Bureau d'économie théorique et appliquée (BETA)
UMR 7522

Documents de travail

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Document de Travail n° 2015 – 18

Juillet 2015

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A 'Jump' in the Stochasticity of the Solow-Swan Growth Model

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Abstract

We characterize 'Solow-Swan' economic growth model in a stochastic environment. Our interest basically lies in modelling arrival of uncommon or stochastic shocks in both physical capital and labour, introducing discontinuities in the growth of these variables. These characterizations are completed by employing a Jump process to the Solow-Swan model. Interesting dynamics of capital and labor growth emerge from our investigation.

Key Words: Stochastic Solow-Swan growth, Brownian motion, Jump process..
JEL Classifications: E13, C60, O41, L1, C1, D2

1 Introduction

Solow-Swan (1956) economic growth model has been extensively celebrated in economic growth literature, the properties and extension of which have been intuitively investigated in, for instance, Barro and Sala-i-Martin (2004). A number of recent research (e.g., Binder and Pesaran, 1999; Stachurski, 2002; Barossi-Filho et al., 2005) have also focused on modelling this growth model in a stochastic environment. The main motivation of the need for a stochastic version of Solow-Swan model arises due to the necessity of reproducing the statistical characteristics of business cycle fluctuations in actual economies. Indeed, this approach to growth modelling in economics has had a tremendous impact on the way we think about the analysis of effects of the different exogenous shocks in an economy. It is possible, by using the modelling approach to analyze which among the possible shocks is more likely to produce a given statistical characteristic of the solution, or which one is more useful in order for a model to replicate a given statistical regularity observed in actual time series data. In addition, one can characterize the role of economic policy in determining the dynamics of relevant variables, as well as the co-movements between them.

Following on the leads of the recent literature, in this note we aim to expand the existing work by introducing a Jump process in the growth of the main determinants of income. Our broad idea is to investigate the stability of the economic growth system under stochastic disturbances when these disturbance bring sudden changes in the growth process. As is well-known the Solow-Swan model is an exogenous growth model, an economic model of long-run economic growth set within the framework of neoclassical economics (see [3], pp26). It attempts to explain long-run economic growth by looking at capital accumulation, labor or population growth, and increases in productivity, commonly referred to as technological progress. The key assumption of the neoclassical growth model is that capital is subject to diminishing returns in a closed economy. More specifically, it incudes two aspects: The total output $Y(t)$ is determined by the overall input; The factor accumulation does not depend on decisions of economic entities. The main difference with the deterministic model is: we assume that the factor accumulation is affected by some stochastic disturbance such that the Ordinary differential equation (ODE) can be transformed into the Stochastic differential equation (SDE). Then we can do the stability analysis for the stochastic system.

Moreno et al. (2011) discuss the usefulness of jump process to reflect how economic variables respond to the arrival of sudden information. While analyzing the dynamics of the model, the authors find that the degree of serial autocorrelation is related to the occurrence and magnitude of abnormal information. In addition, the authors provide some useful approximations in a particular case that considers exponential-type decay. Empirically, the authors propose a GMM approach to estimate the parameters of the model and present an empirical application for the stocks included in the Dow Jones Averaged Index. Our findings seem to confirm the presence of shot-noise effects in 73% of the stocks and a strong relationship between the shot-noise process and the autocorrelation pattern embedded in data.

Jumps were initially analyzed by Merton (1976) for modeling the arrival of uncommon information at financial markets, introducing discontinuities in the stock charts. Jumps account adequately situations such as, for example, the sudden reaction of stock prices to unexpected news about a company, the consequences of extreme fluctuations in supply and demand in electricity markets, or the failure and thus abandonment of a R&D firms investment project, see Pennings and Sereno (2011). Not surprisingly, the empirical evidence about jumps is vast, see Andersen et al. (2002), Eraker et al. (2003), and Escribano et al. (2011).

The rest of the paper is planned as follows. In Section 2, we characterize Solow-Swan model by Brownian Motion. Section 3 introduces Jump process in Solow-Swan framework, whereby stability analysis and properties of the determinants of growth are discussed. Section 4 investigates stationary features of capital stock. Finally, Section 5 concludes with main findings.

