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## Models of currency crises with self-fulfilling features: A comment

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**Abstract:** Much attention has been paid to models of currency crisis with self-fulfilling features and the concept of multiple equilibria developed in the 1990s. They aim at explaining currency crisis without apparent fundamental disequilibrium. They are also useful to render account for currency crisis unpredictability. This paper re-examines an illustrative model of Obstfeld (1996), in which high unemployment may cause an exchange-rate crisis with self-fulfilling features. By completing the algebraic demonstration, this paper shows that there are less equilibria than conjectured.

Keywords: Self-fulfilling currency crisis; Fixed exchange rate; Multiple equilibria.

JEL classification numbers: F33; E58.

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## 1. Introduction

There was a largely shared perception that neither in the Exchange Rate Mechanism (ERM) nor in Mexico, the currency crises were predictable based on criteria developed by the first generation of currency crisis models (Krugman 1979, and Flood and Garber 1984).

To reflect this perception, models of currency crisis with self-fulfilling features were notably developed by Obstfeld (1994, 1996, 1997) and Sachs *et al.* (1996) among others. In these models, currency crises are often explained as a result of the incompatibility between accommodative domestic fiscal and monetary policy and fixed exchange rate under high degree of capital mobility.

These models of the second generation are generally characterized by the existence of multiple equilibria, hence making market expectations determinant in instigating successful currency attacks without that the fundamentals are particularly deteriorated.<sup>1</sup> While not being able to explain the ulterior Asian currency and financial crises, they have popularised the concept of self-fulfilling multiple equilibria which is intellectually attractive for many researchers who work on currency crisis.

The attractiveness of models of currency crisis with self-fulfilling features is that they allow establishing the similitude between a run on a bank and speculation against a currency. They both create objective economic conditions that make bankruptcy or liability devaluation more likely. As a result, even pegged exchange rates that could be sustained indefinitely in the absence of a speculative attack can succumb to adverse market sentiment.

Some currency crisis models of the third generation are strongly influenced by these models of currency crisis with self-fulfilling features and have also generated multiple

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<sup>1</sup> See Masson (2001) for a survey.

equilibria thanks to intuitive approach or static treatment of dynamic model (see, e.g., Krugman 1999, and Aghion *et al.* 2000).<sup>2</sup>

However, the multiple equilibrium solutions are not robust to the introduction of small degree of uncertainty in the information about fundamentals (Morris and Shin, 1998)<sup>3</sup> or opacity about the central bank's preferences (Dai, 2009a).

Without introducing new assumptions, the present paper will show that the existence of three equilibria conjectured by Obstfeld (1996) is not robust to rigorous algebraic demonstration.

The model of Obstfeld aims at giving an example where the underlying macroeconomic 'fundamentals' are far from irrelevant to the outcome of speculative attack, since they can determine the range of possible equilibria. The existence of multiple equilibria provides an economic foundation for the coordination game of currency-market traders who do not neglect the fact that the changing macroeconomic fundamentals alter the degree of discomfort a government will suffer because of an attack.

Notably, Obstfeld wants to show that only extreme values of fundamentals are either consistent with long-run fixity of the exchange rate or are not. There is also a large middle ground over which fundamentals are neither so strong as to make a successful attack impossible, nor so weak as to make it inevitable. In this case speculators may or may not coordinate on an attack equilibrium. Unfortunately, the solution of the model is not completely carried out by Obstfeld and the results are based on conjecture and rhetoric rather than on sound economic analysis.

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<sup>2</sup> Even if the model of Krugman (1999) allows grabbing quite easily the causes of 1997's Asian currency and financial crises, his intuitive and static approach of a dynamic model with balance-sheet effect has yielded a misleading result, i.e. there are three equilibria with the lowest equilibrium corresponding to zero investment. As it is shown by Dai (2009b), there is not always a lowest equilibrium with zero investment.

