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Learning, optimal monetary delegation and stock prices dynamics

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Abstract

This paper studies how learning affects the interactions between monetary policy and stock prices. Learning modifies the intertemporal trade-off of the central bank by giving to the latter the possibility to manipulate private expectations. The result of this manipulation is not socially optimal since it reduces excessively the stabilization bias. To remedy this, the government should appoint a central banker that is less conservative than the society. The turnover rate in the stock market is the key factor that determines the interactions between monetary policy (hence delegation) and stock prices. A positive turnover rate means that the presence of stocks in the households' portfolios distorts the optimal consumption path. This type of distortion compensates somehow these induced by learning. The central bank should be more conservative to avoid the effect of distortions on social welfare induced by learning than in the absence of stocks.

Keywords: adaptive learning, stabilization bias, inflation penalty, optimal monetary delegation, central bank conservatism, stock prices.

JEL Classification: C62, D83, D84, E52, E58.

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1 Introduction

The recent economic downturn has highlighted the crucial role played by financial assets in the transmission mechanism of monetary policy. The high volatility of macroeconomic variables triggered by the financial crisis urges policy makers to better understand the implications of assets price for monetary policy decisions. As Bernanke (2010) underlines in its speech on the implications of the financial crisis for economics, "understanding the relationship between financial and economic stability in a macroeconomic context is a critical unfinished task for researchers." The relationship between the volatility of asset prices and monetary policy and how to deal with this volatility using monetary policy instruments are issues far from resolved.

The early literature studying the feedback effects between the volatility of stock prices and monetary policy, and the implications for macroeconomic stability of a response of monetary policy to asset prices is quite controversial.[1] Bernanke and Gertler (1999, 2001) advocate a flexible inflation targeting approach to tackle price and financial stability issues given the link between inflation and macroeconomic fundamentals of stock-price dynamics, while underlining that reaction to stock price generates a perverse effect disturbing the output dynamics. Cecchetti et al. (2000) suggest that a central bank (CB) concerned with stabilizing inflation is likely to achieve superior performance by adjusting its policy instruments not only in response to its forecasts of future inflation and the output gap, but also to asset prices. Cecchetti (2003) has provided evidence that the Federal Reserve's communications and policies were influenced by the Internet bubble as it was in progress. Bullard and Schaling (2002), and Carlstrom and Fuerst (2007) show that if policymakers place significant weight on the asset price component of the Taylor-type policy rule, other things equal, they will face indeterminacy of rational expectations (RE) equilibrium in the presence of sticky prices.

The global financial crisis has revived this debate among both policymakers and academic researchers. Nisticò (2012) shows that an interest-rate rule reacting to deviations of stock prices from the flexible-price equilibrium could also incur risks of endogenous instability in a New Keynesian model with non-Ricardian features. Gali (2014) shows, in an overlapping-

[1] See Cecchetti et al. (2002) and Gilchrist and Leahy (2002) for a survey of this early literature.

generations model, that “leaning against the wind” policies may raise the volatility of asset bubbles. However, based on a New Keynesian model with a cost channel of monetary policy transmission and a constant turnover between long-time traders and newcomers in market activities, Airaudo et al. (2013) suggest that a mild response to stock prices in the policy rule can restore equilibrium determinacy and therefore rule out non-fundamental volatility. Assenza et al. (2015) confirm the previous result in a model allowing for the cost channel by showing that there is a significant stabilizing role for asset price targeting, but at the cost of higher inflation volatility.

In general, most studies show that response of monetary policy to stock prices restricts the policy space where the RE equilibrium is determinate and can lead to fluctuations of expectations that are sunspot-driven and self-fulfilling. However, the policy space ensuring the determinate equilibrium under RE can be enlarged when private agents implement an adaptive learning process to form expectations. The adaptive learning algorithm allows not only to select among multiple RE equilibria but also to introduce a short-run dynamics for economic variables that is different from the one under RE. Indeed, this type of algorithm has shown its adequacy in financial and economic survey data (Lanne et al. 2008, Markiewicz and Pick 2014, Trehan 2015, Slobodyan and Wouters 2016). Given that learning assumption better describes the behavior of private agents in expectations formation, taking account of learning in monetary policy decision could improve macroeconomic performance.

Several studies have examined the consequences of including asset prices in monetary policy reaction function when agents form expectations using adaptive learning. Airaudo (2013) finds that interest rate rules granting a positive response to stock prices can enlarge the policy space where the equilibrium is determinate and learnable when the degree of asset-market participation is sufficiently large to generate an inverted aggregate demand channel of monetary policy transmission. Airaudo et al. (2015) share the previous view and show further that the Taylor principle ceases to be necessary. In constrast, Machado (2013) shows that a direct monetary policy response to stock prices through interest rate rules is not effective when agents are learning and this result is robust to heterogeneity in agents' beliefs.

This paper contributes to the literature on monetary policy by examining the implications of adaptive learning and optimal policy response to stock prices for CB accountability. Accountability is a major concern for central banking since it is necessary to circumvent the credibility problems that can arise with discretionary monetary policy. A simple inflation targeting regime might not be enough to tackle the issue of CB accountability. The latter can be solved either by the conservative central banker approach due to Rogoff (1985) or the inflation contracting approach through a linear inflation contract (Persson and Tabellini 1993, and Walsh 1995) or an inflation penalty on deviations from inflation target (Walsh 2003). According to Svensson (1997), these approaches can be seen as types of inflation targeting regimes that modify either the CB's preferences or incentives to eliminate inflation bias and stabilization bias. To deal with accountability issue, we adopt a CB loss function similar to the one in Walsh (2003) and consider a delegation game in which the government imposes inflation penalty on the CB for deviations from inflation target. More precisely, when the inflation target set by the government is not achieved, the CB has to undergo an inflation penalty that is increasing in the standard deviation of inflation from its target. A positive inflation penalty has the same implications for monetary policy as appointing a conservative central banker who is more focused on inflation stabilization.[2]

One major issue of this paper is about how the CB should conduct optimal policy and how the equilibrium is affected by stock prices when private agents are learning. In contrast, Machado (2013) and Airaudo et al. (2015) mainly consider exogenous instrumental interest rate rules and focus on issues of E-Stability. Our paper is also closely related to Molnár and Santoro (2014) who study the dynamic effect of learning, and André and Dai (2017a) who examine the implications of learning for optimal monetary delegation in the standard New Keynesian model without stock prices.

The main results obtained in this paper are the following: 1) Learning reinforces the deviations of inflation and the output gap from their RE equilibrium levels when the CB has

[2]Under monetary discretion, inflation or stabilization bias can be eliminated or reduced by appointing as head of the CB a person who is more averse to inflation than society as a whole. For the society, delegating monetary policy to such a conservative central banker is equivalent to putting a lower priority or weight on output stabilization.

to deal with the consequences of stock prices fluctuations for the macroeconomic stability. 2) A positive inflation penalty aggravates these deviations under the learning assumption. 3) An increase in turnover rate in the stock market generally attenuates (reinforces) the effect of learning (inflation penalty rate) on the effects of inflation expectations and technology shocks. 4) To maximize social welfare, the government should set a negative penalty rate that decreases with learning gain. An increase in turnover rate will make the optimal inflation penalty rate less negative for a given learning gain.

The remainder of the paper is organized as follows. The second section briefly presents the microeconomic foundations of the structural model. In the third section, we solve the model under the RE hypothesis. In the fourth section, the laws of motion for inflation, the output gap, stock prices and the interest rate are solved under learning. The fifth section examines the choice of optimal inflation penalty rate under learning. The sixth section discusses some potential extensions of the model. The last section concludes.

2 A structural New Keynesian model with stock prices

The economy is described by a simple microfounded general equilibrium framework developed in previous contributions such as Nisticò (2012) and Airaudo et al. (2015), who extend a discrete-time stochastic version of Yaari (1965) and Blanchard (1985)'s OLG-perpetual youth model to include risky equities and adapt it to a New Keynesian framework.[3] In essence, the private sector includes cohorts of non-Ricardian representative households and a continuum of monopolistically competitive firms that are uniformly distributed over the unit interval. Instead of RE, it is assumed that private agents form expectations using adaptive learning algorithm.

2.1 The consumers

An indefinite number of cohorts, composed of non-Ricardian households, live in the economy. Each cohort faces a constant probability ν of "dying" before the beginning of next period. For

[3]See Nisticò (2012) for the micro-foundations.

simplicity, we assume that entry and exit rates are equal and the total population is normalized to unity. Thus, in each period, a fraction ν of the population leaves the economy, and a new cohort of size ν enters. The economy is characterized by a constant turnover in the financial market between incoming agents without assets and the long-time agents holding assets.

The lifetime utility for the representative agent of the cohort that enters the market at time $j \leq t$ (henceforth, the j^{th}cohort) is:

$$E_t \sum_{k=0}^{\infty} \beta^k (1-\nu)^k \left[\ln C_{j,t+k} + \delta \ln (1 - N_{j,t+k})\right] \tag{1}$$

where $\beta \in (0,1)$, $\nu \in [0,1)$ and $\delta > 0$, $C_{j,t}$ represents consumption, $N_{j,t}$ the working time, and $(1 - N_{j,t})$ the leisure time. Future utility is discounted by, besides the impatience of agent, uncertain lifetime in the market represented by $(1-\nu)$, i.e., the probability of being still present in the market between two consecutive periods. E_t is the expectation operator.

There are two financial assets: the state-contingent bonds issued by the government and risky equity issued by monopolistically competitive firms in the intermediate good sector. At the end of period t, the j^{th} cohort representative agent keeps a portfolio of contingent claims, with a stochastic nominal payoff in $t+1$ equal to $B_{j,t+1}$ and a continuum of risky equity shares , i.e., $S_{j,t+1}(i)$ for $i \in [0,1]$. The i^{th} firm issues a risky share at the real price $Q_t(i)$. The nominal financial wealth for long-time traders (in the case where $j < t$) inherited from the previous period includes the nominal payoffs on contingent claims $B_{j,t}$, and the price plus the dividend obtained by each share, i.e., $Q_t(i) + D_t(i)$ for $i \in [0,1]$. Thus, the nominal financial wealth is defined as:

$$A_{j,t} = B_{j,t} + P_t \int_0^1 \left[Q_t(i) + D_t(i)\right] S_{j,t}(i) di, \text{ for } j < t. \tag{2}$$

The financial wealth $A_{j,t}$ is entrusted to a life insurance company through an insurance contract, as defined in Yaari (1965) and Blanchard (1985). Financial wealth carried over from the previous period also pays off the gross return on the insurance contract. Since such a contract redistributes among survived agents the financial wealth of the ones who die and in proportion to one's current wealth, total personal financial wealth is therefore accrued by a factor of $\frac{1}{1-\nu}$. At the period t, the total financial wealth of the j^{th} cohort $\Omega_{j,t}$ is given by $A_{j,t}$ with $j < t$, weighted by the factor $\frac{1}{1-\nu}$ representing the gross return per unit on the insurance contract while the financial wealth of newcomers is simply $A_{j,t}$ with $j = t$. We have thus

$$\Omega_{j,t} \quad = \quad \left\{ \begin{array}{ll} \frac{A_{j,t}}{1-\nu} & \text{for } j < t \\ A_{j,t} = 0 & \text{for } j = t \end{array} \right\} \tag{3}$$

The heterogeneity of wealth across cohorts is a key feature of the model. The assumption of constant turnover in markets as well as the absence of bequests and any wealth-equalizing fiscal transfer ensure a non-degenerate distribution of financial wealth across cohorts and hence this heterogeneity as in Blanchard (1985). Such heterogeneity is responsible for the structural linkage between stock prices and real activity through the aggregate demand.

The representative consumer of the j^{th} cohort at period t attempts to maximize (1) subject to the budget constraint:

$$P_t C_{j,t} + E_t \left\{ \mathcal{F}_{t,t+1} B_{j,t+1} \right\} + P_t \int_0^1 Q_t(i) S_{j,t+1}(i) di \leq W_t N_{j,t} - P_t T_{j,t} + \Omega_{j,t} \tag{4}$$

where $E_t \left\{ \mathcal{F}_{t,t+1} B_{j,t+1} \right\}$ is the portfolio of state-contingent claims, $\mathcal{F}_{t,t+1}$ is the common stochastic discount factor, and P_t the price index of final output. The household pays lump-sum taxes $P_t T_{j,t}$ to the government and earns $W_t N_{j,t}$ from working in the productive sector. The optimal decision of the household must be consistent with a standard non-Ponzi game condition (or transversality condition), i.e., $\lim\limits_{k \to \infty} E_t \left[\mathcal{F}_{t,t+k} \left(1 - \nu\right)^k \mathit{\Omega}_{j,t+k} \right] = 0$.[4] Differentiating the first-order conditions of the representative household's maximization problem, subject

[4]This condition is sufficient to warrant the existence of an optimal path under the usual assumptions imposed on a time-varying utility function.

to (4), yields:

$$\begin{aligned}\frac{\delta C_{j,t}}{1-N_{j,t}} &= \frac{W_t}{P_t}, &(5)\\ \mathcal{F}_{t,t+1} &= \frac{\beta C_{j,t}P_t}{C_{j,t+1}P_{t+1}}, &(6)\\ P_t Q_t(i) &= E_t\left\{\mathcal{F}_{t,t+1}P_{t+1}\left[Q_{t+1}(i)+D_{t+1}(i)\right]\right\}, \ \ \forall i\in[0,1]. &(7)\end{aligned}$$

Equation (5) is the intra-temporal optimality condition with respect to consumption and leisure and shows that the marginal substitution rate between consumption and leisure is equal to real wage. Equation (6) represents the inter-temporal condition with respect to bonds and gives the definition of the stochastic discount factor. Equation (7), resulting from the inter-temporal condition with respect to stock prices, is the pricing equation for the equity share that the i^{th} firm issues.

Introducing the individual wealth defined in (2)-(3) and (7), the binding budget constraint for the representative agent can be rewritten as a stochastic difference equation in $\Omega_{j,t}$:

$$P_t C_{j,t} + (1-\nu)E_t\left\{\mathcal{F}_{t,t+1}\Omega_{j,t+1}\right\} = W_t N_{j,t} - P_t T_{j,t} + \Omega_{j,t}. \tag{8}$$

By iterating $\Omega_{j,t+1}$ forward and using the non-Ponzi game condition, (8) yields that individual consumption is a linear function of financial and non-financial wealth. Denote non-financial wealth by $H_{j,t} \equiv E_t \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k}(1-\nu)^k \left(W_{t+k}N_{j,t+k} - P_{t+k}T_{j,t+k}\right)$. It follows:

$$P_t C_{j,t} = \left[1-\beta(1-\nu)\right]\left(\Omega_{j,t}+H_{j,t}\right) \tag{9}$$

where the term $[1-\beta(1-\nu)]$ represents the marginal propensity to consume out of total wealth.

2.2 The producers

The supply side of the economy is composed of a retail sector and a wholesale sector. Prices in the retail sector are perfectly flexible. The retail sector is characterized by perfect competition and produces the final consumption good Y_t out of a continuum of intermediate goods, with a production technology described by $Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)} di\right]^{\epsilon/\epsilon-1}$, where $\epsilon > 1$ stands for the elasticity of substitution between any two varieties of intermediate goods. The optimal demand for the intermediate good i is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$, and $P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{1/(1-\epsilon)}$ is the price of the final good.

