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Efficient and fair allocation of aid *

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Abstract

This paper proposes a model of aid allocating which aims to equalize the opportunity between recipient countries to reduce the poverty, in particular the millennium development goal of reducing the poverty by half. The model also takes into account the natural deficit which is defined by the gap between the growth rate required to reach this millennium goal and the actual growth rate observed in the recipient country. The resulting optimal aid allocation is computed using the estimation of the growth equation. The latter takes into account effects of aid and structural handicaps which are represented by the economic vulnerability index and lack of human capital. We also perform a simulation study which shows a substantial difference between the aid allocation obtained with the Collier-Dollar (2002) criterion and that obtained with our model.

Keywords: Efficiency; equity; development aid; growth deficit; vulnerability

JEL Classification: D61; D63; F35; I30; O19

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1 Introduction

This papers fits in the debate on allocation of foreign aid in developing countries. We use a normative approach to determine an efficient and fair distribution of aid in the line of few existing studies such as Collier and Dollar (2001, 2002), Wood (2008), Llavador and Roemer (2001), and Cogneau and Naudet (2007), Carter (2014), etc.[1]

In their seminal papers, Collier and Dollar (2001, 2002) adopt a utilitarian vision by maximizing a social welfare function which is the sum of utilities of aid-recipient countries. A country's utility is measured in terms of number of poor which is a decreasing function of aid. More precisely, Collier and Dollar (2001, 2002) estimate the aid allocation that maximizes the reduction of number of poor in recipient countries. The reduction of poverty depends on several factors such as economic growth, initial poverty, and growth elasticity of poverty reduction. Economic growth is in turn influenced by aid (with a decreasing marginal effect), institutional quality, and policy quality. Consequently, the aid allocation reducing the poverty is determined by the initial poverty of recipient countries, their institutional quality, and their policy quality. The latter two factors (institutional and policy quality) are usually assimilated to the effectiveness principle. Compared to the observed allocation of aid, the Collier and Dollar's allocation gives more aid to the poorest countries implementing the highest policy quality (high CPIA). In the same vein, Wood (2008) incorporates an intertemporal aspect in his analysis and takes into account not only initial poverty but also future poverty in aid donors' objective function. We observe that we can adopt the same effectiveness principle by considering other variables than policy quality in marginal effectiveness of aid. For example, in Guillaumont and Chauvet (2001), the growth function depends on aid and the interaction between aid and economic vulnerability (which is measured by the instability of export of goods and services). The authors find that aid is more efficient in countries with high economic vulnerability.

This utilitarian approach of Collier and Dollar, merely corresponding to the minimization of the total number of poor people in recipient countries, is criticized for its lack of consideration for fairness. Indeed, Llavador and Roemer (2001) and Cogneau and Naudet (2007) propose an alternative way in calculating the optimal allocation of aid based on the Rawlsian principle. Both studies define an aid allocation satisfying the objective of equal opportunities. The idea is to analyze how aid can be distributed in order to equalize growth opportunities of recipient countries. Therefore, aid donors should give an allocation that compensates countries for bad initial circumstances so that the final differences in outcomes between countries will be only imputed to differences in their efforts, not to their initial circumstances.

In Llavador and Roemer (2001), the effort variable is defined by economic management which is the weighted average of three macroeconomic markers: budget surplus relative to GDP, inflation, and trade openness. The initial circumstances or

[1]Other empirical analysis of aid allocation rules emphasizes two main characteristics in recipient countries: their need for assistance, measured by their income per capita, and their absorption constraint, i.e. their ability to use aid effectively, measured by the World Bank's Country Performance Rating, see for example, Easterly (2007), Easterly and Pfutze (2008), Knack, Rogers and Eubank (2011), etc.

initial disadvantages of country i are defined as the component of the growth rate which is not explained by effort or aid. Llavador and Roemer (2001) find for a panel data of 55 countries over the period 1970-1993 that the equal-opportunity allocation of aid is more egalitarian than both the utilitarian allocation and the observed allocation.[2] They also propose a criterion of equal opportunity related to the risk of poverty and then determine an aid allocation that minimizes the poverty difference between recipient countries in 2015.

The study of Cogneau and Naudet (2007) devises a way of allocating aid that also includes the equal opportunity by using another method. The authors criticize the results of Llavador and Roemer (2001). Indeed, the aid allocation derived by Llavador and Roemer (2001) is paradoxically in favor of countries with high macroeconomic performances such as South Korea, Indonesia, and Thailand (low inflation, small budget deficit, and major open trade) to the detriment of countries with bad circumstances such as Nicaragua and Zambia. The Cogneau and Naudet (2007)'s analysis separates effort and circumstances of recipient countries as in Llavador and Roemer (2001). Using the same framework than Collier and Dollar, their aid allocation however shares out poverty risks more fairly among the world's population, and their results show that donors should give more aid to poorest countries than the currently observed aid allocation.

Recently, Carter (2014) proposes an aid allocation rules maximizing recipient welfare rather than economic growth and taking into account the absorptive capacity of recipient countries (measured by the World Bank's Country Performance Rating). Donors target a range of development outcomes by putting more weight to aid-funded consumption, and less weight to economic growth. The division of aid between consumption and investment results from the maximizing of households' utility in recipient countries. In this setting, the objective of maximizing welfare in recipient countries may give us an optimal allocation of aid with more aid to countries where it is least able to stimulate economic growth.

Our paper aims to design an optimal allocation of aid within a utilitarian framework where aid donors maximize the sum of utilities of recipient countries. The difference between our paper and the Collier and Dollar (2002)'s paper stems from two major points. Firstly, we think that aid policy should include uneven economic conditions between countries and then to compensate for them with foreign aid. Therefore, as in Guillaumont, McGillivray, and Pham (2015) our analysis accounts for structural handicaps to growth. More precisely, we introduce economic vulnerability and lack of human capital in the growth equation. This consideration is based on the assumption that a country with a low human capital and/or a strong economic vulnerability may encounter some difficulties to formulate a high quality of economic and social policy, inducing low possibility to achieve its development goal. Secondly, we believe that aid policy could be determined on a fair way. Unlike Llavador and Roemer (2001) and Cogneau and Naudet (2007), we propose an alternative way to model this fairness. We posit the utility of country i as a function of the gap between its current growth rate (depending on the amount of aid received and structural handicaps), on the one hand, and its expected (or targeted) growth rate, on the other hand. The latter represents the growth rate that country

[2]The utilitarian objective is the maximization of the growth rate of GDP of 55 countries.

i has to reach if it want to achieve a certain development goal. This gap somehow represents a natural deficit of growth. Hence, an efficient and fair allocation of aid should have the purpose to reduce the poverty in recipient countries by accounting the specific conditions (structural handicaps) and the natural deficit of growth in these countries.

This paper is organized as follows. Section 2 presents the theoretical model. Section 3 presents estimation results about the economic growth equation. Section 4 discusses the efficient and fair allocation of aid as well as the marginal effect of aid. Section 5 concludes.

2 A model of efficient and fair allocation of aid

In this section, we determine the aid allocation which equalizes the opportunity between countries to reach a development goal. Taking as example the Millennium Development Goal (MDG), proposing to reduce by half the poverty by 2015 compared to 1990, it is all about approaching the most possible to the MDG by giving to each recipient country the same probability to lift out of the poverty by half.[3] This leads to consider the growth rate required to reach this millennium goal for each recipient country and eventually to compensate the gap between this one and the effective growth rate. A high gap between these two growth rates in an country can be used as an argument, among others, to justify the aid allocated. Such a principle involves the consideration of both effectiveness and fairness in designing the optimal aid allocation. For this purpose, we assume that the utility of recipient country i, U_i, which corresponds to the number of poor that can be reduced by economic growth, is defined by

$$U_i = -\eta_i h_i N_i \left[\alpha u(g_i) + (1-\alpha) v \left(\frac{g_i}{g_i^*} \right) \right] \tag{1}$$

where h_i is a measure of poverty (such as the percentage of country i's population living below 2 dollars (in PPP) per day), $\eta_i = \frac{\partial h_i y_i}{\partial y_i h_i}$ is the elasticity of poverty reduction with respect to per capita income y_i, assumed to be a negative constant in Collier and Dollar (2001, 2002), and N_i is the population size.