2 Stochastic Solow Model with the Brownian Motion

2.1 Construct

We initially consider the neoclassical production function. A neoclassical production function $F(K_t, L_t)$ satisfies in the general the following properties:

1. The function $F(\cdot)$ is a linear homogenous function;
2. Let $f(k_t) = F(k_t, 1) = y$, where $k_t = \frac{K_t}{L_t}$, then $f(k_t)$ satisfies
$$f'(k_t) > 0, f''(k_t) < 0, f(k_t) > k_t f'(k_t); \tag{2.1}$$
3. Inada condition:
$$f(0) = f'(\infty) = 0, \quad f'(0) = f(\infty) = \infty, \tag{2.2}$$

where Y_t is the flow of output produced at time t, K_t is the physical capital, such as machines, building, pencils, etc.; L_t represents the labor, including the number of workers and the amount of time they work, as well as their physical strength, skills, and health; k_t is the per capital capital; y_t is output per worker.

Let $F(\cdot)$ be the Cobb-Douglas function, then

$$F(K_t, L_t) = AK_t^{\alpha} L_t^{\beta},$$

where $A > 0$ is the level of technology, $\alpha \in (0, 1)$, $\beta = 1 - \alpha$.

In the deterministic model, the net increase in the stock of physical capital at a point in time equals gross investment less depreciation:

$$\dot{K}_t = sY_t - \delta K_t, \;\; L_t = nL_t \tag{2.3}$$

where $\dot{K}_t = dK_t/dt$ denote the net increase, $s \in (0,1)$ is a constant saving rate, $\delta \in [0,1]$ is the depreciation, the population grows at a constant, exogenous rate $n \geq 0$. The fundamental differential equation of the Solow-Swan model is

$$\dot{k_t} = sf(k_t) - (n+\delta)k_t. \tag{2.4}$$

Assume that the growth rate of K_t and L_t are affected by some random disturbance, (2.4) can be written as:

$$dK_t = (sY_t - \delta K_t)dt + K_t dB_t^K \tag{2.5}$$

and

$$dL_t = nL_t dt + L_t dB_t^L \tag{2.6}$$

where B_t^K and B_t^L are given Brownian Motions. The perturbations of K_t and L_t are results from independent effects of larger number of small factors, so we can use Brownian Motions to describe disturbances in (2.5) and (2.6). We denote the variance of B_t^K by $\sigma_K dt$, B_t^L by $\sigma_L dt$, and $cov(dB_t^K, dB_t^L) := \sigma_{KL} dt$.

By the Itô lemma, we have

$$dk_t = (sf(k_t) - \mu k_t)dt + k_t dB_t, \tag{2.7}$$

where

$$\mu = n + \delta + \sigma_{KL} - \sigma_L^2$$

and

$$dB_t = dB_t^K - dB_t^L.$$

Therefor (2.7) is a stochastic Solow-Swan Model, it is a autonomous function in terms of k_t. From (2.7), we can easily address:

1. Per capita capital k_t is a homogenous diffusion process which the drift and the diffusion coefficients are $sf(k_t) - \mu k_t$ and $k_t^2(\sigma_K^2 - 2\sigma_{KL} + \sigma_L^2)$ respectively;

2. The Markov property of k_t shows that: the economic status at the presents can forecast the trend for the future. Namely, if $s > t$, $k_t = k$, then the probability density of k_s is determined by $k_t = k$. Meanwhile, by using Kolmogorov equation, the transition function $p(t, k, k_1)$ can be obtained. If $s > t$, $\forall (a, b)$,

$$\mathbb{P}\big(a < k_s < b | k_t = k\big) = \int_a^b p(s-t, k, y)dy;$$

3. From (2.7), the expected growth rate of k_t is

$$\begin{aligned} \psi_k := \mathbb{E}(\frac{dk_t}{k}) &= \frac{sf(k_t)}{k} - \mu \\ &= sf(k_t) - (n + \delta + \sigma_{KL} - \sigma_L^2). \end{aligned} \tag{2.8}$$

If the stochastic disturbance is gone, we have the deterministic growth rate g_k :

$$g_k = \frac{\dot{k}_t}{k} = \frac{sf(k_t)}{k_t} - (n + \delta). \tag{2.9}$$

From (2.8) and (2.9),

$$\psi_k - g_k = \sigma_{KL} - \sigma_L^2.$$

This means when $\sigma_{KL} > \sigma_L^2$, the exitances of the stochastic disturbance can raise the growth. On the contrary, $\sigma_{KL} < \sigma_L^2$, the exitances of the stochastic disturbance reduce the growth. Precisely, if the magnitude of L_t is bigger then K_t, then $\sigma_{KL} < \sigma_K \sigma_L < \sigma_L^2, \psi_t < g_t$, therefore the growth rate will decrease. In exceptional circumstances, L_t is disturbed but K_t does not, that is $\sigma_L^2 > 0$, $\sigma_K^2 = 0$, in this case the growth rate will go down. Oppositely if K_t is disturbed but L_t does not, $\sigma_L^2 = 0$, $\sigma_K^2 > 0$, then the growth rate will remain the same.

