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Playing the game the others want to play: Keynes' beauty contest revisited*

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Abstract

In Keynes' beauty contest, agents make choices by referring to their expectations of some fundamental value and of the conventional value to be set by the market. In doing so, agents respond to fundamental and strategic motives, respectively. The prevalence of either motive is usually set exogenously. Our contribution is to consider whether agents favor one of the two motives when the relative weights put on them are taken as strategic variables. We show that the strategic motive tends to prevail over the fundamental one, yielding a disconnection of agents' actions from the fundamental. This is done in a simple valuation game emphasizing the role of public information. We then extend the same result to competition between the owners of two firms, by using a delegation game in which informational issues are embedded into a broader microfounded setting.

Keywords: beauty contest, dispersed information, public signals, coordination, competition.

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1 Introduction

The beauty contest metaphor used by Keynes to characterize the working of financial markets displays the existence of a dual motivation in agents' decision making: there is a *fundamental motive* making agents strive to predict the fundamental value of some financial asset and there is a *strategic motive* making them seek to predict the conventional value eventually set by the market. There is no reason for the two values to coincide and, in Keynes' view, the working of stock markets, rather than imposing a balance between the two motives, tends to favor the strategic motive. Stock markets exhibit not only mutual coordination, but also common interest in coordination. Indeed, professional investors and speculators are not so much concerned with forecasting fundamentals as with "*anticipating what average opinion expects the average opinion to be*" (Keynes, 1936, ch.12, p.156). If others attach little importance to coordinating on each other's actions, and mainly focus their expectations on fundamentals, there is no compelling reason for one to attach importance to coordination either. By contrast, the more others attach importance to coordinating on each other's predictions, mainly basing their expectations on convention, the more one feels dependent on convention and concerned with the strategic motive. In this sense, each agent wants to play the game the others want to play.

While the beauty contest mainly captures the opposition between investment and speculation in stock markets, the same reasoning may apply to competition in a homogeneous product market. Because the law of one price prevails in such a market, coordinating under imperfect information on the same price as the others, even through a poorly accurate public signal, becomes predominant for a firm over being the sole to set a price close to the fundamental. In particular, setting a price higher than the prices chosen by the competitors may be so costly as to dissuade a firm to even approach the demand price.

The aim of our paper is to revisit Keynes' beauty contest in a setup that captures the strategic complementarity leading to the choice to play a pure strategic game, and then to embed such a mechanism into a more complex microfounded setting in which that choice will also depend on structural parameters interacting with informational issues. Our contribution is first to consider the tradeoff between the strategic and the fundamental motives not as structural but as resulting from strategic decisions. Strong strategic complementarity of those decisions may then end up in the full eviction of the fundamental motive and the emergence of sunspots. Second, we want to show that the context to which this phenomenon belongs extends in fact from the prediction of financial values to competition in oligopolistic output markets. We thus propose successively a *valuation game* applied to financial markets and a *delegation game* played by the owners of two firms in an output market. The former is dominated by the role of information: the players' objective is to minimize the cost associated with imperfect and dispersed information, responsible for deviations from the fundamental and conventional values. The latter embeds these informational issues into a more complex microfounded setting, in which a similar expected information cost is just a component of the expected profit to be maximized.

The valuation game is directly based on Morris and Shin (2002) (henceforth MS). In this famous representation of the Keynesian beauty contest, agents' actions consist in choosing a value which is a compromise between the anticipated *fundamental value* and the anticipated *conventional value* (the average of all the agents' actions). Under perfect information, agents can easily coordinate on the fundamental value, so that the fundamental and coordination motives coincide. By contrast, under imperfect information, agents receive public and private signals about the unknown fundamental value. Information being imperfect, agents have to form expectations on the fundamental, and information being dispersed, agents may find it difficult to coordinate. Dispersed information generates a conflict between matching the fundamental value and matching the conventional value, which expresses itself in the information cost.

While the terms of the tradeoff between the fundamental and the coordination motives are exogenously given in MS, we argue that players may be interested in manipulating the weights put on the fundamental and the strategic motives, for instance by choosing the shares of their portfolios they allocate to investment and speculation, respectively. The exogeneity of the relative weight put on the strategic motive leaves open the issue of the potential disconnection of actions from the fundamental value. We extend MS model to a two-stage game, in which agents first choose the weight they attribute to the strategic and fundamental motives before making the choice of the value that best matches the preferred combination of fundamental and strategic motives. We show that there is an incentive for agents to favor the strategic over the fundamental motive. More precisely, we show that the coordination activity (speculation) prevails in the valuation game as coordination on a public signal entails a lower cost than predicting an unknown fundamental. Information is the driving force for the coordination loss to be dominated by the fundamental loss: as agents put more weight on the coordination motive, they rely more on public information to estimate the average action, making it easier to coordinate on the convention. The strategic choice to privilege the convention results in the limit in a total disconnection between the valuation activity and the fundamental.[1]

The valuation game illustrates how the information structure may result in the dominance of the strategic over the fundamental motive, and ultimately in the full disconnection of agents' actions from fundamentals. We extend this analysis to examine whether the same mechanism carries through to a larger model where the strategic choice behind the pursuit of the strategic vs. the fundamental motive is given microfoundations and depends on the structural parameters of the model. We therefore propose a delegation game embedding

[1]The interaction between real and financial sectors is properly modelled in Angeletos, Lorenzoni and Pavan (2010). In the real sector, entrepreneurs invest in a new technology and base their investment decisions on their expectation of the future financial market valuation of their capital. In the financial sector, traders observe entrepreneurs' activity as a signal of the profitability of investment opportunities. The authors show how information spillovers between real and financial sectors amplify higher order uncertainty and exacerbate the disconnection from fundamentals. Here, we disregard the interaction between the real and financial sectors, concentrating on the stock market, and so obtain a much simpler game. Still, by relying on the agents' choice of the weights to be put on the fundamental and the strategic motives, our model captures the (full) disconnection between activity and fundamentals.

a similar mechanism in an Industrial Organization context. Previous IO applications of beauty contest games can be found in Angeletos and Pavan (2007) or Myatt and Wallace (2012, 2015). Our analysis differs from theirs in at least two respects. First, following MS, these applications treat the weight put on the strategic vs. the fundamental motive as determined by the model structure. Instead, as in our valuation game, we consider two stages, in the first of which firms' owners manipulate the relative weights attributed to the fundamental and the strategic motives. Second, as in the literature following Singh and Vives (1984), these applications assume linear demand for differentiated products and oppose Cournot and Bertrand competition, exhibiting quantities as strategic substitutes and prices as strategic complements, respectively. While we keep linear demand for differentiated products, we do not focus on the opposition between Cournot and Bertrand competition. Instead, relying on the delegation game introduced under perfect information by Miller and Pazgal (2001), in which that opposition becomes irrelevant,[2] we focus on the preliminary choice by firms' owners of the degree of cooperation to be later adopted by firms' managers in their conduct towards the competitors.[3] The first stage choice of the degree of cooperation also affects the weights put on the fundamental and strategic motives in our extension of the game to the context of imperfect and dispersed information.[4]

In this context, the marginal impact on the expected profit of a change in the degree of cooperation can be decomposed into a marginal *cooperation benefit*, which does not depend upon informational issues, and a marginal *expected information cost*, which vanishes under perfect information. This information cost is precisely the focus of the valuation game, where the structural part of the delegation game is ignored. Putting together the cooperation benefit and the information cost leads to new insights. The expected information cost deters firms from cooperating, but less and less so as competition intensifies as a consequence of less and less product differentiation. Eventually, in the limit of product homogeneity, the threat of a price disparity does justice to the message of the beauty contest: in spite of a possibly high expected information cost, the coordination motive evicts the fundamental motive, deletes the role of private signals, and opens the way to the emergence of sunspots.

The valuation game, offering a simplified representation of stock markets activity and illustrating the dominance of the coordination over the fundamental motive, with the con-

[2]As emphasized by Miller and Pazgal, the delegation game dilutes the opposition between price and quantity competition, because "*if the owners have sufficient power to manipulate their managers' incentives, the equilibrium outcome is the same regardless of how the firms compete in the second stage*" (2001, p. 284).

[3]The significant point is here the ability to commit in the first stage to adopt some conduct in the second, not the specific second stage payoff that we take from Miller and Pazgal (2001). See *e.g.* d'Aspremont and Dos Santos Ferreira (2009, s.5) for another specification.

[4]A natural alternative way to microfound the choice of the weights to be attributed to the fundamental and the strategic motives would be to assume that firms manipulate, at least to some extent, the degree of differentiation of their products, by locating appropriately in the space of product characteristics. Notice that this degree of differentiation is in particular the sole determinant of those weights in Myatt and Wallace (2012, s.2). However, as less product differentiation increases the relative weight on the strategic motive but at the same time decreases market power, profit maximization would not favor the predominance of the strategic motive. For such result to be attained, we would have to consider a different first stage objective, for instance consumer surplus maximization, source of a possible result reversal (see Myatt and Wallace, 2015).

sequent disconnection of actions from fundamentals, is examined in section 2. The delegation game, developing a microfounded extension of this setup to output markets, where informational issues are incorporated into a duopolistic competition model, is presented and analyzed in section 3. Section 4 concludes the paper.

2 The beauty contest: *investment* vs. *speculation* in stock markets

The opposition between *investment* and *speculation* in stock markets[5] can directly be captured by the model of MS describing a version of Keynes' beauty contest parable. MS formulate a *valuation game* in which agents' decisions have to meet both a fundamental and a strategic motive: their decision consists in choosing an action close to the fundamental value and to the conventional value set by the market. However, while the weight agents put on each motive of MS's valuation game is fully exogenous, we consider that it is a strategic variable. We therefore extend MS framework to consider a two-stage game in which agents first choose the weight they attribute to the strategic (and fundamental) motive(s) before making a decision, i.e. choosing a value, that matches the fundamental and/or the conventional value. This model accounts for the potential disconnection between speculation and enterprise in a very simple manner.

2.1 A two-stage valuation game

There is a finite number n of agents. The utility function of individual i is the negative of a quadratic loss function with three components. The first component (the fundamental motive) is a standard quadratic loss increasing in the distance between i's chosen value (action) a_i and the fundamental value θ. The second component (the coordination motive) is the 'beauty contest' term: the loss is increasing in the distance between i's chosen value (action) a_i and the average action of all players $\frac{1}{n}\sum_j a_j$ (the conventional value). The third component (the competition motive) is a loss decreasing in the variance of the action profile: an individual is happy to better match the average action than the others do.[6] Both coordination and competition components compound a strategic motive in the sense that, contrary to the fundamental motive, they imply interactions with other players. More explicitly, the utility function of agent i is given by:

[5] In his terminology, Keynes opposes *speculation* to *enterprise* (Keynes, 1936, p.158).