<sup>3</sup> See Allegret and Cornand (2006) for a survey of the literature about the pros and cons of information transparency on the exchange rate market and their effect on the occurrence of speculative attacks.

I will show in the following that there is not such middle ground, using exactly the same model and same assumptions. I will demonstrate that there are only two equilibria instead of three. One of these two equilibria is the equilibrium under free float and the other could be a devaluation or a revaluation equilibrium according to the values of parameters characterising the economy, the preference of the government as well as the costs attached to devaluation or revaluation.

In the next section, I present the model used by Obstfeld as well as his algebraic and graphic solutions. In section 3, by completing the algebraic demonstration, I prove that the intermediate equilibrium does not exist. I conclude in the final section.

## 2. The model and the multiple equilibria solution of Obstfeld

The fully-articulated model of currency crisis analysed by Obstfeld (1996) is based on Obstfeld (1994, 1997), in which the government's objectives are explicitly spelled out. In this model, Obstfeld has shown that there are three equilibria which can be ranked by the degree of market scepticism in the current exchange rate, and the consequent worsening of employment conditional on the current parity's maintenance. Collapse may be a sure thing, but it needs not be: different equilibria entail different *probabilities* of collapse. The model's basic framework is drawn from Barro and Gordon (1983), but assumes an open economy and identifies the price of foreign currency with the domestic price level.

The government minimizes the loss function:

$$L = (y - y^*)^2 + \beta \varepsilon^2 + C(\varepsilon), \quad (1)$$

where  $y$  is output,  $y^*$  the government's output target,  $\varepsilon = e - e_{-1}$  the change in the exchange rate ( $e$ , the price of foreign currency expressed in terms of domestic currency),  $C(\varepsilon)$  a cost function which is zero when the fix parity is maintained and positive (negative) if the national

currency is devaluated (revaluated). Variables represented by lower-case Roman letters are expressed in terms of natural logarithms. Output is determined by the expectations-augmented Phillips curve:

$$y = \bar{y} + \alpha(\varepsilon - \varepsilon^e) + u, \quad (2)$$

where  $\bar{y}$  is the ‘natural’ output level,  $\varepsilon^e$  is domestic price-setters’ expectation of  $\varepsilon$  based on *lagged* information, and  $u$  is an i.i.d. mean-zero shock. The assumption  $y^* > \bar{y}$  causes a dynamic inconsistency problem which, aside from being needed for multiple equilibria, provides a reason why a rational government might try to tie its hands by submitting to exchange-rate realignment costs. The expected variation of the exchange rate  $\varepsilon^e$  is taken to be time-invariant, which is the case in equilibrium given the model’s assumptions.

The government chooses the exchange rate  $e$  after observing  $u$  (unlike private price-setters), but any upward change in the rate (devaluation) leads to  $C(\varepsilon) = \bar{c}$  in (1), whereas downward changes (revaluation) cost the government  $C(\varepsilon) = \underline{c}$ .<sup>4</sup>

A government which abandons the fixed exchange rate regime will face two kinds of loss, the loss due to variations of output and inflation around of their respective target under the flexible exchange rate regime and the credibility cost due to the abandon of the fixed exchange rate regime. Consider first the loss due to variations of output and inflation under flexible exchange rate regime, so that we can ignore the term  $C(\varepsilon)$  in (1). The government minimises the loss function (1) with  $C(\varepsilon)$  being set at zero subject to the constraint given by equation (2). Taking  $\varepsilon^e$  as predetermined, the government chooses:

$$\varepsilon = \frac{\alpha(y^* - \bar{y} + u) + \alpha^2 \varepsilon^e}{\alpha^2 + \beta}, \quad (3)$$

achieving an output level of

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<sup>4</sup> The fixed cost of a parity change could be a political cost due to reneging on a promise to fix the exchange rate (e.g., an ERM commitment).

