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On the role of money growth targeting under inflation targeting regime

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Abstract: The mainstream inflation-targeting literature makes the strong assumption that the central bank can exactly target the interest rate which affects investment and consumption decisions and hence the money supply plays no role in the monetary policy strategy. This assumption is equivalent to admitting the perfect credibility of inflation target announced by the central bank, the perfect functioning of money and financial markets and that the central bank is willing to inject as much liquidity as the economic agents demand. Neither of these assumptions corresponds to the reality. In effect, the inflation expectations can not be easily anchored by the cheap talk of central bankers. On the other hand, the central bank may have many difficulties to target, in a context of financial instability, the interest rates which affect the real and financial decisions of private agents. We suggest that under inflation-targeting regime, money and credit markets vehicle the inflation expectations that can be anchored with a well-specified feedback money growth rule. The latter, in contrast to the Friedman's *k* percent money growth rule, can help managing the inflation expectations in a manner to guarantee the dynamic stability of the economy. Furthermore, the model can be easily used to discuss the implications of the zero interest rate policy and the quantitative easing policy.

Key words: Interest rate rule, imperfect money and credit markets, inflation targeting, monetary targeting, inflation expectations, Friedman's *k* percent money growth rule, feedback money growth rule, macroeconomic stability, zero interest rate policy, quantitative easing policy.

JEL Classification: E43, E44, E51, E52, E58.

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1. Introduction

Over the last eighteen years, inflation targeting is increasingly becoming popular monetary policy regime among the central banks in the world since its initial adoption by the central bank of New Zealand in 1990. It has gained more and more popularity among academic economists and central bankers in a context of rapid financial liberalization and innovations, which causes the reported unstable relationship in the short run between monetary aggregates and inflation and hence the failure of monetary targeting. That leads Mishkin (1999) to present the inflation targeting, which can be implemented via an (optimal) interest rate rule, as being more effective in the control of inflation than the monetary targeting and thus the natural successor of the latter.

The emergence of inflation targeting in practice and in theory is clearly related to research on the interest rate rules since the 1990s to reflect best the fact that the central bank of the United States conducts the monetary policy by choosing the federal funds rate (Goodfriend (1991)), a very short term nominal interest rate, and that central banks of other industrialized countries have a similar behavior, including Bundesbank. The latter was considered since a long time as the classical example of a central bank which targets the monetary aggregates. This research were stimulated in particular by the discovery of Taylor (1993) which shows that simple interest rate rules seem to coincide quantitatively with the behavior of the Federal Reserve over various periods. Conceptual questions, concerning the determination of macroeconomic variables for a given interest rate rule, were tackled within various theoretical frameworks (Clarida, Gali and Gertler (1999), King (2000)).

All that research shares the same consensus, namely that the money and hence the credit do not have any crucial and constructive role to play in the monetary policy. In other words, the money market is only useful for determining the supply of money which responds

endogenously to the demand of money, and hence can be largely ignored in making monetary policy decisions (Woodford (1998), Rudebusch and Svensson (1999)). This consensus forged since ten years has substituted to the one forged by Milton Friedman, according to which inflation is "always and everywhere a monetary phenomenon". However, the new consensus is confronted to timid but incessant empirical and theoretical contests as well as the new challenges revealed by the present financial and economic crises.

The experience of the 1970s showed that the inflation expectations of the public can lose their anchor in a context of high oil prices and depreciating US dollars. Monetary targeting such as Milton Friedman's k percent money growth rule was progressively abandoned by central banks in favor of implicit interest rate rules like that discovered by Taylor (1993).[1] To stabilize the inflation expectations, monetary authorities proactively increase (reduce) nominal interest rate when the evidence suggests that inflation will rise above (respectively fall below) some numerical objective.

By recommending the adoption of an interest rate rule by the central bank, economists advocate that the supply of money is automatically determined by the demand. In other words, the monetary authority implicitly confer to the private sector the following message: any quantity of money that you wish at given nominal interest rate will be provided. Under these conditions, a badly specified interest rate rule could lead to the existence of multiple equilibriums or Wicksellian-type dynamic instability. An important lesson of this literature is that, to avoid the existence of multiple equilibriums as well as Wicksellian-type dynamic instability, it is necessary that the interest rate rule reacts in a sufficiently strong manner to the current or expected rate of inflation.

Recently, by considering some of the leading arguments for assigning an important role to tracking the growth of monetary aggregates when making decisions about monetary policy,

Woodford (2008) concludes that none of them provides an indisputable reason to assign a big role to monetary aggregates in the conduct of monetary policy. For him, ignoring money does not mean returning towards conceptual framework that allows the high inflation of the 1970s. He also rejects the view according to which the models of inflation determination with no role for money are incomplete, or inconsistent with elementary economic principles. However, he stresses the importance of avoiding the traps which are the bad estimate of output gap and the ignorance of endogenous nature of inflation expectations. Consequently, it is necessary for the central bank to use all the sources of information to judge if the interest rate policy is consistent with the expected future trend of the economy (Svensson and Woodford (2005), Woodford (2008)).

Within the framework of IS-LM model (neo-classical synthesis or New Keynesian), there exist two conventional strategies to use equation LM as a not so indispensable accessory in the literature relating to the interest rate rules (King (2000)). The first uses LM to specify money supply rules and to compare them with interest rate rules. The second describes the monetary policy strategy using the interest rate rule and LM is used to determine the endogenous money supply. Within a static framework where the central bank adopts an interest rate rule, LM does not interact with other equations and has thus no importance in the absence of real balance effect. Moreover, the last effect is more hypothetical than real.

However, as argued by Romer (2000), one area in which both the IS-LM and IS-MP approaches (where MP stands for monetary policy, i.e. interest rate rule) may have simplified too far is in their treatment of financial markets. In both approaches, the only feature of financial markets that matters for the demand for goods is 'the' real interest rate that monetary policy can powerfully and directly influence as the central bank desires. In practice, the demand for goods depends on interest rates that the central bank may not be able to control

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[1] In an interview, Milton Friedman admits that the use of the quantity of money as target was not a success

directly and tenuously as well as the level of credit which is available at those rates. An analysis, which takes more carefully account of the impacts of various developments in financial markets on the demand for goods as well as the mechanism through which the monetary policy affects these interest rates and the level of credit, would highlight many of the difficulties and uncertainties of actual policy-making.

Recent experiences of monetary policy in the context of unprecedented financial and economic crises have shown that monetary policy defined in terms of interest rate rule may lose its effectiveness in stabilizing the financial markets and the economy. The zero interest rate policy coupled with the quantitative easing policy practiced previously by the central bank of Japan and now by the Fed implies that monetary aggregates have important role to play. The question is why the central bank, paying special attention to monetary aggregates during financial crisis, must neglect them when the economy is booming. In this respect, the European central bank (ECB), with its two-pillar monetary policy strategy, may be better inspired.

In this paper, by adopting a more complicated view of financial markets, we will supply some new arguments in favor of the use of monetary aggregates under inflation targeting regime and show how to do it. The approach that we adopt in this paper is narrowly related to some previous works (Dai (2007), Dai and Sidiropoulos (2009)) which consider also the use of money growth rules under the inflation-targeting regime and where the money market is considered as a link between different economic agents and a coordination device for their inflation expectations. One important point of the present paper is that the target of interest rate does not automatically become the interest rate practiced by lenders on the credit market. The transmission mechanism from the repo interest rate, to money market interest rate and then to the lending interest rate can break down due to macroeconomic and financial

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(London (2003)).

instability. When the monetary and financial markets are imperfect, the central bank can lose its control over the inflation expectations due to the fact that it controls with imperfect precision the liquidity that private banking and financial sectors can also expand or reduce depending on expectations about future inflation and output.

In the next section, we discuss why the role of money is important in the monetary policy strategy and why the inflation-targeting literature cannot neglect the money and discard the monetary targeting without adopting some very strong assumptions about the functioning of money and financial markets. In the section after, we examine some lessons from the monetary targeting experiences and argue that inflation targeting can be submitted to similar problems. In the section 4, we incorporate money and credit markets in a simple New Keynesian model and specify a monetary policy strategy which consists to add a feedback money growth rule to the inflation targeting regime. In section 5, we discuss why the feedback money growth rule can be useful, in comparison with Friedman's k percent rule, in stabilizing dynamically the inflation expectations, under backward-looking or forward-looking solution. In section 6, we discuss how the framework can be easily used to discuss the zero interest rate and quantitative easing policies. We conclude in the last section.

2. Why the money may be useful and important?

Even though the proponents of inflation targeting do not deny the long-run relationship (correlation) between monetary aggregates and inflation, they tend to neglect or deny the causality relationship which runs from monetary aggregates to inflation and hence the role of money as efficient instrument of controlling inflation. Under inflation targeting regime, the causality of the relationship can be inverted since the supply of money is endogenous and

automatically adapts to the evolutions of output and inflation. The inversion of causality cannot be achieved without making some implicit and explicit strong assumptions.

Notably, in the inflation-targeting literature, it is assumed that an independent and transparent central bank without inflation bias can credibly anchor the inflation expectations of private sector by fixing nominal interest rate. This assumption is equivalent to assume that the money market and financial markets are perfectly functioning and hence can be put into a black box without loss of information. From many points of view, these assumptions are very questionable.