[6] This decomposition into fundamental, coordination and competition components is due to Cornand and Heinemann (2008).

$$
\begin{aligned}
u_i(\mathbf{a},\theta;r_i) &= -(1-r_i)\underbrace{(a_i-\theta)^2}_{\text{fundamental motive}} - r_i\underbrace{(L_i-\bar{L})}_{\text{strategic motive}} \qquad (1)\\
&= -(1-r_i)\underbrace{(a_i-\theta)^2}_{\text{fundamental motive}} \underbrace{- r_i\left(a_i-\frac{1}{n}\sum_j a_j\right)^2}_{\text{coordination motive}} \underbrace{+ r_i\frac{1}{n}\sum_j\left(a_j-\frac{1}{n}\sum_j a_j\right)^2}_{\text{competition motive}},
\end{aligned}
$$

where $\mathbf{a}$ is the profile of the chosen value over all agents (the action profile), r_i is the weight agent i has decided to put on the strategic motive, and

$$
L_i = \frac{1}{n}\sum_j (a_i-a_j)^2, \qquad \bar{L} = \frac{1}{n}\sum_j L_j. \qquad (2)
$$

The timing of the game is as follows. First, each agent i chooses the share r_i of his portfolio he wants to allocate to speculation rather than to investment, hence the weight he puts on the strategic motive relative to the fundamental motive. Second, each agent i chooses a_i by combining his information so as to minimize the loss incurred in deviating from the target fundamental and conventional values, in other words so as to minimize the information cost of his action.

The framework we adopt in this section is very close to that of MS. The only differences compared to MS are the finite number of agents (in this respect we are close to Martimort and Stole, 2011) and the addition to the game of a first stage in which agent i chooses $r_i \in [0,1]$.[7]

Under perfect information, each agent i knows with certainty the fundamental state θ and chooses at the second stage $a_i^* = \theta$, implying that any $r_i \in [0,1]$ is an equilibrium of the first stage. Indeed, under perfect information, there is no information cost and no conflict between the motives, which coincide all the three. This is no more the case under imperfect and dispersed information. Following the literature in the vein of MS, we assume that, after choosing r_i and before choosing a_i, each agent i receives two signals on the unknown fundamental value θ: a public signal, common to all agents, with a normally distributed error term $y = \theta + \eta$, such that $\eta \sim N(0, 1/\alpha)$, and a private signal $x_i = \theta + \varepsilon_i$, such that $\varepsilon_i \sim N(0, 1/\beta)$. The ε_i's are identically and independently distributed across agents and independently distributed from η.[8] Now, under imperfect information, there is an information cost, and dispersed imperfect information generates a conflict between the

[7]In MS, $r_i = r$ is set exogenously, and $0 \leq r < 1$, while in Martimort and Stole (2011), the weights r_i are heterogeneous but exogenous.

[8]Notice here that, because of the existence of a first stage, our model presents a major technical difference compared to the literature in the vein of MS. While the fundamental is unknown in the MS one-stage version of the game, its value is realized. Of course, uncertainty remains as the signals the agents receive on this realization are not perfectly informative. By contrast, in the first stage of our two-stage version of the game, the realization of the fundamental has not yet been drawn, and the fundamental is perfectly random, which implies the necessity of considering expectations on the fundamental and of taking into account its mean square value, as we shall see later.

fundamental and the coordination motives. While the conditions of the tradeoff between the different motives are given in MS, we allow players to manipulate the tradeoff's conditions by letting them choose the weights on the different motives. In so doing, we assume that $\alpha > 0$ and $\beta < \infty$, so that the public signal never ceases to be informative and the private signal never becomes fully informative on the fundamental. These assumptions insure that the public signal is always relevant (on the fundamental).

2.2 The second stage equilibrium under dispersed information

We solve the model backwards, starting by the second stage and taking the r_i's as given. The maximization problem of any agent i is: $\max_{a_i} \mathbb{E}\left(u_i\left(\mathbf{a}, \theta; r_i\right) \mid x_i, y\right)$. The first order condition yields

$$a_i = \left(1 - r_i\right) \mathbb{E}_i\left(\theta\right) + r_i \mathbb{E}_i\left(\frac{1}{n}\sum_j a_j\right), \tag{3}$$

where $\mathbb{E}_i(.) = \mathbb{E}(.|x_i, y)$ is the expectation operator conditional on the signals received, and $\mathbb{E}_i(\theta) = (\alpha y + \beta x_i)/(\alpha + \beta)$.

To derive $\mathbb{E}_i\left(\frac{1}{n}\sum_j a_j\right)$ and following MS, we assume that any agent j adopts the same linear strategy: $a_j = \kappa_j y + \left(1 - \kappa_j\right) x_j + \lambda_j s$, where s is a sunspot,[9] the realization of which is known by the agents. We write $\kappa = \frac{1}{n}\sum_j \kappa_j$ and $\lambda = \frac{1}{n}\sum_j \lambda_j$, so that

$$\begin{aligned} \mathbb{E}_i\left(\frac{1}{n}\sum_j a_j\right) &= \kappa y + (1-\kappa)\mathbb{E}_i(\theta) + \lambda s \\ &= \frac{\kappa\beta + \alpha}{\alpha + \beta} y + \left(1 - \frac{\kappa\beta + \alpha}{\alpha + \beta}\right) x_i + \lambda s. \end{aligned} \tag{4}$$

Inserting (4) in (3), the optimal action writes:

$$\begin{aligned} a_i &= \left(1 - r_i\right) \mathbb{E}_i(\theta) + r_i\left(\kappa y + (1-\kappa)\mathbb{E}_i(\theta) + \lambda s\right) \\ &= \frac{\alpha + \kappa\beta r_i}{\alpha + \beta} y + \left(1 - \frac{\alpha + \kappa\beta r_i}{\alpha + \beta}\right) x_i + r_i \lambda s. \end{aligned} \tag{5}$$

Identifying coefficients κ and λ, and writing $r = \frac{1}{n}\sum_j r_j$, we obtain $\kappa = \frac{\alpha + \kappa\beta r}{\alpha + \beta}$, hence $\kappa = \frac{\alpha}{\alpha + \beta(1-r)}$, and $\lambda = r\lambda$, so that $\lambda = 0 \quad$ or $\quad r = 1$. Plugging the expression of κ into (4) yields:

$$\mathbb{E}_i\left(\frac{1}{n}\sum_j a_j\right) = \frac{\alpha(\alpha + \beta(2-r))}{(\alpha+\beta)(\alpha + \beta(1-r))} y + \left(1 - \frac{\alpha(\alpha + \beta(2-r))}{(\alpha+\beta)(\alpha + \beta(1-r))}\right) x_i + \lambda s. \tag{6}$$

[9] As we allow for $r = 1$, contrary to MS, we specify a linear rule that includes sunspots.

Using (6) to re-write (3) delivers:

$$
\begin{aligned}
a_i &= (1-r_i)\frac{\alpha y+\beta x_i}{\alpha+\beta} \\
&\quad +r_i\left(\frac{\alpha(\alpha+\beta(2-r))}{(\alpha+\beta)(\alpha+\beta(1-r))}y+\left(1-\frac{\alpha(\alpha+\beta(2-r))}{(\alpha+\beta)(\alpha+\beta(1-r))}\right)x_i+\lambda s\right) \\
&= \kappa_i y+(1-\kappa_i)\,x_i+r_i\lambda s, \text{ with } \kappa_i\equiv\frac{\alpha}{\alpha+\beta}\left(1+\frac{\beta r_i}{\alpha+\beta(1-r)}\right). \qquad (7)
\end{aligned}
$$

Depending on whether $r=1$ or $r<1$, the solution to the maximization problem of any agent i is given by[10]

$$
a_i=\frac{1}{\alpha+\beta}\left(\alpha\left(1+\frac{\beta r_i}{\alpha+\beta(1-r)}\right)y+\beta\left(1-\frac{\alpha r_i}{\alpha+\beta(1-r)}\right)x_i\right) \quad \text{if} \quad r<1 \qquad (8)
$$

and

$$
a_i=\frac{1}{\alpha+\beta}((\alpha+\beta r_i)y+\beta(1-r_i)x_i)+r_i\lambda s \quad \text{if} \quad r=1. \qquad (9)
$$

Thus, the second stage equilibrium is an action profile $\mathbf{a}^*(\mathbf{r})$ depending on the profile of the weights r_j's chosen by each agent j at the first stage, which is such that for any i, the equilibrium value $a_i^*(r_i,r)$ depends on r_i and on the average r of the weights chosen by all the agents. By (8) and (9), we can distinguish two cases: $r<1$ and $r=1$ (implying $r_i=1$ and $\lambda_i=\lambda$ for any i). We accordingly formulate the following lemma.

Lemma 1 *The second-stage equilibrium is unique for $0\le r<1$, with $a_i^*(x_i,y,r_i,r)$ given by (8). If $r=1$, there is a continuum of equilibria, with $a_i^*=y+\lambda s$, for any $\lambda\ge 0$ and any sunspot s.*

Proof. Follows directly from equations (8) and (9). □

2.3 The subgame perfect equilibrium

To derive the subgame perfect equilibrium, we maximize $\mathbb{E}(u_i(\mathbf{a}^*(\mathbf{r}),\theta;r))\equiv -G(r_i,r;n)$ (we minimize the expected information cost G) with respect to r_i, focusing on two useful benchmarks: $n\to\infty$ (which approximates the continuum of agents assumed by MS) and $n=2$ (which allows to make a direct parallel with the duopoly case developed in Section 3). Appendix A shows that, when $n\to\infty$, the information cost function $G(\cdot,r;n)$ is strictly concave, if we assume $\alpha/\beta>0$. Hence, it is minimized either at $r_i=0$ or at $r_i=1$. Appendix A further shows that $\lim_{n\to\infty}G(0,r;n)>\lim_{n\to\infty}G(1,r;n)$. As to the case when $n=2$, Appendix A shows that, if α and β are both finite, we obtain for any $r_i<1$, $G(r_i,r;2)>0=G(1,r;2)$. Hence, in both cases the unique subgame perfect equilibrium is the symmetric equilibrium $r_i^*=r_j^*=1$ for any i and j. We state this result in the following proposition.