Country i's utility is a function of its growth rate of per capita income, $u(g_i)$ with $u' > 0$ and $u'' \leq 0$, and a function of the ratio between the actual growth rate and the growth rate expected to achieve the MDG, $v(g_i/g_i^*)$ with $v' > 0$ and $v'' \leq 0$. Parameter $\alpha \in [0, 1]$ represents the weight associated to the growth objective and $1 - \alpha$ is the weight relative to 'natural deviation' from the expected (or targeted) growth rate g_i^*. When $\alpha = 1$ and $u(g_i) = g_i$ we exactly recover the model of Collier and Dollar (2001, 2002) where $U_i = -\eta_i h_i N_i g_i$. This expression gives the number of the poor that can be reduced by economic growth. However, this utility function, which is linear in the growth rate, implies that the marginal utility of growth U_g (also interpreted as marginal reduction of poverty) is constant. In this respect, the specification in (1) appears more general as it allows for a nonconstant marginal reduction of poverty. More precisely, we assume that each country's utility

[3] It is evident that we can analyze other goals or other deadlines.

is increasing and concave with respect to economic growth rate g_i. Its first and second derivative are given by

$$U_g = \frac{\partial U_i}{\partial g_i} = -\eta_i h_i N_i \left[\alpha u'(g) + (1-\alpha) v' \left(\frac{g_i}{g_i^*}\right) \frac{1}{g_i^*}\right] > 0, \quad (2)$$

$$U_{gg} = \frac{\partial^2 U}{\partial g_i^2} = -\eta_i h_i N_i (\alpha u''(g) + (1-\alpha) v'' \left(\frac{g_i}{g_i^*}\right) \frac{1}{g_i^{*2}} \leq 0. \quad (3)$$

We observe that marginal utility of growth, U_g, is decreasing with the ratio g_i/g_i^* (in others words, the marginal effect of growth is stronger in countries where the natural deficit is higher, i.e. with lower g_i/g_i^*). This decreasing effect is more important if the curvature of v is higher (or the relative risk aversion of v is higher).

We also assume that the targeted growth rate g_i^* is independent of aid. It can be determined as follows. Adopting the MDG with the number of the poor reduced by haft between 1990 and 2015 means that the cumulated objective of poverty variation is -50% in 25 years. Let x denote the annual reduction rate of the poverty, hence

$$h_i N_i (1 - 0.5) = h_i N_i (1 - x)^{25}. \quad (4)$$

We can easily find that x is equal to 0.0273. As $g_i^* \eta_i = -x$, the targeted growth rate g_i^* depends on the objective of annual poverty reduction rate (x_i) and the elasticity of poverty reduction with respect to per capita income (η_i),

$$g_i^* = -x/\eta_i. \quad (5)$$

Therefore, if $\eta_i = -2$ as in Collier and Dollar (2001, 2002), we obtain $g_i^* = 0.0273/2 = 0.01365$. However, following Bourguignon (2002) the elasticity of poverty reduction with respect to income per capita is not constant across countries. We can therefore estimate η_i and then g_i^* as a function of the development level of a country and of the degree of inequality of the income distribution.

As the growth rate g_i depends on the amount of aid A_i, let us consider the following optimization program which consists in choosing aid allocation maximizing the sum of utilities of n countries under constraints on the total amount of aid, on the one hand, and positiveness of aid, on the other hand,

$$\max_{\{A_i\}_{i=1}^n} \sum_i^n -\eta_i h_i N_i \left[\alpha u\left(g_i(A_i)\right) + (1-\alpha) v \left(\frac{g_i(A_i)}{g_i^*}\right)\right] \quad \text{(P1)}$$

s.t.

$$\sum_i^n A_i y_i N_i = \bar{A} \quad (6)$$

$$A_i \geq 0, \forall i \quad (7)$$

where the variable y_i is per capita income. A_i is the amount of aid, measured as a percentage of total GDP of i. $\bar{A}$ is the total amount of aid available for allocating among all recipient countries. The constraint written in equation (6) indicates the sum of aid allocated to all recipient countries is equal to the total amount of available aid.

The Lagrangian of problem (P1) can be written as follows.

$$L(A) = \sum_i^n -\eta_i h_i N_i \left[\alpha u\left(g_i(A_i)\right) + (1-\alpha) v\left(\frac{g(A_i)}{g_i^*}\right)\right] \quad (8)$$

$$-\lambda\left(\sum_i^n A_i y_i N_i - \bar{A}\right) - \sum_{i=1}^n \mu_i A_i, \quad (9)$$

where λ and μ_i, $i = 1, ..., n$ correspond respectively to the Lagrange multipliers of the constraint on the total available amount of aid and the positiveness of aid. The vector of candidates $\hat{A} \equiv (\hat{A}_1, ..., \hat{A}_n)$, and multipliers $\hat{\lambda}$, and $\hat{\mu}_i$ must satisfy the following first order conditions (FOC), $\forall i = 1, ..., n$:

$$\frac{\partial L\left(\hat{A}\right)}{\partial \hat{A}_i} = U_g(\hat{A}_i) g_A(\hat{A}_i) - \hat{\lambda} y_i N_i - \mu_i = 0 \quad (10)$$

$$\hat{A}_i \geq 0,\ \hat{\mu}_i \geq 0, \qquad \hat{\mu}_i \hat{A}_i = 0 \quad (11)$$

$$\sum_i^n \hat{A}_i y_i N_i = \bar{A} \quad (12)$$

where g_A is the marginal effect of aid on the growth rate and U_g the marginal effect of growth on country i's utility, which is given by equation (2).

Condition (11) is about the complementarity between $\hat{A}_i$ and $\hat{\mu}_i$, i.e. $\hat{\mu}_i = 0$ if $\hat{A}_i \geq 0$, and $\hat{\mu}_i > 0$ if $\hat{A}_i = 0$. Hence, for country i such that $\hat{A}_i \geq 0$, equation (10) gives $U_g(A_i) g_A(A_i) = \hat{\lambda} y_i N_i$ because $\hat{\mu}_i = 0$. Combining this with condition (12) will gives the values for $\hat{A}_i > 0$ and $\hat{\lambda}$. Finally, given $\hat{A}_i > 0$ and $\hat{\lambda}$, $\hat{\mu_j}$ can be recovered from equation (10) which only applies to country j such that $\hat{A}_j = 0$, i.e. $\mu_j = U_g(0) g_A(0) - \hat{\lambda} y_j N_j$. This sketch about the solution of these FOCs appears simple. However, its implementation is cumbersome because it needs some combinatory calculations. Fortunately, certain softwares can help us to solve this problem.[4]

Given the discussion above, we can state the following proposition:

Proposition 1 *Considering the optimization program (P1) where each country's utility is increasing and concave with economic growth (i.e. conditions (2) and (3)), the optimal (efficient and fair) allocation of aid* $\{\hat{A}_i\}_{i=1}^n$ *must verify the following conditions:*

(i) $\hat{A}_i = 0$ *if* $U_g(\hat{A}_i) g_A(\hat{A}_i) = \hat{\lambda} y_i N_i + \hat{\mu}_i$ *and* $\hat{\mu}_i > 0$,

(ii) $\hat{A}_i > 0$ *if* $U_g(\hat{A}_i) g_A(\hat{A}_i) = \hat{\lambda} y_i N_i$ *and* $\hat{\mu}_i = 0$,

(iii) $\sum_i^n \hat{A}_i y_i N_i = \bar{A}$,

[4] We will subsequently see in the simulation of the optimal aid allocation, based on estimation results of the growth equation and a parametrization of the theoretical model, we can use Matlab (function `fmincon`) or R (package `Rsolnp`) to find the solution of the optimization problem. In our case, the implementation of these different packages give generally the same results.

where $\hat{\mu}_i \geq 0$ is the multipliers associated to the positiveness of aid and $\hat{\lambda}$ is the multiplier (or shadow value) associated to the total amount of aid.

It should be noted that the multiplier λ can be viewed as the *marginal efficiency of aid* as in Collier and Dollar (2001, 2002). We observe from point (ii) of Proposition 1 that for any country i which receives a strictly positive amount of aid (i.e. $\hat{A}_i > 0$), we have

$$\hat{\lambda} = \frac{U_g(\hat{A}_i)g_A(\hat{A}_i)}{y_i N_i}. \tag{13}$$

In terms of our specification, this expression is equivalent to

$$\hat{\lambda} = \frac{-\eta_i h_i N_i(\alpha u' + (1-\alpha)v'/g_i^*)g_A(\hat{A}_i)}{y_i N_i}. \tag{14}$$

In the case of the Collier and Dollar' (2001, 2002) model, i.e. $\alpha = 1$ and $u(g_i) = g_i$, this multiplier becomes

$$\hat{\lambda}_{CD} = \frac{-\eta_i h_i N_i g_A(\hat{A}_i)}{y_i N_i}. \tag{15}$$

Collier and Dollar (2001, 2002) precisely defined this quantity as the number of the poor reduced by an increase of the total amount of aid by one unit.

From equation (14), we can compute for our model the equivalent number of the poor in country i that can be reduced by an increase of total amount of aid by one unit (here in millions of dollars) as

$$\hat{\lambda}_i = \frac{\hat{\lambda}}{\alpha u' + (1-\alpha)v'/g_i^*}. \tag{16}$$

We observe that while $\hat{\lambda}$ is constant (i.e. there is a unique solution for this, see also Proposition 1) $\hat{\lambda}_i$ can be different between countries receiving a positive amount of aid (because of different values of $\alpha u' + (1-\alpha)v'/g_i^*$). The only case for which the marginal efficiency of aid is the same for every recipient countries corresponds to the Collier-Dollar model, i.e. $\alpha = 1$ and $u(g_i) = g_i$ implying $\hat{\lambda}_i = \hat{\lambda}_{CD}, \forall i$.