There is a continuum of firms indexed by i, for $i \in [0, 1]$ in the wholesale sector. To produce the i^{th} variety of a continuum of differentiated intermediate goods, the wholesale representative firm i hires labor from a competitive labor market. Once produced, these goods are sold to retailers. To the difference of the retail sector, the wholesale sector is characterized by monopolistic competition and nominal rigidities in price setting. Firms use a simple linear technology: $Y_t(i) = Z_t N_t(i)$, where the aggregate total factor productivity Z_t is stochastic.

Nominal rigidities are introduced via the Calvo's staggered price setting where opportunities to adjust occur following an exogenous Poisson process. Each firm in the wholesale sector optimally updates its price with probability $(1 - \theta)$ in any given period t by maximizing the expected present discounted value of profits. Firms have the same real marginal costs given by $MC_t = (1 - l)\frac{W_t}{Z_t P_t}$, where l is the government's labor subsidy that is assumed to be zero henceforth.[5] The i^{th} firm sets the optimal price $P_t^*(i)$ to maximize

$$E_t \sum_{k=0}^{\infty} \theta^k \mathcal{F}_{t,t+k} Y_{t+k}(i) \left(P_t^*(i) - P_{t+k} MC_{t+k}\right),$$

given the demand constraint $Y_{t+k}(i) = \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$. The larger is the elasticity of substitution ϵ, the smaller the market power for monopolistically competitive wholesale firms. Therefore, real dividends distributed by the i^{th} firm and the related stock prices are inversely related to ϵ. The price setting problem in the wholesale sector implies that $P_t^*(i) = P_t^*$, i.e., all firms that have the opportunity to update their price will set the same price P_t^*.

[5] As indicated in Airaudo et al. (2015), the labor subsidy is set to equate the real wage to the marginal productivity of labor. This rather standard assumption in the New Keynesian literature ensures that the steady-state Frisch elasticity of labor is independent of the elasticity of substitution between goods. Eliminating such subsidy will not change the main results.

2.3 Aggregation

In each period, a fraction ν of each cohort leaves the economy. The population size remains constant over time given that the fraction of the population that exits is substituted by an equally sized cohort of newcomers. The size at period t of the cohort that entered the market at earlier stage ($j \leq t$) is $\nu\,(1-\nu)^{t-j}$. The aggregator is thus defined as: $X_t = \sum_{j=-\infty}^{t} \nu\,(1-\nu)^{t-j} X_{j,t}$ for $X = C,\, N,\, B,\, T,\, H$.

Denote by $D_t \equiv \int_0^1 D_t(i)di$, and $Q_t \equiv \int_0^1 Q_t(i)S(i)di$ the aggregate dividends and the capitalization of all stocks, respectively. The aggregation across cohorts using equations (5), (7) and (9) leads to:

$$\frac{\delta C_t}{1-N_t} = \frac{W_t}{P_t} \tag{10}$$

$$Q_t = E_t\left\{\mathcal{F}_{t,t+1}\Pi_{t+1}\left[Q_{t+1}+D_{t+1}\right]\right\}, \tag{11}$$

$$P_tC_t = \left[1-\beta\,(1-\nu)\right](\Omega_t + H_t)\,, \tag{12}$$

where aggregate wealth Ω_t is defined as $\Omega_t \equiv B_t + P_t\int_0^1 \left[Q_t(i)+D_t(i)\right] S_t(i)di$ and $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

The aggregate binding budget constraint is:

$$P_tC_t + E_t\left\{\mathcal{F}_{t,t+1}\Omega_{t+1}\right\} = W_tN_t - P_tT_t + \Omega_t. \tag{13}$$

Finally, (12)-(13) and the definition of H_t allow defining a stochastic difference equation of aggregate consumption:

$$\frac{\beta(1-\nu)}{1-\beta(1-\nu)}P_tC_t = \nu E_t\left\{\mathcal{F}_{t,t+1}\Omega_{t+1}\right\} + \frac{1-\nu}{1-\beta(1-\nu)}E_t\left\{\mathcal{F}_{t,t+1}P_{t+1}C_{t+1}\right\}. \tag{14}$$

The first term on the right-hand side of (14) stands for the financial wealth effects. Such effects vanish as the probability of exiting the market (ν) tends to zero. For simplicity, we assume hereafter that $S_t(i) = 1$ for all $i \in [0,1]$, and state-contingent bonds are in zero net supply in every period, i.e., $B_t = 0$. These assumptions, together with a constant stock of shares issued by all firms, yield: $\Omega_t = P_t(Q_t + D_t)$.

Forwarding the last equation into the next period and using (11), we get $E_t\left\{\mathcal{F}_{t,t+1}\Omega_{t+1}\right\} = E_t\left\{\mathcal{F}_{t,t+1}P_{t+1}(Q_{t+1}+D_{t+1})\right\} = P_tQ_t$. Combining this result and (14) leads to

$$\frac{\beta(1-\nu)}{1-\beta(1-\nu)}C_t = \nu Q_t + \frac{1-\nu}{1-\beta(1-\nu)}E_t\left\{\mathcal{F}_{t,t+1}\Pi_{t+1}C_{t+1}\right\}, \tag{15}$$

The first term νQ_t on the right-hand side of (15) links the stock market to the real side of the economy. Following Airaudo et al. (2015), it is referred to as the financial wealth channel. Thus, a positive turnover $\nu > 0$ means that the financial wealth distorts the consumption path.

We assume that

$$E_t\left\{\mathcal{F}_{t,t+1}\right\} = \frac{1}{R_t}, \tag{16}$$

with R_t denoting the riskless nominal interest rate so that there is no arbitrage between stocks and state-contingent bonds when the financial market is in equilibrium.

The market clearing conditions in this economy are verified when the consumption equals the output gap, i.e., $C_t = Y_t$, and when the aggregate labor demand for wholesale firms is equal to the labor supply. Since dividends are equal to corporate profits in the wholesale sector, then $D_t = Y_t(1-MC_t)$.

2.4 The linearized model

The linearized model is obtained by log-linearizing the previous equilibrium conditions around the unique non-stochastic steady state. Lower case letters denote percentage deviations of the original variable from its respective steady-state value, i.e., $y = \log(Y_t/Y)$. The reduced structural model is composed of equations representing the demand-side, supply-side, and financial side of the economy.

The New Keynesian Philips curve is obtained using the log-linearized form of the solution of the optimal price setting problem in the intermediate good sector, the definition of real marginal costs and equation (10):

$$\pi_t = \tilde{\beta} E_t^* \pi_{t+1} + \kappa (1+\chi)(x_t - z_t) \tag{17}$$

where E_t^* is the expectation operator that stipulates that private agents' expectations are conditional on information set available at time t, with the asterisk indicating that these agents can form RE or not, π_t the inflation rate, $\tilde{\beta} \equiv \frac{\beta}{1+\psi}$ with $\beta \in (0,1)$, $x_t = y_t - y_n$ the output gap between the actual output and the flexible price equilibrium output, and z_t an *i.i.d.* technology shock. The coefficient $\kappa (1+\chi)$, with $\kappa \equiv \frac{(1-\theta)\left(1-\theta\tilde{\beta}\right)}{\theta}$ and $\chi \equiv \frac{N}{1-N}$, measures the output-gap elasticity for inflation and captures the effects of the output gap on real marginal costs and thus on inflation. The composite parameter χ is the inverse of the steady-state Frisch elasticity of labor supply.

Log-linearizing (15), (16), and $C_t = Y_t$ and arranging the resulting equations give the IS equation:

$$x_t = \frac{1}{1+\psi} E_t^* x_{t+1} + \frac{\psi}{1+\psi} q_t - \frac{1}{1+\psi}(r_t - E_t^* \pi_{t+1}) + u_t, \tag{18}$$

where q_t represents the deviation of stock prices from their flexible-price equilibrium value, and u_t an *i.i.d.* aggregate demand shock. The term $\frac{\psi}{1+\psi} q_t$ captures the financial wealth channel. The strength of this channel depends on the composite coefficient ψ, which is a function of the turnover rate ν:

$$\psi(\nu) \equiv \nu \frac{1-\beta(1-\nu)}{(1-\nu)\epsilon} \frac{1+r}{r} > 0, \tag{19}$$

where r stands for the steady-state real interest rate and is strictly increasing in ν.[6], $\psi(\nu)$ has the following properties:

$$\psi(0) = 0 \text{ and } \psi'(\nu) > 0 \tag{20}$$

From the properties described in (20), we can easily deduce the role played by the turnover rate ν in (18). If the turnover rate in the markets is null, i.e., $\nu = 0$, we get $\psi = 0$, meaning that (18) will simply take the standard form in a canonical New Keynesian model. A positive

[6] The solution of r is determined by Airaudo et al. (2015) in Appendix A and is given by $r = \frac{\epsilon(1-\nu)(1-\beta)+\nu[1-\beta(1-\nu)]+\sqrt{\Psi}}{2\beta(1-\nu)\epsilon}$ with $\Psi = \{\epsilon(1-\nu)(1-\beta) + \nu[1-\beta(1-\nu)]\}^2 + 4\beta\epsilon\nu(1-\nu)[1-\beta(1-\nu)]$.

turnover rate ($\nu > 0$) signifies that financial wealth distorts the optimal consumption path.

The stock-price dynamics stems from the linearized form of (11) that is combined with the linearized form of (10) and (15)-(16), while using $D_t = Y_t(1 - MC_t)$ and $\frac{MC_t}{p_t} = \frac{W_t}{Z_t P_t}$:

$$q_t = \tilde{\beta} E_t^* q_{t+1} - \eta E_t^* x_{t+1} - (r_t - E_t^* \pi_{t+1}) + v_t, \tag{21}$$

where $\eta \equiv (1 - \tilde{\beta}) [(\epsilon - 1)(1 + \chi) - 1]$, and v_t is an *i.i.d.* stock-price shock. According to (21), current stock prices q_t are positively related to their future expected value $E_t^* q_{t+1}$, but negatively to the real interest rate $(r_t - E_t^* \pi_{t+1})$ and the future expected output $E_t^* x_{t+1}$. The last relationship is due to the fact that dividends are counter-cyclical in the New Keynesian framework assuming flexible wages according to Milani (2008).[7]

To complete the model, we assume that private agents form expectations using learning algorithms in accordance with the learning literature pioneered by Marcet and Sargent (1989). . Such an assumption is motivated by observed limited rationality among private agents, who have a restricted knowledge about the process governing the evolution of endogenous variables, a limited access to the information necessary for expectations formation as well as a lack of skills to well use such information. It implies that, to improve their decisions, private agents recursively estimate a Perceived Law of Motion (PLM) in the sense of Evans and Honkapohja (2001), which is consistent with the law of motion that the CB follows under RE, by using the deterministic learning algorithms that follow Marcet and Nicolini (2003):

$$E_t^* \pi_{t+1} \equiv a_t = a_{t-1} + \gamma_t(\pi_{t-1} - a_{t-1}), \tag{22}$$

$$E_t^* x_{t+1} \equiv b_t = b_{t-1} + \gamma_t(x_{t-1} - b_{t-1}), \tag{23}$$

$$E_t^* q_{t+1} \equiv s_t = s_{t-1} + \gamma_t(q_{t-1} - s_{t-1}), \tag{24}$$

where $0 < \gamma_t < 1$ denotes a deterministic sequence of learning gains specifying the speed

[7] In Nisticò's (2012) model, flexible wages would generate countercyclical profits and dividends and hence explain the negative sign before η. However, the sign before η could be positive if labor rigidities were allowed for. This is contrary to the conventional wisdom, which has it that stock returns and subsequent growth rates of real activity are positively linked (Fama 1990). Empirically, this relationship has collapsed in some countries in the early 1980s (Binswanger 2004), while it is supported by Tsouma (2009) using more recent data.

of integration of new data into private agents' expectations with exogenous initial values a_0 and b_0. The fact that $\gamma_t > 0$ makes it possible for policymakers to manipulate future expectations through monetary policy. Underlying the adaptive learning mechanism is the idea that inflation, output-gap and stock-prices expectations are increasing in the past period inflation, output gap and stock prices, respectively.[8] To keep the model analytically tractable, this paper focuses on the case where the learning gain is constant, i.e., $\gamma_{t+1} = \gamma_t = \gamma$. It is found that private agents would be more inclined to use a constant-gain learning algorithm if they believe in possible structural changes to happen in the near future because this kind of learning is more suitable to time-varying environments (Evans and Honkapohja 2009).[9]

The government and the society share a common loss function defined in terms of inflation and output-gap volatility:

$$L_t^S = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i (\pi_{t+i}^2 + \alpha x_{t+i}^2), \tag{25}$$

The government's loss function represents a weighted average of the variance of inflation and the output gap around their respective target. Both targets are assumed to be zero.[10] The government delegates monetary policy decisions to the CB and inflicts an inflation penalty with a rate τ on standard deviation of inflation from its target. This penalty can be seen as a clause in a non-linear contract between the government and the CB. It aims to modify the incentives for the CB as does the linear inflation contract due to Persson and Tabellini (1993) and Walsh (1995) even though these two types of contracts have quite different implications. To keep consistency in monetary policy decisions, it is assumed that $1+\tau > 0$, i.e., the penalty rate that might be positive or negative must be such that it will not change the fact that a deviation of inflation from its targets constitutes a loss for the CB.

The CB conducts discretionary policy to minimize the conditional expectation of its loss function:

[8] The learning process embodied in (22)-(24) is limited by the fact that agents focus on past information and forecast with one period ahead since according to Preston (2005), long-horizon expectations matter for monetary policy decision.

[9] Berardi and Galimberti (2013) highlight that most economic agents might nevertheless adopt decreasing-gain learning as a first approach.

[10] Under RE, setting an output-gap target different from zero gives rise to an inflation bias that could be offset by the optimal inflation contract formulated by Persson and Tabellini (1993) and Walsh (1995), or the non-linear inflation penalty (Walsh 2003).

$$L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[(1+\tau) \pi_{t+i}^2 + \alpha_x x_{t+i}^2 \right]. \tag{26}$$

The CB does not have an objective of financial stability. However, in this model, monetary policy decisions must take into account the deviation of stock prices q_t due to the presence of the financial wealth effect already embodied in the IS equation (18).

The government and the CB play a Stackelberg game. The government sets first the inflation penalty rate. Knowing the latter, the CB implements optimal monetary policy.

3 Benchmark equilibrium under RE

This section examines, under the hypothesis of RE, the optimal monetary policy, the equilibrium solutions of endogenous variables, and how the presence of stock prices affects monetary policy delegation. The results presented in this section will be compared with those obtained under the assumption of adaptive learning.