Although a typical interest rate rule (Taylor rule or optimal interest rate rule) can be effective in anchoring the inflation expectations in certain models, the result is not robust with the modifications, weak but empirically plausible, of model specifications. In effect, there is considerable uncertainty about the correct model specifications (Benhabib, Schmitt-Grohe and Uribe (2001, 2002a, b), Christiano and Rostagno (2001), Carlstrom and Fuerst (2002, 2005)). Sharing this concern, Christiano, Motto and Rostagno (2007) describe two examples which illustrate in different manners how the money and the credit can be useful in the conduct of monetary policy. Their first example presents a channel for monetary policy on the supply side and creates the possibility that the inflation expectations lose their anchoring. Illustrated with the help of an IS-LM model augmented of a supply curve, this example shows how the monitoring of money and credit can help anchoring the inflation expectations of private agents. Their second example, which recapitulates the analysis of Christiano, Ilut, Motto and Rostagno (2007), shows that a monetary policy which concentrates too narrowly on inflation can, in an unintended way, to contribute to reduce the welfare via cycles of expansion and depression in the real and financial variables.

Being aware of the importance of money and credit markets, Benjamin Friedman (2003) also worries about abandoning the role of money and the analytical tool which is curve LM.

He argues that such an abandonment makes more difficult to take into account how the functioning of banking system (and with it credit markets more generally) can affect the monetary policy and also leaves open the fundamental question in the way in which the central bank manages to fix the interest rate in the first place. Similarly, for Goodhart (2007), the central banks must still give attention to the monetary aggregates, in particular the growth rate of the bank credit allocated to the private sector.

Government and central bank might have incentive to care about the stability of monetary aggregates since there is an empirically proved strong long term relationship between inflation and money growth. Söderström (2005) demonstrates how a target for money growth can be beneficial for an inflation-targeting central bank acting under discretion. As the growth rate of money is closely related to the change in the interest rate and the growth of output, delegating a money growth target to the central bank makes discretionary policy more inertial, leading to better social outcomes. In comparing this delegation scheme with other schemes suggested in the literature, he finds that stabilizing money growth around a target can be a sensible strategy for monetary policy, although other delegation schemes are often more efficient.

In a dynamic context, central bank can lose the capacity of controlling inflation expectations with only the instrument of interest rate at its disposition. Independent central bank can verbally persuade the public that it has the firm intention of attaining its inflation target by building its credibility and by making its objective, preferences and operational procedures, data and economic model transparent to the public. But due to economic uncertainty, model uncertainty or operational errors, the central bank cannot always attain its objective. This result may induce some doubts of the public about the future realization of inflation target. Outside of equilibrium, the central bank has the risk of losing its persuasion

power and all the verbal persuasion efforts may not be sufficient to convince the public to adhere entirely to its monetary policy strategy (Dai (2007)).

Furthermore, temporary but persistent shocks[2] could make difficult the conduct of monetary policy only based on the control of interest rate. Workers, who have a finite horizon, could claim a compensation of fall in their purchasing power as soon as they observe a rise in the rate of inflation without awaiting the cancellation of current inflationary shocks by future shocks. In addition, the rational expectations based on information restrained to the goods market and the Philips curve, as it is generally admitted in the research on inflation targeting, could irrelevantly reflect the expectation behavior of private agents. It is contrary to the essential idea of the rational expectations hypothesis, which stipulates that the private agents use any information available to form their expectations. Consequently, one can reasonably suppose that information concerning the money and credit markets is used by the private sector. Considering the money market as device of coordination of private inflation expectations, Dai and Sidiropoulos (2003, 2005, 2009), Dai (2006, 2007), and Dai, Sidiropoulos and Spyromitros (2007) provide theoretical justifications of the utility of this market other than only determining in an endogenous way the money supply within a typical framework of inflation targeting.

These theoretical concerns also found some empirical echoes which show that money is not superfluous. Milton Friedman (2005), using data covering three periods of expansion in the USA and Japan, proves that the quantity of money exerts a determining effect on national revenue and stock prices. Hafer, Haslag and Jones (2007) discover that there is a significant statistical relationship between the delayed values of the money and the output, even when delayed values of real interest rate and output are taken into account. Adding the money into a dynamic IS model, Hafer and Jones (2008) find that money growth usually helps predicting

the GDP and that the predictive power of short-term real interest rate is much lower than previous works have suggested. Their results imply that the omission of the money seems to come at a high cost for dynamic IS models like that employed by Rudebusch and Svensson (1999).

To understand the lacunae in the mainstream literature studying inflation targeting or interest rate rules, consider the traditional IS-LM diagram. By assuming that money is endogenous, the inflation targeting literature assumes that IS, LM and MP curves cross by divine coincidence at the same equilibrium point (Figure 1). The MP curve represents monetary policy stipulated in terms of interest rate rule and can be represented by a horizontal line if the interest rate rule does not depend on output. In effect, by assuming that money supply automatically adjusts to demand, whatever is the interest rate chosen by the central bank, LM moves to cut the two other curves (IS and MP) at their point of intersection.
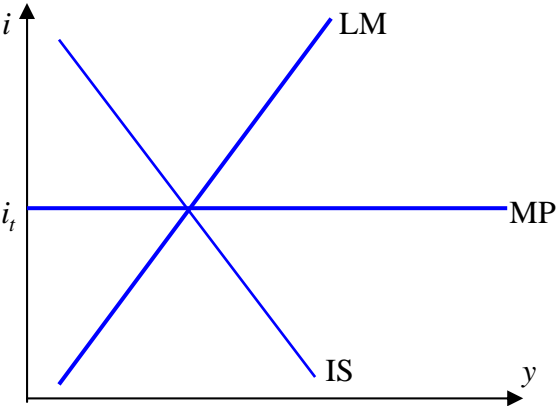
Figure 1: Money supply is perfectly elastic.

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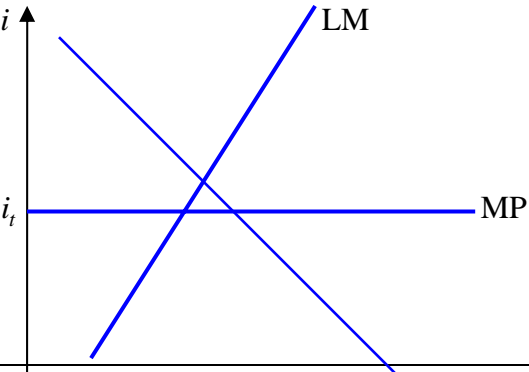
[2] Random shocks can also take the appearance of persistent shock when the same shock repeats itself consecutively.

IS   *y*
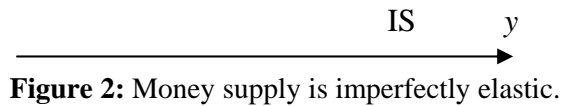
Figure 2: Money supply is imperfectly elastic.

By assuming that money supply adjusts imperfectly to demand, given the inflation expectations, it is possible that these three curves do not cross at the same point as illustrated in Figure 2 after that some shocks have disturbed the initial equilibrium and dislocated these three curves. The resulting disequilibrium could lead private agents to modify their inflation expectations, allowing IS, LM and MP curves to shift so that, after some dynamic adjustment, these three curves cross again at the same equilibrium point.

As we have seen in the recent developments in financial markets, the central bank cannot easily control the interest rates (which correspond to the intersection point between IS and LM curves in Figure 2) at which the interbank loans and other lending are made. It can neither easily control the inflation expectations, since fear of deflation and hyperinflation simultaneously appears in the actual financial turmoil. Many central banks (some of them are explicit inflation targeter) have massively injected the central liquidity to stabilize the financial markets. However, they do not do a symmetrical work during the boom period, i.e. reducing the liquidity when the economy is expanding too quickly. This demonstrates why the neglect of money in the inflation-targeting framework is one of the lacunae in its theoretical foundations.

The mainstream theories of inflation targeting assume that money and financial markets are perfect, and the central bank is perfectly credible and transparent. In the absence of inflationary bias and persistent shocks, these assumptions lead to the equality between the inflation target and the expected rate of inflation as well as the equality between the repo interest rate fixed by the central bank and the interest rate determined on financial markets.

We contest the view popular in the inflation-targeting literature according to which the central bank can be assumed to be completely credible in the sense that its inflation target

automatically becomes the nominal anchor for current and future periods, thus evacuating the possibility that central bank could lack credibility and means to control inflation expectations. In effect, this credibility cannot be always ensured due to the existence of financial and economic uncertainty. For example, when there are major and persistent supply shocks inducing an inflationary pressure, the central bank might have difficulty to explain why it cannot fight inflation without provoking either a fall of employment or a fall in real wages during an extending period.

The inflation-targeting literature focuses on the imperfections on the supply-side of goods and services and is completely unaware of those on money and financial markets. In order to ignore the later, it is implicitly assumed that these markets, in particular the money market, are perfectly functioning. As all financial assets are implicitly assumed to be perfectly substitutable, controlling only the repo interest rate is equivalent to controlling all other lending interest rates. Consequently, the curve of money supply coincides with that of money demand and one can completely ignore the existence of money and financial markets in the theoretical construction of inflation targeting.

The assumption of imperfect money and financial markets allows understanding better the functioning of the economy and how a monetary policy is implemented. In effect, the central liquidity is not accessible at unlimited quantity because the central banks limit the quantity, the quality and the types of assets accepted as collateral as well as the types of financial institutions which have direct access to the central liquidity. That implies that there could be a potential imbalance (excess of liquidity or crisis of illiquidity) on the money market. Central bank's interventions defined in terms of injection or withdrawal of liquidity become essential. These interventions have the advantage of being more flexible than the instrument of interest rate because the latter must generally follow a well defined trend and is only modifiable (except in the event of financial crisis) with much longer intervals separating two interest rate

decisions. The failure of transmission mechanism running from the repo interest rate to other interest rates as well as the zero bound for nominal interest rate could greatly limit the possibility of actions through fixing the nominal interest rate for central banks adopting inflation targeting. A central bank too aggressive in reducing the repo interest rate can quickly find itself without interest rate instrument and hence the means of sufficiently reducing the lending interest rates and anchoring the inflation expectations.