We can now turn to sufficient condition for the solution of the optimization program (P1). As the constraint is linear, the sufficient condition for $\{\hat{A}_i\}_{i=1}^n$ being the solution of the optimization problem (P1) is that the objective function is concave with $\{\hat{A}_i\}_{i=1}^n$. In other words, the second order matrix

$$\begin{pmatrix} \frac{\partial U^2}{\partial^2 A_1} & \frac{\partial U^2}{\partial A_1 A_2} & \cdots & \frac{\partial U^2}{\partial A_1 \partial A_n} \\ \frac{\partial U^2}{\partial A_2 \partial A_1} & \frac{\partial U^2}{\partial A_2^2} & \cdots & \frac{\partial U^2}{\partial A_2 \partial A_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial U^2}{\partial A_n \partial A_1} & \frac{\partial U^2}{\partial A_n \partial A_2} & \cdots & \frac{\partial U^2}{\partial A_n^2} \end{pmatrix}$$

must be negative semi-definite. We observe that this matrix is diagonal and its diagonal elements are given by

$$\frac{\partial^2 U\left(\hat{A}\right)}{\partial \hat{A}_i^{\,2}} = U_{gg}(\hat{A}_i)g_A^2(\hat{A}_i) + U_g(\hat{A}_i)g_{AA}(\hat{A}_i) \tag{17}$$

Hence, the negative semi-definiteness of this matrix corresponds to the negativity of (17) or, in other words,

$$\frac{g_{AA}(\hat{A}_i)}{g_A^2(\hat{A}_i)} \leq -\frac{U_{gg}(\hat{A}_i)}{U_g(\hat{A}_i)}, \ \forall i. \tag{18}$$

The right-hand side term $(-U_{gg}/U_g)$, which is always positive, represents the curvature of the utility function with respect to the growth rate. It is also known as the coefficient of absolute risk aversion at g. The left-hand side term (g_{AA}/g_A^2) corresponds to the curvature of the growth rate with respect to aid. This inequality merely means that the absolute risk aversion of the objective function at g should be sufficiently high in order to warranty that $\{\hat{A}_i\}_{i=1}^n$ is the solution of the maximization problem. Hence, we can state the following proposition on the sufficient condition:

Proposition 2 *If the absolute risk aversion of the utility function with respect to the growth rate is sufficiently high as given by equation (18) (i.e.* $g_{AA}(\hat{A}_i)/g_A^2(\hat{A}_i) \leq -U_{gg}(\hat{A}_i)/U_g(\hat{A}_i)$, $\forall i$*) the allocation of aid* $\{\hat{A}_i\}_{i=1}^n$ *defined in Proposition 1 corresponds to the maximum of the optimization program (P1).*

We remark that Proposition 2 automatically holds if $g_{AA}(\hat{A} \leq 0$. It also holds even with some positive g_{AA} as long as condition (18) is satisfied.

3 Estimation of the growth equation

3.1 Econometric specification

This section describes the estimation of the growth equation, which will serve as the basis for simulating in the next section the optimal aid allocation derived from the theoretical model. Let us turn to the growth equation. Collier and Dollar (2001, 2002) consider that economic growth depends on policy quality and aid with decreasing marginal effects. This implies that countries with high performance in economic policies and institutions will receive more aid than others. However, such a studies ignore initial disadvantages of recipient countries. Those who are under poverty line because of their bad institutional qualities, economic vulnerability, and economic policies are unfairly taken into account. In order to avoid this drawback, it is then necessary, from a development perspective, to make aid fairer by considering structural handicaps to growth of recipient countries, in particular their economic vulnerability and their lack of human capital. Indeed, as underlined in Guillaumont, McGillivray and Wagner (2015), a country with a low human capital level is likely to have a low score of performance in spite of its great efforts. It may encounter difficulties to formulate a high quality of economic and social policies, then to achieve economic development. In addition, Guillaumont and Chauvet (2001), Chauvet and Guillaumont (2003) show that economic vulnerability has a negative impact on economic growth. Allocating more aid to countries with a low human capital and a strong economic vulnerability is therefore a way to compensate for their initial disadvantages which can prevent the objective of poverty reduction during their development process.

The growth equation is estimated by using a panel data framework (see, e.g., Islam, 1995, Caselli, Esquivel and Lefort, 1996, Durlauf, Jognson and Temple, 2005). We assume that the growth rate depends not only on aid, policy quality, but also on structural handicaps reflected by the degree of economic vulnerability and the lack of human capital. More precisely, the growth rate of country i in period t is a function of the degree of economic vulnerability (V_{it}), lack of human capital (H_{it}), level of net receipts of aid relative to GDP (A_{it}), its squared term (A_{it}^2), the interaction of aid with others variables like economic vulnerability, policy quality, human capital, and other exogenous factors (X_{it}). Following the literature (Islam 1995, Caselli et al. 1996, Durlauf et al. 2005, etc.), the growth equation can be then written in a panel data framework as

$$\begin{aligned}\ln y_{it} &= \rho \ln y_{i,t-\tau} + \beta_V V_i + \beta_{AV} A_i V_i + \beta_H H_i + \\ &\quad + \beta_A A_i + \beta_{AA} A_i^2 + \mu_i + \nu_t + \varepsilon_{it} \qquad (19)\end{aligned}$$

where $\ln y_{it}$ is the log real GDP per capita in international prices PPP 2005. The set of covariates includes ratio of aid to GDP (A), its squared term (A^2), economic vulnerability (V), lack of human capital (H), interactions of aid with V and H. The terms μ_i represents the country fixed effects. The term ν_t represents the time effects which are merely treated as time dummies.[5]

3.2 Data

The data employed to assess equation (19) are collected from the World Development Indicators of the World Bank, the Human Assets Index (HAI) and the Economic Vulnerability Index of the FERDI. The WDI database provides information on aid, GDP, population, etc. Aid and GDP are expressed in terms of real dollars, international prices PPP 2005, measured in millions dollars. These series are employed to compute GDP per capita (y) and the share of aid in GDP (A). We use the series on HAI, which is a composite index of education and health (see Closset, Feindouno and Goujon, 2014), to compute H as a measure of lack of human capital. For this purpose, we simply compute $H = (100 - HAI)/100$. The economic vulnerability index (V), which is is one of several criteria used by the United Nations Committee for Development Policy in identifying the least developed countries, was proposed by the FERDI (see Cariolle, 2011, for more details). Following Cariolle (2011), the vulnerability index encompasses the main determinants of structural vulnerability that can harm economic growth and poverty reduction in developing countries. The principal components entering the definition of economic vulnerability index are: (1) population, (2) share of agriculture, forestry and fisheries in GDP, (3) exports concentration, (4) remoteness from main world markets, adjusted for landlockness,

[5] We also include a variable representing the quality of policies, Polity 4 (as in the existing literature), and its interaction with aid. However, they add nothing to the results as they are statistically not significant.

(5) instability of exports receipts, (6) instability of agricultural production, and (7) homelessness due to natural disasters. Values of lack of human capital H and economic vulnerability V are comprised between 0 and 1.

As in the existing literature we use data with some time span interval in order to avoid business cycles. We adopt the 4-year interval ($\tau = 4$) as in Collier and Dollar (2002). More precisely, following Caselli et al. (1996) and Durlauf et al. (2005), among others, y_{it} (and then $y_{i,t-\tau}$) correspond to GDP per capita observed in 1975, 1979, 1983, 1987, 1991, 1995, 1999, 2003, 2007, and 2011. Variables A, V, and H are defined as the averages over the 4 years preceding t, i.e. 1975-1978, 1979-1982, 1983-1986, 1987-1990, 1991-1994, 1995-1998, 1999-2002, 2003-2006, 2007-2010. The resulting data are an unbalanced panel data sample covering 92 countries and period 1983-2010 (8 waves of 4-year interval: 1983-1986, 1987-1990,..., 2007-2010). Definition and descriptive statistics on variables are summarized in Table 1.