$$y = \bar{y} + \frac{\alpha^2(y^* - \bar{y}) - \beta u - \alpha\beta\varepsilon^e}{\alpha^2 + \beta},$$

and a policy loss of

$$L^{FLEX} = \frac{\beta}{\alpha^2 + \beta} (y^* - \bar{y} + u + \alpha\varepsilon^e)^2.$$

If the government chooses to maintain the fixed exchange rate, the loss instead is:

$$L^{FIX} = (y^* - \bar{y} + u + \alpha\varepsilon^e)^2.$$

Since the government is initially engaged in a peg, it faces two policy options: 1) To stay in the peg; 2) To abandon the peg by adopting the flexible exchange rate after a devaluation or revaluation. In the presence of fixed costs  $C(\varepsilon)$  linked to the abandon of the peg, the government chooses option 2 only when  $u$  is so high that  $L^{FLEX} + \bar{c} < L^{FIX}$  or so low that  $L^{FLEX} + \underline{c} < L^{FIX}$ . Devaluation thus occurs for  $u > \bar{u}$ , and revaluation for  $u < \underline{u}$ , where

$$\bar{u} = \frac{1}{\alpha} \sqrt{\bar{c}(\alpha^2 + \beta)} - y^* + \bar{y} - \alpha\varepsilon^e,$$

$$\underline{u} = -\frac{1}{\alpha} \sqrt{\underline{c}(\alpha^2 + \beta)} - y^* + \bar{y} - \alpha\varepsilon^e.$$

Departing from the exchange rate peg, the rational expectations of next period's  $\varepsilon$  given  $\varepsilon^e$  will depend on the probabilities of devaluation and revaluation. Obstfeld considers that  $u$  is uniformly distributed on  $[-\mu, \mu]$ . Then, the rational expectation of next period's  $\varepsilon$ , given price setters' expectation  $\varepsilon^e$ , is

$$E(\varepsilon) = E(\varepsilon|u < \underline{u}) \Pr(u < \underline{u}) + E(\varepsilon|u > \bar{u}) \Pr(u > \bar{u}).$$

Using the property of the uniform law of probability distribution yields:

$$\Pr(u < \underline{u}) + \Pr(u > \bar{u}) = 1 - \frac{\bar{u} - \underline{u}}{2\mu},$$

$$E(u|u < \underline{u}) = \frac{-\mu + \underline{u}}{2}, \quad (\text{expected value of shocks for } u < \underline{u});$$

$$\Pr(u < \underline{u}) = \frac{\underline{u} - (-\mu)}{2\mu}, \quad (\text{probability of having } u < \underline{u});$$

$$E(u|u > \bar{u}) = \frac{\mu + \bar{u}}{2}, \quad (\text{expected value of shocks for } u > \bar{u});$$

$$\Pr(u > \bar{u}) = \frac{\mu - \bar{u}}{2\mu}, \quad (\text{probability of having } u > \bar{u}).$$

Using these properties and equation (3), we obtain the expectations of speculators ( $E(\varepsilon)$ ) under the exchange rate peg as function of price setters' expectations as follows:

$$\begin{aligned} E(\varepsilon) &= E(\varepsilon|u < \underline{u})\Pr(u < \underline{u}) + E(\varepsilon|u > \bar{u})\Pr(u > \bar{u}) \\ &= \frac{\alpha(y^* - \bar{y}) + \alpha^2\varepsilon^e}{\alpha^2 + \beta} [\Pr(u < \underline{u}) + \Pr(u > \bar{u})] + \frac{\alpha}{\alpha^2 + \beta} [E(u|u < \underline{u})\Pr(u < \underline{u}) + E(u|u > \bar{u})\Pr(u > \bar{u})] \\ &= \frac{\alpha}{\alpha^2 + \beta} \left[ \left(1 - \frac{\bar{u} - \underline{u}}{2\mu}\right)(y^* - \bar{y} + \alpha\varepsilon^e) - \frac{\bar{u}^2 - \underline{u}^2}{4\mu} \right]. \end{aligned} \quad (4)$$

