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Robust Monetary Policy under Model Uncertainty and Inflation Persistence

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Abstract

This paper examines the relationship between the preference for robustness of central bank (when it fears that its model is misspecified), the inflation persistence and the output cost of disinflation. Using a simple monetary game model in which higher preference for robustness of central bank is positively associated with the inflation persistence and thus negatively with the speed of disinflation, this paper shows that the output cost of disinflation is higher when the less the central bank believes that its reference model is robust.

Keywords : Model uncertainty, Robust control, Minmax policies, Inflation persistence, Sacrifice ratio.

JEL classification : E50, E52, E58.

1 Introduction

Over the last decade, there is a rapidly growing literature to explain the high degree of inflation persistence observed in the data. Recently, a research network of economists from the national central banks of the euro area and the European Central Bank (ECB) has been investigating the empirical evidence for inflation persistence, its determinants and implications for monetary policy (see Angeloni *et al.*, 2003 for a summary), and alternative interpretations are proposed to explain and remedy the high inflation persistence found in post-war U.S. data (Taylor, 2000a and Cogley and Sargent, 2001).

In this context, several authors (Fuhrer and Moore, 1995; Fuhrer, 2000; Calvo *et al.*, 2001; Christiano *et al.*, 2005 and Blanchard and Gali, 2007) have proposed

different mechanisms to build inflation persistence into the deep structure of the economy. Another point of view suggests that the degree of inflation persistence is not only an inherent structural characteristic of industrial economies, but rather may be due to changes in the orientation of monetary policy (Sargent, 1999; Taylor, 2000b; Goodfriend and King, 2001 and Westelius, 2005). Thus, in an environment with independence of central bank and transparency of monetary policy, inflation expectations may become contained and, hence, price and wage setters may be less inclined to change their contracts in response to shocks. In this respect, the way that economic agents perceive major changes in the policy regime, such as central bank independence and transparency of monetary policy, has important implications for the short-run impact of monetary policy changes. In effect, the speed at which the economy reacts to a change in its environment may be slow and the economy may not move quickly to a new equilibrium with lower price expectations.

On the other hand, the degree of inflation persistence represents a key parameter of the monetary transmission mechanism and thus, it has important implications for the ability of monetary policy to stabilize inflation relative to output as well as a significant impact on the output cost of disinflation or sacrifice ratio (Jordan, 1999 and Diana and Sidiropoulos, 2004). Therefore, the question of how monetary policy should be set optimally when the structure of the economy exhibits inflation persistence is crucial for monetary policy authorities.

However, the above literature is focused on how the characteristics of the central banks may influence the inflation persistence without taking into account the fact that policymakers do not have a complete knowledge of the true structure of the economy. Thus, without the possibility to have a complete description of reality, a policymaker is likely to prefer basing policy on principles that are also valid if the assumptions on which the model is founded differ from reality. Actually, models rest on a set of assumptions that may or may not be good approximations of true economies. In other words, policy prescriptions should be robust to reasonable deviations from the benchmark model.

A relevant example seems to be the euro area for which such an examination seems particularly important. First, the euro area is a new and relatively unexplored entity and, hence, the ECB faces substantial uncertainty about the characteristics of the aggregate euro area inflation process. Second, the mixed empirical evidence based on data for individual euro area member states provides no clear indication of what type of model should be chosen for modelling the aggregate inflation process. Hence, monetary policy rules should ideally be designed to perform reasonably well under a range of alternative models of inflation determination.

The growing literature on monetary policy robustness has been developed into three directions. The first one leads to what has been called robustly optimal instrument rules (Svensson and Woodford, 2004; Giannoni and Woodford, 2003a, 2003b). As these instrument rules do not depend on the specification of the generating processes of exogenous disturbances in the model, they are, therefore, robust to misspecification in these processes. The second one, initiated by Hansen and Sargent (2001, 2003, 2007), corresponds to robust control approach to the decision problem of agents who face model uncertainty. In the sense of Hansen and Sargent, robust monetary policies are designed to perform well in worst-case scenarios. These policies arise as the equilibrium in a game between the monetary authorities and an evil agent who chooses model misspecification to make the authorities look as bad as possible. The third approach to robust control is called structured Knightian uncertainty where the uncertainty is assumed to be located in one or more specific parameters of the model, but where the true values of these parameters are known only to be bounded between minimum and maximum conceivable values (Onatski and Stock, 2002; Giannoni, 2002, 2007; Tetlow and von zur Muehlen, 2004).

This paper investigates the degree of persistence characterising the inflation process when the monetary policy-maker is faced with uncertainty about the model. In this economic environment, the objective of our paper is to examine analytically the effects of increased model uncertainty, in the sense of Hansen and Sargent (2007), on the inflation persistence and the sacrifice ratio using a simple Barro-Gordon economy model and where inflation persistence is introduced through the data generating process for the structural shocks hitting the economy. We show that inflation persistence is greater when the central bank has higher preference for model robustness.

The paper is structured as follows. Section 2 presents the basic model. Section 3 introduces misspecification and solves the equilibrium for the worst case model. Section 4 analyzes the relationship between inflation persistence and preference for robustness and the effects of this latter on the sacrifice ratio. Section 5 concludes.

