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# Could competition always raise the risk of bank failure?*

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## Abstract

The debate between the 'competition-fragility' and 'competition-stability' views has been centered upon the risk of banks' loan portfolios. In this paper, we shift the focus of the debate from the riskiness of loan portfolios to the riskiness of operational costs net of the income of non-traditional banking activities, banks' default resulting from negative aggregate profits. We consider a simple model in which, due to purely idiosyncratic risks, portfolio diversification would eliminate the risk of banks' default if those net operational costs were negligible or were known with certainty. We show that more competition always raises the risk of bank default, non-monotonicity being excluded as an equilibrium outcome under free oligopolistic competition between profit maximizing banks. However, the same result obtains in fact under systemic risk, even under non-stochastic net operation costs, a situation which we explore in a slightly different model. We show further that, under liquidity shortness, a higher intensity of competition in the loan market can result in an increase of deposit rates, rather than a decrease of loan rates.



JEL: G21, D43, L13.

Keywords: Bank failure, oligopolistic competition in the loan market.

# 1 Introduction

The conventional wisdom concerning the relation between bank competition and bank stability is that more of the former undermines the latter. Erosion of market power reduces banks' charter values, leading banks to make riskier choices (Marcus 1984, Keeley 1990). Also, more competition, if associated with a higher number of banks and lower market shares, leads to less portfolio diversification, enhancing risk (Matutes and Vives 1996). However, Boyd and De Nicolò (2005) challenged this view by taking into account the borrowers' standpoint. They claim that higher loan rates directly imply a higher risk of borrowers' bankruptcy and, by moral hazard, further create an incentive for borrowers to make riskier choices, as already analyzed by Stiglitz and Weiss (1981). With two opposite effects, one may wonder whether one of them dominates the other, and whether the relationship between intensity of competition and risk of bank failure is increasing, decreasing or non-monotonic. Martinez-Miera and Repullo (2010), by analyzing how changes in the number of banks and consequent changes in loan rates modify the riskiness of the loan portfolios held by the banks, provided indeed a U-shaped relationship between that number and the probability of bank failure.[1]

The result obtained by Martinez-Miera and Repullo is very much dependent upon banks' ability to diversify large loan portfolios. When borrowers' risks are perfectly correlated, diversification is impossible, so that the risk of bank default is entirely determined by the risk of entrepreneurs' default, a situation vindicating the non-conventional result of Boyd and De Nicolò. On the contrary, when borrowers' risks are independent, portfolio diversification completely rules out banks' failures. In intermediate cases, the favourable effect of stronger competition on bank stability working through borrowers' decisions combines with an unfavourable effect working through lower profit margins, hence "lower revenues from performing loans, which provide a buffer against loan losses" (Martinez-Miera and Repullo 2010, p. 3646). It is shown that the favourable effect is relatively stronger when the number of banks is low and the loan rate high, which may end up in the U-shaped relationship.

Our purpose is not to challenge this result, just to put it in perspective. This we will do in two ways. First, we want to take into account the fact that bank activities do not reduce to deposit-loan intermediation and are not

[1]For further relevant references on the relation between bank competition and bank stability, see the surveys of the related theoretical and empirical literatures by Berger *et al.* (2009), Schaeck *et al.* (2009), Fungáčová and Weill (2013) and Jimenez *et al.* (2013).

exclusively fund-based, but extend to several stakeholder and fee-based activities. As a consequence, the risk of bank failure should not be assessed on the basis of the sole degree of potential diversification of the loan portfolio, in complete ignorance of non-traditional "purely fee-driven lines of business (e.g., asset management, securities brokerage, M&A advising) [which] can place equity capital at risk from operating losses, as volatile revenue streams may not cover their related fixed costs of operation during down years" (DeYoung and Torna 2013, p. 400). More precisely, although the risk of banks' default certainly depends on the overall riskiness of their more or less diversified loan portfolios, with good loans providing a buffer against bad ones, it also depends on the capacity for these portfolios to generate an aggregate profit covering losses which originate in operating costs and non-traditional risky activities.

Second, we want to give a more prominent place to competition. In most of the literature devoted to the influence of competition on the risk of bank failure, tougher competition is reduced to the resulting lower loan rates or higher deposit rates, according to the market in which banks compete. De Nicolò and Lucchetta (2009), for instance, take directly the exogenous deposit interest rate as an index of the bank market power, refraining "from modeling any specific bargaining game generating certain levels of rents" (p.7). In any case, endogenizing interest rates by just making them depend upon the number of banks does not add much to the analysis.

By contrast, we want to place bank competition at the core of the analysis. This means that the temptation should be resisted to simply index bank competition by observed loan and deposit interest rates, or to just identify intensity of competition and market structure, itself reduced, independently of the specific loan or deposit market, to the number of banks. More competition may admittedly result from entry in the banking sector, but also from increased competitive toughness as displayed, say, in Bertrand as opposed to Cournot competition. The hesitation observed in the empirical literature between alternative ways of measuring competition, by referring to Lerner indices and indices of conduct versus concentration indices (see *e.g.* Bikker and Haaf 2002) is an expression of the diversity of dimensions of competition. These diverse dimensions are not stressed enough in the theoretical literature.