The imperfect money and financial markets hypothesis also gives a better account of the dynamic of inflation expectations. The evolutions of the expected rate of inflation deduced from the difference of return between the indexed and un-indexed obligations show that the inflation expectations are not as static as predicts the mainstream inflation targeting literature. Some introductive teachings treat the expected rate of inflation even as fixed in the presence of stochastic shocks (Romer (2000), Walsh (2002)). Using information from money market and financial markets generally allows improving the inflation expectations of private sector compared to the case where private sector uses only information extracted from the interest rate rule, the Philips curve and the goods market equilibrium condition as it is admitted in the literature of interest rate rules and inflation targeting.

By assuming imperfect financial markets, we admit that the target of lending interest rate, decided by the central bank and expressed as optimal interest rate rule and function of other variables in the inflation-targeting regime, cannot be directly fixed and is not always realized due to malfunctioning of money and financial markets or shocks affecting these markets. In effect, the central bank fixes the repo interest rate, which is determined by taking account of inflation and output targets and economic model (including money and financial markets). A modification of repo interest rate allows inducing a change in the interbank money market interest rate, affecting hence the lending interest rate determined on the credit or debt market at which firms and consumers can borrow. If this transmission mechanism is perturbed by

exogenous shocks or endogenous instability, adopting monetary targeting under inflation-targeting regime may have some advantages in terms of monitoring the inflation expectations and controlling the money market interest rate and the lending interest rate. There is then some good reasons for the inflation-targeting central bank, by designing an appropriate money growth rule, to flexibly monitor the level of liquidity in the monetary market and hence in the economy (i.e. to target other interest rates) and to control the inflation expectations in order to ensure the dynamic stability for the economy.

The design of appropriate money growth rule is determinant for the success of monetary targeting since the renowned k percent money growth rule of Milton Friedman is not successful. In effect, in an experiment of overlapping generations economies, Marimon and Sunder (1995) found no evidence that a 'simple' rule such as a constant growth of the money supply, can help coordinate agents' beliefs and help stabilize the economy.

3. Monetary targeting versus inflation targeting

The strategy of monetary targeting (or targeting of monetary aggregates) comprises three elements: reliance on information conveyed by a monetary aggregate to conduct monetary policy, announcement of targets for monetary aggregates, and some accountability mechanism to preclude large and systematic deviations from the monetary targets (Mishkin (2002)).

Monetary targeting is generally associated with the monetarism. Even though the monetarism represents an important advance over prior conventional wisdom and the lessons learned from the monetarist controversy are not to forget, it has lost its steam in modern development of monetary theory and policy. Woodford (2008) argues that the most important

of these lessons,[3] and the ones that are of continuing relevance to the conduct of policy today, are not dependent on the thesis of the importance of monetary aggregates. In other words, the ECB's continuing emphasis on the prominent role of money in its deliberations is not theoretically well justified. It is explained by the concern not to ignore the lessons of the monetarist controversies of the 1960s and 1970s.

The monetary targeting experiences in major industrial countries are mitigated (Bernanke and Mishkin (1992), Mishkin and Posen (1997), Mishkin (2002)). It is found that while Germany and Switzerland could be considered successful monetary targeters, the monetary targeting was not particularly successful in the United States, Canada and the United Kingdom. There are two interpretations for why monetary targeting in these three countries was not successful in controlling inflation. The first is that because monetary targeting, as part of the central bank game playing, was not pursued seriously, it never had a chance to be successful. The second is that growing instability of the relationship between monetary aggregates and goal variables such as inflation (or nominal income) due to financial liberalization and innovations introduced since 1980s meant that this strategy was doomed to failure and indeed should not have been pursued seriously.

However, it is not by pure chance that the Bundesbank (and to some degree the National bank of Swiss) that took on board monetarist teachings to the greatest extent, had the best performance with regard to inflation control in the 1970s and 1980s. The monetary aggregate chosen by the Germans was central bank money, which is less affected by the ulterior financial liberalization and innovations. The key fact which may explain the success of monetary targeting regimes in Germany and Switzerland is that the targeting regimes were

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[3] The most important lessons from the monetarism, according to Woodford (2008), are that monetary policy can do something about inflation, the central bank can reasonably be held accountable for controlling inflation and a verifiable commitment by the central bank to a non-inflationary policy is important.

very far from a Friedman-type monetary targeting rule.[4] The latter implies that a monetary aggregate is kept on a constant-growth-rate path and is the primary focus of monetary policy. In effect, the Bundesbank, could miss its targets in the short-run by allowing growth outside of its target ranges for periods as long as two to three years while subsequently reversing overshoots of its targets.

Monetary targeting frameworks in Germany and Switzerland are best viewed as a mechanism for transparently communicating the strategy of monetary policy that focused on long-run considerations and the control of inflation, and as a means for increasing the accountability of the central bank. The success stories of monetary targeting, in the case of Bundesbank and Swiss National Bank, are explained by some economists[5] as due to that the monetary policy is actually closer in practice to inflation targeting than it is to Friedman-like monetary targeting and thus might best be thought of as "hybrid" inflation targeting. The Bundesbank's monetary targeting is quite similar to inflation targeting as it announced inflation target and transparently communicated to the public and market participants. Central bankers (Freedman, 1996; King, 1996) have also noted the close similarity in the use of central bank instruments and the reaction of central banks to news and shocks under inflation forecast and monetary targeting. That suggests that choice of one or other monetary regime does not seem to matter much for the day-to-day conduct of monetary policy. Empirically, inflation targeting seems to have made little if any difference for inflation and interest rate dynamics (or conduct of interest rate policy) in the countries that adopted this strategy in the 1990s (Groeneveld *et al.*, 1998; Almeida and Goodhart, 1998). However, using real-time data, Gerberding *et al*. (2005) find that the Bundesbank took its monetary targets seriously, but also responded to deviations of expected inflation and output growth from target.

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[4] Issing (1996) noted: "one of the secrets of success of the German policy of money-growth targeting was that ... it often did not feel bound by monetarist orthodoxy as far as its more technical details were concerned."

The recent debate about monetary policy strategy generally opposes monetary targeting to inflation targeting and questions over whether the ECB has to move to full-fledged inflation targeting. Alesina *et al.* (2001) argue that it is hard to see why the growth rate of M3 should have a special role and the ECB could improve its policy by adopting inflation targeting. The empirical study based on US data by Rudebusch and Svensson (2002) has revealed that monetary targeting is quite inefficient, yielding both higher inflation and output variability and therefore, there is no support for the prominent role given to money growth in the Eurosystem's monetary policy strategy. Evans and Honkapohja (2003) have shown that Friedman's *k*-percent money supply rule performs poorly in terms of welfare compared to optimal interest rate rule.[6] For Laubach (2003), monetary targeting facilitates communication of the central bank's type. But, this advantage is outweighed for most parameter values by the advantage of inflation targeting in terms of inflation control. Cabos *et al.* (2003), using German data from the end of the Bretton Woods system until 1997, support also that control problems involved in targeting broad or narrow money are larger than for direct inflation targets. Moreover, Gersbach and Hahn (2003) suggest that inflation targeting is superior to monetary targeting as it makes it easier for central banks to commit to low inflation.

In a quite consensual manner, the monetary targeting is too quickly pushed out as a credible monetary policy strategy. In practice, the ECB continues to attach importance to monetary analysis in its two-pillar strategy. But it is theoretically contested by proponents of inflation targeting. One prominent and systematic attack of the two-pillar strategy is recently given by Woodford (2008).

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[5] See Clarida and Gertler (1997), Bernanke and Mishkin (1997), Bernanke and Mihov (1997), Laubach and Posen (1997), Clarida et *al.* (1998), Mishkin (1999, 2002) and Svensson (1999a, b, 2000).

[6] However, Friedman's rule can generate equilibriums that are determinate and stable under learning. In the contrary, open-loop interest rate rules are subject to indeterminacy and instability problems. Minford et *al.* (2003) compare Friedman's *k*-percent money supply rule with Taylor's rule to see how they are different.

Lessons learned from monetary targeting only indicate that the instability of the relationship between monetary aggregates and goal variables (inflation and nominal income) make monetary targeting problematic, but not necessarily the failure of monetary targeting. In the presence of instability relationship between instrument and goal variables, a central bank with high credibility can successfully stabilize inflation and output through monetary targeting if it is flexible, transparent and accountable (Mishkin (2002)). However, that includes too many conditions and explains why many economists argue against monetary targeting (Mishkin (1999)).

One must not be surprised that the argument used against monetary targeting can be returned against inflation targeting which uses nominal interest rate as instrument. In a context of financial instability, it is difficult to ignore the developments in money and financial markets and consider they have no influence on the monetary policy strategy which can be theoretically formulated only on the base of New-Keynesian Phillips curve and IS curve. In effect, the relationship between nominal interest rate that the central bank can directly control and goal variables such as inflation and output gap can be very instable. If this is the case, it will become difficult for central banks to credibly and transparently communicate their monetary policy strategy in the framework of inflation targeting. Furthermore, inflation targeting regimes, focusing on inflation and output targets, could lead to very ample movements in interest rates and consequently in monetary and financial aggregates and asset prices, creating difficulties for ulterior monetary policy decisions. That is actually the case in many developed and emerging market economies where many financial and real bubbles burst simultaneously.

However, we do not argue against the use of inflation targeting regime. We consider that it is possible to combine monetary targeting with it as in Dai (2007) and Dai and Sidiropoulos (2009). Using this new monetary policy strategy, central banks could simultaneously dispose

of two policy instruments (repo interest rate and money supply) to affront an increasingly uncertain economic environment. More instruments could allow stabilizing the economy with less social costs, particularly in the presence of possible dynamic instability or multiplicative uncertainties (Brainard (1967)).

