Bureau d'économie théorique et appliquée (BETA)

UMR 7522

Documents de travail

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Money growth rule and macro-financial stability under inflation-targeting regime

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Abstract: Recent financial crises and central banks' interventions to ensure liquidity on the monetary markets around the world have shown that using interest rate as instrument of monetary policy can be insufficient. Using an aggregate dynamic macro-economic model, we study how to combine inflation targeting with monetary targeting to warrant macro-economic and financial stability. A commitment to a long-run money growth rate corresponding to the inflation target could reinforce the credibility of central bank announcements and the role of inflation target as strong and credible nominal anchor for private inflation expectations. We show that, using Friedman's k-percent money growth rule to help anchoring inflation expectations under inflation-targeting regime can generate dynamic instability in output, inflation, assets prices as well as real money demand. Alternatively, a well-specified monetary targeting rule that responds negatively to the evolution of expected inflation allows achieving macro-economic and financial stability.

Key words: inflation targeting, monetary targeting, stock prices, macro-economic and financial stability, Friedman's k-percent money growth rule.

JEL Classification: **E41**, E44, E52, E58.

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1. Introduction

Over the last decade, many central banks have adopted a new framework for conducting monetary policy known as inflation targeting. Mishkin (1999) presents this monetary policy regime as a successor to monetary targeting and more efficient in controlling inflation than the latter, given the breakdown of the relationship between monetary aggregates and goal variables such as inflation. Furthermore, the current consensus in the inflation-targeting literature is that money and credit have essentially no constructive role to play in monetary policy. In other words, the money market is only useful for determining the supply of money which responds endogenously to the demand of money, and hence can be largely ignored in making monetary policy decisions (Woodford, 1998; Rudebusch and Svensson, 1999). This consensus forged since ten years has substituted to the one forged by Milton Friedman, according to which inflation is "always and everywhere a monetary phenomenon".

The experience of the 1970s showed that the inflation expectations of the public can lose their anchor in a context of high oil prices and depreciating US dollars. Monetary targeting such as Milton Friedman's k percent money growth rule was progressively abandoned by central banks in favour of implicit interest rate rules like that discovered by Taylor (1993). To stabilize inflation expectations, monetary authorities proactively increase (reduce) nominal interest rate when the evidence suggests that inflation will rise above (respectively fall below) some numerical objective.

Although a typical nominal interest rate rule (Taylor rule or optimal rule) may be effective at anchoring inflation expectations in some models, the finding is not robust to small, empirically plausible, changes in model specification. This is of concern because there is considerable uncertainty about the correct model specification (Benhabib, Schmitt-Grohe and Uribe (2001, 2002a, b), Carlstrom and Fuerst (2002, 2005) and Christiano and Rostagno

(2001)). Sharing this concern, Christiano, Motto and Rostagno (2007) describe two examples that illustrate in different ways how money and credit may be useful in the conduct of monetary policy. Their first example introduces a supply-side channel for monetary policy and creates the possibility for inflation expectations to lose their anchor. Illustrated with the help of the IS-LM model augmented by a supply curve, this example shows how monitoring money and credit can help anchor private sector expectations about inflation. Their second example, which summarizes the analysis of Christiano, Ilut, Motto and Rostagno (2007), shows that a monetary policy that focuses too narrowly on inflation may inadvertently contribute to welfare reducing boom-bust cycles in real and financial variables. Benjamin Friedman (2003) worried also about abandoning the role of money and the analytic of the LM curve since that makes it more difficult to take into account how the functioning of the banking system (and with it the credit markets more generally) matters for monetary policy and also leaves open the underlying question of how the central bank manages to fix the chosen interest rate in the first place. Considering the money market as coordination device of private inflation expectations, Dai and Sidiropoulos (2003, 2005), Dai (2006, 2007), Dai, Sidiropoulos and Spyromitros (2007) provide some theoretical justifications of the utility of this market other than only determining endogenously money supply in a typical inflation-targeting framework. Söderström (2005) demonstrates how a target for money growth can be beneficial for inflation-targeting central banks acting under discretion. Since the money growth rate is closely related to the change in the interest rate and the growth of real output, delegating a money growth target to the central bank can be a sensible strategy for monetary policy by making discretionary policy more inertial, leading to better (if not best) social outcomes.

These theoretical concerns have found some empirical echoes. Milton Friedman (2005), using data covering three booming periods in US and Japan, shows that what happens to the

quantity of money has a determinative effect on what happens to national income and to stock prices. Hafer, Haslag and Jones (2007) find that money is not redundant, notably there is a significant statistical relationship between lagged values of money and the output gap, even when lagged values of real interest rates and lagged values of the output gap are accounted for. Hafer and Jones (2008), adding money to a dynamic IS model, discover that evidence from six countries indicates that money growth usually helps predict the GDP gap and that the predictive power of a short-term real interest rate is much weaker than previous works suggest. Their results suggest that, for dynamic IS models such as that used by Rudebusch and Svensson (1999), the omission of money appears to come at a high cost.

In practice, few central banks are reported to exclusively use monetary aggregates to manage their monetary policy. The rare success stories of monetary targeting in the case of Bundesbank and/or Swiss National Bank are explained as due to that their monetary policy is actually closer in practice to inflation targeting than it is to Friedman-like monetary targeting (Clarida and Gertler, 1996; Bernanke and Mishkin, 1997; Bernanke and Mihov, 1997; Laubach and Posen, 1997; Clarida et al., 1998; Mishkin, 1999, 2002) and thus might best be thought of as "hybrid" inflation targeting. Political considerations, i.e. the need to demonstrate continuity with the policies of the Bundesbank, apparently have dictated that the ECB pays attention to monetary aggregates as well in its two pillars strategy. Many observers have interpreted the ECB's two-pillar monetary strategy, where the "economic pillar" resembles an implicit form of inflation-targeting and the "monetary pillar" a weak type of monetary targeting approach, as a bridge between the monetary targeting strategy of the old Bundesbank and the more up-to-date inflation targeting approach (Bernanke et al., 1999; Svensson, 2000b; Rudebusch and Svensson, 2002; Mayer, 2006). This "misinterpretation" (Assenmacher-Wesche and Gerlach, 2006) has lead to the criticism of the framework for being inconsistent and lacking clarity. The controversial debate is arguably due to the fact that

the ECB provides neither an explicit representation of the inflation process nor an explanation for why it necessitates a two-pillar framework. In other words, it lacks a theory justifying the simultaneous use of monetary and inflation targeting.

Economists at the Bank of Japan (BOJ) argue that, in the case of Japan, a simple announcement of inflation target without instruments would not convince market participants to change their inflation expectations (Ito, 2004). Under inflation targeting, there are no clear instruments to get out of deflation resulting from a combination of the burst of asset price bubbles, fragile financial system and inappropriate past monetary policies. On March 19, 2001, the BOJ embarked, with some success, on an unprecedented monetary policy experiment under very low nominal interest rate, commonly referred to as "quantitative easing", in an attempt to stimulate the nation's stagnant economy (Spiegel, 2006).