Table 1: Definition of variables and descriptive statistics

Variable	Obs	Mean	Std. Dev.	Min.	Max.
Growth rate of GDP per capita (g)	838	1.618	4.543	-28.546	27.271
log GDP per capita ($\ln y$)	971	7.969	0.958	4.6134	10.051
Economic vulnerability index (V)	923	0.413	0.123	0.130	0.730
Lack of human capital (H)	1007	0.428	0.243	0.012	0.965
Aid (A, in GDP share)	958	0.091	0.125	0	1.451
Poverty headcount, 1.25$/day	759	0.193	0.223	0	0.926
Poverty headcount, 2$/day	759	0.325	0.287	0	0.985

3.3 Estimation results

The equation above is a dynamic panel data model which can be estimated by using the system-GMM method of Blundell and Bond (1998). We note that this model considers two sets of regressors: (i) strictly exogenous regressors (including time dummies) and (ii) predetermined regressors (including $\ln y_{i,t-\tau}$, A, V, and H). Outliers are excluded from the estimations. The estimation results by using the system GMM are reported in Table 2.[6] Results of the within fixed effects estimator

[6]The difference-GMM method of Arellano and Bond (1991) was also considered but proved less satisfactory than the system GMM through specification tests. As noted in Roodman (2009), when performing the system GMM, all strictly exogenous regressors are used as instrument in both transformed equation and levels equation. Predetermined regressors are also valid instruments for the levels equation since they are assumed to be uncorrelated with the contemporaneous error term. Moreover, we use all available lags of the predetermined variables in levels as instruments for the transformed equation and the contemporaneous first differences as instruments in the levels equation. Finally, following Roodman (2009), we specify one instrument for each variable and lag distance (rather than one for each time period, variable, and lag distance) in order to reduce the bias that can happen in small samples with increased number of instruments.

are also reported for comparison purpose.[7] Specification tests (Arellano-Bond tests for AR(1) and AR(2) in the regression residuals, Sargan and Hansen overidentifying restrictions tests) and tests for exogeneity of instruments are generally verified.

We observe that results given by system GMM and within FE estimators are comparable. More precisely, the lagged term of GDP per capita is highly significant and has a positive effect on the current level of GDP per capita, which prove the usefulness of the dynamic setting. The linear term of aid has a negative effect whereas the effect of its squared term is positive. While economic vulnerability has no direct effect on income, its interaction term with aid has a significant and positive effect. This finding means that aid and economic vulnerability are complementary in the growth equation. In other words, when vulnerability of a country is high, aid should be directed to this country in order to compensate this weakness. Finally, lack of human capital is a handicap for growth as its coefficient is statistically significant and negative.

As the average annual growth rate of country i between periods $t-\tau$ and t is given by $(1/\tau)(\ln y_{it} - \ln y_{i,t-\tau})$, it can therefore computed from the estimated coefficients of equation (19) as follows:

$$g_{it} \equiv (1/\tau)(\ln y_{it} - \ln y_{i,t-\tau}) = ((\hat{\rho}-1)/\tau)\ln y_{i,t-\tau} + W'_{it}\hat{\beta}/\tau, \tag{20}$$

where W includes all right-hand side variables of equation (19), except $y_{i,t-\tau}$. This is the growth rate we will use in the subsequent simulations of efficient and fair allocation of aid. We remark that the negative coefficient associated to A^2 in the growth equation (see Table 2) corresponds to a negative value of $(\hat{\rho}-1)/\tau$. Hence, this result confirms the sufficient conditions of our optimization problem ($g_{AA} \leq 0$).

By using the estimated coefficients obtained from the growth equation above, we compute the growth-aid relation for the 2008 data, which will be used in our simulation exercises below. We observe from Figure 1 that this relation has an increasing pattern.

4 Simulation of the optimal aid allocation

In order to assess the theoretical allocation, we need an analytically tractable model. For simplicity'sake, we specify $u(g_i) = g_i$ and $v(g_i/g_i^*) = (g_i/g_i^*)^\gamma$ with $\gamma > 0$. Observe that $\gamma > 0$ measures the curvature of v with respect to ratio g_i/g_i^*. In this formulation, $1-\gamma$ represents the relative 'risk' aversion to 'natural deficit' in terms of growth in case of $g_i < g_i^*$. It should be noted that we do not exclude the case where $g_i > g_i^*$ as it is possible that some countries encounter this situation. A reduction in γ (or equivalently, an increase of the relative risk aversion), represented by an increase of the curvature of the function, means that aid donors are more averse to natural deficit (in case of $g_i < g_i^*$) or to 'low natural deviation' (in case of $g_i < g_i^*$). This implies that, aid donors give more weight to the countries that have lower ratio g_i/g_i^* than other countries. In other words, a reduction in γ implies

[7] We do not present the GLS random effects estimator here as it is dominated by the within fixed effects estimator according to the Hausman test.

Table 2: Estimation results of the growth equation

Variable	Within FE		System GMM	
	Coefficient	Std. Err.	Coefficient	Std. Err.
$\ln y_{i,t-\tau}$	0.683^{**}	0.024	0.943^{**}	0.020
Aid	-0.219	0.392	-0.883	0.620
Squared aid	-1.019^{*}	0.620	-1.324^{**}	0.496
Vulnerability	0.101	0.115	-0.206^{**}	0.087
Aid × Vulnerability	1.503^{**}	0.740	2.455^{*}	1.263
Lack of human capital	-0.527^{**}	0.141	-0.259^{**}	0.074
Period 87-90	-0.085^{**}	0.029	-0.032^{*}	0.018
Period 91-94	-0.099^{**}	0.025	-0.029	0.022
Period 95-98	-0.087^{**}	0.023	-0.020	0.016
Period 99-02	-0.058^{**}	0.020	0.003	0.014
Period 03-06	-0.067^{**}	0.016	-0.025^{*}	0.013
Period 07-10	0.026^{*}	0.014	0.062^{**}	0.010
Intercept	2.740^{**}	0.227	0.704^{**}	0.195
Number of observations	700		693	
Number of countries	109		109	
Existence of fixed effects, $F(108, 572)$	4.50^{**}			
Hausman test, random vs. fixed effects, $\chi^2(12)$	151.12^{**}			
Arellano-Bond test, AR(1)			-4.80^{**}	
Arellano-Bond test, AR(2)			-1.44	
Sargan test of overid. restrictions, $\chi^2(50)$			130.63^{**}	
Hansen test of overid. restrictions, $\chi^2(50)$			64.35^{*}	

Notes. Dependent variable: $\ln y_{it}$. Results obtained with the within fixed effects estimator and the one-step system GMM estimator of Blundell and Bond (1998) using robust standard errors. * and ** denote significance at the 10% and 5% levels, respectively. Strictly exogenous regressors include time dummies. Predetermined regressors are $\ln y_{i,t-\tau}$, Aid, Vulnerability, Lack of human capital.

that aid donors give priority to countries with more natural deficit that others in case of $g_i < g_i^*$ or lower 'natural deviation' in case of $g_i > g_i^*$, other things being equal. We think that the case $\gamma > 1$ should be excluded as it is not fair to favor the countries that have $g_i > g_i^*$ to the detriment of the countries that have a natural deficit ($g_i < g_i^*$). With $\gamma \in (0, 1)$, function $v\,(g_i/g_i^*)$ is then increasing and concave with respect to the natural deviation. Figure 2 describes the behavior of function v following different values of γ.

We use the estimation results of the growth equation above to simulate the

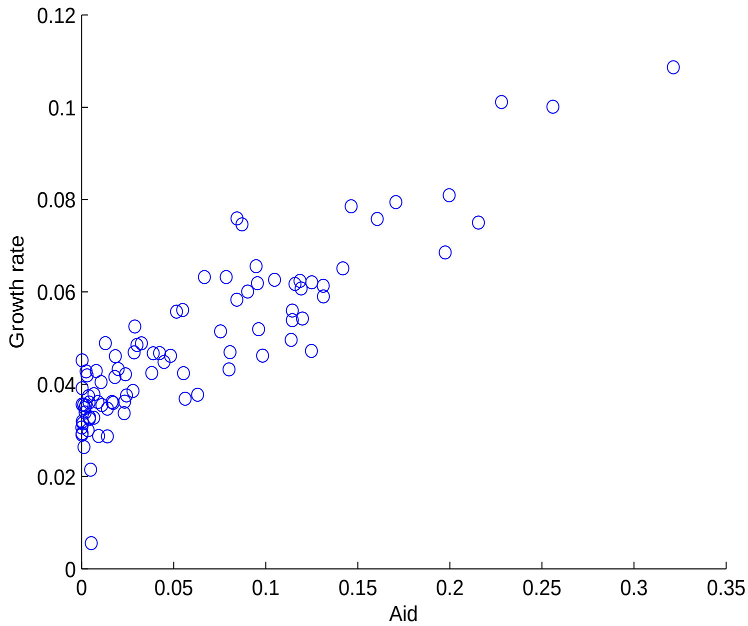


Figure 1: The estimated relation between growth and aid for the 2008 data.