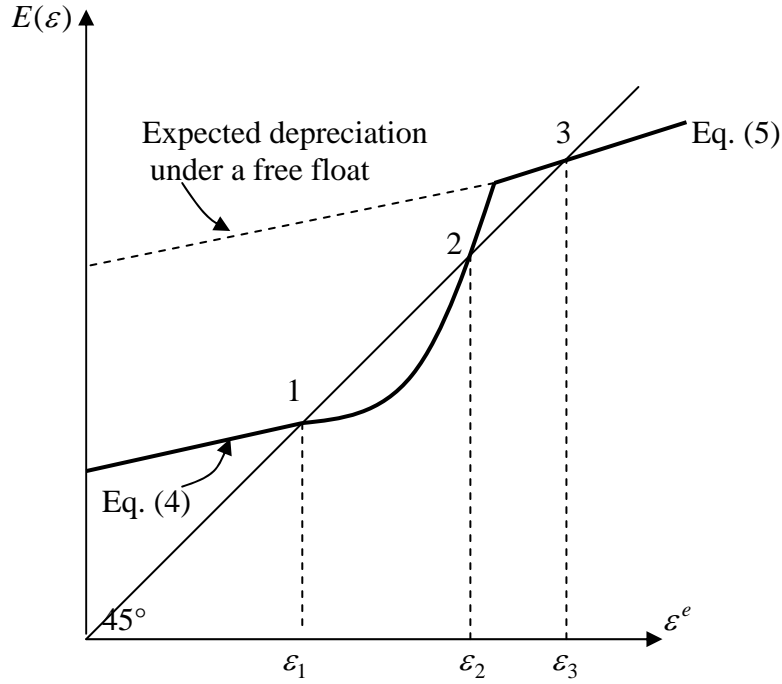
Under the floating exchange rate, using equation (3), the rational expectation of next period's  $\varepsilon$  by the speculators, given price setters' expectations  $\varepsilon^e$ , is:

$$E(\varepsilon) = \frac{\alpha(y^* - \bar{y}) + \alpha^2\varepsilon^e}{\alpha^2 + \beta}. \quad (5)$$

In full equilibrium,  $E(\varepsilon) = \varepsilon^e$ . To find out the fix points of equation (4), Obstfeld graphs it together with a 45° line and conjectures that there 'may' be three equilibria after some discussions about the derivatives of  $\bar{u}$ ,  $\underline{u}$  and  $E(\varepsilon)$  with respect to  $\varepsilon^e$ .

Obstfeld makes then the following analysis. Once  $\varepsilon^e$  has risen high enough that  $\bar{u}$  is stuck at  $-\mu$ , the government's reaction function is simply (3) and depreciation expectations are the same as under a freely flexible exchange rate. Fig. 1 shows how there can be three equilibrium expected depreciation rates,  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$ , corresponding to three different devaluation probabilities and realignment magnitudes conditional on devaluation.





**Fig. 1.** There *may* be three equilibrium expected depreciation rates, the highest of which is same as under a free float.<sup>5</sup>

Equilibrium 3 ( $\varepsilon_3$ ), in which  $\bar{u} = -\mu$ , must entail the same mean depreciation rate  $\varepsilon^e = \frac{\alpha}{\beta}(y^* - \bar{y})$  as the freely flexible exchange-rate equilibrium. Fig. 1 shows that this last equilibrium has to exist, even when  $\bar{c} > 0$ , if there are to be multiplicities at all. By using the definition of  $\bar{u}$  and equation (5) and assuming that  $\bar{u} < -\mu$ , it is easy to show that the formal parameter restriction required is:

$$\frac{\alpha^2 + \beta}{\beta}(y^* - \bar{y}) - \mu \geq \frac{1}{\alpha} \sqrt{\bar{c}(\alpha^2 + \beta)}. \quad (6)$$

which is more likely to hold if the devaluation cost  $\bar{c}$  is low, the slope  $\alpha$  of the Phillips curve is high, inflation aversion  $\beta$  is low, and the credibility distortion  $y^* - \bar{y}$  is big. Condition (6) means that if market operators expect the floating exchange-rate average depreciation rate, it will materialize the fixed cost of devaluation notwithstanding.