Recent massive interventions on monetary markets by several important central banks across the world following the breakdown of US *subprime* market, which brought international financial turmoil since the summer of 2007, put in evidence the usefulness of controlling directly monetary aggregates in order to control the interest rates at which private agents can borrow. Some economists (Mayer, 2006) have been alarming since some times about the pitfalls of inflation targeting: it has been blind to the emergence of record international current account imbalances and inflated asset prices.

The criticism of neglecting asset prices in the inflation-targeting framework is certainly not valid on the theoretical plan. In fact, the large swings in asset prices and economic activity in the United States, the European Union, Japan, and other countries over the past several years have brought focus on the role of asset prices in the monetary strategy such as inflation targeting. The essential question is not about whether the central bank objective function should include asset prices. Instead, it is concerned with how an inflation-targeting central bank can most effectively fulfil its objectives. Bernanke and Gertler (1999, 2001) suggest that

monetary policy should not respond to changes in asset prices, except in so far as they signal changes in expected inflation. A more general case can be made for central banks to react to asset prices in the normal course of policy making without trying to target asset prices (e.g. Cecchetti, Genberg, Lipsky, and Wadhwani, 2000; Cecchetti, Genberg, and Wadhwani, 2003; Filardo, 2000, 2004; Bean, 2003; Disyatat, 2005; Akram and Eitrheim, 2008)[1].

However, we doubt that a Taylor rule or an optimal nominal interest rate rule is sufficient to simultaneously anchor inflation expectations, ensure macro-financial stability and stabilize output and inflation around their respective targets. The fact that inflation-targeting central banks abstain from intervening on the liquidity of interbank markets in good times might explain *ex-post* why financial booms have gone too far in many industrial and emerging markets economies during the recent economic expansion. Therefore, a more systematic monitoring of monetary aggregates, not only during crisis times, may be a better approach.

We propose in this paper to study, in an IS-LM model[2] including a Phillips curve and stock prices, how an inflation-targeting regime may gain in credibility and macro-financial stability by being accompanied by a monetary targeting rule. In the inflation-targeting literature, one important implicit assumption is that central bank's announcements are perfectly credible so that its inflation target is exactly equal to expected inflation rate of private sector as shocks are assumed to be i.i.d.. In fact, a central bank has no other control over inflation expectations than just trying different tactics of persuasion through the

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[1] For a synthesis of the previous literature about the relation between asset prices and monetary policies, see Gilchrist and Leahy (2002).

[2] Instead of adopting the more fashionable New-Keynesian model (see e.g. Clarida, Gali and Gertler, 1999), we have adopted a more traditional IS-LM model and that for two kinds of reasons. First, we want to link the new inflation targeting regime to the old monetary targeting which is discussed generally in the IS-LM framework. Second, as discussed by Blanchard (2008), the adoption of the New Keynesian model has also important costs: notably, while tractable, the first two equations of the New Keynesian model are patently false (more obviously so than those in the more loosely specified IS-LM model). More precisely, the aggregate demand equation ignores the existence of investment, and relies on an intertemporal substitution effect in response to the interest

implementation of complex operational instruments, procedures and communication techniques. But, there is not any reason that shocks are always i.i.d., central bank is always credible and knows exactly the true economic model, and private agents believe always in central bank's announcements. Furthermore, speculative inflation bubbles cannot be excluded in dynamic framework by assuming rational expectations. For the inflation target to be an anchor in all circumstances for private inflation expectations, the credible commitment of central bank to a long-run inflation rate seems necessary. Targeting money growth according to Friedman's k-percent money growth rule[3] might be, at a first sight, an effective way to keep inflation expectations in check. This kind of quantitative limitation of money supply can be put in place without major difficulty since the central bank can serve only partially the demand of banks for liquidity at its announced nominal interest rate. It can also restrict the types of banks which have access to the central liquidity facility or the types of assets which be exchanged for the central liquidity.

The remainder of the paper is organized as follows. In the next section, we present a theoretical model in which stock prices play a role. In the section after, we characterize the optimal reaction function of the inflation-targeting central bank. In the fourth section, we analyze the dynamic stability of the economy under Friedman's k-percent money growth rule. The fifth section examines an alternative monetary targeting rule. The final section concludes.

2. The Model

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rate, which is hard to detect in the data on consumers. The inflation equation implies a purely forward looking behaviour of inflation, which again appears strongly at odds with the data.

[3] Some recent studies compare Friedman's k-percent money supply rule with interest rate rule (Evans and Honkapohja, 2003; Minford, Perugini and Srinivasan, 2003).

We consider a simple continuous time model in order to develop an example where the money is useful to ensure macro-financial stability and where the Friedman's *k*-percent money growth rule is not stabilizing when the central bank monitors also nominal interest rate. The economy is described by an inflation adjustment equation, an aggregate spending relationship linking output to real interest rate and stock prices, and two conditions for equilibrium in financial markets (money, bonds and shares). Inflation is governed by an expectational Phillips curve of the form:

$$\pi = \pi^e + \alpha (y - y^*) + \varepsilon_\pi, \qquad \alpha > 0, \qquad (1)$$

where $\pi \left(\equiv \frac{dp}{dt} \right)$ denotes the inflation rate which is the time derivation of the log of the general price level (p), π^e the expected inflation rate, y the current output, y^* the natural rate of output and ε_π an inflationary shock.

The aggregate demand for goods (y^d) depends positively on the current revenue (y), negatively on the expected real interest rate ($i - \pi^e$), and positively on the real value of shares on the stock market (q) as follows:

$$y^d = cy - \rho(i - \pi^e) + \varphi q + u_d, \qquad 0 < c < 1, \ \rho, \varphi > 0 ,$$

where i denotes the nominal interest rate and u_d a demand shock. The condition for equilibrium in the goods market, $y = y^d$, leads to the following equation:

$$y = -\beta(i - \pi^e) + \gamma q + \varepsilon_d, \qquad \beta, \gamma > 0 , \qquad (2)$$

where $\beta = \frac{\rho}{1-c}$, and $\gamma = \frac{\varphi}{1-c}$ and $\varepsilon_d = \frac{u_d}{1-c}$. The parameters β and γ represent respectively the interest elasticity of demand for goods (ρ) and the marginal propensity to spend wealth (φ), both normalised by the marginal propensity to save the current revenue ($1 - c$). Theoretically and empirically, it is recognized that stock prices play a significant role in determining

aggregate demand.[4] First of all, being part of net wealth, it can affect households' consumption. Second, determining the value of the existing capital relative to its replacement cost (Tobin's q theory of investment), it affects firms' investment level. Third, being net worth and used as collateral, it affects the firms' balance-sheet position and so the risk premium to accept for obtaining funds on the capital market. A fourth mechanism linking stock market with aggregate demand could be the household liquidity effect: an increase in stock prices can imply an increase in the net wealth of households, which in turn increases consumption spending. A final mechanism is referred to as the confidence channel. The confidences of consumers, even these who do not own any share, and that of entrepreneurs, even when their companies are not quoted on the stock market, are positively related to the stock prices.