2 The model

As in Diana and Sidiropoulos (2004), we consider a simple monetary game model extended to allow for persistent stochastic supply-side shocks (Rogoff, 1985) and indexed wage contracts (Ball, 1994; Gray, 1976). Output is given by using a Cobb-Douglas production function to transform the sole variable input, homogeneous labor, in combination with other, fixed, factors of production. Thus, the relationship between output and employment at the aggregate level is described by the following log-linear production function:

$$y_t = \alpha l_t + u_t, \quad 0 < a < 1, \tag{1}$$

where y_t is the log of output, l_t is the log of employment and the parameter u_t represents a random supply-side shock to production technology (to be specified below). Firms decide on labor demand, l_t^d , and output by maximizing their profits, i.e.,

$$l_{t}^{d} = \ln \arg \max_{L_{t}} \left\{ P_{t} Y_{t} - W_{t} L_{t} \mid Y_{t} = L_{t}^{\alpha} \cdot \exp(u_{t}) \right\},$$
(2)

where capital letters denote the according non-logarithmic variables. Thus, by equalizing the marginal product of labor to the real wage, the labor demand function is

$$l_t^d = \tilde{l} - \frac{1}{1 - \alpha} (w_t - p_t - u_t), \quad \tilde{l} > 0,$$
(3)

where $l = ln(\alpha)/(1 - \alpha)$, w_t is the log of nominal wage and p_t the log of output price in time t. All workers are members of the economy-wide union, with the available supply labour, l_t^s , given by:

$$l_t^s = \tilde{l} - \delta + \eta (w_t - p_t), \quad \delta > 0, \quad \eta \ge 0, \tag{4}$$

where the intercept term in (4) is not set equal to that of the demand for labour because we assume that the labour supply is affected by distortions in the labour market, captured by the parameter δ . Equating (3) and (4), and assuming, without any loss of generality, that $\eta = 0$ (i.e., desired supply of labor is assumed to be completely inelastic), we obtain:

$$\widehat{w}_t = p_t + \delta(1 - \alpha),\tag{5}$$

where \widehat{w}_t is the market-clearing or competitive equilibrium nominal wage that would arise in the absence of nominal wage contracts and leads to the following competitive equilibrium output level : $\widehat{y}_t = \widetilde{y} - \kappa + u_t$, with $\widetilde{y} = \alpha \widetilde{l}$ and $\kappa = \alpha \delta$.

The treatment of wage determination, like that of Ball (1994), follows Gray (1976). Wage contracts are negotiated and signed at the beginning of each period, prior to the observation of the disturbances. These contracts specify a base wage, $E_{t-1}\hat{w}_t$, set at the expected market-clearing value, i.e., such that $E_{t-1}l_t^d = l_t^s$, and an indexation parameter, γ , relating the actual nominal wage to unexpected movements in the price level, $(p_t - E_{t-1}p_t)$, following the indexing rule:

$$w_t = E_{t-1}\widehat{w}_t + \gamma(p_t - E_{t-1}p_t), \quad 0 \le \gamma \le 1,$$
 (6)

where E_{t-1} is the rational expectation operator and γ is the indexing parameter. For $\gamma = 1$, wages are fully indexed, for $0 < \gamma < 1$, wages are partially indexed and for $\gamma = 0$, there is no indexation. Thus, a moral hazard problem arises, justifying the incentive of workers to index their nominal wages to unexpected price movements.

Once contracts are signed, workers are committed to supplying whatever amount of labor firms demand, and employment, l_t , is purely demand determined. The output is determined only by the level of employment since capital is assumed fixed. Integrating thus the equation (6) into (3) and using (1), we obtain the following aggregate output supply function:

$$y_t = (\tilde{y} - \kappa) + \xi (1 - \gamma)(\pi_t - E_{t-1}\pi_t) + (1 + \xi)u_t , \ \xi = \alpha/(1 - \alpha) > 0, \quad (7)$$

where $\pi_t (= p_t - p_{t-1})$ is the inflation rate and $E_{t-1}\pi_{t-1} (= E_{t-1}p_t - p_{t-1})$ is the expected inflation rate. Finally, the parameter u_t , representing a random supply-side shock in this model, is assumed to pursue the following process:

$$u_t = \phi u_{t-1} + \epsilon_t, \quad 0 \le \phi \le 1, \tag{8}$$

where ϕ is the degree of autocorrelation in random supply-side shocks and ϵ_t is a normally distributed random variable with zero mean and a variance varying with ϕ so as to standardize the variance of u_t at σ_u^2 , i.e., $\epsilon_t \sim N[0, (1 - \phi^2)\sigma_u^2 I]$. Note that the specification of the way in which supply-side shocks evolve over time in (8) is motivated by the choice of a simpler way to introduce the inflation persistence in this model (see Bleaney, 2001; Diana and Sidiropoulos, 2004) than through models including the inertia that overlapping wage contracts impart to the inflation rate (Taylor, 1980). In this context, the autocorrelation coefficient ϕ shows that a random productivity shock is persistent and, since u_t depends on u_{t-1} , this shock will be transmitted forward in time generating thus an inflation persistence.

3 Robust monetary policy

To design the robust monetary policy, the central bank takes into account a certain degree of model misspecification by minimizing its objective function in the worst possible model within a given set of plausible models.

3.1 Introducing misspecification

We clarify here the monetary policy by assuming that the central bank sets the inflation rate, π_t , to minimize a standard objective function that is quadratic in deviations of the output and inflation from their targets levels :

$$\min_{\{\pi_t\}} \mathcal{E}^{\infty}_{t=0} \beta^t \left[(y_t - y^*)^2 + \lambda \pi_t^2 \right], \quad \lambda > 0$$
(9)

where y^* and π^* are respectively the output and inflation targets, with π^* normalized to zero without any loss of generality. The parameter λ denotes the central bank's weight on inflation stabilization relative to the output stabilization and the output target, $y^* = \hat{y}_t + \kappa$, is expressed in percentage points above the natural output, \hat{y}_t . However, the central bank has an uncertainty about model misspecification. Even if the model (7) is seen as the most likely model, the central bank admits that this reference model may be misspecified. For that reason, it requests to design its monetary policy to be robust against deviations from the reference model. To formalize these uncertainties about model misspecification, we follow Hansen and Sargent (2007) and introduce in equation (7) a specification error, designated by v_t . Thus, the misspecified model is given by

$$y_t = (\tilde{y} - \kappa) + \xi (1 - \gamma)(\pi_t - E_{t-1}\pi_t) + \phi (1 + \xi)u_{t-1} + \epsilon_t + v_t$$
(10)

The two disturbances terms have different properties. The term u_t is assumed to be a random error with a prior known stochastic properties, whilst v_t represents, in the spirit of robust control, a totally ambiguous model misspecification error, in the sense that the policymaker is not able to assign any prior probability distribution to v_t . The model with $v_t = 0$ represents the reference model, while the models with $v_t \neq 0$ represent candidate models surrounding the reference model. In this context, as the central bank is assumed to be unable to provide a probability distribution over different deviations from the reference model, it instead designs its monetary policy to be optimal in the worst possible outcome within a neighborhood of reference model. Hence, the central bank's doubts for misspecification may be formalized by assuming that the worst-case specification errors are chosen by a fictitious *evil agent* to maximize central bank loss subject to some constraints specified below.

