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Keynes' beauty contest revisited »**

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Playing the game the others want to play:
Keynes' beauty contest revisited

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Abstract

In Keynes' beauty contest, agents have to choose actions in accordance with an expected fundamental value and with the conventional value expected to be set by the market. In doing so, agents respond to a fundamental and to a coordination motive respectively, the prevalence of either motive being set exogenously. Our contribution is to consider whether agents favor the fundamental or the coordination motive as the result of a strategic choice that generates a strong strategic complementarity of agents' actions. We show that the coordination motive tends to prevail over the fundamental one, which yields a disconnection of activity away from the fundamental. A valuation game and a competition game are provided as illustrations of this general framework.

Keywords: beauty contest, financial markets, indeterminacy, oligopolistic competition, strategic complementarities.

JEL codes: D43 - D84 - E12 - E44 - L13.

1 Introduction

Keynes' beauty contest metaphor of financial markets is characterized by a dual motivation in agents' decision making: there is a *fundamental motive* making agents strive to predict the fundamental value of some financial asset and there is a *coordination motive* making them seek to predict the conventional value eventually set by the market. There is no reason for the two values to coincide, and in Keynes' view the working of stock markets, rather than imposing a balance between the two motives, tends to favor the coordination

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motive. Stock markets exhibit not only mutual coordination, but also common interest in coordination. Indeed, "*most [of professional investors and speculators] are, in fact, largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public,*" in other words, not with forecasting fundamentals, but with "*anticipating what average opinion expects the average opinion to be*" (Keynes, 1936, ch.12, p.154 and p.156). If others attach little importance to coordinating on others' actions, and mainly focus their expectations on fundamentals, an agent has no reason to attach importance to coordination either. By contrast, the more others attach importance to coordinating on others' predictions, mainly basing their expectations on convention, the more an agent feels himself dependent on convention and concerned by the coordination motive.[1] In this sense, each agent wants to play the game the others want to play.

The aim of our paper is to revisit Keynes' beauty contest parable in a very simple set-up that captures the strategic complementarity resulting in the choice to play a pure coordination game. Our contribution is to consider the "*energies and skill*" (Keynes, 1936, p.154) occupied by the agents in the pursuit of the coordination vs. the fundamental motive not as structural but as the result of strategic decisions. As these decisions are strong strategic complements, the coordination motive tends to evict the fundamental motive, leading to a disconnection of anticipations from fundamentals and to the emergence of sunspots. Another point we want to stress is that the prediction of financial asset values is not the only activity in which the coordination motive tends to prevail over the fundamental motive as a consequence of a strong strategic complementarity. Competition in oligopolistic output markets may also end up in a chase after the competitors' prices that have to be matched or beaten, independently of the state of demand, if a firm wants to survive. We thus provide two illustrations of the beauty contest outcome: a *valuation game* applied to financial markets and a *competition game* applied to oligopolistic output markets.

The first illustration is a valuation game directly based on Morris and Shin (2002). In this famous representation of the Keynesian beauty contest, agents receive public and private signals about some unknown fundamental value. Agents' actions consist in choosing a value which is a compromise between the anticipated *fundamental value* and the anticipated *conventional value* (the average action). Keynes refers to "*the term* speculation *for the activity of forecasting the psychology of the market, and the term* enterprise *for the activity of forecasting the prospective yield of assets over their whole life*" (Keynes, 1936, p.158). The relative weight put on the coordination motive in the agents' utility function is not a decision variable in the model of Morris and Shin but is set exogenously. This exogeneity leaves open the issue of the potential disconnection of actions from the fundamental value. We consider instead

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[1] We find this idea in the following quotation of Keynes (1936, p.156): "*If the reader interjects that there must surely be large profits to be gained from the other players in the long run by a skilled individual who, unperturbed by the prevailing pastime, continues to purchase investments on the best genuine long-term expectations he can frame, he must be answered, first of all, that there are, indeed, such serious-minded individuals and that it makes a vast difference to an investment market whether or not they predominate in their influence over the game-players. But we must also add that there are several factors which jeopardise the predominance of such individuals in modern investment markets. Investment based on genuine long-term expectation is so difficult to-day as to be scarcely practicable.*"

the relative weight each agent puts on the coordination motive as a strategic variable, and show that there is an incentive for agents to devote their energy to speculation rather than to enterprise. As implicitly stated by Keynes, the prevalence of the former activity over the latter is the result of strategic complementarities inherent to agents' activity of speculation: "*as the organisation of investment markets improves, the risk of the predominance of speculation does [...] increase. In one of the greatest investment markets in the world, namely, New York, the influence of speculation (in the above sense) is enormous. [...W]hen he purchases an investment, the American is attaching his hopes, not so much to its prospective yield, as to a favourable change in the conventional basis of valuation, i.e. [...] he is, in the above sense, a speculator.*" We show that the activity of speculation always prevails in the valuation game because coordination on public information entails less cost than valuing an unknown fundamental. In this stock market example, information is the driving force for the coordination loss to be dominated by the fundamental loss: as agents put more weight on the coordination motive, they rely more on public information to estimate the average action, making it easier to coordinate on the convention. The strategic choice to privilege the convention results in the limit in a total disconnection between the valuation activity and the fundamental.[2]

While information issues are predominant in Keynes' beauty contest metaphor, there are other circumstances in which the coordination motive may dominate (by becoming less costly than) the fundamental motive. Our second illustration, based on an oligopolistic competition game in which firms may adopt strategies mixing an aggressive Bertrand-like conduct and a compromising Cournot-like conduct, strikingly echoes the disconnection between speculation and enterprise activities that characterize the valuation game. By focusing on the minimum price to beat, or at least to match so as to stay in the market, firms eventually become more interested in conjecturing their competitors' price decisions than in forecasting the fundamental shocks that affect their customers' demand. Industrial Organization applications of beauty contest games have already been proposed by Angeletos and Pavan (2007) or Myatt and Wallace (2012) and Myatt and Wallace (2014). These applications have also contrasted Cournot and Bertrand competition, but they concerned differentiated product markets where quantities are strategic substitutes and prices strategic complements.[3] It is however in the context of homogeneous product markets, the one both Cournot and Bertrand had originally in mind, that the opposition between the two kinds of competition offers the best illustration of the beauty contest parable. Cournot competitors confront a residual demand, affected by fundamental shocks in addition to their rivals'

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[2]Note that our model is not based on the interaction between real and financial sectors. Angeletos et al. (2010) properly model the interaction between these two sectors. In the real sector, entrepreneurs invest in a new technology and base their investment decisions on their expectation of future market valuation of their capital in the financial market. On the financial market, traders observe entrepreneurs' activity as a signal of the profitability of the investment opportunity. The authors show how information spillovers between real and financial sectors amplify higher-order uncertainty and exacerbate the disconnection from fundamentals. By relying on agents' choice about the weight to put on the fundamental vs. the coordination motive, our model is much simpler, but captures the (full) disconnection between fundamentals and activity.

[3]Moreover, contrary to our own approach, these papers view Cournot and Bertrand competition as mutually exclusive regimes, and treat the weight put in the coordination relative to the fundamental motive as determined by the model structure.

output decisions. By contrast, Bertrand competitors set their prices referring exclusively to their rivals' price decisions. We show, in a model where firms may mix their Cournot and Bertrand conducts, and where the probabilities of adopting one or the other are strategic decisions, that these decisions display a strong strategic complementarity leading to the eventual exclusion of the Cournot conduct, and hence to the disconnection from fundamentals.

While the valuation and the competition games, if not isomorphic, exhibit closely related structures, the reasons differ between them for the strong strategic complementarity in agents' choices of the weight to be put in the coordination vs. the fundamental motive. These reasons lie, in the case of the valuation game, in the coordinating role of public (as opposed to private) information, and in the case of the competition game, in the reciprocal nature of aggressive conduct. This suggests that the ultimate result of the beauty contest – the full disconnection of activities from the fundamental – goes beyond informational issues.