It can be clearly seen that, the perturbation of K_t and L_t has different influences for the economic growth. L_t has relatively bigger impact than K_t.

2.2 Stability Analysis

We define a steady state as a situation in which the various quantities grow at constant rate (see [3] pp.33). In the deterministic Solow Model, the steady state corresponds to $\dot{k}_t = 0$, the corresponding value of k_t is denoted by k_t^*, that is $sf(k_t^*) = (n + \delta)k_t^*$, $t \to 0$, $k_t \to k^*$. We say $k_t \equiv k_t^*$ is asymptotically stable in $(0, \infty)$ globally.

The stability of the system will change along with the appearance of the stochastic disturbance. Firstly, (2.7) does not have the steady state apart from $k_t = 0$ since k_t^* is eliminated by the stochastic disturbance terms. Secondly $k_t = 0$ in (2.7) is not obvious, apparently $k_t = 0$ in (2.4) is not a steady state, so it is not comparable. We will consider the exponential stability of the system in this section. As almost surely exponentially stable can easily apply asymptotically stable in global, so we will focus on this kind of stable on the stochastic system.

Next we will use lyapunov function (see [8] pp.109) to analyze the stability of (2.7). Suppose that $D = \mathbb{R}_+$, $V(k_t) = k_t^2$, we have

$$\sup_{k>0} \left[\frac{sf(k_t)}{k} + \sigma^2 - 2\mu \right] < 2\sigma^2$$

and

$$\inf_{k>0} \left[\frac{sf(k_t)}{k} + \sigma^2 - 2\mu \right] > 2\sigma^2.$$

They implies:

$$\sup_{k>0} \frac{sf(k_t)}{k} < \frac{\sigma^2 + 2\mu}{2s} \tag{2.10}$$

and

$$\inf_{k>0} \frac{sf(k_t)}{k} > \frac{\sigma^2 + 2\mu}{2s}. \tag{2.11}$$

Let $\varphi(k_t) := \frac{f(k_t)}{k}$, from (2.1) and (2.2), we can get

$$k_t^2 \varphi'(k_t) = k_t f'(k_t) - f(k_t), \quad k_t > 0.$$

Thus

$$\lim_{k \to 0} \varphi(k_t) = \lim_{k \to 0} f'(k_t) = \infty,$$

and

$$\lim_{k \to \infty} \varphi(k_t) = \lim_{k \to \infty} f'(k_t) = 0.$$

Therefore we have

$$\sup_{k>0} \varphi(k_t) = \varphi(0) = \infty, \quad \inf_{k>0} \varphi(k_t) = \varphi(\infty) = 0.$$

The discriminant condition (2.10) and (2.11) can be re-written as

$$\sigma^2 + 2\mu > \infty \tag{2.12}$$

$$\sigma^2 + 2\mu < 0. \tag{2.13}$$

Obviously, (2.12) is not possible. From (2.13) we can obtain

$$2(n + \delta) + \sigma_K^2 < \sigma_L^2. \tag{2.14}$$

It means: if $\sigma_L^2 < 2(n+\delta) + \sigma_K^2$, (2.7)'s solution is not exponentially stable in $[0, \infty]$ a.s.. In other words, per capita capital exponential grows from the initial point, the trajectory of the solution that starts from $D = [0, \infty)$ but also eventually coverage in D.

Therefore, (2.14) is the criterion of the exponential instability of the zero solution of (2.7).

3 Role of Jump Process in Stochastic Solow-Swan Model

The stochastic processes in continuous-time with independent stationarity increments is called Lévy processes. The most well known examples of Lévy processes are Brownian motion and the Poisson process. We have studied the stochastic modelling with the Brownian motion in Section 2. In this section, we will use the Jump-type Lévy processes to represent the stochastic disturbances in the Solow-Swan Model. First we will build the Stochastic Solow-Swan Model with the Jump processes, then do the stability analysis.