Thus, the worst-case model is the model in which the central bank selects the inflation rate to minimize its loss function while the evil agent selects the specification errors to maximize loss. This is the outcome that the central bank worries the most and against which it desires monetary policy to be robust. On the other hand, a more likely outcome of the model is one where the central bank sets policy and agents form expectations to reflect misspecification in the worst-case model. However, when there is no such misspecification, the reference model turns out to be correct.

3.2 Setting up the control problem

In this context, the central bank allocates, according to its preference for robustness, a budget, h, to the evil agent, that is used to create misspecification in equation (7). The standard robust control problem would have a common budget constraint on misspecification in all equations of the model. This budget constraint is then given by

$$\mathbf{E}_{t=0}^{\infty}\beta^{t}\left(\upsilon_{t}\right)^{2} \leq h_{t}^{2},\tag{11}$$

where the parameter h_t bounds the square of the central banks specification error v_t^2 . Thus, the size of the distortion term v_t must be bounded as the central bank's reference model remains an approximation of the real world system. Following Hansen and Sargent (2007), the robust monetary policy is obtained by solving the minmax problem

$$\min_{\{\pi_t\}} \max_{\{h_t\}} E_{t=0}^{\infty} \beta^t \left[(y_t - y^*)^2 + \lambda \pi_t^2 - \theta v_t^2 \right],$$
(12)

subject to the misspecified model (10) end the evil agent's budget constraint in (11). The central bank thus sets the inflation rate to minimize the value of its loss function, while the evil agent sets its controls to maximize the central bank's loss. The Lagrangian for this problem is given by

$$E_{t=0}^{\infty} \beta^{t} \left[(y_{t} - y^{*})^{2} + \lambda \pi_{t}^{2} - \theta v_{t}^{2} \right] - \mu_{t} \left[y_{t} - (\tilde{y} - \kappa) - \xi (1 - \gamma) (\pi_{t} - E_{t-1} \pi_{t}) - \phi (1 + \xi) u_{t-1} - \epsilon_{t} - v_{t} \right] (13)$$

where μ_t is the Lagrange multiplier on the constraint (10) and the parameter θ denotes the desire to be robust. This parameter is related to the evil agents budget h, and determines the set of models available to the evil agent against which the policymaker indicates the degree of model uncertainty, as well as the central bank's preference for robustness. As h approaches zero, the parameter θ approaches to infinity ($\theta \to \infty$), and the degree of misspecification approaches zero. This represents the case without model uncertainty. Inversely, a smaller value of θ means an increasing degree of model uncertainty inducing greater preference for robustness.

3.3 Optimality conditions

Assuming that neither the central bank nor the evil agent has access to any commitment mechanism, we take expectations as given in the optimization and look for a discretionary equilibrium. From the first-order conditions we derive the following optimality conditions relating output, y_t , inflation, π_t , and the degree of misspecification, v_t , to each other:

$$y_t - y^* = -\left[\frac{\lambda}{(1-\gamma)\xi}\right]\pi_t,\tag{14}$$

$$\upsilon_t = \frac{1}{\theta} \left(y_t - y^* \right). \tag{15}$$

Combining equations (14) and (15) we get

$$\pi_t = -\frac{\theta}{\lambda} \xi (1 - \gamma) \upsilon_t. \tag{16}$$

An interesting implication of these results is that the optimal inflation-output trade-off in equation (14) is not affected by the presence of model uncertainty illustrated by the central bank's preference for robustness θ (see Walsh, 2004). On the other hand, equation (14) shows that the optimal monetary policy leans against the wind, reducing the output when inflation is high. The coefficient

 $[\lambda/(1-\gamma)\xi]$ of the optimal trade-off illustrates that if the central bank assigns a large weight on inflation stabilization (λ) or if monetary policy has stronger effects on inflation through the output (γ is large and ξ is small), the optimal trade-off is steeper, so the central bank reduces output more when inflation is high.

We consider also that the worst-case specification error v_t in equation (16) is larger in absolute value when inflation is far away from steady state. This error tends to push inflation even further away, through specification errors in the aggregate output supply function (10). This specification error forces the central bank to move the output further to achieve the desired trade-off between inflation and the output [see equation (14)]. When the output y_t is below its target y^* , the misspecification is tend to lower the production by reinforcing a negative supply shock, and on the other hand, increases the inflation rate. As long as the central bank wants to be robust (so $\theta < \infty$), the policymaker will fear misspecification in this equation.

3.4 Solving the worst-case model

To find a closed-form solution for the robust control problem, we will look for the worst-case solution for the endogenous variables, π_t and y_t , and the worst possible degree of misspecification or the evil agent's instrument, v_t . This equilibrium solution illustrates the central bank's worst fears of misspecification and therefore helps us to understand the design of the robust monetary policy.