Section 2 presents a very simple general formulation of strategic complementarities in choosing to play a coordination game, which captures the disconnection of actions from fundamentals. Sections 3 and 4 respectively develop applications of this general set-up to stock markets in a valuation game and to oligopolistic markets in a competition game. Finally, section 5 concludes the paper.

2 General framework

In this section, following Keynes' beauty contest metaphor of financial markets, we consider that economic agents' decisions are characterized by a dual motive, namely a fundamental and a coordination motive. Each agent chooses how much weight he wishes to attribute to either motive before making a decision that matches the chosen motive combination.

2.1 Utility function

There is a continuum of agents indexed on the unit interval $[0, 1]$, playing a two-stage game. The utility function of individual i has two components, with weights r_i and $1 - r_i$:

$$U(a_i, \bar{a}, \theta; r_i) \equiv (1 - r_i)v_1(a_i, \bar{a}, \theta) + r_i v_2(a_i, \bar{a}). \tag{1}$$

The first component, corresponding to the fundamental motive, is a function of the underlying unknown fundamental state θ, his action a_i and the average action $\bar{a} = \int_j a_j dj$. The second component, corresponding to the coordination motive, is a function of i's action a_i and the average action $\bar{a}$. Both functions v_1 and v_2 are assumed quadratic, concave in a_i (strictly for at least one of them). Agent i chooses $r_i \in [0, 1]$ (the weight he puts on the coordination motive) before making a decision $a_i \in \mathbb{R}$.

2.2 Second stage equilibrium

We solve the model backwards, starting by the second stage and taking r_i as given. The maximization problem of any agent i is:

$$\max_{a_i} \mathbb{E}\left(U\left(a_i, \bar{a}, \theta; r_i\right)\right).$$

Because of the quadratic specification of U, we can rely on certainty equivalents, and obtain the solution to this problem as the best response $\hat{a}_i = \hat{a}\left(\bar{a}, \bar{\theta}, r_i\right)$, depending on the mathematical expectation $\mathbb{E}\left(\theta\right) = \bar{\theta}$, rather than on the whole distribution of θ. The fixed point $\bar{a}^* = \int_i \hat{a}\left(\bar{a}^*, \bar{\theta}, r_i\right) di$ is the equilibrium, denoted $\bar{a}^*\left(\bar{\theta}, \mathbf{r}\right)$, where $\mathbf{r}$ represents the whole distribution of the r_i's.

Using differentiability of functions v_1 and v_2, the first order condition is:

$$\frac{\partial \mathbb{E}(U(.))}{\partial a_i} = 0 \Leftrightarrow (1 - r_i)\frac{\partial \mathbb{E}(v_1(a_i, \bar{a}, \theta))}{\partial a_i} + r_i \frac{\partial \mathbb{E}(v_2(a_i, \bar{a}))}{\partial a_i} = 0. \tag{2}$$

To get equilibrium uniqueness, a sufficient condition is that the slope of the best response $\hat{a}_i\left(\bar{a}, \bar{\theta}, r_i\right)$ with respect to aggregate activity $\bar{a}$ is < 1. As v_1 and v_2 are twice differentiable, the slope of the best response is given by:

$$\frac{\partial \hat{a}_i\left(\cdot\right)}{\partial \bar{a}} = -\frac{U_{a_i \bar{a}}\left(\cdot\right)}{U_{a_i a_i}\left(\cdot\right)} = -\frac{\frac{\partial^2 \left[(1-r_i)\mathbb{E}v_1(\cdot) + r_i \mathbb{E}v_2(\cdot)\right]}{\partial a_i \partial \bar{a}}}{\frac{\partial^2 \left[(1-r_i)\mathbb{E}v_1(\cdot) + r_i \mathbb{E}v_2(\cdot)\right]}{\partial a_i^2}} = -\frac{(1 - r_i)\frac{\partial^2 v_1(\cdot)}{\partial a_i \partial \bar{a}} + r_i \frac{\partial^2 v_2(\cdot)}{\partial a_i \partial \bar{a}}}{(1 - r_i)\frac{\partial^2 v_1(\cdot)}{\partial a_i^2} + r_i \frac{\partial^2 v_2(\cdot)}{\partial a_i^2}}, \tag{3}$$

neglecting the effect of a_i on $\bar{a}$, since there is a continuum of agents.

Thus, a sufficient condition for uniqueness is:

$$-(1 - r_i)\left(\frac{\partial^2 v_1\left(\cdot\right)}{\partial a_i^2} + \frac{\partial^2 v_1\left(\cdot\right)}{\partial a_i \partial \bar{a}}\right) > r_i\left(\frac{\partial^2 v_2\left(\cdot\right)}{\partial a_i^2} + \frac{\partial^2 v_2\left(\cdot\right)}{\partial a_i \partial \bar{a}}\right). \tag{4}$$

2.3 First stage equilibrium

At the first stage, the choice of r_i is the solution to the problem:

$$\max_{r_i \in [0,1]} \mathbb{E}[U(\hat{a}\left(\bar{a}^*, \bar{\theta}, r_i\right), \bar{a}^*\left(\bar{\theta}, r\right), \theta, r_i)],$$

giving the best response for the individual. Now, the derivative of the objective function of this problem with respect to the strategic variable (taking into account the fact that we can rely on certainty equivalents, and that $\bar{a}^*$ is insensitive to a change in r_i) is

$$\frac{\partial U(\hat{a}\left(\bar{a}^*, \bar{\theta}, r_i\right), \bar{a}^*\left(\bar{\theta}, r\right), \bar{\theta}, r_i)}{\partial r_i} = \underbrace{\frac{\partial U\left(\cdot\right)}{\partial a_i}}_{=0} \frac{\partial \hat{a}(\cdot)}{\partial r_i} + \frac{\partial U\left(\cdot\right)}{\partial r_i}$$

$$= -v_1(\hat{a}\left(\bar{a}^*, \bar{\theta}, r_i\right), \bar{a}^*\left(\bar{\theta}, r\right), \bar{\theta}) + v_2(\hat{a}\left(\bar{a}^*, \bar{\theta}, r_i\right), \bar{a}^*\left(\bar{\theta}, r\right)).$$

If $v_2 (\cdot) > v_1 (\cdot)$ for any $r_i \in [0,1]$, that is, if the coordination motive is always more rewarding than the fundamental motive at the second stage equilibrium, the objective function of the individual's problem at the first stage is naturally increasing in r_i, so that we obtain the corner solution $r_i = 1$ to this problem. Moreover, as individuals differ solely through the weights r_i, we obtain a perfect equilibrium with $r_j = 1$ for any j.

2.4 Equilibrium indeterminacy at the second stage

The case where $r_j = 1$ for all j at a perfect equilibrium creates conditions for equilibrium indeterminacy at the second stage, as the fundamental motive becomes fully irrelevant for the individuals' utility, leaving us confronted with a pure coordination game. Consider the uniqueness condition (4). When r_i reaches its limit 1, the condition becomes $\partial^2 v_2 (\cdot) / \partial a_i^2 + \partial^2 v_2 (\cdot) / \partial a_i \partial \bar{a} < 0$, an inequality which can be violated when the actions, as arguments of the function v_2, are strategic complements. This is for instance the case if $\partial^2 v_2 (\cdot) / \partial a_i \partial \bar{a} = -\partial^2 v_2 (\cdot) / \partial a_i^2$, implying that the optimal strategy for agent i is to mimick the average behaviour of the other agents, and leading to a best response curve lying on the first diagonal, hence to a continuum of the second stage equilibria.[4] A striking example of this case is the price matching policy, in the context of Bertrand competition. As such policy may be asymmetric, applying only to downward price movements, we may have to take into account in this case discontinuities of the second derivatives.