In the following, we shall use a quite simple partial equilibrium model in which a discrete number of banks are oligopolistic competitors in the loan market and perfect competitors in the deposit market. Non-traditional activities are the source of an income which is exogenous relative to deposit-loan intermediation and which must be deducted from any fixed costs generated by this intermediation. The existence of such exogenous and stochastic net costs creates the possibility of bank default, even in the context of pure idiosyncratic entrepreneurs' risks that we will first assume for simplicity. Moreover, those net costs induce scale economies in the basic banking activity, an effect which adds to the risk-shifting effect (here due to adverse selection rather than moral hazard), both working against the profit margin effect that provides a buffer against bad loans and which is at the basis of the conventional wisdom. Our main result is that, in spite of the two countervailing effects, more competition always raises the risk of bank failure, since the margin effect can be dominated only for levels of the loan interest rate higher than its collusive value, hence unobservable under free (oligopolistic) competition. Of course, should the loan market be regulated, such high levels of the interest rate would not be excluded. This result stands under idiosyncratic risk, once we take into account stochastic net costs, exogenous with respect to deposit-loan intermediation, but extends in fact to a situation of systemic risk.

At the empirical level there is no consensus on the impact of bank competition on bank stability. Indeed, the empirical literature provides evidence both in favor of the 'competition-fragility' and of the 'competition-stability' views. However, our result, in agreement with conventional wisdom, that more competition increases the risk of bank failure, is supported by empirical works that use either the Lerner index, like Berger *et al.* (2009), Turk Ariss (2010), Fungáčová and Weill (2013), Jimenez *et al.* (2013) or the non-structural H-statistic, like Schaeck *et al.* (2009). These measures, although not ideal, are certainly more sophisticated than mere concentration indices (*e.g.* the number of banks, the share of the $k$ most important in assets, or the Herfindahl index).[2]

We present our model in section 2, and discuss the relationship between competition for loans and the risk of bank failure in section 3. We extend the analysis to a situation of systemic risk in section 4. Section 5 concludes.

[2] Schaeck *et al.* (2009) find that "competition and concentration capture different characteristics of banking systems, meaning that concentration is an inappropriate proxy for competition" (p. 711).

## 2 The model

Our model involves two markets, the *loan market* and the *deposit market*, and three sets of agents, a continuum of unit mass of *entrepreneurs*, $n$ *banks* and a continuum of mass $\overline{L}$ of *depositors*.

### 2.1 Entrepreneurs

Each entrepreneur $i \in [0,1]$ is endowed with an effort capacity and with a technological knowledge allowing her to productively exert an effort. This technological knowledge materializes as a project $i$ that can be operated in period 0 at a fixed scale (normalized to one) and with an effort $e \in \{0,1\}$, so as to yield in period 1 an output $e + s_i$ in state $s_i$. This state is the value taken by an independent random variable $S_i$ describing an *idiosyncratic risk*.

As there is no initial capital endowment, the entrepreneur must borrow from a bank, at date 0, one unit of capital in order to operate her project. Having obtained a loan at a non-negative *loan interest rate* $R$, she owes to the bank at date 1 the principal 1 plus the interest $R$. However, by limited liability, she will actually pay $\min\{1+R, e+s_i\}$, the debt being only partially recovered by the bank in case of default.

We assume that the disutility $v_i$ per unit of effort (expressed in monetary terms) is uniformly distributed among entrepreneurs of both classes over the same interval $[0, \overline{v}]$. For entrepreneur $i$ with disutility $v_i$, the utility in state $s_i$ of her project is equal to

$$u_i(e, s_i, R) = \max\{e + s_i - (1+R), 0\} - v_i e. \tag{1}$$

Entrepreneurs belong to two classes of mass $a$ and $b = 1 - a$, according to the type of projects they are endowed with. We consider two types of projects: a type of risky projects $(x_a, \pi_a)$ entailing a high output $x_a \in (0,1]$ with a low probability $\pi_a \in (0,1)$ of success, and a type of projects $(x_b, \pi_b)$ with the opposite characteristics ($0 < x_b < x_a \leq 1$ and $0 < \pi_a < \pi_b < 1$). Like Stiglitz and Weiss (1981), we assume the mean-preserving property $\pi_a x_a = \pi_b x_b = g$. For both types of projects the output is zero when they do not succeed. We thus obtain the following expression for the expected utility of entrepreneur $i$ endowed with a project of type $k$ ($k = a, b$), hence anticipating a state

$s_i = x_k$ if the project succeeds and $s_i = 0$ if it does not:

$$\begin{aligned} & \mathbb{E}\left[u_i\left(e, s_i, R\right)\right] \qquad (2) \\ = \; & \pi_k \max\left\{e + x_k - \left(1 + R\right), 0\right\} + \underbrace{\left(1 - \pi_k\right) \max\left\{e - \left(1 + R\right), 0\right\}}_{=0} - v_i e, \end{aligned}$$

using the constraint $1 + R \geq 1 \geq e$ on the viable values of $R$.

Clearly, entrepreneur $i$ chooses to make an effort only if the resulting expected utility is larger than the one obtained with no effort: $\mathbb{E}\left[u_i\left(1, s_i, R\right)\right] \geq \mathbb{E}\left[u_i\left(0, s_i, R\right)\right]$. Moral hazard imposes this *incentive compatibility constraint* on loans. Besides, no loan will be accepted by the entrepreneur unless the higher expected utility (the one with $e = 1$) is non-negative: $\mathbb{E}\left[u_i\left(1, s_i, R\right)\right] \geq 0$. This results in a *participation constraint* on loans. However, as $x_b < x_a \leq 1 \leq 1 + R$ by assumption, the expected utility in the case of no effort is nil ($\mathbb{E}\left[u_i\left(0, s_i, R\right)\right] = 0$), so that the participation and the incentive compatibility constraints coincide in our model, as the condition:

$$v_i \leq \pi_k \max\left\{x_k - R, 0\right\}.$$

If $R$ is too high, namely if $R \geq x_a$, the demand for loans vanishes. If $x_b \leq R \leq x_a$, the demand for loans is restricted to the class of entrepreneurs endowed with risky projects, those which may entail a high enough output. If $R \leq x_b < x_a$, we obtain from the incentive compatibility constraint, binding for the marginal entrepreneur of each class, the demand for loans:

$$\begin{aligned} D\left(R\right) &= \underbrace{a \min\left(\frac{g - \pi_a R}{\overline{v}}, 1\right)}_{D_a(R)} + \underbrace{b \min\left(\frac{g - \pi_b R}{\overline{v}}, 1\right)}_{D_b(R)} \qquad (3) \\ &= \frac{g - \left(a\pi_a + b\pi_b\right) R}{\overline{v}}, \end{aligned}$$

assuming for simplicity that $g \leq \overline{v}$.