3.1 Optimal monetary policy

The CB conducts policy under discretion by minimizing the loss function (26) subject to (17), (18) and (21). The first-order conditions of the CB's minimization problem yield the optimal intra-temporal trade-off condition or inflation targeting rule:

$$x_t = -\frac{\kappa(1+\tau)(1+\chi)}{\alpha} \pi_t. \tag{27}$$

The optimal interest rate is obtained using (27) to eliminate x_t in (18):

$$r_t = E_t^* x_{t+1} + E_t^* \pi_{t+1} + \frac{\kappa(1+\chi)(1+\psi)(1+\tau)}{\alpha} \pi_t + \psi q_t + (1+\psi) u_t. \tag{28}$$

According to (28), the optimal policy interest rate should respond to stocks prices. Condition (20) states that ψ is equal to zero in the absence of turnover in the financial market, or equivalently in a model with representative infinitely-lived agent. If this is the case, the interest rate rule collapses to the one without reaction to stock prices. The higher the turnover rate, the more the interest rate will respond to stock prices given that ψ increases with ν. The following proposition follows from these observations.

Proposition 1. *The optimal interest rate rule responds to stock prices only if the latter distort the optimal consumption path, i.e., the turnover rate in the financial market ν is positive. The higher is ν (i.e., the higher is ψ), the more responsive the policy interest rate to stock prices. When $\nu = 0$ (or equivalently $\psi = 0$) the policy interest rate rule is reduced to the form it takes in the standard New Keynesian model.*

The response of optimal policy interest rate allows offseting the effects of financial shocks. As a result, due to optimal monetary policy, the equilibrium inflation and output gap will not be affected by stock prices despite the existence of financial wealth effect on the aggregate demand.

3.2 Equilibrium solutions and optimal inflation penalty rate

For given inflation expectations $E_t^*\pi_{t+1}$ and inflation penalty rate τ, solving (17), (18), (21) and (27) yields the Actual Laws of Motion (ALMs) that govern the evolution of inflation, the output gap, stock prices, and the interest rate rule that implements these allocations:

$$
\begin{aligned}
\pi_t &= \frac{\alpha\tilde{\beta}}{\Upsilon}E_t^*\pi_{t+1} - \frac{\alpha\kappa\left(1+\chi\right)}{\Upsilon}z_t, && (29)\\
x_t &= -\frac{\tilde{\beta}\kappa\left(1+\chi\right)\left(1+\tau\right)}{\Upsilon}E_t^*\pi_{t+1} + \frac{\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)}{\Upsilon}z_t. && (30)\\
q_t &= -\frac{1+\eta}{1+\psi}E_t^*x_{t+1} - \frac{\tilde{\beta}\kappa\left(1+\chi\right)\left(1+\tau\right)}{\Upsilon}E_t^*\pi_{t+1} + \frac{\tilde{\beta}}{1+\psi}E_t^*q_{t+1}\\
&\quad + \frac{\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)}{\Upsilon}z_t + \frac{1}{1+\psi}v_t - u_t, && (31)\\
r_t &= \frac{1-\eta\psi}{1+\psi}E_t^*x_{t+1} + \frac{1}{\Upsilon}\left[\Upsilon + \tilde{\beta}\kappa\left(1+\chi\right)\left(1+\tau\right)\right]E_t^*\pi_{t+1} + \frac{\psi}{1+\psi}\tilde{\beta}E_t^*q_{t+1}\\
&\quad - \frac{\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)}{\Upsilon}z_t + \frac{\psi}{1+\psi}v_t + u_t. && (32)
\end{aligned}
$$

where $\Upsilon = \alpha + \kappa^2\left(1+\chi\right)^2\left(1+\tau\right)$.

The ALM (32) is an expectations-based reaction function and describes the anticipated utility policy set by a policymaker who ignores how private agents revise their beliefs in the future.[11]

The system of equations (17), (18), (21) and (27) has a unique non-explosive RE equilibrium (REE) solution, called the "minimal state variable" solution (McCallum 1983), in terms of state variable z_t, u_t and v_t. Under RE, with $E_t^* = E_t$, the solution of π_t takes the following form: $\pi_t = \zeta_0 + \zeta_1 z_t + \zeta_2 u_t + \zeta_3 v_t$. Since all shocks are serially uncorrelated, i.e., $E_t z_{t+1} = E_t u_{t+1} = E_t v_{t+1} = 0$, it follows that $E_t \pi_{t+1} = \zeta_0 + \zeta_1 E_t z_{t+1} + \zeta_2 E_t u_{t+1} + \zeta_3 E_t v_{t+1} = \zeta_0$. Using the method of undetermined coefficients yields $E_t \pi_{t+1} = \zeta_0 = 0$, and $\zeta_1 = -\frac{\alpha\kappa(1+\chi)}{\Upsilon}$. Similarly, we can show that $E_t x_{t+1} = E_t q_{t+1} = 0$. Thus, at the REE, we obtain:

$$\pi_t = -\frac{\alpha\kappa\left(1+\chi\right)}{\Upsilon} z_t, \tag{33}$$

$$x_t = \frac{\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)}{\Upsilon} z_t, \tag{34}$$

$$q_t = \frac{\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)}{\Upsilon} z_t - u_t + \frac{1}{1+\psi} v_t, \tag{35}$$

$$r_t = -\frac{\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)}{\Upsilon} z_t + u_t + \frac{\psi}{1+\psi} v_t. \tag{36}$$

At the equilibrium, only the technology shock affects inflation and the output gap. Meanwhile, both stock prices and the interest rate are affected by the stock-price shock and the demand shock besides the technology shock. The optimal response of the interest rate to stock-price and demand shocks offsets their effects on the equilibrium inflation and output gap at the cost of higher interest-rate and stock-price volatility. The higher is the turnover rate, the stronger (lower) is the sensitiveness of the equilibrium interest rate (stock-prices) to the stock-price shock.

Proposition 2. *The equilibrium solutions of inflation and the output gap are only function of the technology shock. The sensitiveness of inflation to the latter is amplified by an increase*

[11] The anticipated utility (Kreps 1998), commonly used in the learning literature, is distinct from expected utility namely for two characteristics. The first is that private agents do not know the true model. The second is that, private agents who are learning about the parameters or the state of the economy choose myopic actions today upon the updating of their information set, and ignore that they will keep on learning in the future. Under RE, private agents know the true model so that their current beliefs reflect all available information and the optimal anticipated utility policy would also maximize expected utility, meaning that anticipated utility coincides with expected utility.

in turnover rate in the financial market if $\alpha > \kappa^2 (1+\chi)^2 (1+\tau)$ *and the same is true for the output gap, stock prices and the interest rate for any parameter values. The responses of stock prices and the interest rate to demand shocks are independent of* ν*. An increase in* ν *increases (decreases) the sensitiveness of the equilibrium interest rate (stock-prices) in response to the stock-price shock.*

Proof. From (19), it follows that $\frac{\partial\psi(\nu)}{\partial\nu} = \frac{1+r}{r}\left[\frac{1}{\epsilon(1-\nu)^2} - \frac{\beta}{\epsilon}\right] > 0$. From the definition of $\tilde{\beta}$ and κ, we obtain $\frac{\partial\tilde{\beta}}{\partial\nu} < 0$ and $\frac{\partial\kappa}{\partial\nu} > 0$. Differentiating π_t, x_t, q_t, and r_t given by (33)-(36) with respect to ν shows that $\frac{\partial^2\pi_t}{\partial z_t\partial\nu} = -\alpha(1+\chi)\Upsilon^{-2}\left[\alpha - \kappa^2(1+\chi)^2(1+\tau)\right]\frac{\partial\kappa}{\partial\nu} < 0$ if $\alpha > \kappa^2(1+\chi)^2(1+\tau)$, $\frac{\partial^2 x_t}{\partial z_t\partial\nu} > 0$, $\frac{\partial^2 q_t}{\partial z_t\partial\nu} > 0$ and $\frac{\partial^2 r_t}{\partial z_t\partial\nu} < 0$; $\frac{\partial^2\pi_t}{\partial u_t\partial\nu} = \frac{\partial^2 x_t}{\partial u_t\partial\nu} = \frac{\partial^2 q_t}{\partial u_t\partial\nu} = \frac{\partial^2 r_t}{\partial u_t\partial\nu} = 0$; $\frac{\partial^2\pi_t}{\partial v_t\partial\nu} = \frac{\partial^2 x_t}{\partial v_t\partial\nu} = 0$, $\frac{\partial^2 q_t}{\partial v_t\partial\nu} < 0$ and $\frac{\partial^2 r_t}{\partial v_t\partial\nu} > 0$.□

To know precisely the effect of these shocks on the equilibrium, it is necessary to determine the optimal inflation penalty rate. The latter is determined by minimizing (25) taking account of the solutions of π_t and x_t given by (33)-(34):

$$\tau = 0. \tag{37}$$

Notice that this result is obtained under the hypothesis of RE and in the absence of inflation bias.

Proposition 3. *The optimal inflation penalty rate is equal to zero and is independent of the turnover rate in the stock market.*

3.3 Dynamic stability and speed of convergence

Examining the ALM that governs the evolution of inflation given by (29) and taking account of the result $\tau = 0$, we find the speed of convergence of inflation is determined by $\Theta \equiv \frac{\alpha\tilde{\beta}}{\alpha+\kappa^2(1+\chi)^2} < 1$. Differentiating Θ with respect to ν and using the fact that $\frac{\partial\tilde{\beta}}{\partial\nu} < 0$ and $\frac{\partial\kappa}{\partial\nu} > 0$ yield $\frac{\partial\Theta}{\partial\nu} < 0$ and hence the following proposition.

Proposition 4. *A positive turnover rate in the stock market accelerates the speed of convergence of inflation in response to a future shock that affects inflation expectations compared to* *its dynamic path in the standard New Keynesian model.*

A higher turnover rate corresponds to a lower discount factor $\tilde{\beta}$, implying that (persistent) shocks affecting future inflation expectations have smaller impacts on the current inflation. Thus, the current inflation converges more quickly to its steady-state value. However, the effect on the speed of convergence of the output gap, stock prices and the interest rate is ambiguous and depends on structural parameter values.

4 Equilibrium with constant-gain learning

This section analyzes how constant-gain learning interacts with monetary delegation and macroeconomic stabilization when stock prices are introduced compared to the benchmark case where private agents form RE. As the learning equilibrium without stock prices is extensively studied in the absence of monetary delegation by Molnár and Santoro (2014) and under delegation by André and Dai (2017a), this section focuses on the difference introduced by the financial wealth channel through which the response of monetary policy to stock prices affects the real economy.

4.1 Optimal inflation targeting rule

When agents are learning, the CB will take account of the effects of its policy decisions on future expectations. Under discretion, monetary policy is therefore conducted to minimize the loss function (26) subject to the usual constraints (17), (18) and (21) as well as the learning algorithms (22)-(24). Substituting $E_t^* \pi_{t+1} = a_t$, $E_t^* x_{t+1} = b_t$ and $E_t^* q_{t+1} = s_t$ into (17), (18) and (21), we write the Lagrangian of the CB's minimization problem as follows:

$$
\begin{aligned}
\mathscr{L}_t^{CB} \quad = \quad & E_t \sum_{i=0}^{+\infty} \beta^i \left\{ \tfrac{1}{2} \left[\alpha x_{t+i}^2 + (1+\tau)\pi_{t+i}^2 \right] \right. \\
& -\lambda_{1,t+i} \left[\pi_{t+i} - \tilde{\beta} a_{t+i} - \kappa \left(1+\chi\right)\left(x_t - z_t\right) \right] \\
& -\lambda_{2,t+i} \left[x_{t+i} - \frac{1}{1+\psi} b_{t+i} - \frac{\psi}{1+\psi} q_{t+i} + \frac{1}{1+\psi} (r_{t+i} - a_{t+i}) - u_{t+i} \right] \\
& -\lambda_{3,t+i} \left[q_{t+i} - \tilde{\beta} s_{t+i} + \eta b_{t+i} + (r_{t+i} - a_{t+i}) - v_{t+i} \right] \\
& -\lambda_{4,t+i} \left[a_{t+i+1} - a_{t+i} - \gamma_{t+i+1} (\pi_{t+i} - a_{t+i}) \right] \\
& -\lambda_{5,t+i} \left[b_{t+i+1} - b_{t+i} - \gamma_{t+i+1} (x_{t+i} - b_{t+i}) \right] \\
& \left. -\lambda_{6,t+i} \left[s_{t+i+1} - s_{t+i} - \gamma_{t+i+1} (q_{t+i} - s_{t+i}) \right] \right\},
\end{aligned}
$$

where $\lambda_{i,t}$, with i=1, 2,...6 are Lagrangian multipliers associated with (17), (18), (21), and (22)-(24), respectively. The first-order conditions are obtained by differentiating the Lagrangian with respect to π_t, x_t, r_t, q_t, a_{t+1}, b_{t+1}, and s_{t+1}:

$$(1+\tau)\pi - \lambda_{1,t} + \lambda_{4,t}\gamma_{t+1} = 0, \tag{38}$$

$$\alpha x_t + \lambda_{1,t}\left(1+\chi\right)\kappa - \lambda_{2,t} + \lambda_{5,t}\gamma_{t+1} = 0, \tag{39}$$

$$\frac{1}{1+\psi}\lambda_{2,t} + \lambda_{3,t} = 0, \tag{40}$$

$$\frac{\psi}{1+\psi}\lambda_{2,t} - \lambda_{3,t} + \lambda_{6,t}\gamma_{t+1} = 0, \tag{41}$$

$$\tilde{\beta}\beta E_t\lambda_{1,t+1} + \tilde{\beta} E_t\lambda_{2,t+1} + \beta E_t\lambda_{3,t+1} - \lambda_{4,t} + \beta E_t\lambda_{4,t+1}(1-\gamma_{t+2}) = 0, \tag{42}$$

$$\tilde{\beta} E_t\lambda_{2,t+1} - \beta\eta E_t\lambda_{3,t+1} - \lambda_{5,t} + \beta E_t\lambda_{5,t+1}(1-\gamma_{t+2}) = 0, \tag{43}$$

$$\tilde{\beta}\beta E_t\lambda_{3,t+1} - \lambda_{6,t} + \beta E_t\lambda_{6,t+1}(1-\gamma_{t+2}) = 0. \tag{44}$$

Using (40), we obtain $\lambda_{3,t} = -\frac{1}{1+\psi}\lambda_{2,t}$. Substituting this result into (41) yields $\lambda_{2,t} = -\lambda_{6,t}\gamma_{t+1}$ and $\lambda_{3,t} = \frac{1}{1+\psi}\lambda_{6,t}\gamma_{t+1}$. These results and (43)-(44) imply that $\lambda_{2,t} = \lambda_{3,t} = \lambda_{5,t} = \lambda_{6,t} = 0$. Given this, we get from (39) that $\lambda_{1,t} = -\frac{\alpha x_t}{(1+\chi)\kappa}$. Combining this result with (38) gives the optimal inflation targeting rule when agents are learning:

$$x_t = -\frac{\kappa\,(1+\chi)\,(1+\tau)}{\alpha}\pi_t - \frac{(1+\chi)\,\kappa}{\alpha}\gamma_{t+1}\lambda_{4,t}. \tag{45}$$

The optimal targeting rule (45) is comparable to (27) except for the last term on the right-hand side of (45) that embodies the effect of learning on optimal monetary policy. The Lagrange multiplier $\lambda_{4,t}$ in (45) stands for, following Molnár and Santoro (2014), the marginal effect of an increase in inflation expectations on welfare loss at time $t+1$. This effect is responsible for the intertemporal trade-off for monetary policy introduced by learning. Since $\gamma_{t+1} > 0$, the sign of $\lambda_{4,t}$ is conditional on the sign of inflation expectations formed in the current period a_t. The possibility of having a positive a_t is as large as that of having a negative one given that we have set for simplicity the inflation target to zero. Consequently, the sign of a_t is conditional on the nature of past shocks. For a positive (negative) a_t, $\lambda_{4,t}$ is positive (negative) because a rise in a_t moves future inflation expectations further away from (closer to) the inflation target and implies lower (higher) social welfare.

