sigma\delta)}{\theta^{\pi}\left[\kappa(1+\sigma\delta)-\sigma\phi\right]}x_t, \tag{13}$$

$$h_t^{x} = \frac{\sigma\phi}{\theta^{x}(1+\sigma\delta)}\pi_t, \tag{14}$$

$$h_t^{e} = -\frac{\phi}{\theta^{e}(1+\sigma\delta)}\pi_t. \tag{15}$$

The optimal interest rate is obtained using (4), (12) and (14)-(15) to eliminate x_t and h_t^x in (3).

$$r_t = \sigma\left[\Gamma + \frac{\sigma\phi}{\theta^{x}(1+\sigma\delta)}\right]\pi_t + (1-\sigma\Gamma)E_t\pi_{t+1} - \sigma\delta(E_te_{t+1}-e_t) + \sigma\varepsilon_t^x, \tag{16}$$

where $\Gamma \equiv \frac{\kappa}{\alpha} - \frac{\sigma\phi}{\alpha(1+\sigma\delta)} > 0$. Equation (16) is an optimal implicit instrument rule in the terminology of Giannonni and Woodford (2003). An increase in the CB's preference for robustness in the IS equation (a decrease in θ^x) implies a more agressive response to inflation. The CB's preference for robustness in the UIP equation affects the policy interest rate through the term $(E_te_{t+1}-e_t)$, which negatively depends on h_t^e that is inversely related to θ^e.

Substituting h_t^{π}, h_t^{x}, and h_t^{e} given by (13)-(15) into (3)-(4) respectively, and then using the resulting equations and equation (12) to obtain the ALMs for inflation, the output gap, the exchange rate and the interest rate under RE yield

$$\pi_t = \frac{(\delta+\sigma^{-1})(-\beta E_t\pi_{t+1} + \phi E_te_{t+1} - \varepsilon_t^{\pi}) - \phi E_tx_{t+1} - \phi\varepsilon_t^x + \phi\sigma^{-1}\varepsilon_t^e}{\triangle}, \tag{17}$$

$$x_t = \frac{\Gamma\left[(\delta+\sigma^{-1})(\beta E_t\pi_{t+1} - \phi E_te_{t+1} + \varepsilon_t^{\pi}) + \phi E_tx_{t+1} + \phi\varepsilon_t^x - \phi\sigma^{-1}\varepsilon_t^e\right]}{\triangle}, \tag{18}$$

$$e_t = \frac{V_1\beta E_t\pi_{t+1} + V_2E_tx_{t+1} - (\delta+\sigma^{-1})V_2E_te_{t+1} + V_1\varepsilon_t^{\pi} + V_2\left(\varepsilon_t^x - \sigma^{-1}\varepsilon_t^e\right)}{\triangle}, \tag{19}$$

$$r_t = \frac{\begin{array}{c} -\left\{(\beta-\phi)V_3+\left(\frac{1}{\sigma}V_2-\frac{\phi^2}{\sigma\theta^e}\right)\right\}E_t\pi_{t+1}-\left(V_2-\frac{\phi}{(1+\sigma\delta)}\frac{\phi}{\theta^e}\right)(E_tx_{t+1}+\varepsilon_t^x) \\ +\phi V_3E_te_{t+1}-V_3\varepsilon_t^\pi+\left[\phi V_3-\delta\left(V_2-\frac{\phi}{(1+\sigma\delta)}\frac{\phi}{\theta^e}\right)\right]\varepsilon_t^e \end{array}}{\triangle}, \quad (20)$$

where $\triangle \equiv -(\delta+\sigma^{-1})\left[1+\frac{\kappa^2}{\alpha}-\frac{1}{\theta^\pi}-\frac{\kappa\sigma\phi}{\alpha(1+\sigma\delta)}\right]+\phi\left[\frac{1}{\sigma}+\frac{\kappa}{\alpha}-\frac{\sigma\phi}{\alpha(1+\sigma\delta)}-\frac{\sigma\phi}{\theta^x(1+\sigma\delta)}\right]$, $V_1 \equiv \frac{\kappa}{\alpha}+\frac{\phi}{(1+\sigma\delta)}\left(\frac{\sigma^{-1}}{\theta^e}-\frac{\sigma}{\theta^x}-\frac{\sigma}{\alpha}\right)$, $V_2 \equiv 1-\frac{1}{\theta^\pi}+\kappa\left[\frac{\kappa}{\alpha}-\frac{\sigma\phi}{\alpha(1+\sigma\delta)}\right]$, and $V_3 \equiv \frac{\kappa}{\alpha}-\left(\frac{\sigma}{\alpha}+\frac{\sigma}{\theta^x}+\frac{\delta}{\theta^e}\right)\frac{\phi}{(1+\sigma\delta)}$. We have $\triangle < 0$, $V_1, V_2, V_3 > 0$, for standard parameters values calibrated by Galí and Monacelli (2005) and Leitemo and Söderström (2008b) , i.e., $\hat{\sigma} = \zeta = 1$, $\eta = 3$, ϑ=0.75, and $\omega = 0.4$, which imply $\kappa = 0.401$, $\phi = 0.057$, $\delta = 0.4$, $\sigma = 1.667$, $\alpha = 0.25$ and $\beta = 0.99$, and for values of θ^x and θ^e ensuring the dynamic stability of the economy. The ALMs (17)-(20) are obtained under a monetary policy regime where the CB does not take into account how private agents revise their beliefs.

To highlight the key differences, regarding the optimal trade-off condition and the CB's worst-case fears for misspecification, between monetary policy in open and closed economies, we compare equations (12)-(15) with the corresponding solutions in the closed economy in the following.

The optimal inflation-output trade-off in (12) is independent of the CB's preference for robustness. The term $\frac{\sigma\phi}{\alpha(1+\sigma\delta)}$ in the composite coefficient associated with π_t is due to the open-economy feature. If the coefficient on the exchange rate in the Phillips curve is null (i.e., $\phi = 0$), the targeting rule (12) is identical to that in the closed economy. In the latter, the optimal trade-off between π_t and x_t only depends on κ and α. If monetary policy has stronger effects on inflation through the output gap (higher κ) or if the CB has a stronger preference for inflation stabilization (smaller α), the CB reduces more aggressively the output gap as inflation rises. If $\phi > 0$, movements in the policy interest rate are transmitted to inflation through both the output gap and the exchange rate. There is an "exchange rate channel": an increase in the policy interest rate yields an appreciation of domestic currency (a decrease in e_t) and thus a higher inflation. As shown by Leitemo and Söderström (2008b), the term $\frac{\sigma\phi}{\alpha(1+\sigma\delta)}$ is less important than $\frac{\kappa}{\alpha}$ so that the openness of the economy does not change the sign of the slope of the Phillips curve but only makes it less steep. In other words, the exchange rate channel attenuates the effects of monetary policy through the demand channel and does not change the fact that the optimal monetary policy leans against the wind, i.e., reduces the output gap when inflation is high.

The worst-case specification errors given by equations (13)-(15) measured in absolute value increase with the deviation of inflation from its steady state value. These specification errors tend to push inflation even further away and thus force the CB to accept a stronger variation in the output gap to achieve the desired trade-off between inflation and the output gap.

In the closed economy, h_t^x is set to zero under optimal robust monetary policy because the policymaker is able to fully offset any misspecification in the IS equation by an appropriate adjustment of the policy interest rate. The CB does not fear such misspecification as long as it does not aim at interest-rate smoothing. In the open economy, the CB cannot entirely offset demand shocks via appropriate changes in the policy interest rate because of the latter's feedback on the exchange rate and hence inflation. Therefore, the exchange rate channel makes the CB fear about misspecification in the IS equation in the worst-case scenario.

Equation (14) indicates that when the interest-rate elasticity of the demand for domestic goods is high (σ is small), the CB can conduct policy to easily offset specification errors in the IS equation and hence worries less about h_t^x. In the opposite, equation (15) shows that higher interest-rate elasticity raises misspecification in the UIP equation. The CB experiences a growing difficulty to change the policy interest rate to counteract the inflationary effect of these specification errors that would have larger costly effects on output as σ decreases.