3 Information in stock markets: *speculation* vs. *enterprise*

The opposition emphasized by Keynes between *speculation* and *enterprise* can directly be captured by the model of Morris and Shin (2002) (henceforth MS) who describe a version of Keynes' beauty contest parable. MS formulate a *valuation game* in which agents' decisions have to meet both a fundamental and a coordination motive: their decision consists in choosing an action close to the fundamental value and to the conventional value set by the market. However, while the weight agents put on each motive of MS's valuation game is fully exogenous, we consider that it is a strategic variable. We therefore extend MS framework to consider a two-stage game in which agents first choose the weight they attribute to the coordination (and fundamental) motive(s) before making a decision, i.e. choosing a value, that matches the fundamental and/or the conventional value. This model accounts for the potential disconnection between speculation and enterprise in a very simple manner.

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[4]Notice that as long as r_i remains smaller than 1, the uniqueness condition (4) relies also on its LHS: it holds unless strategic complementarity is strong for both functions v_1 and v_2.

3.1 Beauty contest framework

There is a continuum of agents indexed on the unit interval $[0, 1]$. The utility function for individual i has two components. The first component is a standard quadratic loss in the distance between the underlying fundamental value θ and i's chosen value (action) a_i. The second component is the 'beauty contest' term: the loss is increasing in the distance between i's chosen value (action) a_i and the conventional value (average action of the whole population) $\bar{a} = \int_0^1 a_j dj$. The utility function of agent i is given by:

$$u_i(\mathbf{a}, \theta; r_i) = -(1 - r_i)(a_i - \theta)^2 - r_i(L_i - \bar{L}), \tag{5}$$

where $\mathbf{a}$ is the profile of the chosen value (action profile) over all agents, r_i is the weight any agent i decides to put on the beauty contest term, and

$$L_i = \int_0^1 (a_i - a_j)^2 dj, \qquad \bar{L} = \int_0^1 L_j dj. \tag{6}$$

Agent i chooses $r_i \in [0, 1]$ before making a decision $a_i \in \mathbb{R}$. The only difference compared to MS is precisely the addition of a first stage in which agent i chooses $0 \leq r_i \leq 1$.[5] This example is very simply nested in our general framework with $v_1(a_i, \bar{a}, \theta) = -(a_i - \theta)^2$ and $v_2(a_i, \bar{a}) = -(L_i - \bar{L}) = -(a_i - \bar{a})^2 + \int_0^1 (a_j - \bar{a})^2 dj$. Notice that, in addition to the loss $-(a_i - \bar{a})^2$ in the distance between the chosen and the conventional values, reflecting the coordination motive, the function v_2 is increasing in the variance $\int_0^1 (a_j - \bar{a})^2 dj$ of the action profile, reflecting the competitive dimension of stock market activity. Indeed, when engaging in speculation, an individual tries to coordinate with the expected average opinion, but would rather compete with poor predictors of that opinion.

3.2 Timing and information structure

The timing of the game is as follows. First, each agent i chooses r_i: he evaluates which motive he favors to maximize his utility (he somehow chooses 'the game' he wants to play). Second, signals are realized: each agent i receives two signals on the unknown fundamental value θ. All agents receive a public signal with a normally distributed error term: $y = \theta + \eta$, with $\eta \sim N(0, 1/\alpha)$. Each agent additionally receives a private signal: $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim N(0, 1/\beta)$, where the ε_i's are identically and independently distributed across agents and independently distributed from η. We assume that $\alpha > 0$ and $\beta < \infty$, so that the public signal never ceases to be informative and the private signal never becomes fully informative on the fundamental. These assumptions insure that the public signal is always relevant (on the fundamental). Third, each agent i chooses a_i: he evaluates how to combine his information to decide on the value that matches the combination of motives he favored.

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[5]In MS, $r_i = r$ is set exogenously and $0 < r < 1$.

3.3 Second stage equilibrium

We solve the model backwards, starting by the second stage and taking the r_i's as given. The solution to the maximization problem

$$\max_{a_i} \mathbb{E}\left(u_i\left(\mathbf{a}, \theta; r_i\right) \mid x_i, y\right)$$

of any agent i is given by

$$a_i = \frac{1}{\alpha + \beta}\left(\alpha\left(1 + \frac{\beta r_i}{\alpha + \beta(1 - r)}\right)y + \beta\left(1 - \frac{\alpha r_i}{\alpha + \beta(1 - r)}\right)x_i\right) \quad \text{if} \quad r < 1 \qquad (7)$$

and

$$a_i = \frac{1}{\alpha + \beta}((\alpha + \beta r_i)y + \beta(1 - r_i)x_i) + r_i S \quad \text{if} \quad r = 1, \qquad (8)$$

where $r = \int_0^1 r_j dj$ and S is a sunspot. See Appendix A for more details about the derivation.[6]

Thus, the second stage equilibrium is an action profile $\mathbf{a}^*\left(\mathbf{r}\right)$ depending on the profile of the weights r_j's chosen by each agent j at the first stage, which is such that for any $i \in [0, 1]$, the equilibrium value $a_i^*\left(r_i, r\right)$ depends on r_i and on the average r of the weights chosen by all the agents. By (7) and (8), we can distinguish two cases ($r < 1$ and $r = 1$) and formulate the following lemma.

Lemma 1 *While the second-stage equilibrium value $a_i^*\left(r_i, r\right)$ is unique for $0 \leq r < 1$, there is a continuum of equilibria for $r = 1$, with any sunspot S yielding a different equilibrium.*

Proof. Follows directly from equations (7) and (8), derived in Appendix A. □

3.4 First stage equilibrium

To derive the sub-game perfect equilibrium, we maximize $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right), \theta; r_i\right)\right)$ with respect to r_i. Appendix B shows that this function is strictly convex in r_i.[7] Hence, the expected utility is maximized either at $r_i = 0$ or at $r_i = 1$. Appendix B further shows that $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right), \theta; 1\right)\right)$ is always larger than $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right), \theta; 0\right)\right)$, so that the unique sub-game perfect equilibrium is the symmetric equilibrium $r_i^* = r_j^* = 1$ for any i and j.[8] We state this result (by also referring to Lemma 1) in the following proposition.

Proposition 1 *The valuation game has a unique sub-game perfect equilibrium $r_i^* = r^* = 1$ for any i. The corresponding second-stage sub-game admits a continuum of equilibrium actions $a_i^* = y + S$, depending upon a sunspot S.*

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[6]Note that when $r_i = r \neq 1$, we are back to the MS case: $a_i = \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$.

[7]Because we excluded the limit cases where $\alpha = 0$ or $\beta = \infty$.

[8]If $\alpha = 0$ or $\beta = \infty$, there is a continuum of (necessarily) symmetric equilibria with $r_i^* = r_j^* \in [0, 1]$.

While the strength of v_1 compared to that of v_2 is not determined in the general framework of section 2, it is always weaker in this beauty contest example: the loss associated with the fundamental motive exceeds the loss associated with the coordination motive, so that agents choose to play the coordination game rather than the fundamental one or any mixture of the two. Looking at the second stage equilibrium action given by (7) is instructive. When r_i increases, agents put more weight on the public signal y, and the more so the higher the precision of the public signal α/β relative to the one of the private signal.[9] A more accurate public signal makes it easier to know what the others do in the second stage and so to coordinate.[10] In this stock market illustration, it is therefore the information structure that is responsible for the loss associated with the coordination motive to become smaller than the loss associated with the fundamental motive.

A discontinuity is involved in Proposition 1. It operates via the first stage equilibrium as soon as $\alpha/\beta > 0$, making r^* become equal to one at equilibrium, which disconnects the second stage valuations from the fundamental and creates the possibility of sunspots. Although this case where actions are disconnected from fundamentals and valuation rests exclusively on convention seems to be sometimes practically relevant, as argued by Keynes, the literature in the vein of MS has excluded it by assumption, as the debate related to the social value of information becomes then trivial.[11]

4   Competition in oligopolistic markets: Bertrand vs. Cournot

The opposition between the two kinds of competition usually associated with the names of Cournot and Bertrand offers a good illustration of the beauty contest parable, with Cournot competitors moved by the fundamental motive and Bertrand competitors driven by the coordination motive. We shall presently defend this idea by using a model inspired by d'Aspremont and Dos Santos Ferreira (2009).