[10]Note that when $r_i=r\neq 1$, we are back to the MS case: $a_i=\frac{\alpha y+\beta(1-r)x_i}{\alpha+\beta(1-r)}$.

Proposition 1 *Take $0 < \alpha < \infty$ and $\beta < \infty$. Then, if $n \to \infty$ or $n = 2$, the valuation game has a unique sub-game perfect equilibrium $r_i^* = r^* = 1$ for any i. By Lemma 1, the corresponding second stage subgame admits a continuum of equilibrium actions $a_i^* = y + \lambda s$, depending upon a sunspot s.*

Proof. See Appendix A. □

The influence of the fundamental motive vanishes in both extreme cases, when $n \to \infty$ and when $n = 2$: indeed, the fundamental motive contributes more heavily than the strategic motive to the loss (or to the information cost) beared by the agents, who consequently choose to play the strategic game rather than the fundamental one or any mixture of the two. Looking at the second stage equilibrium action given by (8) is instructive. When r_i increases, agents put more weight on the public signal y, and the more so the higher the precision of the public signal α/β relative to that of the private signal. A more accurate public signal makes it easier to know what the others do in the second stage and so to coordinate. At the second stage, the competition term plays no strategic role, while at the first stage it reinforces agents' will to choose $r_i = 1$.

A discontinuity is involved in Proposition 1. It operates via the first stage equilibrium as soon as $\alpha/\beta > 0$, making r^* become equal to one at equilibrium, which disconnects the second stage valuations from the fundamental and creates the possibility of sunspots.[11] Although this case where actions are disconnected from fundamentals and valuation rests exclusively on convention seems to be sometimes practically relevant, as argued by Keynes, the literature in the vein of MS has excluded it by assumption, as the debate related to the social value of information becomes then trivial.

Note that the intuition for the case when $n = 2$ is very simple. In this case, the coordination and competition motives merge: matching the average value coincides with matching the other's value. This is due to the particular form of the competition motive. Because there are only two players, when a player increases his own performance, he increases that of the other player at the same time. The effects of the two strategic motives are exactly compensated, and only the fundamental motive remains. The best an agent can do is then to set this motive to 0, which corresponds to a choice of $r_i = 1$. The valuation game in which $n = 2$ is particularly relevant to illustrate how the information structure is responsible for the loss associated with the strategic motive to become smaller than the loss associated with the fundamental motive and thus to illustrate the full disconnection from fundamentals that results. In the next section, we study whether this unambiguous disconnection carries over when we insert this mechanism into a larger, microfounded duopoly setting, in which the information cost iteracts with cooperation issues.

[11] In particular, note that as $\alpha \to 0$, the weight agents put on the public signal is nil only as long as $r < 1$. For $\alpha = 0$, the public signal becomes so noisy that it ceases not only to contain information about the fundamental value but also, in this case, to play its coordinating role. Individuals have then to rely on their private information, possibly feeling more concerned with the fundamental motive. When $r = 1$ though, even if y has no informational content on the fundamental, it insures coordination and plays the role of a sunspot.

3 Competition in a duopolistic market: *cooperation* and *co-ordination*

Let us now move from stock to output markets in order to illustrate some of the properties of beauty contest games that we have emphasized in the preceding section. Industrial Organization applications are already available in this context, for instance in Angeletos and Pavan (2007) and Myatt and Wallace (2012, 2015). Like these authors, we assume linear demand for differentiated products – here only two, for ease of exposition. By contrast however, and in line with our previous analysis, we shall consider a two-stage game, a *delegation game* to be specific, where the players' decisions at the first stage determine the relative weights attributed to the fundamental and the strategic motives.

3.1 A two-stage delegation game

Consider a market for two differentiated commodities where the demand for good i ($i, j = 1, 2$, $i \neq j$) takes the form:

$$q_i = a - p_i + d\left(p_j - p_i\right), \text{with } 0 \leq d \leq \infty, \tag{10}$$

q_i being the quantity of good i demanded at prices p_i and p_j of the two goods.[12] The differentiation parameter d ($d = 0$ for independent goods, $d = \infty$ for perfect substitutes) is an indicator of the intensity of competition in the duopoly. Firms are price setters but, referring to the delegation game of Miller and Pazgal (2001), we assume that the owner of each firm i has sufficient control over the respective manager's conduct to impose a degree of cooperation $\gamma_i \in [0, 1]$ weighting the competitor's profit and turning firm i's payoff at the second stage into

$$\Pi_i\left(p_i, p_j; \gamma_i, a\right) = p_i\left(a - p_i + d\left(p_j - p_i\right)\right) + \gamma_i p_j\left(a - p_j + d\left(p_i - p_j\right)\right), \tag{11}$$

to be maximized in p_i. By taking the extreme values of the two degrees of cooperation, we obtain Bertrand competition for a fully non-cooperative conduct ($\gamma_i = \gamma_j = 0$), and tacit collusion for a fully cooperative conduct ($\gamma_i = \gamma_j = 1$). A continuum of intermediate equilibrium outcomes, in particular the Cournot outcome, is attainable between these extremes.

The first order condition for maximization of this payoff leads to the best reply price

$$p_i = \frac{\frac{a}{2} + d\frac{1+\gamma_i}{2}p_j}{1+d}, \tag{12}$$

a weighted mean of the monopoly price $a/2$ (which will be taken in the following as the fundamental θ) and of the transformed competitor's price (such that p_j is augmented by

[12]Generality is not lost by assuming a demand curve with slope -1 instead of $-b < 0$.

the factor $(1+\gamma_i)/2$, increasing with the degree of cooperation). Notice that the relative weight put into the fundamental decreases with the intensity of competition from 1 (when the products are independent) to 0 (when they are perfect substitutes).

A simple computation (using equations (12) for p_i and p_j) allows us to determine the equilibrium prices at the second stage of the game, when information is perfect:

$$p_i^*\left(\gamma_i,\gamma_j\right)=\frac{1+d+d\frac{1+\gamma_i}{2}}{\left(1+d\right)^2-d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}\frac{a}{2}, \tag{13}$$

for $i,j=1,2$, $i\neq j$. These prices increase with both degrees of cooperation, from their fully non-cooperative Bertrand value $a/\left(2+d\right)$ to the collusive (or monopoly) price $a/2$.

Each firm owner, anticipating these prices, chooses at the first stage the degree of cooperation she wants to impose on her manager's conduct, in order to maximize her own profit, say $\Pi_i^*\left(\gamma_i,\gamma_j\right)=\Pi_i\left(p_i^*\left(\gamma_i,\gamma_j\right),p_j^*\left(\gamma_i,\gamma_j\right);0,a\right)$ for firm i, that is,

$$\Pi_i^*\left(\gamma_i,\gamma_j\right)=p_i^*\left(\gamma_i,\gamma_j\right)\left(a-p_i^*\left(\gamma_i,\gamma_j\right)+d\left(p_j^*\left(\gamma_i,\gamma_j\right)-p_i^*\left(\gamma_i,\gamma_j\right)\right)\right). \tag{14}$$

Maximizing this profit in γ_i is clearly equivalent to minimizing in the same variable its loss with respect to the profit $(1/2)\,\Pi_i^*\left(1,1\right)$ that would be obtained under tacit collusion, namely (denoting $\theta\equiv a/2$, $p_i^*\equiv p_i^*\left(\gamma_i,\gamma_j\right)$ and $\overline{p}^*\equiv\left(p_i^*+p_j^*\right)/2$)

$$(1/2)\,\Pi_i^*\left(1,1\right)-\Pi_i^*\left(\gamma_i,\gamma_j\right)=\underbrace{\left(\theta-p_i^*\right)^2}_{\text{fundamental motive}}+2d\underbrace{\left(\overline{p}^*-p_i^*\right)^2}_{\text{coordination motive}}-2d\underbrace{\overline{p}^*\left(\overline{p}^*-p_i^*\right)}_{\text{competition motive}}, \tag{15}$$

where we retrieve the three motives identified in the valuation game, although in a slightly modified form. In particular, the competition motive adds to the loss of firm i only if its second stage equilibrium price is higher than the one of its competitor, and moderates the loss otherwise.[13] Thus, the coordination and the competition motives do not exactly compensate as they do in the valuation game, when there are only two players.

Should the competition motive vanish, because of a genuine cooperative attitude of firm owners, instead of a cooperative conduct strategically imposed by them on firm managers, the subgame perfect equilibrium would be characterized under perfect information by maximum degrees of cooperation, with tacit collusion at the second stage. As in the valuation game, perfect information would naturally play an essential role in this result, by dissolving the potential conflict between the fundamental and the coordination motives. However, even keeping information perfect, the competition motive destroys in the present game the collusive outcome, entailing the following best reply degree of coopera-

[13] Another modification concerns the weights put into the fundamental and the strategic motives, here 1 and $2d$ ($nd/\left(n-1\right)$ with n players), which do not add to 1. Normalizing them to $1/\left(1+2d\right)$ and $2d/\left(1+2d\right)=r$ would not change the argument, except when evaluating the effects of modifying, strategically or not, the differentiation parameter d.

tion for firm i

$$\gamma_i = \frac{(2+3d)\, d\left(1+\gamma_j\right)}{4\,(1+d)^2 + (2+d)\, d\left(1+\gamma_j\right)}, \tag{16}$$

which is increasing in γ_j but always smaller than 1 as long as d is finite. Strategic complementarity at the first stage is not strong enough to translate into full cooperation. The (symmetric) subgame equilibrium value of the degree of cooperation is indeed

$$\gamma^* = \frac{d}{2+d}, \tag{17}$$

attaining 1 only in the limit case of perfect substitutability of the two products.

3.2 The second stage equilibrium under dispersed information

We shall now assume that the parameter a is stochastic, with mean $\overline{a}$ and variance σ^2, so that $\mathbb{E}\left(\theta^2\right) = \left(\overline{a}^2 + \sigma^2\right)/4$, the sum of the mean square and of the variance of the fundamental. Each manager i takes decisions at the second stage ignoring the realized value θ of the fundamental, but after receiving public and private signals y and x_i, both with mean θ. We adopt the same information structure as in section 2, except that we do not immediately impose symmetry on the quality of private information, the precision β_i possibly differing from the precision β_j. As will become clear, this remains tractable in a two player game and allows us to distinguish the effects on each firm behavior of changes in the precision of its own private signal and in the precision of that of its competitor. Distinguishing these effects is particularly important in a context where we have to disentangle the cooperative and coordinating roles played by the strategic variable γ_i.