According to (14)-(15), a stronger effect of the exchange rate on inflation (larger ϕ) raises the costs of specification errors in both IS and UIP equations. Since the interest rate movements aiming at offsetting these errors have a stronger direct effect on inflation, the CB worries more about misspecification in these equations. Meanwhile, a higher interest-rate elasticity of the demand for domestic goods (larger δ) reduces the costs of specification errors in these equations, implying that IS and UIP equations are less prone to misspecification.

The complexity in the transmission mechanism of monetary policy introduced by the exchange rate channel implies that the CB in an open economy has to face a more difficult intratemporal trade-off than in the closed economy, and the design of monetary policy that is robust against model misspecification becomes more costly. Moreover, the scope for misspecification should be extended to both IS and UIP equations besides the Phillips curve.

3.2 The equilibrium of the worst-case model

The state variables are the shocks ε_t^π, ε_t^x and ε_t^e. The solution of the worst-case model with the method of undetermined coefficients (McCallum1983) is assumed to be function of state variables:

$$\begin{bmatrix} \pi_t \\ x_t \\ e_t \\ r_t \\ h_t^\pi \\ h_t^x \\ h_t^e \end{bmatrix} = \begin{bmatrix} d_\pi^{RE} & d_x^{RE} & d_e^{RE} \\ k_\pi^{RE} & k_x^{RE} & k_e^{RE} \\ j_\pi^{RE} & j_x^{RE} & j_e^{RE} \\ m_\pi^{RE} & m_x^{RE} & m_e^{RE} \\ \hat{d}_\pi^{RE} & \hat{d}_x^{RE} & \hat{d}_e^{RE} \\ \widehat{k}_\pi^{RE} & \hat{k}_x^{RE} & \widehat{k}_e^{RE} \\ \hat{j}_\pi^{RE} & \hat{j}_x^{RE} & \hat{j}_e^{RE} \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}. \tag{21}$$

Eliminating x_t and h_t^π in the Phillips curve (2) using the targeting rule (12) and equation (13), and substituting r_t and h_t^e given respectively by the optimal interest rate rule (16) and equation (15) into the UIP condition (4) yield

$$C\pi_t = \beta E_t\pi_{t+1} - \phi e_t + \varepsilon_t^\pi, \tag{22}$$

$$(1+\sigma\delta)\, e_t = (1+\sigma\delta)\, E_t e_{t+1} - D\pi_t + \sigma\Gamma E_t\pi_{t+1} - \sigma\varepsilon_t^x + \varepsilon_t^e, \tag{23}$$

where $C \equiv \theta^\pi(1+\Gamma\kappa) - 1 > 0$ and $D \equiv \left[\Gamma\sigma + \frac{\phi\sigma^2}{\theta^x(1+\sigma\delta)} + \frac{\phi}{\theta^e(1+\sigma\delta)}\right] > 0$ if θ^j with $j = \pi,\ x,\ e$

are sufficiently large (i.e., when the preference for robustness is sufficiently small).

Using the assumed solution of π_t and e_t given in (21) and the assumption that all shocks are serially uncorrelated, i.e., $E_t\varepsilon^{\pi}_{t+1} = E_t\varepsilon^{x}_{t+1} = E_t\varepsilon^{e}_{t+1} = 0$, it follows that $E_t\pi_{t+1} = d^{RE}_{\pi}E_t\varepsilon^{\pi}_{t+1} + d^{RE}_{x}E_t\varepsilon^{x}_{t+1} + d^{RE}_{e}E_t\varepsilon^{e}_{t+1} = 0$ and that $E_te_{t+1} = j^{RE}_{\pi}E_t\varepsilon^{\pi}_{t+1} + j^{RE}_{x}E_t\varepsilon^{x}_{t+1} + j^{RE}_{e}E_t\varepsilon^{e}_{t+1} = 0$. Substituting $E_t\pi_{t+1} = 0$ and $E_te_{t+1} = 0$ into (22)-(23), solving the resulting system of equations to obtain the solutions of π_t and e_t, and then comparing the latter with the assumed solution of π_t and e_t given in (21), we obtain

$$d^{RE}_{\pi} = \frac{(1+\sigma\delta)}{(1+\sigma\delta)C - \phi D},\ d^{RE}_{x} = \frac{\phi\sigma}{(1+\sigma\delta)C - \phi D},\ d^{RE}_{e} = -\frac{\phi}{(1+\sigma\delta)C - \phi D}, \tag{24}$$

$$j^{RE}_{\pi} = -\frac{D}{(1+\sigma\delta)C - \phi D},\ j^{RE}_{x} = -\frac{C\sigma}{(1+\sigma\delta)C - \phi D},\ j^{RE}_{e} = \frac{C}{(1+\sigma\delta)C - \phi D}, \tag{25}$$

where $(1+\sigma\delta)C - \phi D > 0$ for sufficiently large θ^j, with $j = \pi,\ x,\ e$, implying that there is a lower bound for the degree of model robustness that the CB can introduce into the model. Otherwise, i.e. θ^j are such that $(1+\sigma\delta)C - \phi D < 0$, inflation will decrease following a positive cost-push shock and this is counterfactual.

Using (12) and (24), we get the coefficients of the assumed solution of x_t:

$$k^{RE}_{\pi} = -\Gamma d^{RE}_{\pi} = -\frac{\Gamma(1+\sigma\delta)}{(1+\sigma\delta)C - \phi D}, \tag{26}$$
$$k^{RE}_{x} = -\Gamma d^{RE}_{x} = -\frac{\Gamma\phi\sigma}{(1+\sigma\delta)C - \phi D}, \tag{27}$$
$$k^{RE}_{e} = -\Gamma d^{RE}_{e} = \frac{\Gamma\phi}{(1+\sigma\delta)C - \phi D}. \tag{28}$$

Substituting the final solution of π_t, e_t, $E_t\pi_{t+1}$ and E_te_{t+1} into (16) yields

$$m^{RE}_{\pi} = \sigma\left[Bd^{RE}_{\pi} + \delta j^{RE}_{\pi}\right],\ m^{RE}_{x} = \sigma\left[Bd^{RE}_{x} + \delta j^{RE}_{x} + 1\right],\ m^{RE}_{e} = \sigma\left[Bd^{RE}_{e} + \delta j^{RE}_{e}\right], \tag{29}$$

where $B \equiv \Gamma + \frac{\sigma\phi}{\theta^x(1+\sigma\delta)}$. For sufficiently large θ^j, with $j = \pi, x, e$, i.e., when the CB's preference for robustness is sufficiently low, it is straightforward to show that $m^{RE}_{\pi} > 0$,

$m_x^{RE} > 0$ and $m_e^{RE} > 0$.

Equations (13)-(15) imply that the coefficients in the solution of misspecification h^j with $j = \pi$, x, e, in the CB's worst-case scenario are related to the coefficients in the final solution of inflation, the output gap and the exchange rate given in the above:

$$\hat{d}_j^{RE} = \tfrac{1}{\theta^\pi} d_j^{RE}, \quad (30)$$

$$\widehat{k}_j^{RE} = \frac{\phi\sigma}{\theta^x (1+\sigma\delta)} d_j^{RE}, \quad (31)$$

$$\hat{j}_j^{RE} = -\frac{\phi}{\theta^e (1+\sigma\delta)} d_j^{RE}. \quad (32)$$

The main results derived by Leitemo and Söderström (2008b) under rational expectations for the worst-case model can be summarized as follows.

First, according to (29), the policy interest rate positively responds not only to positive cost-push and output-gap shocks as it is well known in the closed economy but also to positive exchange-rate shocks. In the open economy, monetary policy is tightened to offset the additional inflationary pressure induced by positive realizations of exchange-rate shocks.

Second, the coefficients given in (24)-(25) show that equilibrium inflation rises (falls) following positive cost-push and demand (exchange-rate) shocks as we can expect from the original model without misspecification. Compared to the latter, the effects of these shocks are amplified by the specification errors (see equations (30)-(32)) and are partly offset through the optimal response of the policy interest rate (see equation (29)).