We consider a market with a continuum of unit mass of identical firms supplying a homogeneous product to a continuum of identical customers. In order to introduce oligopolistic competition in this atomless world, we assume that firms are randomly pairwise matched with identical subsets of customers. More specifically, we refer to the following three-stage *competition game*.

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[9]By contrast, for $\alpha = 0$, when the public signal becomes so noisy that it ceases not only to contain information about the fundamental value but also to play its coordinating role, individuals have to rely on their private information, possibly feeling more concerned with the fundamental motive.

[10]This argument corresponds surprisingly well to Keynes' own words: "*It would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain. It is reasonable, therefore, to be guided to a considerable degree by the facts about which we feel somewhat confident, even though they may be less decisively relevant to the issue than other facts about which our knowledge is vague and scanty*" (1936, p.148).

[11]There is also a discontinuity that results from the relative precision α/β of the public signal becoming equal to zero, which annihilates the advantage of the coordination motive. The weight r^* may then be smaller than one at equilibrium, albeit with no consequences on the equilibrium outcome.

4.1 A three-stage game

At the first stage, each competitor i chooses the probability r_i of an aggressive (Bertrand-like) price-beating conduct to be adopted at the third stage of the game. The complementary probability $1 - r_i$ applies to a compromising (Cournot-like) quantity-abiding conduct. Firms are pairwise matched at the second stage, resulting in a continuum of identical duopolies, in which each firm i posts a *list price* p_i and produces an output q_i. At the third stage, each firm i competes with its competitor j by adopting either the Cournot-like conduct, with probability $1 - r_i$, or the Bertrand-like conduct, with probability r_i. We analyze this game by backward induction.

4.1.1 The third stage

We start with the third stage. When compromising, behaving *à la* Cournot, firm i supplies at *list price* p_i, posted at the previous stage, a quantity exceeding neither the quantity q_i produced at the previous stage, nor the *residual demand*, namely $\max [D (p_i; \theta) - q_j, 0]$, where $D (\cdot; \theta)$ is the (decreasing) demand function $(0, \overline{P}) \rightarrow (0, \infty)$, with $\overline{P} \leq \infty$. The parameter θ is the fundamental, which is known at this stage. By contrast, when aggressive, behaving *à la* Bertrand, firm i undercuts the minimum of the two list prices (its own and the one set by its rival) in a proportion $\varepsilon > 0$ (taken as exogenous). It further supplies at *discount price* $\psi = (1 - \varepsilon) \min [p_i, p_j]$ a quantity at most equal to available output q_i and to what it can actually sell, namely $D (\psi; \theta)$ against a compromising competitor, and $\max [D (\psi; \theta) - q_j, D (\psi; \theta) / 2]$ otherwise.[12] Clearly, since supply entails no cost at this stage, one of these bounds will always be binding at equilibrium. Consumers' rationing is however not excluded *a priori*.

Four states of the world are involved, with two Cournot (CC) or two Bertrand (BB) competitors, or with one Cournot and the other Bertrand (CB and BC). The corresponding payoffs (with, say, π_i^{CB} if i behaves *à la* Cournot and j *à la* Bertrand) are the following:

$$\pi_i^{\text{CC}} (p, q; \theta) = \pi_i^{\text{CB}} (p, q) = p_i \min \{q_i, \max [D (p_i; \theta) - q_j, 0]\}; \tag{9}$$

$$\pi_i^{\text{BB}} (p, q; \theta) = \psi \min \{q_i, \max [D (\psi; \theta) - q_j, D (\psi; \theta) / 2]\}; \tag{10}$$

$$\pi_i^{\text{BC}} (p, q; \theta) = \psi \min \{q_i, D (\psi; \theta)\}. \tag{11}$$

4.1.2 The second stage

We next consider the second stage, at which firm i, knowing θ but not the combination of Cournot and Bertrand competition that will prevail at the third stage, anticipates the

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[12] Aggressive firms are here supposed to be involved in some kind of Bertrand-Edgeworth, rather than pure Bertrand, competition. Also, notice that we are assuming efficient (parallel) rationing in our definition of residual demands.

expected profit

$$\begin{aligned}
\Pi_i\left(p, q, r; \theta\right) \;=\; & \left(1 - r_i\right) p_i \min\left\{q_i, \max\left[D\left(p_i; \theta\right) - q_j, 0\right]\right\} \qquad (12) \\
& + r_i r_j \psi \min\left\{q_i, \max\left[D\left(\psi; \theta\right) - q_j, D\left(\psi; \theta\right)/2\right]\right\} \\
& + r_i\left(1 - r_j\right) \psi \min\left\{q_i, D\left(\psi; \theta\right)\right\} - c q_i.
\end{aligned}$$

When behaving *à la* Cournot, firm i is indifferent as concerns the conduct, aggressive or compromising, of its competitor, hence as concerns the probabilities r_j and $1 - r_j$ of finding itself in each one of the two situations. This is not the case when it behaves *à la* Bertrand. However, it can be shown that *equilibrium* choices of firm i are in fact always independent of probabilities r_j and $1 - r_j$, as stated in the following lemma.

Lemma 2 *At an equilibrium $(p^*, q^*) \in \left(0, \overline{P}\right)^2 \times (0, \infty)^2$ of the duopoly sub-game for $r \in [0, 1)^2$, with positive sales of both firms in all states, the following conditions necessarily hold (for $i = 1, 2$ and $j = 1, 2$, $j \neq i$):*

(i) $p_i^* = p_j^* = P^* = D^{-1}\left(q_i^* + q_j^*; \theta\right)$ and,
(ii) $q_i^* \in \arg\max_{q_i} \widehat{\Pi}_i\left(q_i, r_i, p_j^*, q_j^*; \theta\right)$ if we take $\varepsilon \simeq 0$, with

$$\widehat{\Pi}_i\left(q_i, r_i, p_j^*, q_j^*; \theta\right) \equiv \left(\left(1 - r_i\right) D^{-1}\left(q_i + q_j^*; \theta\right) + r_i \min\left[D^{-1}\left(q_i + q_j^*; \theta\right), p_j^*\right] - c\right) q_i.$$

Proof. See Appendix C. □

Referring to this lemma, we see that we must distinguish upward and downward quantity deviations by firm i from the equilibrium value q_i^*. Upward deviations in q_i will be coupled with downward deviations in $p_i = D^{-1}\left(q_i + q_j^*; \theta\right)$, hence in $\psi \simeq p_i$, so that the first order condition for profit maximization at equilibrium must be formulated, in terms of the right derivative, as

$$\frac{\partial_+ \widehat{\Pi}_i\left(q_i^*, r_i, p_j^*, q_j^*; \theta\right)}{\partial q_i} = P^* - c + \frac{q_i^*}{D'\left(P^*; \theta\right)} \leq 0. \qquad (13)$$

By contrast, downward deviations in q_i will still be coupled with, now upward, deviations in $p_i = D^{-1}\left(q_i + q_j^*; \theta\right)$, but will leave $\psi \simeq p_j^* < p_i$ unaffected. The first order condition, formulated in terms of the left derivative, is consequently

$$\frac{\partial_- \widehat{\Pi}_i\left(q_i^*, r_i, p_j^*, q_j^*; \theta\right)}{\partial q_i} = P^* - c + \frac{\left(1 - r_i\right) q_i^*}{D'\left(P^*; \theta\right)} \geq 0. \qquad (14)$$

We thus obtain the following bounds for the Lerner index of market power at equilibrium:

$$\frac{\left(1 - r_i\right) s_i^*}{-\epsilon D\left(P^*; \theta\right)} \leq \frac{P^* - c}{P^*} \leq \frac{s_i^*}{-\epsilon D\left(P^*; \theta\right)}, \qquad (15)$$

where $-\epsilon D\left(P^*; \theta\right) = -D'\left(P^*; \theta\right) P^*/D\left(P^*; \theta\right)$ is the Marshallian elasticity of demand and

$s_i^* = q_i^* / (q_i^* + q_j^*)$ the market share of firm i. The upper bound is the expression of the Lerner index at Cournot equilibrium. The lower bound is the expression of the Lerner index adopted by the *New Empirical Industrial Organization* (see *e.g.* Bresnahan (1989), and Corts (1999)), with $1 - r_i$ as the so-called *conduct parameter*. Making price and quantity decisions while conjecturing the lowest possible profit margin is equivalent for firm i to maximizing the function

$$\widetilde{\Pi}_i \left(q_i, r_i, p_j^*, q_j^*; \theta \right) \equiv \left((1 - r_i) D^{-1} \left(q_i + q_j^*; \theta \right) + r_i p_j^* - c \right) q_i, \tag{16}$$

which leads to a value of the Lerner index equal to its lower bound. This corresponds to anticipating equilibria such that firm i would rather want to increase the discount price ψ, but is constrained by the list price p_j set by its rival.