According to the two signals received, given Π_i as defined by (11), the problem of firm i's manager becomes: $\max_{p_i} \mathbb{E}\left(\Pi_i\left(p_i, p_j; \gamma_i, 2\theta\right) \middle| \, y, x_i\right)$. Referring to the first order condition (12), we may reformulate firm i's best reply as

$$p_i = \frac{\mathbb{E}_i\left(\theta\right) + d\frac{1+\gamma_i}{2}\mathbb{E}_i\left(p_j\right)}{1+d}, \tag{18}$$

where $\mathbb{E}_i\left(\cdot\right) \equiv \mathbb{E}(\cdot \,|\, y, x_i)$ is the expectation operator conditional on the signals received, and $\mathbb{E}_i(\theta) = (\alpha y + \beta_i x_i)/(\alpha + \beta_i)$.[14]

To determine $\mathbb{E}_i\left(p_j\right)$, we assume as in section 2 that both firms follow the same linear strategy: $p_j = \kappa_j y + \kappa_j' x_j + \lambda_j s$, where s is a sunspot, the realization of which is known by

[14] When assuming normal distributions of the public and private signals, we are ignoring the consequences of obtaining values of $\mathbb{E}_i\left(\theta\right)$ that are either negative or too high to ensure a positive demand.

both firms. As $\mathbb{E}_i(x_j) = \mathbb{E}_i(\theta)$, we thus obtain

$$
\begin{aligned}
p_i &= \frac{\frac{\alpha y+\beta_i x_i}{\alpha+\beta_i} + d\frac{1+\gamma_i}{2}\left(\kappa_j y + \kappa_j' \frac{\alpha y+\beta_i x_i}{\alpha+\beta_i} + \lambda_j s\right)}{1+d} \qquad (19)\\
&= \underbrace{\frac{\left(1+d\frac{1+\gamma_i}{2}\kappa_j'\right)\alpha + d\frac{1+\gamma_i}{2}\left(\alpha+\beta_i\right)\kappa_j}{\left(1+d\right)\left(\alpha+\beta_i\right)}}_{\kappa_i} y + \underbrace{\frac{\left(1+d\frac{1+\gamma_i}{2}\kappa_j'\right)\beta_i}{\left(1+d\right)\left(\alpha+\beta_i\right)}}_{\kappa_i'} x_i + \underbrace{\frac{d\frac{1+\gamma_i}{2}\lambda_j}{1+d}}_{\lambda_i} s,
\end{aligned}
$$

for $i, j = 1, 2$, $i \neq j$. Identifying the coefficients κ_i, κ_i' and λ_i, and using the two equations to establish their equilibrium values, we obtain after some tedious but straightforward computations:

$$
\kappa_i' = \frac{\beta_i\left(\left(1+d\right)\left(\alpha+\beta_j\right) + d\frac{1+\gamma_i}{2}\beta_j\right)}{\left(1+d\right)^2\left(\alpha+\beta_i\right)\left(\alpha+\beta_j\right) - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta_i\beta_j}, \qquad (20)
$$

$$
\begin{aligned}
\kappa_i &= \frac{\alpha}{\beta_i}\kappa_i' + \frac{d\frac{1+\gamma_i}{2}}{1+d}\kappa_j = \frac{\left(1+d\right)\alpha}{\left(1+d\right)^2\left(\alpha+\beta_i\right)\left(\alpha+\beta_j\right) - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta_i\beta_j}\\
&\quad \times\left(\frac{1+d+d\frac{1+\gamma_i}{2}}{\left(1+d\right)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}\left(1+d\right)\left(\alpha+\beta_i+\beta_j\right) - \beta_i\right), \qquad (21)
\end{aligned}
$$

and

$$
\left(1 - \frac{d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}{\left(1+d\right)^2}\right)\lambda_i = 0. \qquad (22)
$$

We may conclude this subsection by the following lemma.

Lemma 2 *(i) Take $d < \infty$ or $(\gamma_1, \gamma_2) \neq (1, 1)$. Then, conditionally on the realization of the public signal y and the private signals x_i and x_j, the second stage equilibrium price of good i in the delegation game is uniquely determined by $p_i^*\left(x_i, y, \gamma_i, \gamma_j\right) = \kappa_i y + \kappa_i' x_i$ ($i, j = 1, 2$, $i \neq j$), with κ_i and κ_i' given by equations (21) and (20), respectively. (ii) If $d = \infty$ and $\gamma_1 = \gamma_2 = 1$, there is a continuum of equilibria conditional on the realization of the public signal y and of a sunspot s, such that the second stage equilibrium price of good i in the delegation game is determined by $\widetilde{p}_i^*\left(y, s, \lambda\right) = \kappa y + \lambda s$, with any $\kappa \in [0, 1]$ and $\lambda \geq 0$.*

Proof. Result (i) follows directly from equations (19) to (22). Equation (22) implies in this case $\lambda_i = \lambda_j = 0$. In case (ii), $\kappa_i' = \kappa_j' = 0$ by (20) and $\kappa_i = \kappa_j = \kappa \in [0, 1]$ by (21), indeterminacy being ascribable to the presence of the product $d^2\left(1 - \left(1+\gamma\right)^2/4\right)$ in the denominator of the first term inside the parentheses. Also, λ_i and λ_j are multiplied by 0 in the LHS of equation (22) and may consequently be positive. However, they must be equal, otherwise the two prices would be different, implying (with $d = \infty$) demands equal to $\pm\infty$. □

3.3 Induced changes in the weights on the public and private signals

In this subsection, we will consider the changes in the weights κ_i and κ_i' on the public and private signals, respectively, that are induced either by changes in the quality of public and private information, or by changes in the degrees of cooperation. First, consider the sum of those weights

$$\kappa_i + \kappa_i' = \frac{1 + d + d\frac{1+\gamma_i}{2}}{(1+d)^2 - d^2 \frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}} \equiv K_i\left(\gamma_i, \gamma_j; d\right). \tag{23}$$

We see that it is independent of the information quality characterized by $\left(\alpha, \beta_i, \beta_j\right)$. Changes in the precision of the public and private signals can however modify the relative weights $\kappa_i / \left(\kappa_i + \kappa_i'\right)$ and $\kappa_i' / \left(\kappa_i + \kappa_i'\right)$. In order to allow a comparison with the corresponding weight in the valuation game, let us express the relative weight put on the public signal as

$$\frac{\kappa_i}{\kappa_i + \kappa_i'} = \frac{\alpha}{\alpha + (1 - r_i)\,\beta_i}, \tag{24}$$

where, according to (21) and (23),

$$r_i \equiv \frac{d\frac{1+\gamma_i}{2}}{1+d} \frac{\left(1 + d + d\frac{1+\gamma_j}{2}\right)(1+d)\left(\alpha + \beta_i\right) + \left(1 + d + d\frac{1+\gamma_i}{2}\right) d\frac{1+\gamma_j}{2}\beta_j}{\left(1 + d + d\frac{1+\gamma_i}{2}\right)(1+d)\left(\alpha + \beta_j\right) + \left(1 + d + d\frac{1+\gamma_j}{2}\right) d\frac{1+\gamma_i}{2}\beta_i}. \tag{25}$$

We find two ratios in this expression. The first is just the weight put into the competitor's price by the best reply of firm i under perfect information (see (12)). The second is equal to 1 under symmetry ($\beta_i = \beta_j$ and $\gamma_i = \gamma_j$), and tends to the reciprocal of the first as $\beta_i \to \infty$. Thus, if firm i has alone access to perfect private information, r_i is equal to 1, so that we cannot conclude that $\alpha / \left(\alpha + (1 - r_i)\,\beta_i\right)$ tends to 0 as $\beta_i \to \infty$. As a matter of fact, it can be checked from (21) and (20) that both κ_i and κ_i' tend in this case to finite positive values. In spite of perfect private information, the public signal will still be taken into account, for the sake of coordination. Only if the competitor's private signal becomes perfectly informative too, will the weight on the public signal tend to 0, coordination resulting spontaneously from perfect private information shared by *both* firms.

As we see, it is important to distinguish the two precisions β_i and β_j. The weight κ_i' on the private signal x_i is naturally more sensitive to changes in the precision β_i of this signal than to those in the precision β_j of the other private signal, even if it is increasing in both precisions. It tends in particular to 0 as $\beta_i \to 0$, whereas $\lim_{\beta_j \to 0} \kappa_i' = \frac{1}{1+d}\frac{\beta_i}{\alpha + \beta_i}$, which remains possibly large under a high precision β_i. The weight κ_i on the public signal is on the contrary more sensitive to changes in the precision β_j than to changes in the precision β_i. Finally, observe that the private signal tends to be completely disregarded, a maximum weight being put into the public signal, as the former becomes totally uninformative or the latter perfectly informative, whatever the precision of the competitor's private signal:

$$\lim_{\alpha \to \infty} \kappa_i' = \lim_{\beta_i \to 0} \kappa_i' = 0 \text{ and } \lim_{\alpha \to \infty} \kappa_i = \lim_{\beta_i \to 0} \kappa_i = K_i\left(\gamma_i, \gamma_j; d\right), \tag{26}$$

with $K_i\left(\gamma_i, \gamma_j; d\right)$ given by (23). By contrast, the limit values of the two weights are reversed when the public signal becomes totally uninformative, or as the precisions of *both* private signals tend to infinity:

$$\lim_{\alpha\to 0} \kappa_i' = \lim_{\beta_i\to\infty,\beta_j\to\infty} \kappa_i' = K_i\left(\gamma_i, \gamma_j; d\right) \text{ and } \lim_{\alpha\to 0} \kappa_i = \lim_{\beta_i\to\infty,\beta_j\to\infty} \kappa_i = 0. \tag{27}$$

We finally consider the consequences of changing the degrees of cooperation. Higher values of γ_i or γ_j determine an increase in the sum $\kappa_i + \kappa_i' = K_i\left(\gamma_i, \gamma_j; d\right)$. Notice that, from the point of view of the first stage of the game, to be analyzed in the next subsection, the expectation of the equilibrium price, formed before reception of the signals x_i and y, is $\mathbb{E}\left(p_i^*\right) = \kappa_i \mathbb{E}\left(y\right) + \kappa_i' \mathbb{E}\left(x_i\right) = \left(\kappa_i + \kappa_i'\right) \mathbb{E}\left(\theta\right)$, proportional to the sum of the two weights. Thus, increasing γ_i exerts a positive effect through $K_i\left(\cdot, \gamma_j; d\right)$ on the expected price, which can be seen as a *cooperation effect*, absent in the valuation game, where $\kappa_i + \kappa_i' \equiv 1$.[15] In addition to this effect, increasing γ_i has also, by (25), a positive effect on r_i and hence on $\kappa_i / \left(\kappa_i + \kappa_i'\right)$, which can be seen as a *coordination effect*.