As bonds and shares are considered as imperfect substitutes in the portfolios of private agents, the arbitrage between bonds and shares implies the following equality in the short run between the expected yield of the bonds ($i - \pi^e$) and that of shares minus risk premium ($\frac{\varepsilon_q}{q}$):

$$i - \pi^e = \frac{\dot{q}^e}{q} + \frac{\psi y}{q} - \frac{\varepsilon_q}{q} \,. \tag{3}$$

In equation (3), the expected yield for shares is composed by the expected rate of capital gains or losses $\frac{\dot{q}^e}{q}$ and the rate of distributed dividends $\frac{\psi y}{q}$. The share of profits in national income is assumed to be constant and represented by the parameter ψ. The term ψy represents the firms' profits, which by assumption are entirely redistributed. Equation (3) can be rewritten in the form of dynamic equation in assuming perfect foresight ($\dot{q}^e = \dot{q}$) in the stock market[5]:

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[4] See for example Blanchard (1981), Fama (1981), Fisher and Merton (1984), Barro (1990) and Poterba and Samwick (1995), Poterba (2000), Ludwig and Slok (2002).

[5] Perfect foresight is assumed to simplify the model resolution. It is equivalent to assume that the stock operators have perfect information about the contemporary shocks affecting the model.

$$\dot{q} = (i - \pi^e)q - \psi y + \varepsilon_q. \tag{4}$$

Various factors, such as the formation of "speculative bubbles" or/and an exogenous variation of risk aversion associated to financial assets, can be at the origin of variations of ε_q.[6]

Finally, the money market equilibrium is characterized by

$$m - p = l_1 y - l_2 i + \varepsilon_m, \qquad l_1, l_2 > 0, \tag{5}$$

where m represents the log of nominal money supply. The real money demand (right hand of the above equation) depends on real income, nominal interest rate and an exogenous shock affecting the money market (ε_m) that is contemporary and can be perfectly observed by market participants and the central bank during the current period.

Taking the time derivative and denoting $\dot{m} = \mu$ and $\dot{p} = \pi$, equation (5) is rewritten as:

$$\mu - \pi = l_1 \dot{y} - l_2 \dot{i} + \dot{\varepsilon}_m. \tag{6}$$

Consider the steady state equilibrium (where $\dot{\pi}^e = \dot{\pi} = \dot{q} = \dot{i} = 0$ and $\dot{y} = \dot{y}^*$, i.e. when the effects of permanent shocks are entirely carried out) or average equilibrium[7] which is a useful reference point for stabilization policy makers and market participants. If we assume $\dot{y}^* = 0$, equation (6) implies that, in order to increase the credibility of the inflation targeting regime and hence the chance of stabilizing current and expected inflation rates as well as output closely around their respective target level, monetary authorities could set a growth rate of money supply consistent with the positive inflation target. One example is:

$$\mu = \overline{\mu} + \dot{\varepsilon}_m, \quad \text{with } \overline{\mu} = \pi^T > 0, \tag{7}$$

where $\overline{\mu}$ is the (average) long-run money growth rate consistent with the inflation target $\overline{\mu} = \pi^T$. This is a variant of Friedman's k-percent money growth rule. This monetary

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[6] See Schiller (1981), Blanchard, Rhee and Summers (1993).

[7] The average equilibrium is a useful benchmark when shocks are stochastic and temporary only.

targeting rule could be considered as a warrant against major deviations of current and expected inflation rate from the inflation target and thus reinforce the belief of private sector that monetary authorities will be more successful in implementing their interest rate policy consistent with the inflation target. For von Hagen (1999), this kind of monetary targeting is a signal that the central bank is independent and fighting against price instability, and a means to define the role of monetary policy vis-à-vis other players in the macroeconomic policy game, and to structure the internal monetary policy debate. In the absence of monetary targeting, inflation targeting might not have perfect credibility in the sense that private agents don't automatically use inflation target as nominal anchor. Instead, private agents use information extracted from current market conditions to revise their expected future inflation rate. In effect, to believe in the inflation target, private agents must believe that the random shocks must conceal their inflation consequences in their respective time horizon. As their time horizons are far from infinite and the effects of i.i.d. shocks cannot be mutually compensated, they might be incited to use alternative method to formulate their inflation expectations which correspond better to their horizon of decision during which current inflation rate could be systematically different from expected inflation due to permanent, persistent or even stochastic shocks[8]. If this is the case, private agents could anticipate an inflation rate different from the inflation target announced by the central bank. Thus, without other warrant, inflation targeting will not necessarily offer the nominal anchor for private inflation expectations as assumed in the inflation-targeting literature.

Monetary authorities systematically act to minimize fluctuations of output around the natural level of output, y^*, and inflation around the inflation target, π^T. The nominal interest rate is treated as the principal instrument of monetary policy. Central bank is assumed to minimize the following loss function:

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[8] The random nature of shocks does not exclude the same kind of shock arrives repetitively for several times.

$$E_t \int_0^\infty L(t)\exp(-\theta t)dt, \quad \text{with} \quad L(t) = \frac{1}{2}[\lambda(y - y^*)^2 + \kappa(\pi - \pi^T)^2], \quad \lambda, \kappa, \theta > 0, \quad (8)$$

where E_t is the expectation operator. Preference parameters λ and κ denote respectively the weights that monetary authorities assign to output and inflation targets. θ is a discount factor. The minimization of the loss function (8) summarizes the essence of flexible inflation targeting: the central bank should strive to maintain inflation as close to a clearly specified target level as possible, while at the same time limiting fluctuations of real economic activity. In practice, inflation-targeting central banks actually all pursue flexible inflation targeting since the consequences for the economy of strict inflation targeting are simply undesirable (Svensson, 1997, 2000a). Empirical evidence recently provided by Collins and Siklos (2004) suggests that countries with explicit inflation targets were not overly aggressive toward inflation. Flexible inflation targeting shows up in less policy activism, gradualism in returning the inflation back to target, and in aiming at the inflation target at a somewhat longer horizon.

We complete our model description by the following time sequence of events: 1. Workers and financial operators form their inflation expectations in order, respectively, to negotiate current wages and to decide lending and investment; 2. Shocks realise; 3. Central bank fixes nominal interest rate following an optimal interest rate rule; 4. Firms decide their production and prices; 5. Workers and financial operators revise their inflation expectations and central bank tries to influence this revision with a money growth rule.