By limited liability, entrepreneurs only pay interest if projects are successful, so that a lower probability of success $\pi_a$ implies a smaller response to interest rate variations. Thus, as $R$ increases, the demand for loans from entrepreneurs endowed with risky projects $D_a\left(R\right)$ decreases less than the demand from the complementary subset of entrepreneurs. As a consequence, the proportion of risky projects

$$\frac{D_a\left(R\right)}{D\left(R\right)} = a \frac{g - \pi_a R}{g - \left(a\pi_a + b\pi_b\right) R} \qquad (4)$$

is an increasing function of $R$, as can be easily seen by computing the elasticity of this proportion with respect to $R$:

$$\epsilon_R \left( \frac{D_a(R)}{D(R)} \right) = \frac{(a\pi_a + b\pi_b) R}{g - (a\pi_a + b\pi_b) R} - \frac{\pi_a R}{g - \pi_a R} > 0, \tag{5}$$

since $a\pi_a + b\pi_b > \pi_a$.

Thus, the quality of loans, as measured by the probability of success of the "representative entrepreneur"

$$P(R) = \frac{D_a(R)}{D(R)}\pi_a + \frac{D_b(R)}{D(R)}\pi_b, \tag{6}$$

declines as $R$ increases: $P'(R) < 0$. This is the *risk-shifting effect* of the variation of the loan interest rate $R$, here reduced to its adverse selection component (cf. Theorem 2 of Stiglitz and Weiss, 1981). It vanishes at the limit of equal probabilities of success of the two types of projects: $\pi_a = \pi_b = \pi$.

## 2.2 Banks

Each bank $j \in \{1, ..., n\}$ is endowed with a license to operate, endures positive fixed operating costs and generates an exogenous random income from other, non-traditional, banking activities. We denote the excess of exogenous operating costs over the income of those activities by $\phi_j$, a random variable which can take positive or negative values (when operating costs are respectively higher or lower than that net income).

The cost of the loan granted to an entrepreneur is the *deposit interest rate* $r$, which we take now as exogenous, leaving its adjustment to subsection 3.3. Each bank $j$ chooses, together with its supply of loans $l_j \in \mathbb{R}_+$, a loan interest rate $R_j$. In this section, we shall however provisionally take as exogenous the loan interest rate $R \in [r, x_b]$, such that the equality of supply and demand for loans is satisfied:

$$\sum_{j=1}^{n} l_j = D(R), \tag{7}$$

with $D(R)$ as given by equation (3).

Assuming a very large number of projects with idiosyncratic risks and also perfect symmetry among banks, we can immediately take the entrepreneurs' default rate, according to the law of large numbers, as given by the probability of default $1-P(R)$ of the "representative entrepreneur" (see equation (6)), which is also the proportion of failing projects in each bank's portfolio. Using this rate, and recalling that the bank obtains 1 (instead of $1+R$) from an unsuccessful entrepreneur, we can determine the *gross profit per loan*

$$\Pi(R,r) = P(R)(1+R) + (1-P(R)) - (1+r) = P(R)R - r, \quad (8)$$

which depends on $R$, negatively through $P(R)$ (the *risk-shifting effect*), and positively through a direct *margin effect*: a higher $R$ increases the bank profit per loan $R-r$ when entrepreneurs succeed, providing a buffer to cover the loss per loan $-r$ when they do not.

We next define the value $\widehat{\phi}(R,r,n)$ as the maximum potential net cost that is compatible with no bank default for symmetric profiles of $n$ banks when the loan and the deposit interest rates are $R$ and $r$, respectively. It is determined by the *zero profit condition*, $\Pi(R,r)\,l - \phi = 0$, so that, using (7), we obtain:

$$\widehat{\phi}(R,r,n) \equiv \Pi(R,r)\frac{D(R)}{n}, \quad (9)$$

which is also the gross profit of each bank's loan portfolio for a symmetric profile. For any bank $j$, a net cost $\phi_j$ larger than $\widehat{\phi}(R,r,n)$ will lead to bank $j$'s default. If the probability distribution of the net costs is given by the distribution function $F$, the probability of any bank's default is consequently $1-F\left(\widehat{\phi}(R,r,n)\right)$, decreasing in $\widehat{\phi}(R,r,n)$.

We see from (8) and (9) that, in addition to the positive *risk-shifting effect* and the negative *margin effect* of an increase in the loan interest rate $R$ on the probability of bank's default, both working through the gross profit per loan $\Pi(R,r)$, we must take into account a positive *scale effect* operating through the demand for loans $D(R)$, because of the increasing returns induced by (positive) net costs. As a last remark, notice that an increase in the number $n$ of banks has, in addition to its possible indirect effects through $R$, a direct positive *congestion effect* on the risk of bank failure.[3] Both the opposite risk-shifting and margin effects have been widely discussed in the literature. However, although several papers mention the importance of economies of scale in banking, this is, to our knowledge, the first paper that explicitly

[3]The scale and congestion effects are positive as long as $\Pi(R,r) > 0$. More generally, they have the sign of $\Pi(R,r)$.

takes into account this channel, as well as the congestion one, in shaping the effects of the interest rate $R$ on the probability of a bank's default.