The Lagrange multiplier $\lambda_{4,t}$ can be expressed using (45) as $\lambda_{4,t} = -\frac{\alpha}{\kappa(1+\chi)\gamma_{t+1}}x_t - \frac{(1+\tau)}{\gamma_{t+1}}\pi_t$. Using the latter, the solution of $\lambda_{1,t}$, (42) and $\lambda_{2,t+1} = \lambda_{3,t+1} = 0$, we obtain the optimal intertemporal trade-off condition between stabilizing inflation (the output gap) at period t and at period $t+1$, or optimal inflation targeting rule:

$$(1+\tau)\pi_t + \frac{\alpha}{\kappa\,(1+\chi)}x_t \quad = \quad \frac{\alpha\beta\gamma_{t+1}[1-(1-\tilde{\beta})\gamma_{t+2}]}{\kappa\,(1+\chi)\,\gamma_{t+2}}E_t x_{t+1} + \frac{\gamma_{t+1}\beta(1+\tau)(1-\gamma_{t+2})}{\gamma_{t+2}}E_t\pi_{t+1}. \tag{46}$$

The assumption of constant-gain learning, i.e., $\gamma_{t+2} = \gamma_{t+1} = \gamma$, allows us to rewrite the previous rule as

$$(1+\tau)\pi_t + \frac{\alpha}{\kappa\,(1+\chi)}x_t = \frac{\alpha\beta[1-\gamma(1-\tilde{\beta})]}{\kappa\,(1+\chi)}E_t x_{t+1} + \beta(1+\tau)(1-\gamma)E_t\pi_{t+1}. \tag{47}$$

The Phillips curve (17) and the rule (47) allow determining the equilibrium solution.

4.2 The ALMs for endogenous variables

We solve the model following the methodology of Molnár and Santoro (2014). The ALM for inflation establishing a feedback relationship between current inflation and inflation expectations (and technology shocks) when agents adopt constant-gain learning is given by (Appendices A.1 and A.2):

$$\pi_t = c_\pi^{cg} a_t + d_\pi^{cg} z_t, \tag{48}$$

with

$$\begin{aligned} c_\pi^{cg} &= \frac{-p_1 - \sqrt{p_1^2 - 4p_2 p_0}}{2p_2} > 0, & (49)\\ d_\pi^{cg} &= -\frac{\alpha\kappa(1+\chi)}{\Upsilon + \alpha\gamma^2\tilde{\beta}^2(1+\psi)\left(\tilde{\beta} - c_\pi^{cg}\right) + \tilde{\beta}\gamma(1+\psi)(1-\gamma)\left(\alpha\tilde{\beta} - c_\pi^{cg}\Upsilon\right)} < 0, & (50) \end{aligned}$$

where

$$\begin{aligned} p_0 &= \alpha\tilde{\beta}\left\{1 - \tilde{\beta}(1+\psi)(1-\gamma)\left[1 - \gamma\left(1 - \tilde{\beta}\right)\right]\right\} > 0, \\ p_2 &= \tilde{\beta}\gamma(1+\psi)\left\{\alpha\left[1 - \gamma\left(1 - \tilde{\beta}\right)\right] + \kappa^2(1+\chi)^2(1-\gamma)(1+\tau)\right\} > 0, \\ p_1 &= -\kappa^2(1+\chi)^2(1+\tau)\left[1 - \tilde{\beta}(1+\psi)(1-\gamma)\right] \\ &\quad -\alpha(1-\tilde{\beta})\left\{1 - \tilde{\beta}(1+\psi)\left[1 - \gamma\left(1 - \tilde{\beta}\right)\right]\right\} - p_0 - p_2 < 0. \end{aligned}$$

The feedback coefficient for inflation expectations in the ALM for inflation is positive whereas the one for the technology shocks is negative. It is shown that $0 < c_\pi^{cg} < 1$ and $d_\pi^{cg} < 0$ (Appendix A.3). The response of monetary policy to stock prices can offset their effect on the ALM for inflation but not the structural effects of stock prices on the Phillips curve, reflected by the fact that $\tilde{\beta} = \frac{\beta}{1+\psi} < \beta$ and $\psi > 0$ are present in both solutions of c_π^{cg} and d_π^{cg} when the turnover rate is positive. Notice that $\tilde{\beta} = \beta$ and $\psi = 0$ when the turnover rate is equal to zero.

The time horizon within which private agents' beliefs converge to RE is determined by the learning gain. The latter induces the persistence of inflation even when shocks are stochastic and affects the possibility of intertemporal trade-off for the CB.

If $\gamma = 0$, inflation expectations are constant over time with $a_t = a_{t-1}$ and $b_t = b_{t-1}$. We observe the convergence of the feedback coefficients towards the REE, by comparing the coefficients in (29) and those given below:

$$c_\pi^{cg} = \frac{\alpha\tilde{\beta}}{\Upsilon}, \tag{51}$$

$$d_\pi^{cg} = -\frac{\alpha\kappa(1+\chi)}{\Upsilon}. \tag{52}$$

The coefficients in the ALM for inflation given by (51)-(52) are equal to the corresponding ones in (29), i.e., the ALM for inflation under RE.

For $\gamma = 1$, i.e., inflation expectations are static with $a_t = \pi_{t-1}$ and $b_t = x_{t-1}$, we have

$$c_\pi^{cg} = \frac{\alpha\tilde{\beta}^3(1+\psi)+\Upsilon-\sqrt{\left[\alpha\tilde{\beta}^3(1+\psi)+\Upsilon\right]^2-4\alpha^2\tilde{\beta}^3(1+\psi)}}{2\alpha\tilde{\beta}^2(1+\psi)}, \tag{53}$$

$$d_\pi^{cg} = -\frac{\alpha\kappa(1+\chi)}{\Upsilon+\alpha\tilde{\beta}^2(1+\psi)\left(\tilde{\beta}-c_\pi^{cg}\right)}. \tag{54}$$

When the learning gain γ hits 1, inflation is self-sustained because private agents' inflation expectations are only depending on past inflation.

Inserting π_t given by (48) into (17) yields the ALM for the output-gap:

$$x_t = c_x^{cg} a_t + d_x^{cg} z_t, \tag{55}$$

where $c_x^{cg} = -\frac{\tilde{\beta}-c_\pi^{cg}}{\kappa(1+\chi)}$ and $d_x^{cg} = 1 + \frac{d_\pi^{cg}}{\kappa(1+\chi)}$.

The feedback coefficient on inflation expectations in (55) is negative while the feedback coefficient on the technology shock is positive but smaller than 1. It is straightforward to find the corresponding values of c_x^{cg} and d_x^{cg} for the limit cases, i.e., $\gamma = 0$ or $\gamma = 1$ using (51)-(54). We notice that since κ is very small, the impact of a positive inflation penalty rate on the feedback coefficients on inflation expectations and the technology shocks in the ALM for the output gap is substantially greater than that on corresponding feedback coefficients in the ALM for inflation.

Combining (18), (21)-(24), (48) and (55) allows obtaining the ALM for the interest rate:

$$r_t = c_r^{cg} a_t + \delta_x b_t + \delta_q s_t + u_t + \delta_v v_t + d_r^{cg} z_t \tag{56}$$

with $c_r^{cg} = 1 + \frac{\tilde{\beta} - c_\pi^{cg}}{\kappa(1+\chi)}$, $\delta_x = \frac{1-\psi\eta}{1+\psi}$, $\delta_q = \frac{\psi\tilde{\beta}}{1+\psi}$, $\delta_v = \frac{\psi}{1+\psi}$, and $d_r^{cg} = -1 - \frac{d_\pi^{cg}}{\kappa(1+\chi)}$.

We find the ALM for stock prices using the ALMs (48), (55) and (56) combined with (18) and (21):

$$q_t = c_q^{cg} a_t + \omega_x b_t + \omega_q s_t + \omega_u u_t + \omega_v v_t + d_q^{cg} z_t, \tag{57}$$

where $c_q^{cg} = -\frac{\tilde{\beta} - c_\pi^{cg}}{\kappa(1+\chi)} = c_x^{cg}$, $\omega_x = -\left(\frac{1+\eta}{1+\psi}\right)$, $\omega_q = \frac{\tilde{\beta}}{(1+\psi)}$, $\omega_u = -1$, $\omega_v = \frac{1}{(1+\psi)}$, and $d_q^{cg} = 1 + \frac{d_\pi^{cg}}{\kappa(1+\chi)}$.

4.3 The effects of learning and turnover rate on feedback coefficients

The effect of learning gain on the feedback coefficient on inflation expectations in the ALM for inflation is obtained by differentiating (49) with respect to γ as (see Appendix A.4):

$$\frac{\partial c_\pi^{cg}}{\partial \gamma} = \frac{\alpha\tilde{\beta} - \Upsilon c_\pi^{cg}}{\alpha\tilde{\beta}^2 p_2 \sqrt{p_1^2 - 4p_2p_0}} \left(p_0 \frac{\partial p_1}{\partial \gamma} - p_1 \frac{\partial p_0}{\partial \gamma}\right),$$

where

$$\begin{aligned}
\frac{\partial p_2}{\partial \gamma} &= \alpha\tilde{\beta}\left(1+\psi\right)\left[1 - 2\gamma(1-\tilde{\beta})\right] + \tilde{\beta}\kappa^2\left(1+\chi\right)^2\left(1-2\gamma\right)\left(1+\psi\right)\left(1+\tau\right), \\
\frac{\partial p_0}{\partial \gamma} &= \alpha\tilde{\beta}^2\left(1+\psi\right)\left[\left(1-\tilde{\beta}\right)\left(1-\gamma\right) + 1 - \gamma\left(1-\tilde{\beta}\right)\right] > 0, \\
\frac{\partial p_1}{\partial \gamma} &= -2\tilde{\beta}\left(1-\gamma\right)\left(1+\psi\right)\left[\alpha + \kappa^2\left(1+\chi\right)^2\left(1+\tau\right)\right] - 2\alpha\gamma\tilde{\beta}^3\left(1+\psi\right) < 0.
\end{aligned}$$

Differentiating (50) with respect to γ yields

$$\frac{\partial d_\pi^{cg}}{\partial \gamma} = \frac{\alpha\tilde{\beta}(1+\psi)\kappa(1+\chi)\left\{2\alpha\gamma\tilde{\beta}\left(\tilde{\beta} - c_\pi^{cg}\right) + (1-2\gamma)\left(\alpha\tilde{\beta} - c_\pi^{cg}\Upsilon\right) - \gamma\left[\alpha\gamma\tilde{\beta} + (1-\gamma)\Upsilon\right]\frac{\partial c_\pi^{cg}}{\partial \gamma}\right\}}{\Upsilon + \alpha\gamma^2\tilde{\beta}^2(1+\psi)\left(\tilde{\beta} - c_\pi^{cg}\right) + \tilde{\beta}\gamma(1+\psi)(1-\gamma)\left(\alpha\tilde{\beta} - c_\pi^{cg}\Upsilon\right)}.$$

Given the link between the ALM for inflation and the ALMs for the output gap, stock prices and the interest rate, it is easy to find that $\frac{\partial c_x^{cg}}{\partial \gamma} = -\frac{\partial c_r^{cg}}{\partial \gamma} = \frac{\partial c_q^{cg}}{\partial \gamma} = \frac{1}{\kappa(1+\chi)}\frac{\partial c_\pi^{cg}}{\partial \gamma}$ and

$\frac{\partial d_x^{cg}}{\partial \gamma} = -\frac{d_r^{cg}}{\partial \gamma} = \frac{\partial d_q^{cg}}{\partial \gamma} = \frac{1}{\kappa(1+\chi)} \frac{\partial d_\pi^{cg}}{\partial \gamma}$.

Proposition 4. *For a positive learning gain, the feedback coefficient on inflation expectations in the ALMs for inflation, the output gap and stock prices (the interest rate) is lower (higher) than under RE, for a given inflation penalty rate. The higher is the learning gain, the greater the deviation of these coefficients from their corresponding ones obtained under RE if*

$$-1 < \tau < \frac{\alpha\tilde{\beta}\left[2 - \tilde{\beta}^2 (1+\psi)\right] - \alpha}{\kappa^2 (1+\chi)^2} - 1 \tag{58}$$

and[12]

$$0 \leq \nu < \bar{\nu}. \tag{59}$$

For the feedback coefficients on the technology shock, the effect of learning is reversed under these conditions.

Proof. Proposition 4 follows from differentiating c_π^{cg}, d_π^{cg}, c_x^{cg}, d_x^{cg}, c_q^{cg}, d_q^{cg}, c_r^{cg} and d_r^{cg}, using their respective definition, with respect to γ. See Appendix A.4.□

Proposition 4 gives the conditions under which $\frac{\partial c_x^{cg}}{\partial \gamma} = -\frac{\partial c_r^{cg}}{\partial \gamma} = \frac{\partial c_q^{cg}}{\partial \gamma} = \frac{1}{\kappa(1+\chi)} \frac{\partial c_\pi^{cg}}{\partial \gamma} < 0$, and $\frac{\partial d_x^{cg}}{\partial \gamma} = -\frac{d_r^{cg}}{\partial \gamma} = \frac{\partial d_q^{cg}}{\partial \gamma} = \frac{1}{\kappa(1+\chi)} \frac{\partial d_\pi^{cg}}{\partial \gamma} > 0$. In the case of the ALM for inflation, $\frac{\partial c_\pi^{cg}}{\partial \gamma} < 0$ means that a higher learning gain leads to a smaller response of inflation to inflation expectations. Since $c_\pi^{cg} > 0$, the higher the learning gain is, the larger the deviation of the feedback coefficient on inflation expectations from its RE level. For $\frac{\partial d_\pi^{cg}}{\partial \gamma} > 0$, the feedback coefficient on the technology shock increases with the learning gain. As d_π^{cg} is negative, a positive technology shock reduces less inflation as the learning gain increases.

This proposition focuses on the effect of learning for a given turnover rate. According to it, the aggressiveness of the interest rate policy increases with the learning gain if conditions (58)-(59) are verified. By being aggressive, monetary policy improves the stabilization of inflation but deteriorates the stabilization of the output gap and increases the volatility of stock prices.