4. The model

The economy is described by a stylized new-Keynesian model (Clarida, Gali and Gertler (1999)):

$$\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + \varepsilon_t^\pi, \qquad \text{with } 0 < \beta < 1, \ \lambda > 0, \tag{1}$$

$$x_t = E_t x_{t+1} - \varphi(i_t^c - E_t \pi_{t+1}) + \varepsilon_t^x, \qquad \text{with } \varphi > 0, \tag{2}$$

where π_t $(\equiv p_t - p_{t-1})$ denotes the rate of inflation, p_t the general price level, x_t the output gap (i.e., the log deviation of output from its flexible-price level), i_t^c the nominal lending interest rate at which non-financial private sector can borrow from banks.

Equation (1) represents the New-Keynesian Phillips curve, where the rate of inflation is related to the expected future rate of inflation ($E_t \pi_{t+1}$) and current marginal cost, which is affected by the output gap. The inflation shock, ε_t^π, is due to productivity disturbances.

Equation (2), an expectational IS curve, relates the current output gap to the expected future output gap ($E_t x_{t+1}$), the real lending interest rate. The real lending interest rate is defined as the difference between the nominal lending interest rate and the expected future rate of inflation. In this model, we assume that the individual saver can save at i_t^c if she directly buys bonds emitted by firms, which offer a rate of return equal to i_t^c. We assume that savers also save in a deposit account bearing no interests at banks and hence their

intertemporal arbitrage between saving and present consumption depends only on i_t^c. The demand shock, ε_t^x, reflects either productivity disturbances which affect the flexible-price level of output or, equivalently, changes in the natural real interest rate.

The model is completed with money and credit market equilibrium conditions in the spirit of Bernanke and Blinder (1988):

$$m_t - p_t \equiv b_t + h i_t^m - p_t = l_1 x_t - l_2 i_t^m + \varepsilon_t^l, \qquad \text{with } l_1, l_2 > 0, \qquad (3)$$

$$-\kappa_1 i_t^m + \kappa_2 i_t^c = \delta_1 x_t - \delta_2 i_t^c + \varepsilon_t^c, \qquad \text{with } \kappa_1, \kappa_2, \delta_1, \delta_2 > 0, \qquad (4)$$

where i_t^m is the rate of interest determined on the money market at which the banks can refinance, m_t represents the money supply, b_t is the base money that the central bank can control, ε_t^l is a random shock that could incorporate both money supply and money demand shocks, ε_t^c is a random shock that could incorporate both credit supply and credit demand shocks. The lending interest rate, i_t^c, at which the banks lend, and firms and households borrow is determined on the credit market for given i_t^m.

Some modifications relative to the model of Bernanke and Blinder have been introduced to reflect recent developments in money and financial markets. Notably, ample disturbances affect the money market interest rate and the lending interest rate applied to the borrowing of non-financial private agents. We do not include public bonds in this model. The private bonds are assumed to be a perfect substitute to bank lending. Furthermore, in this model, it is the rate of interest i_t^m (instead of the rate of return on the public bonds) which determines the demand and supply of liquidity on the money market. By simplification assumption, i_t^m does not affect consumption and investment decisions.

Despite these simplifications, by giving a special attention to money and credit markets, we can quite realistically examine how the interest rate decision of the central bank makes its way into the economy and how the inflation expectations adjust in this process.

In equation (3), it is assumed that the central bank has not a total control over the money supply. Financial market innovations (e.g. credit derivatives) and private equity activities could create important endogenous liquidity (hi_t^m) that cannot be moped up by monetary policies except when higher interest rate undermines economic growth, curtail the flow of investor funds to "alternatives" and widen risk spreads in debt markets.[7] Instead, if the central bank desires, control can be exercised over a narrow monetary aggregate such as monetary base, and its variations are then associated with these in broader measures of money supply. The money supply is endogenous but it is imperfectly elastic as the banking system will increase or decrease the internal money in taking account of money market interest rate as well as collateral, and will not satisfy the money demand whenever it appears. The link between the money supply and the base, used here as a second policy instrument besides nominal interest rate, is given by $m = b + hi_t^m$, where b is the (log) monetary base, and money multiplier ($m - b$ in log terms) is assumed to be an increasing function of money market interest rate (i.e. $h > 0$), a money-multiplier disturbance is not explicitly considered but is implicitly incorporated in the shock ε_t^l (Modigliani *et al*. (1970), McCallum and Hoehn (1983), Walsh (1999)).

Equation (4) gives the condition for clearing the credit market. The supply of loans decreases with i_t^m and increases with i_t^c. The demand of loans decreases with i_t^c. The dependence of load demand on x_t captures the transactions demand for credit, which might arise, for example, from working capital or liquidity considerations.

Taking the difference between equation (3) and its equivalent in period $t-1$ yields:

$$\mu_t + h\Delta i_t^m - \pi_t = l_1\Delta x_t - l_2\Delta i_t^m + \Delta\varepsilon_t^l, \tag{5}$$

where $\mu_t = \Delta b_t = b_t - b_{t-1}$. Equation (5) implies that, in average or at steady state, the monetary base growth rate $\bar{\mu}$ must be equal to the current and expected rates of inflation, adjusted for the growth rate of output, i.e., $\bar{\mu} = \bar{\pi} + l_1\Delta x^* = \bar{\pi}^e + l_1\Delta x^*$. An inflation-targeting central bank can reinforce the credibility in its capacity of monitoring the inflation expectations by keeping an average long-term growth rate of monetary base consistent with its inflation target, i.e. $\bar{\mu} = \pi^T + \Delta x^*$. However, monetary base targeting must not be considered as a unique and independent strategy for achieving price stability by stabilizing inflation around a given inflation target since it faces, as shown by Svensson (1999a), an unpleasant choice between being either inefficient and transparent or efficient and non-transparent.

The model is closed with the specification of central bank's objective function. The latter translates the behavior of the target variables into a welfare measure to guide the policy choice. We assume that this objective function is over the target variables x_t and π_t, and takes the form:

$$L^{CB} = \frac{1}{2}E_t\sum_{i=0}^{\infty}\beta^i[\alpha x_{t+i}^2 + (\pi_{t+i} - \pi^T)^2], \tag{6}$$

where the parameter α is the relative weight on output deviations. The central bank's loss depends on output gap variability around of zero and inflation variability around of its constant target π^T which can be zero or positive. Since x_t is the output gap, the loss function takes potential output as the target.

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[7] See "In the new liquidity factories, buyers must still beware" by M. El-Erian, president and chef executive of the Harvard Management Company, published in *Financial Times*, March 22, 2007.

The minimization of loss function (6) taking account of the Phillips curve given by equations (1) leads to the following targeting rule in the sense of Svensson (2002):

$$x_t = -\frac{\lambda}{\alpha}(\pi_t - \pi^T),$$ (7)

This rule is also valid for the next period, hence we have:

$$E_t x_{t+1} = -\frac{\lambda}{\alpha}(E_t \pi_{t+1} - \pi^T).$$ (8)

Using equations (1)-(2) and targeting rules (7)-(8), we obtain the following instrument rule in the sense of Svensson (2002):

$$i_t^{cT} = \frac{\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)}{\alpha\varphi(\alpha+\lambda^2)}E_t\pi_{t+1} + \frac{\lambda^3}{\alpha\varphi(\alpha+\lambda^2)}\pi^T + \frac{\lambda}{\varphi(\alpha+\lambda^2)}\varepsilon_t^\pi + \frac{1}{\varphi}\varepsilon_t^x.$$ (9)

The optimal target of lending interest rate, i_t^{cT}, corresponding to the central bank's optimization solution, must react positively to the expected future rate of inflation if $\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)>0$. It reacts positively to variations in π^T, and shocks ε_t^x and ε_t^π. The lending interest rate defined by equation (9) is not an instrument directly manipulated by the central bank. Given its target, the central bank must use some instruments to achieve it. One instrument is the repo interest rate that we have not explicitly modeled in this paper and we assume that it is the same as the money market interest rate as the central bank is resolute to counterbalance any financial perturbation that can cause the money market interest rate to deviate significantly from the repo interest rate.

In the inflation-targeting literature, it is assumed that the central bank directly controls the interest rate affecting the goods demand. In practice, controlling the interest rate which affects economic agents' decisions is quite indirect and hence submitted to many disturbances on money and credit markets. Generally, central banks manipulate the repo interest rate in order to influence the money market interest rate and then through the mechanism of arbitrage the lending interest rate on the credit market, which is determinant for consumption and

investment. Under normal financial market conditions, the money market interest rate is almost identical to the repo interest rate if the banking sector is considered as sound and transparent. Similarly, the lending interest rate in the credit market is not far higher than that practiced on the money market for short-term borrowing between banks. Consequently, it is indifferent to fix the repo interest rate so that $i_t^m = i_t^{cT}$ (if the central bank believes that the credit market is perfect) or $i_t^c = i_t^{cT}$. Then using equation (4), we can determine the other interest rate which is not targeted in the first place.

However, disturbances in financial and corporate sectors can create dislocation on financial markets and enlarge the difference between the repo interest rate, the money market interest rate and the lending interest rate. Furthermore, absorbing negative disturbances in goods market may require a low lending interest rate which may not be within the reach of the central bank due to negative financial market disturbances and the zero-bound for the nominal repo interest rate. Non-orthodox monetary policy, such as the quantitative easing policy, must be used to ease the tension on the money market or more audaciously the credit market through strengthening banks' balance sheet and/or buying private debts on the credit market by the central bank or Treasury.