<sup>5</sup> Fig. 1. corresponds to Fig. 3 in Obstfeld (1996).

The government is powerless to enforce its preferred equilibrium should market expectations coordinate around an inferior one. Then Obstfeld deduces that some seemingly minor random event (a sunspot) could shift the exchange rate from a position where it is only vulnerable to very bad  $u$  realizations to one where output is so low in the absence of devaluation that even moderate shocks will induce the authorities to realign. The government could also be forced into an immediate effective float.

The strength of ‘fundamentals’, as reflected in the government’s tastes and the economy’s structure, affects the multiplicity of equilibria. A fall in the natural output level  $y$  shifts the vertical intercept of Eq. (4) upward. Thus, as  $\bar{y}$  falls from  $y^*$ , the corresponding expectations schedule shifts from a single low-inflation intersection with the  $45^\circ$  line, to three, and back to a unique equilibrium in which devaluation is a probability 1 event.

One minor problem with the above discussion due to Obstfeld is that condition (6) is obtained under the condition  $\bar{u} < -\mu$ . Since he initially assumes that  $u$  is uniformly distributed on  $[-\mu, \mu]$ , condition (6) cannot be true since this assumption implies that  $\bar{u} \in [-\mu, \mu]$ .

In the following, I will focus on a much serious error arises in Obstfeld’s intuitive conjecture based on equation (4). After having intuitively discussed why the conjecture made by Obstfeld is unfounded, I will formally demonstrate it.

### 3. Existence of two fix-points instead of three

In the absence of credibility costs associated with revaluation or devaluation, the threshold under which a revaluation is possible is the same threshold over which a devaluation is possible, i.e.  $\underline{u} = \bar{u}$ .

Since the central bank is assumed to have an output objective which is higher than the natural level,  $\underline{u}$  and  $\bar{u}$  are drawn to the left. Consequently, in the absence of credibility costs, the probability of expected devaluation is superior to that of expected revaluation under the law of uniform distribution.

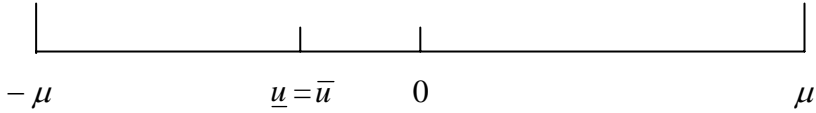


Fig. 2. Threshold of devaluation and revaluation.

The introduction of credibility costs increases the threshold above which a devaluation is possible, but reduces the threshold under which a revaluation is possible.

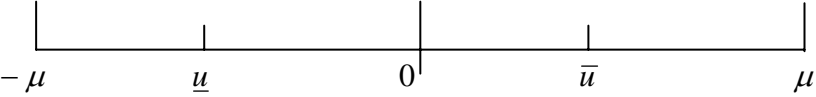


Fig. 3. The effect of credibility costs on the thresholds of devaluation or revaluation.

Since the cost of devaluation is higher than that of revaluation,  $\bar{u}$  moves more to the right than  $\underline{u}$  moves to the left. Therefore, the expected value of a revaluation for  $u < \underline{u}$  may dominate or not the expected value of devaluation for  $u > \bar{u}$ .

The above discussion shows that  $(\bar{u} - \underline{u})$  is independent of  $\varepsilon^e$  and hence  $(\bar{u}^2 - \underline{u}^2)$  is a linear function of  $\varepsilon^e$ . Consequently, there are no reason for  $E(\varepsilon)$ , determined by equation (4), to increase exponentially as shown in Fig. 1.