Conditions (58)-(59) are sensitive to the turnover rate. For $\epsilon = 2.3$, $\beta = 0.99$, we obtain the threshold for ν such that $\bar{\nu} < 0.082$. Thus, the condition (59) is generally verified since

[12] $\bar{\nu}$ is the solution that verifies $\nu \frac{1-\beta(1-\nu)}{(1-\nu)\epsilon} \frac{1+r(\nu)}{r(\nu)} < \beta - 1 + \beta\sqrt{1-\beta}$, where $r(\nu)$ is defined in footnote 6.

the value for ν is clearly smaller than $\bar{\nu}$ according to Airaudo et al. (2015). In the absence of stock prices, i.e., $\psi = \nu = 0$, the right-hand side of (58) will be largely positive, meaning that Proposition 4 is in general true for all values of γ.

Using the parameter values $\alpha = 0.048$, $\beta = 0.99$, $\sigma = 0.157$, $r = 0.01$, $\theta = 0.9$, $\chi = 1$, $\epsilon = 2.3$, and $\tau = 0$, we draw the relationship between the feedback coefficients in the ALMs for three values of ν, i.e., $\nu = 0$, $\nu = 0.025$, and $\nu = 0.05$, in Figure 1.

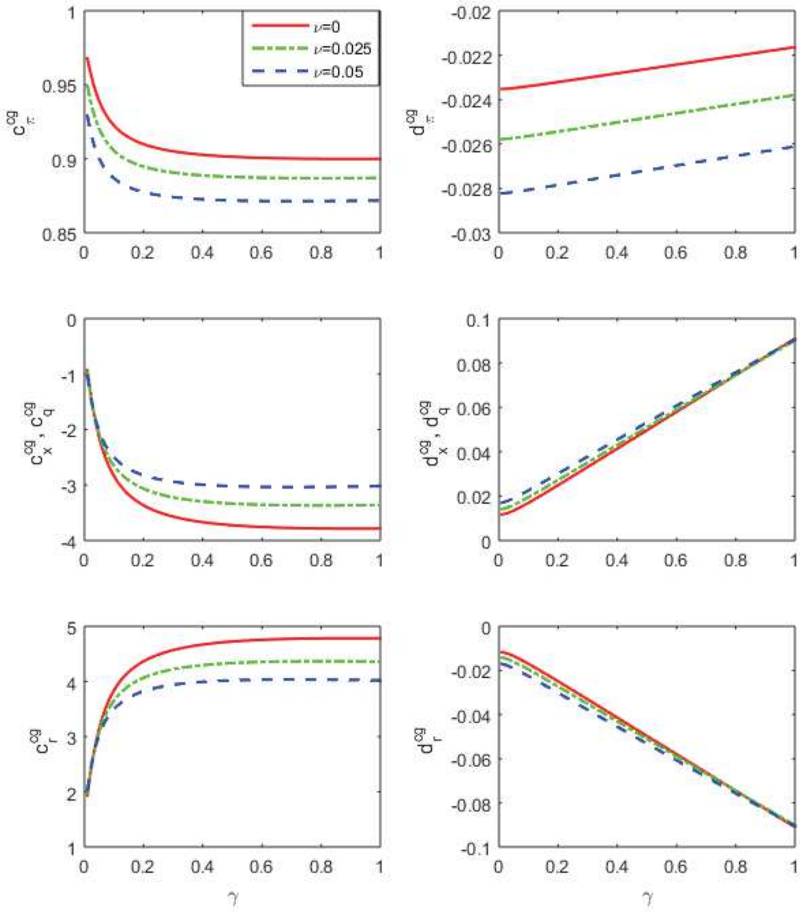

Figure 1: Feedback coefficients on inflation expectations and technology shocks in the ALMs with $\tau = 0$.

It follows from Figure 1 that in the absence of inflation penalty ($\tau = 0$), an increase in turnover rate reduces (increases) the feedback coefficients on the technology shock in the

ALMs for inflation and the interest rate (the output gap and stock prices) for all values of learning gain. Higher turnover rate implies a decrease in the feedback coefficient on inflation expectations in the ALM for inflation for any learning gain. An increase in turnover rate clearly increases (decreases) the feedback coefficient on inflation expectations in the ALMs for the output gap and stocks prices (the interest rate) for $\gamma > 0.05$. For smaller learning gains, the effect of turnover rate is insignificant and reversed for these three feedback coefficients. Simulations for $\tau > 0$ shows similar effect of an increase in turnover rate.

For $\tau < 0$, the previous effects are still valid for most feedback coefficients except that an increase in turnover rate could either raise or reduce the feedback coefficients on the technology shock in the ALMs for the output gap, stock prices and the interest rate, depending on learning gain and turnover rate. More precisely, an increase in turnover rate raises (reduces) the feedback coefficients on the technology shock in the ALMs for the output gap and stock prices (the interest rate) if the learning gain is sufficiently low with the threshold depending on the level of turnover rate. These results are summarized in the following proposition.

Proposition 5. *For positive inflation penalty rates, an increase in turnover rate significantly mitigates the aggressiveness of monetary policy in response to inflation expectations as the learning gain is sufficiently high ($\gamma > 0.05$) and amplifies it in response to the technology shock particularly when the learning gain is relatively low ($\gamma < 0.85$). An increase in turnover rate thus generally attenuates the responses of inflation, the output gap and stock prices to a change in inflation expectations ($\gamma > 0.05$) but amplifies those to the technology shock ($\gamma < 0.85$).*

4.4 The effects of inflation penalty rate on feedback coefficients

Imposing an inflation penalty is a mean to induce the CB to improve the trade-off between inflation and the output gap. The effect of an increase in inflation penalty rate on the ALMs could be obtained by differentiating the feedback coefficients in (48) and (55) with respect to τ (Appendix A.5):

$$\frac{\partial c_{\pi}^{cg}}{\partial \tau} = \kappa \left(1+\chi\right) \frac{\partial c_{x}^{cg}}{\partial \tau} = \frac{c_{\pi}^{cg}\left(p_2 \frac{\partial p_1}{\partial \tau} - p_1 \frac{\partial p_2}{\partial \tau}\right) - p_0 \frac{\partial p_2}{\partial \tau}}{p_2 \sqrt{p_1^2 - 4 p_2 p_0}} < 0,$$

$$\frac{\partial d_{\pi}^{cg}}{\partial \tau} = \kappa\left(1+\chi\right)\frac{\partial d_{x}^{cg}}{\partial \tau} = \frac{\alpha\tilde{\beta}\kappa(1+\psi)(1+\chi)\left\{2\alpha\gamma\tilde{\beta}\left(\tilde{\beta}-c_{\pi}^{cg}\right)+(1-2\gamma)\left(\alpha\tilde{\beta}-c_{\pi}^{cg}\Upsilon\right)-\gamma\left[\alpha\gamma\tilde{\beta}+(1-\gamma)\Upsilon\right]\frac{\partial c_{\pi}^{cg}}{\partial\gamma}\right\}}{\Upsilon+\alpha\gamma^{2}\tilde{\beta}^{2}(1+\psi)\left(\tilde{\beta}-c_{\pi}^{cg}\right)+\tilde{\beta}\gamma(1+\psi)(1-\gamma)\left(\alpha\tilde{\beta}-c_{\pi}^{cg}\Upsilon\right)} > 0$$

where $\frac{\partial p_1}{\partial \tau} = -\kappa^2\left(1+\chi\right)^2\left[1-\tilde{\beta}(1-\gamma)^2\left(1+\psi\right)\right] < 0$, and $\frac{\partial p_2}{\partial \tau} = \gamma\tilde{\beta}\kappa^2\left(1+\psi\right)\left(1+\chi\right)^2(1-\gamma) > 0$.

For $\gamma = 0$, the CB cannot influence private agents' expectations by varying the actual inflation given that private expectations are stationary according to (22). Since the possibility for the CB to manipulate private expectations is null, the effect of inflation penalty will be smaller in this case than when $\gamma > 0$.

When $\gamma = 1$, an increase in inflation penalty makes the largest impact (compared to any other learning gain) on the feedback effects of inflation expectations and the technology shocks in the ALMs for endogenous variables. Numerical simulations show that for $\gamma > 0.20$, the impact of inflation penalty on the feedback coefficients on inflation expectations (the technology shocks) in the ALMs is very close to (still far from) the ones obtained when $\gamma = 1$.

The feedback coefficients on inflation expectations (the technology shock) in the ALMs for inflation, the output gap and stock prices, i.e., c_{π}^{cg}, c_{x}^{cg} and c_{q}^{cg} (d_{π}^{cg}, d_{x}^{cg} and d_{q}^{cg}), are decreasing (increasing respectively) in inflation penalty rate while the reversed effect is observed for the feedback coefficients in the ALM for the interest rate (Appendix A.5). Figure 2 shows how a change in τ makes these coefficients vary. More specifically, an increase in both inflation penalty rate and learning gain raises the aggressiveness of the interest rate policy in response to a change in inflation expectations and the technology shock, reducing (increasing) thus the deviations of inflation (the output gap and stock prices) compared to the REE. An increase in inflation penalty rate reduces the volatility of inflation and thus the stabilization bias but at the cost of increasing the volatility of the output gap and stock prices. An increase in learning gain strengthens these trends and more so as the inflation penalty rate rises. The effect of learning is null when $\tau = 0$. We remark that the feedback coefficients on inflation expectations are very sensitive to learning gain in all ALMs, contrary to the feedback coefficients on the technology shock.

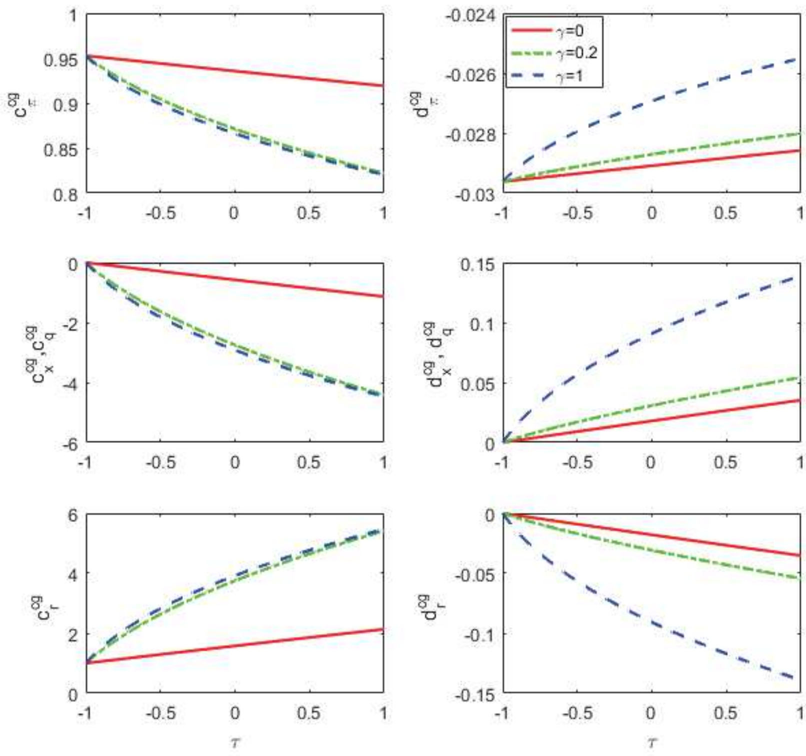

Figure 2: Feedback coefficients on inflation expectations and the technology shock in the ALMs for a positive turnover rate ($\nu = 0.025$).

For standard parameter values, Figure 3 shows how the turnover rate affects the effects of inflation penalty rate for a given learning gain. An increase in turnover rate clearly weakens (reinforces) the amplification (attenuation) effect of inflation penalty rate on the feedback coefficients in the ALMs for the output gap, stock prices and the interest rate (inflation).

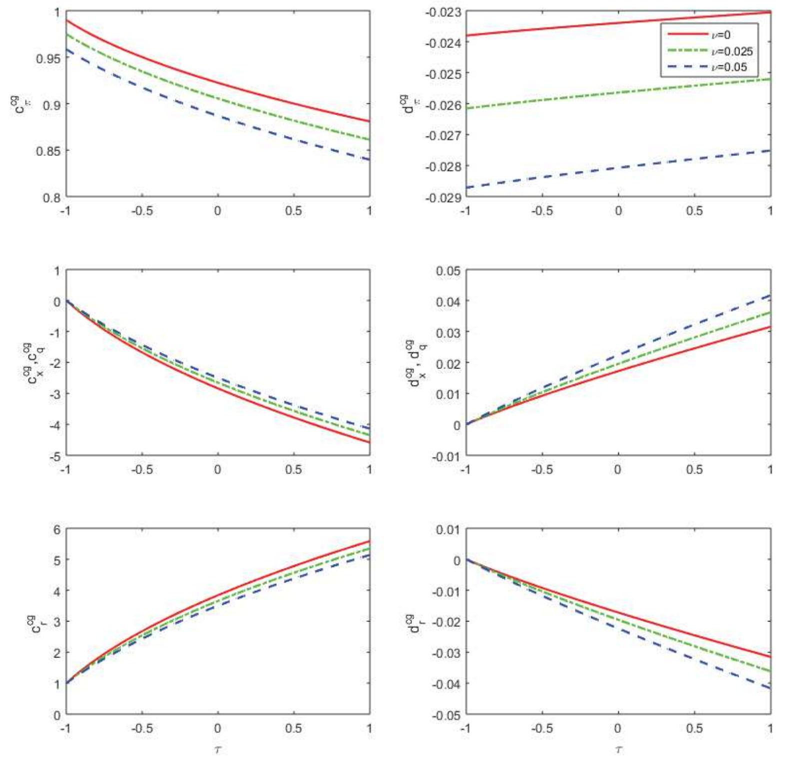

Figure 3: The effect of inflation penalty for a given learning gain, i.e, $\gamma = 0.1.$, and various turnover rates.

Figure 4 illustrates how the turnover rate affects the deviation of the feedback coefficients in the ALMs from their respective RE equilbrium level. An increase in turnover rate generally reduces the deviations of the feedback coefficients in the ALMs except for the feedback coefficient on the technology shock in the ALM for inflation. As shown in Figure 4, the effect of turnover rate is particularly important for the output gap, stock prices and the interest rate while it is almost insignificant for inflation.

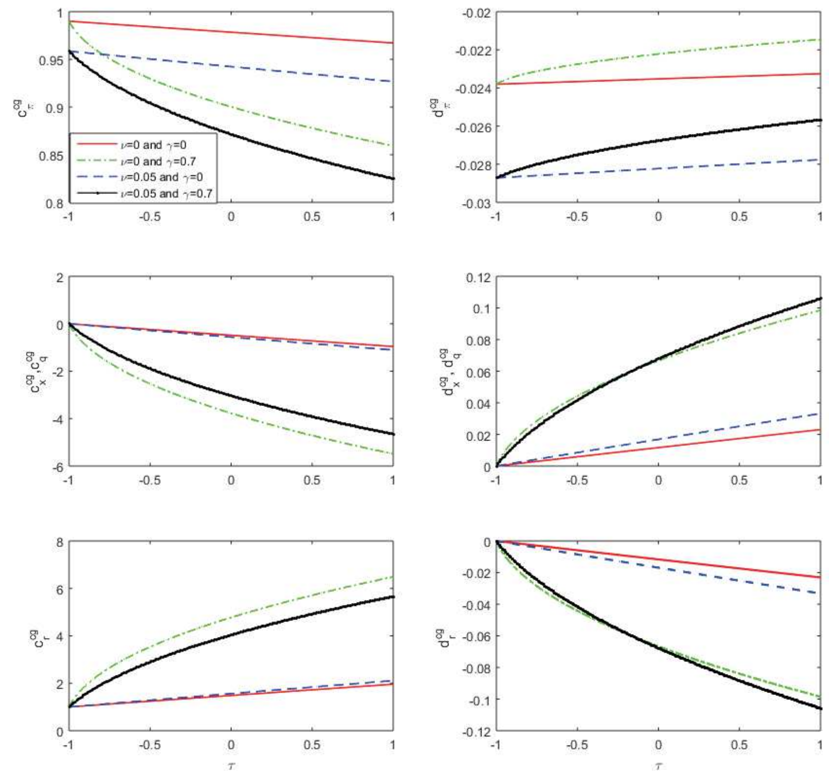

Figure 4: The impact of the turnover rate on the combined effect of inflation penalty rate and learning gain.