We begin by looking for the optimal robust policy rule. The central bank sets its policy instrument π_t in order to minimize the expected value of the loss function (12), taking $E_{t-1}\pi_t$ and u_{t-1} as given, and after observing the currentperiod supply shock. This yields :

$$\pi_t = \frac{\theta \xi (1-\gamma)}{\lambda (\theta-1) + \theta \xi^2 (1-\gamma)^2} \left[\kappa + \xi (1-\gamma) E_{t-1} \pi_t - \xi \left(\phi u_{t-1} + \epsilon_t \right) \right].$$
(17)

Then, assuming rational expectations for the private agents, we get :

$$E_{t-1}\pi_t = \left(\frac{\theta}{\theta-1}\right)\frac{\xi}{\lambda}(1-\gamma)(\kappa-\xi\phi u_{t-1}).$$
(18)

The next step is to find the solutions of the worst-case model. Using equations (17), (18) and (10), we will thus find a solution for the endogenous variables π_t and y_t , as:

$$\pi_t = \left(\frac{\theta}{\theta - 1}\right) \frac{\xi}{\lambda} (1 - \gamma) (\kappa - \xi \phi u_{t-1}) - \frac{\theta \xi^2 (1 - \gamma)}{\lambda (\theta - 1) + \theta \xi^2 (1 - \gamma)^2} \epsilon, \quad (19)$$

$$y_t = \tilde{y} - \left(\frac{\theta}{\theta - 1}\right)\kappa + \left(1 + \frac{\xi\theta}{\theta - 1}\right)\phi u_{t-1} + \left[1 + \frac{\theta\lambda\xi}{\lambda(\theta - 1) + \theta\xi^2(1 - \gamma)^2}\right]\epsilon_t, \quad (20)$$

and the worst possible degree of model misspecification will be given by

$$\upsilon_t = -\left(\frac{1}{\theta - 1}\right)\left(\kappa - \xi\phi u_{t-1}\right) + \frac{\lambda\xi}{\lambda(\theta - 1) + \theta\xi^2(1 - \gamma)^2}\epsilon_t.$$
 (21)

The above solution for the worst-case model is the reduced form under the worst possible case of misspecification. In this context, the evil agent chooses the specification errors to be as damaging as possible, and the optimal policy rule of the central bank and the expectations of the private sector reflect this misspecification. Equations (17) and (18) are both the reaction functions describing the central bank's policy rule and private agents' expectations of inflation. Both of them are dependent on the preference for robustness (θ) as the central bank fears misspecification in the model. Since the private sector shares the doubt of central bank about the reference model, it takes into account this uncertainty when forming their expectations. Equations (19) and (20) show that the equilibrium solutions for π_t and y_t depend not only on the central bank's preference for robustness (θ) and supply-side shocks of the current period, ϵ_t , but also that of the last period, u_{t-1} . Equation (21) determines the worst possible degree of misspecification or the evil agent's instrument, v_t , which is restricted to respond to the same variables as the policymaker.

Using the above solution of the worst-case model, we next analyze how an increase in central bank preference for robustness (that is, a decrease in θ) affects the economy on the equilibrium. We consider modest preference for robustness (so that, the worst-case model misspecification is not easily identified by the policymaker) and we analyze the effects of small decreases in θ starting from $\theta = \infty$. Thus to understand the effects of robustness on monetary policy we first study the worst-case model for inflation, output and inflation expectations. We establish the following propositions:

Proposition 1 In the worst-case model, a stronger preference for robustness of the central bank against misspecification (i.e., a decrease in θ) increases the sensitivity of inflation (π_t), as well as the sensitivity of output (y_t) to the supply-side shocks (ϵ_t).

Proof. Using equation (19) of the worst-case model to obtain the effects of a change in ϵ_t on the value of inflation π_t , we obtain:

$$\frac{\partial \pi_t}{\partial \epsilon_t} = -\frac{\theta \xi^2 (1-\gamma)}{\lambda(\theta-1) + \theta \xi^2 (1-\gamma)^2} \quad \text{and} \quad \frac{\partial y_t}{\partial \epsilon_t} = \frac{\theta \lambda \xi}{\lambda(\theta-1) + \theta \xi^2 (1-\gamma)^2} .$$
(22)

Then, the effect of a decrease in θ on the absolute value of the above derivatives (22) is:

$$-\frac{\partial \left|\frac{\partial \pi_t}{\partial \epsilon_t}\right|}{\partial \theta} = \frac{\lambda \xi^2 (1-\gamma)}{\left[\lambda(\theta-1) + \theta \xi^2 (1-\gamma)^2\right]^2} > 0,$$
(23)

$$-\frac{\partial \left|\frac{\partial y_t}{\partial \epsilon_t}\right|}{\partial \theta} = \frac{\xi \lambda^2}{\left[\lambda(\theta-1) + \theta \xi^2 (1-\gamma)^2\right]^2} > 0.$$
(24)

As $\theta > 1$ and $\gamma \in (0, 1)$, we get $\partial \pi_t / \partial \epsilon_t < 0$, $\partial y_t / \partial \epsilon_t < 0$, and $-\partial \left| \frac{\partial \pi_t}{\partial \epsilon_t} \right| / \partial \theta < 0$, $-\partial \left| \frac{\partial y_t}{\partial \epsilon_t} \right| / \partial \theta > 0$.

Proposition 2 In the worst-case model, a stronger preference for robustness of the central bank against the model misspecification (i.e., a decrease in θ) increases the expected inflation, increases the sensitivity of expected inflation to the inflation bias ($\kappa = \alpha \delta$) due to the labor market distortions (δ), but decreases the sensitivity of expected inflation to the central bank's preference for inflation target (λ).