In effect, Obstfeld has not exploited the algebraic implications of equation (4). Instead, he conjectures that “there may be three equilibrium expected depreciation rates, the highest of which is same as under a free float”. That is also the title of his Fig. 3. Unfortunately, this conjecture is wrong. We can check it easily by substituting the definition of  $\bar{u}$  and  $\underline{u}$  into equation (4). It follows:

$$E(\varepsilon) = \frac{\alpha}{\alpha^2 + \beta} \left[ \frac{\left(1 - \frac{\left[\frac{1}{\alpha}\sqrt{\bar{c}(\alpha^2 + \beta) - y^* + \bar{y} - \alpha\varepsilon^e}\right] - \left[-\frac{1}{\alpha}\sqrt{\underline{c}(\alpha^2 + \beta) - y^* + \bar{y} - \alpha\varepsilon^e}\right]}{2\mu}\right)(y^* - \bar{y} + \alpha\varepsilon^e)}{\left[-\frac{\left[\frac{1}{\alpha}\sqrt{\bar{c}(\alpha^2 + \beta) - y^* + \bar{y} - \alpha\varepsilon^e}\right]^2 - \left[-\frac{1}{\alpha}\sqrt{\underline{c}(\alpha^2 + \beta) - y^* + \bar{y} - \alpha\varepsilon^e}\right]^2}{4\mu}}\right]}. \quad (7)$$

After some simplifications, equation (7) becomes:

$$E(\varepsilon) = \frac{\alpha}{\alpha^2 + \beta} \left[ (y^* - \bar{y} + \alpha\varepsilon^e) - \frac{(\bar{c} - \underline{c})(\alpha^2 + \beta)}{4\alpha^2\mu} \right]. \quad (8)$$

The equilibrium solution of  $E(\varepsilon)$ , i.e. the fix-point where  $E(\varepsilon) = \varepsilon^e$ , can be obtained by solving equation (8):

$$E(\varepsilon) = \varepsilon^e = \frac{\alpha}{\beta} \left[ (y^* - \bar{y}) - \frac{(\bar{c} - \underline{c})(\alpha^2 + \beta)}{4\alpha^2\mu} \right]. \quad (9)$$

The second solution is the flexible exchange rate, given by equation (5). If the government decides to devalue, it is this equilibrium that will be finally attained after some adjustment of the speculators' expectations.

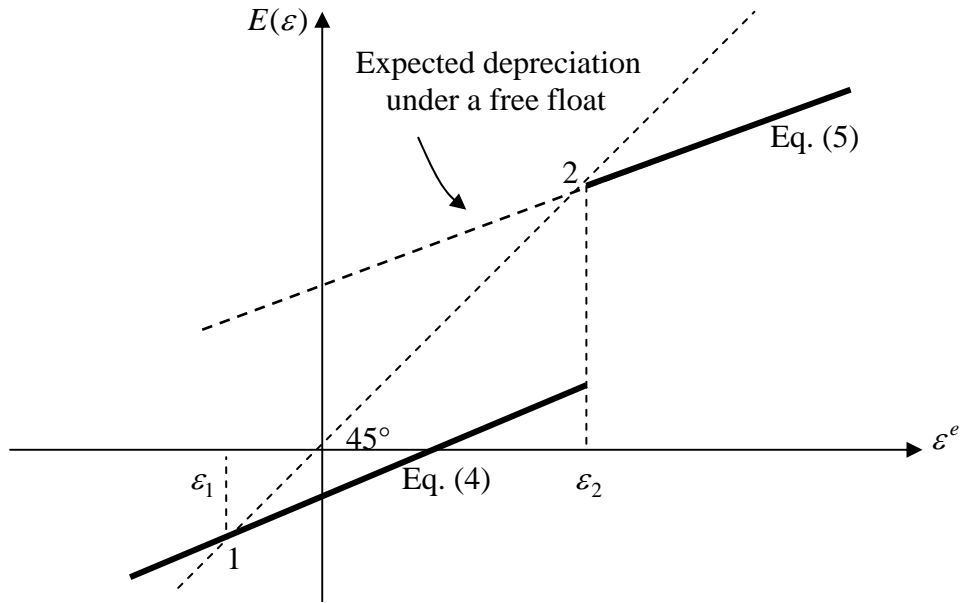
The intermediate equilibrium disappears since the relationship between  $E(\varepsilon)$  and  $\varepsilon^e$  represented by equation (4) is linear.