During the last decade, even though inflation-targeting central banks' principal objectives, i.e. stabilization of inflation and output gap, are relatively well achieved, too much disequilibrium on the financial markets has been accumulated, translating into bubbles in real and financial asset prices. One reason for this to happen repeatedly is that central banks do not give anymore attention to the increase in the quantity of money (or liquidity) and credits. However, they pay a particularly great attention to these aggregates when the financial system and the real economy are facing with the risk of collapsing. This asymmetrical behavior with regard to quantity of money and credits is at the origin of dramatic financial shocks that we live actually, with devastating effects on the real economy.

To avoid large self-inflicted financial shocks in the future without rejecting the recent advances in the central banking such as inflation targeting which puts accents on central bank's independence and transparency, one solution is to combine the inflation targeting with an appropriate monetary targeting (Dai (2007), Dai and Sidiropoulos (2009)) through the specification of appropriate money growth rules which are compatible with the dynamic stability of inflation expectations and asset prices. As we do not include asset prices in this simple model, we consider in the following how a well-specified feedback money growth rule can stabilize the inflation expectations, hence helping to stabilize the economy.

By manipulating only the repo interest rate to indirectly affect the lending interest rate, the central bank has no credible instrument of anchoring the inflation expectations besides the cheap talk about its firm intention to attain its inflation target. The real challenge appears whenever the economy is outside of equilibrium. When the rate of inflation moves away from the target announced by the central bank, verbal persuasion via the publication of the minutes, the monthly reports, the data, the procedures of decisions as well as the models used could be not enough to convince the public to adhere to the monetary policy of the central bank. Temporary but persistent shocks could make further difficult the conduct of monetary policy based only on the control of interest rate. In this context, private agents may find rational to lose some precious time to collect all information about the economy to form their inflation expectations instead of using only the Phillips curve, IS curve and central bank's targeting rule, not to say that using Phillips curve is also submitted to instability of the relationship in the long-run and some important pitfalls.

In this context, the central bank might be able to more effectively anchor the inflation expectations by controlling the liquidity available in the financial system. Since the constant

money growth rule has been considered as failure in stabilizing inflation and inflation expectations, we consider a feedback rule which reacts to output gap.[8]

The central bank desires that the private sector believes in its objective even though shocks can deviate the realized rate of inflation from its inflation target. Knowing that the non-financial private sector scrutinizes the money and financial markets to find out the market inflation expectations before determining its own ones, the central bank, concerned with ensuring its credibility, controls the rate of growth of the money supply (in the narrow definition) at a level, which in average is consistent with its inflation target. A money (base) growth rule can be specified to neutralize the shocks affecting the money market (if they are observable, if not, we exclude them from the feedback money growth rule) and to react to variation of output gap:

$$\mu_t = \eta \Delta x_t + \overline{\mu} + \Delta \varepsilon_t^l, \qquad \text{with } \overline{\mu} = \pi^T + l_1 \Delta \overline{x}. \tag{10}$$

This rule is similar to the one considered by Taylor (1985), McCallum (1988a, b), Judd and Motley (1991), Hess, Small and Brayton (1993), and Feldstein and Stock (1994). It is a variant of the Friedman's k percent rule, adjusted for shocks affecting the money market and variation of output gap. It is implemented thanks to a kind of rationing or limitation of access to the central liquidity by the commercial banks. It is a more flexible instrument rule since the quantity of liquidity can be modified at every instant between two interest rate decisions by the central bank. It complements well the interest rate as another powerful instrument of monetary policy. In this monetary policy strategy of duo instruments, one admits that the money growth rate is maintained in the medium and long term at a level compatible with the

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[8] For alternative feedback money growth rules in a framework which combines monetary targeting and inflation targeting, see Dai (2007).

inflation target of the central bank.[9] The most important thing is that the value of η must be chosen to ensure the dynamic stability of the adjustment process of inflation expectations.

By specifying a money growth rule, we introduce the possibility of endogenous and more complex adjustment of inflation expectations in this model. These expectations are not only concerned with inflation dynamics reflected in the Phillips curve and IS curve, but also these reflected in information concerning money and financial markets.

The monetary targeting rule given by equation (10) implies that it is not necessary for the central bank to scrupulously make the inflation target equal to the growth rate of a chosen monetary aggregate, which may be subjected to exogenous shocks or even disturbances due to speculative behaviors of financial operators. When the monetary targeting rule is well specified, the inflation target of the central bank is always realizable when the effects of shocks disappear. Although the expected and realized rates of inflation can be temporarily different from the inflation target, their difference will decreases since the dynamic stability is embedded in the economy through an appropriate control of money growth rate. Without this control, an exogenous change in the inflation expectations could lead the economy to deviate far from the equilibrium corresponding to the inflation and output objectives announced by the central bank. If the inflation target represents a potential nominal anchor of the economy, the control of money growth rate makes it more credible in the eyes of private agents and provides a kind of additional nominal anchor for their inflation expectations. Private agents could revise as fine as possible their inflation expectations given the state of the economy by using all available information, including that concerning the money and credit markets.

5. The dynamics of inflation expectations under inflation-targeting regime with feedback money growth rule

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[9] Feldstein and Stock (1996) suggest that, with periodic adjustment, a monetary aggregate can be a useful

Equilibrium solutions of endogenous variables can be easily determined once the expected future rate of inflation is determined. Consequently, we will not give more details about these solutions and will instead focus on the dynamic stability in the adjustment process of the expected rate of inflation.

We assume that the central bank can target the optimal lending interest rate i^{cT} and exclude the possibility that the zero-bound for nominal interest rate is attained. Using equations (1)-(2) and the targeting rule (7), we solve the current rate of inflation and output gap as function of the expected future rate of inflation, inflation target, and supply shocks as follows:

$$\pi_t = \frac{\alpha\beta}{\alpha+\lambda^2}\mathrm{E}_t\pi_{t+1} + \frac{\lambda^2}{\alpha+\lambda^2}\pi^T + \frac{\alpha}{\alpha+\lambda^2}\varepsilon_t^\pi, \tag{11}$$

$$x_t = -\frac{\beta\lambda}{\alpha+\lambda^2}\mathrm{E}_t\pi_{t+1} + \frac{\lambda}{\alpha+\lambda^2}\pi^T - \frac{\lambda}{\alpha+\lambda^2}\varepsilon_t^\pi. \tag{12}$$

Equation (11) can also be interpreted as a difference equation of inflation rate which relates the current rate of inflation to the expected future rate of inflation and inflation shocks. It can be solved in a forward-looking manner. Consequently, in the inflation-targeting literature, the inflation dynamic is not interesting to examine and hence generally neglected. One criticism that can be addressed to the use of equation (11) to obtain solution of the expected rate of inflation is that rational economic agents are not so rational since they will neglect all information coming from money and credit markets.

Given the target of lending interest rate, i_t^{cT}, the central bank must determine the money market interest rate that shall be attained through rule-based (and discretionary if necessary)

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intermediate target.

variations in monetary base. Using equation (4), where we substitute i_t^c by i_t^{cT} given by instrument rule (9) and eliminate x_t with the help of equation (12), we obtain:

$$i_t^m = \frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)} \mathrm{E}_t\pi_{t+1} + \frac{\lambda[(\kappa_2+\delta_2)\lambda^2-\alpha\varphi\delta_1]}{\alpha\varphi\kappa_1(\alpha+\lambda^2)} \pi^T$$
$$+ \frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)} \varepsilon_t^\pi + \frac{\kappa_2+\delta_2}{\varphi\kappa_1} \varepsilon_t^x - \frac{1}{\kappa_1} \varepsilon_t^c. \tag{13}$$

Following a variation of inflation target and potential output, a supply shock or a shock affecting the goods or credit markets (or eventually the money market if the monetary shock is not perfectly counterbalanced as is assumed in the money growth rule (10)), the private sector will revise its inflation expectations. In practice, shocks affecting money and credit markets can generate major dislocations on goods and labor markets as well as modification in inflation expectations. These shocks affect the goods markets through their effect on the lending interest rate at which the banks or investors lend to firms and households. And through this channel, they affect the level of current output, employment and inflation. Anyone who wants to form good inflation expectations cannot neglect these developments since inadequate monetary policy response to these shocks can put the economy either on diverging inflationary or deflationary paths for long times.

It is to notice that, in some introductive studies to inflation targeting (Romer, 2000; Walsh, 2002), by excluding equation LM, the model made up of AS-AD-MP equations cannot generate a dynamic adjustment process for the expected rate of inflation. In effect, the expected rate of inflation is given by the inflation target of the central bank whatever is the nature of the random shock affecting the economy. The model becomes primarily static. In other words, the endogenous variables instantaneously find their equilibrium value. The instantaneous adjustment is far from being the case in practice, in particular when the rate of inflation is weak. In effect, empirical studies show inertia in the adjustment of the rate of inflation (Gordon, 1997). Consequently, even the inflation-targeting central bank is very

credible, private agents have no reason to stick to its announced inflation target when forming their inflation expectations.

To reflect in the inflation-targeting framework the empirically observed inflation dynamics, one must either introduce persistent shocks or formulate an *ad hoc* difference equation of inflation in order to introduce a more realistic dynamic analysis of the current or expected rates of inflation.[10]

The way that we borrow here is very different from that adopted by the literature on the interest rate rules and the inflation-targeting. We admit in particular that the private agents use all information at their disposal to rationally form their inflation expectations, including that provided by the money and credit markets. This is translated by the use of equations (4)-(5) in our analysis of the dynamic adjustment of the expected rate of inflation, which also takes account of other equations of the model, including the reaction function of the central bank.