Proof. To establish this proposition, we use (18) to derive the effects of a decrease in θ on the value of expected inflation as:

$$-\frac{\partial \left(E_{t-1}\pi_t\right)}{\partial \theta} = \frac{\xi(1-\gamma)}{\lambda^2(\theta-1)} \left(\kappa - \xi \phi u_{t-1}\right) > 0 \tag{25}$$

As $\theta > 1$ and $\gamma \in (0, 1)$, and assuming that $\kappa > \xi \phi u_{t-1}$, we obtain $-\partial (E_{t-1}\pi_t) / \partial \theta > 0$. Then, the effects of a decrease in θ on the absolute values of $\partial (E_{t-1}\pi_t) / \partial \lambda$ and $\partial (E_{t-1}\pi_t) / \partial \delta$ are given by

$$-\frac{\partial \left|\frac{\partial (E_{t-1}\pi_t)}{\partial \lambda}\right|}{\partial \theta} = -\frac{\xi(1-\gamma)}{\lambda^2(\theta-1)^2} \left(\kappa - \xi \phi u_{t-1}\right) < 0$$
(26)

$$-\frac{\partial \left|\frac{\partial (E_{t-1}\pi_t)}{\partial \delta}\right|}{\partial \theta} = \frac{\alpha \xi (1-\gamma)}{\lambda (\theta-1)^2} > 0.$$
(27)

$$-\frac{\partial \left|\frac{\partial (E_{t-1}\pi_t)}{\partial \phi}\right|}{\partial \theta} = -\frac{\xi^2 (1-\gamma) u_{t-1}}{\lambda (\theta-1)^2} < 0$$
(28)

Note that in the above results a positive sign implies that the variable in question becomes more sensitive to that particular shock when robustness increases, and vice versa. A lower value of the parameter θ corresponds to a stronger fear of the central bank for model misspecification. In this context, the above results (23) and (24) show that the less central bank considers that its reference model is robust (or central bank has a stronger preference for robustness), the higher the sensitivity of inflation to the supply-side shocks is. These results reveal that the robust central bank fears that inflation and output gap are more sensitive to shocks and therefore more volatile than in the reference model, as the worst-case misspecification increases the volatility of all variables (see Leitemo and Söderström, 2008). Equation (25) shows that a fear of model misspecification leads to a higher expected inflation. As the central bank considers that its reference model is less robust (stronger preference for robustness) and since the private sector shares the doubt of central bank about the reference model, it takes into account this doubt when forming its expectations of inflation. On the other hand, (26) shows that the less the central bank considers that its reference model is robust (a stronger preference for robustness), the higher is the sensitivity of expected inflation to the central bank's preference for the inflation target (or aversion for inflation) is. That is, the robust central bank fears that expected inflation is more volatile than in the reference model and responds more aggressively to variations on the weight assigned to the inflation target.

4 Monetary policy and inflation persistence

The main focus of our next analysis concerns the relationship between the central bank's preference for robustness and the inflation persistence as well as the resulting output cost of disinflation.

4.1 Introducing inflation persistence

We attempt here to determine the relationship between the parameter of preference for robustness θ and the degree of inflation persistence. The inflation persistence, captured by the relation : $\pi_t = \rho_{\pi} \pi_{t-1}$, may be calculated by using the correlation coefficient ρ_{π} between π_t and π_{t-1} , as :

$$\rho_{\pi} = \operatorname{Cov} \left(\pi_t, \pi_{t-1} \right) / \operatorname{Var} \left(\pi_t \right).$$
(29)

To determine $\operatorname{Var}(\pi_t) \equiv E\left[(\pi_t - \bar{\pi})^2\right]$ and $\operatorname{Cov}(\pi_t, \pi_{t-1}) \equiv E\left[(\pi_t - \bar{\pi})(\pi_{t-1} - \bar{\pi})\right]$, the unconditional mean of the inflation $\bar{\pi}$ can be written as :

$$\bar{\pi} = \frac{\theta}{\lambda(\theta - 1)} \xi(1 - \gamma) \kappa.$$
(30)

Due to the overly ambitious output target of the policymaker, κ , the unconditional inflation is not zero. In fact, this inflation bias depends on the degree of model uncertainty. Combining equations (19) and (30), the difference of inflation rate from its mean is

$$\pi_t - \bar{\pi} = -\theta \xi^2 (1 - \gamma) \left[\frac{\phi}{\lambda(\theta - 1)} u_{t-1} + \frac{1}{\lambda(\theta - 1) + \theta \xi^2 (1 - \gamma)^2} \varepsilon_t \right], \quad (31)$$

and the unconditional variance of inflation is of the form

$$\operatorname{Var}(\pi_t) = \left[\frac{\theta\xi^2(1-\gamma)}{\lambda(\theta-1)}\right]^2 \frac{\left\{\phi^2 \left[\lambda(\theta-1) + \theta\xi^2(1-\gamma)^2\right]^2 + \lambda^2(\theta-1)^2\right\}\sigma_u^2}{\left[\lambda(\theta-1) + \theta\xi^2(1-\gamma)^2\right]^2}.$$
 (32)

The variability of the inflation is determined principally by the variance of the supply shock σ_u^2 premultiplied by a coefficient depending upon the structural and preference parameters.

To obtain the covariance of the inflation rates π_t and π_{t-1} , we use equation (19) for the period t-1 and, from equation (8), the fact that $\phi u_{t-2} = u_{t-1} - \epsilon_{t-1}$. Finally, considering that the unconditional mean of π_{t-1} is also equal to $\bar{\pi}$, and assuming that $E(\epsilon_t \epsilon_{t-1}) = E(\epsilon_{t-1} u_{t-1}) = E(\epsilon_t u_{t-1}) = 0$, we obtain:

$$\operatorname{Cov}(\pi_t, \pi_{t-1}) = \left[\frac{\theta \xi (1-\gamma)\xi}{\lambda(\theta-1)}\right]^2 \phi \sigma_u^2.$$
(33)

Finally, using equations (29), (32) and (33), it follows the correlation coefficient (or inflation persistence), ρ_{π} , between π_t and π_{t-1} , as:

$$\rho_{\pi} = \frac{\phi \Omega^2}{\phi^2 \Omega^2 + \lambda^2 (\theta - 1)^2},\tag{34}$$

where $\Omega = \lambda(\theta - 1) + \theta \xi^2 (1 - \gamma)^2 > 0$. It is easy to verify that the correlation coefficient is non negative $(\rho_{\pi} \ge 0)$. To ensure also that $\rho_{\pi} \le 1$, we need : $(\theta - 1)^2 \ge \phi \Omega^2 / [\lambda^2 (1 + \phi)]$. Solving this inequality yields

$$\theta \ge \left(1 - \sqrt{\phi/(1+\phi)}\right) \left\{1 - \sqrt{\phi/(1+\phi)} \left[1 + \xi^2 (1-\gamma)^2/\lambda\right]\right\}^{-1}.$$
 (35)

This condition determines another lower bound of the central bank's preference for robustness θ which depends on the structural parameters (ξ, γ, ϕ) and the relative weight assigned to inflation stabilization λ . Therefore, the degree of inflation persistence ρ_{π} is determined by the central bank's preference parameters (i.e. preference for inflation stabilization λ , and the preference for robustness θ) and the structural parameters $(\gamma, \xi \text{ and } \phi)$.