Two cases are distinguished in the following.

**Case 1.** According to equation (8), for  $\varepsilon^e \leq 0$ , the equilibrium value of  $E(\varepsilon)$  would be negative if:

$$y^* - \bar{y} < \frac{(\bar{c} - \underline{c})(\alpha^2 + \beta)}{4\alpha^2\mu}. \quad (10)$$

In the Fig. 4, an equilibrium with revaluation ( $\varepsilon_1$ ) and an equilibrium with devaluation ( $\varepsilon_2$ ) are represented. The public believes in a revaluation, even if the government has incentive to devalue in the event of large inflation shock.

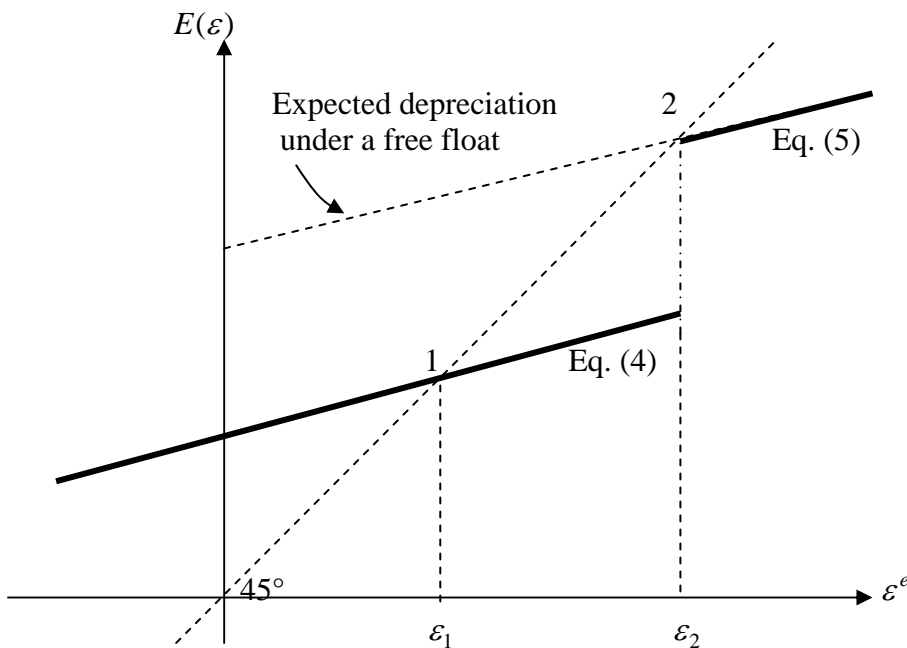


**Fig. 4.** Two equilibria: coexistence of beliefs of devaluation and revaluation.

*Case 2.* Equation (8) implies that, for  $\varepsilon^e \geq 0$ , the value of  $E(\varepsilon)$  would be positive if:

$$y^* - \bar{y} > \frac{(\bar{c} - c)(\alpha^2 + \beta)}{4\alpha^2\mu}. \quad (11)$$

Equation (4) could imply the existence of two equilibria of devaluation (Fig. 5).



**Fig. 5.** Two equilibria of devaluation.

## 5. Conclusion

By reconsidering a well-known example of model with self-fulfilling features studied by Obstfeld (1996), I have shown that the solution of three devaluation equilibria that he has conjectured is not robust to rigorous algebraic demonstration. By completing the algebraic demonstration without introducing new assumptions, I have found that there are only two equilibria. One is a devaluation equilibrium corresponding to the free float and another could be a revaluation or devaluation equilibrium.

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