3. The optimal monetary policy rule

The optimal monetary policy is the solution to the minimization of the loss function (8). Since the central bank takes the expected inflation as given and does not try to directly

influence the stock prices with monetary policy instruments[9], the economic constraints useful for its optimization problem are equations (1) and (2). That implies that the central bank's optimisation problem is static. The first-order condition is given by

$$\lambda \, (y - y^*)\frac{\partial y}{\partial \pi} = -\kappa(\pi - \pi^T).$$

(9)

Using equation (1) to obtain $\frac{\partial y}{\partial \pi} = \frac{1}{\alpha}$ as π^e is given and inserting it in the condition (9) lead to the following central bank's optimal targeting rule:

$$y = y^* - \frac{\kappa\alpha}{\lambda}(\pi - \pi^T),$$

(10)

which, using equation (2), leads to the optimal interest rate rule of the central bank:

$$i = \pi^e + \frac{1}{\beta}[\gamma q + \frac{\kappa\alpha}{\lambda}(\pi - \pi^T) + \varepsilon_d - y^*].$$

(11)

According to equation (11), it is optimal for the central bank to adjust the nominal interest rate upward to fully reflect the expected inflation, the gap between current inflation and the inflation target, as well as increase in stock prices and increase in the output gap due to a positive demand shock. There is no major difficulty in including stock prices in the central bank reaction function, since it is easily observable and observed in an instantaneous way[10]. Stock prices at every moment convey information contained in a set of data provided by individual investors having a more upstream knowledge about the origin and the nature of shocks than the central bank. Generally, stock prices tend to react quickly to new information. Reacting to the evolution of asset prices undoubtedly gives an advantage to the central bank so it can react quickly and stay in tune with the evolution of the economy. Otherwise, it takes

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[9] Central bankers are generally opposed to targeting asset prices. See for example, Gramlich (2001), Poole (2001) and Trichet (2002). They argue that it is difficult to determine the equilibrium stock prices.

[10] Some operational difficulties might result from the short-run volatility of stock price. One way to avoid the repercussion of stock price volatility into the interest rate is to use a moving average with stronger weights being given to more recent developments of stock prices.

a risk while trying to base its monetary policy decision only on useful information all collected by itself. The presence of stock prices in the optimal interest rate rule reveals that monetary authorities are not urged, when applying the optimal monetary policy rule, either to know the nature of these shocks or to judge on the modification of risk premium and the presence or not of mania or speculative bubbles phenomena. Indeed, one of the arguments advanced by some academics and central bankers as to why a monetary authority might not directly react to asset prices is that monetary authority cannot judge the presence or not of speculative bubbles.

In the inflation targeting literature, it is generally assumed that inflationary shocks are i.i.d. and hence π^e is equal to the central bank's inflation target:

$$\pi^e = \pi^T .$$ (12)

If private agents believe that shocks are i.i.d. and use mathematical expectations to anticipate future inflation rate, then, according to the consensus in the inflation targeting literature, money can be largely ignored in making monetary policy decisions. Some recent findings (e.g. Christiano, Motto and Rostagno (2007), Hafer, Haslag and Jones (2007)) reject the redundancy hypothesis of money. They imply that the behaviours of money supply and demand might have important influence on inflation expectations and hence on macroeconomic and financial stability. To account for that, we assume that money and financial markets provides useful information to coordinate inflation expectations of private agents (Dai and Sidiropoulos (2003, 2005) and Dai (2006, 2007)). In other words, we assume that private agents believe that they can form better inflation expectations in taking account of current events and inflation expectations reflected by the money market (implicitly the bonds market). Then, the time-consistent private expected inflation becomes an endogenous variable. Under the above assumption, LM curve can play an important role in the solution of

the model. Therefore, monetary targeting is not redundant and must not be simply reduced to inflation targeting as in the literature discussing German monetary targeting[11].

4. The dynamic effects of Friedman's *k*-percent money growth rule

As expected inflation rate is determined before current inflation rate and revenue, its dynamic trajectory can be more easily studied in a reduced dynamic system where the values of π and y are substituted by their solution in terms of expected inflation rate, exogenous variables and shocks. Once, the dynamic trajectory of π^e is solved, we can determine these of π and y. Equations (1)-(2) and (11) enable us to solve inflation rate and output as follows:

$$\pi = \frac{\lambda}{\lambda + \kappa\alpha^2}\pi^e + \frac{\kappa\alpha^2}{\lambda + \kappa\alpha^2}\pi^T + \frac{\lambda}{\lambda + \kappa\alpha^2}\varepsilon_\pi \ , \tag{13}$$

$$y = y^* - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\pi^e + \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\pi^T - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\varepsilon_\pi . \tag{14}$$

Equations (13) and (14) are not final solutions for inflation rate and output, which can only be obtained after having solved expected inflation rate. Departing from an initial equilibrium where $\pi^e = \pi^T$, changes in inflation expectations will entail variations in output and current inflation. The interaction between expected inflation rate and other variables might induce complex dynamics. As we have argued before, there are good reasons that economic agents will not blindly believe in the announced inflation target π^T. Consequently, simply using equation (13) to estimate the expected inflation rate, which leads to the result given in equation (12), is misleading for the central bank as well as for private agents.

Systematic revision of inflation expectations under the light of the information flow conveyed by the money market reflects a higher degree of rationality of market participants.

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[11] See e.g. Bernanke and Mihov (1997) and Mishkin (1999, 2002) among others.

Consequently, the inclusion of a monetary targeting rule defined by equation (7) implies that money supply would significantly affect the determination of current price level and inflation rate. That is due to the fact that a money growth rule can play a role in influencing the revision of inflation expectations in this model.

In economies with developed financial markets, sophisticated financial instruments (such as inflation-indexed bonds, interest rate options, swaps or futures) are traded and implicitly convey market expectations about future inflation. These complex financial instruments are not modelled in this simple model. However, we can obtain a proxy for these instruments in order to estimate expected inflation by adopting the assumption, according to which money market and so financial markets are coordination devices for economic agents trying to form good and consensual inflation expectations. It follows that private agents learn directly from information conveyed by the money market to determine the expected rate of inflation and its future evolution. Using equations (6), (7), (11), (13) and (14), we derive the following differential equation of π^e (Appendix A):

$$\dot{\pi}^e = \frac{1}{\Omega}\pi^e - \frac{(\lambda + \kappa\alpha^2)l_2\gamma}{\beta\lambda\Omega}\dot{q} - \frac{(\lambda + \kappa\alpha^2)}{\lambda\Omega}\bar{\mu} - \frac{\kappa\alpha^2}{\lambda\Omega}\pi^T - \frac{1}{\Omega}\varepsilon_\pi \ , \qquad (15)$$

where $\Omega = \dfrac{\beta l_1\kappa\alpha + l_2\beta(\lambda + \kappa\alpha^2) + l_2\kappa\alpha}{\beta\lambda} > 0$.