The presence of conflicting effects of a change in $R$ suggests the possibility of obtaining an inverse U-shaped graph of the function $\widehat{\phi}(\cdot, r, n)$. Under our assumptions, this function is indeed quadratic concave. We just represent two examples of this function in Figure 1, with $R \in [0, x_b]$ on the horizontal axis and $\widehat{\phi}(\cdot, 0, 5)$ on the vertical axis.[4] The dashed curve, which remains increasing for larger values of $R$, represents the case of a single (average) type of projects (with output $x = \sqrt{x_a x_b}$ and probability of success $\pi = \sqrt{\pi_a \pi_b}$), hence with no risk-shifting effect.

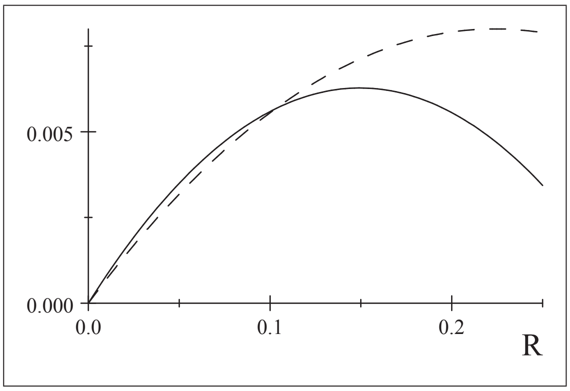


Figure 1: Maximum net cost with no bank default

Notice that the curves represented in Figure 1 would be U-shaped in the space $\left(R, 1 - F\left(\widehat{\phi}(R, r, n)\right)\right)$, with the probability of bank failure on the vertical axis.

## 2.3 Depositors

Each depositor is endowed with one unit of money, which he can deposit in some bank so as to be able to spend, in period 1, $1 + r$ money units ($r \geq 0$). We suppose that more and more potential depositors actually deposit their money endowments as the deposit interest rate $r$ increases, resulting in an increasing deposit supply function $L : [0, \overline{r}] \to \left[0, \overline{L}\right]$. As a limit case, this

[4] The parameter values used in Figure 1 are: $r = 0$, $a = 1/2$, $\overline{v} = 0.25$, $n = 5$, and $\pi_a = 1/4 = x_b$, $\pi_b = 4/5 = x_a$ for the solid curve, $\pi_a = \pi_b = x_a = x_b = \sqrt{1/5}$ for the dashed curve.

supply function may be rigid, with $L(r) = 0$ for any $r < \overline{r}$ and $L(\overline{r}) = \overline{L}$ (with possibly $\overline{r} = 0$). The deposit interest rate $r$ is adjusted parametrically, so as to balance supply $L(r)$ and demand $\sum_{j=1}^{n} l_j$. Recall that the deposit interest rate is restricted to belong to the interval $[0, x_b]$ because for $r < 0$ the deposit supply would be zero, and for $r > x_b$ no bank would be able to remunerate depositors, since we have assumed that the $R_j$'s are chosen in the interval $[r, x_b]$.

# 3 Competition for loans and the risk of bank failure

In section 2, we have provisionally treated the loan interest rate as an exogenous parameter when considering the different effects of its variations on the risk of bank failure. In this section, we shall on the contrary place bank competition in the foreground, treating the loan interest rate as a strategic variable, whose equilibrium value decreases as competition for loans becomes more intense.

## 3.1 Intensity of competition in the loan market

In the context of Cournot competition under perfect product substitutability (the one to which refer Martinez-Miera and Repullo 2010), the degree of concentration (the market share $1/n$ with symmetric profiles) may be used as an inverse index of intensity of competition. Intensity of competition encompasses however other dimensions: it would decrease should product substitutability become more and more imperfect, within the relevant sector as well as across sectors, and it would increase with changes in the regime of competition, when switching from Cournot to Bertrand. In the following, we will focus on the last dimension, examining a homogeneous oligopoly in which competitive toughness varies continually from Cournot to Bertrand (see d'Aspremont and Dos Santos Ferreira 2009).

We have assumed in section 2 that bank $j$ chooses the pair $(R_j, l_j) \in [r, x_b] \times [0, \infty)$ so as to maximize its gross profit $\Pi(R_j, r)\, l_j$. This maximization is supposed to be performed under two constraints:

$$R_j \leq \min_{j' \neq j} R_{j'} \quad (10)$$

$$R_j \leq D^{-1}\left(l_j + \sum_{j' \neq j} l_{j'}\right). \quad (11)$$

The first inequality is a competitiveness constraint, imposing a ceiling on the loan interest rate, equal to the minimum of the values set by the other suppliers of the same homogeneous service. The second inequality is the usual Cournot condition, also imposing a ceiling on the loan interest rate, now determined by the inverse demand for loans when each bank takes as given the aggregate volume of loans that its competitors intend to grant.