4.1.3 The first stage

At the first stage, firm i must decide, before it is matched with another firm and before the value of the fundamental is revealed, the probabilities of aggressive and compromising conduct that it will assume at the third stage. To do that, firm i has to anticipate the payoff expected at the next stage. To keep things tractable, we assume a linear inverse demand function $D^{-1} \left(q_i + q_j; \theta \right) = b + \theta - (q_i + q_j)$, with $b > 0$ and θ an exogenous demand shifter (with zero mathematical expectation), the fundamental.

The first order condition for maximization of $\widetilde{\Pi}_i \left(\cdot, r_i, p_j^*, q_j^*; \theta \right)$ at q_i^*, referring to equation (16) and taking into account Lemma 2(i), which states that $D^{-1} \left(q_i^* + q_j^*; \theta \right) = P^* = p_j^*$, gives us the equilibrium value

$$q_i^* = \frac{P^* - c}{1 - r_i} \equiv (P^* - c) R_i, \tag{17}$$

where $R_i = 1/(1 - r_i) \in [1, \infty]$ is introduced just in order to simplify notations. As $q_j^* = (P^* - c) R_j$, we can compute the equilibrium price P^* that is consistent with the choices R_i and R_j,

$$P^* = b + \theta - (P^* - c)(R_i + R_j) = \frac{b + \theta + c(R_i + R_j)}{1 + R_i + R_j}, \tag{18}$$

and then the corresponding payoff of firm i

$$F(R_i, R_j; \theta) \equiv (P^* - c)^2 R_i = \left(\frac{b + \theta - c}{1 + R_i + R_j} \right)^2 R_i. \tag{19}$$

Differentiating with respect to R_i, we obtain:

$$\frac{\partial F(R_i, R_j; \theta)}{\partial R_i} = \frac{(b + \theta - c)^2}{(1 + R_i + R_j)^3} (1 - R_i + R_j), \tag{20}$$

a derivative that changes sign, from positive to negative, at $R_i = 1 + R_j$, or $r_i = 1/(2 - r_j)$.

Hence, there is a strong strategic complementarity in the choice of the probability of engaging in Bertrand competition: each firm would always want to be fiercer than its opponents. The only possible equilibrium at this stage is consequently symmetric and such that $R_i = \infty$, or $r_i = 1$, for all i.

4.2 Predominance of the coordination motive and equilibrium indeterminacy

The result of uniqueness of the sub-game perfect equilibrium with $r_i = 1$ for all i is quite independent of the fundamental. Of course, we might complete our analysis by taking into account the information problem underlying the formation of expectations about the fundamental θ and about the competitors' average actions $\bar{p}$ and $\bar{q}$, along the lines of the preceding section. We want however to stress that there is more than information considerations underlying the predominance of the coordination motive. In order to allow an interpretation of this phenomenon in the terms of the previous sections, consider the profit function $\widetilde{\Pi}_i$ of firm i, when the competitor's price and quantity are replaced by the corresponding averages:

$$\widetilde{\Pi}_i\left(q_i, r_i, \bar{p}, \bar{q}; \theta\right) \equiv \left((1 - r_i)\left(b + \theta - (q_i + \bar{q})\right) + r_i \bar{p} - c\right) q_i. \tag{21}$$

Here, the profit margin results from the weighted mean of two prices: the demand price $b + \theta - (q_i + \bar{q})$ and the average list price $\bar{p}$. The former, fitting Cournot competition, contains a reference to the fundamental θ, while the latter, fitting Bertrand competition, refers exclusively to the potential competitors' decisions. By Lemma 2(i), we know that any firm's price and quantity decisions are linked at equilibrium by the relation $p_j^* = b + \theta - \left(q_{-j}^* + q_j^*\right)$, so that we can reduce to the single variable $\bar{q}$ the average decision made by the competitors, using the constraint $\bar{p} = b + \mathbb{E}\left(\theta\right) - 2\bar{q} = b - 2\bar{q}$. We thus obtain the payoff

$$U_i\left(q_i, r_i, \bar{q}, \theta\right) = (1 - r_i)\underbrace{\left(b - c + \theta - (q_i + \bar{q})\right) q_i}_{v_1(q_i, \bar{q}, \theta)} + r_i \underbrace{\left(b - c - 2\bar{q}\right) q_i}_{v_2(q_i, \bar{q})}. \tag{22}$$

If we consider the expected payoff (with $\mathbb{E}\left(\theta\right) = 0$), we see that $v_2\left(q_i, \bar{q}\right) > v_1\left(q_i, \bar{q}, 0\right)$, creating an incentive to increase r_i, as soon as $q_i > \bar{q}$, a condition that has the more chances to be satisfied the larger r_i.

The strong strategic complementarity in the choice by each firm i of the probability r_i of adopting an aggressive Bertrand-like conduct leads in the limit to the exclusion of a compromising Cournot-like conduct and, by the same token, of any influence of the fundamental on the choice of price strategies.[13] This strategic complementarity is self-destructive since Bertrand profits, to be expected in the limit, are zero.

There is however a discontinuity at the limit, since second stage equilibria for any pair (i, j) of firms, conditional on $r_i = r_j = 1$, are indeterminate, as stated in the following

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[13]Of course, the choice of quantity strategies $q_i = q_j = D\left(\psi; \theta\right)/2$ is still dependent on the fundamental.

lemma.

Lemma 3 *If $\varepsilon \simeq 0$, the equilibria $(p^*, q^*) \in \left(0, \overline{P}\right)^2 \times (0, \infty)^2$ of the duopoly sub-game when both firms i and j adopt with certainty an aggressive, Bertrand-like, conduct ($r_i = r_j = 1$) are symmetric and indeterminate, such that the price can take any value between its Bertrand and Cournot equilibrium prices.*

Proof. See Appendix D. □

We are now ready to formulate the following proposition, synthesizing our preceding analysis.

Proposition 2 *The competition game has a unique sub-game perfect equilibrium $r_i^* = r^* = 1$ for any i. Each one of the corresponding duopoly sub-games admits a continuum of equilibrium prices $P^* \in [c, (b + 2c)/3]$, between Bertrand and Cournot prices.*

5 Concluding remarks

Although inherent to Keynes' beauty contest metaphor, the idea that participants to financial markets exhibit a common interest in coordination *per se* has not yet received sufficient attention. The main contribution of this paper is to approach as strategic decisions the weights put by those participants in the coordination (rather than the fundamental) motive (in *speculation* rather than *enterprise*, to employ Keynes' own terms). These strategic decisions display a strong strategic complementarity that ends up in the complete dominance of the coordination motive, evicting the fundamental motive and hence resulting in a disconnection of market activity from fundamentals. Such disconnection opens the door to indeterminacy and the emergence of sunspots.

Two illustrations are provided: a stock market example formalized as a valuation game and an oligopolistic market example formalized as a competition game. Whereas both illustrations have in common the same beauty contest structure, the reasons for the coordination motive to dominate the fundamental motive differ. While it is the public nature of information that generates a strong strategic complementarity in the valuation game, it is the reciprocal nature of competitive aggressiveness that drives the same result in the competition game.