Finally, in the worst-case model, a stronger preference for robustness (against misspecification in any equation) increases the sensitivity of inflation, output, and the exchange rate to all shocks. This result arises from the CB's worst fears of misspecification and hence its fears of more volatile inflation, output gap, and exchange rate than in the reference model. Such fears are translated into the design of robust monetary policy and then into the equilibrium values of endogenous variables. This result stands in contrast to the one in the closed-economy model. Using (13)-(15), we can easily deduce that for $\phi = 0$, we have

$h_t^x = h_t^e = 0$ for $h_t^\pi \neq 0$ and $\pi_t \neq 0$. In the closed economy (equivalent to $\phi = 0$), the CB that desires to conduct robust policy fears only that inflation is more volatile than in the reference model, but not that the output gap is more volatile. This is because demand shocks do not modify a trade-off for the CB in the closed economy since the policy interest rate can be used as intensively as it is necessary to fully offset the effects of such shocks without affecting the CB loss. In the open economy, due to the presence of the real exchange rate in the Phillips curve, all shocks change the trade-off for monetary policy, inducing the CB to fear that all variables are more volatile compared to the reference model. Therefore, in the open economy, the CB should be more precautious than in the closed economy, and hence more aggressive when setting the policy interest rate.

4 The ALMs under learning

This section studies how constant-gain learning interacts with the conduct of monetary policy and affects the equilibrium compared to the misspecified benchmark model where private agents form RE. Since the worst-case closed economy model is extensively studied under learning by André and Dai (2018), this section focuses on the difference introduced by opening the economy.

4.1 The min-max problem and the manipulation of private expectations

The learning behavior of private agents gives rise to an intertemporal trade-off and hence leads the CB to embed expectations interactions in policy decisions. Solving for a discretionary monetary policy amounts to solving the Lagrangian of the min-max problem for the CB while substituting $E_t^*\pi_{t+1} = a_t$, $E_t^*x_{t+1} = b_t$ and $E_t^*e_{t+1} = z_t$ into (3)-(4). The CB's optimization problem is as follows:

$$
\begin{aligned}
\min_{\Psi}\max_{h_t^j} \mathscr{L}_t^{CB} \quad = \quad & E_t \sum_{i=0}^{+\infty} \beta^i \{ \tfrac{1}{2} \left[\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi {h_{t+i}^\pi}^2 - \theta^x {h_{t+i}^x}^2 - \theta^e {h_{t+i}^e}^2 \right] \\
& -\lambda_{1,t+i} \left[\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} + \phi e_{t+i} - h_{t+i}^\pi - \varepsilon_{t+i}^\pi \right] \\
& -\lambda_{2,t+i} \left[x_{t+i} - b_{t+i} + \sigma^{-1}(r_{t+i} - a_{t+i}) + \delta\left(z_{t+i} - e_t\right) - h_{t+i}^x - \varepsilon_{t+i}^x \right] \\
& -\lambda_{3,t+i} \left[e_{t+i} - z_{t+i} + (r_{t+i} - a_{t+i}) - h_{t+i}^e - \varepsilon_{t+i}^e \right] \\
& -\lambda_{4,t+i} \left[a_{t+1+i} - a_{t+i} - \gamma_{t+i}(\pi_{t+i} - a_{t+i}) \right] \\
& -\lambda_{5,t+i} \left[b_{t+1+i} - b_{t+i} - \gamma_{t+i}(x_{t+i} - b_{t+i}) \right] \\
& -\lambda_{6,t+i} \left[z_{t+1+i} - z_{t+i} - \gamma_{t+i}(e_{t+i} - z_{t+i}) \right] \}. \qquad (33)
\end{aligned}
$$

where $\Psi \equiv \{r_t,\, \pi_t,\, x_t,\, e_{t,},\, a_{t+1},\, b_{t+1},\, z_{t+1}\}$, $j = \pi,\, x\,, e$, and $\lambda_{n,t}$, with $n = 1,\ 2, ...6$ are Lagrange multipliers that are respectively associated with (2)-(4), and (8)-(10).

Differentiating the Lagrangian (33) with respect to r_t, π_t, x_t, e_t, a_{t+1}, b_{t+1}, z_{t+1}, h_t^π, h_t^x and h_t^e leads to the following FOCs

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial r_t} = 0 \;\Rightarrow\; -\sigma^{-1}\lambda_{2,t} - \lambda_{3,t} = 0, \qquad (34)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial \pi_t} = 0 \;\Rightarrow\; \pi_t - \lambda_{1,t} + \gamma\lambda_{4,t} = 0, \qquad (35)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial x_t} = 0 \;\Rightarrow\; \alpha x_t + \kappa\lambda_{1,t} - \lambda_{2,t} + \gamma\lambda_{5,t} = 0, \qquad (36)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial e_t} = 0 \;\Rightarrow\; -\phi\lambda_{1,t} + \delta\lambda_{2,t} - \lambda_{3,t} + \gamma\lambda_{6,t} = 0, \qquad (37)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial a_{t+1}} = 0 \;\Rightarrow\; -\lambda_{4,t} + \beta E_t\left[\beta\lambda_{1,t+1} + \sigma^{-1}\lambda_{2,t+1} + \lambda_{3,t+1} + \lambda_{4,t+1}\left(1-\gamma\right)\right] = 0, \qquad (38)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial b_{t+1}} = 0 \;\Rightarrow\; -\lambda_{5,t} + \beta E_t\left[\lambda_{2,t+1} + \lambda_{5,t+1}\left(1-\gamma\right)\right] = 0, \qquad (39)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial z_{t+1}} = 0 \;\Rightarrow\; -\lambda_{6,t} + \beta E_t\left[-\delta\lambda_{2,t+1} + \lambda_{3,t+1} + \lambda_{6,t+1}\left(1-\gamma\right)\right] = 0, \qquad (40)$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial h_t^{\pi}} = 0 \;\Rightarrow\; \lambda_{1,t} = \theta^{\pi} h_t^{\pi}, \tag{41}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial h_t^{x}} = 0 \;\Rightarrow\; -\theta^{x} h_t^{x} + \lambda_{2,t} = 0, \tag{42}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial h_t^{e}} = 0 \;\Rightarrow\; -\theta^{e} h_t^{e} + \lambda_{3,t} = 0. \tag{43}$$

The FOC (34) implies that $\lambda_{3,t+1} = -\sigma^{-1}\lambda_{2,t+1}$. Substituting $\lambda_{3,t+1}$ into (40) gives $-\lambda_{6,t} + \beta E_t\left[-(\delta+\sigma^{-1})\lambda_{2,t+1} + \lambda_{6,t+1}(1-\gamma)\right] = 0$. From the previous equation and (39), we find that a possible set of solutions for $\lambda_{5,t}$ and $\lambda_{6,t}$ verifies the following condition:

$$\lambda_{6,t} = -(\delta+\sigma^{-1})\lambda_{5,t}. \tag{44}$$

After having substituted $\lambda_{6,t}$ given by (44) into (37), and $\lambda_{3,t}$ by $\lambda_{3,t} = -\sigma^{-1}\lambda_{2,t}$, we deduce from the resulting equations and the FOC (36) that: $\lambda_{1,t} = -\frac{\alpha(1+\sigma\delta)}{\theta^{\pi}[\kappa(1+\sigma\delta)-\sigma\phi]}x_t$ and $\lambda_{5,t} = \frac{1}{\gamma}\lambda_{2,t} + \frac{\alpha\sigma\phi}{\gamma[\kappa(1+\delta\sigma)-\sigma\phi]}x_t$. Using the expression of $\lambda_{1,t}$ into (35) yields that $\lambda_{4,t} = -\frac{1}{\gamma}\pi_t - \frac{\alpha(1+\delta\sigma)}{\gamma[\kappa(1+\delta\sigma)-\sigma\phi]}x_t$.

We now look for the intertemporal trade-off condition implied by the FOC (34) and (38). Substituting $\lambda_{1,t}$ and $\lambda_{4,t}$ obtained in the above into (38) and using the FOC (34) to eliminate $\lambda_{2,t+1}$ and $\lambda_{3,t+1}$ lead to the intertemporal optimal trade-off condition for the CB between stabilizing inflation and the output gap in periods t and $t+1$:

$$\pi_t + \frac{\alpha(\delta+\sigma^{-1})}{[\kappa(\delta+\sigma^{-1})-\phi]}x_t = \beta(1-\gamma)E_t\pi_{t+1} + \frac{(\delta+\sigma^{-1})\left[\alpha\beta^2\gamma + \alpha\beta(1-\gamma)\right]}{[\kappa(\delta+\sigma^{-1})-\phi]}E_t x_{t+1}. \tag{45}$$

One major difference with the misspecified benchmark model with RE is that constant-gain learning introduces into the CB's decision process an intertemporal trade-off in addition to the intratemporal one that already exists under the RE hypothesis. This intertemporal trade-off is reflected by the terms associated with $E_t\pi_{t+1}$ and $E_t x_{t+1}$ at the right-hand side of (45).