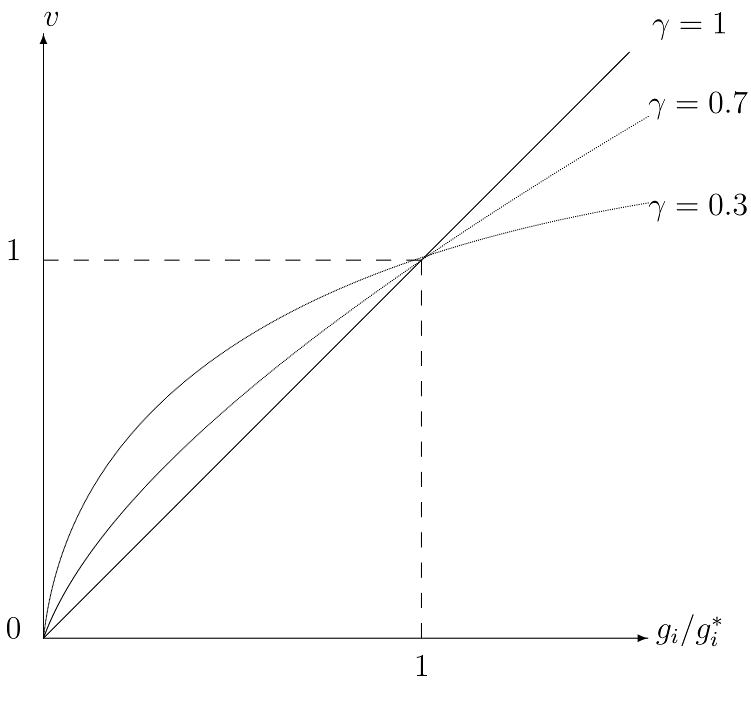


Figure 2: Behavior of $v(g_i/g_i^*) = (g_i/g_i^*)^\gamma$ with different values of γ $(0 < \gamma < 1)$.

amount of aid resulted from our theoretical model. We also compare these simulations with the observed data in 2008 which cover more than 90 countries.[8] Our

[8]We use the 2008 data because that contains a much higher number of countries than other years.

simulations cover the optimal allocation of aid as proposed by the model of Collier and Dollar (2002) and that of our model.

We consider the following assumptions about parameters for two models: (i) the headcount poverty rate based on two types of poverty line, 2 dollars per day and 1.25 dollars per day, which correspond to two sets of countries, 93 and 91 respectively, (ii) the elasticity of poverty reduction with respect to income per capita is the same for all countries, $\eta = -2$ (like Collier and Dollar, 2002). The model of Collier and Dollar (2002) corresponds to $\alpha = 1$. In this case, the value of γ has no effect on the results. Regarding our model, we consider two cases: (i) $\alpha = 0.7$, $\gamma = 0.3$ and (ii) $\alpha = 0.7$, $\gamma = 0.7$. Moreover, as our model requires the value of the targeted growth rate g^*, we set it as given in Section 2, i.e. $g^* = 0.01365, \quad \forall i$. Simulation results on optimal allocation of aid using estimation value of economic growth rate are reported in Tables 3-6. Our results are manifold.

1. Our first result concerns the number of countries receiving aid to development. While the totality of countries considered in our exercises (93 countries with the 2\$/day poverty line and 91 countries with the 1.25\$/day poverty line) actually received a positive amount of aid (in GDP share) in 2008, the optimal allocation shortlists only around one third of them. Given that our study consider structural handicaps to growth of recipient countries by taking into account their economic vulnerability and their lack of human capital, this result corroborates the reality.

2. The second result concerns the difference between Collier-Dollar allocation of aid and our allocation. Our proposed allocation cover more countries than that of Collier and Dollar (2002). More precisely, taking the case of the 2 dollars per day poverty line (see Tables 3-4), the Collier-Dollar solution (i.e. when $\alpha = 1$) will cover 33 countries. When taking into account the natural deviationin economic growth as our proposed model, i.e. $\alpha = 0.7$, the number of countries receiving a positive aid increases to 38 and 39 for $\gamma = 0.7$ and $\gamma = 0.3$, respectively. A similar result can be found when the 1.25 dollars per day poverty line is considered (see Tables 5-6). Indeed, while the number of countries receiving a positive aid is 30 for the Collier-Dollar solution ($\alpha = 1$), it increases in our proposed model to 32 and 34 for $\gamma = 0.7$ and $\gamma = 0.3$, respectively.

   In summary, compared to the Collier-Dollar solution, additional countries associated to our model are: (i) Angola, Cambodia, Congo Republic, Mauritania, and Senegal for the case with the 2\$ poverty line and $\gamma = 0.7$, (ii) Angola, Cambodia, Congo Republic, Mauritania, Senegal, Yemen for the 2\$ poverty line and $\gamma = 0.3$, (iii) Angola and Congo Republic for the 1.25\$ poverty line and $\gamma = 0.7$, and (iv) Angola, Congo Republic, Mauritania, and Senegal for the 1.25\$ poverty line and $\gamma = 0.3$.

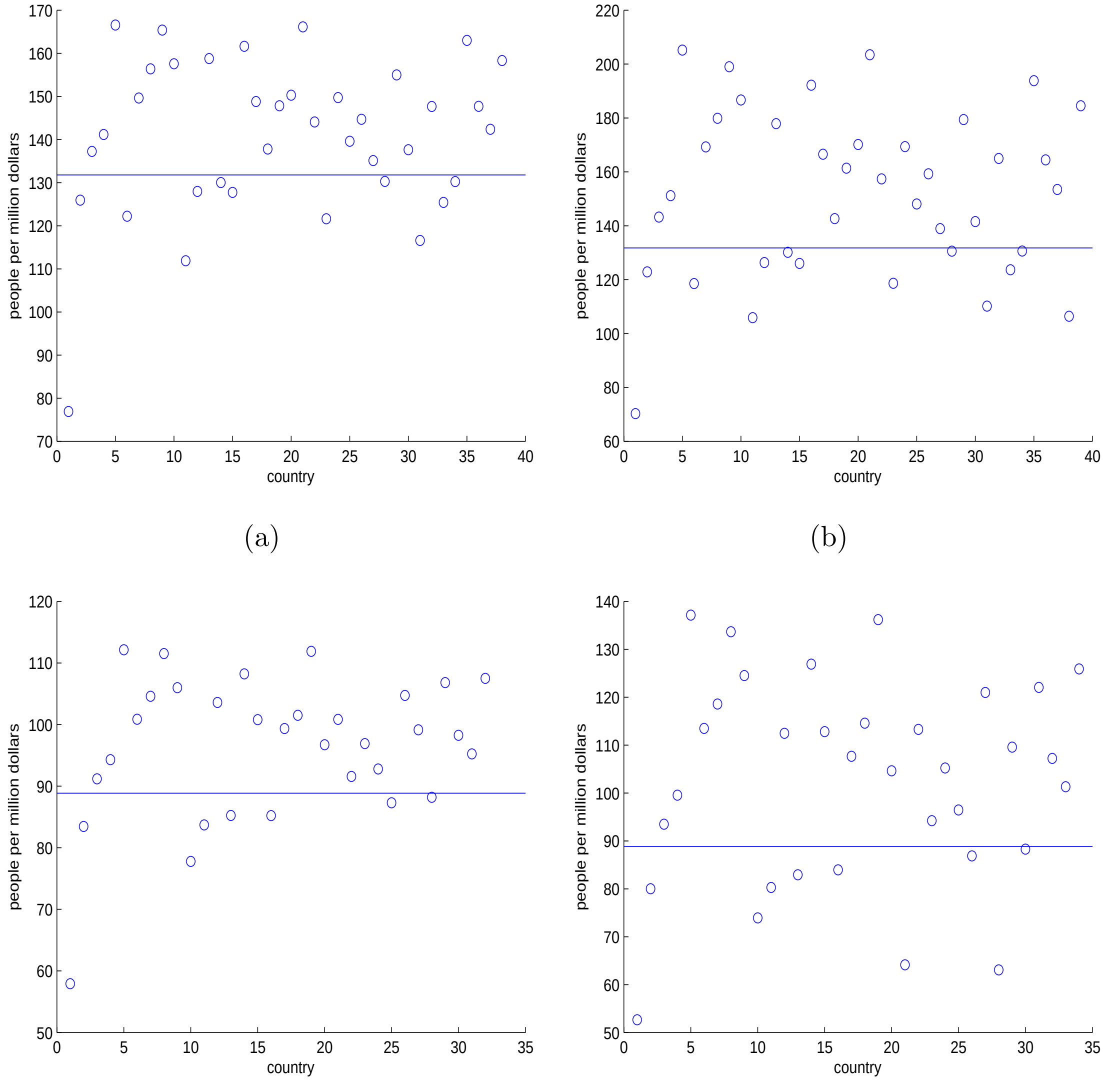
3. The third result relates to the countries covered by the Collier-Dollar solution and our proposed one. For most of them, our solution gives an aid amount lower than the Collier-Dollar solution. In particular, our proposed model taking into the donors' aversion to natural deviation indicates that when the 2\$

poverty line is considered 12 countries (Angola, Bangladesh, Cambodia, Congo Republic, Ethiopia, Ghana, Guinea, Mauritania, Papua New Guinea, Senegal, Sudan and Tanzania) and 13 countries (the previous 12 countries plus Yemen) receive more aid, respectively in the case of $\gamma = 0.7$ and $\gamma = 0.3$. When using the 1.25$ poverty line, the figures corresponding to $\gamma = 0.7$ and $\gamma = 0.3$ are respectively 8 countries (Angola, Bangladesh, Congo Republic, Ethiopia, Guinea, Lao PDR, Papua New Guinea, and Tanzania) and 10 countries (the previous 8 countries plus Mauritania and Senegal). This result means that taking into account the donors' sensitivity with respect to natural growth deficit $(1-\gamma)$ will significantly modify the aid allocation which is favorable to countries with low ratio g_i/g_i^* to the detriment of others.[9]