The dynamic behaviour of the economy can be summarized in a system constituted of the two first-order differential equations (4) and (15) which allows to study the dynamic adjustment paths of other endogenous variable after having studied that of expected inflation π^e and stock prices q. A linear approximation of these equations at the neighbourhood of the steady state or average equilibrium characterized by ($\bar{q}$, $\bar{\pi}^e$) yields (Appendix A):

$$\begin{bmatrix} \dot{\pi}^e \\[2ex] \dot{q} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\Omega} - \dfrac{\alpha\kappa\gamma l_2}{\beta\lambda\Omega}(\psi + \dfrac{\bar{q}}{\beta}) & -\dfrac{\gamma l_2(\lambda+\kappa\alpha^2)}{\beta^2\lambda\Omega}(2\gamma\bar{q} + \dfrac{\kappa\alpha\bar{\pi}^e}{\lambda+\kappa\alpha^2}) \\[3ex] \dfrac{\kappa\alpha}{\lambda+\kappa\alpha^2}(\psi + \dfrac{\bar{q}}{\beta}) & \dfrac{1}{\beta}(2\gamma\bar{q} + \dfrac{\kappa\alpha\bar{\pi}^e}{\lambda+\kappa\alpha^2}) \end{bmatrix}\begin{bmatrix} (\pi^e - \bar{\pi}^e) \\[2ex] (q - \bar{q}) \end{bmatrix}. \qquad (16)$$

Stock prices are considered to adjust more quickly than expected inflation rate. Stock prices, quoted in continuous time on a centralized market, are much more flexible than goods prices and wages. Thus they are free to make discrete instantaneous jumps in response to "news" concerning all previously unanticipated current or future changes in exogenous variables and policy instruments. Therefore, the stock prices (q) clearing an efficient financial market, are considered as a non-predetermined variable. On the other hand, in a low inflation environment, inflation rate and hence expected inflation rate (π and π^e), resulting from a relatively slow adjustment of goods prices and wages due to different factors (e.g. menu costs, overlapping or long term contracts, efficient wages etc.), is considered as a predetermined variable[12]. This distinction is essentially based on relative speed of adjustment of these two variables.

The steady state equilibrium is characterized by the condition $\dot{q} = \dot{\pi}^e = 0$. At the steady state, equations (1), (2), (4), (6), (7) and (11) determine the equilibrium values of endogenous variables as follows:

$$\bar{y} = y^*, \ \bar{\pi} = \bar{\pi}^e = \bar{\mu} = \pi^T,$$

$$\bar{q} = \frac{y^* + \sqrt{y^{*2} + 4\gamma\beta\psi y^*}}{2\gamma} \ \text{and} \ \bar{i} = \pi^T + \frac{-y^* + \sqrt{y^{*2} + 4\gamma\beta\psi y^*}}{2\beta}.$$

Denote by A the stability matrix for the system (16). The nature of the paths taken by the expected inflation and the stock prices in their dynamic adjustment to the steady state equilibrium depends in effect on the signs of the determinant and the trace of A:

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[12] See e.g. Buiter and Panigirtzoglou (2003) for a similar assumption concerning inflation rate.

$$\det(A) = \frac{1}{\beta\Omega}(2\gamma\bar{q} + \frac{\kappa\alpha\bar{\pi}^e}{\lambda + \kappa\alpha^2}),$$

$$\text{tr}(A) = \frac{1}{\Omega} - \frac{\alpha\kappa\gamma l_2}{\beta\lambda\Omega}(\psi + \frac{\bar{q}}{\beta}) + \frac{1}{\beta}(2\gamma\bar{q} + \frac{\kappa\alpha\bar{\pi}^e}{\lambda + \kappa\alpha^2}).$$

Proposition 1: The dynamic system (16) under optimal interest rate rule (11) combined with monetary targeting rule such as (7) (Friedman's k-percent money supply rule) is unstable under the condition $\frac{\bar{q}}{\beta} > \psi$.

Proof: The determinant is positive as $\Omega > 0$, $\bar{q} > 0$ and the inflation target is generally positive, i.e. $\bar{\pi}^e = \bar{\pi}^T > 0$. If the sign of the trace of the stability matrix is also positive, then there will be no stable eigenvalue and the economic system will be unstable. To discover the condition under which the trace is positive, we develop the positive trace condition:

$$\text{tr}(A) = \frac{1}{\Omega} - \frac{\alpha\kappa l_2\gamma}{\beta\lambda\Omega}(\psi + \frac{\bar{q}}{\beta}) + \frac{1}{\beta}(2\gamma\bar{q} + \frac{\kappa\alpha\bar{\pi}^e}{\lambda + \kappa\alpha^2}) > 0. \tag{17}$$

Substituting Ω by its definition given previously into (17), using $\bar{\pi}^e = \bar{\pi}^T$ and rearranging the terms, we obtain:

$$1 + \frac{\alpha\kappa l_2\gamma}{\beta\lambda}(\frac{\bar{q}}{\beta} - \psi) + \frac{2\alpha\kappa l_1\gamma\bar{q} + 2l_2(\lambda + \kappa\alpha^2)\gamma\bar{q}}{\beta\lambda} + \frac{[l_1\kappa\alpha + l_2(\lambda + \kappa\alpha + \kappa\alpha^2)]\kappa\alpha\bar{\pi}^T}{\beta\lambda(\lambda + \kappa\alpha^2)} > 0. \tag{18}$$

One immediately observes that there are much more positive terms at the right than at the left side of the inequality (18). A sufficient condition for (18) to be true (or the trace to be positive) is that:

$$\frac{\bar{q}}{\beta} > \psi. \tag{19}$$

Q.E.D.

The parameter $\beta = \rho/(1-c)$ represents the ratio of the interest elasticity of the goods demand relative to the marginal propensity to save. It is to notice that $\overline{q}$ depends positively on the parameters ψ and β as shown by its equilibrium solution. For an economy with low interest elasticity of the demand for goods, high saving rate and relatively high stock market capitalisation, the condition (19) tends to be satisfied. The sufficient condition of macro-economic and financial instability (19) is relatively easy to be checked when stock markets are highly developed. In effect, one of the important financial developments since 1980 is the rapidly increasing role played by the stock markets in industrial as well as in emerging market economies, even though the debt markets continue to increase.

As the condition (19) is only a sufficient condition, its violation does not automatically imply that the system is stable. In this case, we must re-examine the trace condition (18) to determine the stable or unstable nature of the system.

Two extreme cases are interesting to examine. Consider firstly the case where the central bank puts zero weight on the inflation target, i.e. κ tends to zero. As a result, the condition (17) is always checked independently of the value of ψ and the economy is unstable.