Replacing $\lambda_{1,t} = -\frac{\alpha(1+\sigma\delta)}{\theta^{\pi}[\kappa(1+\sigma\delta)-\sigma\phi]} x_t$ into (41) yields for time t and $t+1$:

$$h_t^{\pi} = -\frac{\alpha\left(1+\sigma\delta\right)}{\theta^{\pi}\left[\kappa\left(1+\sigma\delta\right)-\sigma\phi\right]} x_t. \tag{46}$$

Notice that there is not a simple relationship between h_t^{π} and π_t under learning.

The FOCs (34) and (42)-(43) can be arranged to obtain $\frac{\lambda_{2,t}}{\lambda_{3,t}} = \frac{\theta^x h_t^x}{\theta^e h_t^e} = -\sigma$, implying

$$h_t^e = -\frac{\theta^x}{\sigma\theta^e} h_t^x, \tag{47}$$

meaning that the coefficients in the solution of h_t^e are proportional to those in the solution of h_t^x.

We now replace h_t^{π} given by (46) and $E_t^* \pi_{t+1} = a_t$ into the Phillips curve (2) to obtain

$$\pi_t = \beta a_t + \left[\kappa - \frac{\alpha\left(1+\sigma\delta\right)}{\theta^{\pi}\left[\kappa\left(1+\sigma\delta\right)-\sigma\phi\right]}\right] x_t - \phi e_t + \varepsilon_t^{\pi}. \tag{48}$$

Then substituting into (45) the expressions of x_t and $E_t x_{t+1}$ that are drawn from (48) while using $a_{t+1} = a_t + \gamma(\pi_t - a_t)$ implied by the learning algorithm (8) give:

$$E_t\pi_{t+1} = A_{11}\pi_t + A_{12}a_t + A_{13}E_t e_{t+1} + A_{14}e_t + P_1\varepsilon_t^{\pi}, \tag{49}$$

where

$$A_{11} = \frac{\kappa\theta^{\pi}\left[\kappa(1+\sigma\delta)-\sigma\phi\right] - \alpha(1-\theta^{\pi})(1+\sigma\delta) + \theta^{\pi}\alpha\gamma\beta^2(1+\sigma\delta)\left[1-\gamma(1-\beta)\right]}{\beta\left(1-\gamma\right)\left\{\kappa\theta^{\pi}\left[\kappa(1+\sigma\delta)-\sigma\phi\right]-\alpha(1+\sigma\delta)\right\}+\theta^{\pi}\alpha(1+\sigma\delta)\left[1-\gamma(1-\beta)\right]},$$

$$A_{12} = \frac{\beta\theta^{\pi}(1-\gamma)(1+\sigma\delta)\left[\alpha\beta^2\gamma+\alpha\beta(1-\gamma)\right] - \alpha\beta\theta^{\pi}(1+\sigma\delta)}{\beta\left(1-\gamma\right)\left\{\kappa\theta^{\pi}\left[\kappa(1+\sigma\delta)-\sigma\phi\right]-\alpha(1+\sigma\delta)\right\}+\theta^{\pi}\alpha(1+\sigma\delta)\left[1-\gamma(1-\beta)\right]},$$

$$A_{13} = \frac{-\phi\theta^{\pi}(1+\sigma\delta)\left[\alpha\beta^2\gamma+\alpha\beta(1-\gamma)\right]}{\beta\left(1-\gamma\right)\left\{\kappa\theta^{\pi}\left[\kappa(1+\sigma\delta)-\sigma\phi\right]-\alpha(1+\sigma\delta)\right\}+\theta^{\pi}\alpha(1+\sigma\delta)\left[1-\gamma(1-\beta)\right]},$$

$$A_{14} = \frac{\alpha\phi\theta^{\pi}(1+\sigma\delta)}{\beta\left(1-\gamma\right)\left\{\kappa\theta^{\pi}\left[\kappa(1+\sigma\delta)-\sigma\phi\right]-\alpha(1+\sigma\delta)\right\}+\theta^{\pi}\alpha(1+\sigma\delta)\left[1-\gamma(1-\beta)\right]},$$

$$P_1 = -\frac{\alpha(1+\sigma\delta)\theta^{\pi}}{\beta(1-\gamma)\{\kappa\theta^{\pi}[\kappa(1+\sigma\delta)-\sigma\phi]-\alpha(1+\sigma\delta)\}+\theta^{\pi}\alpha(1+\sigma\delta)[1-\gamma(1-\beta)]}.$$

Inserting $r_t - E_t^*\pi_{t+1}$ given by (4), h_t^e given by (47), $E_t^* x_{t+1} = b_t$ and $E_t^* e_{t+1} = z_t$ into the IS equation (3) and rearranging the terms lead to

$$x_t = b_t - \left(\delta + \sigma^{-1}\right)(z_t - e_t) + \frac{\theta^x}{\sigma^2\theta^e} h_t^x - \sigma^{-1}\varepsilon_t^e + h_t^x + \varepsilon_t^x. \tag{50}$$

Using (36) and (39), $\lambda_{5,t} = \frac{1}{\gamma}\lambda_{2,t} + \frac{\alpha\sigma\phi}{\gamma[\kappa(1+\delta\sigma)-\sigma\phi]}x_t$, and (42) to eliminate the Lagrange multipliers, we get

$$h_t^x = -\frac{\alpha\sigma\phi}{\theta^x[\kappa(1+\sigma\delta)-\sigma\phi]}x_t + \beta E_t h_{t+1}^x + \frac{\alpha\beta\sigma\phi(1-\gamma)}{\theta^x[\kappa(1+\sigma\delta)-\sigma\phi]}E_t x_{t+1}. \tag{51}$$

Substituting $E_t^*\pi_{t+1} = a_t$, $E_t^* e_{t+1} = e_t$ and h_t^e given by (47) into (4) yields

$$r_t = a_t + z_t - e_t - \frac{\theta^x}{\sigma\theta^e} h_t^x + \varepsilon_t^e. \tag{52}$$

We use the system of equations (8)-(10) and (48)--(52) to solve for the equilibrium solutions of a_t, b_t, z_t, π_t, x_t, r_t, e_t and h_t^x and then using (46) and (47) to obtain the equilibrium solutions of h_t^{π} and h_t^e. Since it is impossible to obtain reasonably simple analytical solutions, we will numerically simulate the model using calibrations proposed by Galí and Monacelli (2005) and Leitemo and Söderström (2008b) for the baseline framework of a small open economy.

Notice that in the closed economy, the equilibrium with RE is identical to the equilibrium when the learning gain γ is equal to zero (André and Dai 2018). This is because, when $\gamma = 0$, the expected values of π_{t+1} and x_{t+1} are exogenous and equal to their past values, and they must be equal to the steady-state values (identical to those under RE so that $a_t = b_t = E_t\pi_{t+1} = E_t x_{t+1} = 0$) for the model to converge to the steady state equilibrium. In the open economy, setting $\gamma = 0$ and $a_t = b_t = E_t\pi_{t+1} = E_t x_{t+1} = 0$, we find that the intertemporal optimal trade-off condition (45) will be identical to (12), implying the equilibrium solution obtained is the same as the one with RE.

4.2 The degree of model robustness ensuring determinacy

As we cannot derive the explicit condition to be imposed on the degree of model robustness to ensure determinacy, we simulate the model by taking values for the structural parameters from Galí and Monacelli (2005): $\hat{\sigma} = \zeta = 1$, $\eta = 3$, ϑ=0.75, $\beta = 0.99$, and $\omega = 0.4$. This implies that $\kappa = 0.401$, $\phi = 0.057$, $\delta = 0.4$, and $\sigma = 1.667$ according to Leitemo and Söderström (2008). We set the relative weight on output stabilization in the CB's loss function to $\alpha = 0.25$.[13]

We use the system of equations (8)-(10) and (48)-(52) to simulate the lower bound to be imposed on θ^{π}, θ^{x} and θ^{e} for the robust monetary policy not to induce indeterminacy under RE and for different values of γ in the interval (0, 1). The results about the stability condition obtained in the open economy are not allowing a direct comparison with the those in the closed economy obtained by André and Dai (2018) since several parameters take the standard parameter values (i.e., $\kappa = 0.024$, $\sigma = 0.157$, and $\alpha = 0.048$) in the literature of closed-economy New-Keynesian models, and are very different from the calibrations values used here (i.e., $\kappa = 0.401$, $\sigma = 1.667$ and $\alpha = 0.25$).