4. We also compute the marginal efficiency of aid, i.e. number of the poor reduced by increasing the total aid amount of 1 million dollars. The figures for the Collier-Dollar solution ($\hat{\lambda}_{CD}$) is 131.78 and 88.85 people per million dollars respectively for the 2$ poverty line and the 1.25$ poverty line. Regarding our model, this quantity (denoted as $\hat{\lambda}_i$, see also Section 2) varies between countries that receive a positive amount of aid. The last column of each of Tables 3-4 reports details on this. The mean values of these $\hat{\lambda}_i$ for the case of 2$ poverty line are 140.96 and 151.89 people per million dollars for $\gamma = 0.7$ and $\gamma = 0.3$, respectively. Those for the case of 1.25$ poverty line are respectively 96.42 and 102.59 people per million dollars. Figure 3 gives a full picture about the distribution of $\hat{\lambda}_i$. We remark that Angola is the country where aid is the lowest efficient among countries receiving a positive amount of aid. The numbers of people lifting from poverty (regarding the 2$ poverty line) are 77 and 70 for $\gamma = 0.7$ and $\gamma = 0.3$ respectively. If the poverty line is 1.25$/day, these numbers are respectively 58 people and 53 people for $\gamma = 0.7$ and $\gamma = 0.3$. In contrast, Burundi is the country where aid is the most efficient. Indeed, for the 2$ poverty line, there are 167 people (in the situation of $\gamma = 0.7$) and 205 people ($\gamma = 0.3$) that can escape poverty. When using the 1.25$ poverty line, these figures are respectively 112 people and 137 people per million dollars.

5. Finally, a reduction in value of γ, or equivalently an increase in the aversion to natural deviation $(1-\gamma)$, significantly impacts the optimal aid allocation and raises the average marginal efficiency of aid. In other words, by giving more priority to countries with a low ratio g_i/g_i^*, aid donors choose an efficient and fair allocation which can help lifting more people from poverty, the average number of people increases from 140.96 to 151.89 for the case of 2$ poverty line, and from 96.42 to 102.59 for the case of 1.25$ poverty line.

[9] Angola is a striking example of this result. Indeed, when the 2$/day poverty line is considered, its growth rate computed at the optimal aid allocation is 0.0078 and 0.0174 for $\gamma = 0.7$ and $\gamma = 0.3$, respectively. When the 1.25$/day poverty line is used, these figures are 0.0116 and 0.0211 respectively for $\gamma = 0.7$ and $\gamma = 0.3$.



(a) (b) (c) (d)

Figure 3: Marginal efficiency of aid, $\hat{\lambda}_i$. (a) poverty line = 2\$/day, $\gamma = 0.7$, (b) poverty line = 2\$/day, $\gamma = 0.3$, (c) poverty line = 1.25\$/day, $\gamma = 0.7$, (d) poverty line = 1.25\$/day, $\gamma = 0.3$. The horizontal lines correspond to the Collier-Dollar case, $\hat{\lambda}_{CD} = 131.78$ for poverty line = 2\$ per day and = 88.85 for poverty line = 1.25\$/day.

Table 3: Actual and optimal allocations of aid, headcount poverty rate based on 2$ per day poverty line, $\alpha = 0.7$, $\gamma = 0.7$

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7, \gamma = 0.7$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7, \gamma = 0.7$
Algeria	0.0019	0	0	0
Angola	0.0052	0	0.0134	76.91
Argentina	0.0004	0	00	0
Bangladesh	0.0239	0.0336	0.0412	125.93
Belize	0.0199	0	0	0
Benin	0.0961	0.1575	0.1496	137.24
Bhutan	0.0806	0	0	0
Bolivia	0.0389	0	0	0
Botswana	0.0562	0	0	0
Brazil	0.0003	0	0	0
Burkina Faso	0.1199	0.2199	0.2094	141.15
Burundi	0.3214	0.4652	0.4507	166.55
Cambodia	0.0755	0	0.0156	122.21
Cameroon	0.0234	0	0	0
Cape Verde	0.1464	0	0	0
Central African Rep.	0.1312	0.3108	0.2990	149.62
Chad	0.0631	0.3545	0.3216	156.39
Chile	0.0006	0	0	0
China	0.0003	0	0	0
Colombia	0.0041	0	0	0
Comoros	0.0785	0.3741	0.3361	165.38
Congo, Dem. Rep.	0.1707	0.3805	0.3743	157.56
Congo, Rep.	0.0553	0	0.0001	111.86
Costa Rica	0.0023	0	0	0
Cote d'Ivoire	0.0279	0	0	0
Djibouti	0.1313	0	0	0
Dominican Republic	0.0035	0	0	0
Ecuador	0.0044	0	0	0
Egypt, Arab Rep.	0.0106	0	0	0
El Salvador	0.0111	0	0	0
Ethiopia	0.1248	0.1140	0.1174	127.95
Fiji	0.0129	0	0	0
Gabon	0.0049	0	0	0
Gambia	0.0948	0.2812	0.2171	158.77
Ghana	0.0515	0.0125	0.0165	130.01
Guatemala	0.0140	0	0	0
Guinea	0.0983	0.0960	0.1004	127.73
Guinea-Bissau	0.1605	0.3781	0.3478	161.61
Guyana	0.0872	0	0	0

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Table 3 – continued from previous page

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7, \gamma = 0.7$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7, \gamma = 0.7$
Haiti	0.1419	0.2694	0.2523	148.81
Honduras	0.0424	0	0	0
India	0.0017	0	0	0
Indonesia	0.0025	0	0	0
Iran	0.0003	0	0	0
Iraq	0.1187	0	0	0
Jamaica	0.0067	0	0	0
Jordan	0.0326	0	0	0
Kenya	0.0448	0	0	0
Lao PDR	0.0955	0.1045	0.0905	137.81
Lesotho	0.0667	0.1696	0.1405	147.81
Madagascar	0.0902	0.2735	0.2586	150.28
Malawi	0.2280	0.4000	0.3785	166.12
Malaysia	0.0007	0	0	0
Maldives	0.0301	0	0	0
Mali	0.1144	0.2466	0.2356	144.05
Mauritania	0.1251	0	0.0335	121.61
Mexico	0.0001	0	0	0
Morocco	0.0166	0	0	0
Mozambique	0.2155	0.3009	0.2883	149.73
Namibia	0.0244	0	0	0
Nepal	0.0549	0.1227	0.1135	139.59
Nicaragua	0.1192	0	0	0
Niger	0.1144	0.2831	0.2746	144.70
Nigeria	0.0066	0.1341	0.1283	135.10
Pakistan	0.0093	0	0	0
Panama	0.0013	0	0	0
Papua New Guinea	0.0381	0.0681	0.0717	130.29
Paraguay	0.0080	0	0	0
Peru	0.0038	0	0	0
Philippines	0.0003	0	0	0
Rwanda	0.1996	0.3159	0.2954	154.99
Sao Tome & Principe	0.2559	0.0684	0.0516	137.62
Senegal	0.0801	0	0.0270	116.57
Seychelles	0.0140	0	0	0
Sierra Leone	0.1974	0.2981	0.2858	147.66
South Africa	0.0042	0	0	0
Sri Lanka	0.0184	0	0	0
St. Lucia	0.0181	0	0	0
Sudan	0.0483	0.0174	0.0383	125.41
Suriname	0.0286	0	0	0

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Table 3 – continued from previous page

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7$, $\gamma = 0.7$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7$, $\gamma = 0.7$
Swaziland	0.0232	0	0	0
Syrian Arab Rep.	0.0030	0	0	0
Tanzania	0.1138	0.1133	0.1149	130.24
Timor-Leste	0.0844	0.3070	0.2700	162.98
Togo	0.1048	0.2361	0.2209	147.70
Tunisia	0.0088	0	0	0
Turkey	0.0015	0	0	0
Uganda	0.1159	0.1804	0.1682	142.37
Uruguay	0.0011	0	0	0
Venezuela	0.0002	0	0	0
Vietnam	0.0290	0	0	0
Yemen	0.0172	0	0	0
Zambia	0.0843	0.3294	0.2978	158.32
		$\hat{\lambda}_{CD}$=131.78	$\hat{\lambda}$=1450.95	average $\{\hat{\lambda}_i\}$=140.96

Notes: Simulations are run under the assumption $\eta = -2$ (elasticity of poverty reduction with respect to income per capita). The model of Collier and Dollar (2002) corresponds to $\alpha = 1$. The last two columns correspond to our model with $\alpha = 0.7$, $\gamma = 0.7$, $g^* = 0.01365$, $\forall i$. The last row reports the marginal efficiency of aid of the Collier and Dollar' model ($\hat{\lambda}_{CD}$) and our model ($\hat{\lambda}$ and the average of $\hat{\lambda}_i$).