An interior solution $(R_j, l_j)$ to bank $j$'s problem, with $r < R_j < x_b$ and $l_j > 0$, will satisfy both constraints as equalities. The corresponding first order conditions can consequently be expressed as

$$\frac{\partial \Pi(R_j, r)}{\partial R_j} l_j - \lambda_j - \mu_j = 0 \quad (12)$$

$$\Pi(R_j, r) + \frac{\mu_j}{D'(R_j)} = 0, \quad (13)$$

with non-negative Lagrange multipliers $\lambda_j$ and $\mu_j$. As in section 2, let us confine our analysis to symmetric profiles. We then obtain from these first order conditions (using equation (8) and denoting $R_j = R$, $l_j = l$, $\lambda_j = \lambda$ and $\mu_j = \mu$ for any $j$):

$$\frac{P(R)R - r}{P(R)R} = \underbrace{\frac{\mu}{\lambda + \mu}}_{\theta} \frac{1/n}{-\epsilon_R D(R)} \left(1 + \epsilon_R P(R)\right), \quad (14)$$

where $\epsilon_R D(R)$ and $\epsilon_R P(R)$ are the elasticities with respect to $R$ of the demand $D(R)$ and of the probability of entrepreneurial success $P(R)$, respectively. Equation (14), allowing to determine the equilibrium value of $R$, can be seen as an extended formula for the *Lerner index of market power*, namely the relative margin of the expected price $P(R)R$ over the marginal cost $r$.

The second ratio on the right hand side of equation (14) is the usual Cournot's degree of monopoly, with the market share $1/n$ in the numerator and the absolute value of demand elasticity $-\epsilon_R D(R)$ in the denominator.

The first ratio $\mu/(\lambda+\mu) \equiv \theta$ is the conduct parameter of the *New Empirical Industrial Organization* literature (see Bresnahan 1989 and Corts 1999). Notice that it increases as the implicit cost $\mu$ imposed by the consensual participation constraint, accomodating the rivals' loan targets, increases relative to the implicit cost $\lambda$ imposed by the more confrontational competitiveness constraint, reflecting the conflicting interests of all the competitors. We may accordingly see $\theta$ as an index of *competitive softness* displayed at a particular equilibrium, and taking values between 0 (for Bertrand equilibrium) and 1 (for Cournot equilibrium).[5] The third and last term expresses the risk-shifting effect, which reduces the profit margin (since $\epsilon_R P(R) < 0$).

The right-hand side of equation (14) is decreasing in $R$. Indeed, by referring to equations (3) to (6), we can easily compute:

$$\frac{1}{-\epsilon_R D(R)} = \frac{g}{(a\pi_a + b\pi_b) R} - 1 \tag{15}$$

and

$$\epsilon_R P(R) = -\frac{(\pi_b - \pi_a)^2 abg}{g(a\pi_a + b\pi_b) - (a\pi_a^2 + b\pi_b^2) R} \frac{R}{g - (a\pi_a + b\pi_b) R}, \tag{16}$$

both decreasing functions of $R$. As to the left-hand side of equation (14), it is constant for $r = 0$, otherwise increasing for admissible values of $R$ (such that $1 + \epsilon_R P(R) > 0$, ensuring a positive Lerner index). Thus, the equilibrium value of $R$, uniquely determined by equation (14), increases as $n/\theta$ declines towards 1, its floor. This ratio may be taken as an index of *intensity of competition* in the loan market: it varies inversely with the degree of *concentration* (as measured by $1/n$, the individual bank market share, which is also the Herfindahl index) and with the *competitive softness* (as indexed by $\theta$). Notice that the effects of changes in structure (through $n$) and conduct (through $\theta$) are indistinguishable at this stage. Notice also that competition may well be intensified through conduct (a lower $\theta$) even under higher concentration (a lower $n$).

[5] Competitive softness is endogenous. It parameterizes a particular equilibrium in a large set of (symmetric) oligopolistic equilibria. Referring to a given value of $\theta$ which indexes a specific conduct is however not different in nature from referring to the Cournot or Bertrand regimes, which are just limit cases of our parameterization. Following the convention used in the *NEIO* literature, we denote by $\theta$ the conduct parameter $\mu/(\lambda+\mu)$, viewed as an index of competitive softness; d'Aspremont and Dos Santos Ferreira (2009) refer instead to the complementary index $\lambda/(\lambda+\mu)$ of competitive *toughness*, also denoted by $\theta$ in their paper.

## 3.2 Loan interest rates: are they regulated or do they stem from competition?

We are now in a position allowing to determine the maximum value of the loan interest rate $R$ attainable through competition: it corresponds to the softest possible conduct $\theta = 1$ and to the lowest possible number of competitors $n = 2$. As a matter of fact, we can even consider the monopoly case $n = 1$, implying $\theta = 1$ (since the competitiveness constraint ceases then to be active, so that $\lambda = 0$). The monopoly solution $\widehat{R}$, associated with an index of intensity of competition at its floor, coincides with the collusive solution: $\widehat{R} = \arg\max_R \widehat{\phi}(R, r, n)$, whatever $n$. This solution is implementable if banks view themselves as confronting a constant share $1/n$ of demand, rather than the residual demand (as in Cournot). Thus, a viable loan interest rate $R$ must belong to the interval $\left[r, \widehat{R}\right]$.