3.4 The subgame perfect equilibrium

3.4.1 Equilibrium determination

At the first stage, the owner of firm i maximizes in γ_i its expected profit, namely $\mathbb{E}\left(\Pi_i^*\left(\gamma_i, \gamma_j\right)\right)$, Π_i^* being defined by (14) with $a = 2\theta$ and $p_i^*\left(\gamma_i, \gamma_j\right) = \kappa_i\left(\gamma_i, \gamma_j\right) y + \kappa_i'\left(\gamma_i, \gamma_j\right) x_i$. The weights κ_i and κ_i', given by (21) and (20) respectively, do not depend upon the random values θ, x_i, x_j and y. Also, recall that $x_i = \theta + \varepsilon_i$, $x_j = \theta + \varepsilon_j$ and $y = \theta + \eta$, with ε_i, ε_j and η normally and independently distributed with mean 0 and variances $1/\beta_i$, $1/\beta_j$ and $1/\alpha$, respectively. Firm i's expected profit can accordingly be formulated as

$$\mathbb{E}\left(\Pi_i^*\right) = \mathbb{E}\left(\theta^2\right) \left[\begin{array}{c} \underbrace{\left(\kappa_i + \kappa_i'\right)\left(2 - (1+d)\left(\kappa_i + \kappa_i'\right) + d\left(\kappa_j + \kappa_j'\right)\right)}_{F_i\left(\gamma_i, \gamma_j\right)} \\ -\underbrace{\left(\frac{\kappa_i}{\mathbb{E}\left(\theta^2\right)\alpha}\left(\kappa_i + d\left(\kappa_i - \kappa_j\right)\right) + \frac{\kappa_i'}{\mathbb{E}\left(\theta^2\right)\beta_i}\left(1+d\right)\kappa_i'\right)}_{G_i\left(\gamma_i, \gamma_j\right)} \end{array} \right]. \tag{28}$$

By (21) and (20), κ_i and κ_i' ($i = 1, 2$) are homogeneous of degree 0 in $\left(\alpha, \beta_i, \beta_j\right)$, so that, when maximizing $\mathbb{E}\left(\Pi_i^*\right)$ or, equivalently, $\mathbb{E}\left(\Pi_i^*\right)/\mathbb{E}\left(\theta^2\right)$, we may substitute the normalized precisions $\mathbb{E}\left(\theta^2\right)\alpha$, $\mathbb{E}\left(\theta^2\right)\beta_i$ and $\mathbb{E}\left(\theta^2\right)\beta_j$ to the original precisions. In other words, we may take from now on $\mathbb{E}\left(\theta^2\right) = 1$ without changing the argument.

The first term inside the brackets, $F_i\left(\gamma_i, \gamma_j\right) \equiv \Pi_i\left(\kappa_i + \kappa_i', \kappa_j + \kappa_j'; 0, 2\right)$, is the profit that firm i would obtain at prices $\kappa_i + \kappa_i'$ and $\kappa_j + \kappa_j'$ if θ were expected to be equal to 1 with certainty. It only depends upon the sums of the weights on the private and public signals,

[15] In the delegation game, we would obtain $\kappa_i + \kappa_i' = 1$ by imposing $\gamma_i = \gamma_j = 1$, in other words by assuming full cooperation (tacit collusion) from the start.

not on their distribution, and these sums, as we have seen, are independent of the precisions α, β_i and β_j of the three signals. As already mentioned in the previous subsection, the impact on Π_i, through prices $\kappa_i + \kappa_i' = K_i\left(\gamma_i, \gamma_j; d\right)$ and $\kappa_j + \kappa_j' = K_j\left(\gamma_i, \gamma_j; d\right)$, of changes in γ_i and γ_j can be viewed as a (strategic) cooperation effect, absent in the valuation game, where $\kappa_i + \kappa_i' \equiv 1 = K_i\left(1, 1; d\right)$. Correspondingly, the derivative

$$\frac{\partial F}{\partial \gamma_i} = \frac{\partial \Pi_i}{\partial K_i}\frac{\partial K_i}{\partial \gamma_i} + \frac{\partial \Pi_i}{\partial K_j}\frac{\partial K_j}{\partial \gamma_i} \tag{29}$$

can be seen as the *marginal cooperation benefit*, which is completely independent of the information quality.

The second term inside the brackets, $G_i\left(\gamma_i, \gamma_j\right)$ may be viewed as the *expected information cost*, arising because the second stage equilibrium price is expected to differ both from the fundamental and from the competitor's price. This information cost plays a role similar to the loss function of the valuation game. By referring to equation (15), it can be checked that, abstracting from the terms in $\kappa_i + \kappa_i'$ and $\kappa_j + \kappa_j'$ which appear in $F_i\left(\gamma_i, \gamma_j\right)$ and concentrating on $G_i\left(\gamma_i, \gamma_j\right)$, the term $\kappa_i^2/\alpha + \kappa_i'^2/\beta_i$ corresponds to the fundamental motive and its complementary term $d\left(\kappa_i\left(\kappa_i - \kappa_j\right)/\alpha + \kappa_i'^2/\beta_i\right)$ to the strategic motive, combining its coordination and competition components.[16] The information cost vanishes when information becomes perfect, either because the precision α of the public signal tends to infinity (since $\lim_{\alpha\to\infty}\kappa_i' = \lim_{\alpha\to\infty}\kappa_i/\alpha = 0$), or because *both* precisions β_i and β_j tend to infinity (since $\lim_{\beta_i,\beta_j\to\infty}\kappa_i'/\beta_i = \lim_{\beta_i,\beta_j\to\infty}\kappa_i = 0$). As already observed, perfect private information, shared by the two players, spontaneously entails free coordination, in addition to the knowledge of the fundamental. Otherwise, if β_i tends alone to infinity, the cost $(1+d)\,\kappa_i'^2/\beta_i$ associated with the private signal tends to 0, but the complementary cost associated with the public signal remains, unless this signal becomes completely uninformative (since $\lim_{\alpha\to 0}\kappa_i = 0$). In the limit case of a vanishing information cost, expected profit maximization requires the marginal benefit of cooperation $\partial F_i/\partial\gamma_i$ to be zero, and we are back to the optimal degree of cooperation given by (16).

3.4.2 Symmetric subgame perfect equilibria: existence and uniqueness

In order to ensure computational tractability, we shall from now on assume symmetry in the precision of the private signals ($\beta_i = \beta_j = \beta$) and accordingly look for symmetric subgame perfect equilibria, such that $\gamma_i = \gamma_j = \gamma$. Under symmetry, we can formulate the following lemma.

Lemma 3 *Take $\mathbb{E}\left(\theta^2\right) = 1$ and assume symmetry in the precision of both private signals ($\beta_i =$*

[16] The term $d\left(\kappa_i\left(\kappa_i - \kappa_j\right)/\alpha + \kappa_i'^2/\beta_i\right)$ is the sum of

$$\frac{d}{2}\left(\frac{\left(\kappa_i - \kappa_j\right)^2}{\alpha} + \frac{\kappa_i'^2}{\beta_i} + \frac{\kappa_j'^2}{\beta_j}\right) \text{ and } \frac{d}{2}\left(\frac{\kappa_i^2 - \kappa_j^2}{\alpha} + \frac{\kappa_i'^2}{\beta_i} - \frac{\kappa_j'^2}{\beta_j}\right),$$

which represent the coordination and competition motives, respectively.

$\beta_j = \beta$). The first order condition for expected profit maximization with a symmetric profile, such that $\gamma_i = \gamma_j = \gamma$, can be expressed as

$$\frac{\partial F_i(\gamma,\gamma)}{\partial \gamma_i} = f(\gamma,d) \leq g(\gamma,d,\alpha,\beta) = \frac{\partial G_i(\gamma,\gamma)}{\partial \gamma_i}, \tag{30}$$

with equality if $\gamma > 0$, where

$$f(\gamma,d) = \frac{d^2}{2\left(1+d-d\frac{1+\gamma}{2}\right)^3}\left(1 - \frac{2(1+d)\frac{1+\gamma}{2}}{1+d+d\frac{1+\gamma}{2}}\right) \tag{31}$$

and

$$g(\gamma,d,\alpha,\beta) = \frac{(1+d)^2}{(1+d)^2(\alpha+\beta)^2 - \left(d\frac{1+\gamma}{2}\beta\right)^2}\frac{d}{(1+d)(\alpha+\beta) - d\frac{1+\gamma}{2}\beta} \tag{32}$$
$$\times \left(\begin{array}{c} \frac{\beta^2}{(1+d)(\alpha+\beta)-d\frac{1+\gamma}{2}\beta}\left(\frac{d\frac{1+\gamma}{2}\alpha}{\left(1+d-d\frac{1+\gamma}{2}\right)^2} + \alpha + \beta\right) \\ +\frac{(1+d)(\alpha+2\beta)\alpha}{2\left(1+d-d\frac{1+\gamma}{2}\right)^3}\left(1+(1+d)\frac{1+d-d\frac{1+\gamma}{2}}{1+d+d\frac{1+\gamma}{2}}\right) \end{array} \right).$$

Proof. See Appendix B. □

Let us now analyze the impact of a change in the precision of the public signal on the equilibrium value of the degree of cooperation. As already emphasized, the marginal cooperation benefit $f(\gamma,d)$ does not depend upon the information quality, and changes sign, from positive to negative, at $\gamma^* = d/(2+d)$, the equilibrium value of γ_i and γ_j in the game with perfect information. As to the marginal information cost $g(\gamma,d,\alpha,\beta)$, it is always non-negative. We represent in Figure 1 as thick curves the graphs of the marginal cooperation benefit $f(\cdot,10)$ (the hump shaped curve) and of the marginal information cost $g(\cdot,10,10,10)$ (the increasing curve). The thin solid and dashed curves represent the graphs of $g(\cdot,10,10,2)$ and $g(\cdot,10,2,10)$, respectively.