We admit that the expected rate of inflation is a predetermined variable so that it cannot instantaneously adjust to its equilibrium value.[11] This assumption is based on the fact that, in an environment of weak inflation, the adjustment of prices and wages and consequently that of inflation depend much on the price and wages contracts negotiated in the past. Due to the relatively high adjustment costs compared to the rate of inflation, the instantaneous adjustment of prices and wages is not an advantageous action for private agents. The majority of them adopt a partial adjustment as what is generally allowed in the New Keynesian models. The expected rate of inflation, which partially reflects the evolution of current

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[10] One can for example organize contests of forecasts where the best forecasters are rewarded, which makes it possible to have regular information on the inflation expectations of private agents.

[11] Assume that the current and expected rates of inflation are predetermined variable is compatible with the forward-looking rational expectations. Indeed, in dynamic models with rational expectations, the adjustment of current and expected rates of inflation takes into account the impacts of all news on the equilibrium, even if they are not instantaneously and completely reflected in the expected inflation. See Buiter and Panigirtzoglou (2003) for a similar assumption. This type of behaviour is more often observed in an environment of weak inflation.

inflation, must behave in the same manner. The expected rate of inflation, as implicitly reflected in the prices of inflation-indexed bonds, could show more stable evolution than the actual rate of inflation. Indeed, the rise of the rate of inflation in the euro area in 2008 involves only a moderate rise in the expected rate of inflation, translating the confidence of financial operators in the monetary policy of the ECB in the medium and long term, knowing that the ECB uses also the pillar of monetary analysis in its decision of monetary policy.

We examine in the following the dynamic stability of inflation expectations adjustment process under forward-looking or backward-looking solutions respectively.

Forward-looking expectations

Economic agents expect that the economy will behave in period $t+1$ in a similar manner as in the current period t. The only difference is that they cannot estimate exactly future monetary policy and shocks affecting different markets. Taking one period forward equations (5) and (10)-(13), using them together with equations (12)-(13) to eliminate other endogenous variables except expected rates of inflation, and taking expectations of resulting difference equation, we obtain (Appendix):

$$\mathrm{E}_t \pi_{t+2} = \frac{\Omega}{\Omega - \alpha^2 \beta \varphi \kappa_1} \mathrm{E}_t \pi_{t+1} + \frac{\Theta_t}{\Omega - \alpha^2 \beta \varphi \kappa_1}, \tag{14}$$

with

$$\Omega = \alpha\beta\varphi\lambda\kappa_1(l_1 - \eta) + \alpha\beta\varphi\lambda\delta_1(h + l_2) + (h + l_2)(\kappa_2 + \delta_2)[\alpha\varphi(\alpha + \lambda^2) + \lambda(\alpha\beta - \alpha - \lambda^2)],$$

$$\Theta_t = \begin{bmatrix} -\alpha\varphi\kappa_1(\alpha + \lambda^2)\bar{\mu} - \alpha\varphi\lambda^2\kappa_1\pi^T \\ +[\alpha\lambda(\kappa_2 + \delta_2 + \delta_1\varphi)(h + l_2) + \alpha\varphi\kappa_1(l_1 - \eta)]\varepsilon_t^\pi \\ +\alpha(\alpha + \lambda^2)(h + l_2)(\kappa_2 + \delta_2)\varepsilon_t^x - \alpha\varphi(\alpha + \lambda^2)(h + l_2)\varepsilon_t^c \end{bmatrix}.$$

However, it is not good working assumption in episodes of high inflation or hyperinflation where the current and expected rates of inflation adjust very quickly.

Recursively using equation (14) and assuming that future shocks are randomly distributed, the forward-looking solution of the expected rate of inflation for period $t+1$ is given by:

$$E_t \pi_{t+1} = \left[\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega} \right]^n E_t \pi_{t+1+n} - \frac{1}{\Omega} \Theta_t .$$ (15)

Given that the rate of inflation in period $t+1+n$ when $n \to \infty$ is well controlled and thus not explosive, a convergent solution of $E_t \pi_{t+1}$ satisfying equation (15) exists if the eigenvalue has a modulus superior to unity, which is translated by:

$$\left| \frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega} \right| < 1 .$$ (16)

If $\eta = 0$, i.e. the money growth rule defined by equation (10) is reduced to Friedman's k percent rule adjusted for change responding to shocks affecting the money market, we have $\Omega_0 > 0$, with $\Omega_0 = \alpha \beta \varphi \lambda \kappa_1 l_1 + \alpha \beta \varphi \lambda \delta_1 (h + l_2) + (h + l_2)(\kappa_2 + \delta_2)[\alpha \varphi (\alpha + \lambda^2) + \lambda(\alpha \beta - \alpha - \lambda^2)]$. Then according to inequality (16), the expected rate of inflation is converging to its equilibrium solution if $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 > 0$.

For $\eta > 0$, the stability condition defined by inequality (16) is always satisfied if we have simultaneously $\Omega - \alpha^2 \beta \varphi \kappa_1 > 0$ and $\Omega > 0$. These conditions are satisfied at the same time, if η is small enough so that $\Omega - \alpha^2 \beta \varphi \kappa_1 > 0$. That implies:

$$\eta < l_1 + \Phi - \frac{\alpha}{\lambda} .$$ (17)

where $\Phi = \frac{(h + l_2)\{\alpha \beta \lambda \varphi \delta_1 + (\kappa_2 + \delta_2)[\alpha \varphi (\alpha + \lambda^2) + \lambda(\alpha \beta - \alpha - \lambda^2)]\}}{\alpha \beta \lambda \varphi \kappa_1}$. Condition (17) means that the central bank must conceive a monetary targeting rule which do not respond excessively to the variation of output if $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 > 0$. A feedback rule has in this case the advantage of increasing the

modulus of the positive eigenvalue (i.e. reducing $\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega}$ since $(\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega})'_\eta =$

$-\frac{\alpha^3 \beta^2 \varphi^2 \lambda \kappa_1^2}{\Omega^2} < 0$) and increasing the speed of convergence.

Consider now the plausibility of the case where we have $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 < 0$, i.e.:

$$\alpha \beta \varphi \kappa_1 (\alpha - \lambda l_1) > (h + l_2)\{\alpha \beta \lambda \varphi \delta_1 + (\kappa_2 + \delta_2)[\alpha \varphi (\alpha + \lambda^2) + \lambda (\alpha \beta - \alpha - \lambda^2)]\}. \text{ (alphacondi)}$$

Condition (alphacondi) can be checked if the weight assigned by the central bank to the output target, α, is sufficiently high (so that $(\alpha - \lambda l_1) > 0$), κ_1 is sufficiently large, λ sufficiently small, and the terms $(h + l_2)$ and $(\kappa_2 + \delta_2)$ are sufficiently small. We notice that the terms κ_1 and $(h + l_2)$ represent respectively how the supply and demand on the money and credit markets are sensible to the money market interest rate, and term $(\kappa_2 + \delta_2)$ represents how the supply and demand on the credit market are sensible to the lending interest rate.

If $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 < 0$, then Friedman's k percent rule help for anchoring the inflation expectations if $\frac{\alpha^2 \beta \varphi \kappa_1}{2} < \Omega < \alpha^2 \beta \varphi \kappa_1$. If $\Omega < \frac{\alpha^2 \beta \varphi \kappa_1}{2}$, the economy will not converge to the equilibrium and hence indeterminate.

Consider now the design of the feedback money growth rule by defining an interval η if $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 < 0$. The latter implies that $l_1 + \Phi - \frac{\alpha}{\lambda} < 0$.

Case 1: Choosing η so that $\Omega > 0$, $\Omega - \alpha^2 \beta \varphi \kappa_1 < 0$ and condition (16) are satisfied simultaneously. Condition $\Omega > 0$ implies anther interval for η, i.e.:

$$\eta < l_1 + \Phi. \tag{18}$$

Then condition (16) implies $\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega} > -1$, that leads to $2\Omega > \alpha^2 \beta \varphi \kappa_1$ and hence:

$$\eta < l_1 + \Phi - \frac{\alpha}{2\lambda}. \tag{19}$$

Furthermore, condition $\Omega - \alpha^2 \beta \varphi \kappa_1 < 0$ yields:

$$\eta > l_1 + \Phi - \frac{\alpha}{\lambda}. \tag{20}$$

Combining conditions (18)-(20) leads to:

$$l_1 + \Phi - \frac{\alpha}{\lambda} < \eta < l_1 + \Phi - \frac{\alpha}{2\lambda}. \tag{21}$$

Condition (21) suggests that if $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 < 0$, then a feedback money growth rule with η neither too high nor too low help to anchor the inflation expectations. However, choosing a value for η in the interval defined in condition (21) could reduce more or less the speed of convergence to the equilibrium. As $\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega} > -1$ and an increase in η will reduce the value of $\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega}$ and the lattar nearer to -1, it is better to choose a value for η nearer the inferior limit given in inequality (21) in order to ensure a speedier convergence to the equilibrium.

Case 2: Choosing η so that we have simultaneously $\Omega > 0$ and $\Omega - \alpha^2 \beta \varphi \kappa_1 > 0$. These two conditions imply that $\eta < l_1 + \Phi - \frac{\alpha}{\lambda}$ (see condition (17)). The condition $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 < 0$ implies that $l_1 + \Phi - \frac{\alpha}{\lambda} < 0$ and hence $\eta < 0$. Since $\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega}$ decrease when η increases, by choosing a negative value for η which is inferior but as close as possible to $l_1 + \Phi - \frac{\alpha}{\lambda}$, we make the dynamic adjustment process stable and be able to increase at maximum the speed of convergence to the equilibrium as the central bank may desire.