Table 4: Actual and optimal allocations of aid, headcount poverty rate based on 2$ per day poverty line, $\alpha = 0.7$, $\gamma = 0.3$

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7$, $\gamma = 0.3$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7$, $\gamma = 0.3$
Algeria	0.0019	0	0	0
Angola	0.0052	0	0.0495	70.28
Argentina	0.0004	0	0	0
Bangladesh	0.0239	0.0336	0.0452	122.89
Belize	0.0199	0	0	0
Benin	0.0961	0.1575	0.1410	143.20
Bhutan	0.0806	0	0	0
Bolivia	0.0389	0	0	0
Botswana	0.0562	0	0	0
Brazil	0.0003	0	0	0
Burkina Faso	0.1199	0.2199	0.1981	151.18
Burundi	0.3214	0.4652	0.4346	205.17
Cambodia	0.0755	0	0.0255	118.51
Cameroon	0.0234	0	0	0
Cape Verde	0.1464	0	0	0
Central African Rep.	0.1312	0.3108	0.2861	169.25
Chad	0.0631	0.3545	0.2901	179.85
Chile	0.0006	0	0	0
China	0.0003	0	0	0
Colombia	0.0041	0	0	0
Comoros	0.0785	0.3741	0.2980	199.01
Congo, Dem. Rep.	0.1707	0.3805	0.3672	186.69
Congo, Rep.	0.0553	0	0.0211	105.87
Costa Rica	0.0023	0	0	0
Cote d'Ivoire	0.0279	0	0	0
Djibouti	0.1313	0	0	0
Dominican Republic	0.0035	0	0	0
Ecuador	0.0044	0	0	0
Egypt, Arab Rep.	0.0106	0	0	0
El Salvador	0.0111	0	0	0
Ethiopia	0.1248	0.1140	0.1188	126.32
Fiji	0.0129	0	0	0
Gabon	0.0049	0	0	0
Gambia	0.0948	0.2812	0.1717	177.88
Ghana	0.0515	0.0125	0.0162	130.14
Guatemala	0.0140	0	0	0
Guinea	0.0983	0.0960	0.1023	125.99
Guinea-Bissau	0.1605	0.3781	0.3168	192.14
Guyana	0.0872	0	0	0

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Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7$, $\gamma = 0.3$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7$, $\gamma = 0.3$
Haiti	0.1419	0.2694	0.2346	166.52
Honduras	0.0424	0	0	0
India	0.0017	0	0	0
Indonesia	0.0025	0	0	0
Iran	0.0003	0	0	0
Iraq	0.1187	0	0	0
Jamaica	0.0067	0	0	0
Jordan	0.0326	0	0	0
Kenya	0.0448	0	0	0
Lao PDR	0.0955	0.1045	0.0792	142.67
Lesotho	0.0667	0.1696	0.1159	161.34
Madagascar	0.0902	0.2735	0.2427	170.16
Malawi	0.2280	0.4000	0.3550	203.44
Malaysia	0.0007	0	0	0
Maldives	0.0301	0	0	0
Mali	0.1144	0.2466	0.2236	157.40
Mauritania	0.1251	0	0.0439	118.66
Mexico	0.0001	0	0	0
Morocco	0.0166	0	0	0
Mozambique	0.2155	0.3009	0.2746	169.37
Namibia	0.0244	0	0	0
Nepal	0.0549	0.1227	0.1035	148.06
Nicaragua	0.1192	0	0	0
Niger	0.1144	0.2831	0.2651	159.28
Nigeria	0.0066	0.1341	0.1216	138.94
Pakistan	0.0093	0	0	0
Panama	0.0013	0	0	0
Papua New Guinea	0.0381	0.0681	0.0710	130.57
Paraguay	0.0080	0	0	0
Peru	0.0038	0	0	0
Philippines	0.0003	0	0	0
Rwanda	0.1996	0.3159	0.2739	179.40
Sao Tome & Principe	0.2559	0.0684	0.0403	141.55
Senegal	0.0801	0	0.0436	110.16
Seychelles	0.0140	0	0	0
Sierra Leone	0.1974	0.2981	0.2723	164.95
South Africa	0.0042	0	0	0
Sri Lanka	0.0184	0	0	0
St. Lucia	0.0181	0	0	0
Sudan	0.0483	0.0174	0.0441	123.64
Suriname	0.0286	0	0	0

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Table 4 – continued from previous page

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7$, $\gamma = 0.3$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7$, $\gamma = 0.3$
Swaziland	0.0232	0	0	0
Syrian Arab Rep.	0.0030	0	0	0
Tanzania	0.1138	0.1133	0.1145	130.62
Timor-Leste	0.0844	0.3070	0.2334	193.83
Togo	0.1048	0.2361	0.2049	164.47
Tunisia	0.0088	0	0	0
Turkey	0.0015	0	0	0
Uganda	0.1159	0.1804	0.1555	153.46
Uruguay	0.0011	0	0	0
Venezuela	0.0002	0	0	0
Vietnam	0.0290	0	0	0
Yemen	0.0172	0	0.0121	106.38
Zambia	0.0843	0.3294	0.2666	184.52
		$\hat{\lambda}_{CD}$=131.78	$\hat{\lambda}$=440.18	average $\{\hat{\lambda}_i\}$=151.89

Notes: Simulations are run under the assumption $\eta = -2$ (elasticity of poverty reduction with respect to income per capita). The model of Collier and Dollar (2002) corresponds to $\alpha = 1$. The last two columns correspond to our model with $\alpha = 0.7$, $\gamma = 0.3$, $g^* = 0.01365$, $\forall i$. The last row reports the marginal efficiency of aid of the Collier and Dollar' model ($\hat{\lambda}_{CD}$) and our model ($\hat{\lambda}$ and the average of $\hat{\lambda}_i$).

Table 5: Actual and optimal allocations of aid, headcount poverty rate based on 1.25$ per day poverty line, $\alpha = 0.7$, $\gamma = 0.7$

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7, \gamma = 0.7$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7, \gamma = 0.7$
Algeria	0.0019	0	0	0
Angola	0.0052	0	0.0274	57.94
Argentina	0.0004	0	0	0
Bangladesh	0.0239	0.0102	0.0220	83.45
Belize	0.0199	0	0	0
Benin	0.0961	0.1373	0.1317	91.19
Bhutan	0.0806	0	0	0
Bolivia	0.0389	0	0	0
Botswana	0.0562	0	0	0
Brazil	0.0003	0	0	0
Burkina Faso	0.1199	0.2058	0.1959	94.30
Burundi	0.3214	0.4773	0.4660	112.13
Cambodia	0.0755	0	0	0
Cameroon	0.0234	0	0	0
Cape Verde	0.1464	0	0	0
Central African Rep.	0.1312	0.3229	0.3128	100.87
Chad	0.0631	0.3423	0.3089	104.58
Chile	0.0006	0	0	0
China	0.0003	0	0	0
Colombia	0.0041	0	0	0
Comoros	0.0785	0.3830	0.3472	111.53
Congo, Dem. Rep.	0.1707	0.3888	0.3843	105.99
Congo, Rep.	0.0553	0	0.0145	77.79
Costa Rica	0.0023	0	0	0
Cote d'Ivoire	0.0279	0	0	0
Djibouti	0.1313	0	0	0
Dominican Republic	0.0035	0	0	0
Ecuador	0.0044	0	0	0
Egypt, Arab Rep.	0.0106	0	0	0
El Salvador	0.0111	0	0	0
Ethiopia	0.1248	0.0601	0.0699	83.72
Fiji	0.0129	0	0	0
Gabon	0.0049	0	0	0
Gambia	0.0948	0.2369	0.1776	103.58
Ghana	0.0515	0	0	0
Guatemala	0.0140	0	0	0
Guinea	0.0983	0.0820	0.0883	85.25
Guinea-Bissau	0.1605	0.3663	0.3346	108.23
Guyana	0.0872	0	0	0

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Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7$, $\gamma = 0.7$	Marginal efficiency $\hat{\lambda}_i$ (people/\$ million) $\alpha = 0.7$, $\gamma = 0.7$
Haiti	0.1419	0.2909	0.2760	100.79
Honduras	0.0424	0	0	0
India	0.0017	0	0	0
Indonesia	0.0025	0	0	0
Iran	0.0003	0	0	0
Iraq	0.1187	0	0	0
Jamaica	0.0067	0	0	0
Jordan	0.0326	0	0	0
Kenya	0.0448	0	0	0
Lao PDR	0.0955	0.0073	0.0238	85.22
Lesotho	0.0667	0.1684	0.1400	99.34
Madagascar	0.0902	0.2902	0.2775	101.50
Malawi	0.2280	0.4103	0.3914	111.90
Maldives	0.0301	0	0	0
Mali	0.1144	0.2429	0.2321	96.73
Mauritania	0.1251	0	0	0
Mexico	0.0001	0	0	0
Morocco	0.0166	0	0	0
Mozambique	0.2155	0.3076	0.2961	100.84
Namibia	0.0244	0	0	0
Nepal	0.0549	0.0767	0.0705	91.57
Nicaragua	0.1192	0	0	0
Niger	0.1144	0.2689	0.2598	96.90
Nigeria	0.0066	0.1679	0.1591	92.80
Pakistan	0.0093	0	0	0
Panama	0.0013	0	0	0
Papua New Guinea	0.0381	0.0634	0.0690	87.31
Paraguay	0.0080	0	0	0
Peru	0.0038	0	0	0
Philippines	0.0003	0	0	0
Rwanda	0.1996	0.3339	0.3163	104.74
Sao Tome & Principe	0.2559	0	0	0
Senegal	0.0801	0	0	0
Seychelles	0.0140	0	0	0
Sierra Leone	0.1974	0.2933	0.2809	99.14
South Africa	0.0042	0	0	0
Sri Lanka	0.0184	0	0	0
St. Lucia	0.0181	0	0	0
Sudan	0.0483	0	0	0
Suriname	0.0286	0	0	0
Swaziland	0.0232	0	0	0