References

Angeletos, G.-M., Lorenzoni, G., and Pavan, A. (2010). Beauty contests and irrational exuberance: a neoclassical approach. *mimeo*.

Angeletos, G.-M. and Pavan, A. (2007). Efficient use of information and social value of information. *Econometrica*, 75:1103–1142.

Bresnahan, T. (1989). Empirical studies of industries with market power. In *R. Schmalensee and R.D. Willig (eds.), Handbook of Industrial Organization*, volume II. Elsevier B.V., pages 1011–1057.

Corts, K. (1999). Conduct parameters and the measurement of market power. *Journal of Econometrics*, 88:227–250.

d'Aspremont, C. and Dos Santos Ferreira, R. (2009). Price-quantity competition with varying toughness. *Games and Economic Behavior*, 65:62–82.

Keynes, J. M. (1936). Chap. 12: The state of long-term expectation. In *The General Theory of Employment, Interest, and Money*.

Morris, S. and Shin, H. S. (2002). Social value of public information. *American Economic Review*, 92:1521–1534.

Myatt, D. P. and Wallace, C. (2012). Endogenous information acquisition in coordination games. *Review of Economic Studies*, 79:340–374.

Myatt, D. P. and Wallace, C. (2014). Cournot competition and the social value of information. *Journal of Economic Theory*, forthcoming.

6 Appendix

6.1 Appendix A - Derivation of the second stage equilibrium of the valuation game

The maximization problem of any agent i is: $\max_{a_i} \mathbb{E}\left(u_i\left(\mathbf{a}, \theta; r_i\right) \mid x_i, y\right)$. The first order condition yields

$$a_i = (1 - r_i)\, \mathbb{E}\left(\theta \mid x_i, y\right) + r_i \mathbb{E}\left(\overline{a} \mid x_i, y\right), \tag{23}$$

where $\mathbb{E}(.|x_i, y) = \mathbb{E}_i(.)$ is the expectation operator conditional on the signals received. To determine the optimal action (23), we need the expressions of $\mathbb{E}_i(\theta) = (\alpha y + \beta x_i)/(\alpha + \beta)$ and $\mathbb{E}_i(\overline{a})$. To derive $\mathbb{E}_i(\overline{a})$, following MS, we assume that any other agent j follows the same linear strategy: $a_j = (1 - \kappa_j)\, y + \kappa_j x_j + \lambda_j S$, where S is a sunspot.[14] We write $\kappa = \int_0^1 \kappa_j dj$ and $\lambda = \int_0^1 \lambda_j dj$, so that

$$
\begin{aligned}
\mathbb{E}\left(\overline{a} \mid x_i, y\right) &= (1 - \kappa)y + \kappa\mathbb{E}_i(\theta) + \lambda S \\
&= \left(1 - \frac{\kappa\beta}{\alpha + \beta}\right)y + \frac{\kappa\beta}{\alpha + \beta}x_i + \lambda S.
\end{aligned}
\tag{24}
$$

---

[14] As we allow for $r = 1$, contrary to MS, we specify a linear rule that includes sunspots.

Inserting (24) in (23), the optimal action writes:

$$
\begin{aligned}
a_i &= (1 - r_i)\,\mathbb{E}_i(\theta) + r_i\left((1 - \kappa)y + \kappa\mathbb{E}_i(\theta) + \lambda S\right) \\
&= \left(1 - \frac{\beta(1 - r_i + r_i\kappa)}{\alpha + \beta}\right)y + \frac{\beta(1 - r_i + r_i\kappa)}{\alpha + \beta}x_i + r_i\lambda S.
\end{aligned}
$$

Identifying coefficients κ and λ, we obtain $\kappa = \frac{\beta(1 - r + r\kappa)}{\alpha + \beta} \implies \kappa = \frac{\beta(1-r)}{\alpha + \beta(1-r)}$ and $\lambda = r\lambda \implies$ $\lambda = 0$ or $r = 1$. Plugging the expression of κ into (24) yields:

$$
\mathbb{E}\left(\bar{a} \mid x_i, y\right) = \left(1 - \frac{\beta^2(1-r)}{(\alpha + \beta)(\alpha + \beta(1-r))}\right)y + \frac{\beta^2(1-r)}{(\alpha + \beta)(\alpha + \beta(1-r))}x_i + \lambda S. \qquad (25)
$$

Using (25) to re-write (23) yields:

$$
\begin{aligned}
a_i &= (1 - r_i)\frac{\alpha y + \beta x_i}{\alpha + \beta} + r_i\left(\left(1 - \frac{\beta^2(1-r)}{(\alpha + \beta)(\alpha + \beta(1-r))}\right)y + \frac{\beta^2(1-r)}{(\alpha + \beta)(\alpha + \beta(1-r))}x_i\right) \\
&= \gamma_i y + (1 - \gamma_i)\,x_i + r_i\lambda S, \text{ with } \gamma_i \equiv \frac{\alpha}{\alpha + \beta}\left(1 + \frac{\beta r_i}{\alpha + \beta(1-r)}\right).
\end{aligned}
$$

We now distinguish two cases depending on whether $r = 1$ or $r < 1$. If $r < 1$ (implying $\lambda = 0$),

$$
a_i = \frac{1}{\alpha + \beta}\left(\alpha\left(1 + \frac{\beta r_i}{\alpha + \beta(1-r)}\right)y + \beta\left(1 - \frac{\alpha r_i}{\alpha + \beta(1-r)}\right)x_i\right).
$$

If $r = 1$ and taking $\lambda = 1$ (without loss of generality),

$$
a_i = \frac{1}{\alpha + \beta}((\alpha + \beta r_i)y + \beta(1 - r_i)x_i) + r_i S.
$$

6.2 Appendix B - Derivation of the first stage equilibrium and proof of Proposition 1

To derive the sub-game perfect equilibrium, we maximize $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right), \theta; r_i\right)\right)$ with respect to r_i. More explicitly, we take the utility function (5) of agent i

$$
u_i(\mathbf{a}, \theta; r_i) = -(1 - r_i)(a_i - \theta)^2 - r_i(a_i - \bar{a})^2 + r_i\int_0^1 (a_j - \bar{a})^2 dj
$$

and the second stage equilibrium value for $r < 1$

$$
a_i^*\left(r_i, r\right) = \gamma_i y + (1 - \gamma_i)\,x_i, \text{ with } \gamma_i \equiv \frac{\alpha}{\alpha + \beta}\left(1 + \frac{\beta r_i}{\alpha + \beta(1-r)}\right).
$$

Thus, the expected utility

$$\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right) = -\left(1-r_i\right)\mathbb{E}\left(a_i^*\left(r_i,r\right)-\theta\right)^2 - r_i\mathbb{E}\left(a_i^*\left(r_i,r\right)-\overline{a}\right)^2$$
$$+r_i\mathbb{E}\left(\int_0^1\left(a_j^*\left(r_j,r\right)-\overline{a}\right)^2 dj\right)$$

has a derivative with respect to r_i:

$$\frac{\partial\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right)}{\partial r_i} = \mathbb{E}\left(a_i^*\left(r_i,r\right)-\theta\right)^2 - \mathbb{E}\left(a_i^*\left(r_i,r\right)-\overline{a}\right)^2 + \mathbb{E}\left(\int_0^1\left(a_j^*\left(r,r\right)-\overline{a}\right)^2 dj\right)$$
$$-2\underbrace{\left(\left(1-r_i\right)\mathbb{E}\left(a_i^*\left(r_i,r\right)-\theta\right)+r_i\mathbb{E}\left(a_i^*\left(r_i,r\right)-\overline{a}\right)\right)}_{=0}\frac{\partial a_i^*\left(r_i,r\right)}{\partial r_i}.$$