Assuming that $\theta^{\pi} = \theta^{x} = \theta^{e}$, we have simulated their value under which the economy is indeterminate. Table 1 shows that the thresholds of θ^{π}, θ^{x}, θ^{e}, denoted as $\underline{\theta}^{\pi}$, $\underline{\theta}^{x}$, $\underline{\theta}^{e}$, ensuring the determinacy of the equilibrium exponentially increase with the learning gain. This is to compare with a threshold for θ^{π} equal to 12.80 when $\gamma = 0.99$ in the closed-economy, using open-economy parameters.

[13] Here, we adopt $\alpha = 0.25$ following Leitemo and Söderström. This is higher than the value set by Galí and Monacelli who consider an objective function derived as a second-order approximation to the household's utility, the value of $\alpha = (1-\vartheta)(1-\beta\vartheta)(1+\eta)/(\epsilon\vartheta)$, where ϵ is the elasticity of substitution across the differentiated domestic goods. Using $\epsilon = 6$, they obtain $\alpha = 0.0572$.

γ	0	0.01	0.05	0.10	0.20	0.50	0.99
$\underline{\theta}^{\pi},\underline{\theta}^{x},\underline{\theta}^{e}$	1.8139	331.393	5528.009	20304.969	77582.851	471216.826	1829645.999

Table 1: The thresholds for the central bank's preference for robustness.

As previously discussed, the learning equilibrium with $\gamma = 0$ is identical to the equilibrium with RE given that the initial expectations must be equal to the steady state expectations.

Summarizing the results reported in Table 1 leads to the following proposition.

Proposition 1: *The degree of openness of the economy puts an additional constraint on the conduct of robust monetary policy. The more the economy is open to the rest of the world, the less wide the set of worst-case scenarios against which the monetary policy should be robust. Moreover, the set of worst-case scenarios drastically decreases with the learning gain.*

This result demonstrates that the hypothesis of constant-gain learning in an open economy substantially reduces the possibility for the CB to conduct robust policy. As shown in Table 1, the threshold for the CB's preference for robustness that ensures dynamic stability for $\gamma = 0.01$ is $\theta^{\pi} = \theta^{x} = \theta^{e} = 331.393$. Even in this case, the misspecification that is allowed in the Phillips curve, the IS equation and the UIP condition is related to cost-push, demand and exchange-rate shocks, and is given respectively as follows: $h_t^x = 0.000117\varepsilon_t^{\pi}$, $h_t^e = -0.000070\varepsilon_t^{\pi}$, $h_t^{\pi} = 0.002043\varepsilon_t^{\pi}$; $h_t^x = -0.000004\varepsilon_t^e$, $h_t^e = 0.000002\varepsilon_t^e$, $h_t^{\pi} = -0.000069\varepsilon_t^e$; $h_t^x = 0.000006\varepsilon_t^x$, $h_t^e = -0.000004\varepsilon_t^x$, $h_t^{\pi} = 0.000114\varepsilon_t^x$. This means that under learning, the misspecification is insignificant for all shocks and all equations of the model, and even more so as γ rises to a value between 0.2 and 0.5, i.e. the interval commonly used in the learning literature. For $\gamma = 0$, which is a proxy for the case of RE equilibrium, we have approximately $\underline{\theta}^{\pi},\underline{\theta}^{x},\underline{\theta}^{e} = 1.8139$. This comparison suggests that the CB should be less afraid of model misspecification under learning than under RE. Two factors explain this difference. The first is that learning is one kind of model misspecification so that to deal with the fact that private agents are learning, the CB should have less concern for the worst-case scenarios. The second is that the openness of the economy increases the volatility of inflation and the output gap not only in response to cost-push shocks but also to demand and exchange-rate shocks according to numerical simulations, meaning that adding too much model misspecification would feed too much volatility into the economy.

The key factor for which the thresholds of $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e}$ are so sensitive to an increase in γ is the openness of the economy. Some simulation exercises can show that the threshold for $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e}$ increases significantly as ϕ rises, and to a lesser extent as δ decreases. This is because a rise in ϕ deteriorates the intertemporal trade-off and a decrease in δ makes it less effective for the CB to use the interest rate policy to react to an exchange-rate shock, making the economy more volatile and dissuading thus the CB to introduce very bad worst-case scenarios.

Notice that in the closed economy, for a set of different parameter values, André and Dai (2018) find that the threshold of θ^{π} is 83.33 for $\gamma < 1$, compared to a threshold of $\theta^{\pi} = 45.45$ under RE. André and Dai have assumed that the misspecification in the IS equation is set to zero. This assumption is justified since Leitemo and Söderström (2008a) have found that the CB would optimally set $h^{x} = 0$ and Dai and Spyromitros (2012) confirm this result even when asset prices are included into the closed-economy model.

4.3 Learning effects on the equilibrium

To the difference of a closed economy, we cannot explicitly obtain the ALMs for inflation, the output gap, the exchange rate and the interest rate that depend on private expectations of inflation, the output gap, the exchange rate, and cost-push, demand and exchange-rate shocks. With the help of Dynare, we simulate the equilibrium solutions using previously given parameters values. Note that we have simulated the model with values of $\theta^{\pi} = \theta^{x} = \theta^{e}(= 1867240)$ ensuring the determinacy of the equilibrium for $\gamma \in (0, 1)$. Dynare gives transition functions of inflation, the output gap, the exchange rate and the interest rate with one period lag as follows:

$$\begin{bmatrix}\pi_t\\ x_t\\ e_t\\ r_t\end{bmatrix} = \begin{bmatrix}d_\pi^{cg} & d_x^{cg} & d_e^{cg}\\ k_\pi^{cg} & k_x^{cg} & k_e^{cg}\\ j_\pi^{cg} & j_x^{cg} & j_e^{cg}\\ m_\pi^{cg} & m_x^{cg} & m_e^{cg}\end{bmatrix}\begin{bmatrix}\pi_{t-1}\\ x_{t-1}\\ e_{t-1}\end{bmatrix} + \begin{bmatrix}d_a^{cg} & d_b^{cg} & d_z^{cg}\\ k_a^{cg} & k_b^{cg} & k_z^{cg}\\ j_a^{cg} & j_b^{cg} & j_z^{cg}\\ m_a^{cg} & m_b^{cg} & m_z^{cg}\end{bmatrix}\begin{bmatrix}a_{t-1}\\ b_{t-1}\\ z_{t-1}\end{bmatrix} + \begin{bmatrix}d_{\varepsilon^\pi}^{cg} & d_{\varepsilon^x}^{cg} & d_{\varepsilon^e}^{cg}\\ k_{\varepsilon^\pi}^{cg} & k_{\varepsilon^x}^{cg} & k_{\varepsilon^e}^{cg}\\ j_{\varepsilon^\pi}^{cg} & j_{\varepsilon^x}^{cg} & j_{\varepsilon^e}^{cg}\\ m_{\varepsilon^\pi}^{cg} & m_{\varepsilon^x}^{cg} & m_{\varepsilon^e}^{cg}\end{bmatrix}\begin{bmatrix}\varepsilon_t^\pi\\ \varepsilon_t^x\\ \varepsilon_t^e\end{bmatrix}. \tag{53}$$