3.1 Basic framework

Let $\{N_t^K\}_{t\geq 0}$ and $\{N_t^L\}_{t\geq 0}$ two poisson processes with intensity measures λ_1, λ_2, we further assume that $\langle N_t^K, N_t^L\rangle = \lambda_1\lambda_2 t$ where $\langle N_t^K, N_t^L\rangle$ stands for the quadratic process of N_t^K and N_t^L. The Poisson distribution with associated parameter λ is:

$$\mathbb{P}(N_t^K(\omega) = n) = e^{-\lambda_1 t}\frac{(\lambda_1 t)^n}{n!}, \quad n = 1, 2, \ldots,$$

and

$$\mathbb{P}(N_t^L(\omega) = n) = e^{-\lambda_2 t}\frac{(\lambda_2 t)^n}{n!}, \quad n = 1, 2, \ldots.$$

Consider

$$dK_t = (sY_t - \delta K_t)dt + K_t dN_t^K \tag{3.1}$$

and

$$dL_t = nL_t dt + L_t dN_t^L \tag{3.2}$$

Due to the Itô formula, we obtain from (3.2)

$$d(\frac{1}{L_t}) = -\frac{n}{K_t}dt - \frac{1}{2L_t}dN_t^L. \tag{3.3}$$

We also have

$$d\langle K_t, \frac{1}{L_t}\rangle = \frac{K_t}{2L_t}\lambda_1\lambda_2 dt \tag{3.4}$$

Also by the Itô formula, we deduce that

$$dk_t = d(\frac{K_t}{L_t}) = \frac{1}{L_t}dK_t + K_t d(\frac{1}{L_t}) + d\langle K_t, \frac{1}{L_t}\rangle. \tag{3.5}$$

Substitute (3.1), (3.3),(3.4) into (3.5), we can get

$$dk_t = [sf(k_t) - \delta k_t - nk_t - \frac{k_t}{2}\lambda_1\lambda_2]dt + k_t dN_t^K - \frac{k_t}{2}dN_t^L. \tag{3.6}$$

Therefor (3.6) is a stochastic Solow-Swan Model with jumps. From (3.6), the expected growth rate of k_t is

$$\phi_k = \mathbb{E}\big(\frac{dk_t}{k}\big) = \frac{sf(k_t)}{k_t} - \delta - n - \frac{1}{2}\lambda_1\lambda_2 + \lambda_1 - \frac{1}{2}\lambda_2 \tag{3.7}$$

Recall in the deterministic model, the growth rate g_k :

$$g_k = \frac{\dot{k}_t}{k} = \frac{sf(k_t)}{k_t} - (n+\delta).$$

From (2.9) and (3.7), we have

$$\phi_k - g_k = -\frac{1}{2}\lambda_1\lambda_2 + \lambda_1 - \frac{1}{2}\lambda_2.$$

This means when $2\lambda_1 - \lambda_1\lambda_2 - \lambda_2 > 0$ the exitances of the stochastic disturbance driven by jumps can raise the growth. On the contrary, $2\lambda_1 - \lambda_1\lambda_2 - \lambda_2 < 0$ the growth will be reduced. If $\lambda_1 = \lambda_2 = 0$ the stochastic disturbance is gone, then $\phi_k = g_k$.

3.2 Stability Analysis

As the same deification in Section 2, the steady state level of capital stock is the stock of capital at which investment and depreciation just offset each other, that $\dot{K}_t = 0$.

In order to analyze the stability of (3.6). we will employ lyapunov function. Assume $D = \mathbb{R}_+$, $V(k_t) = k_t^2$, we have

$$dk_t^2 = k_t^2 \big[\frac{sf(k_t)}{k_t} - 2\delta - 2n - 2\lambda_1\lambda_2 + 3\lambda_1 - \frac{3}{4}\lambda_2 \big] \\ + \text{ Martingale.}$$

By the definition of lyapunov function, if

$$\sup_{k>0} \left(\frac{sf(k_t)}{k_t} - 2\delta - 2n - 2\lambda_1\lambda_2 + 3\lambda_1 - \frac{3}{4}\lambda_2 \right) < 0,$$