Case 3: Fixing η so that $\Omega < 0$, i.e.

$$\eta > l_1 + \Phi. \tag{22}$$

Then, condition (16) yields $\frac{\Omega - \alpha^2 \beta \varphi \kappa_1}{\Omega} < 1$. The last condition cannot be checked since it leads to $\Omega < \Omega - \alpha^2 \beta \varphi \kappa_1$ or $-\alpha^2 \beta \varphi \kappa_1 > 0$.

The above discussion of dynamic stability conditions shows that a well-specified feedback money growth rule is necessary for stabilizing inflation expectations when the money and credit markets have certain characteristics. This explains why when financial innovations perturb the relationship between monetary aggregates and goal variables, simple approaches of monetary targeting which accompanies the fixation of repo interest rate by the central bank are not successful. In contrast, a well-specified feedback rule can stabilize dynamically the economy and increase the speed of convergence to the equilibrium.

Backward-looking expectations

We assume that private agents do not automatically take the central bank's announced inflation target as a credible anchor of inflation expectations. We admit the view according to which, in relatively unstable environments, they base their forecasts more on observed fluctuations than on the announcements of stabilizing monetary policies (Marimon and Sunder (1995)). The backward-looking solution of the expected rate of inflation corresponds well to this view.

Taking the difference of equations (12)-(13), we obtain:

$$\Delta x_t = -\frac{\beta\lambda}{\alpha+\lambda^2}\Delta \mathrm{E}_t \pi_{t+1} + \frac{\lambda}{\alpha+\lambda^2}\Delta\pi^T - \frac{\lambda}{\alpha+\lambda^2}\Delta\varepsilon_t^{\pi},\qquad(23)$$

$$\Delta i_t^m = \frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}\Delta \mathrm{E}_t \pi_{t+1} + \frac{\lambda[(\kappa_2+\delta_2)\lambda^2-\alpha\varphi\delta_1]}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}\Delta\pi^T$$
$$+\frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)}\Delta\varepsilon_t^{\pi} + \frac{\kappa_2+\delta_2}{\varphi\kappa_1}\Delta\varepsilon_t^{x} - \frac{1}{\kappa_1}\Delta\varepsilon_t^{c}.\qquad(24)$$

Knowing that $\Delta\pi^T = 0$ and using equations (10)-(11) and (23)-(24) to eliminate π_t, μ_t, Δx_t and Δi_t^m in equation (5), we obtain the following difference equation for the expected rate of inflation after some arrangements of terms:

$$\mathrm{E}_t \pi_{t+1} = \frac{\Omega}{\Omega-\alpha^2\beta\varphi\kappa_1}\mathrm{E}_{t-1}\pi_t + \frac{\Psi_t}{\Omega-\alpha^2\beta\varphi\kappa_1},\qquad(25)$$

$$\text{with } \Psi_t = \begin{bmatrix} -\alpha\varphi\kappa_1(\alpha+\lambda^2)\overline{\mu} - \alpha\varphi\kappa_1(\alpha+\lambda^2)l_1\Delta\overline{x} + \alpha\varphi\kappa_1\lambda^2\pi^T \\ -[\alpha\lambda(h+l_2)(\kappa_2+\delta_2) + \alpha\varphi\lambda\kappa_1(l_1-\eta) + \alpha\varphi\lambda\delta_1(h+l_2) - \alpha^2\varphi\kappa_1]\Delta\varepsilon_t^\pi \\ -\alpha(h+l_2)(\alpha+\lambda^2)(\kappa_2+\delta_2)\Delta\varepsilon_t^x + \alpha\varphi(\alpha+\lambda^2)(h+l_2)\Delta\varepsilon_t^c \end{bmatrix}.$$

Under backward-looking expectations, the adjustment process of the expected rate of inflation is stable if the modulus of the eigenvalue is inferior to unity:

$$\left| \frac{\Omega}{\Omega - \alpha^2\beta\varphi\kappa_1} \right| < 1. \tag{26}$$

When $\eta = 0$, we have always $\Omega_0 > 0$. If $\Omega_0 - \alpha^2\beta\varphi\kappa_1 > 0$, the stability condition (26) cannot be checked if we maintain $\eta = 0$ since $\Omega_0 > \Omega_0 - \alpha^2\beta\varphi\kappa_1$. The Friedman's k percent money growth rule will fail to stabilize the inflation expectations around the inflation target announced by the central bank.

In contrast, the feedback money growth rule can stabilize the inflation expectations if η is specified so that $\Omega < 0$ and hence $\Omega - \alpha^2\beta\varphi\kappa_1 < 0$. Consequently, we have $\frac{\Omega}{\Omega-\alpha^2\beta\varphi\kappa_1} < 1$ and hence the stability condition (26) is satisfied. To ensure $\Omega < 0$ and hence the dynamic stability under backward-looking expectations, η must satisfy the following condition:

$$\eta > l_1 + \Phi. \tag{27}$$

By increasing η to a value far greater than $l_1 + \Phi$, the speed of convergence could be reduce since $\frac{\Omega}{\Omega-\alpha^2\beta\varphi\kappa_1}$ will be nearer to 1. So, we must keep η superior but as close as possible to $l_1 + \Phi$.

Another case to consider is that when $\eta = 0$, we have simultaneously $\Omega_0 > 0$ and $\Omega_0 - \alpha^2\beta\varphi\kappa_1 < 0$. Specifying η so that $\Omega > 0$, $\Omega - \alpha^2\beta\varphi\kappa_1 < 0$ and condition (26) are simultaneously checked. The condition $\Omega > 0$ implies condition (18). The condition $\Omega - \alpha^2\beta\varphi\kappa_1 < 0$ is always true if η is not too small, i.e.:

$$\eta > l_1 + \Phi - \frac{\alpha}{\lambda}. \tag{28}$$

The condition $\Omega_0 - \alpha^2 \beta \varphi \kappa_1 < 0$ implies that the right hand of inequality (28) is negative. The stability condition (26) implies that we must have $\frac{\Omega}{\Omega - \alpha^2 \beta \varphi \kappa_1} > -1$, which is equivalent to $2\Omega > \alpha^2 \beta \varphi \kappa_1$. That yields:

$$\eta < l_1 + \Phi - \frac{\alpha}{2\lambda}. \tag{29}$$

Combining conditions (18) and (28)-(29) leads to the interval of η for which the dynamic stability is ensured:

$$l_1 + \Phi - \frac{\alpha}{\lambda} < \eta < l_1 + \Phi - \frac{\alpha}{2\lambda}. \tag{30}$$

Condition (30) is the same as condition (21) and implies that the growth rate must be defined over an interval with the lower limit being negative. Increasing the value of η will increase the modulus of the eigenvalue (i.e. increase $\frac{\Omega}{\Omega - \alpha^2 \beta \varphi \kappa_1}$ since $(\frac{\Omega}{\Omega - \alpha^2 \beta \varphi \kappa_1})'_\eta = \frac{\alpha^3 \beta^2 \varphi^2 \lambda \kappa_1^2}{(\Omega - \alpha^2 \beta \varphi \kappa_1)^2} > 0$) and hence the speed of convergence. Consequently, in order to increase the speed of convergence, η must be chosen to be as near as possible to $l_1 + \Phi - \frac{\alpha}{2\lambda}$.

Under backward-looking expectations, the dynamic stability of the economy can be ensured with a feedback money growth rule which responds positively and sufficiently to variations of output if the money and credit markets and the structure of the real economy have certain characteristics (i.e. $\Omega < 0$ and $\Omega - \alpha^2 \beta \varphi \kappa_1 < 0$). However, the stability must be ensured by a feedback money growth rule which can react positively or negatively in a well-defined interval to variations of output if other economic and financial conditions prevail (i.e. $\Omega > 0$ and $\Omega - \alpha^2 \beta \varphi \kappa_1 < 0$).

6. Zero interest rate policy and quantitative easing policy

Recent experiences of monetary policy at the Fed have shown that after having brought down the repo interest rate to zero, the quantitative easing policy is one of the last options that the central bank can use in a context of financial turmoil. There is no conceptual difficulty for discussing these two monetary policies in our framework,[12] since we consider already the money and credit markets while assuming that the central bank practices inflation targeting. In this section, we just briefly discuss how our framework allows examining such issues without fully carrying out the dynamic analysis as we have done before.

We have admitted that the central bank targets the lending interest rate, but cannot directly fix this latter. It must make the desired change in the lending interest rate through its action on the money market by fixing the repo interest rate, which is not explicitly considered in our model and is implicitly assimilated to the money market interest rate. In effect, the repo interest rate and the money market interest rate can be assimilated if the financial institutions are solid and transparent. Otherwise, a risk premium might be applied to money market interest rate. However, the central bank or the Treasury can reduce the premium to zero by implicitly or explicitly guaranteeing all lending on the money market or by supplying an amount of liquidity as large as demanded by financial operators.

In this model, the zero interest rate policy becomes necessary if the money market interest rate defined by equation (13) becomes zero or negative:

$$
i_t^m = \frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}\mathrm{E}_t\pi_{t+1} + \frac{\lambda[(\kappa_2+\delta_2)\lambda^2-\alpha\varphi\delta_1]}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}\pi^T
$$
$$
+ \frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)}\varepsilon_t^\pi + \frac{\kappa_2+\delta_2}{\varphi\kappa_1}\varepsilon_t^x - \frac{1}{\kappa_1}\varepsilon_t^c \leq 0. \tag{31}
$$

Since there is zero bound for the nominal interest rate, then the central bank must fix:

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[12] For a review of Japanese experience of zero interest rate policy coupled with quantitative easing policy, see Spiegel (2006).

$$i_t^m = 0 .$$ (32)

As the zero interest rate policy cannot allow the realization of the optimal lending interest rate due to malfunctioning of the money and credit markets, the effective lending interest rate determined by the credit market will be superior to the target of lending interest rate determined by equation (9). Consequently, the targeting rule (9) will not be effective. Inflation expectations dynamics and equilibrium solutions of economic variables are determined by equations (1)-(5), (10) and (32).