They differ from the ALMs as defined by Evans and Honkapohja (2001). The latter are defined in terms of current values of private expectations and shocks.[14] We establish an equivalence between these transition functions and the ALMs using (8)-(10). The latter yield $a_{t-1} = \frac{1}{(1-\gamma)}a_t - \frac{\gamma}{(1-\gamma)}\pi_{t-1}$, $b_{t-1} = \frac{1}{(1-\gamma)}b_t - \frac{\gamma}{(1-\gamma)}x_{t-1}$, $z_{t-1} = \frac{1}{(1-\gamma)}z_t - \frac{\gamma}{(1-\gamma)}e_{t-1}$. Substituting a_{t-1}, b_{t-1} and z_{t-1} by their expressions into the transition functions (53), we obtain

$$\begin{bmatrix}\pi_t\\ x_t\\ e_t\\ r_t\end{bmatrix} = \begin{bmatrix}\tilde{d}_\pi^{cg} & \tilde{d}_x^{cg} & \tilde{d}_e^{cg}\\ \tilde{k}_\pi^{cg} & \tilde{k}_x^{cg} & \tilde{k}_e^{cg}\\ \tilde{j}_\pi^{cg} & \tilde{j}_x^{cg} & \tilde{j}_e^{cg}\\ \tilde{m}_\pi^{cg} & \tilde{m}_x^{cg} & \tilde{m}_e^{cg}\end{bmatrix}\begin{bmatrix}\pi_{t-1}\\ x_{t-1}\\ e_{t-1}\end{bmatrix} + \begin{bmatrix}\tilde{d}_a^{cg} & \tilde{d}_b^{cg} & \tilde{d}_z^{cg}\\ \tilde{k}_a^{cg} & \tilde{k}_b^{cg} & \tilde{k}_z^{cg}\\ \tilde{j}_a^{cg} & \tilde{j}_b^{cg} & \tilde{j}_z^{cg}\\ \tilde{m}_a^{cg} & \tilde{m}_b^{cg} & \tilde{m}_z^{cg}\end{bmatrix}\begin{bmatrix}a_t\\ b_t\\ z_t\end{bmatrix} + \begin{bmatrix}d_{\varepsilon^\pi}^{cg} & d_{\varepsilon^x}^{cg} & d_{\varepsilon^e}^{cg}\\ k_{\varepsilon^\pi}^{cg} & k_{\varepsilon^x}^{cg} & k_{\varepsilon^e}^{cg}\\ j_{\varepsilon^\pi}^{cg} & j_{\varepsilon^x}^{cg} & j_{\varepsilon^e}^{cg}\\ m_{\varepsilon^\pi}^{cg} & m_{\varepsilon^x}^{cg} & m_{\varepsilon^e}^{cg}\end{bmatrix}\begin{bmatrix}\varepsilon_t^\pi\\ \varepsilon_t^x\\ \varepsilon_t^e\end{bmatrix}. \tag{54}$$

where $\tilde{\ell}_\pi^{cg} \equiv \frac{(1-\gamma)\ell_\pi^{cg}-\gamma\ell_a^{cg}}{1-\gamma}$, $\tilde{\ell}_x^{cg} \equiv \frac{(1-\gamma)\ell_x^{cg}-\gamma\ell_b^{cg}}{1-\gamma}$, and $\tilde{\ell}_e^{cg} \equiv \frac{(1-\gamma)\ell_e^{cg}-\gamma\ell_z^{cg}}{1-\gamma}$, $\tilde{\ell}_n^{cg} \equiv \frac{\ell_n^{cg}}{1-\gamma}$ with $\ell = d, k, j, m$, and $n = a, b, z$. We have numerically checked that the absolute values of the composite coefficients on π_{t-1}, x_{t-1} and e_{t-1} in the above equation are extremely close to zero and more precisely they are generally smaller than 1×10^{-5} for $\gamma \in (0, 1)$ so that the terms associated with π_{t-1}, x_{t-1} and e_{t-1} are negligible and equations in (54) can be considered as the ALMs of endogenous variables.

Proposition 2. *In the open economy, the ALMs for inflation, the output gap, the exchange rate and the interest rate are function of expected inflation, expected output gap, expected exchange rate, and cost-push, demand and exchange-rate shocks.*

[14]For example, the ALM for inflation would take the following form: $\pi_t = \tilde{d}_a^{cg}a_t + \tilde{d}_b^{cg}b_t + \tilde{d}_z^{cg}z_t + \tilde{d}_{\varepsilon^\pi}^{cg}\varepsilon_t^\pi + \tilde{d}_{\varepsilon^x}^{cg}\varepsilon_t^x + \tilde{d}_{\varepsilon^e}^{cg}\varepsilon_t^e$. Despite this difference, equations in (53) allow us to see clearly the effects of learning on the equilibrium values of endogenous variables.

This result considerably contrasts with the one obtained in the closed economy. Regarding the ALM for the interest rate, the difference is also remarkable since in a closed economy model, it depends on expected inflation, expected output gap, and cost-push shocks (André and Dai, 2018). Indeed, the demand shocks can be entirely offset by an optimal adjustment of the interest rate. However, this is not the case in the open economy since an adjustment of the interest rate affects not only the aggregate demand but also the exchange rate while the latter affects the Phillips curve and hence the CB's intratemporal and intertemporal trade-offs.

The effects of learning on the feedback coefficients in the ALM for inflation are illustrated in Figure 1.

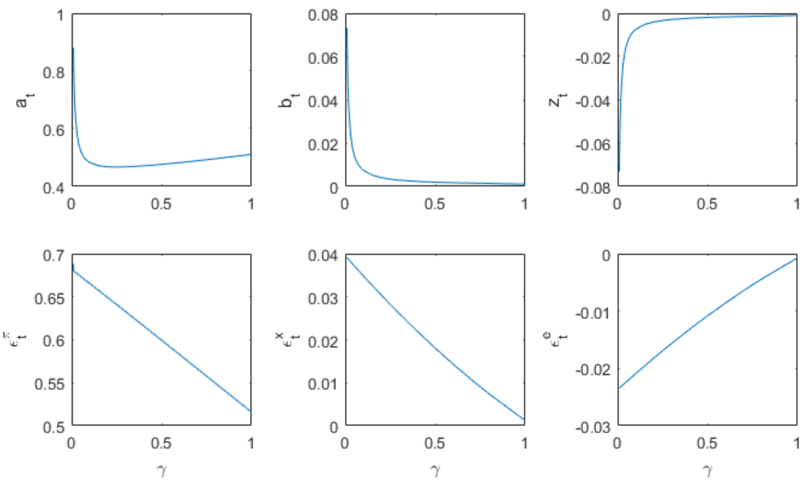

Figure 1: The effect of learning on the feedback coefficients in the ALM for π_t.

We notice that $\tilde{d}_a^{cg} > 0$, $\tilde{d}_b^{cg} > 0$, $\tilde{d}_z^{cg} < 0$, $d_{\varepsilon^\pi}^{cg} > 0$, $d_{\varepsilon^x}^{cg} > 0$, and $d_{\varepsilon^e}^{cg} < 0$. Compared to the equilibrium with RE that can be proxied by $\gamma = 0$, an increase in the learning gain always attenuates the response of inflation to b_t, z_t, ε_t^π, ε_t^x and ε_t^e, $\forall \gamma \in (0,\, 1)$ except for a_t whose coefficient sharply decreases for small values of γ and continues to decrease until $\gamma = 0.25$, and slightly increases with γ for $\gamma > 0.25$.

Figure 2 displays the effects of learning on the feedback coefficients in the ALM for the output gap and shows that $\tilde{k}_a^{cg} < 0$, $\tilde{k}_b^{cg} < 0$, $\tilde{k}_z^{cg} > 0$, $k_{\varepsilon^\pi}^{cg} < 0$, $k_{\varepsilon^x}^{cg} < 0$, and $k_{\varepsilon^e}^{cg} > 0$.

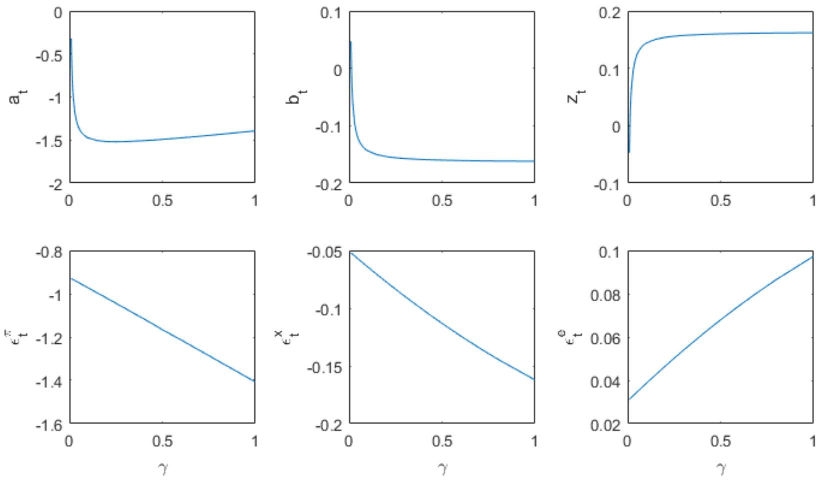

Figure 2: The effect of learning on the feedback coefficients in the ALM for x_t.