Consider then the case where the central bank becomes a strict targeter in the sense of Svensson (1997, 2000a), i.e. λ tends to zero. Taking this into account, the condition (18) becomes:

$$\alpha l_2 \gamma (\frac{\overline{q}}{\beta} - \psi) + 2\alpha l_1 \gamma \overline{q} + 2\alpha^2 l_2 \gamma \overline{q} + (l_1 + l_2)\overline{\pi}^T + l_2 \alpha \overline{\pi}^T > 0 . \tag{20}$$

A quick examination of the inequality (20) shows that the condition (19) is always a sufficient condition to generate macro-economic and financial instability.

Generally, when the condition (19) is satisfied, as expected inflation rate and stock prices are on trajectories diverging from the equilibrium after a shock affecting the economic

system, equations (13)-(14) imply that current inflation rate and output will follow also divergent trajectories. In this context, money demand is also instable. This result is interesting in the sense that instability in money demand is observed when central banks use more intensely interest rate as instrument of monetary policy while keeping simple monetary targeting rule.[13]

To understand why macro-economic and financial instability may arise, one can imagine the undesirable effect resulting from an aggressive reaction of nominal interest rate to an inflation shock. Given inflation expectations, higher nominal interest rate reduces real money demand and aggregate demand for goods. The reduction of the latter implies also a smaller real money demand. If the money growth rate is constant, a reduction of the real money demand and the equilibrium condition for the money market imply a higher inflation rate. Economic agents could anticipate this inflationary pressure. In particular, workers could ask higher nominal wages for the following periods. That will generate further inflationary pressures. In this respect, many emerging market economies (i.e., Latin American countries during the 1980s) and transition economies (i.e., Eastern European countries in 1990s) provide examples where a sharp increase in nominal interest rate does not permit to reduce inflation expectations and inflation rate.

5. A stabilizing monetary targeting rule

The instability result of *k*-percent monetary targeting rule under inflation-targeting regime is due to the fact that the money growth rate is given when the interest rate rule is

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[13] When the economy is dynamically unstable, we may be unable to distinguish a situation where the relationship between the demand of money and the nominal interest rate is unstable and the other one where the relationship between these two variables is stable but the stability of relationship is not discernable in data due to dynamic instability.

tightening to react to inflationary pressures. The remedy to this instability problem is to fine-tune monetary targeting rule so that it reacts in harmony with nominal interest rate rule.

An example of monetary targeting rules[14] remedying the instability property of the Friedman's *k*-percent money growth rule without modifying the steady state value of endogenous variables is to tie negatively money growth rate to the variation of expected inflation as following:

$$\mu = \overline{\mu} - \varphi \dot{\pi}^e + \dot{\varepsilon}_m, \quad \text{with } \overline{\mu} = \pi^T, \tag{21}$$

The central bank uses the same model and information as the private agents to extract data about the variation of expected inflation rate ($\dot{\pi}^e$). In modern economy, this data will be easily obtainable since there are many financial instruments (interest rate options, forward or futures contracts, inflation indexed bonds etc.) that convey inflation expectations of market participants for future periods.[15]

In taking account of the money growth rule (21), the linearized dynamic system is modified as follows (Appendix B):

$$\begin{bmatrix} \dot{\pi}^e \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \Theta - \dfrac{\Theta \alpha \kappa \gamma l_2}{\beta \lambda}(\psi + \dfrac{\overline{q}}{\beta}) & -\dfrac{\Theta \gamma l_2 (\lambda + \kappa \alpha^2)}{\beta^2 \lambda}(2\gamma \overline{q} + \dfrac{\kappa \alpha \overline{\pi}^e}{\lambda + \kappa \alpha^2}) \\ \dfrac{\kappa \alpha}{\lambda + \kappa \alpha^2}(\psi + \dfrac{\overline{q}}{\beta}) & \dfrac{1}{\beta}(2\gamma \overline{q} + \dfrac{\kappa \alpha \overline{\pi}^e}{\lambda + \kappa \alpha^2}) \end{bmatrix} \begin{bmatrix} (\pi^e - \overline{\pi}^e) \\ (q - \overline{q}) \end{bmatrix}, \tag{22}$$

where $\Theta = \dfrac{\lambda}{\lambda \Omega - (\lambda + \kappa \alpha^2)\varphi}$. The determinant and the trace of the stability matrix are respectively:

$$\det(A) = \dfrac{\lambda}{\beta[\lambda \Omega - (\lambda + \kappa \alpha^2)\varphi]}(2\gamma \overline{q} + \dfrac{\kappa \alpha \overline{\pi}^e}{\lambda + \kappa \alpha^2}),$$

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[14] For other examples of stabilizing money growth rules, see Dai (2007). Furthermore, we can consider a monetary targeting rule with asset prices as determinant.

[15] See e.g. ECB (2000) for discussions about the methods used to extract information about inflation expectations from these kinds of financial instruments.

$$\text{tr}(A) = \frac{\lambda}{\lambda\Omega - (\lambda + \kappa\alpha^2)\varphi}[1 - \frac{\alpha\kappa\gamma l_2}{\beta\lambda}(\psi + \frac{\overline{q}}{\beta})] + \frac{1}{\beta}(2\gamma\overline{q} + \frac{\kappa\alpha\overline{\pi}^e}{\lambda + \kappa\alpha^2}).$$

Since there is one predetermined variable and one non-predetermined variable, for the system to be saddle-point stable whatever is the degree of flexibility of the inflation-targeting regime, it is sufficient to have one eigenvalue with negative real part. It is sufficient then to define the parameter φ in the manner that the determinant becomes negative. This possibility is resumed in the following proposition.

Proposition 2: i) The dynamic system (22) under the optimal interest rate rule (11) combined with monetary targeting rule (21) is saddle-point stable under the condition $\frac{\lambda\Omega}{\lambda+\kappa\alpha^2} < \varphi$. ii) The minimal value of φ ($\varphi_{\min} = \frac{\lambda\Omega}{\lambda+\kappa\alpha^2}$) compatible with saddle-point equilibrium decreases with λ and β, increases with κ, l_1 and l_2. It increases with α if $\frac{\lambda}{\kappa} > \alpha^2$ or decreases with it in the contrary case.

Proof: To demonstrate the part i) of Proposition 2, it is sufficient to show that the determinant is negative when $\frac{\lambda\Omega}{\lambda+\kappa\alpha^2} < \varphi$. This is true since $\overline{q}$ is positive and $\overline{\pi}^e$ is equal to $\overline{\pi}^T$ which is generally superior to zero. If the condition $\frac{\lambda\Omega}{\lambda+\kappa\alpha^2} < \varphi$ is checked, there is only one eigenvalue with negative real part. Since we have one predetermined variable and one non-predetermined variable, there is then a unique path converging to stationary equilibrium.