To determine the sign of this derivative we need to calculate $\mathbb{E}\left(a_i^*\left(r_i,r\right)-\theta\right)^2$ and $\mathbb{E}\left(a_i^*\left(r_i,r\right)-\overline{a}\right)^2$:

$$\mathbb{E}\left(a_i^*\left(r_i,r\right)-\theta\right)^2 = \mathbb{E}\left(\gamma_i\underbrace{\left(y-\theta\right)}_{\eta}+\left(1-\gamma_i\right)\underbrace{\left(x_i-\theta\right)}_{\varepsilon_i}\right)^2 = \gamma_i^2\frac{1}{\alpha}+\left(1-\gamma_i\right)^2\frac{1}{\beta}$$
$$= \frac{\left(\alpha+\beta\left(1-r\right)\right)^2+\alpha\beta r_i^2}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)^2},$$

$$\mathbb{E}\left(a_i^*\left(r_i,r\right)-\overline{a}\right)^2 = \mathbb{E}\left(\begin{array}{c}\frac{1}{\alpha+\beta}\left(\alpha\left(1+\frac{\beta r_i}{\alpha+\beta\left(1-r\right)}\right)y+\beta\left(1-\frac{\alpha r_i}{\alpha+\beta\left(1-r\right)}\right)x_i\right)\\ -\frac{1}{\alpha+\beta}\left(\alpha\left(1+\frac{\beta r}{\alpha+\beta\left(1-r\right)}\right)y+\beta\left(1-\frac{\alpha r}{\alpha+\beta\left(1-r\right)}\right)\theta-\frac{\alpha\beta\int_0^1 r_j\varepsilon_j dj}{\alpha+\beta\left(1-r\right)}\right)\end{array}\right)^2$$

$$= \left(\frac{\alpha\beta}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)}\right)^2\mathbb{E}\left(\left(r_i-r\right)\eta+\left(\frac{\alpha+\beta\left(1-r\right)-\alpha r_i}{\alpha}\right)\varepsilon_i-\int_0^1 r_j\varepsilon_j dj\right)^2$$

$$= \left(\frac{\alpha\beta}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)}\right)^2\left(\frac{\alpha\beta\left(r_i-r\right)^2+\left(\alpha+\beta\left(1-r\right)-\alpha r_i\right)^2}{\alpha^2\beta}+\mathbb{E}\left(\left(\int_0^1 r_j\varepsilon_j dj\right)^2\right)\right).$$

By putting together the different terms of $\partial\mathbb{E}\left(u_i\left(\cdot\right)\right)/\partial r_i$, we obtain after a straightforward computation:

$$\frac{\partial\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right)}{\partial r_i} = \alpha\frac{\alpha+\beta+2\beta\left(r_i-r\right)}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)^2}-\left(\frac{\alpha\beta}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)}\right)^2\mathbb{E}\left(\int_0^1 r_j\varepsilon_j dj\right)^2$$
$$+\mathbb{E}\left(\int_0^1\left(a_j^*\left(r,r\right)-\overline{a}\right)^2 dj\right).$$

This derivative is increasing in r_i, so that the expected utility $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right)$ is strictly convex in r_i. Its maximum can only be attained if $r_i=0$ or if $r_i=1$. The condition for $r_i=1$ to maximize the expected utility is $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;1\right)\right)-\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;0\right)\right)>0$, that

16

is,

$$\mathbb{E}\left(a_i^*\left(0,r\right)-\theta\right)^2 - \mathbb{E}\left(a_i^*\left(1,r\right)-\overline{a}\right)^2 + \mathbb{E}\left(\int_0^1 \left(a_j^*\left(r_j,r\right)-\overline{a}\right)^2 dj\right)$$

$$= \frac{\alpha\left(\alpha+2\beta\left(1-r\right)\right)}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)^2} + \mathbb{E}\left(\int_0^1 \left(a_j^*\left(r_j,r\right)-\overline{a}\right)^2 dj\right)$$

$$- \left(\frac{\alpha\beta}{\left(\alpha+\beta\right)\left(\alpha+\beta\left(1-r\right)\right)}\right)^2 \mathbb{E}\left(\int_0^1 r_j\varepsilon_j dj\right)^2 > 0.$$

When the profile of other agents' strategies is symmetric ($r_j = r$ for $j \neq i$), the terms $\mathbb{E}\left(\int_0^1 \left(a_j^*\left(r,r\right)-\overline{a}\right)^2 dj\right)$ and $\int_0^1 r_j\varepsilon_j dj = r\int_0^1 \varepsilon_j dj$ are equal to zero, so that the preceding condition is satisfied.[15] There is a unique sub-game perfect symmetric equilibrium, with $r = 1$.

Can we obtain other sub-game perfect equilibria, which are asymmetric? First, observe that it is still true that the expected utility $\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;r_i\right)\right)$ is strictly convex in r_i, so that asymmetric equilibria must be such that there is a subset of individuals, say $[0,r] \subset [0,1]$, which choose the weight $r_j = 1$, the complementary subset $[r,1]$ choosing $r_j = 0$ ($\int_0^1 r_j dj = \int_0^r dj = r$). Then, $\mathbb{E}\left(\int_0^1 r_j\varepsilon_j dj\right)^2 = \mathbb{E}\left(\int_0^r \varepsilon_j dj\right)^2 \leq \mathbb{E}\left(r\int_0^r \varepsilon_j^2 dj\right) = r\int_0^r \mathbb{E}\left(\varepsilon_j^2\right) dj = r^2/\beta$, using the Cauchy-Schwarz inequality and the independency of the ε_j's. As a consequence, given the positivity of the variance $\mathbb{E}\left(\int_0^1 \left(a_j^*\left(r_j,r\right)-\overline{a}\right)^2 dj\right)$ when equilibria are asymmetric, we obtain from (26):

$$\mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;1\right)\right) - \mathbb{E}\left(u_i\left(\mathbf{a}^*\left(\mathbf{r}\right),\theta;0\right)\right) > \frac{\alpha\left(\left(\alpha+\beta\right)\left(\alpha+2\beta\left(1-r\right)\right)-\alpha\beta r^2\right)}{\left(\alpha+\beta\right)^2\left(\alpha+\beta\left(1-r\right)\right)^2}$$

$$\geq \frac{\alpha^3}{\left(\alpha+\beta\right)^2\left(\alpha+\beta\left(1-r\right)\right)^2} > 0.$$

So, no individual i would ever choose $r_i = 0$.[16]

6.3 Appendix C - Proof of Lemma 2

As Π_i is piecewise linear in q_i, we obtain, at an equilibrium (p^*, q^*) with positive sales for both firms in all the states, $q_i^* = D\left(p_i^*;\theta\right) - q_j^*$ or $q_i^* = \max\left[D\left(\psi^*;\theta\right) - q_j^*, D\left(\psi^*;\theta\right)/2\right]$.[17] The case $q_i^* = D\left(\psi^*;\theta\right)$ is excluded, since it implies zero sales for a firm j behaving *à la* Cournot.

We first show by contradiction that equilibrium list prices cannot be different. Suppose, say, that $p_j^* < p_i^*$. There are two possible cases.

---

[15]Except in the limit cases where $\alpha = 0$ (the public signal ceases to be informative) or $\beta = \infty$ (the private signal becomes fully informative on the fundamental). In these cases, since $\partial\mathbb{E}\left(u_i\left(\cdot\right)\right)/\partial r_i = 0$, individual i is indifferent about the choice of r_i and there is a continuum of sub-game perfect symmetric equilibria with $r \in [0,1]$.

[16]Again except in the extreme cases where $\alpha = 0$ or $\beta = \infty$ and when facing a symmetric profile of the r_j's.

[17]We suppose that, when locally indifferent as concerns its output decision, each firm chooses the maximum quantity that can be sold.