4.2 The effects of central bank robustness

We discuss in this section the effects of central bank's preference for robustness on the degree of inflation persistence and the sacrifice ratio. In this respect, we establish the following propositions:

Proposition 3 The increase in the preference for robustness of the central bank against the model misspecification (i.e., a decrease in θ) increases the inflation persistence (ρ_{π}) .

Proof. To establish this result, we take the first derivative of ρ_{π} with respect to θ from equation (34). This yields

$$\frac{\partial \rho_{\pi}}{\partial \theta} = -\frac{2\phi \left[\lambda(\theta-1) + \theta\xi^2(1-\gamma)^2\right]^2 \lambda^2(\theta-1)}{\left\{\phi^2 \left[\lambda(\theta-1) + \theta\xi^2(1-\gamma)^2\right]^2 + \lambda^2(\theta-1)^2\right\}^2}.$$
(36)

As $\theta > 1$ and $\gamma \in (0, 1)$, from the above derivative (36), we obtain $\frac{\partial \rho_{\pi}}{\partial \theta} < 0$.

The intuition behind the above result (36) is that as the central bank has a stronger preference for robustness (i.e., a decrease in θ), or the less the central bank believes that its reference model is robust, the higher the inflation persistence will be. In fact, inflation persistence may arise for several reasons: On the one hand, the inertia due to a slow adjustment of inflation expectations. According to our previous result (25) in Proposition 2, a fear of model misspecification leads to a higher expected inflation and thus a greater inflation persistence. Moreover, as we have shown in Proposition 1, an increase in the preference for robustness positively affects the variability of inflation and therefore, the inflation persistence Second, other determinants such as the inertia of imperfect credibility due to a low degree of central bank's aversion for inflation (designed by the parameter λ), a higher degree of shock persistence when the supply-side shocks are highly correlated (designed by the parameter ϕ), or the inertia that wage and price contracts captured here by a low degree of wage indexation (parameter γ). In this context, we attempt to establish the relationship between the preference for robustness and these determinants of inflation persistence. The following propositions can be derived.

Proposition 4 A stronger preference for robustness against the model misspecification (i.e., a decrease in θ) decreases the sensitivity of inflation persistence (ρ_{π}) to the central bank's aversion of inflation (λ).

Proof. To set-up this result, we take from equation (34) the first derivative of ρ_{π} with respect to λ and we get

$$\frac{\partial \rho_{\pi}}{\partial \lambda} = -\frac{2\phi \theta A^2 \left[\lambda(\theta-1) + \theta A^2\right] \lambda(\theta-1)^2}{\left[\lambda^2(\theta-1)^2(1+\phi^2) + \phi^2 \theta^2 A^4 + 2\lambda \phi^2 \theta A^2(\theta-1)\right]^2} < 0.$$
(37)

As $\theta > 1$ and $A \equiv \xi(1 - \gamma) > 0$ since $\gamma \in (0, 1)$, we obtain : $\partial \rho_{\pi} / \partial \lambda < 0$. Then, the effect of a decrease in θ on the value of the above derivative (37) is given by

$$-\frac{\partial \left|\frac{\partial \rho_{\pi}}{\partial \lambda}\right|}{\partial \theta} = -\frac{2\phi\lambda A^{2}(\theta-1)\left\{\lambda^{3}(\theta-1)^{3}(1+\phi^{2})+2\theta\lambda^{2}A^{2}(\theta-1)^{2}\right\}}{\left[\lambda^{2}(\theta-1)^{2}(1+\phi^{2})+\phi^{2}\theta^{2}A^{4}+2\lambda\phi^{2}\theta A^{2}(\theta-1)\right]^{3}} -\frac{2\phi\lambda A^{2}(\theta-1)\left\{\phi^{2}\theta^{2}A^{4}\left[2\theta A^{2}+3\lambda(\theta-1)\right]\right\}}{\left[\lambda^{2}(\theta-1)^{2}(1+\phi^{2})+\phi^{2}\theta^{2}A^{4}+2\lambda\phi^{2}\theta A^{2}(\theta-1)\right]^{3}}.$$
 (38)

As $\theta > 1$ and $A \equiv \xi(1 - \gamma) > 0$ since $\gamma \in (0, 1)$, we obtain : $-\partial \left| \frac{\partial \rho_{\pi}}{\partial \lambda} \right| / \partial \theta < 0$.

Proposition 5 An increase in the preference for robustness of the central bank against the model misspecification (i.e., a decrease in θ) decreases the sensitivity of inflation persistence (ρ_{π}) to the degree of supply-side shock persistence (ϕ).