The above condition can guarantee the system is exponentially stable, otherwise

$$\inf_{k>0} \left(\frac{sf(k_t)}{k_t} - 2\delta - 2n - 2\lambda_1\lambda_2 + 3\lambda_1 - \frac{3}{4}\lambda_2 \right) > 0,$$

the system is not exponentially stable. They implies:

$$\sup_{k>0} \frac{sf(k_t)}{k_t} < \frac{2\delta + 2n + 2\lambda_1\lambda_2 - 3\lambda_1 + \frac{3}{4}\lambda_2}{2s} \tag{3.8}$$

and

$$\inf_{k>0} \frac{sf(k_t)}{k_t} > \frac{2\delta + 2n + 2\lambda_1\lambda_2 - 3\lambda_1 + \frac{3}{4}\lambda_2}{2s} \tag{3.9}$$

Let $\varphi(k_t) := \frac{f(k_t)}{k}$, from (2.1) and (2.2), we can get

$$k_t^2 \varphi'(k_t) = k_t f'(k_t) - f(k_t), \quad k_t > 0.$$

Thus

$$\lim_{k\to 0} \varphi(k_t) = \lim_{k\to 0} f'(k_t) = \infty,$$

and

$$\lim_{k\to \infty} \varphi(k_t) = \lim_{k\to \infty} f'(k_t) = 0.$$

Therefore we have

$$\sup_{k>0} \varphi(k_t) = \varphi(0) = \infty, \quad \inf_{k>0} \varphi(k_t) = \varphi(\infty) = 0.$$

The discriminant condition (3.8) and (3.9) can be re-written as

$$2\delta + 2n + 2\lambda_1\lambda_2 - 3\lambda_1 + \frac{3}{4}\lambda_2 > \infty \tag{3.10}$$

and

$$2\delta + 2n + 2\lambda_1\lambda_2 - 3\lambda_1 + \frac{3}{4}\lambda_2 < 0. \tag{3.11}$$

Apparently, (3.10) does not hold. Under the criterion of (3.11), that

$$2(\delta + n) < 3\lambda_1 - 2\lambda_2\lambda_2 - \frac{3}{4}\lambda_2$$

The stochastic system is exponentially unstable.

4 Characterization of stationary distribution of k_t

In the deterministic Solow-Swan model, k_t finally goes to steady state k_t^*. In a similar way, in the stochastic model, k_t will go to a non-zero random variable as $t \to \infty$. If this random variable is continuous, then we can apply apply it's probability density function (PDF) denoted by $\pi(\cdot)$. Here $\pi(\cdot)$ is named the stationary distribution of k_t. Merton [9] and Bourguigono [5] first propose to use a definition of the stationary distribution in the theory of economic growth. In this Section, based on Merton's model, we have some improvement in order to compute the stationary distribution of k_t in the stochastic Solow Model.

Now let us discuss $\pi(\cdot)$. Recall k_t is from (2.7), the production function is k_t^α $(0 < \alpha < 1)$, by the Kolmogorov forward equation, we have

$$\varphi(k_t) = \frac{s(1 - k_t^{-\alpha'})}{\alpha'\sigma^2} - \frac{\mu}{\sigma^2}\ln k_t,$$

where $\alpha' = 1 - \alpha$. Then

$$\begin{aligned}\frac{1}{\pi(k_t)} &= k_t^2 e^{2\varphi(k_t)} \int_0^\infty x^{-2-\frac{2\mu}{\sigma^2}} e^{-\beta k_t^{-\alpha'}} dx \\ &= \frac{\Gamma(\omega)}{\alpha'}\beta^{-\omega} k_t^{2+2\mu/\sigma^2} \exp(-\beta k_t^{-\alpha'}),\end{aligned}$$

where $\beta = \frac{2s}{\alpha'\alpha^2}$, $\omega = \frac{2\mu+\sigma^2}{\sigma^2\alpha'}$.

Therefore,

$$\pi(k_t) = \frac{\alpha'}{\Gamma(\omega)}\beta^\omega k_t^{-2-2\mu/\sigma^2} \exp(-\beta k_t^{-\alpha'}).$$

We need $\omega > 0$, which implies the following condition

$$2(n + \delta) + \sigma_K^2 > \sigma_L^2. \tag{4.1}$$

Here (4.1) is the condition which can make k_t converges to stationary distribution.