Compared to the RE equilibrium, a higher learning gain implies an attenuation in the response of the output gap to b_t, z_t, ε_t^π, ε_t^x and ε_t^e, $\forall\gamma \in (0, 1)$ while the coefficient on a_t sharply decreases for small values of γ and then slightly decreases until $\gamma = 0.25$, and moderately increases with γ fmaking more complexor $\gamma > 0.25$.

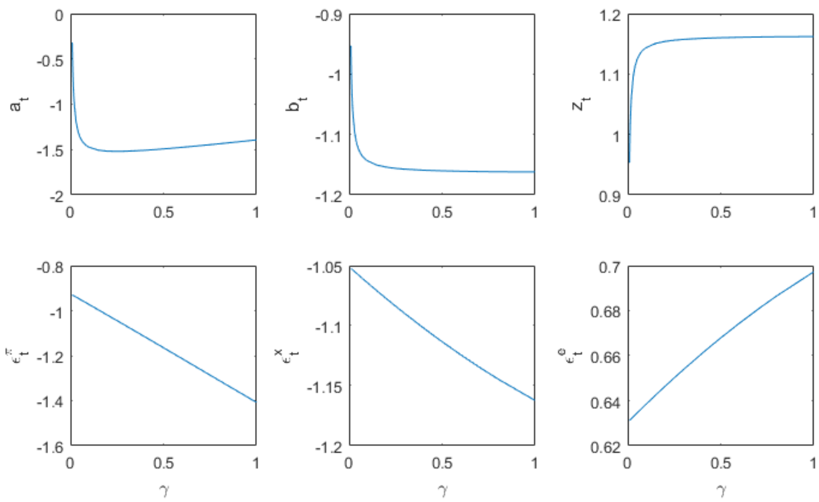

Figure 3: The effect of learning on the feedback coefficients in the ALM for e_t.

It follows from Figure 3 depicting the effects of learning on the feedback coefficients in the ALM for the exchange rate that $\tilde{j}_a^{cg} < 0$, $\tilde{j}_b^{cg} < 0$, $\tilde{j}_z^{cg} > 0$, $j_{\varepsilon^\pi}^{cg} < 0$, $j_{\varepsilon^x}^{cg} < 0$, and $j_{\varepsilon^e}^{cg} > 0$. An increase in the learning gain strengthens (attenuates) the response of the exchange rate to a_t for $\gamma \leq 0.25$ ($\gamma > 0.25$), and b_t , z_t , ε_t^{π}, ε_t^{x} and ε_t^{e} for any γ. Notice that the response of the exchange rate to b_t and z_t is amplified by an increase in γ but the amplification effect decelerates with γ for $\gamma > 0.25$.

Figure 4 shows the effects of learning on the feedback coefficients in the ALM for the interest rate. We remark that $\tilde{m}_a^{cg} > 0$, $\tilde{m}_b^{cg} > 0$, $\tilde{m}_z^{cg} < 0$, $m_{\varepsilon^{\pi}}^{cg} > 0$, $m_{\varepsilon^{x}}^{cg} > 0$ and $m_{\varepsilon^{e}}^{cg} > 0$. An increase in learning gain leads the CB to amplify (attenuate) the response of the interest rate to a_t for $\gamma \leq 0.25$, b_t, z_t, ε_t^{π}, ε_t^{x} for any γ (to a_t for $\gamma > 0.25$, and ε_t^{e} for any γ).

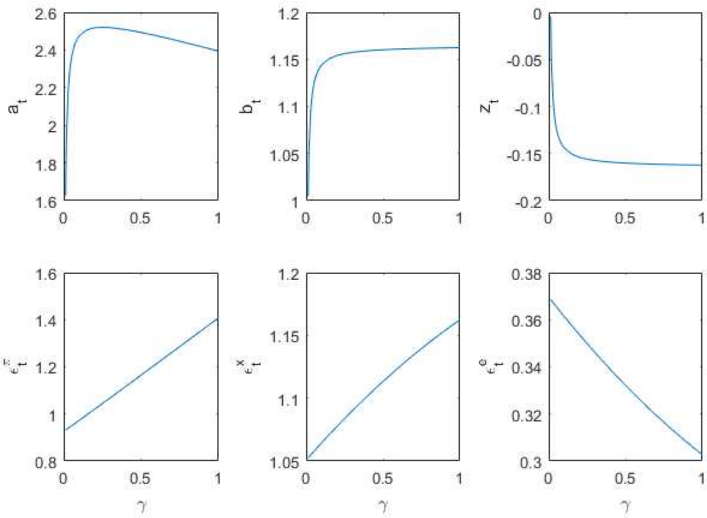

Figure 4: The effect of learning on the feedback coefficients in the ALM for r_t.

Proposition 3. *Adaptive learning makes robust monetary policy less (more) accommodative compared to RE in its response to cost-push and demand (exchange-rate) shocks. In the worst-case model, the fact that the CB exploits the intertemporal trade-off resulting from the learning behavior of private agents globally leads an attenuation (amplification) in the response of inflation (the output gap, the exchange rate) to inflation, output-gap and exchange-rate expectations, and cost-push, demand and exchange-rate shocks.*

Private agents' learning behavior modifies the intratemporal trade-off the CB faces and offers the latter the possibility to manipulate private expectations through its policy. In general, the higher the value of learning, the more (less) aggressive the monetary policy should be in response to cost-push and demand (exchange rate) shocks. An increas in both ε_t^π and ε_t^x is inflationary while an increase in ε_t^e decreases π_t, feeding thus into higher (lower) future expected inflation, hence calling for a more (less) aggressive policy for positive ε_t^π and ε_t^x (ε_t^e).

4.4 Effects of robustness on the equilibrium

In the closed economy model, under RE, an increase in the CB's preference for robustness against inflation misspecification (decrease in θ^π) results in a more aggressive monetary policy compared to the model without misspecification, while an increase in the CB's preference for robustness against output-gap misspecification has no implication for monetary policy (Leitemo and Söderström 2008a, Dai and Spyromitros 2012).

In the open economy model with RE, Leitemo and Söderström (2008b) have investigated the effect of an increase in the CB's preference against misspecification in model's equations and have found that a stronger preference for robustness against inflation and output-gap misspecification makes monetary policy respond more aggressively to inflation and output shocks, but less aggressively to exchange rate shocks, whereas a stronger preference for robustness against exchange rate misspecification has the opposite effects.

Adaptive learning imposes a much more restrictive constraint on monetary policy robustness to ensure the dynamic stability of the equilibrium than under RE. This is also true in a closed economy model but to a lesser extent (André and Dai 2018). Opening the economy with learning agents will sharply reinforce the constraint on the preference for robustness against a particular misspecification, and more so as the learning gain increases. The misspecification that the malevolent agent can introduce becomes insignificant and is hence quite insensitive to the change in the preference for robustness.

In Figures 5-8, simulations show the evolution of feedback coefficients of inflation, the output gap, the exchange rate and the interest rate according to θ_j, $j = \pi$, x, e, for three values of learning gain, i.e., $\gamma = 0.01$,$\gamma = 0.2$,$\gamma = 0.99$, represented by the red line, dotted green line, and dashed blue line, respectively. The red and green lines begin with relatively small values of θ whereas the blue line begins with a very high value of θ since this is required for the determinacy of the equilibrium.