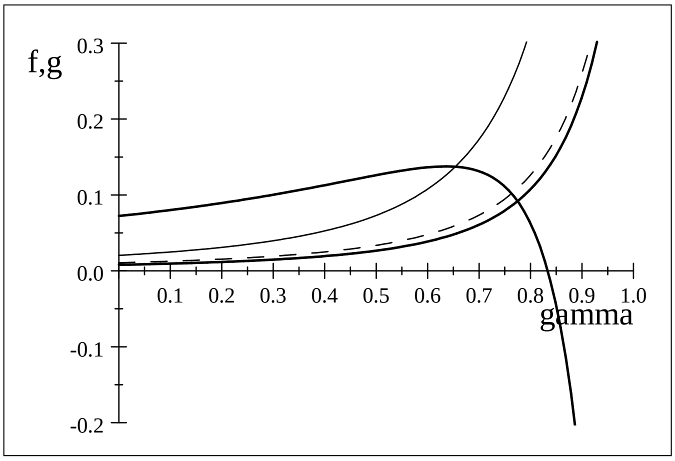

Figure 1 - The equilibrium value of γ as determined by the marginal cooperation benefit and the marginal information cost

By increasing the marginal information cost, a decrease in the precision of either signal diminishes the equilibrium value of the degree of cooperation, but the effect is stronger when it is the quality of the public signal that is affected.

The following lemma implies that the marginal information cost curve can cut at most once (from below) the marginal cooperation benefit curve.

Lemma 4 *Take $0 < \alpha < \infty$, $0 < \beta_i = \beta_j = \beta < \infty$ and $d < \infty$. The elasticity with respect to γ of the marginal information cost $g(\gamma, d, \alpha, \beta)$ is larger than the corresponding elasticity of the marginal cooperation benefit $f(\gamma, d)$ for any $\gamma < \gamma^* = d/(2+d)$.*

Proof. See Appendix B. □

As a consequence, we obtain a unique symmetric subgame perfect equilibrium, as stated in the following proposition.

Proposition 2 *Assume symmetry in the precision of both private signals ($\beta_i = \beta_j = \beta$) and imperfect substitutability of the two commodities ($d < \infty$). Then the delegation game has a unique symmetric subgame perfect equilibrium, with a degree of cooperation $\gamma \leq \gamma^* = d/(2+d)$. The degree of cooperation tends to its maximum value γ^* as information becomes perfect ($\alpha \to \infty$ or $\beta \to \infty$), and takes a zero value for a low enough precision of both public and private signals (e.g. $\alpha \to 0$ and $\beta \leq 4(1+1/d)^2$, or $\beta \to 0$ and $\alpha \leq 1 + 6/d + 4/d^2 < 4(1+1/d)^2$).*

Proof. Since, by Lemma 4, $\epsilon_\gamma g(\gamma, d, \alpha, \beta) > \epsilon_\gamma f(\gamma, d)$,[17] the graph of $g(\cdot, d, \alpha, \beta)$ can intersect at most once, from below, the graph of $f(\cdot, d)$. As $g(\gamma, d, \alpha, \beta)$ is non-negative for any γ and $f(\cdot, d)$ changes sign, from positive to negative, at γ^*, the intersection of the two graphs, verifying the FOC for interior maximization of the expected profits at a symmetric configuration of the degrees of cooperation, can only occur at some $\gamma \leq \gamma^*$. Besides, if $g(0, d, \alpha, \beta) \geq f(0, d)$, no intersection occurs at $\gamma > 0$, and we get a corner solution, at $\gamma = 0$, for that maximization. It is easy to compute:

$$g(0, d, 0, \beta) = \frac{(1+d)^2 d}{\left(1+\frac{1}{2}d\right)^3 \left(1+\frac{3}{2}d\right)} \frac{1}{\beta} \geq \frac{d^3}{4\left(1+\frac{1}{2}d\right)^3 \left(1+\frac{3}{2}d\right)} = f(0, d), \tag{33}$$

implying $\beta \leq 4(1+1/d)^2$, and

$$g(0, d, \alpha, 0) = \frac{\left(2+3d+\frac{1}{2}d^2\right) d}{2\left(1+\frac{1}{2}d\right)^3 \left(1+\frac{3}{2}d\right)} \frac{1}{\alpha} \geq \frac{d^3}{4\left(1+\frac{1}{2}d\right)^3 \left(1+\frac{3}{2}d\right)} = f(0, d), \tag{34}$$

implying $\alpha \leq 1 + 6/d + 4/d^2 < 4(1+1/d)^2$. □

3.5 The influence of competitive intensity

We conclude our analysis by considering the consequences of higher competitive intensity, associated with a larger value of the differentiation parameter d. This is an important

[17] We denote $\epsilon_x f(x, y)$ the partial elasticity of f with respect to x, at (x, y).

issue for our purpose. As well known, an indefinite increase in product substitutability augments the need for cooperation, so as to avoid the Bertrand outcome, with the eventual vanishing of profits. In addition, such an increase also augments the need for coordination: as d becomes higher and higher, the average profit is more and more eroded by the divergence between the competitors' prices rather than by their distance from the fundamental.

To be explicit, the ceiling $\gamma^* = d/(2+d)$ imposed on the degree of cooperation clearly increases as a result of such higher values of d. The subgame perfect equilibrium value of γ, if positive, also increases, as illustrated by Figure 2, where the thin curves are taken from Figure 1 and correspond to $d = 10$ (with $\alpha = \beta = 10$), whereas the thick curves result from $d = 50$ (the increasing thick curve represents the graph of $g(\cdot, 50, 10, 10)$, and the hump shaped thick curve represents the graph of $f(\cdot, 50)$).

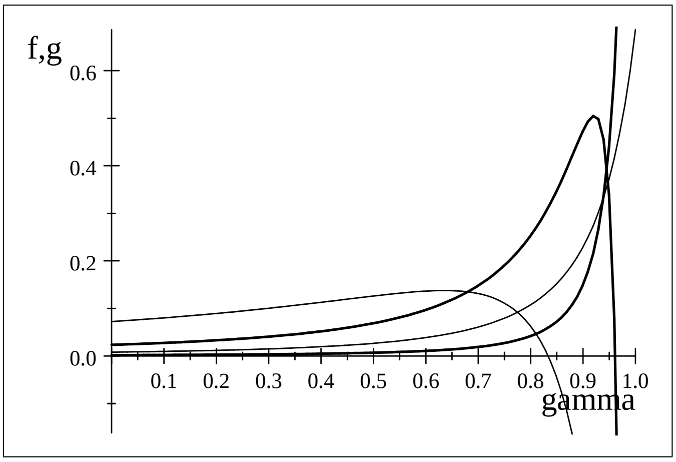

Figure 2 - Influence of a higher competitive intensity on the equilibrium value of γ

What about the limit case of perfect substitutability of the two products, when $d \to \infty$? As the curve representing the marginal information cost can cut only once, from below, the curve representing the marginal cooperation benefit, the condition to eventually obtain $\gamma = 1$ is simply that $\lim_{d\to\infty,\gamma\to 1} f(\gamma, d) \geq \lim_{d\to\infty,\gamma\to 1} g(\gamma, d, \alpha, \beta)$. We first compute

$$\lim_{d\to\infty} \frac{f(\gamma, d)}{g(\gamma, d, \alpha, \beta)} = \underbrace{\frac{\left(\alpha + \frac{3+\gamma}{2}\beta\right)\left(\alpha + \frac{1-\gamma}{2}\beta\right)^3}{\left(\frac{1-\gamma}{2}\right)^2 (3+\gamma)(\alpha+\beta)\beta^2 + \alpha\left(\alpha + \frac{1-\gamma}{2}\beta\right)(\alpha + 2\beta)}}_{h(\gamma,\alpha,\beta)}, \tag{35}$$

and then take the limit as $\gamma \to 1$ of this expression, not to be smaller than 1:

$$\lim_{\gamma\to 1} h(\gamma, \alpha, \beta) = \alpha \geq 1. \tag{36}$$

The condition $\alpha \geq 1$ ensures that both firms maximize their profits at the limit case of homogeneous products by choosing full cooperation ($\gamma_i = \gamma_j = 1$), and hence full coordination through the public signal ($r_i = r_j = 1$, implying $\kappa'_i = \kappa'_j = 0$). In order to be sure that such choice corresponds to a subgame perfect equilibrium, we must however check that the expected profit is positive at the first stage. From (28) we see that the condition for non-negativity of the expected profit is then, for $\kappa_i = \kappa_j = \kappa$, $\kappa(2-\kappa) - \kappa^2/\alpha \geq 0$, or $\alpha \geq \kappa/(2-\kappa)$, a condition always verified for any $\kappa \in [0,1]$, provided $\alpha \geq 1$.

In order to interpret the preceding condition on α, recall that we have assumed that $\mathbb{E}(\theta^2) = 1$ in order to simplify notations, but that we are in fact dealing with the precision $\mathbb{E}(\theta^2)\alpha = \mathbb{E}(\theta^2)/\mathbb{E}(\eta^2)$, so that the condition on the precision α of the public signal should be viewed as a condition $\mathbb{E}(\eta^2) \leq \mathbb{E}(\theta^2)$. Since $\mathbb{E}(y^2) = \mathbb{E}(\theta^2) + \mathbb{E}(\eta^2)$, this condition states that the variance of the noise affecting the public signal should not contribute to more than a half of the expected mean square of the signal.

In the limit case of product homogeneity, by choosing $\gamma_i = \gamma_j = 1$ and $r_i = r_j = 1$, hence $\kappa'_i = \kappa'_j = 0$, both firms will disregard their private signals and perfectly coordinate on the public signal. This signal is assumed to be unbiased but, if its precision is too poor, it may lead to negative profits by inducing prices that are too distant from the fundamental. Equilibria ruled by a precise sunspot s, with $p_i = \lambda s$ (by Lemma 2), may then be preferable even if the sunspot is biased. Thus, at the limit of perfect substitutability of the two products, when full cooperation and perfect coordination of the two firms prevail, the disconnection from the fundamental may be complete (if $\mathbb{E}(s) \neq \mathbb{E}(\theta)$) without entailing zero or negative expected profits, provided the variance of the sunspot is low enough.