Now let us assume (4.1) is satisfied and

$$\omega_\tau = \frac{2\mu + \tau'\sigma^2}{\alpha'\sigma^2} = \omega - \frac{\tau}{\alpha'}.$$

Obviously, $\omega = \omega_0$, suppose k_t has stationary distribution, then

$$\begin{aligned}\mathbb{E}(k_t^{\tau}) &= \int_0^{\infty} k_t^{\pi}\pi(k_t)dk_t \\ &= \frac{\alpha'}{\Gamma(\omega)}\beta^{\omega}\int_0^{\infty} k_t^{\tau-2-2\mu/\sigma^2}\exp(-\beta k_t^{-\alpha'})dk_t \\ &= \beta^{\frac{\tau}{\alpha'}}\frac{\Gamma(\omega_{\tau})}{\Gamma(\omega)}.\end{aligned}$$

Particularly, let $\tau = 1$, $\tau = \alpha$, $\tau = -\alpha'$, we have

$$\begin{aligned}&\bar{k}_t = \mathbb{E}(k_t) = \beta^{\frac{1}{\alpha'}}\Gamma(\omega_1)/\Gamma(\omega), \\ &\bar{y}_t = \mathbb{E}(k_t^{\alpha}) = \beta^{\frac{\alpha}{\alpha'}}\frac{\Gamma(\omega_{\alpha})}{\Gamma(\omega)} = \beta^{\frac{\alpha}{\alpha'}}\frac{\Gamma(\omega_1+1)}{\Gamma(\omega)} = \beta^{\frac{\alpha}{\alpha'}}\frac{\omega_1\Gamma(\omega_1)}{\Gamma(\omega)} = \frac{\mu}{s}\bar{k}_t, \\ &\mathbb{E}(\frac{y_t}{k_t}) = \mathbb{E}(k_t^{-\alpha'}) = \beta^{-1}\frac{\Gamma(\omega_{-\alpha'})}{\Gamma(\omega)} = \beta^{-1}\frac{\Gamma(\omega+1)}{\Gamma(\omega)} = \beta^{-1}\omega = \frac{2\mu+\sigma^2}{2s}.\end{aligned}$$

In the deterministic system, due to $f(k_t) = k^{\alpha}$, the steady state of k_t is:

$$k_t^* = \Big(\frac{s}{n+\delta}\Big)^{\frac{1}{\alpha'}}.$$

Assume $n+\delta > 0$, now we can compare $\bar{k}_t$ and k_t^*. Set $\varepsilon = \frac{1}{\alpha'} = \frac{1}{1-\alpha} > 1$ and $x = \frac{2\mu}{\alpha'\sigma^2} = \omega_1$, then we have

$$\begin{aligned}\frac{\bar{k}_t}{k_t^*} &= \Big(\frac{2n+2\delta}{\alpha'\sigma^2}\Big)^{\varepsilon}\frac{\Gamma(x)}{\Gamma(x+\varepsilon)} \\ &= (\frac{n+\delta}{\mu})^{\varepsilon}\frac{x^{\varepsilon}\Gamma(x)}{\Gamma(x+\varepsilon)}.\end{aligned}$$

When $\sigma^2 \to 0$, $(\frac{n+\delta}{\mu})^{\varepsilon} \to 1$; By Stirling Approximation, we have $\frac{x^{\varepsilon}\Gamma(x)}{\Gamma(x+\varepsilon)} \to 1$ as long as $x \to \infty$. Therefor we can say when $\sigma^2 \to 0$, $\bar{k}_t \to k_t^*$.

This conclusion can prove that when the stochastic terms disappear, $\bar{k}_t = k_t^*$, the steady state of k_t^* (in deterministic model) is coincident with $\bar{k}_t$ (in stochastic model).

5 Conclusion

In this note, we modelled Solow-Swan growth mechanism in a stochastic environment. We allowed both the growth of capital and labor to be characterized by Brownian motion as well as by a Jump process. These characterizations closely approximate real life growth phenomena where economies are persistently subject to some form of stochastic jump by endogenizing sudden information/shocks which may alter the long-term growth dynamics of the model. Jump process and its stability analysis in Solow-Swan growth offer interesting insights regarding the persistence of output growth which may is a feature of endogenous growth mechanism. The Jump process we have introduced may be captured in empirical analysis as an endogenous shift or exogenous break which leave long-term permanent effect on growth processes.

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