Proof. To set-up this result, we take from equation (34) the first derivative of ρ_{π} with respect to ϕ and we obtain

$$\frac{\partial \rho_{\pi}}{\partial \phi} = \frac{\left[\lambda^2 (\theta - 1)^2 (1 - \phi^2) - \theta^2 \xi^4 (1 - \gamma)^2\right] \Omega^2}{\left\{\phi^2 \left[\lambda (\theta - 1) + \theta \xi^2 (1 - \gamma)^2\right]^2 + \lambda^2 (\theta - 1)^2\right\}^2},\tag{39}$$

Assuming that $\lambda^2(\theta - 1)^2(1 - \phi^2) > \theta^2 \xi^4(1 - \gamma)^2$ (for example, when $\gamma \to 1$), we obtain: $\frac{\partial \rho_{\pi}}{\partial \phi} > 0$. Then, the effect of a decrease in θ on the value of the above derivative is given by

$$-\frac{\partial \left|\frac{\partial \rho_{\pi}}{\partial \phi}\right|}{\partial \theta} = -\frac{2\lambda^{2}(\theta-1)\left[\theta^{2}\xi^{4}(1-\gamma)^{4} + \lambda^{2}(\theta-1)^{2}\left(3\phi^{2}-1\right)\right]\Omega^{2}}{\left\{\phi^{2}\left[\lambda(\theta-1) + \theta\xi^{2}(1-\gamma)^{2}\right]^{2} + \lambda^{2}(\theta-1)^{2}\right\}^{3}} < 0.$$
(40)

As $\theta > 1$ and $\gamma \in (0, 1)$, when $3\phi^2 > 1 \Rightarrow \phi > \sqrt{1/3}$, we obtain: $-\partial \left| \frac{\partial \rho_{\pi}}{\partial \phi} \right| / \partial \theta < 0$.

Proposition 6 The increase in the preference for robustness of the central bank (i.e., a decrease in θ) decreases the sensitivity of inflation persistence (ρ_{π}) to the wage indexation parameter (γ).

Proof. To establish this result, we recall that the equation (34) may be transformed as

$$\rho_{\pi} = \left(\phi + \frac{\lambda^2}{\phi}g(\theta)\right)^{-1} \quad \text{with} \quad g(\theta) = \left[\frac{(\theta - 1)}{\lambda(\theta - 1) + \theta\xi^2(1 - \gamma)^2}\right]^2 \tag{41}$$

Then, we get respectively the first derivative of ρ_{π} with respect to g and the first derivative of g with respect to γ as

$$\frac{\partial \rho_{\pi}}{\partial g} = -\frac{\lambda^2 \phi}{\left[\phi^2 + \lambda^2 g\right]^2} < 0, \quad \frac{\partial g}{\partial \gamma} = \frac{4\theta \xi^2 (1-\gamma)(\theta-1)^2}{\left[\lambda(\theta-1) + \theta \xi^2 (1-\gamma)^2\right]^3} > 0$$

Using the above results we get :

$$\frac{\partial \rho_{\pi}}{\partial \gamma} = \frac{\partial \rho}{\partial g} \cdot \frac{\partial g}{\partial \gamma} < 0 \tag{42}$$

Finally, the effect of a decrease in θ on the value of the sensitivity of inflation persistence to wage indexation $(\partial \rho_{\pi}/\partial \gamma)$ is given by

$$-\frac{\partial \left|\frac{\partial \rho_{\pi}}{\partial \gamma}\right|}{\partial \theta} = -\frac{4\theta \xi^2 (1-\gamma)(\theta-1) \left[2\theta \xi^2 (1-\gamma)^2 + \lambda(\theta-1)\right]}{\left[\lambda(\theta-1) + \theta \xi^2 (1-\gamma)^2\right]^4} < 0$$
(43)

As $\theta > 1$ and $\gamma \in (0, 1)$, for the above derivative we obtain : $-\partial \left| \frac{\partial \rho_{\pi}}{\partial \gamma} \right| / \partial \theta < 0$.

According to results (37), (39) and (42) respectively in the three above propositions, the inflation persistence may arise from several reasons, including: i) the inertia that imperfect credibility (due to a low degree of central bank independence) may impart to inflation, ii) the inertia that supply-side productivity shocks may impart to inflation, or iii) the inertia that nominal wage indexation imparts to inflation. The first result reveals a negative relationship between the relative weight assigned by the central bank on inflation stabilization (or central bank's aversion to inflation, λ) and the inflation persistence. The second result illustrates a positive relationship between the inertia of supply-side productivity shocks and the inflation persistence. In other words, a lower degree of the inertia of supply-side productivity shocks (i.e. a lower ϕ) leads to a lower degree of the inflation persistence. The third result shows a negative relationship between the degree of wage indexation (parameter γ) and the inflation persistence or a positive relationship between the inertia of supply-side productivity persistence.

Further, we show respectively in (38), (40) and (43) that the above three results on inflation persistence are deteriorated when the less the central bank believes that its reference model is robust. That is, as the central bank has a stronger preference for robustness (i.e., a decrease in θ), the lower the sensitivity of the above three effects on inflation persistence will be. These results confirm the outcome of proposition 3, so that an increase in central bank's preference for robustness positively affects inflation persistence.

The intuition behind these results is the following. Normally, an increasing inflation aversion (greater value of λ) stabilizes better inflation and decreases the inflation persistence. However, the presence of model uncertainty leads to a higher inflation, which reduces the stabilization effects of a greater value of inflation aversion. Thus, the inflation persists in spite of the increasing inflation aversion. The wage indexation effects on inflation persistence may also be affected by an increase in the preference for model robustness. Once more, the introduction of model uncertainty to our analysis, leading to more aggressive results with regard to inflation, diminishes the stabilization effects of wage indexation. The increase in the preference for robustness of the central bank (i.e., a decrease in θ) decreases the sensitivity of inflation persistence (ρ_{π}) to the wage indexation parameter (γ).

4.3 Speed and cost of disinflation

We examine now the effects of central bank robustness on the output cost of disinflation or the sacrifice ratio through its effects on inflation persistence (or speed of disinflation). The sacrifice ratio is computed here by first adding up all output losses during the phase of disinflation and then dividing this sum by the achieved reduction of inflation. The following proposition can be established.

Proposition 7 The increase in the preference for robustness of the central bank (i.e., a decrease in θ), by increasing the degree of inflation persistence (closely connected to the speed of disinflation), increases the output cost of disinflation or sacrifice ratio.