4 Conclusion

Although inherent to Keynes' beauty contest metaphor, the idea that participants to financial markets exhibit a common interest in coordination *per se* has not yet received sufficient attention. The main contribution of this paper is to approach as strategic variables the weights put by those participants in the coordination (rather than the fundamental) motive. These strategic variables display a strong strategic complementarity that ends up in the complete dominance of the coordination motive, evicting the fundamental motive and hence resulting in a disconnection of market activity from fundamentals. While this disconnection between fundamentals and agents' actions is trivial in the case where all the weight is exogenously put on the coordination motive, it becomes crucial in a context where agents may manipulate the weights on each motive. Such disconnection opens the door to indeterminacy and the emergence of sunspots.

We have developed a valuation game focusing on how the information cost due to imperfect and dispersed information may be the source of the disconnection between fundamentals and activity. We have also proposed a delegation game that shows how this result may carry over to a broader context, in which the information cost interacts with the structural form of the agents' payoff function. The latter game gives some substance to the idea that the beauty contest parable in its strengthened interpretation goes beyond speculation in financial markets, as it finds a natural application in a IO context. It further suggests that our disconnection result is robust to its insertion into a framework in which the choice to play the game the others want to play is given microfoundations.

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Appendix A - Derivation of the subgame perfect equilibrium of the valuation game and proof of Proposition 1

To derive the subgame perfect equilibrium, we maximize $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right)$ with respect to r_i. More explicitly, we take the utility function (1) of agent i

$$u_i(\mathbf{a},\theta;r_i) = -(1-r_i)\left(a_i-\theta\right)^2 - r_i\left(a_i - \frac{1}{n}\sum_j a_j\right)^2 + r_i\frac{1}{n}\sum_j\left(a_j - \frac{1}{n}\sum_j a_j\right)^2, \quad (37)$$

and the second stage equilibrium value for $r<1$,

$$a_i^*\left(r_i,r\right) = \kappa_i y + \left(1-\kappa_i\right)x_i, \text{ with } \kappa_i \equiv \frac{\alpha}{\alpha+\beta}\left(1+\frac{\beta r_i}{\alpha+\beta(1-r)}\right). \quad (38)$$

Thus, denoting $\kappa = \frac{1}{n}\sum_j \kappa_j$, the expected utility writes

$$\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right) = -\left(1-r_i\right)\mathbb{E}\left(a_i^*\left(r_i,r\right)-\theta\right)^2 - r_i\mathbb{E}\left(a_i^*\left(r_i,r\right)-\frac{1}{n}\sum_j a_j^*(r_j,r)\right)^2 \quad (39)$$

$$+r_i\mathbb{E}\left(\frac{1}{n}\sum_j\left(a_j^*\left(r_j,r\right)-\frac{1}{n}\sum_k a_k^*(r_k,r)\right)^2\right)$$

$$= \quad -\left(1-r_i\right)\mathbb{E}\left(\kappa_i\eta+\left(1-\kappa_i\right)\varepsilon_i\right)^2 - r_i\mathbb{E}\left(\kappa_i\eta+\left(1-\kappa_i\right)\varepsilon_i-\frac{1}{n}\sum\nolimits_j\left(\kappa_j\eta+\left(1-\kappa_j\right)\varepsilon_j\right)\right)^2$$

$$+r_i\mathbb{E}\left(\frac{1}{n}\sum_j\left(\kappa_j\eta+\left(1-\kappa_j\right)\varepsilon_j-\frac{1}{n}\sum\nolimits_k\left(\kappa_k\eta+\left(1-\kappa_k\right)\varepsilon_k\right)\right)^2\right)$$

$$= \quad -\left(1-r_i\right)\left(\frac{\kappa_i^2}{\alpha}+\frac{\left(1-\kappa_i\right)^2}{\beta}\right) - r_i\left(\frac{\left(\kappa_i-\kappa\right)^2}{\alpha}+\left(1-\frac{2}{n}\right)\frac{\left(1-\kappa_i\right)^2}{\beta}+\frac{1}{n^2}\sum_j\frac{\left(1-\kappa_j\right)^2}{\beta}\right)$$

$$+r_i\left(\frac{1}{n}\sum_j\frac{\left(\kappa_j-\kappa\right)^2}{\alpha}+\left(1-\frac{1}{n}\right)\frac{1}{n}\sum_j\frac{\left(1-\kappa_j\right)^2}{\beta}\right)$$

$$= \quad -\left(1-r_i\right)\left(\frac{\kappa_i^2}{\alpha}+\frac{\left(1-\kappa_i\right)^2}{\beta}\right) - r_i\left(\begin{array}{c}\frac{\left(\kappa_i-\kappa\right)^2}{\alpha}-\frac{1}{n}\sum_j\frac{\left(\kappa_j-\kappa\right)^2}{\alpha}\\ +\left(1-\frac{2}{n}\right)\left(\frac{\left(1-\kappa_i\right)^2}{\beta}-\frac{1}{n}\sum_j\frac{\left(1-\kappa_j\right)^2}{\beta}\right)\end{array}\right).$$

Replacing κ, κ_i and κ_j by their values, and denoting $r=\frac{1}{n}\sum_j r_j$ and $r_Q=\sqrt{\frac{1}{n}\sum_j r_j^2}$ the arithmetic and quadratic means of the r_j's, respectively, we obtain the following expression for the expected information cost $G\left(r_i,r,r_Q;n\right)=-\,\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right)$:

$$G\left(r_i,\mathbf{r}_{-i};n\right)=\frac{1}{\alpha+\beta}+\underbrace{\frac{\alpha\beta}{\left(\alpha+\beta\right)^2\left(\alpha+\beta(1-r)\right)^2}}_{b(r)}r_i \quad (40)$$

$$\times\underbrace{\left(\begin{array}{c}\left(\alpha+\beta\right)\left(1-r_i\right)r_i-\frac{\alpha+\beta}{\alpha\beta}\left(\alpha+\beta(1-r)\right)^2\\ +\beta\left(\left(r_i-r\right)^2-\left(r_Q^2-r^2\right)\right)\\ +\left(1-\frac{2}{n}\right)\left(\alpha\left(r_i^2-r_Q^2\right)-2\left(\alpha+\beta(1-r)\right)\left(r_i-r\right)\right)\end{array}\right)}_{c(r_i,r,r_Q;n)}$$

We concentrate in Proposition 1 into two benchmark cases: $n\to\infty$ and $n=2$.

Proof. [Proof of Proposition 1]

First benchmark case: $n\to\infty$.

In this case, we may take the two means r and r_Q as unaffected by changes in r_i, and refer exclusively to the partial derivative of $G\left(r_i,r,r_Q;n\right)$ with respect to r_i. Using

$$\frac{\partial c\left(r_i,r,r_Q;n\right)}{\partial r_i}=\alpha+\beta+\frac{2}{n}\left(2\alpha r_i-2\left(\alpha+\beta(1-r)\right)\right), \quad (41)$$

we obtain the limit:

$$\lim_{n\to\infty}\frac{\partial G(r_i,r,r_Q;n)}{\partial r_i} = b(r)\left(\lim_{n\to\infty}c(r_i,r,r_Q;n)+r_i\lim_{n\to\infty}\frac{\partial c(r_i,r,r_Q;n)}{\partial r_i}\right) \tag{42}$$
$$= -b(r)(\alpha+\beta)\left(2(r_i-r)+\frac{(\alpha+\beta(1-r))^2}{\alpha\beta}+r_Q^2\right),$$

which is decreasing in r_i (as $\alpha/\beta>0$ by assumption). Hence, $\lim_{n\to\infty}G(\cdot,r,r_Q;n)$ is strictly concave and its minimum can only be attained at $r_i=0$ or $r_i=1$. As

$$\lim_{n\to\infty}G(1,r,r_Q;n)-\lim_{n\to\infty}G(0,r,r_Q;n)=b(r)\lim_{n\to\infty}c(1,r,r_Q;n) \tag{43}$$
$$= -(\alpha+\beta)b(r)\left(\frac{(\alpha+\beta(1-r))^2}{\alpha\beta}+(1-r)+\left(r_Q^2-r\right)\right)<0$$

(again since $\alpha/\beta>0$), the solution is always $r_i=1$.

Second benchmark case: $n=2$.

With $n=2$, the difference between the coordination and competition motives is nil, which simplifies the expression of $G(r_i,r,r_Q;2)$:

$$G(r_i,r,r_Q;2)=\frac{1-r_i}{\alpha+\beta}\left(1+\frac{\alpha\beta r_i^2}{(\alpha+\beta(1-r))^2}\right), \tag{44}$$

which is positive for any $r_i\in[0,1)$ (provided α and β remain finite) and nil for $r_i=1$. Consequently, the expected information cost is minimized at $r_i=1$. □

Appendix B - Derivation of the symmetric subgame perfect equilibrium of the delegation game: Proofs of Lemma 3 and Lemma 4