To show the part ii) of Proposition 2, we derive $\varphi_{\min}$ with respect to different parameters as follows:

$$\frac{d\varphi_{\min}}{d\lambda} = \frac{-\alpha\kappa(\beta l_1 + l_2)}{\beta(\lambda + \kappa\alpha^2)^2} < 0 \; ; \; \frac{d\varphi_{\min}}{d\kappa} = \frac{\alpha\lambda(\beta l_1 + l_2)}{\beta(\lambda + \kappa\alpha^2)^2} > 0 \; ;$$

$$\frac{d\varphi_{min}}{d\beta} = \frac{-l_2\kappa\alpha}{\beta^2(\lambda + \kappa\alpha^2)} < 0 \; ; \; \frac{d\varphi_{min}}{dl_1} = \frac{\kappa\alpha}{\lambda + \kappa\alpha^2} > 0 \; ; \; \frac{d\varphi_{min}}{dl_2} = 1 \; ;$$

$$\frac{d\varphi_{min}}{d\alpha} = \frac{(\beta l_1 \kappa + l_2 \kappa)(\lambda - \kappa\alpha^2)}{\beta(\lambda + \kappa\alpha^2)^2} \begin{cases} > 0 \\ < 0 \end{cases} \text{if} \begin{cases} \frac{\lambda}{\kappa} > \alpha^2 \\ \frac{\lambda}{\kappa} < \alpha^2 \end{cases}. \qquad\qquad \text{Q.E.D.}$$

When $\frac{\lambda\Omega}{\lambda+\kappa\alpha^2} < \varphi$, the money growth rule (21) allows reducing sufficiently the money growth rate to equilibrate the money market following shocks that lead to an initial rise in current and expected future inflation. This result implies, according to the interest rate rule (11), an increase in nominal and real interest rates, hence involving a reduced real money demand. As the money growth rate is reduced sufficiently, no further increase in inflation rate but the contrary is then justifiable since the surplus of liquidity in the economy is eliminated.

The minimal value of φ compatible with saddle-point equilibrium diminishes with the weight assigned to output stabilization (greater λ) and increases if the central bank worries more about the realization of the inflation target (greater κ). It varies also with parameters (β, α, l_1 and l_2) reflecting the economic and financial characteristics of the underlying economy. If β has higher values and l_1 and l_2 smaller values, the central bank can give smaller value to φ in formulating its monetary targeting rule. The relation between α and φ_{min} depends on the relative weight the central bank assigns to the output target. The relation is positive when the relative weight is high, i.e. $\frac{\lambda}{\kappa} > \alpha^2$, *vice versa*. We notice further that, higher β corresponds to more important financial development, smaller α to a more flexible labour market, smaller l_1 to a more efficient transaction and payment system and smaller l_2 to lesser interest elasticity of the money demand.

Consequently, more financial developments, more efficient transaction and payment system, decreasing interest elasticity of the money demand allow the central bank to link

more weakly the money growth rate to the rate of change of the expected inflation while guaranteeing macro-economic and financial stability. Meanwhile, more labour market flexibility will have the same effect only if the central bank is a relatively strict inflation targeter.

The convergence to the saddle-point stable equilibrium depends on the capacity of markets operators to coordinate their expectations so that the stock prices are instantly adjusted to the level that allows putting the endogenous variables on the unique converging path. We doubt that this is always possible. In particular, financial operators could make bad guesses about the equilibrium price for stock prices. Small errors in the private expectations might turn into positive or negative speculative bubbles in asset prices as well as in expected inflation. Consequently, other endogenous variables could also be on diverging paths.

Therefore, it could be a judicious political option to equip the economy with a stable equilibrium by eliminating the unstable eigenvalue of the stability matrix. To make this possible, the value of φ must be chosen to ensure that the determinant and the trace of stability matrix of system are positive and negative respectively. The following proposition gives the conditions under which the economy is stable.

Proposition 3: The dynamic system (22) under the optimal interest rate rule (11) combined with monetary targeting rule such as (21) is stable if $\dfrac{\lambda}{\kappa} < \dfrac{\alpha\gamma\lambda_2(\beta\psi+\overline{q})}{\beta^2}$ and

$$\frac{\beta^2\lambda-\alpha\kappa\gamma\lambda_2(\beta\psi+\overline{q})}{\beta[2\gamma\overline{q}(\lambda+\kappa\alpha^2)+\kappa\alpha\overline{\pi}^e]} + \frac{\lambda\Omega}{\lambda+\kappa\alpha^2} < \varphi < \frac{\lambda\Omega}{\lambda+\kappa\alpha^2}\,.$$

Proof: To ensure that the system to be stable, the two eigenvalues of the stability matrix must have both a negative real part. That needs the determinant and the trace of the stability matrix to be positive and negative respectively. The positive determinant condition implies that

$$\frac{\lambda\Omega}{\lambda + \kappa\alpha^2} > \varphi. \tag{23}$$

The negative trace condition leads to

$$\frac{\beta^2\lambda - \alpha\kappa\gamma l_2(\beta\psi + \bar{q})}{\beta[2\gamma\bar{q}(\lambda + \kappa\alpha^2) + \kappa\alpha\bar{\pi}^e]} + \frac{\lambda\Omega}{\lambda + \kappa\alpha^2} < \varphi. \tag{24}$$

There is only one case where conditions (23) and (24) could be simultaneously realised, i.e. when the following condition is satisfied:

$$\frac{\lambda}{\kappa} < \frac{\alpha\gamma l_2(\beta\psi + \bar{q})}{\beta^2}. \tag{25}$$

Q.E.D.

The condition (25) implies that a smaller value of φ than suggested by the proposition 2 is compatible with the macro-economic and financial stability only if the central bank is not putting too high relative weight on the output target. In other words, the central bank is a relatively strict inflation targeter. Taking account of the equilibrium solution of $\bar{q}$, the right side of the inequality (25), denoted by ζ, is impacted by parameter changes as following:

$$\frac{\partial\zeta}{\partial\alpha} = \gamma l_2(\frac{\psi}{\beta} + \frac{y^* + \Phi}{2\gamma\beta^2}) > 0; \quad \frac{\partial\zeta}{\partial\gamma} = \frac{\alpha l_2\psi}{\beta} + \frac{\alpha l_2\psi y^*}{\beta\Phi} > 0;$$

$$\frac{\partial\zeta}{\partial\beta} = -\frac{\alpha\gamma l_2}{\beta^2}(\psi + \frac{y^{*2} + 3\gamma\beta\psi y^* + y^*\Phi}{\gamma\beta\Psi}) < 0; \quad \frac{\partial\zeta}{\partial l_2} = \frac{\alpha\gamma}{\beta}(\psi + \frac{y^* + \Phi}{2\gamma\beta}) > 0;$$

$$\frac{\partial\zeta}{\partial\psi} = \frac{\alpha\gamma l_2}{\beta}(1 + \frac{y^*}{\Phi}) > 0; \quad \frac{\partial\zeta}{\partial y^*} = \frac{\alpha\gamma l_2}{2\gamma\beta^2}(1 + \frac{y^* + 2\gamma\beta\psi}{\Phi}) > 0;$$

where $\Phi = \sqrt{y^{*2} + 4\gamma\beta\psi y^*}$. These partial derivatives show that higher labour market flexibility (α), higher ratio of the marginal propensity to spend wealth relative to the marginal propensity to save the current revenue ($\gamma = \varphi/(1-c)$), higher interest elasticity of the money demand (l_2), higher part of distributed profits relative to the output (ψ) and higher

potential output y^* allow having a more flexible inflation-targeting regime while keeping the equilibrium stable. In the contrary, a higher ratio of the interest elasticity of demand for goods relative to the marginal propensity to save ($\beta = \rho/(1-c)$) will have inverse effects by reducing the maximal degree of flexibility, compatible with the existence of a stable equilibrium, that a inflation-targeting central bank could implement.