Let us now come back to the inverse U-shaped graph of the function $\widehat{\phi}(\cdot, r, n)$ introduced in section 2. What are the consequences for this relationship of endogenizing $R$? Consider the elasticity of this function. By equations (8) and (9), we have:

$$\begin{aligned} \epsilon_R \widehat{\phi}(R, r, n) &= \epsilon_R \Pi(R, r) + \epsilon_R D(R) \\ &= \frac{P(R) R}{P(R) R - r}(1 + \epsilon_R P(R)) + \epsilon_R D(R). \end{aligned} \tag{17}$$

If we now apply formula (14) for the Lerner index of market power, we obtain:

$$\epsilon_R \widehat{\phi}(R, r, n) = -\epsilon_R D(R)\left(\frac{n}{\theta} - 1\right). \tag{18}$$

As $-\epsilon_R D(R) > 0$ and $n/\theta \geq 1$, we may conclude that the function $\widehat{\phi}(\cdot, r, n)$ is necessarily increasing for admissible values of $R \in \left[r, \widehat{R}\right]$. In other words, the decreasing region of the graph of $\widehat{\phi}(\cdot, r, n)$ can only be attained for regulated values of the loan interest rate, which are not observable as the result of any regime of competition.[6]

To conclude, outside the case of regulated interest rates in the loan market, an increase in the intensity of competition $n/\theta$ can only increase, through

[6] This must *a fortiori* also be the case for the decreasing region of the graph of the expected gross profit per loan $\Pi(R, r) = P(R) R - r$, which is not affected by the scale effect.

a decline in $R$, the probability $1-F\left(\widehat{\phi}\left(R,r,n\right)\right)$ of bank failure. Of course, if the loan market becomes more competitive because of lower concentration, we should take into account the direct congestion effect of a higher $n$, which *reinforces* the increase in the risk of bank failure.

## 3.3 Liquidity shortness

We now consider perfectly competitive adjustments of the deposit market, beginning with the case of a rigid supply function such that $L\left(r\right)=0$ for any $r<\overline{r}$ and $L\left(\overline{r}\right)=\overline{L}$, with $\overline{r}\geq 0$. Clearly, the preceding analysis applies (with $r=\overline{r}$) as long as $nl<\overline{L}$. Otherwise, $r$ (and ultimately $R$) must be adjusted upwards so as to ensure the equalities of loan demand and supply and deposit demand and supply $D\left(R\right)=nl=\overline{L}$.

To be explicit, we must have

$$R=D^{-1}\left(\overline{L}\right)=\frac{g-\overline{v}\overline{L}/p}{a\pi_a+b\pi_b}, \tag{19}$$

$r$ being then determined by equation (14). In a situation of liquidity shortness and rigid deposit supply, more intense competition in the loan market, be it through lower concentration or through lower competitive softness, still decreases market power, but in this context through a heightening effect on $r$, rather than through a depressing effect on $R$. In any case, the profit margin is squeezed and the risk of bank failure aggravated.

Introducing an increasing deposit supply function $L:[0,\overline{r}]\rightarrow\left[0,\overline{L}\right]$, with more and more consumers assumed to deposit their money endowments as the deposit rate $r$ increases, barely modifies the analysis. In this case, a change in the intensity of competition for loans modifies both interest rates: lower concentration or lower competitive softness decreases $R$ and increases $r$.

# 4 Systemic risk

Up to now, we have limited our analysis to the case of idiosyncratic risk. Does systemic risk lead to a significantly different result? In order to answer this question, let us introduce some modifications in our model so as to switch from idiosyncratic to systemic risk. Also, since bank failure will now be possible as a consequence of the sole borrowers' default, we shall take net operational costs as non-stochastic.

### 4.1 Entrepreneurs in the modified model

In addition to her effort capacity, each entrepreneur $i \in [0,1]$ is endowed with a project $(x_i, \pi_i)$ that can be operated in period 0 at a fixed scale (normalized to one) and with an effort $e \in \{0,1\}$, so as to yield in period 1 with probability $\pi_i$ the output $e + x_i$, or with complementary probability $1-\pi_i$ an output $(1-\gamma)\,e$ (with a percentage loss $\gamma \in [0,1)$ when the project fails). More productive projects are riskier, and we assume again the mean-preserving property $\pi_i x_i = g \leq 1$, for any $i$. Systemic risk is described by a random variable $S$ uniformly distributed over the interval $[0,1]$. Given a realization $s$ of this random variable, any project $i$ such that $\pi_i \geq s$ will be successful, unsuccessful if $\pi_i < s$.

We assume the same disutility $v \in (0,g]$ per unit of effort for all entrepreneurs, and a uniform distribution of the characteristic $\pi_i$ of the different projects over the interval $[0,1]$. The investment financing conditions are the same as before. By limited liability, entrepreneurs cannot be forced to pay more than what they realize when their investment projects fail. Thus, entrepreneur $i$'s expected utility is

$$U_i(e,R) = \pi_i \max\{e + x_i - (1+R), 0\} + \underbrace{(1-\pi_i)(1-\gamma)(e-e)}_{=0} - ve. \tag{20}$$

Clearly, she chooses to make an effort only if the resulting expected utility is larger than the one obtained with no effort, which imposes the following *incentive compatibility constraint* on loans: $U_i(1,R) \geq U_i(0,R) = \max\{g - \pi_i(1+R), 0\}$. Besides, no loan will be accepted by the entrepreneur unless the higher expected utility (the one with $e = 1$) is non-negative: $U_i(1,R) \geq 0$. This results in a *participation constraint* on loans, which is however implied by the incentive compatibility constraint. Hence, we must take into account one single condition, which ensures participation with effort:

$$v \leq \max\{g - \pi_i R, 0\} - \max\{g - \pi_i(1+R), 0\} = \left\{ \begin{array}{c} \pi_i \text{ if } \pi_i \leq \frac{g}{1+R} \\ g - \pi_i R \text{ if } \frac{g}{1+R} \leq \pi_i \leq \frac{g}{R} \\ 0 \text{ if } \frac{g}{R} \leq \pi_i \end{array} \right. . \tag{21}$$