Case 1: $q_i^* = D\left(p_i^*;\theta\right) - q_j^*$. This case arises if $r_i\psi^* = r_i\left(1-\varepsilon\right)p_j^* < c$, since firm i would not want to choose a costly q_i^* exceeding the residual demand (what it can sell), nor to choose a price p_i^* low enough to induce a positive excess demand. But exactly the same argument applies to firm j, which leads to the contradiction $D\left(p_i^*,\theta\right) = q_i^* + q_j^* = D\left(p_j^*;\theta\right)$, unless $r_j\psi^* > c$, leading to $q_j^* \geq D\left(\psi^*;\theta\right) - q_i^* = D\left(\psi^*;\theta\right) - D\left(p_i^*;\theta\right) + q_j^*$. This implies $D\left(p_i^*;\theta\right) \geq D\left(\psi^*;\theta\right)$, contradicting $\left(1-\varepsilon\right)p_j^* < p_i^*$. We can consequently discard Case 1.

Case 2: $\max\left[D\left(\psi^*;\theta\right) - q_j^*, D\left(\psi^*;\theta\right)/2\right]$. This arises if $r_i\psi^* = r_i\left(1-\varepsilon\right)p_j^* \geq c$. So, $q_i^* \geq D\left(\psi^*;\theta\right) - q_j^* > D\left(p_j^*;\theta\right) - q_j^*$. This is excluded if $r_j\psi^* < c$, which would lead to $q_i^* = D\left(p_j^*;\theta\right) - q_j^*$ by the argument for Case 1. Thus, $r_j\psi^* \geq c$, and we end up with $q_i^* = q_j^* = D\left(\psi^*;\theta\right)/2 = D\left(\psi^*;\theta\right) - q_j^* > D\left(p_i^*;\theta\right) - q_j^*$. So, firm i, when behaving *à la* Cournot, would not be able to sell all of its output. This situation would trigger the choice of a lower price $p_i \in \left[p_j^*, p_i^*\right)$, unless $p_i^* = \arg\max_{p_i}\left[p_i\left(D\left(p_i;\theta\right) - q_j^*\right)\right]$. But, if this is the case and since $q_i^* = q_j^*$, firm j would want to increase its price. We can consequently also discard Case 2.

Therefore, both cases being discarded, the equality $p_i^* = p_j^* = P^*$ is proved. Two cases must again be considered in order to show that $P^* = D^{-1}\left(q_i^* + q_j^*;\theta\right)$ and to prove (ii).

Case 1: $q_i^* = D\left(P^*;\theta\right) - q_j^*$. This case results from $r_i\psi^* = r_i\left(1-\varepsilon\right)P^* < c$. As $p_i^* = P^* = D^{-1}\left(q_i^* + q_j^*;\theta\right)$, the maximization in $\left(p_i, q_i\right)$ of the payoff function $\Pi_i\left(p_i, p_j^*, q_i, q_j^*, r_i, r_j;\theta\right)$ implies the maximization in the sole strategy variable q_i of the objective function

$$
\begin{aligned}
&\Pi_i^1\left(q_i, q_j^*, p_j^*, r_i;\theta,\varepsilon\right) \equiv \\
&\left(\left(1-r_i\right)D^{-1}\left(q_i + q_j^*;\theta\right) + r_i\left(1-\varepsilon\right)\min\left[D^{-1}\left(q_i + q_j^*;\theta\right), p_j^*\right] - c\right)q_i, \quad (26)
\end{aligned}
$$

which depends on r_j only through the equilibrium values $\left(p_j^*, q_j^*\right)$.

Case 2: $q_i^* = \max\left[D\left(\psi^*;\theta\right) - q_j^*, D\left(\psi^*;\theta\right)/2\right]$. This case results from $r_i\psi^* = r_i\left(1-\varepsilon\right)P^* \geq c$. The subcase in which $r_j\psi^* < c$ is excluded by the above argument: firm j's choice would lead to $q_i^* = D\left(P^*;\theta\right) - q_j^* < D\left(\psi^*;\theta\right) - q_j^*$, contradicting $q_i^* \geq D\left(\psi^*;\theta\right) - q_j^*$. Hence, we must have $q_i^* = q_j^* = D\left(\psi^*;\theta\right)/2$, hence $p_i^* = p_j^* = P^* = D^{-1}\left(q_i^* + q_j^*;\theta\right)/\left(1-\varepsilon\right)$. Firm i would not be able to manipulate the price ψ^* by choosing $p_i > p_j^*$, but it can undercut p_j^* by choosing a price $p_i = D^{-1}\left(q_i + q_j^*;\theta\right)/\left(1-\varepsilon\right)$, with $q_i > q_i^*$. We thus obtain a variant of the preceding objective function, namely

$$
\begin{aligned}
&\Pi_i^2\left(q_i, q_j^*, p_j^*, r_i;\theta,\varepsilon\right) \equiv \\
&\left(\left(1-r_i\right)\frac{D^{-1}\left(q_i + q_j^*;\theta\right)}{1-\varepsilon} + r_i\left(1-\varepsilon\right)\min\left[\frac{D^{-1}\left(q_i + q_j^*;\theta\right)}{1-\varepsilon}, p_j^*\right] - c\right)q_i, \quad (27)
\end{aligned}
$$

to be maximized in its sole variable q_i.

Taking the limit case where $\varepsilon \simeq 0$, we finally obtain:

$$
\Pi_i^1\left(q_i, q_j^*, p_j^*, r_i;\theta, 0\right) = \Pi_i^2\left(q_i, q_j^*, p_j^*, r_i;\theta, 0\right) = \widehat{\Pi}_i\left(q_i, r_i, p_j^*, q_j^*;\theta\right), \quad (28)
$$

completing the proof.

6.4 Appendix D - Proof of Lemma 3

By equation (12), the payoff function of firm i (with $r_i = r_j = 1$ and $\varepsilon \simeq 0$) is

$$\Pi_i\left(p, q, 1, 1; \theta\right) = \psi \min\left\{q_i, \max\left[D\left(\psi; \theta\right) - q_j, D\left(\psi; \theta\right)/2\right]\right\} - cq_i, \qquad (29)$$

where $\psi \simeq \min\left[p_i, p_j\right]$. Take an equilibrium (p^*, q^*) with positive outputs. Clearly, $\psi^* \geq c$. Also, if $q_i^* = D\left(\psi^*; \theta\right) - q_j^* > D\left(\psi^*; \theta\right)/2$ and $q_j^* = D\left(\psi^*; \theta\right) - q_i^* > D\left(\psi^*; \theta\right)/2$ we obtain a contradiction by adding the two inequalities, the sense of which can be reversed with the same consequence. As $q_i^* = D\left(\psi^*; \theta\right) - q_j^*$ is equivalent to $q_j^* = D\left(\psi^*; \theta\right) - q_i^*$, we are left with $q_i^* = D\left(\psi^*; \theta\right)/2 = q_j^*$ as the sole possibility. So, the equilibrium is symmetric in quantities.

Observe that Π_i is constant in p_i as soon as $p_i > p_j$. Thus, potential equilibrium quantity configurations sharing demand equally can only be destroyed unilaterally by downward price deviations from equilibrium price ψ^*. For such deviations, $D\left(\psi; \theta\right) - q_j^* > D\left(\psi; \theta\right)/2$ (an inequality equivalent to $D\left(\psi; \theta\right) > D\left(\psi^*; \theta\right)$, since $q_j^* = D\left(\psi^*; \theta\right)/2$). Hence, the profit of firm i to be maximized in p_i is $\pi_i = \left(p_i - c\right)\left[D\left(p_i; \theta\right) - q_j^*\right]$. If p_i^* maximizes this function, and p_j^* the corresponding function for $q_i^* = q_j^*$ (hence $p_j^* = p_i^* \simeq \psi^*$), we obtain the Cournot equilibrium. Any price ψ^* higher than the Cournot price will trigger a downward price deviation. Of course, if ψ^* is lower than the Cournot price, any firm would like to increase the price but is unable to manipulate it upwards if $p_j^* = p_i^*$ (asymmetric price configurations cannot be sustained). We thus have a continuum of symmetric equilibria, with P^* between the Bertrand price c and the Cournot price (such that $\left(\psi^* - c\right)/\psi^* = 1/2\epsilon D\left(\psi^*; \theta\right)$), and outputs sharing equally the demand $D\left(\psi^*\right)$.