6. Conclusion

In this paper, using an aggregate dynamic macro-economic model with stock market, we have examined macro-economic and financial stability under flexible inflation-targeting regimes including a money growth rule. Used as communication and anchoring device, monetary targeting with a commitment to a long-run growth rate of money supply, identical to the inflation target, could reinforce the credibility of the central bank and the role of inflation target as strong and credible nominal anchor for private inflation expectations.

We have shown that achieving inflation and output stabilisation under a special hybrid inflation-targeting regime, i.e. associating the optimal interest rate rule with Friedman's *k*-percent money growth rate rule, can generate macro-economic and financial instability for these economies characterized by relatively low interest elasticity of the demand for goods, high marginal propensity to save and/or relatively high stock market capitalization.

To ensure the existence of saddle-point equilibrium with a unique converging path independently of the degree of flexibility of the inflation-targeting regime, a solution is to adopt a money growth rule that responds negatively, in a sufficient strong manner, to the variation of expected inflation. When the central bank is suffuciently strict inflation targeter, it is possible to conceive a money growth rule to ensure the existence of stable equilibrium independently of the predetermined or non-predetermined nature of the endogenous variables.

The conception of an appropriate money growth rule needs to take account of the structural

parameters of the economy as well as the central bank's preferences.

Appendix A. Dynamics of expected inflation rate and stock prices

i) The differential equation for stock prices (q):

Substituting in equation (4) the solutions of i, π and y defined respectively by equations (11), (13) and (14) and linearizing the resulting equation in the neighbourhood of steady state give,

$$\dot{q} = \frac{1}{\beta}(2\gamma\overline{q} + \frac{\kappa\alpha\overline{\pi}^e}{\lambda + \kappa\alpha^2})(q - \overline{q}) + \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}(\psi + \frac{\overline{q}}{\beta})(\pi^e - \overline{\pi}^e). \qquad (A.1)$$

ii) The differential equation for expected inflation rate (π^e):

At the end of current period, workers and financial operators revise their inflation expectations for next period using all information available at this moment concerning current inflation rate, output and monetary and financial market conditions. Combining equation (6) and the *k*-percent money growth rule (7) yields,

$$\overline{\mu} + \dot{\varepsilon}_m - \pi = l_1\dot{y} - l_2\dot{i} + \dot{\varepsilon}_m. \qquad (A.2)$$

Private agents use all information concerning the conditions of supply and demand on goods market, and on financial and money markets to form their inflation expectations. Taking time derivative of equations (11), (13) and (14) and assuming $\dot{\varepsilon}_\pi = \dot{\varepsilon}_d = 0$ (i.e. shocks without tendency), $\dot{y}^* = 0$ and $\dot{\pi}^T = 0$, we obtain:

$$\dot{i} = \dot{\pi}^e + \frac{1}{\beta}(\gamma\dot{q} + \frac{\kappa\alpha}{\lambda}\dot{\pi}), \qquad (A.3)$$

$$\dot{\pi} = \frac{\lambda}{\lambda + \kappa\alpha^2}\dot{\pi}^e, \qquad (A.4)$$

$$\dot{y} = -\frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\pi}^e. \qquad (A.5)$$

In substituting $\dot{i}$, $\dot{\pi}$, $\dot{y}$ and π defined respectively by equations (A.3)-(A.5) and (13) in equation (A.2), we find:

$$\dot{\pi}^e = \frac{1}{\Omega}\pi^e - \frac{(\lambda + \kappa\alpha^2)l_2\gamma}{\beta\lambda\Omega}\dot{q} - \frac{1}{\Omega}\bar{\mu} + \frac{1}{\Omega}\varepsilon_\pi, \tag{A.6}$$

where $\Omega = \dfrac{\beta l_1\kappa\alpha + l_2\beta(\lambda + \kappa\alpha^2) + l_2\kappa\alpha}{\beta\lambda} > 0$. Linearizing equation (A.6) in the neighbourhood

of the steady state taking account of equation (A.1) yields:

$$\dot{\pi}^e = \left[\frac{1}{\Omega} - \frac{\alpha\kappa l_2\gamma}{\beta\lambda\Omega}(\psi + \frac{\bar{q}}{\beta})\right](\pi^e - \bar{\pi}^e) - \frac{(\lambda + \kappa\alpha^2)l_2\gamma}{\beta^2\lambda\Omega}(2\gamma\bar{q} + \frac{\kappa\alpha\bar{\pi}^e}{\lambda + \kappa\alpha^2})(q - \bar{q}). \tag{A.7}$$

Appendix B. Dynamics of expected inflation rate under alternative monetary targeting

Substituting the money growth rate defined by (21) into (6) yields

$$\bar{\mu} - \varphi\dot{\pi}^e - \pi = l_1\dot{y} - l_2\dot{i}, \qquad \text{with } \bar{\mu} = \pi^T. \tag{B.1}$$

Substituting $\dot{i}$ and $\dot{y}$ by their expressions given by equations (A.3) and (A.5), and then $\dot{\pi}$ and π by these given by equations (A.4) and (13) in equation (B.1), it follows:

$$\dot{\pi}^e = \frac{\lambda}{\lambda\Omega - (\lambda + \kappa\alpha^2)\varphi}\left[\pi^e - \frac{l_2(\lambda + \kappa\alpha^2)\gamma}{\beta\lambda}\dot{q} - \bar{\mu} + \varepsilon_\pi\right], \tag{B.2}$$

where Ω is defined as before. Linearizing equation (B.2) in the neighbourhood of the steady state taking account of equation (A.1) leads to:

$$\dot{\pi}^e = \Theta\left\{[1 - \frac{\alpha\kappa\gamma l_2}{\beta\lambda}(\psi + \frac{\bar{q}}{\beta})](\pi^e - \bar{\pi}^e) - \frac{\gamma l_2(\lambda + \kappa\alpha^2)}{\beta^2\lambda}(2\gamma\bar{q} + \frac{\kappa\alpha\bar{\pi}^e}{\lambda + \kappa\alpha^2})(q - \bar{q})\right\}. \tag{B.3}$$

where $\Theta = \dfrac{\lambda}{\lambda\Omega - (\lambda + \kappa\alpha^2)\varphi}$.

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