The zero interest rate policy can correspond to a suboptimal equilibrium since it cannot always bring down the lending interest rate to a level which is optimal for the central bank. To make the monetary policy effective, when large negative shocks on financial markets imply a need for the zero interest rate policy, the quantitative easing policy sometimes becomes necessary. The quantitative easing policy, targeting the liquidity in the banking and financial system, is used in order to allow increasing the supply on the credit markets and so to bring down the lending interest rate to its target level. The quantitative easing policy can be directed to the money market only or the money and credit markets simultaneously, it modifies equations (3)-(4) as follows:

$$q^m + b_t + h i_t^m - p_t = l_1 x_t - l_2 i_t^m + \varepsilon_t^l ,$$ (33)

$$q^e - \kappa_1 i_t^m + \kappa_2 i_t^c = \delta_1 x_t - \delta_2 i_t^c + \varepsilon_t^c ,$$ (34)

where q^m and q^e represent the discretionary injection of liquidity on the money and credit markets respectively.

If the quantitative easing policy is fully executed so that the target and effective lending interest rate are equalized (i.e., $i_t^c = i_t^{cT}$) under the zero interest rate policy (i.e., $i_t^m = 0$), the economic system can be described by equations (10)-(12) and (33)-(34). These equations allow examining the adjustment dynamics of inflation expectations and determining the

equilibrium solutions of endogenous variables under the zero interest rate policy coupled with the quantitative easing policy. If the quantitative easing policy is enough aggressive, the liquidity trap accompanied by expectations of deflation is avoidable as a situation of equilibrium.

7. Conclusion

In a closed economy New-Keynesian model in which we introduce imperfect money and credit markets, we have shown that inflation-targeting central banks have good reasons to use monetary targeting together with inflation targeting. Considering that the cheap talk of central bankers is not sufficient to ensure the announced inflation target as credible nominal anchor of private inflation expectations, and that the money growth rate can be adjusted more flexibly to answer to shocks affecting real as well as money and financial markets between two interest rate decisions by the central bank, we defend the idea that the quantity of money must be regulated with a rule but not in the way conceived by Milton Friedman who proposes a k percent money growth rule. This view find strong support in recent financial and economic turmoil where many central banks massively inject liquidity in the financial system to avoid the collapse of financial and economic system and where using interest rate rule is not anymore sufficient. Another support is found in the long term relationship between money and inflation found in empirical study, which is not a simple correlation but a causal relationship in this sense that a high growth rate of money supply will systematically lead to high inflation rate. A strict control of money supply allows always controlling the inflation rate in the medium and long term.

The model is sufficiently rich so it allows illustrating the complex transmission mechanism of monetary policy, which may be perturbed by malfunctioning of the money and

credit markets or shocks affecting these markets. We can use it to explain why the central bank cannot control directly (and hence always perfectly) the lending interest rate which affects effectively the investment and consumption by modifying only the repo interest rate. We have shown that the inflation expectations are not easily controlled in this framework as it is optimistically conceived in the mainstream literature on interest rate rules and inflation-targeting. As the central bank controls the growth of money supply by limiting the access to the central liquidity in order to ensure good functioning of money and credit markets, private agents will give much attention to this strategy and consider that future inflation depends narrowly on the monetary growth rule as well as developments in money and credit markets. A well-conceived money growth rule, according to the type of expectations (forward-looking or backward-looking) and depending on the structural parameters of the economy, can help anchor the inflation expectations and increase the speed of convergence to the equilibrium. Finally, our framework can be easily used to discuss the recent developments in the monetary policy strategy of the Fed which adopts quantitative easing policy after having adopted zero interest rate policy.

Appendix: The difference equation under forward-looking expectations

Taking one period forward equations (5) and (10)-(13) yields:

$$\mu_{t+1} + h(i_{t+1}^m - i_t^m) - \pi_{t+1} = l_1(x_{t+1} - x_t) - l_2(i_{t+1}^m - i_t^m) + (\varepsilon_{t+1}^l - \varepsilon_t^l), \tag{A1}$$

$$\mu_{t+1} = \eta(x_{t+1} - x_t) + \overline{\mu} + (\varepsilon_{t+1}^l - \varepsilon_t^l). \tag{A2}$$

$$\pi_{t+1} = \frac{\alpha\beta}{\alpha + \lambda^2} E_{t+1}\pi_{t+2} + \frac{\lambda^2}{\alpha + \lambda^2}\pi^T + \frac{\alpha}{\alpha + \lambda^2}\varepsilon_{t+1}^\pi, \tag{A3}$$

$$x_{t+1} = -\frac{\beta\lambda}{\alpha + \lambda^2} E_{t+1}\pi_{t+2} + \frac{\lambda}{\alpha + \lambda^2}\pi^T - \frac{\lambda}{\alpha + \lambda^2}\varepsilon_{t+1}^\pi, \tag{A4}$$

$$i_{t+1}^m = \frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)} E_{t+1}\pi_{t+2} + \frac{\lambda[(\kappa_2+\delta_2)\lambda^2-\alpha\varphi\delta_1]}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}\pi^T$$
$$+ \frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)}\varepsilon_{t+1}^\pi + \frac{\kappa_2+\delta_2}{\varphi\kappa_1}\varepsilon_{t+1}^x - \frac{1}{\kappa_1}\varepsilon_{t+1}^c. \tag{A5}$$

Using equations (12)-(13) and (A4)-(A5) to calculate the following difference equations:

$$i_{t+1}^m - i_t^m = \frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}(E_{t+1}\pi_{t+2}-E_t\pi_{t+1}) + \frac{\lambda[(\kappa_2+\delta_2)\lambda^2-\alpha\varphi\delta_1]}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}\Delta\pi^T$$
$$+ \frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)}(\varepsilon_{t+1}^\pi - \varepsilon_t^\pi) + \frac{\kappa_2+\delta_2}{\varphi\kappa_1}(\varepsilon_{t+1}^x - \varepsilon_t^x) - \frac{1}{\kappa_1}(\varepsilon_{t+1}^c - \varepsilon_t^c), \tag{A6}$$

$$x_{t+1} - x_t = -\frac{\beta\lambda}{\alpha+\lambda^2}(E_{t+1}\pi_{t+2}-E_t\pi_{t+1}) + \frac{\lambda}{\alpha+\lambda^2}\Delta\pi^T - \frac{\lambda}{\alpha+\lambda^2}(\varepsilon_{t+1}^\pi - \varepsilon_t^\pi). \tag{A7}$$

Taking account of the assumption of constant inflation target, i.e. $\Delta\pi^T = 0$, then using equations (A2)-(A3) and (A6)-(A7) to eliminate other endogenous variables except the expected rates of inflation in equation (5b), we obtain:

$$(h+l_2)\left[\begin{array}{c}\frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}(E_{t+1}\pi_{t+2}-E_t\pi_{t+1}) + \frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)}(\varepsilon_{t+1}^\pi - \varepsilon_t^\pi) \\ + \frac{\kappa_2+\delta_2}{\varphi\kappa_1}(\varepsilon_{t+1}^x - \varepsilon_t^x) - \frac{1}{\kappa_1}(\varepsilon_{t+1}^c - \varepsilon_t^c)\end{array}\right]$$
$$-\frac{\alpha\beta}{\alpha+\lambda^2}E_{t+1}\pi_{t+2} + \frac{\lambda^2}{\alpha+\lambda^2}\pi^T + \frac{\alpha}{\alpha+\lambda^2}\varepsilon_{t+1}^\pi = (l_1-\eta)\left[-\frac{\beta\lambda}{\alpha+\lambda^2}(E_{t+1}\pi_{t+2}-E_t\pi_{t+1}) - \frac{\lambda}{\alpha+\lambda^2}(\varepsilon_{t+1}^\pi - \varepsilon_t^\pi)\right] - \bar{\mu}. \tag{A8}$$

Taking mathematical expectations of equation (A8)conditional on information available at time t and assuming that the best estimates at time t of future shocks are zero, i.e. $E_t\varepsilon_{t+1}^l = E_t\varepsilon_{t+1}^\pi = E_t\varepsilon_{t+1}^c = E_t\varepsilon_{t+1}^x = 0$, we obtain:

$$(h+l_2)\left[\begin{array}{c}\frac{(\kappa_2+\delta_2)[\alpha\varphi(\alpha+\lambda^2)+\lambda(\alpha\beta-\alpha-\lambda^2)]+\alpha\beta\lambda\varphi\delta_1}{\alpha\varphi\kappa_1(\alpha+\lambda^2)}(E_t\pi_{t+2}-E_t\pi_{t+1}) \\ -\frac{\lambda(\kappa_2+\delta_2+\delta_1\varphi)}{\varphi\kappa_1(\alpha+\lambda^2)}\varepsilon_t^\pi - \frac{\kappa_2+\delta_2}{\varphi\kappa_1}\varepsilon_t^x + \frac{1}{\kappa_1}\varepsilon_t^c\end{array}\right] -\frac{\alpha\beta}{\alpha+\lambda^2}E_t\pi_{t+2} + \frac{\lambda^2}{\alpha+\lambda^2}\pi^T$$
$$= (l_1-\eta)\left[-\frac{\beta\lambda}{\alpha+\lambda^2}(E_t\pi_{t+2}-E_t\pi_{t+1}) + \frac{\lambda}{\alpha+\lambda^2}\varepsilon_t^\pi\right] - \bar{\mu}. \tag{A9}$$

Simplifying and rearranging the terms in equation (A9) leads to the difference equation (14).

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