Proof. To establish this result, we consider the following sacrifice ratio definition

$$sr = -\sum_{i=1}^{\infty} (\bar{y} - y_{t+i}) / \Delta \pi, \qquad (44)$$

We first compute the size of the disinflation $\Delta \pi$ between t and t + 1: $\Delta \pi = \pi_{t+1} - \pi_t$. Starting from a situation with an initial inflation π_t equal to its mean $\bar{\pi}$ and assuming that the inflation rate follows an AR(1) process: $\pi_{t+1} = \rho_{\pi}\pi_t = \rho_{\pi}\bar{\pi}$, we can write:

$$\Delta \pi = -\left(1 - \rho_{\pi}\right)\bar{\pi},\tag{45}$$

where we consider the average inflation rate as given when policymaker determines the inflation rate in a specific period. Using respectively equations (44) and (45), we obtain:

$$\partial \left(sr \right) / \partial \rho_{\pi} = \sum \left(\bar{y} - y_{t+i} \right) / \left(\rho_{\pi} - 1 \right)^2 \bar{\pi} > 0 .$$
(46)

Combining the above result with the Proposition 3 yields :

$$\frac{\partial \left(sr\right)}{\partial \theta} = \frac{\partial \left(sr\right)}{\partial \rho_{\pi}} \cdot \frac{\partial \rho_{\pi}}{\partial \theta} < 0.$$
(47)

As $\partial \rho_{\pi} / \partial \theta < 0$ from (36) and $\partial (sr) / \partial \rho_{\pi} > 0$ from (46), we get : $\partial (sr) / \partial \theta > 0$.

According to this result, the model predicts a cost of disinflation which is increasing with the degree of the central bank's preference for robustness. In particular, the intuition behind this result is that a higher preference for robustness of central bank is positively associated with the inflation persistence and thus negatively associated with the speed of disinflation. Therefore, this paper shows that the output cost of disinflation is higher when the central bank believes less that its reference model is robust and thus its preference for robustness is higher.

| Tab | le 1: P | aram | eter | val | ues for | the mo | de | el ca | libra | tion |
|-----|---------|------|----------|-----|-----------------|-----------|----|--------|-------|------|
| | Model | | | | Shocks | | | Policy | | |
| | ϕ | ξ | γ | _ | ε_t | u_{t-1} | | λ | k | • |
| | 0.77 | 1.5 | 0.5 | | 0.001 | 0.01 | | 2 | 1 | |

4.4 A numerical example

To demonstrate the analytical results more intuitively, we provide the following numerical simulations. All parameters values are presented in Table 1.

We first compute the degree of inflation persistence as a function of the degree of model uncertainty. Figure 1 shows how the degree of inflation persistence varies with different levels of the preference for robustness.¹ The decreasing curve implies the negative relationship between ρ and θ . As the degree of model uncertainty increases (smaller values of θ), the inflation persistence increases as well. Moreover, when θ takes small values, a trivial decrease of this latter induces a rather large increase in the degree of inflation persistence. Whereas, as the values of θ are higher, the inflation persistence is less sensitive to that variations. As shown in Figure 1, the curve becomes less steeper for higher values of θ . This observation yields an interesting implication. If both the private agents and the central bank rather believe in the reference model (relative great values of θ), a small increase in the preference for robustness does not significantly affect the inflation persistence. While their doubts about the reference model turn out to be more important (relative small values of θ), the inflation persistence becomes more sensitive to changes in the preference for robustness.

Another factor that influences the process of inflation dynamics is the exogenous wage indexing parameter γ . It is thus of interest to examine the effect of wage indexing behavior on the disinflation rate. For this purpose, we repeat the simulation with the same numerical value for λ , ϕ and ξ , letting the indexing parameter vary. Figure 2 shows the evolutions of the degree of inflation persistence under different indexing parameters, taking account of model uncertainty. Under the same degree of model uncertainty, inflation exhibits a higher inertia if the wage indexing rate is relatively low. According to the optimal conditions (14) and (16), a small value of wage indexing worsens the output-inflation trade-off (i.e. $1 - \gamma$ becomes greater) and strengthens the misspecification. Consequently, inflation become more sluggish under a low indexing wage setting behavior.

Using the expression of output y_t given by Eq.(20), we can obtain

$$\bar{y} - y_{t+i} = \frac{1}{\theta - 1}k - \left(1 + \frac{\xi\theta}{\theta - 1}\right)\phi u_{t-1} - \left[1 + \frac{\theta\lambda\xi}{\lambda(\theta - 1) + \theta A^2}\right]\varepsilon_t.$$
 (48)

Inserting the latter into equation (44), together with (30) and (45), we can com-

¹When choosing the numerical values for θ , we have verified that the lower bound condition (35) are satisfied under this parameter set.

pute the sacrifice ratio. Figure 3 plots the changes in the sacrifice ratio when the preference for robustness decreases (an increase in θ) using the parameter values given in Table 1. The decreasing curve is in accordance with the negative relationship between sr and θ shown by equation (47).

5 Concluding remarks

This paper examines the relationship between the preference for robustness of central bank against the model misspecification and the inflation persistence or the speed of disinflation. We use a simple monetary game model in which a stronger preference for robustness of the central bank is positively associated with the inflation persistence and therefore negatively related with the speed of disinflation. In this framework, we have shown that the output cost of disinflation (or sacrifice ratio), associated positively with the inflation persistence, will be higher when the preference for robustness is higher and thus less the central bank believes that its reference model is robust. The policy implication lurking behind this finding is that a central banker who faces model uncertainty, should design and implement his robust monetary policy taking into account that the inflation persistence will be higher.

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Figure 1: Inflation persistence under model uncertainty, (the impact of θ)



Figure 2: Inflation persistence under model uncertainty, with different indexing parameters



Figure 3: Sacrifice ratio under model uncertainty