The simulation results (Figures 3 and 4) lead to the following proposition.

Proposition 6. *The more conservative the central banker is, the more aggressive is the interest rate policy and the lower (higher) the feedback coefficients on inflation expectations (on the technology shock) in the ALMs for inflation, the output gap, stock prices. An increase in learning gain raises the aggressiveness of the interest rate policy, and reduces (increases) the volatility of inflation (the output gap and stock prices) due to changes in inflation expectations and the technology shock, and increasingly so as the degree of CB conservatism* (*or inflation penalty rate* τ) *increases. A higher turnover rate generally leads to smaller deviations of the feedback coefficients in the ALMs, except for the feedback coefficient on the technology shock in the ALM for inflation.*

On the one hand, learning reinforces the effect of CB conservatism on inflation stabilization but less than on the stabilization of the output gap. On the other hand, the optimal response of monetary policy to stock prices implies a more aggressive interest rate policy that reduces more drastically inflation than it increases the output gap. Moreover, a positive turnover rate in the stock market reduces the effect of learning. All effects taken into account, the CB increasingly improves inflation stabilization but deteriorates output-gap stabilization as its degree of conservatism rises.

These effects have important implications for the optimal choice of inflation penalty rate by the government and how the CB manages future inflation expectations. By reducing the feedback effect of inflation expectations on current inflation, a positive inflation penalty could have a contractionary effect on future inflation expectations, with its importance depending on learning gain according to (22). This reduces the possibility for the CB to control future inflation expectations through the feedback between inflation and inflation expectations. A higher inflation penalty rate induces the CB to focus more on reducing inflation volati1ity while accepting a higher volatility of the output gap in the event of positive technology shocks, making the stabilization bias smaller. However, learning reduces the benefit of setting the inflation penalty rate at higher level since it decreases inflation volatility while increasing the volatility of the output gap. These effects are attenuated by an increase in turnover rate in the stock market. Thus, one would expect that a decrease in inflation penalty rate could improve social welfare by allowing the CB to correct a too severe deviation of the output gap, induced by learning, from its RE equilbrium level.

5 Optimal inflation penalty rate

The feedback coefficients in the ALMs for inflation and the output gap depend on inflation penalty rate, turnover rate and learning gain. Consequently, the contribution of inflation and output-gap volatility to the social welfare loss are function of these parameters. Inserting π_t

and x_t given by (48) and (55) into the social loss function (25) yields

$$L_t^s = \frac{1}{2}\sum_{i=0}^{+\infty}\beta^i\left\{\left[(c_\pi^{cg})^2+\frac{\alpha(c_\pi^{cg}-\tilde{\beta})^2}{\kappa^2(1+\chi)^2}\right]E_t(a_{t+i}^2)+\left[(d_\pi^{cg})^2+\frac{\alpha\left[d_\pi^{cg}+\kappa(1+\chi)\right]^2}{\kappa^2(1+\chi)^2}\right]E_t(z_{t+i}^2)\right\} \tag{60}$$

Given that $\tilde{\beta} < c_\pi^{cg}$ and $d_\pi^{cg} < 1$, the volatility of inflation (the output gap) is decreasing (increasing respectively) in inflation penalty rate.

For $\gamma = 0$, using $c_x^{cg} = -\frac{\tilde{\beta}-c_\pi^{cg}}{\kappa(1+\chi)}$, $d_x^{cg} = -\frac{d_\pi^{cg}+\kappa(1+\chi)}{\kappa(1+\chi)}$ and (51)-(52), the social loss function (60) is reduced to

$$\begin{aligned} L_t^s &= \frac{1}{2}\sum_{i=0}^{+\infty}\beta^i\left\{\alpha\tilde{\beta}^2\left[\frac{\alpha(1-\alpha)+\kappa^2(1+\chi)^2\left[\alpha-2\alpha(1+\tau)+(1+\tau)^2\right]}{\kappa^2(1+\chi)^2\Upsilon^2}\right]E_t(a_{t+i}^2)\right. \\ &\left. +\alpha^2\left[\frac{\alpha+\kappa^2(1+\chi)^2}{\Upsilon^2}-\frac{\left[2-\alpha-\kappa^2(1+\chi)^2(1+\tau)\right]}{\Upsilon}\right]E_t(z_{t+i}^2)\right\} \end{aligned} \tag{61}$$

The minimization of (61) leads the government to set the optimal inflation penalty rate at $\tau = 0$. This result shows that when private agents' expectations are stationary (absence of learning), the government sets the same optimal inflation penalty rate as under RE. This is explained by the fact that in both cases, inflation expectations are always equal to the inflation target in the absence of any average inflation bias, the government cannot influence inflation expectations and hence current inflation by imposing an inflation penalty on the CB. Indeed, in both cases, the CB cannot make an intertemporal trade-off by manipulating private future expectations. As a result, the imposition of an inflation penalty different from zero worsens social welfare.

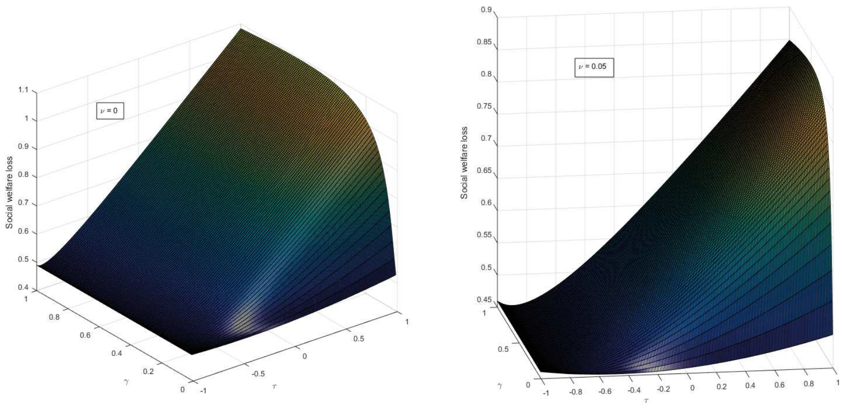

Figure 5: The welfare social loss varying with τ and γ

When $\gamma > 0$, the social loss function (60) is too complex to allow for an analytical solution of τ. Given that τ in each period is time independent, we can compute the optimal level of τ by examining the unconditional (or average) expected social loss function in one period. We proceed to numerically simulate the social loss function by setting $\alpha = 0.048$, $\beta = 0.99$, $\sigma = 0.157$, $r = 0.01$, $\chi = 0.25$, $\epsilon = 2.3$, $\theta = 0.8752$, $var(a) = 0.5$, and $var(z) = 0.5$ for $\gamma \in (0,1)$, and $\tau \in (-1,1)$ and two level of turnover rates, i.e., $\nu = 0$ and $\nu = 0.05$. It results from Figure 5 two findings. First, the social loss is substantially lower for $\nu = 0.05$ compared to the case where $\nu = 0$. Second, the optimal level of inflation penalty rate is negative when private agents are learning and decreases at a faster pace for $\nu = 0$ than when $\nu = 0.05$.

Proposition 7. *If the learning gain in the learning algorithms* (22)-(24) *is equal to zero, i.e.,* $\gamma = 0$, *the government sets the optimal inflation penalty rate to zero, for any turnover rate in the stock market. For positive learning gains* $\gamma \in (0,1)$, *the government sets an optimal inflation penalty rate that decreases with* γ *such that* $\tau \in (-1,0)$. *When the turnover rate rises, the optimal inflation penalty rate becomes less negative, and increasingly so as the learning gain* γ *tends to unity.*

Table 1 shows numerically how changes in turnover rate and learning gain affect the optimal

	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 1$
$\nu = 0$	$\tau = 0$	$\tau = -0.9232$	$\tau = -0.959$
$\nu = 0.025$	$\tau = 0$	$\tau = -0.8945$	$\tau = -0.9278$
$\nu = 0.5$	$\tau = 0$	$\tau = -0.8639$	$\tau = -0.8942$

Table 1: Optimal penalty rate according to turnover rates and learning gains

inflation penalty rate.

To understand why, when private agents are learning, appointing a central banker with the same preferences as the society is not socially optimal, we must know how learning affects the way monetary policy is conducted. The fact that private agents use learning algorithms to correct expectations errors makes it possible for the CB to manipulate their future expectations. However, from the social point of view, the equilibrium under adaptive learning is not optimal because inflation and output-gap expectations based on past information deviate from correct expectations formed with the knowledge of the distribution law of cost-push shocks. Imposing a positive inflation penalty rate does not improve social welfare since it amplifies the deviations of the feedback coefficients in the ALMs for inflation and the output gap under adaptive learning from the corresponding one under RE, aggravating the problem of an excessive reduction of stabilization bias under learning during the transition to the steady-state equilibrium. The only possible choice for the government is to set a negative inflation penalty rate to prompt the CB to mimic the REE. This is equivalent to delegate monetary policy to a liberal central banker. An increase in turnover rate in the stock market offsets partially the effect of learning by reducing quite significantly the level of inflation as well as the deviation of feedback coefficients in the ALM for the output gap from their corresponding levels at the RE equilbrium (see Figure 4), and increasingly so as the learning gain rises. This explains why the inflation penalty rate is less negative when the turnover rate is positive than in its absence.

6 Discussion

The previous results show that the turnover rate in the stock market substantially affects monetary delegation and economic dynamics in a New Keynesian model when agents are learning. While this paper gives interesting insights into the issue of CB accountability when the CB optimally reacts to stock prices, it can be completed by future studies that examine important issues not examined in the above.

One interesting complementary study to this paper is to examine the delegation framework with linear inflation contract as in Walsh (1995). Linear inflation contracts have been extensively studied in Barro-Gordon and New Keynesian frameworks under RE hypothesis. André and Dai (2017b) have shown that the conception of linear inflation contract when agents are learning is radically different from the one conceived under RE. Extending the model to include stock prices could add further insights about monetary delegation in a context where financial stability becomes a major concern for CBs around the world.

Another important extension is to consider that the CB has an objective of financial stability as in Machado (2013). Since the global financial crisis and the great recession of 2008-09, a heated debate has arisen among economists and central bankers about how to stabilize the financial market and to deal with the consequences of financial instability in the course of conducting monetary policy. Indeed, financial stability can affect the trade-off between inflation and the output gap and has become a crucial issue for monetary policy. Since stabilizing the financial market helps reduce the risk of economic instability, it is reasonable to include an objective of financial stabilization into the CB's objective function. However, the society and hence the government might be still focusing on social welfare. This divergence between the CB's objectives and these of the society should lead the government to reconsider monetary delegation. Including an objective of financial stability implies on the one hand a relative reduction of the weight on inflation stabilization and on the other hand a better stabilization of inflation given that the objective of financial stability calls for a more aggressive interest rate policy, and has therefore an ambiguous effect on the optimal degree of CB conservatism. It will be interesting to examine how both linear and non-linear inflation contracts are affected by the objective of financial stability when agents are learning.

Our paper only considers the solution converging to the equilibrium and hence the absence of bubbles in stock prices. Recent experiences in stock prices have shown that the speculation in the stock market can lead stock prices to excessively deviate from their fundamental values and finally to inevitable crash. The CB can either adopt a "mopping up after" strategy that focuses on dealing on the consequences of bubble bursting on the economy or the "leaning against the wind" strategy that tries to prevent the formation of bubbles (Issing 2009). The optimal choice among these two strategies could depend on the way agents form their expectations. Monetary delegation could be substantially affected by such choice and the speed of learning.

7 Conclusion

This paper studies how stock prices affect monetary delegation when agents are learning. The key determinant of the interactions between monetary policy (hence delegation) and stock prices is the turnover rate in the stock market.

Both under rational expectations or learning, optimal interest rate policy reacts to stock prices only if the turnover rate in the stock market is positive, meaning that holding stocks by households distorts the optimal consumption path. The responsiveness of optimal monetary policy to stock prices increases with the turnover rate. This type of policy ensures the dynamic stability of the economy. Given that optimal policy fully offsets stock-price shocks and hence their effects on the equilibrium, such shocks do not affect equilibrium inflation and output gap.

Compared to the rational expectations equilibrium, learning increases the aggressiveness of the interest rate policy, thus reducing (increasing) the feedback effect of inflation expectations on inflation (the output gap and stock prices) while the impact is reversed for a positive technology shock. Such deviations increase with learning gain and inflation penalty rate. An increase in turnover rate generally attenuates the responses of inflation, the output gap and stock prices and the interest rate to a change in inflation expectations and the technology shock.

Given the distortions introduced by learning on the dynamics of the economy, the government should impose an optimal inflation penalty rate that is negative and decreases with the learning gain. When the learning gain approaches zero, the optimal inflation penalty rate is equal to zero as under rational expectations, but tends to minus unity as the learning gain tends to unity. A higher turnover rate in the stock market, by generally attenuating the effects of learning on the deviations of the feedback effects of inflation expectations and the technology from the ones under RE, will make the optimal inflation penalty rate less negative for a given learning gain.

A APPENDIX

A.1 Finding the law of motion under learning

A.1.1 The ALM for inflation

Using (17) and the fact that $\tilde{\beta} \equiv \frac{\beta}{1+\psi} \Rightarrow \beta = (1+\psi)\,\tilde{\beta}$, we rewrite (47) as

$$E_t\pi_{t+1} = A_{11,t}\pi_t + A_{12,t}a_t + P_{1,t}z_t, \tag{A.1}$$

with

$$A_{11} \equiv \frac{\alpha + \alpha\gamma\tilde{\beta}^2(1+\psi)[1-\gamma(1-\tilde{\beta})] + \kappa^2(1+\chi)^2(1+\tau)}{(1+\psi)\tilde{\beta}\{\alpha[1-\gamma(1-\tilde{\beta})] + \kappa^2(1+\chi)^2(1-\gamma)(1+\tau)\}}, \tag{A.2}$$

$$A_{12} \equiv -\alpha\tilde{\beta}\frac{1-\tilde{\beta}(1+\psi)\,(1-\gamma)\,[1-\gamma(1-\tilde{\beta})]}{(1+\psi)\,\tilde{\beta}\{\alpha[1-\gamma(1-\tilde{\beta})] + \kappa^2(1+\chi)^2(1-\gamma)(1+\tau)\}}, \tag{A.3}$$

$$P_1 \equiv \frac{\alpha\kappa(1+\psi)}{(1+\psi)\,\tilde{\beta}\{\alpha[1-\gamma(1-\tilde{\beta})] + \kappa^2(1+\chi)^2(1-\gamma)(1+\tau)\}}, \tag{A.4}$$

where P_1 is the coefficient on the technology shock z_t.