We can now establish the demand for loans:

$$
\begin{aligned}
D(R) &= \max\left\{\frac{g}{1+R}-v,0\right\}+\max\left\{\min\left\{\frac{g-v}{R},1\right\}-\max\left\{\frac{g}{1+R},v\right\},0\right\} \\
&= \left\{\begin{array}{c} 1-v \text{ if } 0\leq R\leq g-v \\ \frac{g-v(1+R)}{R} \text{ if } g-v\leq R\leq \frac{g-v}{v} \\ 0 \text{ if } \frac{g-v}{v}\leq R \end{array}\right. . \qquad (22)
\end{aligned}
$$

Notice that entrepreneur $i$ is a demander if she is endowed with a project $(x_i,\pi_i)$ such that $\pi_i\in[v,1]$ when $0\leq R\leq g-v$, or $\pi_i\in[v,(g-v)/R]$ when $g-v\leq R\leq (g-v)/v$. Hence, for a realized state $s\in[v,1]$, the mass of successful projects, those characterized by $\pi_i\geq s$, is $1-s$ and $(g-v)/R-s$, when $R$ belongs to the intervals $[0,g-v]$ and $[g-v,(g-v)/v]$, respectively. The proportion of successful projects in the aggregate loan portfolio $D(R)$ is consequently, in state $s\in[0,1]$,

$$
p(R,s)=\left\{\begin{array}{c} \min\left\{\frac{1-s}{1-v},1\right\} \text{ if } 0\leq R\leq g-v \\ \min\left\{\max\left\{\frac{g-v-sR}{g-v-vR},0\right\},1\right\} \text{ if } g-v\leq R\leq \frac{g-v}{v} \end{array}\right. , \qquad (23)
$$

which is constant in $R$ if $R\leq g-v$ and decreasing in $R$ if $v<s<(g-v)/R<1$, an expression of the *risk-shifting effect* (again obtained through adverse selection).

## 4.2 Banks in the modified model

Contrary to the case of idisyncratic risk, where large size and diversification of loan portfolios allowed to eliminate risk, banks can now only refer to *expected values*. They know that the proportion of successful projects in their portfolios is $p(R,s)$, so that in the case of realization of state $s$, they expect to obtain a gross profit per loan equal to

$$
\begin{aligned}
\widetilde{\Pi}(R,r,s) &= p(R,s)(1+R)+(1-p(R,s))(1-\gamma)-(1+r) \\
&= p(R,s)(R+\gamma)-(r+\gamma). \qquad (24)
\end{aligned}
$$

Assuming an exogenous net operational cost $\phi$ (now taken as non-stochastic and applying uniformly to anyone of the $n$ banks), and referring to the zero profit condition under a symmetric profile (for $n\phi/D(R)\leq R-r$)

$$
p(R,s^*)(R+\gamma)-(r+\gamma)=\frac{n\phi}{D(R)}, \qquad (25)
$$

we obtain, using equations (22) and (23), the following expression for $s^*$, the threshold state value that triggers bank default (occurring for any $s > s^*$):

$$s^*(R, r, n) = \begin{cases} 1 - \frac{(1-v)(r+\gamma)+n\phi}{R+\gamma} & \text{if} \quad 0 \leq R \leq g - v \\ \frac{1}{R+\gamma}\left((g-v)\left(1-\frac{r}{R}\right) + (r+\gamma)v - n\phi\right) & \text{if} \quad g - v \leq R \leq \frac{g-v}{v} \end{cases}. \tag{26}$$

The probability of bank default is consequently $1 - s^*(R, r, n)$, decreasing in $R$ if $R < g - v$ (the dominating effect is the margin effect), and increasing in $R$ (at least for small $r$) if $R > g - v$ (the dominating effect is the risk-shifting effect). We thus obtain a U-shaped relationship (or rather a V-shaped relationship, because of non-differentiability at the point $g - v$) between the loan interest rate and the probability of bank failure.

## 4.3 Free oligopolistic competition

Bank $j$'s objective is to maximize its expected gross profit

$$\Pi(R_j, r)\, l_j = \left(\int_0^1 \widetilde{\Pi}(R_j, r, s)\, ds\right) l_j = \left(\underbrace{\int_0^1 p(R_j, s)\, ds}_{P(R_j)} (R_j + \gamma) - (r + \gamma)\right) l_j \tag{27}$$

under the competitiveness constraint (10) on $R_j$ and the demand constraint (11) on $(R_j, l_j)$, as in section 3. It will not be necessary to repeat the analysis performed ih that section. It suffices to recall that no loan interest rate higher than the monopoly (or collusive) solution $\widehat{R} = \arg\max_R \Pi(R, r)\, D(R)$ can be observed in equilibrium.

Since, by (23), the probability of borrowers' success is

$$\begin{aligned} P(R) &= \begin{cases} \int_0^v ds + \int_v^1 \frac{1-s}{1-v} ds \text{ if } 0 \leq R \leq g - v \\ \int_0^v ds + \int_v^{(g-v)/R} \frac{g-v-sR}{g-v-vR} ds \text{ if } g - v \leq R \leq \frac{g-v}{v} \end{cases} \\ &= \begin{cases} \frac{1+v}{2} \text{ if } 0 \leq R \leq g - v \\ \frac{g-v(1-R)}{2R} \text{ if } g - v \leq R \leq \frac{g-v}{v} \end{cases}, \end{aligned} \tag{28}$$

we obtain by (22) and (24) the following expected aggregate gross profit (for the whole set of banks):

$$\begin{aligned}\widehat{\phi}(R,r,1) &= \Pi(R,r)\,D(R) \qquad (29)\\ &= \begin{cases} \left(\frac{1+v}{2}(R+\gamma)-(r+\gamma)\right)(1-v) & \text{if} \quad 0\le R\le g-v \\ \left(\frac{g-v(1-R)}{2R}(R+\gamma)-(r+\gamma)\right)\frac{g-v(1+R)}{R} & \text{if} \quad g-v\le R\le \frac{g-v}{v} \end{cases}.\end{aligned}$$