Proof. [Proof of Lemma 3] From the expression (28) of the expected profit and using (23), we obtain the first order condition for its maximization: $\partial F_i(\gamma_i,\gamma_j)/\partial\gamma_i\le\partial G_i(\gamma_i,\gamma_j)/\partial\gamma_i$, with equality if $\gamma_i>0$. The marginal cooperation benefit is

$$\frac{\partial F_i(\gamma_i,\gamma_j)}{\partial\gamma_i}=\frac{dK_j}{2}\left(\frac{(2-2(1+d)K_i+d(K_j-K_i))(1+d)}{(1+d)^2-d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}+dK_iK_j\right), \tag{45}$$

where K_i and K_j are functions of (γ_i,γ_j) given by (23), and the marginal information cost is

$$\frac{\partial G_i(\gamma_i,\gamma_j)}{\partial\gamma_i}=2(1+d)\left(\frac{\kappa_i}{\alpha}\frac{\partial\kappa_i}{\partial\gamma_i}+\frac{\kappa_i'}{\beta}\frac{\partial\kappa_i'}{\partial\gamma_i}\right)-d\left(\frac{\kappa_i}{\alpha}\frac{\partial\kappa_j}{\partial\gamma_i}+\frac{\kappa_j}{\alpha}\frac{\partial\kappa_i}{\partial\gamma_i}\right), \tag{46}$$

where we are taking $\mathbb{E}(\theta^2)=1$. By (20) and (21), we have:

$$\frac{\kappa_i}{\alpha}=\frac{1+d}{(1+d)^2(\alpha+\beta)^2-d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2}\left(\frac{(1+d)\left(1+d+d\frac{1+\gamma_i}{2}\right)}{(1+d)^2-d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}(\alpha+2\beta)-\beta\right), \tag{47}$$

$$\frac{\partial \kappa_i}{\partial \gamma_i} = \tag{48}$$
$$\frac{(1+d)\frac{d^2}{2}\frac{1+\gamma_j}{2}\alpha\beta^2}{\left((1+d)^2(\alpha+\beta)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2\right)^2}\left(\frac{(1+d)\left(1+d+d\frac{1+\gamma_i}{2}\right)}{(1+d)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}(\alpha+2\beta) - \beta\right)$$
$$+\frac{(1+d)^2(\alpha+2\beta)\alpha}{(1+d)^2(\alpha+\beta)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2}\frac{(1+d)\frac{d}{2}\left(1+d+d\frac{1+\gamma_j}{2}\right)}{\left((1+d)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\right)^2},$$

$$\frac{\partial \kappa_j}{\partial \gamma_i} = \tag{49}$$
$$\frac{(1+d)\frac{d^2}{2}\frac{1+\gamma_j}{2}\alpha\beta^2}{\left((1+d)^2(\alpha+\beta)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2\right)^2}\left(\frac{(1+d)\left(1+d+d\frac{1+\gamma_j}{2}\right)}{(1+d)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}}(\alpha+2\beta) - \beta\right)$$
$$+\frac{(1+d)^2(\alpha+2\beta)\alpha}{(1+d)^2(\alpha+\beta)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2}\frac{\frac{d^2}{2}\frac{1+\gamma_j}{2}\left(1+d+d\frac{1+\gamma_j}{2}\right)}{\left((1+d)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\right)^2},$$

$$\frac{\kappa_i'}{\beta} = \frac{(1+d)(\alpha+\beta) + d\frac{1+\gamma_i}{2}\beta}{(1+d)^2(\alpha+\beta)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2}, \tag{50}$$

$$\frac{\partial \kappa_i'}{\partial \gamma_i} = \frac{(1+d)\frac{d}{2}\left((1+d)(\alpha+\beta) + d\frac{1+\gamma_j}{2}\beta\right)(\alpha+\beta)\beta^2}{\left((1+d)^2(\alpha+\beta)^2 - d^2\frac{1+\gamma_i}{2}\frac{1+\gamma_j}{2}\beta^2\right)^2}. \tag{51}$$

Imposing symmetry ($\gamma_i = \gamma_j = \gamma$), we then obtain for the marginal cooperation benefit (by (23))

$$\frac{\partial F_i(\gamma_i,\gamma_j)}{\partial \gamma_i} = \frac{d^2}{2\left(1+d-d\frac{1+\gamma}{2}\right)^3}\left(1 - \frac{2(1+d)\frac{1+\gamma}{2}}{1+d+d\frac{1+\gamma}{2}}\right) \equiv f(\gamma,d), \tag{52}$$

and for the marginal information cost

$$\begin{aligned}
\frac{\partial G_i(\gamma,\gamma)}{\partial \gamma_i} &= \frac{\kappa}{\alpha}\left((2+d)\frac{\partial \kappa_i}{\partial \gamma_i} - d\frac{\partial \kappa_j}{\partial \gamma_i}\right) + 2(1+d)\frac{\kappa_i'}{\beta}\frac{\partial \kappa_i'}{\partial \gamma_i} \\
&= \frac{(1+d)^2}{(1+d)^2(\alpha+\beta)^2 - \left(d\frac{1+\gamma}{2}\beta\right)^2}\frac{d}{(1+d)(\alpha+\beta) - d\frac{1+\gamma}{2}\beta} \\
&\quad \times \left(\begin{array}{c} \frac{\beta^2}{(1+d)(\alpha+\beta)-d\frac{1+\gamma}{2}\beta}\left(\frac{d\frac{1+\gamma}{2}\alpha}{\left(1+d-d\frac{1+\gamma}{2}\right)^2} + \alpha+\beta\right) \\ +\frac{(1+d)(\alpha+2\beta)\alpha}{2\left(1+d-d\frac{1+\gamma}{2}\right)^3}\left(1+(1+d)\frac{1+d-d\frac{1+\gamma}{2}}{1+d+d\frac{1+\gamma}{2}}\right) \end{array}\right) \\
&\equiv g(\gamma,d,\alpha,\beta).
\end{aligned} \tag{53}$$

□

Proof. [Proof of Lemma 4] According to equation (53), the function $g(\cdot,d,\alpha,\beta)$ can be expressed as

$$A(x)(B(x)+C(x)), \text{ with } x=\frac{1+\gamma}{2}, \tag{54}$$

and $A(x)$, $B(x)$ and $C(x)$ all positive for $0<\alpha<\infty$ and $0<\beta<\infty$. The elasticity of this expression is[18]

$$\epsilon A(x)+\frac{B(x)\,\epsilon B(x)+C(x)\,\epsilon C(x)}{B(x)+C(x)}, \tag{55}$$

with

$$\epsilon A(x) = \frac{(1+\alpha/\beta)(1+d)+3dx}{(1+\alpha/\beta)^2(1+d)^2-(dx)^2}dx, \tag{56}$$

$$\epsilon B(x) = \left(\begin{array}{c}\frac{1}{(1+\alpha/\beta)(1+d-dx)+(\alpha/\beta)dx}\\ +\frac{(\alpha/\beta)\frac{1+d+dx}{1+d-dx}}{(1+\alpha/\beta)(1+d-dx)^2+(\alpha/\beta)dx}\end{array}\right)dx, \tag{57}$$

$$\epsilon C(x) = \left(3\frac{1+d+dx}{1+d-dx}-1\right)\frac{dx}{1+d+dx}-\frac{d^2x}{(2+d)(1+d)-d^2x}. \tag{58}$$

Denoting $\widehat{f}(x)\equiv f(2x-1,d)$, we also have

$$\epsilon\widehat{f}(x)=\left(3\frac{1+d+dx}{1+d-dx}-1\right)\frac{dx}{1+d+dx}-\frac{2(1+d)x}{1+d-(2+d)x}. \tag{59}$$

We want to prove that

$$\epsilon A(x)+\frac{B(x)\,\epsilon B(x)+C(x)\,\epsilon C(x)}{B(x)+C(x)}-\epsilon\widehat{f}(x)>0 \tag{60}$$

or, equivalently, that

$$B(x)\left(\epsilon A(x)+\epsilon B(x)-\epsilon\widehat{f}(x)\right)+C(x)\left(\epsilon A(x)+\epsilon C(x)-\epsilon\widehat{f}(x)\right)>0. \tag{61}$$

Take first $\epsilon A(x)+\epsilon B(x)-\epsilon\widehat{f}(x)$. The elasticity of $B(x)$ is always positive. The remaining terms are

$$\begin{aligned}\epsilon A(x)-\epsilon\widehat{f}(x) &= \frac{dx}{(1+\alpha/\beta)(1+d)-dx} \\ &\quad +\frac{2(dx)^2}{(1+\alpha/\beta)^2(1+d)^2-(dx)^2}-\left(\frac{1+d+dx}{1+d-dx}-1\right)\frac{dx}{1+d+dx} \\ &\quad -2\frac{1+d+dx}{1+d-dx}\frac{dx}{1+d+dx}+\frac{2(1+d)x}{1+d-(2+d)x}\end{aligned} \tag{62}$$

[18] We denote by $\epsilon f(x)$ the elasticity of f.

$$
\begin{aligned}
= \quad & \frac{dx}{(1+\alpha/\beta)(1+d)-dx} \\
& \underbrace{-\frac{2(dx)^2}{(1+d)^2-(dx)^2}\frac{(\alpha/\beta)(2+\alpha/\beta)(1+d)^2}{(1+\alpha/\beta)^2(1+d)^2-(dx)^2}}_{m(x)} \\
& +\frac{2x}{1+d-(2+d)x}\frac{1+d+dx}{1+d-dx},
\end{aligned}
$$

where the sole negative term is $m(x)$. Thus, we have just to prove the positivity of

$$
\begin{aligned}
& C(x)\left(\epsilon A(x)+\epsilon C(x)-\epsilon\widehat{f}(x)\right)-B(x)\,m(x) \qquad (63)\\
= \quad & \beta^2\frac{(\alpha/\beta)(2+\alpha/\beta)(1+d)\left((1+d)(2+d)-d^2x\right)}{2(1+d-dx)^2\left((1+d)^2-(dx)^2\right)} \\
& \times\left(\begin{array}{c}
\frac{(1+\alpha/\beta)(1+d)+3dx}{(1+\alpha/\beta)^2(1+d)^2-(dx)^2}dx \\
+\left(3\frac{1+d+dx}{1+d-dx}-1\right)\frac{dx}{1+d+dx}-\frac{d^2x}{(2+d)(1+d)-d^2x} \\
-\left(3\frac{1+d+dx}{1+d-dx}-1\right)\frac{dx}{1+d+dx}+\frac{2(1+d)x}{1+d-(2+d)x} \\
-\frac{4(dx)^2}{(1+\alpha/\beta)^2(1+d)^2-(dx)^2}\frac{(1+\alpha/\beta)(1+d-dx)^2+(\alpha/\beta)dx}{(1+\alpha/\beta)(1+d)-dx}\frac{1+d}{(1+d)(2+d)-d^2x}
\end{array}\right) \\
= \quad & \beta^2\frac{(\alpha/\beta)(2+\alpha/\beta)(1+d)\left((1+d)(2+d)-d^2x\right)}{2(1+d-dx)^2\left((1+d)^2-(dx)^2\right)} \\
& \times\left(\begin{array}{c}
\frac{dx}{(1+\alpha/\beta)(1+d)+dx}+\left(\frac{2(1+d)x}{1+d-(2+d)x}-\frac{d^2x}{(2+d)(1+d)-d^2x}\right) \\
+\frac{4(dx)^2}{(1+\alpha/\beta)^2(1+d)^2-(dx)^2}\left(1-\frac{(1+\alpha/\beta)(1+d-dx)^2+(\alpha/\beta)dx}{(1+\alpha/\beta)(1+d)-dx}\frac{1+d}{(1+d)(2+d)-d^2x}\right)
\end{array}\right).
\end{aligned}
$$

The first term inside the parentheses is clearly positive, and it is straightforward to check that the other two terms are positive too. □