It follows from the proposition 1 from Blanchard and Kahn (1980) that the solution of the ALM for inflation takes the following form:

$$\pi_t = c_\pi^{cg}a_t + d_\pi^{cg}z_t. \tag{A.5}$$

With the help of (22) and (A.5), we obtain:

$$E_t\pi_{t+1} = c_\pi^{cg}\left[(1-\gamma)a_t + \gamma\pi_t\right] \tag{A.6}$$

Using (A.6) to eliminate $E_t\pi_{t+1}$ in (A.1) and arranging terms give:

$$\pi_t = \frac{A_{12} - c_\pi^{cg}(1-\gamma)}{c_\pi^{cg}\gamma - A_{11}} a_t + \frac{P_1}{c_\pi^{cg}\gamma - A_{11}} z_t. \tag{A.7}$$

Comparing this with (A.5) yields

$$c_\pi^{cg} = \frac{A_{12} - c_\pi^{cg}(1-\gamma)}{c_\pi^{cg}\gamma - A_{11}}, \tag{A.8}$$

$$d_\pi^{cg} = \frac{P_1}{c_\pi^{cg}\gamma - A_{11}}. \tag{A.9}$$

A.1.2 The ALMs for the output gap, the interest rate and stock prices

To obtain the ALM for the output gap, we combine the Philips Curve (17) and the ALM for inflation (A.5) as

$$x_t = c_x^{cg} a_t + d_x^{cg} z_t, \tag{A.10}$$

where $c_x^{cg} = -\frac{\tilde{\beta} - c_\pi^{cg}}{\kappa(1+\chi)}$ and $d_x^{cg} = \frac{d_\pi^{cg} + \kappa(1+\chi)}{\kappa(1+\chi)}$.

Using the IS equation (18) to obtain an expression of q_t and equaling it to the one given by stock-price equation (21), while substituting x_t given by (A.10), we obtain

$$r_t = c_r^{cg} a_t + \delta_x b_t + \delta_q s_t + u_t + \delta_v v_t + d_r^{cg} z_t, \tag{A.11}$$

with $c_r^{cg} = 1 + \frac{\tilde{\beta} - c_\pi^{cg}}{\kappa(1+\chi)}$, $\delta_x = \frac{1-\psi\eta}{1+\psi}$, $\delta_q = \frac{\psi\tilde{\beta}}{1+\psi}$, $\delta_v = \frac{\psi}{1+\psi}$, and $d_r^{cg} = -1 - \frac{d_\pi^{cg}}{\kappa(1+\chi)}$.

Using (21) and (A.11), we find the ALM for stock prices:

$$q_t = c_q^{cg} a_t + \omega_x b_t + \omega_q s_t + \omega_u u_t + \omega_v v_t + d_q^{cg} z_t,$$

where $c_q^{cg} = -\frac{\tilde{\beta} - c_\pi^{cg}}{\kappa(1+\chi)} = c_x^{cg}$, $\omega_x = -\frac{1+\eta}{1+\psi}$, $\omega_q = \frac{\tilde{\beta}}{1+\psi}$, $\omega_u = -1$, $\omega_v = \frac{1}{1+\psi}$ and $d_q^{cg} = 1 + \frac{d_\pi^{cg}}{\kappa(1+\chi)}$.

A.2 The dynamic stability under learning

Putting (22) and (A.1) into matrix form leads to

$$E_t y_{t+1} = A y_t + P z_t, \tag{A.12}$$

where

$$y_t \equiv [\pi_t,\, a_t]\,,\; A \equiv \begin{bmatrix} A_{11} & A_{12} \\ \gamma & 1-\gamma \end{bmatrix},\; \text{and}\; P \equiv \begin{bmatrix} P_1 \\ 0 \end{bmatrix}.$$

The dynamics of b_t and s_t can be expressed in terms of π_t and a_t so that if the latter converge, b_t and s_t also converge. System (A.12) has two boundary conditions: a_0 and $\lim_{s\to\infty} |E_t \pi_{t+s}| < \infty$.

Both the trace and determinant of A are positive. Since a_t is predetermined and π_t is non-predetermined, the matrix A must have one real eigenvalue inside and another outside the unit circle to ensure the dynamic stability of the economy. Denote by μ_1 and μ_2 the two real eigenvalues, it is sufficient to show that $(1-\mu_1)(1-\mu_2) < 0$ or alternatively:

$$\mu_1 + \mu_2 > 1 + \mu_1\mu_2. \tag{A.13}$$

Using the fact that $\mu_1 + \mu_2$ is equal to the trace of A_1 and $\mu_1\mu_2$ equal to its determinant, and substituting $(\mu_1 + \mu_2)$ by $(A_{11} + 1 - \gamma)$, and $\mu_1\mu_2$ by $(A_{11}(1-\gamma) - \gamma A_{12})$ into (A.13) with A_{11} and A_{12} given respectively by (A.2) and (A.3), we obtain after rearranging terms:

$$\kappa^2 (1+\tau)\,(1+\chi)^2\,[1-\tilde{\beta}(1+\psi)(1-\gamma)] + \alpha(1-\tilde{\beta})\{1-\tilde{\beta}(1+\psi)[1-\gamma(1-\tilde{\beta})]\} > 0. \tag{A.14}$$

Since $\psi > 0$ and $\tilde{\beta} \equiv \frac{\beta}{1+\psi} < 1$, it is easy to check that $\tilde{\beta}\,(1+\psi)\,(1-\gamma) < 1$ and $\tilde{\beta}(1+\psi)[1-\gamma(1-\tilde{\beta})] < 1$. As a result, (A.14) is verified for any ψ and hence for any ν.

The verification of (A.14) proves that the matrix A has an eigenvalue inside and one outside the unit circle. Since we are not interested in explosive solutions corresponding to a bubbly economy, we look for the non-explosive solution among infinite stochastic sequences of c_π^{cg} satisfying (A.8), i.e., the one corresponding to the eigenvalue of A within the unit circle.

A.3 The non-explosive solution of the feedback coefficients in the ALM for inflation

Rewriting (A.8) as $c_\pi^{cg} c_\pi^{cg} \gamma - c_\pi^{cg} A_{11} - A_{12} + c_\pi^{cg}(1-\gamma) = 0$ and substituting A_{11} and A_{12} by their respective expression, we obtain:

$$p_2 (c_\pi^{cg})^2 + p_1 c_\pi^{cg} + p_0 = 0 \tag{A.15}$$

with

$$\begin{aligned} p_0 &= \alpha\tilde{\beta}\{1-\tilde{\beta}(1+\psi)(1-\gamma)[1-\gamma(1-\tilde{\beta})]\} > 0, \\ p_1 &= \tilde{\beta}(1+\psi)(1-\gamma)\{\alpha[1-\gamma(1-\tilde{\beta})] + \kappa^2(1+\chi)^2(1-\gamma)(1+\tau)\} - \alpha \\ &\quad -\alpha\gamma\tilde{\beta}^2(1+\psi)[1-\gamma(1-\tilde{\beta})] - \kappa^2(1+\chi)^2(1+\tau), \\ p_2 &= \tilde{\beta}\gamma(1+\psi)\{\alpha[1-\gamma(1-\tilde{\beta})] + \kappa^2(1+\chi)^2(1-\gamma)(1+\tau)\} > 0. \end{aligned}$$

We can rewrite p_1 as

$$p_1 = -\kappa^2(1+\chi)^2(1+\tau)\left[1-\tilde{\beta}(1+\psi)(1-\gamma)\right] - \alpha(1-\tilde{\beta})\left\{1-\tilde{\beta}(1+\psi)\left[1-\gamma(1-\tilde{\beta})\right]\right\} - p_0 - p_2 < 0.$$

Then, it follows straightforwardly that the discriminant of the polynomial (A.15) is positive.

To find the nature of the solutions of c_π^{cg} in (A.15), we rewrite the latter as:

$$c_\pi^{cg} = -\frac{p_0 + p_2 (c_\pi^{cg})^2}{p_1} \equiv f(c_\pi^{cg}) \tag{A.16}$$

It is straightforward to see that $f(0) = -\frac{p_0}{p_1} > 0$ and $f(1) = \frac{p_0+p_2}{-p_1} < 1$ given that $-p_1 > p_0 + p_2 > 0$. This implies $f(c_\pi^{cg}) : [0,1] \to (0,1)$. Given that $f'(c_\pi^{cg}) = -\frac{2p_2}{p_1} c_\pi^{cg} > 0$ for $c_\pi^{cg} \in [0,1]$, $f(c_\pi^{cg})$ is strictly increasing in this interval. Consequently, applying the theorem of Brouwer, we deduce that there is one unique solution of c_π^{cg} inside the unit interval:

$$c_\pi^{cg} = \frac{-p_1 - \sqrt{p_1^2 - 4p_2p_0}}{2p_2}. \tag{A.17}$$

The other solution $c_{\pi}^{cg} = \frac{-p_1+\sqrt{p_1^2-4p_2p_0}}{2p_2}$ is greater than unity and is to be excluded to avoid that inflation follows an explosive path.

Substituting A_{11} and P_1 into (A.9) leads to

$$d_{\pi}^{cg} = -\frac{\alpha\kappa\left(1+\chi\right)}{\Upsilon + \alpha\gamma^2\tilde{\beta}^2\left(1+\psi\right)\left(\tilde{\beta} - c_{\pi}^{cg}\right) + \tilde{\beta}\gamma\left(1+\psi\right)\left(1-\gamma\right)\left(\alpha\tilde{\beta} - \Upsilon c_{\pi}^{cg}\right)}. \tag{A.18}$$

Furthermore, we can show that $f(c_{\pi}^{cg}):\ [0; \frac{\alpha\tilde{\beta}}{\Upsilon}] \rightarrow (0; \frac{\alpha\tilde{\beta}}{\Upsilon})$ with $\Upsilon \equiv \alpha + \kappa^2\left(1+\chi\right)^2\left(1+\tau\right)$. Knowing that $f(0) > 0$ and substituting c_{π}^{cg} by $\frac{\alpha\tilde{\beta}}{\Upsilon}$ into (A.16), we find

$$f(\tfrac{\alpha\tilde{\beta}}{\Upsilon}) \quad = \quad \frac{\frac{\alpha\tilde{\beta}}{\Upsilon}\left(\frac{\Upsilon}{\alpha\tilde{\beta}}p_0 + \frac{\alpha\tilde{\beta}}{\Upsilon}p_2\right)}{-p_1}. \tag{A.19}$$

Using $p_0 = \frac{\alpha\tilde{\beta}-\Upsilon}{\alpha\tilde{\beta}}p_0 + \frac{\Upsilon}{\alpha\tilde{\beta}}p_0$ and the definition of p_0, p_1, and p_2 given above, we find after some fastidious arrangements that

$$-p_1 \quad = \quad \tilde{\beta}p_2 + \frac{\Upsilon}{\alpha\tilde{\beta}}p_0 \tag{A.20}$$

Substituting the above expression of $-p_1$ into (A.19), we obtain:

$$f(\tfrac{\alpha\tilde{\beta}}{\Upsilon}) \quad = \quad \frac{\frac{\alpha\tilde{\beta}}{\Upsilon}\left\{\frac{\Upsilon}{\alpha\tilde{\beta}}p_0 + \frac{\alpha\tilde{\beta}}{\Upsilon}p_2\right\}}{\frac{\tilde{\beta}\kappa^2(1+\tau)(1+\chi)^2}{\Upsilon}p_2 + \frac{\Upsilon}{\alpha\tilde{\beta}}p_0 + \frac{\alpha\tilde{\beta}}{\Upsilon}p_2} < \frac{\alpha\tilde{\beta}}{\Upsilon}.$$

Since $f'(c_{\pi}^{cg}) = -\dfrac{2p_2}{p_1}c_{\pi}^{cg} > 0$ for $c_{\pi}^{cg} \in [0,1]$, $f(c_{\pi}^{cg})$ is strictly increasing in $\left(0;\ \frac{\alpha\tilde{\beta}}{\Upsilon}\right)$. This characteristic and the fact that $f(c_{\pi}^{cg}):\ [0;\ \frac{\alpha\tilde{\beta}}{\Upsilon}] \rightarrow (0;\ \frac{\alpha\tilde{\beta}}{\Upsilon})$ prove the existence of a unique non-explosive solution for c_{π}^{cg} such that $0 < c_{\pi}^{cg} < \frac{\alpha\tilde{\beta}}{\Upsilon}$. This implies that $d_{\pi}^{cg} < 0$.

The case where $\gamma = 0$. Substituting $\gamma = 0$ into (A.2)-(A.4) and using the results in (A.8)-(A.9), we obtain:

$$c_{\pi}^{cg} = \frac{\alpha\tilde{\beta}}{\Upsilon} \tag{A.21}$$

$$d_\pi^{cg} = -\frac{\alpha\kappa\,(1+\chi)}{\Upsilon}. \tag{A.22}$$

The case where $\gamma = 1$. Inserting $\gamma = 1$ into (A.2)-(A.4) and combining the results with (A.8)-(A.9) lead to

$$c_\pi^{cg} = \frac{\alpha\tilde{\beta}^3\,(1+\psi) + \Upsilon - \sqrt{[\alpha\tilde{\beta}^3\,(1+\psi) + \Upsilon]^2 - 4\alpha^2\tilde{\beta}^3\,(1+\psi)}}{2\alpha\tilde{\beta}^2\,(1+\psi)}, \tag{A.23}$$

$$d_\pi^{cg} = -\frac{\alpha\kappa\,(1+\chi)}{\Upsilon + \alpha\tilde{\beta}^2\,(1+\psi)\,(\tilde{\beta} - c_\pi^{cg})}. \tag{A.24}$$

A.4 The effect of learning gain

Differentiating the solution of c_π^{cg} given by (A.17) ensuring a non-explosive evolution of inflation with respect to γ gives

$$\frac{\partial c_\pi^{cg}}{\partial\gamma} = \frac{p_2\left(\frac{-p_1-\sqrt{p_1^2-4p_2p_0}}{\sqrt{p_1^2-4p_2p_0}}\right)\frac{\partial p_1}{\partial\gamma} + \frac{2p_2p_2}{\sqrt{p_1^2-4p_2p_0}}\frac{\partial p_0}{\partial\gamma} + \left(p_1 + \frac{p_1^2-2p_2p_0}{\sqrt{p_1^2-4p_2p_0}}\right)\frac{\partial p_2}{\partial\gamma}}{2p_2^2}$$

Using (A.20) and its derivative $\frac{\partial p_1}{\partial\gamma} = -\tilde{\beta}\frac{\partial p_2}{\partial\gamma} - \frac{\Upsilon}{\alpha\tilde{\beta}}\frac{\partial p_0}{\partial\gamma}$ to transform the above derivative, and after fastidious arrangements of terms, we finally obtain

$$\frac{\partial c_\pi^{cg}}{\partial\gamma} = \frac{1-\frac{\Upsilon}{\alpha\tilde{\beta}}c_\pi^{cg}}{\tilde{\beta}p_2\sqrt{p_1^2-4p_2p_0}}H. \tag{A.25}$$

where $H \equiv p_0\frac{\partial p_1}{\partial\gamma} - p_1\frac{\partial p_0}{\partial\gamma}$. The fact that $c_\pi^{cg} < \frac{\alpha\tilde{\beta}}{\Upsilon}$ yields $1 - \frac{\Upsilon}{\alpha\tilde{\beta}}c_\pi^{cg} > 1 - \frac{\Upsilon}{\alpha\tilde{\beta}}\frac{\alpha\tilde{\beta}}{\Upsilon} = 0$. To show the conditions under which $\frac{\partial c_\pi^{cg}}{\partial\gamma} < 0$, we consider the case where $H < 0$ for $\gamma = 1$, and show that $\frac{\partial H}{\partial\gamma} > 0$, $\forall\gamma \in (0,1)$.