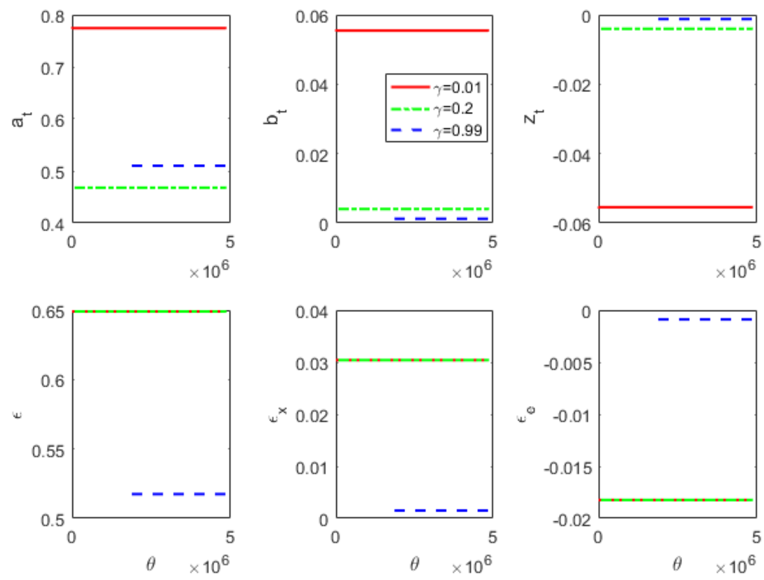

Figure 5: Feedback coefficients in the ALM for inflation.

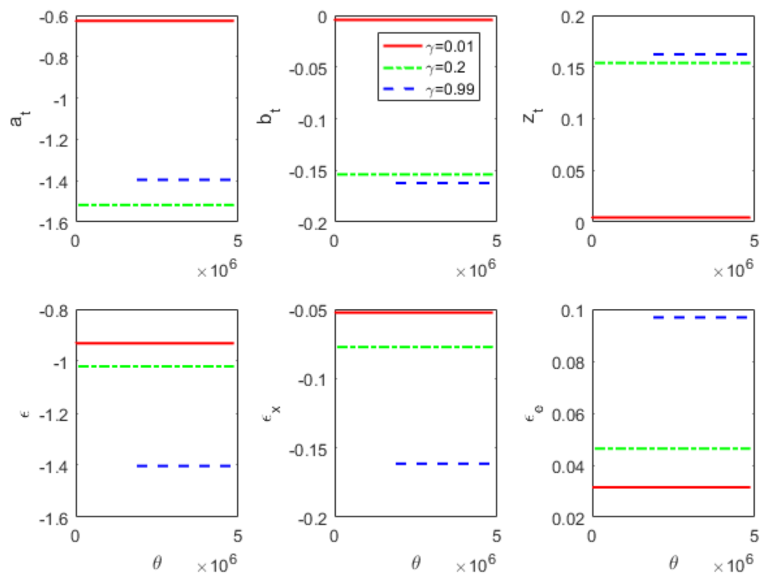

Figure 6: Feedback coefficients in the ALM for the output gap.

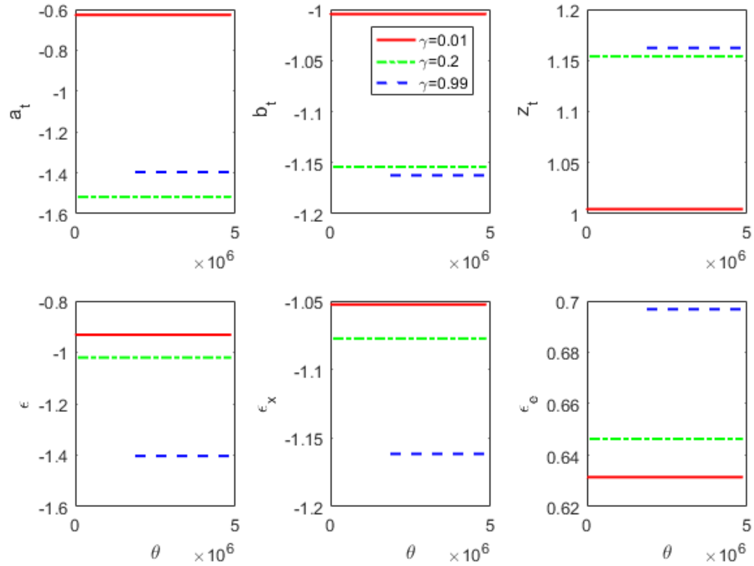

Figure 7: Feedback coefficients in the ALM for the exchange rate.

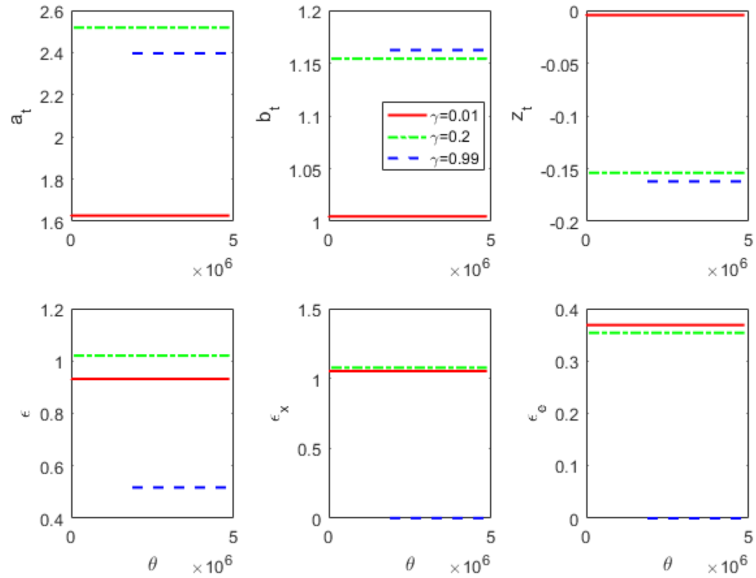

Figure 8: Feedback coefficients in the ALM for the interest rate.

Proposition 4. *The CB's preference for robustness against model misspecification has no significant impact on the value of feedback coefficients once its preference is bounded to ensure the dynamic stability of the equilibrium.*

This result suggests that in the open economy, the CB exploiting the intertemporal trade-off allowed by the learning behaviors of private agents, cannot introduce much misspecification that accounts for worst-case scenarios. Furthermore, due to the bound imposed on its preference for robustness, increasing such preference has almost no effect on the dynamics of the economy. This result is obtained only when the exchange rate affects the Phillips curve even for a very small parameter value of ϕ (the value retained in this paper is $\phi = 0.057$).

5 Conclusion

Using a stylized New Keynesian model of a small open economy, our paper finds that conceiving robust monetary policy with the robust control approach is not such a good idea for a central bank that is confronted to challenges arising from uncertain economic environment, openness to foreign trade and capital flows, and the learning behavior of private agents.

These results are expected to hold as long as the central bank influences inflation only through aggregate demand, and interest rate fluctuations in themselves do not affect social loss. The mechanism underlying our main result is that the exchange rate affects the Phillips curve through the wage-setting process,– hence making more complex the intertemporal trade-off for the central bank introduced by learning. This has crucial consequences on the dynamic stability of the economy as well as the interactions between endogenous variables and shocks. Notably, opening the economy reduces very significantly the set of worst-case scenarios against which monetary policy should be robust, and drastically more so as the learning gain rises. In a closed economy or an open economy with a Phillips curve not affected by the exchange rate, the central bank can offset all shocks other than cost-push shocks. Compared to such economies, the equilibrium values of endogenous variables in the type of economy examined in this paper are affected by all sorts of disturbances. We find that misspecification can affect the IS equation and the uncovered interest rate parity, as the central bank cannot costless offset demand shocks without affecting inflation through the exchange rate. However, due to the effect of learning and the openness of the economy, the central bank's preference for robustness against model misspecification has no significant impact on the equilibrium once its preference is bounded to ensure the determinacy of the equilibrium.

The main results obtained in this paper are based on the assumption of constant-gain learning. Nevertheless, agents could start learning with a decreasing gain before adopting a constant gain. The first can be seen as the first step in the expectations process adopted by most economic agents whereas the second is more suitable for time-varying environments. One immediate extension to this paper is to consider that private agents are learning with a gain decreasing over time as studied in Molnár and Santoro (2014). In general, the equilibria under decreasing-gain learning replicate the equilibria under learning with different constant gains, this extension will not change significantly the main results.

This paper focuses on the worst-case model, meaning that the malevolent agent chooses model misspecification to be as damaging as possible, and the central bank's policy rule and private agents' expectations reflect this misspecification. An interesting extension is to examine the case where the central bank uses the robust control approach to design the policy interest rate rule but the economy functions according to an approximating model as in Leitemo and Söderström (2008b). Since only the interest rate rule is disturbed to take account of model misspecification while the true model of the economy remains undisturbed, it seems that the economy has a smaller risk of being destabilized, meaning that the central bank could have a higher preference for model misspecification than in the worst-case model.

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