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Table 5 – continued from previous page

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7, \gamma = 0.7$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7, \gamma = 0.7$
Syrian Arab Rep.	0.0030	0	0	0
Tanzania	0.1138	0.1294	0.1303	88.20
Timor-Leste	0.0844	0.2484	0.2048	106.82
Togo	0.1048	0.2093	0.1931	98.27
Tunisia	0.0088	0	0	0
Uganda	0.1159	0.1693	0.1575	95.24
Uruguay	0.0011	0	0	0
Venezuela	0.0002	0	0	0
Vietnam	0.0290	0	0	0
Yemen	0.0172	0	0	0
Zambia	0.0843	0.3570	0.3298	107.50
		$\hat{\lambda}_{CD}$=88.85	$\hat{\lambda}$=975.46	average $\{\hat{\lambda}_i\}$=96.42

Notes: Simulations are run under the assumption $\eta = -2$ (elasticity of poverty reduction with respect to income per capita). The model of Collier and Dollar (2002) corresponds to $\alpha = 1$. The last two columns correspond to our model with $\alpha = 0.7$, $\gamma = 0.7$, $g^* = 0.01365$, $\forall i$. The last row reports the marginal efficiency of aid of the Collier and Dollar' model ($\hat{\lambda}_{CD}$) and our model ($\hat{\lambda}$ and the average of $\hat{\lambda}_i$).

Table 6: Actual and optimal allocations of aid, headcount poverty rate based on 1.25$ per day poverty line, $\alpha = 0.7$, $\gamma = 0.3$

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7, \gamma = 0.3$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7, \gamma = 0.3$
Algeria	0.0019	0	0	0
Angola	0.0052	0	0.0642	52.65
Argentina	0.0004	0	0	0
Bangladesh	0.0239	0.0102	0.0296	80.01
Belize	0.0199	0	0	0
Benin	0.0961	0.1373	0.1262	93.49
Bhutan	0.0806	0	0	0
Bolivia	0.0389	0	0	0
Botswana	0.0562	0	0	0
Brazil	0.0003	0	0	0
Burkina Faso	0.1199	0.2058	0.1864	99.53
Burundi	0.3214	0.4773	0.4540	137.14
Cambodia	0.0755	0	0	0
Cameroon	0.0234	0	0	0
Cape Verde	0.1464	0	0	0
Central African Rep.	0.1312	0.3229	0.3022	113.48
Chad	0.0631	0.3423	0.2792	118.57
Chile	0.0006	0	0	0
China	0.0003	0	0	0
Colombia	0.0041	0	0	0
Comoros	0.0785	0.3830	0.3123	133.67
Congo, Dem. Rep.	0.1707	0.3888	0.3794	124.50
Congo, Rep.	0.0553	0	0.0332	73.93
Costa Rica	0.0023	0	0	0
Cote d'Ivoire	0.0279	0	0	0
Djibouti	0.1313	0	0	0
Dominican Republic	0.0035	0	0	0
Ecuador	0.0044	0	0	0
Egypt, Arab Rep.	0.0106	0	0	0
El Salvador	0.0111	0	0	0
Ethiopia	0.1248	0.0601	0.0764	80.29
Fiji	0.0129	0	0	0
Gabon	0.0049	0	0	0
Gambia	0.0948	0.2369	0.1418	112.46
Ghana	0.0515	0	0	0
Guatemala	0.0140	0	0	0
Guinea	0.0983	0.0820	0.0925	82.94
Guinea-Bissau	0.1605	0.3663	0.3040	126.91
Guyana	0.0872	0	0	0

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Table 6 – continued from previous page

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7, \gamma = 0.3$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7, \gamma = 0.3$
Haiti	0.1419	0.2909	0.2611	112.82
Honduras	0.0424	0	0	0
India	0.0017	0	0	0
Indonesia	0.0025	0	0	0
Iran	0.0003	0	0	0
Iraq	0.1187	0	0	0
Jamaica	0.0067	0	0	0
Jordan	0.0326	0	0	0
Kenya	0.0448	0	0	0
Lao PDR	0.0955	0.0073	0.0296	83.95
Lesotho	0.0667	0.1684	0.1175	107.66
Madagascar	0.0902	0.2902	0.2644	114.57
Malawi	0.2280	0.4103	0.3716	136.18
Maldives	0.0301	0	0	0
Mali	0.1144	0.2429	0.2212	104.63
Mauritania	0.1251	0	0.0004	64.15
Mexico	0.0001	0	0	0
Morocco	0.0166	0	0	0
Mozambique	0.2155	0.3076	0.2841	113.29
Namibia	0.0244	0	0	0
Nepal	0.0549	0.0767	0.0645	94.23
Nicaragua	0.1192	0	0	0
Niger	0.1144	0.2689	0.2504	105.21
Nigeria	0.0066	0.1679	0.1510	96.46
Pakistan	0.0093	0	0	0
Panama	0.0013	0	0	0
Papua New Guinea	0.0381	0.0634	0.0706	86.85
Paraguay	0.0080	0	0	0
Peru	0.0038	0	0	0
Philippines	0.0003	0	0	0
Rwanda	0.1996	0.3339	0.2984	121.00
Sao Tome & Principe	0.2559	0	0	0
Senegal	0.0801	0	0.0022	63.07
Seychelles	0.0140	0	0	0
Sierra Leone	0.1974	0.2933	0.2683	109.57
South Africa	0.0042	0	0	0
Sri Lanka	0.0184	0	0	0
St. Lucia	0.0181	0	0	0
Sudan	0.0483	0	0	0
Suriname	0.0286	0	0	0
Swaziland	0.0232	0	0	0

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Table 6 – continued from previous page

Country	Actual aid (%GDP)	Optimal aid (%GDP) $\alpha = 1$	Optimal aid (%GDP) $\alpha = 0.7$, $\gamma = 0.3$	Marginal efficiency $\hat{\lambda}_i$ (people/$ million) $\alpha = 0.7$, $\gamma = 0.3$
Syrian Arab Rep.	0.0030	0	0	0
Tanzania	0.1138	0.1294	0.1302	88.27
Timor-Leste	0.0844	0.2484	0.1679	122.06
Togo	0.1048	0.2093	0.1778	107.21
Tunisia	0.0088	0	0	0
Uganda	0.1159	0.1693	0.1464	101.30
Uruguay	0.0011	0	0	0
Venezuela	0.0002	0	0	0
Vietnam	0.0290	0	0	0
Yemen	0.0172	0	0	0
Zambia	0.0843	0.3570	0.3030	125.89
		$\hat{\lambda}_{CD}$=88.85	$\hat{\lambda}$=293.10	average $\{\hat{\lambda}_i\}$=102.59

Notes: Simulations are run under the assumption $\eta = -2$ (elasticity of poverty reduction with respect to income per capita). The model of Collier and Dollar (2002) corresponds to $\alpha = 1$. The last two columns correspond to our model with $\alpha = 0.7$, $\gamma = 0.3$, $g^* = 0.01365$, $\forall i$. The last row reports the marginal efficiency of aid of the Collier and Dollar' model ($\hat{\lambda}_{CD}$) and our model ($\hat{\lambda}$ and the average of $\hat{\lambda}_i$).

5 Conclusion

In this study, we propose a model of aid allocating which aims to equalize the opportunity between recipient countries to reduce the poverty. The modeling accounts for the 'natural' deficit between the growth rate required to reach a certain target (e.g. the millennium development goal of poverty reduction) and the actual growth rate observed in the recipient country. We show that our proposed aid allocation substantially differs from the allocation obtained following the Collier-Dollar (2002) criterion in terms of number of countries receiving aid, amounts of aid, and number of the poor lifting from poverty. Our results also shed light on the impact of the aversion to growth deficit on the allocation of aid. More precisely, giving more weight to countries with a high growth gap can help lifting more people from poverty.

Further extensions are needed to check these results. We can generalize the simulation to the case where the elasticity of poverty reduction with respect to income differs between countries. The theoretical analysis can be also extended to include the dynamic aspect of aid allocation as development aid may result from a dynamic interaction between donor and recipient countries.

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