The function $\widehat{\phi}(\cdot,r,1)$ is always increasing if $R<g-v$ and decreasing if $R>g-v$ and if $r$ and $\gamma$ are small enough ($r+(1-v)\gamma \le (g-v)/2$ is a sufficient condition). Hence, the expected aggregate gross profit has a maximum at $\widehat{R}=g-v$, which is also the value of $R$ at which the risk-shifting effect becomes dominant (see (26)). In other words, the risk-shifting effect is always dominated by the margin effect, verifying the conventional wisdom, for any equilibrium value of $R$, necessarily smaller than $\widehat{R}$ (if there are at least two competing banks). We represent in Figure 2 the graphs of function $\widehat{\phi}(\cdot,r,n)$ (the inverse V-shaped curve) and of function $1-s^*(\cdot,r,n)$ (the V-shaped curve).[7] They correspond respectively to the bank's expected gross profit (in a symmetric profile) and to the probability of bank's default.

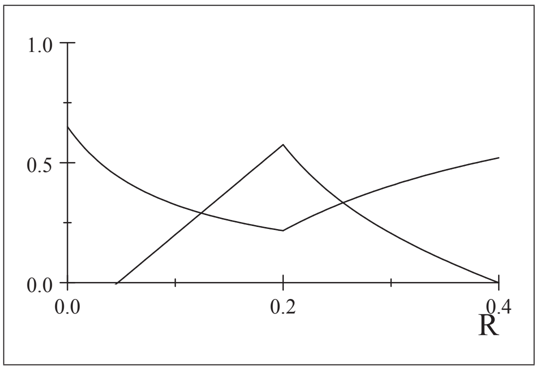


Figure 2: Bank's expected gross profit and probability of default

As long as net operation costs are taken as stochastic, bank failure is not determined by borrowers' default alone. In the modified model, where the net operation cost $\phi$ is non-stochastic, bank failure results from any realization $s$ of the random variable $S$ such that $s>s^*(R,r,n)$. We may in this case

[7]We have used the following parameter values: $g=0.7$, $v=0.5$, $\gamma=0.1$, $r=0.01$, $n\phi=0.01$. The scale in the vertical axis corresponds to the function $1-s^*(\cdot,r,n)$. The scale corresponding to the other function has been modified so as to superpose the two graphs on the same figure.

want to reformulate the bank's objective so as to take into account the bank's limited liability (as Martinez-Miera and Repullo do). To be explicit, bank $j$'s expected gross profit is then computed over the sole states leading to positive net profits, that is, over the interval $[0, s^*(R_j, r, n)]$:

$$\Pi^*(R_j, r, n)\, l_j = \left(\int_0^{s^*(R_j,r,n)} \pi(R_j, r, s)\, ds\right) l_j. \tag{30}$$

We have now a further effect of a change in $R_j$, through $s^*(R_j, r, n)$, on bank $j'$s objective function. Does this effect modify our conclusion? It does not. Indeed, a higher threshold state $s^*$ entails a higher expected profit, but such higher $s^*$ results from a *higher* loan interest rate if $R < \widehat{R} = g - v$ and from a *lower* loan interest rate if $R > \widehat{R} = g - v$. In other words, the effect introduced by taking into account limited liability reinforces the choice of $\widehat{R}$ as the monopoly (or collusive) value, a value which cannot be exceeded in any equilibrium of oligopolistic competition.

# 5 Conclusion

The debate between defenders of the 'competition-fragility' and 'competition-stability' views has been centered upon the risk of banks' loan portfolios, and thus ultimately on the risk of borrowers' default. This approach disregards the complexity of banking activities, of which the deposit-loan intermediation is only a part, and not necessarily the riskier one. In this paper, we have tried to shift the focus of the debate from the riskiness of loan portfolios to the riskiness of operational costs net of the income of non-traditional banking activities. In order to emphasize this shift, we have considerably simplified the analysis by using a very simple model in which, because of purely idiosyncratic risks, portfolio diversification would completely eliminate the risk of banks' default if those net operational costs were negligible or were known with certainty.

In this context, as the bank's default results from its aggregate profit being negative, the corresponding risk is intimately linked to the bank's objective function. As a consequence, in spite of countervailing margin and risk-shifting effects translating into a U-shaped relationship between the loan interest rate and the probability of bank default, the observable relationship is necessarily decreasing, since the increasing segment corresponds to inadmissible rates, higher than their collusive level. Hence, more competition always raises the risk of bank failure in our model.

A further step of our analysis made us investigate if this result is still valid once we consider systemic rather than idiosyncratic risk. Since in this context there is already risk of banks' default due to loan portfolios' riskiness, we assumed non-stochastic net operational costs. We obtained the same result: countervailing margin and risk-shifting effects translate into a U-shaped relationship between the loan interest rate and the probability of bank failure, but the decreasing segment is alone observable as a possible equilibrium outcome of competition for loans.

Another point of our contribution we want to stress is that our model, in spite of belonging to the partial equilibrium brand, takes into account the interdependence of loan and deposit markets, with asymmetric intensities of competition. In situations of liquidity shortness, more competition in the loan market may translate, at least partially, into higher deposit rates rather than lower loan rates. If we add to this the existence of regulated markets, in particular in developing countries, it is clear that variations in the intensity of bank competition should not be inferred from the mere observation of movements in loan rates. The same applies to changes in the bank number, since the intensity of competition cannot be reduced to market structure, ignoring conduct, as we have stressed in this paper.

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