For $\gamma = 1$, we have $\frac{\partial p_0}{\partial\gamma} = \alpha\tilde{\beta}^3\,(1+\psi) > 0$, $\frac{\partial p_1}{\partial\gamma} = -2\alpha\tilde{\beta}^3\,(1+\psi) < 0$, $p_1 = -\alpha\tilde{\beta}^3\,(1+\psi) - \Upsilon$, and $p_0 = \alpha\tilde{\beta}$. It follows that, for $\gamma = 1$,

$$H = -\alpha^2\tilde{\beta}^4\,(1+\psi)\,(2-\beta^2) + \alpha\tilde{\beta}^3\left[\alpha + \kappa^2(1+\tau)\,(1+\psi)\right].$$

We can have $H < 0$ if

$$1+\tau<\alpha\frac{\tilde{\beta}\left[2-\tilde{\beta}^{2}\left(1+\psi\right)\right]-1}{\kappa^{2}\left(1+\chi\right)^{2}}>0, \tag{A.26}$$

which implies, using $\tilde{\beta}=\frac{\beta}{1+\psi}$ that

$$0<\psi<\beta-1+\beta\sqrt{1-\beta}. \tag{A.27}$$

Using (19), we can define a threshold of $\bar{\nu}$ that solve this is equivalent to

$$\nu\frac{1-\beta\left(1-\nu\right)}{\left(1-\nu\right)\epsilon}\frac{1+r(\nu)}{r(\nu)}<\beta-1+\beta\sqrt{1-\beta}. \tag{A.28}$$

Differentiating H with respect to γ yields

$$\frac{\partial H}{\partial\gamma}=p_0\frac{\partial^2 p_1}{\partial^2\gamma}-p_1\frac{\partial^2 p_0}{\partial^2\gamma} \tag{A.29}$$

Differentiating twice p_0 and p_1 with respect to γ, $\forall\gamma\in(0,1)$, leads to

$$\begin{aligned}
\frac{\partial^2 p_0}{\partial^2\gamma} &= -2\alpha\tilde{\beta}^2\left(1-\tilde{\beta}\right)\left(1+\psi\right)<0,\\
\frac{\partial^2 p_1}{\partial^2\gamma} &= 2\alpha\tilde{\beta}\left(1+\psi\right)\left(1-\tilde{\beta}^2\right)+2\tilde{\beta}\left(1+\psi\right)\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)>0.
\end{aligned}$$

Substituting these second derivatives as well as the definition of p_0 and p_1 into (A.29), we can show that, $\forall\gamma\in(0,1)$,

$$\begin{aligned}
\frac{\partial H(\gamma)}{\partial\gamma} &= 2\alpha^2\tilde{\beta}^3\left(1-\tilde{\beta}\right)\left(1+\psi\right)\left\{1-\tilde{\beta}\left(1+\psi\right)\left[1-\gamma\left(1-\tilde{\beta}\right)\right]\right\}\\
&\quad+2\alpha\tilde{\beta}^3\left(1+\psi\right)\kappa^2\left(1+\chi\right)^2\left(1+\tau\right)\left\{1-\tilde{\beta}\left(1+\psi\right)\left(1-\gamma\right)\right\}>0.
\end{aligned}$$

Consequently, given that $H<0$ for $\gamma=1$ under conditions (A.26) and (A.28), and $\frac{\partial H}{\partial\gamma}>0$, $\forall\gamma\in(0,1)$, we deduce from (A.25) that

$$\frac{\partial c_\pi^{cg}}{\partial\gamma}<0.$$

Using d_π^{cg} with respect to γ yields:

$$\frac{\partial d_\pi^{cg}}{\partial \gamma} = \frac{\alpha\tilde{\beta}\left(1+\psi\right)\kappa\left(1+\chi\right)\{2\alpha\gamma\tilde{\beta}(\tilde{\beta}-c_\pi^{cg})+\left(1-2\gamma\right)(\alpha\tilde{\beta}-c_\pi^{cg}\Upsilon)-\gamma[\alpha\gamma\tilde{\beta}+\left(1-\gamma\right)\Upsilon]\frac{\partial c_\pi^{cg}}{\partial \gamma}\}}{\Upsilon+\alpha\gamma^2\tilde{\beta}^2\left(1+\psi\right)(\tilde{\beta}-c_\pi^{cg})+\tilde{\beta}\gamma\left(1+\psi\right)\left(1-\gamma\right)(\alpha\tilde{\beta}-c_\pi^{cg}\Upsilon)}.$$

Using $c_\pi^{cg} < \frac{\alpha\tilde{\beta}}{\alpha+\kappa^2(1+\tau)(1+\chi)^2}$, we find that $2\alpha\gamma\tilde{\beta}(\tilde{\beta}-c_\pi^{cg})+\left(1-2\gamma\right)(\alpha\tilde{\beta}-c_\pi^{cg}\Upsilon) > 2\alpha\gamma\tilde{\beta}(\tilde{\beta}-c_\pi^{cg}) > 0$, it follows that

$$\frac{\partial d_\pi^{cg}}{\partial \gamma} > 0.$$

Using the definition of c_x^{cg}, d_x^{cg}, c_q^{cg}, and d_q^{cg}, c_r^{cg} and d_r^{cg}, it is straightforward to show the sign of their partial derivative with respect to γ.

A.5 The effects of inflation penalty

Differentiating c_π^{cg} given by (A.17) with respect to τ, and using the fact that $p_1 < 0$ implies $p_1 - \sqrt{p_1^2 - 4p_2p_0} < 0$ and $\frac{p_1}{\sqrt{p_1^2-4p_2p_0}} < -1$, as well as $p_2 > 0$, $p_0 > 0$, $\frac{\partial p_1}{\partial \tau} = -\kappa^2\left(1+\chi\right)^2[1-\tilde{\beta}(1-\gamma)^2\left(1+\psi\right)] < 0$, and $\frac{\partial p_2}{\partial \tau} = \gamma\tilde{\beta}\kappa^2\left(1+\psi\right)\left(1+\chi\right)^2\left(1-\gamma\right) > 0$, we obtain

$$\frac{\partial c_\pi^{cg}}{\partial \tau} = \frac{-p_2\left(1+\frac{p_1}{\sqrt{p_1^2-4p_2p_0}}\right)\frac{\partial p_1}{\partial \tau}+\left[\frac{4p_2p_0+2p_0}{\left(p_1-\sqrt{p_1^2-4p_2p_0}\right)\sqrt{p_1^2-4p_2p_0}}\right]\frac{\partial p_2}{\partial \tau}}{2p_2^2} < 0.$$

Using this result and differentiating d_π^{cg} given by (A.18) with respect to τ, we get

$$\frac{\partial d_\pi^{cg}}{\partial \tau} = \frac{\alpha\kappa^3\left(1+\chi\right)^3\left[1-\tilde{\beta}\gamma\left(1+\psi\right)\left(1-\gamma\right)c_\pi^{cg}\right]-\alpha\kappa\tilde{\beta}\gamma\left(1+\chi\right)\left(1+\psi\right)[\alpha\gamma\tilde{\beta}+\left(1-\gamma\right)\Upsilon]\frac{\partial c_\pi^{cg}}{\partial \tau}}{\Upsilon+\alpha\gamma^2\tilde{\beta}^2\left(1+\psi\right)(\tilde{\beta}-c_\pi^{cg})+\tilde{\beta}\gamma\left(1+\psi\right)\left(1-\gamma\right)(\alpha\tilde{\beta}-c_\pi^{cg}\Upsilon)} > 0.$$

References

[1] Airaudo, M. (2013). Monetary policy and stock price dynamics with limited asset market participation. *Journal of Macroeconomics* 36, 1-22.

[2] Airaudo, M., Cardani, R., & Lansing, K.J. (2013). Monetary policy and asset prices with belief-driven fluctuations. *Journal of Economic Dynamics and Control* 37(8), 1453-1478.

[3] Airaudo, M., Nisticò, S., & Zanna, L.F. (2015). Learning, monetary policy, and asset prices. *Journal of Money, Credit and Banking*, 47(7), 1273-1307.

[4] André, M.C., & Dai, M. (2017a). Is central bank conservatism desirable under learning? *Economic Modelling* 60(C), 281-296.

[5] André, M.C., & Dai, M. (2017b). Can inflation contract discipline central bankers when agents are learning? *Working Paper of BETA* n° 2017-25.

[6] Assenza, T., Berardi, M., & Gatti, D.D. (2015). Was Bernanke right? Targeting asset prices may not be a good idea after all. In William A. Barnett , Fredj Jawadi (ed.) *Monetary Policy in the Context of the Financial Crisis: New Challenges and Lessons* (International Symposia in Economic Theory and Econometrics, Volume 24) Emerald Group Publishing Limited, 451 - 496.

[7] Benigno, P., & Paciello, L. (2014). Monetary policy, doubts and asset prices. *Journal of Monetary Economics* 64, 85-98.

[8] Bernanke, B.S., & Gertler, M. (1999). Monetary policy and asset price volatility. *Economic Review,* Federal Reserve Bank of Kansas City, issue Q IV, 17-51.

[9] Bernanke, B.S. (2010). Implications of the Financial Crisis for Economics. *Speech* at the Conference Co-sponsored by the Center for Economic Policy Studies and the Bendheim Center for Finance, Princeton University, Princeton, New Jersey. September 24, 2010.

[10] Binswanger, M. (2004) Stock returns and real activity in the G-7 countries: Did the relationship change during the 1980s? *Quarterly Review of Economics and Finance* 44(2), 237–252.

[11] Blanchard, O. J., & Kahn, C.M. (1980). The solution of linear difference models under rational expectations. *Econometrica* 48(5), 1305–1311.

[12] Blanchard, O.J. (1985). Debt, Deficits, and Finite Horizons. *The Journal of Political Economy* 93(2), 223-247.

[13] Bullard, J., & Schaling, E., (2002). Why the fed should ignore the stock market? *Federal Reserve Bank of St. Louis Review* 84 (2), 35–41.

[14] Carlstrom, C.T., & Fuerst, T.S., (2007). Asset prices, nominal rigidities, and monetary policy. *Review of Economic Dynamics* 10 (2), 256–275.

[15] Cecchetti, S. G., Genberg, H., Lipsky, J., & Wadhwani,S., (2000). Asset prices and central bank policy. Centre for Economic Policy Research.

[16] Cecchetti, S.G. (2003). What the FOMC says and does when the stock market booms. In Asset Prices and Monetary Policy, Proceedings of the Research Conference of the Reserve Bank of Australia (77-96).

[17] Cecchetti, S.G., Genberg, H., & Wadhwani, S. (2002). Asset prices in a flexible inflation targeting framework. NBER Working Paper No. 8970, National Bureau of Economic Research.

[18] Evans, G. W., & Honkapohja, S. (2009). Learning and Macroeconomics. *Annual Review of Economics* 1, 421-449.

[19] Fama, E. (1990). Stock returns, expected returns, and real activity. *Journal of Finance* 45, 1089– 1108.

[20] Gali, J. (2014). Monetary Policy and Rational Asset Price Bubbles. *American Economic Review* 104(3), 721–752.

[21] Gilchrist, S., & Leahy, J.V., (2002). Monetary policy and asset prices. *Journal of Monetary Economics,* 49.

[22] Issing O. (2009). Asset Prices and Monetary Policy. *Cato Journal* 29(1), 45-51.

[23] Kreps, D. (1998). Anticipated Utility and Dynamic Choice. In Jacobs, D.P., Kalai, E., & Kamien M. (eds.), *Frontiers of Research in Economic Theory*, Cambridge University Press, 242-274.

[24] Lanne, M., Luoma, A., & Luoto, J. (2009). A naïve sticky information model of households' inflation expectations. *Journal of Economic Dynamics and Control* 33(6), 1332-1344.

[25] Machado, V.d.G. (2013). Monetary policy rules, asset prices and adaptive learning. *Journal of Financial Stability* 9(3), 251-258.

[26] Markiewicz, A., & Pick, A. (2014). Adaptive learning and survey data. *Journal of Economic Behavior & Organization* 107, 685-707.

[27] McCallum, B. (1983). On Non-Uniqueness in Rational Expectation Models – An Attempt at Perspective. *Journal of Monetary Economics* 11(2), 139–168.

[28] Milani, F. (2008). Learning about the Interdependence between the Macroeconomy and the Stock Market. *Working Paper 070819*, University of California–Irvine, Department of Economics.

[29] Molnár, K. & Santoro, S., 2014. Optimal monetary policy when agents are learning. *European Economic Review* 66(C), 39-62.

[30] Nisticò, S. (2012). Monetary policy and stock-price dynamics in a DSGE framework. *Journal of Macroeconomics,* 34(1), 126-146.

[31] Persson, T., & Tabellini, G. (1993). Designing institutions for monetary stability. *Carnegie Rochester S*eries on Public Policy 39, 33-84.

[32] Preston, B. (2005). Learning about Monetary Policy Rules when Long-Horizon Expectations Matter. *International Journal of Central Banking* 1(2), September.

[33] Rogoff, K. (1985). The optimal degree of commitment to an intermediate monetary target. *Quarterly Journal of Economics* 100, 1169-89.

[34] Slobodyan, S., & Wouters, R. (2016). Adaptive Learning and Survey Expectations of Inflation. Working Paper.

[35] Svensson, L. E. (1997). Optimal inflation targets, conservative central banks, and linear inflation contracts. *American Economic Review* 87, 98-114.

[36] Trehan, B. (2015). Survey measures of expected inflation and the inflation process. *Journal of Money, Credit and Banking*, Blackwell Publishing 47(1), 207-222.

[37] Tsouma, E. (2009) Stock returns and economic activity in mature and emerging markets. *Quarterly Review of Economics and Finance* 49, 668–685.

[38] Walsh, C. E. (1995). Optimal contracts for central bankers. *American Economic Review* 85, 150-167.

[39] Walsh, C. E. (2003). Accountability, Transparency, and Inflation Targeting. *Journal of Money, Credit and Banking* 35(5), 829-849.

[40] Yaari, M., E. (1965). Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. *The Review of Economic Studies* 32(2) 137–150.